Elimination

1. Elimination Using Matrices

Following this formula $Ax = b$ to solve the system of linear equations using row operations to reduce A to a simpler form

Example:

$2x_1 + 4x_2 - 2x_3 = 2$

$4x_1 + 9x_2 - 3x_2 = 8$

$-2x_1 - 3x_2 + 7x_3 = 10$

Represented as:

$$A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}$$

2. Matrix Multiplication

Combining elimination steps can be expressed as matrix multiplication.

$$AB_{ij} = (row\ i\ of\ A) \cdot (column\ j\ of\ B)$$

Properties:

- Associative: A(BC) = (AB)C
- AB $\neq$BA

  Example:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$$

  Result

$$AB = \begin{bmatrix} 1 \cdot 2 + 1 \cdot 3 & 1 \cdot 2 + 1 \cdot 4 \\ 2 \cdot 2 - 1 \cdot 3 & 2 \cdot 2 - 1 \cdot 4 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$$

3. Rules for Matrix Operations

Addition and Scalar Multiplication Rules must following this:

- Have the same dimension to add (or subtract)
- Scalar multiplication must applies the scalar to all element.
  Example:
  For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, 2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

4. The Augmented Matrix

Is the combined between 2 matrices for example A and b into one matrix [Ab] for simultaneous row operations.
Example:
$x + 2y + 2z = 1$
$4x + 8y + 9z = 3$
$3y + 3z = 1$
Augmented matrix:

$$[Ab] = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 4 & 8 & 9 & 3 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

5. Matrix P_{ij} for Row Exchange

A permutation matrix P_{ij} swaps rows i and j.
Example:
$P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

6. Block Matrices

Matrices can be divided into smaller blocks for simplified operations . (Like a Divide and Conquer)
Use for Elimination, Augmentation, or even multiplication on blocks instead of individual elements.
Example:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} B = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}$$

Result

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} \\ A_{21}B_{11} + A_{22}B_{21} \end{bmatrix}$$