Bir Coklu Ikisel Dagilim (Multivar. Binary Distribution) ve Hopfield Agi

$$ P(x;W) = \frac{1}{Z(W)} \exp \bigg[\frac{1}{2} x^T W x \bigg] $$

Olurluk (likelihood)

$$ \prod_{n=1}^{N} P(x^{(n)};W) = \frac{1}{Z(W)} \exp \bigg[\frac{1}{2} x^{(n)^T} W x^{(n)} \bigg] $$

Log olurluk

$$ \ln \big(\prod_{n=1}^{N} P(x^{(n)};W) \big) = \sum_{n=1}^{N} \bigg[\frac{1}{2} x^{(n)^T} W x^{(n)} - \ln Z(W) \bigg] $$

Birazdan w_{ij} uzerinden turev alacagiz, $\ln Z(W)$'nin turevi ne olacak, daha dogrusu $Z(W)$'yi nasil turevi alinir hale getiririz?

$Z(W)$ normalizasyon sabiti olduguna gore, dagilimin geri kalaninin sonsuzlar uzerinden entegrali (ya da toplami) normalizasyon sabitine esittir,

$$ Z(W) = \sum_x \exp \bigg[\frac{1}{2} x^T W x \bigg] $$

$$ \ln Z(W) = \ln \bigg[\sum_x \exp \big(\frac{1}{2} x^T W x \big) \bigg] $$

Log bazli turev alinca log icindeki hersey oldugu gibi bolume gider, ve log icindekinin turevi alinirak bolume koyulur. Fakat log icine dikkatli bakarsak bu zaten $Z(W)$'nin tanimidir, boylece denklemi temizleme sansi dogdu, bolume hemen $Z(W)$ deriz, ve turevi log'un icine uygulariz,

$$ \frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \bigg[\sum_x \frac{\partial}{\partial w_{ij}} \exp \big(\frac{1}{2} x^T W x \big) \bigg] $$

$$ \frac{\partial}{\partial w_{ij}} \exp \big(\frac{1}{2} x^T W x \big) = \frac{1}{2} \exp \big(\frac{1}{2} x^T W x \big) \frac{\partial}{\partial w_{ij}} x^T W x $$

$$ \frac{\partial}{\partial w_{ij}} x^T W x = x_i x_j $$

$$ = \frac{1}{2} \exp \big(\frac{1}{2} x^T W x \big) x_i x_j $$

$$
\begin{aligned}
\frac{\partial}{\partial w_{ij}} \ln Z(W) &= \frac{1}{Z(W)} \Big[\sum_x \frac{1}{2} \exp\big(\frac{1}{2} x^T W x\big) x_i x_j \Big] \\
&= \frac{1}{2} \sum_x \frac{1}{Z(W)} \exp\big(\frac{1}{2} x^T W x\big) x_i x_j \\
&= \frac{1}{2} \sum_x P(x; W) x_i x_j
\end{aligned}
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