

Axioms of Probability

- Total probability of a set of possible worlds is 1

$$\sum_{x \in D_X} p_X(x) = 1$$

- Probability of an event is the sum of probabilities of the worlds in which it holds

$$\Pr[X \subseteq A] \equiv \Pr_X[A] = \sum_{x \in A} p_X(x)$$

- Probability of a disjunction (inclusion-exclusion principle)

$$\Pr[A] + \Pr[B] = \Pr[A \wedge B] + \Pr[A \vee B]$$

- Conditional probability/product rule

$$\Pr[A|B] = \frac{\Pr[A \wedge B]}{\Pr[B]}$$

We can think about \cap
as \wedge and \cup as \vee

Putting it All Together

We want to compute $\Pr[X_1 = a \mid X_2 = b]$

1. Bayes' rule: $\Pr[a \mid b] = \frac{\Pr[a,b]}{\Pr[b]} = \alpha \Pr[a, b]$

2. Total Probability: $\Pr[a, b] = \sum_{x_3 \in X_3} \cdots \sum_{x_n \in X_n} \Pr[a, b, x_3, \dots, x_n]$

3. Bayesian Network Factoring:

$$\sum_{x_3 \in X_3} \cdots \sum_{x_n \in X_n} \prod_j \Pr[x_j \mid Parents(X_j)]$$