# Week 12 Lab: Maximum Likelihood Estimation

PLSC 309

5 April 2019

We learned back in Week 4 that the exponential distribution describes any interval that is interrupted by an event (such as waiting times, geographic distince until something happens, etc.) The parameter for the exponetial distribution is $\lambda$. Using maximum likelihood estimation, derive the best estimator for $\lambda$; the first three steps of which are already done for you below. *YOU MUST SHOW ALL OF YOUR WORK TO RECEIVE FULL CREDIT.* If it is easier for you to do math on paper, you may turn in a hand–written supplement with the math, or take a picture of your work. Simplify your answer as much as possible.

After calculating the MLE for $\lambda$ describe what two assumptions were necessary to compute this quantity and why you had to make those assumptions. Please be specific, just saying "independence" will not receive full credit.

## MLE for Exponential distribution

- Find the pdf for the exponential distribution

$$f(x; \lambda) = \lambda e^{-\lambda} x \tag{1}$$

- Express the probability as a likelihood function

$$L(\lambda) = \prod \lambda^N e^{-\lambda \sum_{i=1}^{N} x_i} \tag{2}$$

- Take the log of the likelihood

$$ln(L(\lambda)) = N ln(\lambda) - \lambda \sum_{i=1}^{N} i \tag{3}$$

- Calculate the derivative of the log–likelihood

$$\frac{d}{d\lambda} ln(L(\lambda)) = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i \tag{4}$$