Vector 2

1. Combination of Vectors

Matrix can be formed by combining vectors Assume the vectors

$$u = \begin{bmatrix}1 \\ 2 \\ 3\end{bmatrix}, v = \begin{bmatrix}4 \\ 5 \\ 6\end{bmatrix}$$

Ex1: A matrix A is the combining form of vector u and v:

$$A = \begin{bmatrix}1 & 4 \\ 2 & 5 \\ 3 & 6\end{bmatrix}$$

2. Matrices

A matrix is a rectangular array of numbers arranged in rows and columns.

Matrix Identifier

- Matrix Times Vector:

  ```
  $A=\begin{bmatrix}1&0&0 \\ -1&1&0 \\ 0&-1&1\end{bmatrix}$
  $x=\begin{bmatrix}x_{1} \\ x_{2} \\ x_{3}\end{bmatrix}$
  $Ax=\begin{bmatrix}x_{1} \\ -x_{1}+x_{2} \\ -x_{2}+x_{3}\end{bmatrix}$
  **Ex1:**
  $A=\begin{bmatrix}1&0 \\ 0&1\end{bmatrix},$
  $x=\begin{bmatrix}3 \\ 5\end{bmatrix}$
  $Ax=\begin{bmatrix}3 \\ 5\end{bmatrix}$
  ```

  This show the combination of columns or **dot products with rows**.
- Types of Matrices:
  - Identity Matrix:
    $$I = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}$$

3. Solving Linear Equations

Equation will represented as this format $Ax = b$
Example:
$x + 2y + 3z = 6$
$2x + 5y + 2z = 4$
$6x - 3y + z = 2$
To solving using the technique called **elimination or matrix inversion**.
Ex1:
$x + 2y = 5$
$2x - 3y = -1$
Represent as $A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}, b = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$. Solve for x.
$Ax = b$

4. Vectors & Linear Equations

Linear Combination

using vectors to form new vectors.
Example:

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = b$$

Independence

If no vector in a set is a linear combination of the others. Describes a set of vectors in a vector space where no vector can be written as a linear combination of the others.

Linearly independent

If a set of vectors is linearly independent, it means they contribute unique information to the space and do not overlap in terms of direction or magnitude.

Example 1

$$c_1 v_1 \$ + c_2 v_2 + \cdots + c_n v_n$$

is:

$$c_1 = c_2 = \cdots = c_n = 0$$

Where:

- $c_1, c_2, \ldots, c_n$ are scalar coefficients.
- 0 is the zero vector.
  If there exists any non-zero c_i that satisfies the equation, the vectors are linearly dependent.

Example 2

Linearly Independent Vectors:

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

In 2D space, these vectors are the standard basis vectors.
$c_1 v_1 + c_2 v_2 = 0 \implies c_1 = 0 \cap c_2 = 0$
Linearly Dependent Vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Here, $v_2 = 2v_1$, so they are linearly dependent.
The equation $c_1 v_1 + c_2 v_2 = 0$ has non-trivial solutions like $c_1 = 2, c_2 = -1$

Summary

- A set of vectors is linearly independent if the only solution to $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$ is $c_1 = c_2 = \cdots = c_n = 0$
- If any non-zero values of $c_1, c_2, c_3, \ldots, c_n$ make the equation true, then the vectors are linearly dependent.

In-short

- Independent - Only c = 0 works.
- Dependent - Non-zero c also works.

5. Idea of Elimination

To simplify the system of equations to find solutions.
Process:
convert the system into an upper triangular matrix.
Back Substitution:
Solve the simplified system starting from the last equation.
Examples:
$x - 2y = 1$
$3x + 2y = 11$
After elimination: $y = 1, x = 3$

6. Breakdowns in Elimination

There will have 2 conditions that might occur during the elimination.

- **No Solution:**
  $x - 2y = 1$
  $3x - 6y = 11 \rightarrow Parallel\ lines$
- **Infinite Solutions**
  $x - 2y = 1$
  $3x - 6y = 3 \rightarrow Same\ line$