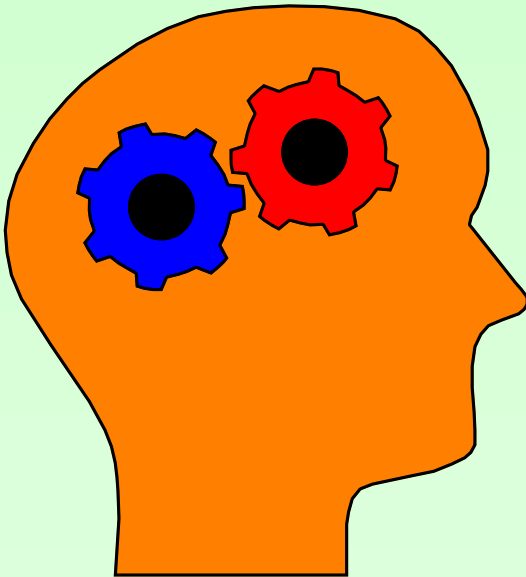




CS2104: Programming Languages Concepts

Lecture 5 : **Other Haskell Features**



*“Comprehension Syntax and
More Type-Classes”*

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Haskell – Other Highlights

- List Comprehension
- Numbers
- Arrays
- Monoid Type Class

Comprehension

Syntactic sugar makes codes more readable

List Comprehension

- List comprehension is a useful *shorthand* for building list data structures:

```
[f x | x <- xs]
```

- Captures a list of all $f\ x$ where x is drawn from xs .
More than one generators are allowed:

```
[(x,y) | x <- xs, y <- ys]
```

- Guards are also permitted. Example :

```
quicksort []      = []  
quicksort (x:xs) = quicksort [y | y <- xs, y < x]  
                  ++ [x]  
                  ++ quicksort [y | y <- xs, y >= x]
```

Translation for List Comprehension

- Given:

`[f x | x <- xs]`

- This get translated to:

`map (\ x -> f x) xs`

- Recall:

`map f [] = []`
`map f (x:xs) = (f x):map f xs`

Translation for List Comprehension

- Given:

```
[f x | x <- xs, x > 5]
```

- This gets translated to:

```
map (\x -> f x) (filter (\x -> x > 5) xs)
```

- Recall:

```
filter f [] = []  
filter f (x:xs) = if (f x) then x : (filter f xs)  
                  else filter f xs
```

Translation for List Comprehension

- Given:

$[(x,y) \mid x \leftarrow xs, y \leftarrow ys]$

- This get translated to:

`concatMap (\ x -> map (\y -> (x,y)) ys) xs`

where:

`concatMap f [] = []
concatMap f (x:xs) = (f x) ++ (concatMap f xs)`

Translation for List Comprehension

- General translation scheme:

$[e \mid x \leftarrow xs]$
 $\rightarrow \text{map } (\backslash x \rightarrow e) \text{ } xs$

$[e \mid x \leftarrow xs, y \leftarrow ys, \text{rest}]$
 $\rightarrow \text{concatMap } (\backslash x \rightarrow [e \mid y \leftarrow ys, \text{rest}]) \text{ } xs$

$[e \mid x \leftarrow xs, \text{test}, \text{rest}]$
 $\rightarrow [e \mid x \leftarrow \text{filter } (\backslash x \rightarrow \text{test}) \text{ } xs, \text{rest}]$

Exercise

$[e \mid x \leftarrow xs] \rightarrow \text{map } (\backslash x \rightarrow e) \text{ } xs$

$[e \mid x \leftarrow xs, y \leftarrow ys, \text{rest}] \rightarrow \text{concatMap } (\backslash x \rightarrow [e \mid y \leftarrow ys, \text{rest}]) \text{ } xs$

$[e \mid x \leftarrow xs, \text{test}, \text{rest}] \rightarrow [e \mid x \leftarrow \text{filter } (\backslash x \rightarrow \text{test}) \text{ } xs, \text{rest}]$

$[(x+x,j) \mid x \leftarrow [1..3], x < 2, j \leftarrow [7..8]] \rightarrow$

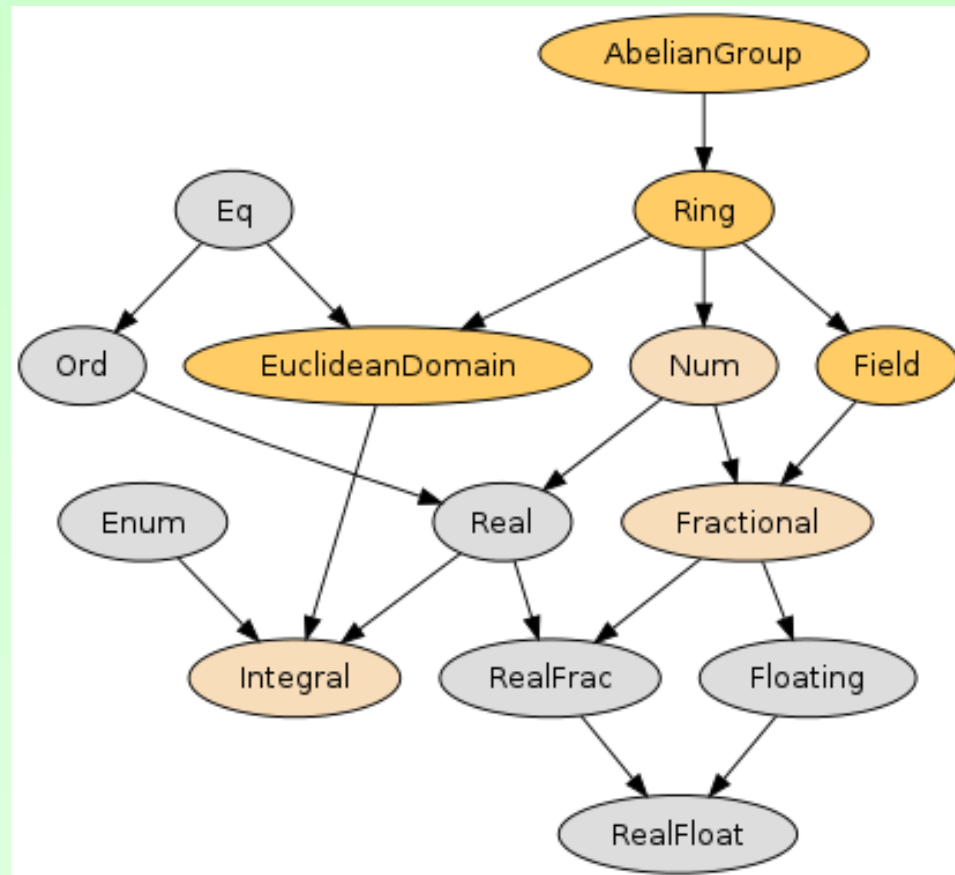
More Type Classes

Recap of Type Classes

- Supports systematic overloading via types.
- So far, mostly first-order type class
- Type sub-classes can be supported (rich hierarchy)
- Captures only type signatures but not properties, such as associativity and commutativity etc

Numbers

Hierarchy in Prelude YAP



Numbers

- `Num` class has the following basic operations: ...

```
class Num a where
    (+), (-), (*)    :: a -> a -> a
    negate, abs      :: a -> a
```

- Division via `div/mod` is supported in `Integral` class, while `(/)` is supported by `Fractional` class.
- The `Floating` class contains trigonometric, logarithmic and exponential functions.

Constructed Numbers

- Standard numeric types `Int`, `Integer`, `Float`, `Double` are primitives and other numeric types are constructed from these
- `Complex` type is under `Floating` but made from `RealFloat`, as follows:

```
data (realFloat a) => Complex a
    = !a :+: !a deriving (Eq,text)
```

```
conjugate :: RealFloat a => Complex a -> Complex a
conjugate (x :+: y) = x :+: (-y)
```

Constructed Numbers

- Another class is `Ratio` :

```
(%)      :: (Integral a) => a => a -> Ratio a
numerator, denominator
          :: Integral a => Ratio a -> a
```

- This is quite different from `Complex`

Arrays

Arrays

- Arrays can be regarded as functions from indices to values, but for efficient retrieval it has to be contiguous and bounded.

```
class (Ord a) => Ix a where
    range      :: (a,a) -> [a]
    index      :: (a,a) -> a -> Int
    inRange    :: (a,a) -> a -> Bool
```

- Possible index types : `Int`, `Integer`, `Char`, `Bool`, tuples of `Ix` type upto length 5
- Possible bounds:
 `(0,9)` for 1-dimensional array with 10 elements
 `((1,1),(100,100))` for 2-dimensional array

Arrays

- `range` enumerates a list of indices in index order.

```
range (0,4)           ⇒      [0,1,2,3,4]
range ((0,0) , (1,2)) ⇒      [(0,0) , (0,1) , (0,2) , (1,0) ,
                               (1,1) , (1,2) ]
```

- `inRange` checks if an index is between a pair of bounds
- `index` calculates zero-origin offset of an index from its bounds

```
index (1,9) 2           ⇒      1
index ((0,0) , (1,2)) (1,1) ⇒      4
```

Array Creation

- Can build an array from an association list with:

```
array :: (Ix a) => (a,a) -> [(a,b)] -> Array a b
```

- An array of squares from 1 to 100

```
squares = array (1,100) [(i,i*i) | i <- [1..100]]
```

- Array subscription done with ! Operator.

```
squares ! 7           ⇒      49
```

- Bounds of an array determined by

```
bounds squares        ⇒      (1,100)
```

Recursive Array

- Arrays may be defined recursively

```
fibs    :: Int -> Array Int Integer
fibs n = a where a = array (0,n) [(0,1), (1,1)] ++
                               [(i, a!(i-2)+a!(i-1) | i <- [2..n]]
```

- Lazy evaluation avoids need to worry on the order of evaluating such recursive arrays. A wavefront example:

```
wavefront :: Int -> Array (Int,Int) Int
wavefront n = a where
    a = array ((1,1), (n,n))
          ([((1,j),1) | j <- [1..n]] ++
           [((i,1),1) | i <- [2..n]] ++
           [((i,j), a!(i,j-1) + a!(i-1,j-1) + a!(i-1,j))
            | i <- [2..n], j <- [2..n]])
```

Accumulation

- While index is unique for array creation, we may wish to accumulate a number of values into each index.

The diagram illustrates the type signature of the `accumArray` function. The signature is written in blue text on a light green background. Four black arrows point from text labels to specific parts of the signature:

- accumulating function** points to `(b->c->b)`.
- initial value** points to `b`.
- bounds** points to `(a,a)`.
- elements to accumulate** points to `[Assoc a b]`.

```
accumArray :: (Ix a) => (b->c->b) -> b -> (a,a)
              -> [Assoc a b] -> Array a b
```

- histogram can calculate occurrences

```
hist :: (Ix a, Integral b) => (a,a) -> [a] -> Array a b
hist bnds is = accumArray (+) 0 bnds [(i,1) | i <- is,
                                             inRange bnds i]
```

```
e17 :: Array Char Int
e17 = hist ('a','z') "This counts the frequencies of each lowercase
letter"
```

Incremental Updates

- An incremental update operation allows a modified array to be returned, e.g. `a // [(i,v), (j,w)]`

```
(//) :: (Ix a) => Array a b -> [(a,b)] -> Array a b
```

- Its index may appear multiple times with the last value taking precedence.
- An example to swap two rows.

```
swapRows :: (Ix a, Ix b, Enum b) => a -> a -> Array (a,b) c
          -> Array (a,b) c
swapRows i i' a = a // [assoc | j <- [jLo..jHi],
                             assoc <- [((i,j),a!(i',j)),
                                         ((i',j),a!(i,j))]]
      where ((iLo,jLo),(iHi,jHi)) = bounds a
```

Algebraic Structures

Semi-Group and Monoids

- We can capture *mathematical* structures as type classes.

```
class SemiGroup a where
  op      :: a -> a -> a
class SemiGroup a => Monoid a where
  unit    :: a
```

- Above declaration only captures the type signatures.
Two properties of monoids are:

```
unit `op` x = x `op` unit = x
(x `op` y) `op` z = x `op` (y `op` z)
```

- But these properties are important but not checked by Haskell. We assume that users ensure them.

Semi-Group and Monoids

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Higher-Order Type Classes

Monoid \longrightarrow Monads