Bir Coklu Ikisel Dagilim (Multivar. Binary Distribution) ve Boltzmann Dagilimi

$$ P(x;W) = \frac{1}{Z(W)} \exp \bigg[\frac{1}{2} x^T W x \bigg] $$

Olurluk (likelihood)

$$ \prod_{n=1}^{N} P(x^{(n)};W) = \frac{1}{Z(W)} \exp \bigg[\frac{1}{2} x^{(n)^T} W x^{(n)} \bigg] $$

Log olurluk

$$ \ln \Big(\prod_{n=1}^{N} P(x^{(n)};W) \Big) = \sum_{n=1}^{N} \bigg[\frac{1}{2} x^{(n)^T} W x^{(n)} - \ln Z(W) \bigg] \tag{1} $$

Birazdan w_{ij} uzerinden turev alacagiz, $\ln Z(W)$'nin turevi ne olacak, daha dogrusu $Z(W)$'yi nasil turevi alinir hale getiririz?

$Z(W)$ normalizasyon sabiti olduguna gore, dagilimin geri kalaninin sonsuzlar uzerinden entegrali (ya da toplami) normalizasyon sabitine esittir,

$$ Z(W) = \sum_x \exp \bigg[\frac{1}{2} x^T W x \bigg] $$

$$ \ln Z(W) = \ln \bigg[\sum_x \exp \big(\frac{1}{2} x^T W x \big) \bigg] $$

Log bazli turev alinca log icindeki hersey oldugu gibi bolume gider, ve log icindekinin turevi alinirak bolume koyulur. Fakat log icine dikkatli bakarsak bu zaten $Z(W)$'nin tanimidir, boylece denklemi temizleme sansi dogdu, bolume hemen $Z(W)$ deriz, ve turevi log'un icine uygulariz,

$$ \frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \bigg[\sum_x \frac{\partial}{\partial w_{ij}} \exp \big(\frac{1}{2} x^T W x \big) \bigg] $$

$$ \frac{\partial}{\partial w_{ij}} \exp \big(\frac{1}{2} x^T W x \big) = \frac{1}{2} \exp \big(\frac{1}{2} x^T W x \big) \frac{\partial}{\partial w_{ij}} x^T W x \tag{2} $$

(2)'in icindeki bolumu acalim,

$$ \frac{\partial}{\partial w_{ij}} x^T W x = x_i x_j $$

Simdi (2)'ye geri koyalim,

$$= \frac{1}{2} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j$$

$$\frac{\partial}{\partial w_{ij}} \ln Z(W) = \frac{1}{Z(W)} \left[\sum_x \frac{1}{2} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j \right]$$

$$= \frac{1}{2} \sum_x \frac{1}{Z(W)} \exp\left(\frac{1}{2} x^T W x\right) x_i x_j$$

$$= \frac{1}{2} \sum_x P(x; W) x_i x_j$$

Artik $\ln Z(W)$'nin turevini biliyoruz. O zaman tum log olurlugun turevi, (1) uzerinde uygularsak,

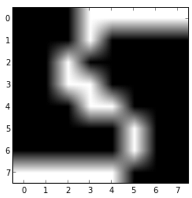
$$\sum_{n=1}^{N} \left[\frac{\partial}{\partial w_{ij}} \frac{1}{2} x^{(n)^T} W x^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \right]$$

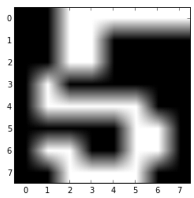
$$\sum_{n=1}^{N} \left[\frac{1}{2} x_i^{(n)^T} x_j^{(n)} - \frac{\partial}{\partial w_{ij}} \ln Z(W) \right]$$

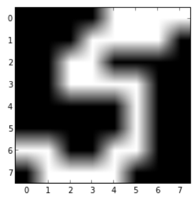
```
Y = np.loadtxt('../../stat/stat_mixbern/binarydigits.txt')
label = np.ravel(np.loadtxt('../../stat/stat_mixbern/bindigitlabels.txt'))
Y5 = Y[label==5]
plt.imshow(Y5[0,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_01.png')

plt.imshow(Y5[1,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_02.png')

plt.imshow(Y5[2,:].reshape((8,8),order='C'), cmap=plt.cm.gray)
plt.savefig('boltzmann_03.png')
```







Information Theory, Inference and Learning Algorithms

http://nbviewer.ipython.org/gist/aflaxman/7d946762ee99daf739f1