

Termination of STLC

(Termination)

For every $\vdash e : T$ there exists a
 v val such that $e \rightarrow^* v$

Recursion + Fixed Point

A recursive function is one that calls itself

This violates termination property because the onus is on the programmer to ensure termination

Non-terminating

> $f = f f$

removing base case violates termination

> $\text{fib } n = \text{if } n=1 \text{ then } 0 \text{ else } f(n-1)$

decreasing is important for termination

> $\text{fac} :: \text{Int} \rightarrow \text{Int}$
 > $\text{fac } n = \text{if } n=0 \text{ then } 1$
 > $\text{else } n * \text{fac}(n-1)$

$\text{fac} := \lambda n. \text{if } n=0 \text{ then } 1$
 $\text{else } n * \text{fac}(n-1)$

$\lambda f. \lambda n. \text{if } n=0 \text{ then } 1$
 $\text{else } n * \underbrace{f(n-1)}_{\text{Int} \rightarrow \text{Int}}$

> $\text{fix} :: (a \rightarrow a) \rightarrow a$
 > $\text{fix } f = f(\text{fix } f)$

> $\text{factF} :: (\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})$
 > $\text{factF } f \ n = \text{if } n=0 \text{ then } 1 \text{ else } n * f(n-1)$

> $\text{fac} :: \text{Int} \rightarrow \text{Int}$
 > $\text{fac} = \text{fix factF}$

fac

fix factF

$\text{factF}(\text{fix factF})$

$\lambda f. \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n * f(n-1) (\text{fix factF})$

$\xrightarrow{\beta} \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n * (\text{fix factF})(n-1)$

$\text{fix } \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n * \text{fac}(n-1)$

> $\text{inc} :: \text{Int} \rightarrow \text{Int}$
 > $\text{inc } n = \text{inc } (n+1)$

> $\text{incF} :: (\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})$
 > $\text{incF } f \ n = f(n+1)$

> $\text{inc} :: \text{Int} \rightarrow \text{Int}$
 > $\text{inc} = \text{fix incF}$

Statics

$$\frac{\Gamma, x:\tau \vdash e:\tau}{\Gamma \vdash \text{fix}(x:\tau.e):\tau} \text{fix}$$

$\text{fix} :: (\tau \rightarrow \tau) \rightarrow \tau$
 $\text{fix } f = f(\text{fix } f)$

↓ Dynamics

D-FIX
$$\frac{}{\text{fix}(x:\tau.e) \mapsto e[\text{fix}(x:\tau.e)/x]}$$

PCF

PCF = (same version of) STLC + fixed points

Partial functions may not be defined for all inputs

> $\text{sqw} :: \text{Int} \rightarrow \text{Maybe Int}$
 $\rightarrow \text{Int}$

> $\text{sqw } 4 = 2$

> $\text{head} :: [a] \rightarrow a$
> $\text{head } (x:xs) = x$

-- undefined for
 $[]$

> $\text{tail} :: [a] \rightarrow [a]$
> $\text{tail } (x:xs) = xs$