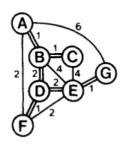
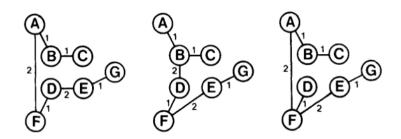
```
G1 = {
  'a': {'b':1, 'f':2, 'g': 6},
  'b': {'a':1, 'c':1},
  'c': {'b':1},
  'd': {'f':1, 'e':2},
  'e': {'d':2, 'g':1},
  'f': {'a':2, 'd':1},
  'g': {'e':1, 'a': 6}
}
```

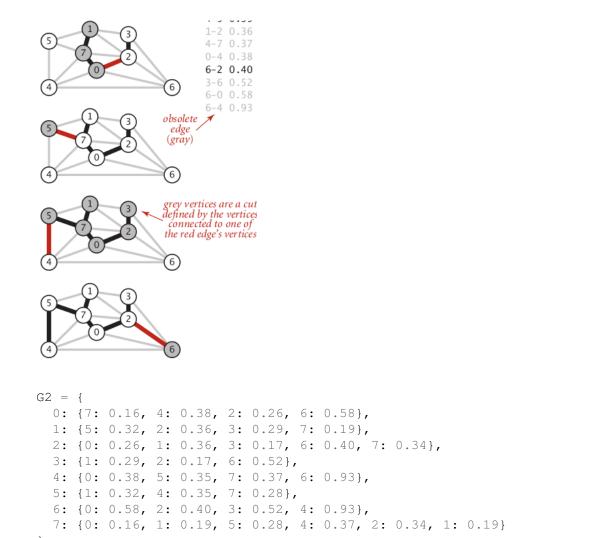




```
def find(C, u):
    if C[u] != u:
        C[u] = find(C, C[u])
                                                   # Path compression
    return C[u]
def union(C, R, u, v):
    u, v = find(C, u), find(C, v)
    if R[u] > R[v]:
                                                    # Union by rank
        C[v] = u
    else:
        C[u] = v
    if R[u] == R[v]:
                                                    # A tie: Move v up a level
        R[v] += 1
def kruskal(G):
    E = [(G[u][v],u,v) \text{ for } u \text{ in } G \text{ for } v \text{ in } G[u]]
    T = set()
    C, R = {u:u for u in G}, {u:0 for u in G} # Comp. reps and ranks
    print sorted(E)
    for _, u, v in sorted(E):
        if find(C, u) != find(C, v):
            T.add((u, v))
            print (u, v)
            union(C, R, u, v)
```

return T

```
print list(kruskal(G1))
  [\,(1,\ 'a',\ 'b'),\ (1,\ 'b',\ 'a'),\ (1,\ 'b',\ 'c'),\ (1,\ 'c',\ 'b'),\ (1,\ 'd',\ 'f'),\ (1,\ 'e',\ 'f'),\ (1,\ 'd',\ 'f'),\ (1,\ 'e',\ 'f'),\ (1,\ 'd',\ 'f'),\ (1,\ 'e',\ 'f'),\ (1,\ (1,\ 'e',\ 'f'),\ (1,\ (1,\ 'e',\ 'f')),\ (1,\ (1,\ (1,\ )e',\ 'f')),\ (1,\ (1,\ )e',\ (1,\ )
  ('a', 'b')
  ('b', 'c')
  ('d', 'f')
  ('e', 'g')
  ('a', 'f')
  ('d', 'e')
  [('d', 'e'), ('e', 'g'), ('d', 'f'), ('b', 'c'), ('a', 'f'), ('a', 'b')]
m = 100 * 1000
print m**2
print np.log(m)
print m*np.log(m)
 10000000000
 11.512925465
 1151292.5465
                                                             (3) next MST edge is red
                                                              (2)
                                                                                                                         graph edges
                                                                                                                           sorted
by weight
                                                                                       MST edge
(black) \
                                                                                                                        0-7 0.16
                                                                                                                        2-3 0.17
                                                                                                                        1-7 0.19
                                                                                                                        0-2 0.26
                                                                                                                         5-7 0.28
                                                                                                                         1-3 0.29
                                                                                                                         1-5 0.32
                                                                                                                         2-7 0.34
                                                                                                                         4-5 0.35
```



```
print list(kruskal(G2))
```

```
[(0.16, 0, 7), (0.16, 7, 0), (0.17, 2, 3), (0.17, 3, 2), (0.19, 1, 7), (0.19, 7, 1), (0, 7) (2, 3) (1, 7) (0, 2)
```

(5, 7) (4, 5)

(2, 6) [(2, 6), (4, 5), (5, 7), (0, 7), (2, 3), (1, 7), (0, 2)]

Sedgewick, R. Algorithms, sf. 409

Sedgewick, R. Algorithms, 4rd Edition, sf. 624

Heatland, Python Algorithms