Bir Coklu Ikisel Dagilim (Multivar. Binary Distribution) ve Hopfield Agi

$$P(x;W) = \frac{1}{Z(W)} \exp \bigg[ \frac{1}{2} x^T W x \bigg]$$

Olurluk (likelihood)

$$\prod_{n=1}^{N} P(x^{(n)};W) = \frac{1}{Z(W)} \exp \bigg[ \frac{1}{2} x^{(n)^T} W x^{(n)} \bigg]$$

Log olurluk

$$\ln \Big( \prod_{n=1}^{N} P(x^{(n)};W) \Big) = \sum_{n=1}^{N} \bigg[ \frac{1}{2} x^{(n)^T} W x^{(n)} - \ln Z(W) \bigg] \tag{1}$$

Birazdan $w_{ij}$ uzerinden turev alacagiz, $\ln Z(W)$'nin turevi ne olacak, daha dogrusu $Z(W)$'yi nasil turevi alinir hale getiririz?

$Z(W)$ normalizasyon sabiti olduguna gore, dagilimin geri kalaninin sonsuzlar uzerinden entegrali (ya da toplami) normalizasyon sabitine esittir,

$$Z(W) = \sum_x \exp \bigg[ \frac{1}{2} x^T W x \bigg]$$

$$\ln Z(W) = \ln \bigg[ \sum_x \exp \big( \frac{1}{2} x^T W x \big) \bigg]$$

Log bazli turev alinca log icindeki hersey oldugu gibi bolume gider, ve log icindekinin turevi alinirak bolume koyulur. Fakat log icine dikkatli bakarsak bu zaten $Z(W)$'nin tanimidir, boylece denklemi temizleme sansi dogdu, bolume hemen $Z(W)$ deriz, ve turevi log'un icine uygulariz,

$$\frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \bigg[ \sum_x \frac{\partial}{\partial w_{ij}} \exp \big( \frac{1}{2} x^T W x \big) \bigg]$$

$$\frac{\partial}{\partial w_{ij}} \exp \big( \frac{1}{2} x^T W x \big) = \frac{1}{2} \exp \big( \frac{1}{2} x^T W x \big) \frac{\partial}{\partial w_{ij}} x^T W x \tag{2}$$

(2)'in icindeki bolumu acalim,

$$\frac{\partial}{\partial w_{ij}} x^T W x = x_i x_j$$

Simdi (2)'ye geri koyalim,

$$ = \frac{1}{2} \exp\big(\frac{1}{2} x^T W x\big) x_i x_j $$

$$ \frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \bigg[ \sum_x \frac{1}{2} \exp\big(\frac{1}{2} x^T W x\big) x_i x_j \bigg] $$

$$ = \frac{1}{2} \sum_x \frac{1}{Z(W)} \exp\big(\frac{1}{2} x^T W x\big) x_i x_j $$

$$ = \frac{1}{2} \sum_x P(x; W) x_i x_j $$

Artik $\ln Z(W)$'nin turevini biliyoruz. O zaman tum log olurlugun turevi, (1) uzerinde uygularsak,

$$ \sum_{n=1}^{N} \bigg[ \frac{\partial}{\partial w_{ij}} \frac{1}{2} x^{(n)^T} W x^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \bigg] $$

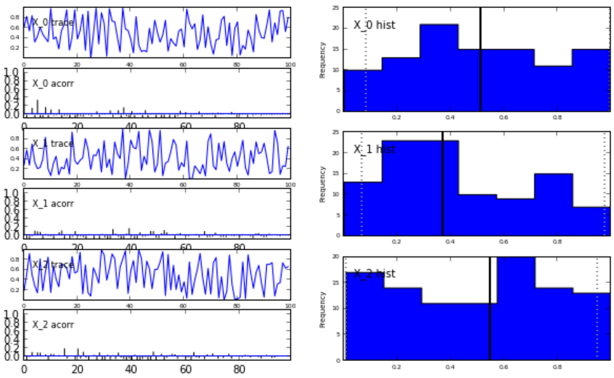
$$ \sum_{n=1}^{N} \bigg[ \frac{1}{2} x_i^{(n)^T} x_j^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \bigg] $$

```
import pymc as pm
@pm.potential
def SNLS(X=X):
    logp = -X[0]**2 / X[1]
    logp += -X[1]**2 / X[2]   # or whatever...
    return logp
X = pm.Uniform('X', 0, 1, value=[0.45, 0.24, 0.68])
m = pm.MCMC([X, SNLS])
m.use_step_method(pm.AdaptiveMetropolis, X)
m.sample(100)
```

```
 [-----------------100%-----------------] 100 of 100 complete in 0.0 sec
```

```
pm.Matplot.plot(X)
```

```
Plotting X_0
Plotting X_1
Plotting X_2
```


X_0 trace
X_0 acorr
X_1 trace
X_1 acorr
X_2 trace
X_2 acorr
X_0 hist
X_1 hist
X_2 hist
Frequency