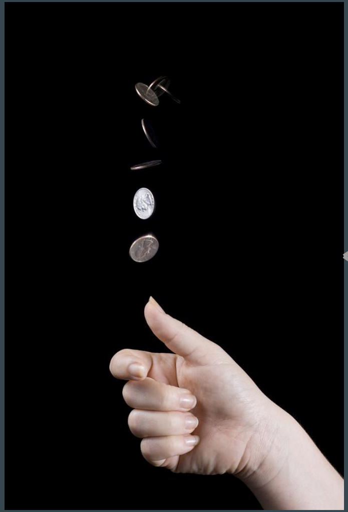
Algebra of Events

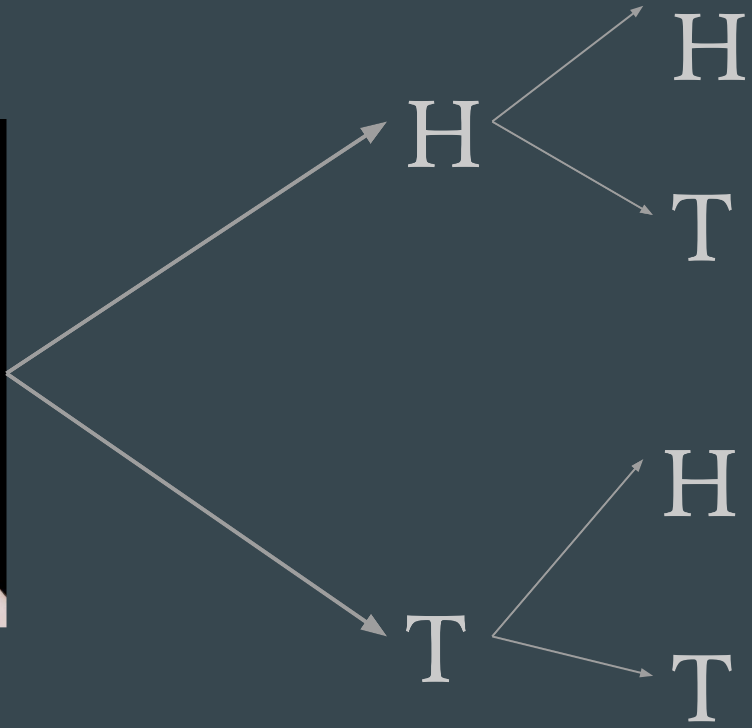
PLSC 309

16 January 2019

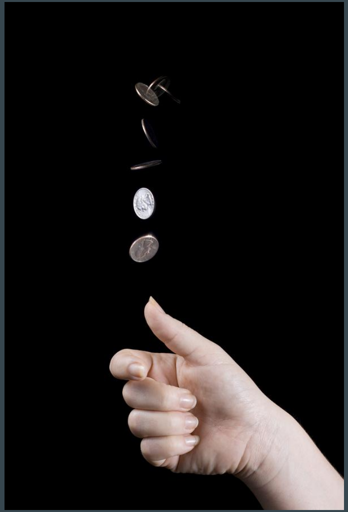
Review of outcomes and events

Flipping a coin twice



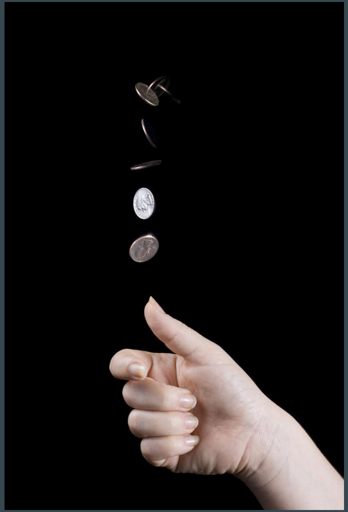


Sigma-notation for two coin flips



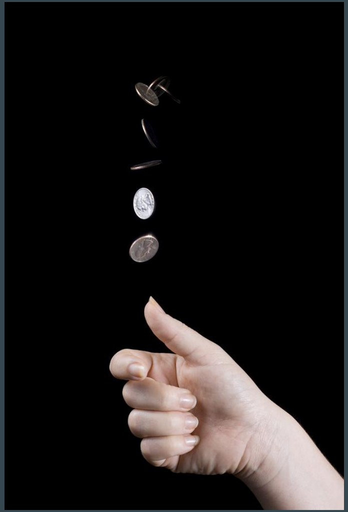
Ω?

Sigma-notation for two coin flips



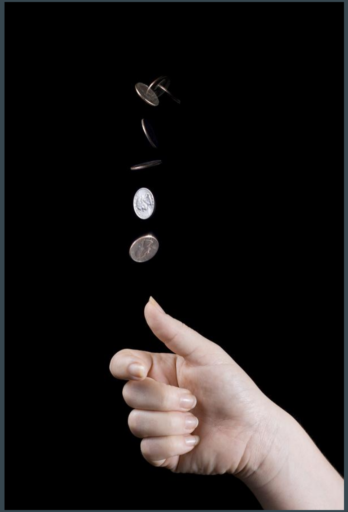
Heads or Tails

Sigma-notation for two coin flips



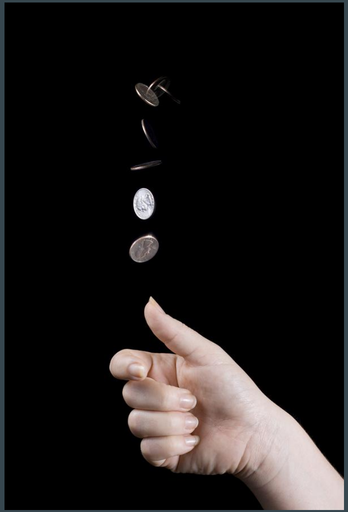
F?

Sigma-notation for two coin flips



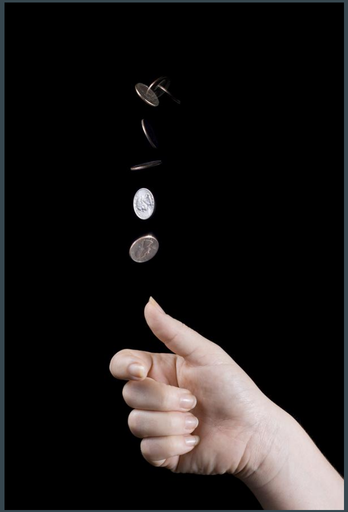
All combinations of the two flips:
HH
HT
TH
TT

Sigma-notation for two coin flips



P?

Sigma-notation for two coin flips



P(HH) = ¼
P(HT) = ¼
P(TH) = ¼
P(TT) = ¼

Welcome to the widget factory

- Factory makes widgets
- Interested in keeping the failure rate down below 5%
- In other words, no more than 5 out of 100 widgets should fail
- Each widget made is an observation
- Our main variable is “failure” which is binary variable (1/0)

Welcome to the widget factory

Welcome to the widget factory

Ω?

Welcome to the widget factory

Widget success or failure

Welcome to the widget factory

F?

Welcome to the widget factory

All combinations that make up >5% failure or <5% failure

Welcome to the widget factory

P?

Welcome to the widget factory

P(>5% failure)
P(<5% failure)

F can change depending on the question

Probability that there is a less than 5% failure rate

All combinations that make up >5% failure or <5% failure

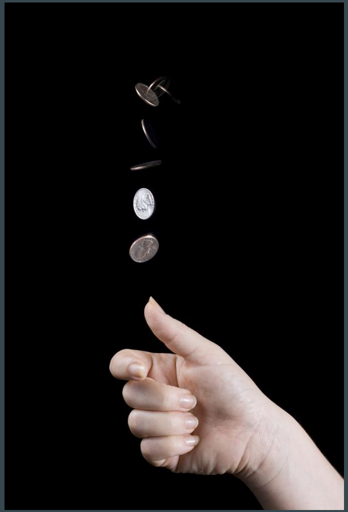
F can change depending on the question

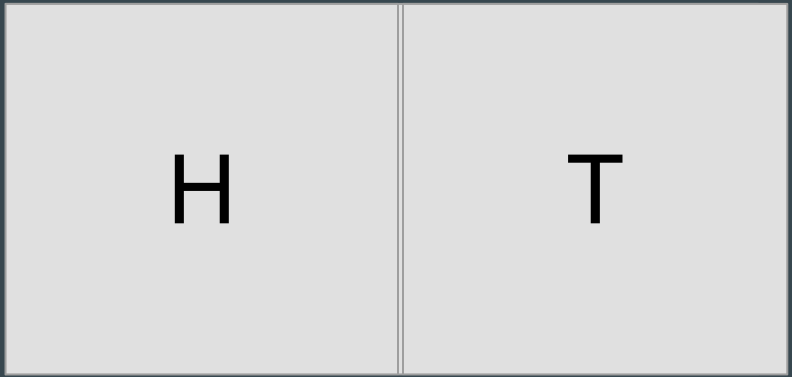
How many failures will I get in 1000 trials? What is the overall failure rate?

All combinations that have 1 failure, 2 failures, 3 failures, …

Sample space □ Event Space □ Probability Space

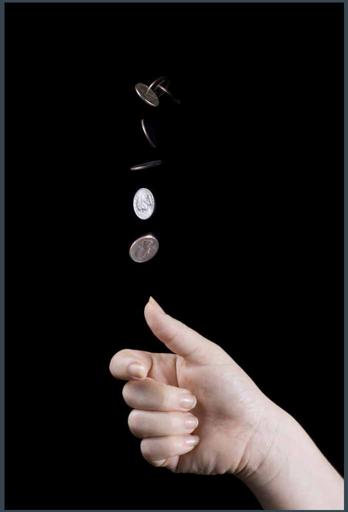
Flip a coin twice





Sample space □ Event Space □ Probability Space

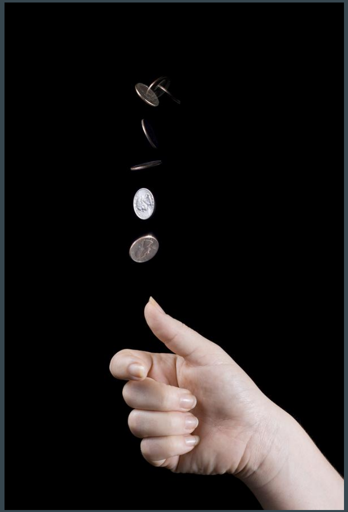
Flip a coin twice



HH	HT
TT	TH

Sample space □ Event Space □ Probability Space

Flip a coin twice



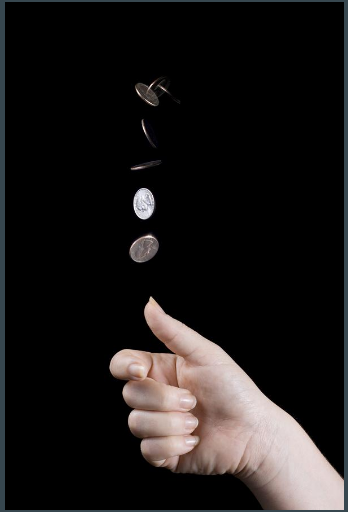
P(HH) = ¼	P(HT) = ¼
P(TT) = ¼	P(TH) = ¼

Sample space □ Event Space □ Probability Space

- Sample space (Ω): each possible outcome
- Event space (F): the combinations of possible outcomes you're interested in
  - A visualization of the set of events
- Probability space (P): the chance the events you're interested in actually happens
  - "Actually happens" is a shorthand way of saying, "If I repeated this an infinite number of times, what percentage of those times would the events happen"

Probability Space

What is the chance I get both a heads and tails in two flips?



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Probability space

Probability Space

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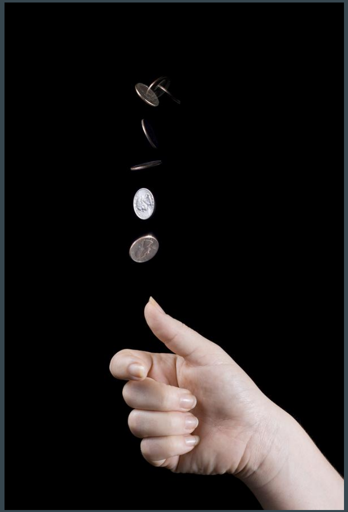
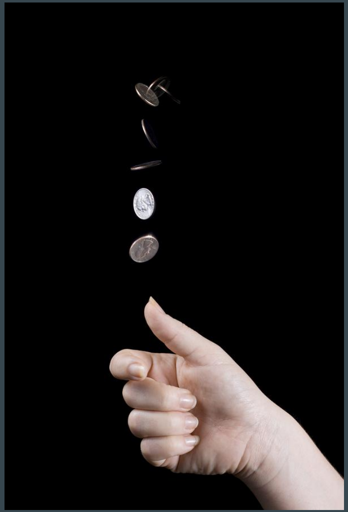


P(Either both heads or both tails) = ½	P(Either heads and tails or tails and heads) = ½

Probability space

Probability Space as a Venn Diagram

What is the chance I get both a heads and tails in two flips?

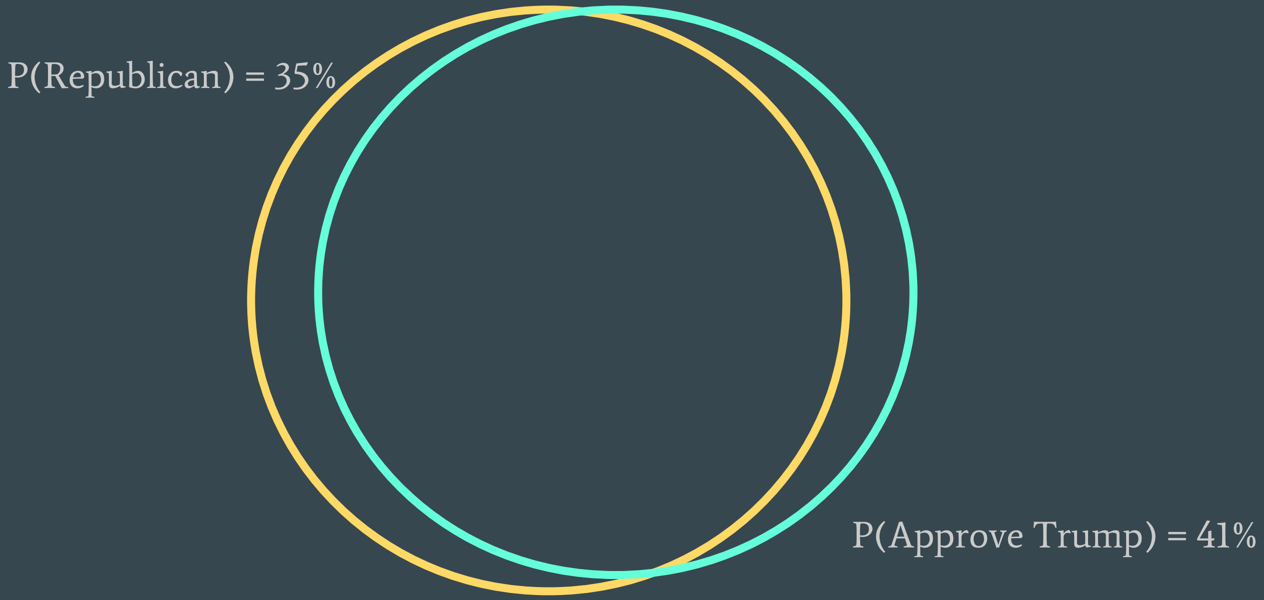


P(Either both heads or both tails) = ½

P(Either heads and tails or tails and heads) = ½

Probability Space as a Venn Diagram

President Trump has a 93% approval rating among Republicans. 35% of Americans identify as republicans. Trump has an approval rating of 41% among all Americans.

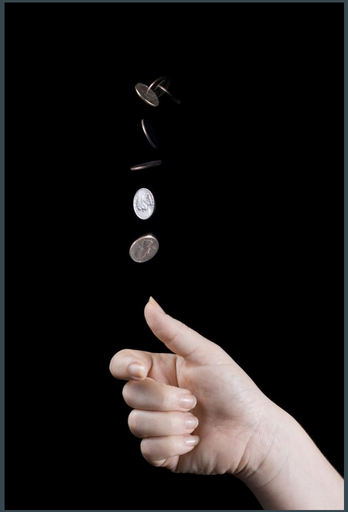


Types of probabilities

- *Marginal probability*: the individual, separate chance that the event happens
    - P(A)
    - P(B)
- *Joint probability:* the chance that all marginal probabilities occur simultaneously
    - P(A and B)
- *Conditional probability:* given that an event happens, how likely is it that another event happens
    - P(A | B)

When there is no joint probability, events are disjoint

What is the chance I get both a heads and tails in two flips?



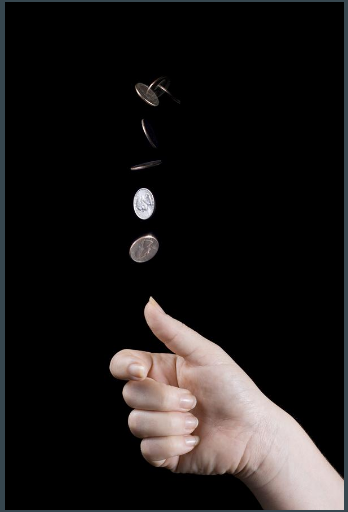
P(Either both heads or both tails) = ½

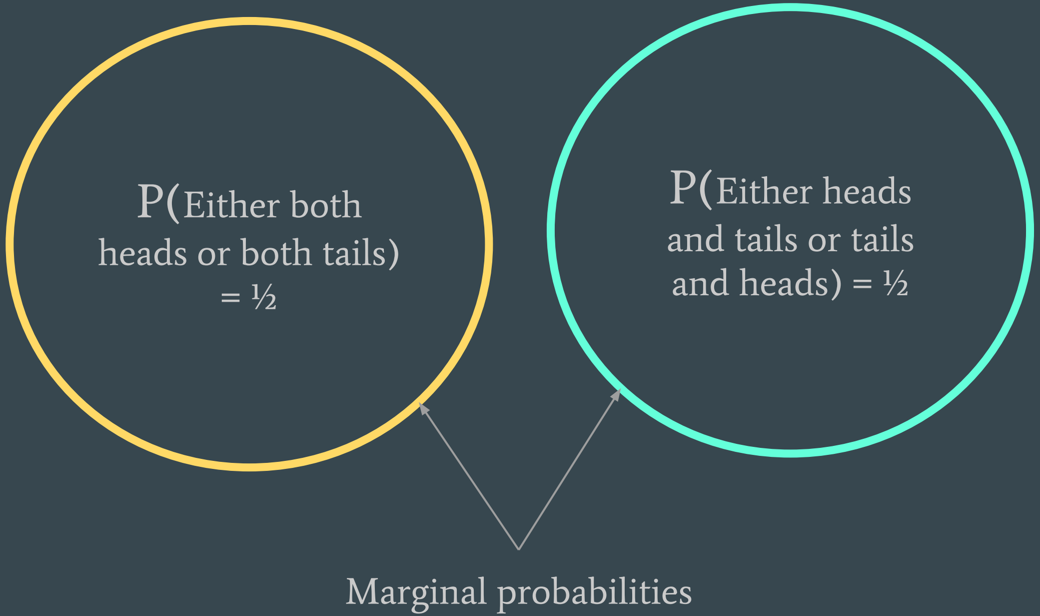
P(Either heads and tails or tails and heads) = ½

Joint probability = 0 (definition of disjoint events)

Marginal probabilities = P(Event)

What is the chance I get both a heads and tails in two flips?



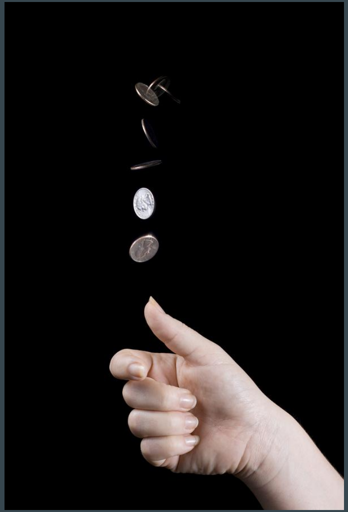


Independent events and joint probability

- Independent events are two or more events whose occurrence does not affect the others
- Independence is always an *assumption*
- The joint probability of two independent events is calculated as:
    - P(A and B) = P(A) * P(B)

Independent Events

What is the chance I first get a heads, and then get a tails?



P(HH) = ¼	P(HT) = ¼
P(TT) = ¼	P(TH) = ¼

P (H) = ½

P(T) = ½

P(H) * P(T) = ¼

Independent Events

3:05 p.m. Eastern	ELO POINT SPREAD	WIN PROB.	SCORE
L.A. Rams		36%	
New Orleans	-4	64%	

6:40 p.m.	ELO POINT SPREAD	WIN PROB.	SCORE
New England		39%	
Kansas City	-3	61%	

- P(Rams win) = 36%
- P(Patriots win) = 39%
- P(Patriots and Rams win) = 36% * 39%

Conditional probabilities

- $P(A|B)$: if B happens, how likely is A?
- Formula for calculating conditional probabilities

$$P(A|B) = P(A \text{ and } B) / P(B)$$

- For independent events, $P(A|B) = P(A)$
- The conditional probability of getting a heads if the first coin is heads

$$P(A|B) = P(A \text{ and } B) / P(B)$$

$$P(A|B) = \tfrac{1}{4} / \tfrac{1}{2}$$

$$P(A|B) = \tfrac{1}{2} = P(A)$$

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
	Mod/Cons Democrat	0.25	0.07	0.02	0.34
	Liberal Democrat	0.18	0.01	0.01	0.20
	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a liberal democrat?

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
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	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a liberal democrat?

P(A) = Believes Earth is not warming

P(B) = Is a Liberal Democrat

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
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	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a liberal democrat?

P(A|B) = P(A and B) / P(B)

? = 0.01 / 0.2

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
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	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a conservative Republican?

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
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	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a conservative Republican?

P(A) = Believes Earth is not warming

P(B) = Is a conservative Republican

Conditional probability calculations

		Response			
		Earth is warming	Not warming	Don't Know Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
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	Total	0.60	0.34	0.06	1.00

What is the probability that a respondent believes the Earth is not warming, given they're a liberal democrat?

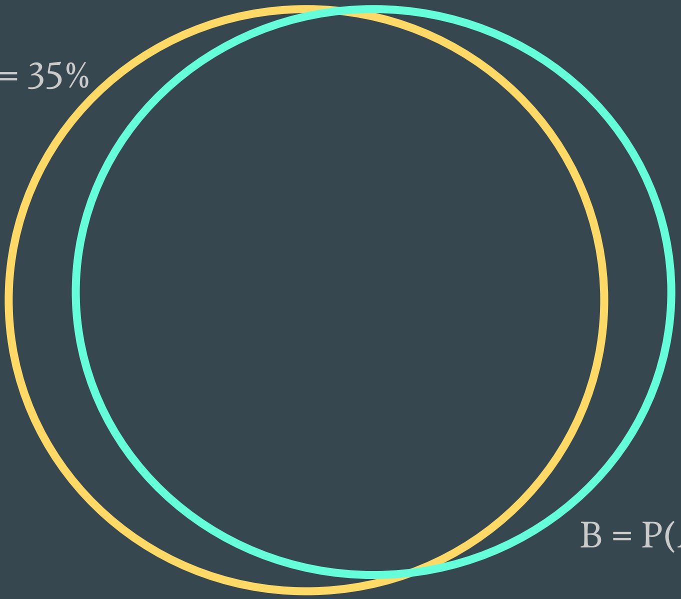
P(A|B) = P(A and B) / P(B)

? = 0.2 / 0.33

Conditional probability calculations

President Trump has a 93% approval rating among Republicans. 35% of Americans identify as republicans. Trump has an approval rating of 41% among all Americans.

A = P(Republican) = 35%



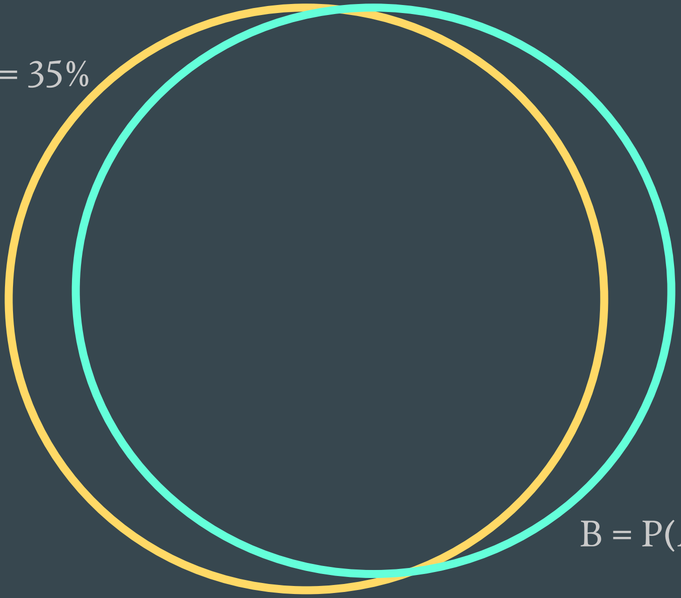
B = P(Approve Trump) = 41%

What is the probability that somebody supports Trump and is a Republican?

Conditional probability calculations

President Trump has a 93% approval rating among Republicans. 35% of Americans identify as republicans. Trump has an approval rating of 41% among all Americans.

A = P(Republican) = 35%



B = P(Approve Trump) = 41%

P(A) = Approves Trump

P(B) = Is Republican

P(A and B) = Is a Republican and approves Trump

Conditional probability calculations

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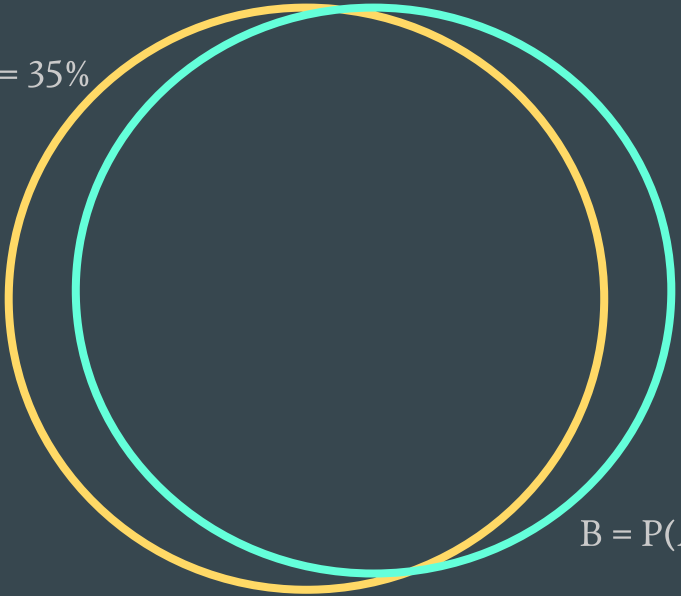

$$P(A|B) = P(A \text{ and } B) / P(B)$$

$$P(A \text{ and } B) = P(A|B) * P(B)$$

Conditional probability calculations

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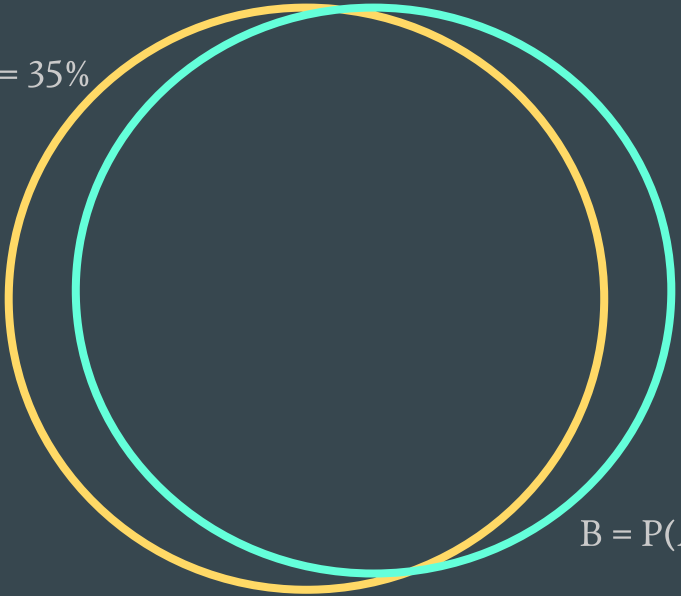


$$P(A \text{ and } B) = P(A|B) * P(B)$$

$$P(A \text{ and } B) = .93 * .41$$

General Multiplication Rule for joint probabilities

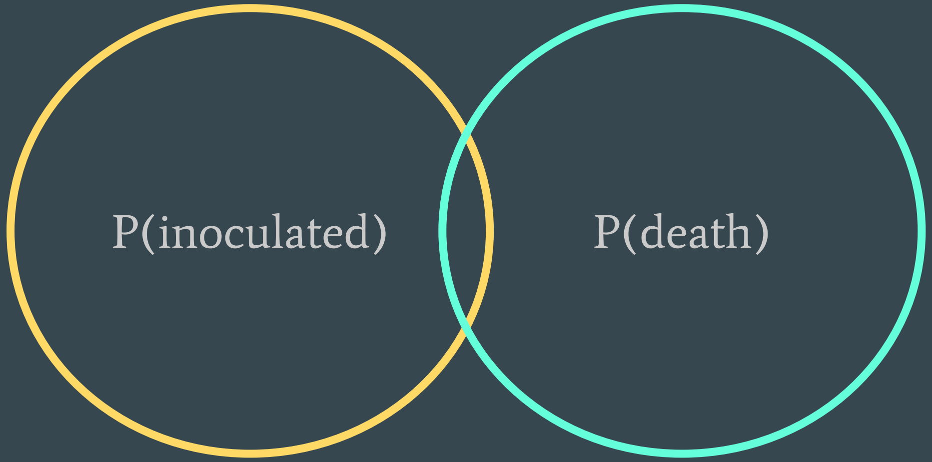
- For independent events, P(A and B) = P(A) * P(B)
- This is actually a simplification for the probability of any combination of events

$$P(A \text{ and } B) = P(A|B) * P(B)$$

- In the case of independent events, P(A|B) = P(A)

Boston smallpox: Venn Diagram

- The probability that multiple events occur



Boston smallpox: marginal and joint probabilities

- A *crosstab* displays joint and marginal probabilities in one table
  - The joint probabilities are the cross-sections
  - The marginal probabilities are the row and column totals
- We can use this to calculate conditional probabilities

		inoculated		
		yes	no	Total
result	lived	0.0382	0.8252	0.8634
	died	0.0010	0.1356	0.1366
	Total	0.0392	0.9608	1.0000

Boston smallpox: calculating conditional probabilities

		inoculated		
		yes	no	Total
result	lived	0.0382	0.8252	0.8634
	died	0.0010	0.1356	0.1366
	Total	0.0392	0.9608	1.0000

What is the probability that a random person in Boston was not inoculated and lived?

P(lived = yes | inoculated = no) = P(lived and not inoculated) / P(not inoculated)

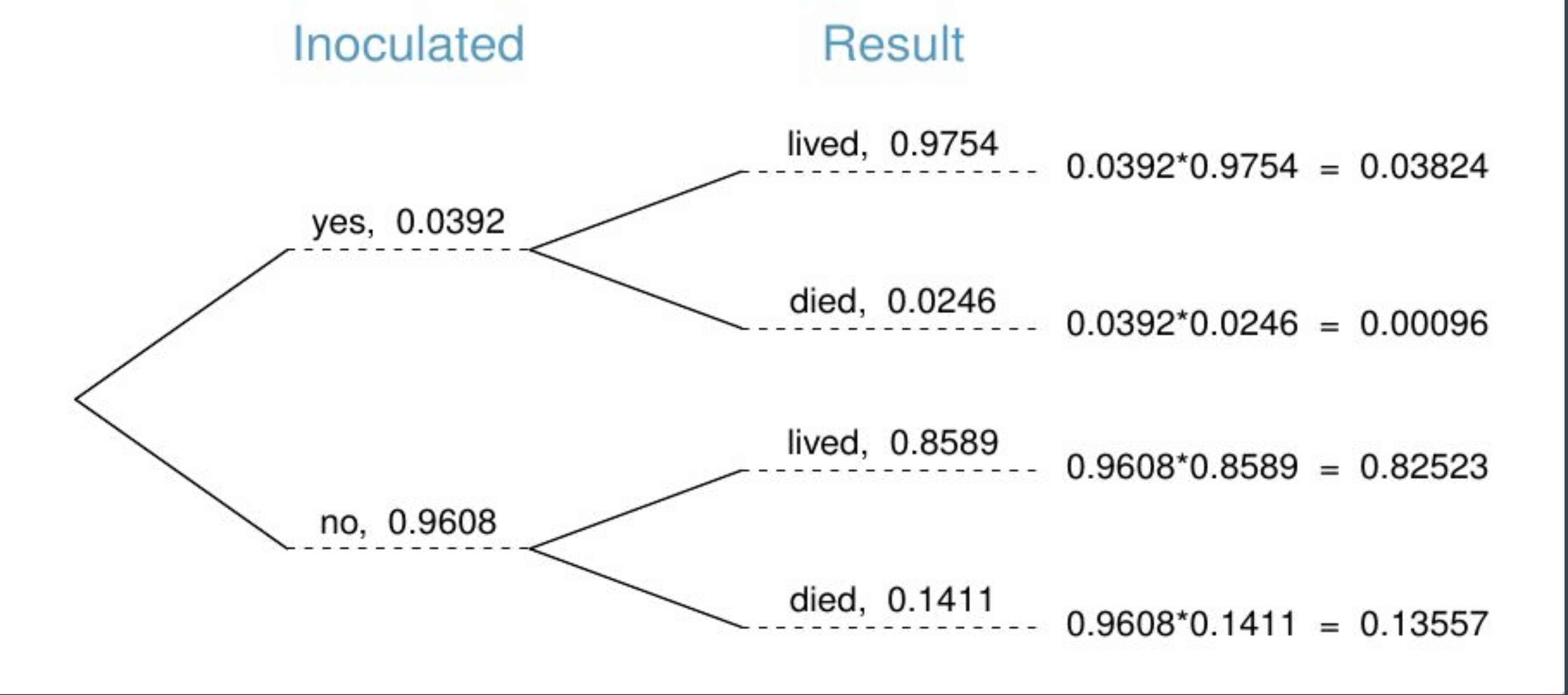
Boston smallpox: calculating conditional probabilities

		inoculated		
		yes	no	Total
result	lived	0.0382	0.8252	0.8634
	died	0.0010	0.1356	0.1366
	Total	0.0392	0.9608	1.0000

What is the probability that a random person in Boston was not inoculated and lived?

P(lived = yes | inoculated = no) = 0.8252 / 0.9608 = 0.8588

Boston smallpox: conditional probabilities



Review

- How to represent probabilities as a venn diagram
- Independent and disjoint events
- *Marginal probability*
  - P(A) or P(B)
- *Joint probability*
  - P(A and B)
- *Conditional Probability*
  - P(A|B)