

Bir Coklu Ikisel Dagilim (Multivar. Binary Distribution) ve Hopfield Agi

$$P(\mathbf{x}; W) = \frac{1}{Z(W)} \exp \left[\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right]$$

Olurluk (likelihood)

$$\prod_{n=1}^N P(\mathbf{x}^{(n)}; W) = \frac{1}{Z(W)^N} \exp \left[\frac{1}{2} \sum_{n=1}^N \mathbf{x}^{(n)T} W \mathbf{x}^{(n)} \right]$$

Log olurluk

$$\ln \left(\prod_{n=1}^N P(\mathbf{x}^{(n)}; W) \right) = \sum_{n=1}^N \left[\frac{1}{2} \mathbf{x}^{(n)T} W \mathbf{x}^{(n)} - \ln Z(W) \right]$$

Birazdan w_{ij} uzerinden turev alacagiz, $\ln Z(W)$ 'nin turevi ne olacak, daha dogrusu $Z(W)$ 'yi nasil turevi alinir hale getiririz?

$Z(W)$ normalizasyon sabiti olduguna gore, dagilimin geri kalaninin sonsuzlar uzerinden entegrali (ya da toplami) normalizasyon sabitine esittir,

$$Z(W) = \sum_{\mathbf{x}} \exp \left[\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right]$$

$$\ln Z(W) = \ln \left[\sum_{\mathbf{x}} \exp \left(\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right) \right]$$

Log bazli turev alinca log icindeki hersey oldugu gibi bolume gider, ve log icindeki turevi alinirak bolume koyulur. Fakat log icine dikkatli bakarsak bu zaten $Z(W)$ 'nin tanimidir, boylece denklemi temizleme sansi dogdu, bolume hemen $Z(W)$ deriz, ve turevi log'un icine uygulariz,

$$\frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \left[\sum_{\mathbf{x}} \frac{\partial}{\partial w_{ij}} \exp \left(\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right) \right]$$

$$\frac{\partial}{\partial w_{ij}} \exp \left(\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right) = \frac{1}{2} \exp \left(\frac{1}{2} \mathbf{x}^T W \mathbf{x} \right) \frac{\partial}{\partial w_{ij}} \mathbf{x}^T W \mathbf{x}$$

(2)'in icindeki bolumu acalim,

$$\frac{\partial}{\partial w_{ij}} \mathbf{x}^T W \mathbf{x} = x_i x_j$$

Simdi (2)'ye geri koyalim,

$$\begin{aligned}
&= \frac{1}{2} \exp \left(\frac{1}{2} \mathbf{x}^T \mathbf{W} \mathbf{x} \right) x_i x_j \\
\frac{\partial}{\partial w_{ij}} \ln Z(\mathbf{W}) &= \frac{1}{Z(\mathbf{W})} \left[\sum_{\mathbf{x}} \frac{1}{2} \exp \left(\frac{1}{2} \mathbf{x}^T \mathbf{W} \mathbf{x} \right) x_i x_j \right] \\
&= \frac{1}{2} \sum_{\mathbf{x}} \frac{1}{Z(\mathbf{W})} \exp \left(\frac{1}{2} \mathbf{x}^T \mathbf{W} \mathbf{x} \right) x_i x_j \\
&= \frac{1}{2} \sum_{\mathbf{x}} P(\mathbf{x}; \mathbf{W}) x_i x_j
\end{aligned}$$

Artık $\ln Z(\mathbf{W})$ 'nin turevini biliyoruz. O zaman tum log olurlugun turevi, (1) uzerinde uygularsak,

$$\sum_{n=1}^N \left[\frac{\partial}{\partial w_{ij}} \frac{1}{2} \mathbf{x}^{(n)T} \mathbf{W} \mathbf{x}^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(\mathbf{W}) \right]$$