Bir Coklu Ikisel Dagilim (Multivar. Binary Distribution) ve Boltzmann Dagilimi

$$P(x; W) = \frac{1}{Z(W)} \exp\left[\frac{1}{2} x^T W x\right]$$

Olurluk (likelihood)

$$\prod_{n=1}^{N} P(x^{(n)}; W) = \frac{1}{Z(W)} \exp\left[\frac{1}{2} x^{(n)^T} W x^{(n)}\right]$$

Log olurluk

$$\mathcal{L} = \ln\big(\prod_{n=1}^{N} P(x^{(n)}; W)\big) = \sum_{n=1}^{N} \left[\frac{1}{2} x^{(n)^T} W x^{(n)} - \ln Z(W)\right] \tag{1}$$

Birazdan $\frac{\partial \mathcal{L}}{\partial w_{ij}}$ turevini alacagiz, o sirada $\ln Z(W)$'nin turevi lazim, daha dogrusu $Z(W)$'yi nasil turevi alinir hale getiririz?

$Z(W)$ normalizasyon sabiti olduguna gore, dagilimin geri kalaninin sonsuzlar uzerinden entegrali (ya da toplami) normalizasyon sabitine esittir,

$$Z(W) = \sum_{x} \exp\left[\frac{1}{2} x^T W x\right]$$

$$\ln Z(W) = \ln\left[\sum_{x} \exp\big(\frac{1}{2} x^T W x\big)\right]$$

Log bazli turev alinca log icindeki hersey oldugu gibi bolume gider, ve log icindekinin turevi alinirak bolume koyulur. Fakat log icine dikkatli bakarsak bu zaten $Z(W)$'nin tanimidir, boylece denklemi temizleme sansi dogdu, bolume hemen $Z(W)$ deriz, ve turevi log'un icine uygulariz,

$$\frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \left[\sum_{x} \frac{\partial}{\partial w_{ij}} \exp\big(\frac{1}{2} x^T W x\big)\right]$$

$$\frac{\partial}{\partial w_{ij}} \exp\big(\frac{1}{2} x^T W x\big) = \frac{1}{2} \exp\big(\frac{1}{2} x^T W x\big) \frac{\partial}{\partial w_{ij}} x^T W x \tag{2}$$

(2)'in icindeki bolumu acalim,

$$\frac{\partial}{\partial w_{ij}} x^T W x = x_i x_j$$

Simdi (2)'ye geri koyalim,

$$= \frac{1}{2} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j$$

$$\frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \left[\sum_x \frac{1}{2} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j \right]$$

$$= \frac{1}{2} \sum_x \frac{1}{Z(W)} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j$$

$$= \frac{1}{2} \sum_x P(x; W) x_i x_j$$

Ustteki son ifadede bir kisaltma kullanalim,

$$\sum_x P(x; W) x_i x_j = < x_i, x_j >_{P(x;W)}$$

Artik $\ln Z(W)$'nin turevini biliyoruz. O zaman tum log olurlugun turevine (1) donebiliriz,

$$\frac{\partial \mathcal{L}}{\partial w_{ij}} = \sum_{n=1}^{N} \left[\frac{\partial}{\partial w_{ij}} \frac{1}{2} x^{(n)^T} W x^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \right]$$

$$= \sum_{n=1}^{N} \left[\frac{1}{2} x_i^{(n)^T} x_j^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \right]$$

$$= \sum_{n=1}^{N} \left[\frac{1}{2} x_i^{(n)^T} x_j^{(n)} - \frac{1}{2} < x_i x_j >_{P(x;W)} \right]$$

1/2 sabitlerini atalim,

$$= \sum_{n=1}^{N} \left[x_i^{(n)^T} x_j^{(n)} - < x_i x_j >_{P(x;W)} \right]$$

Eger

$$< x_i x_j >_{Data} = \frac{1}{N} \sum_{n=1}^{N} x_i^{(n)^T} x_j^{(n)}$$

olarak alirsak, esitligin sag tarafi verisel kovaryansi (empirical covariance) temsil eder. Duzenleyince,

$$N \cdot < x_i x_j >_{Data} = \sum_{n=1}^{N} x_i^{(n)^T} x_j^{(n)}$$

simdi esitligin sag tarafi uc ustteki formule geri koyulabilir,

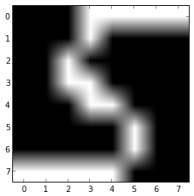
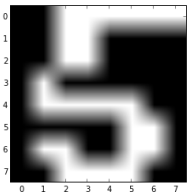
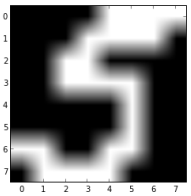
$$\frac{\partial \mathcal{L}}{\partial w_{ij}} = N\left[< x_i x_j >_{Data} - < x_i x_j >_{P(x;W)} \right]$$

Bu bir gradyan guncelleme formulu olarak gorulebilir, ve N yerine bir guncelleme sabiti alinabilir.

```
Y = np.loadtxt('../../stat/stat_mixbern/binarydigits.txt')
label = np.ravel(np.loadtxt('../../stat/stat_mixbern/bindigitlabels.txt'))
Y5 = Y[label==5]
plt.imshow(Y5[0,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_01.png')

plt.imshow(Y5[1,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_02.png')

plt.imshow(Y5[2,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_03.png')
```

```
from pymc import *
X = Uniform'X', zeros(20), ones(20), value=0.5*ones(20))
@pm.potential
def SNLS(X=X):
    logp = -X[0]**2 / X[1]
    logp += -X[1]**2 / X[2]  # or whatever...
    return logp
m = pm.MCMC([X, SNLS])
m.use_step_method(pm.AdaptiveMetropolis, X)
m.sample(100)
print X.trace().sum()
print X.trace()
```

```
 [-----------------100%-----------------] 100 of 100 complete in 0.0 sec929.258052868
[[ 0.53524187  0.5101911   0.66042418 ...,  0.50036561  0.54607666
   0.59754949]
```

```
[ 0.65638942  0.4783261   0.63907422 ...,  0.40159229  0.60001351
  0.73953164]
[ 0.66938509  0.42037807  0.56442453 ...,  0.4538526   0.5867844
  0.6921165 ]
...,
[ 0.61453101  0.78038041  0.87755512 ...,  0.41324804  0.21268127
  0.3127227 ]
[ 0.61453101  0.78038041  0.87755512 ...,  0.41324804  0.21268127
  0.3127227 ]
[ 0.61453101  0.78038041  0.87755512 ...,  0.41324804  0.21268127
  0.3127227 ]]
```

Information Theory, Inference and Learning Algorithms

http://nbviewer.ipython.org/gist/aflaxman/7d946762ee99daf739f1