Vector 1

1. Introduction Vectors

Vectors 2D represent as:

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Vector 3D:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Finding length

to finding the length (magnitude) of the vector by these following formula.

$$||v|| = \sqrt{v_1^2 + v_2^2}$$

Ex1:

If $v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, its magnitude (length) can be found as:

$$||v|| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$$

2. Vectors and Linear Combinations

We can scaling the vector by using the constant make it more larger.

$$Linear\ combination: \ cv + dw = c \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + d \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Ex1:

If $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

For $c = 2 \cap d = 3$

$$cv + dw = 2 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

3. Lengths and Dot Products

The length of of vector is magnitude and the dot product is an operation that combines two vectors to give a scalar value.
The length formula

$$\|v\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

The dot product formula

$$v \cdot w = v_1 \cdot w_1 + v_2 \cdot w_2 + \cdots + v_n \cdot w_n$$

Key Point:

- If $v \cdot w = 0$, the vector are perpendicular

Ex1:
Let v = (1,2) and w = (3,4)
Dot product

$$v \cdot w = 1 \cdot 3 + 2 \cdot 4 = 3 + 8 = 11$$

4. Angles Between Vectors

The angle θ between two vectors v and w is determined using the dot product.
Angle formula:

$$\cos\theta = \frac{v \cdot w}{(\|v\|\|w\|)}$$

Key Point:

- If $\cos\theta = 0$, the vector are perpendicular ($\theta = 90$).
- The Schwarz Inequality ensures that: $|v \cdot w| \leq \|v\|\|w\|$

Ex1:

Let $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Find $\cos\theta$

1. $v \cdot w = 1 \cdot 3 + 3 \cdot 1 = 6$
2. $\|v\| = \sqrt{1^2 + 3^2} = \sqrt{10}$
3. $\|w\| = \sqrt{3^2 + 1^2} = \sqrt{10}$
4. $\cos\theta = \frac{6}{(\sqrt{10} \cdot \sqrt{10})} = \frac{6}{10} = 0.6$
   Thus that $\theta = \cos^{-1}(0.6)$

5. Unit Vector

A unit vector has length of 1 and points in the same direction as a given vector.
Unit Vector formula:

$$u = \frac{v}{\|v\|}$$

Ex1:

Let $v = (3, 4)$

1. Find $\|v\| = \sqrt{3^2 + 4^2} = 5$
2. Unit Vector $u = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$