Logical Agents

AIMA Chapter 7

Assignment 2 (Due: Oct 18, 2359)

- No tutorial restriction to form groups
- Exactly 2 people – make life simple for ________
- Any CSP modelling (including the ones in the textbook!)
- Don’t worry too much about the “timing” aspect
- Explain the algorithm in the report

KB-Agent & Wumpus World

- Agent interacts with the KB using $TELL$ and ASK
- KB is a set of sentences
  - Wumpus initial KB set of rules (and possibly some percepts)

Logic in General

- Logic: formal language for KR, infer conclusions
- Syntax: defines the sentences in the language
- Semantics: define the “meaning” of sentences;
  - i.e., define truth of a sentence in a world
- E.g., language of arithmetic
  - $x + 2 \geq y$ is a sentence; $x2y +>$ is not a sentence
  - $x + 2 \geq y$ is true in a world where $x = 7, y = 1$
  - $x + 2 \geq y$ is false in a world where $x = 0, y = 6$

Entailment

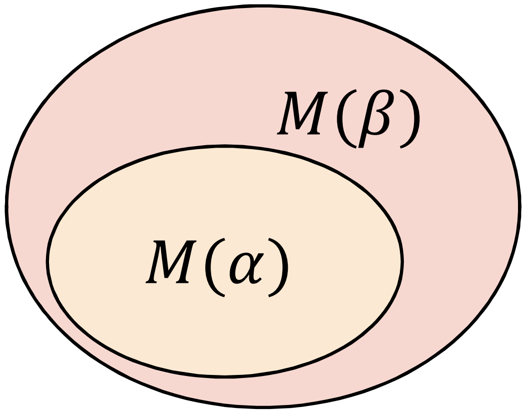
- Modeling: m models α if α is true under m.
  For example, what are models for the following?

$$\alpha = (q \in \mathbb{Z}_+) \wedge (\forall n, m \in \mathbb{Z}_+ : q = nm \Rightarrow n \vee m = 1)$$

- We let $M(\alpha)$ be the set of all models for α
- Entailment means that one thing follows from another:

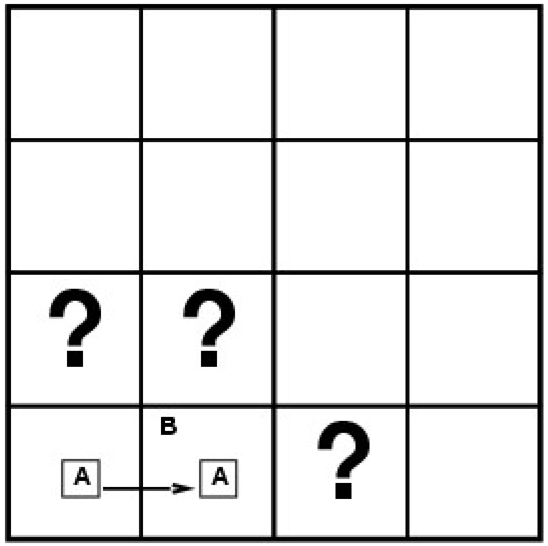
$$\alpha \vDash \beta \text{ or equivalently } M(\alpha) \subseteq M(\beta)$$

- For example:
  $\alpha = (q \text{ is prime})$ entails
  $\beta = (q \text{ is odd}) \vee (q = 2)$.

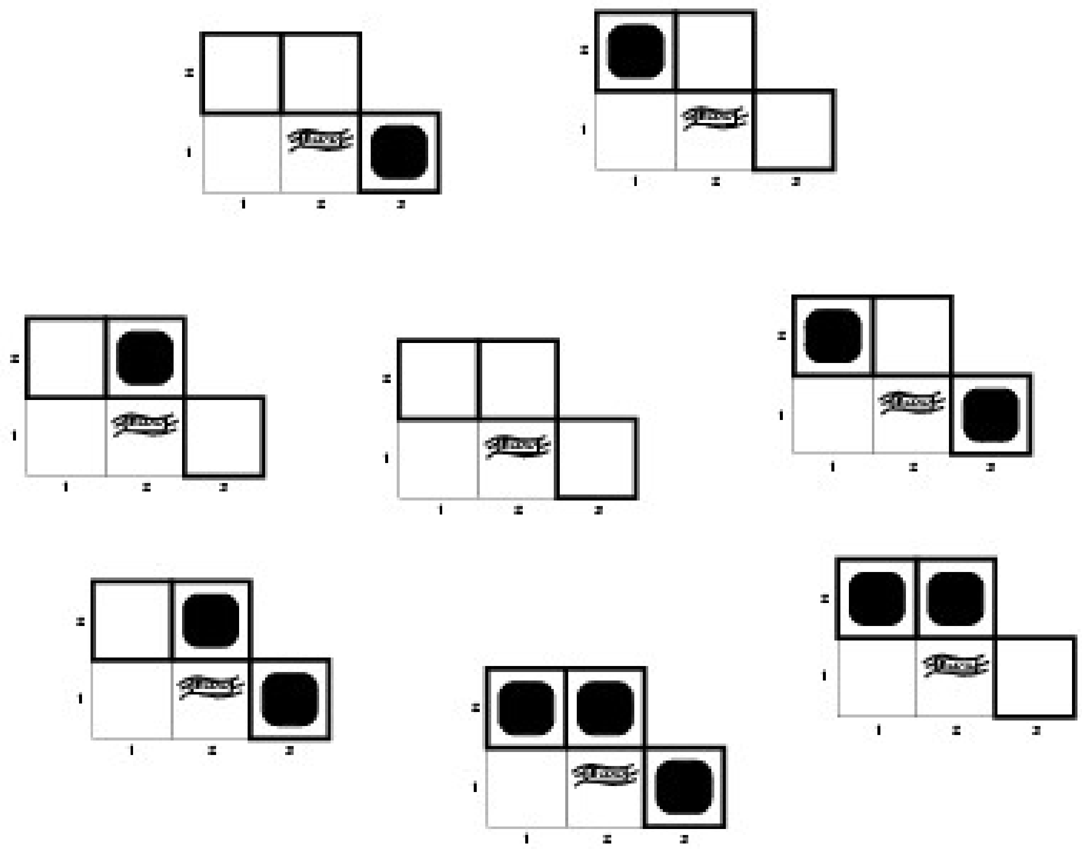


Entailment in the Wumpus World

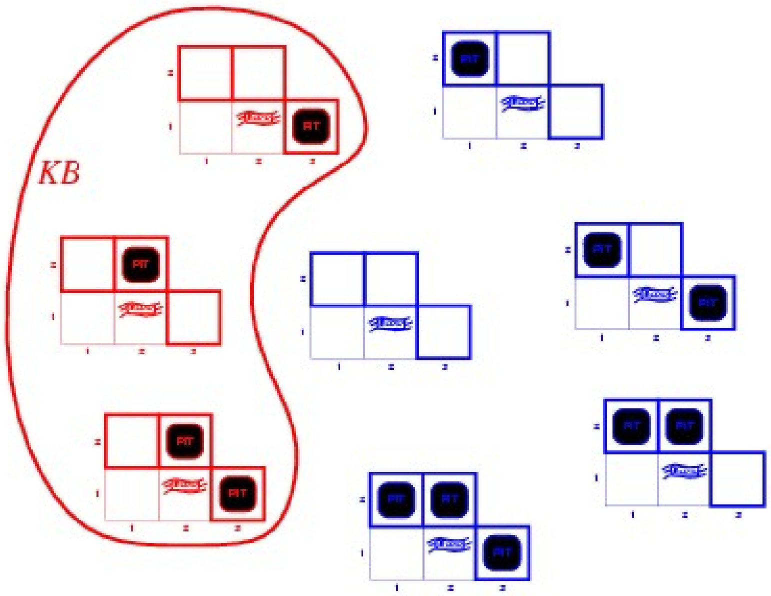
- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for KB assuming only pits
- 3 Boolean choices ⇒ 8 possible models



Wumpus Models

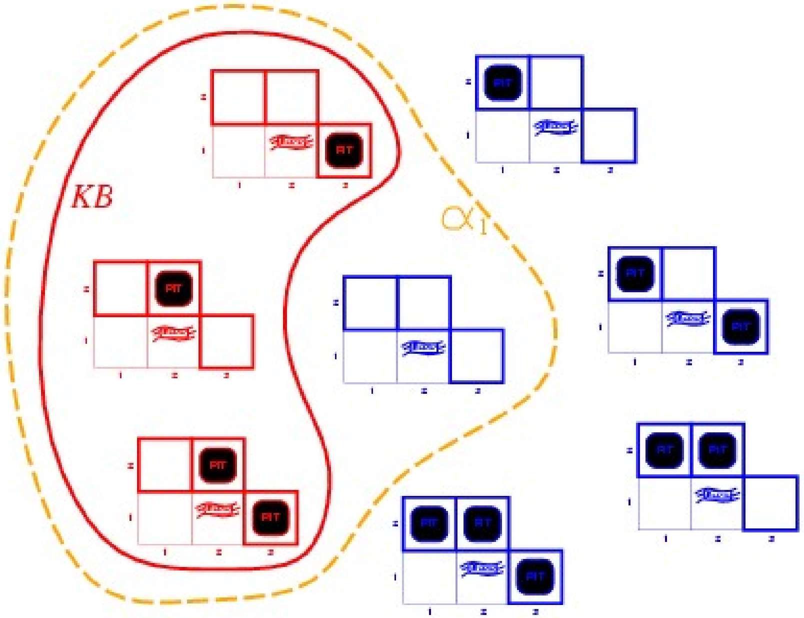


Wumpus Models



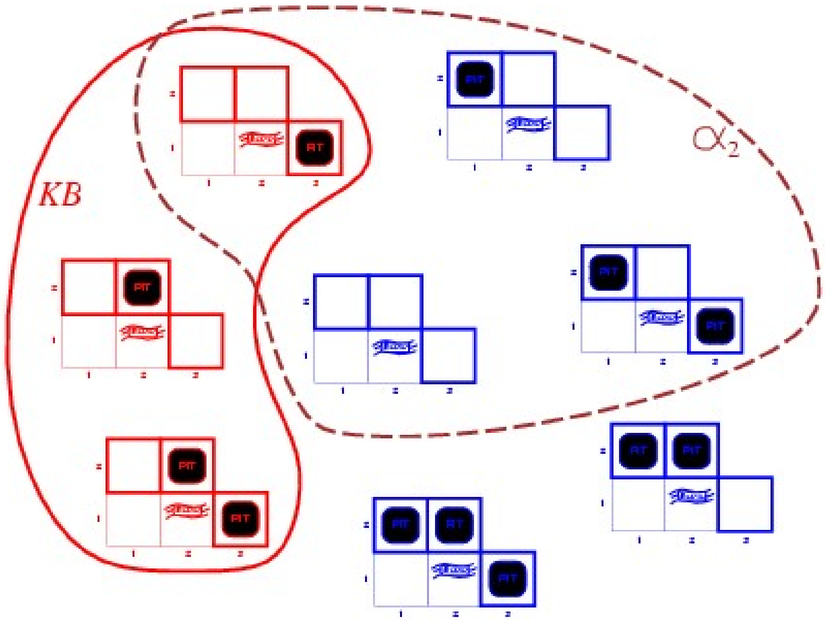
- KB = wumpus-world rules + percepts

Wumpus Models



- KB = wumpus-world rules + percepts
- α_1 = “[1,2] is safe”, $KB \vDash \alpha_1$, proved by model checking
- The agent can infer that [1,2] is safe

Wumpus Models



- KB = wumpus-world rules + percepts
- α_2 = “[2,2] is safe”, $KB \not\models \alpha_2$
- The agent cannot infer that [2,2] is safe (or unsafe)!

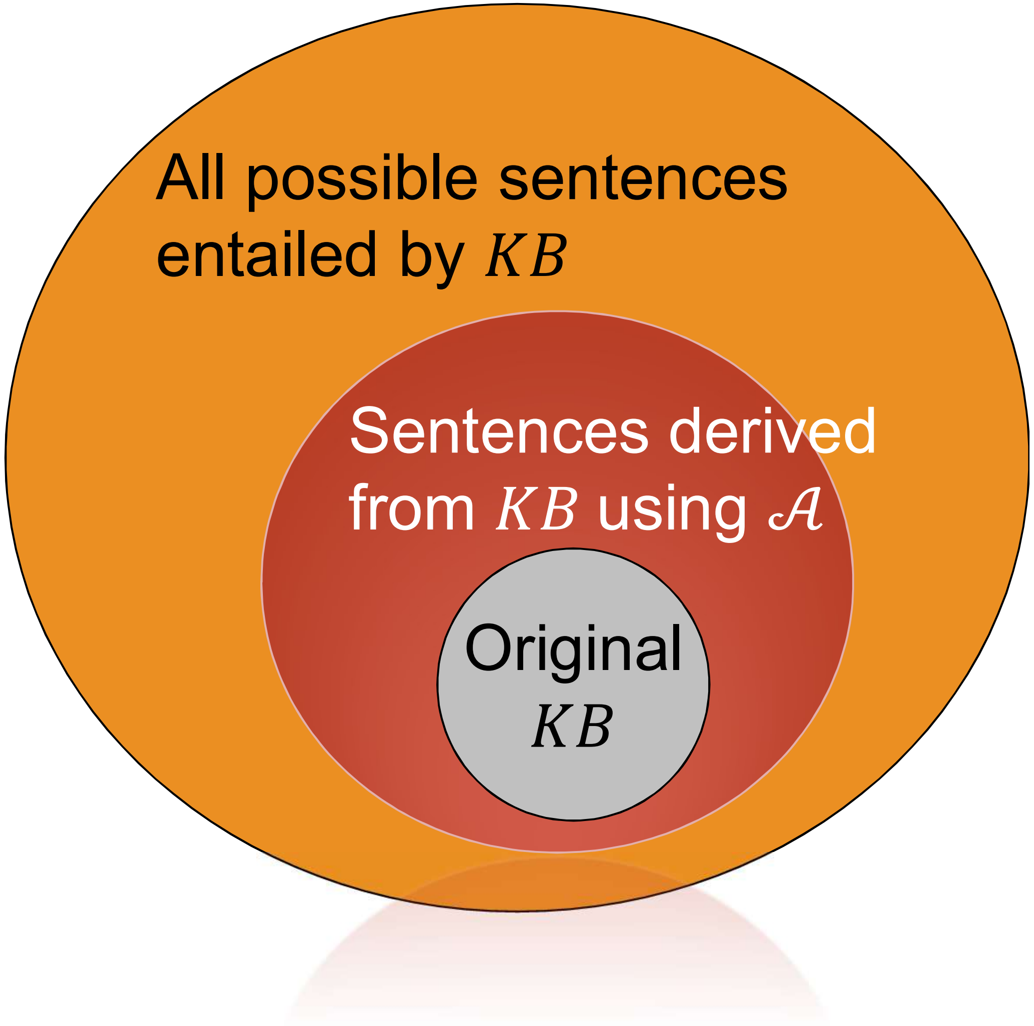
Inference algorithm: is a sentence α is derived from KB?

- Define $KB \vdash_{\mathcal{A}} \alpha$ to be "sentence α is derived from KB by inference algorithm $\mathcal{A}$"
  - $\mathcal{A}$ is **sound** if $KB \vdash_{\mathcal{A}} \alpha$ implies $KB \vDash \alpha$.
    "don't infer nonsense"
  - $\mathcal{A}$ is **complete** if $KB \vDash \alpha$, implies $KB \vdash_{\mathcal{A}} \alpha$.
    "If it's implied, it can be inferred"

Is an inference algorithm complete and sound?

Completeness: $\mathcal{A}$ **is complete if whenever** $KB \vDash \alpha$**, it is also true that** $KB \vdash_{\mathcal{A}} \alpha$

- An incomplete inference algorithm cannot reach all possible conclusions
- Equivalent to completeness in search (chapter 3)



Propositional Logic: Syntax

- A simple logic – illustrates basic ideas
- Defines allowable sentences
- Sentences are represented by symbols e.g. S_1, S_2
- Logical connectives for constructing complex sentences from simpler ones:
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences:
    - $S_1 \wedge S_2$ is a sentence (conjunction)
    - $S_1 \vee S_2$ is a sentence (disjunction)
    - $S_1 \Rightarrow S_2$ is a sentence (implication)
    - $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic: Semantics

A model is then just a **truth assignment to the basic variables**.

If a model has n variables, how many truth assignments are there?

All other sentences' truth value is derived according to logical rules.

$$x_1 = T; x_2 = F; x_3 = T$$

$$(x_1 \land \neg x_2) \Rightarrow \neg\big(x_3 \lor (\neg x_1 \land x_2)\big) =?$$

Knowledge Base for Wumpus World

- $P_{ij} = \text{True} \Leftrightarrow$ there is a pit in $[i, j]$.
- $B_{ij} = \text{True} \Leftrightarrow$ there is breeze in $[i, j]$
- Rules:
  - $R_1: \neg P_{1,1}$
  - $R_4: \neg B_{1,1}$
  - $R_5: P_{2,1}$

KB is true iff $\bigwedge_{k=1,\ldots,5} R_k$ is true

- "Pits cause breezes in adjacent squares"
  - $R_2: B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
  - $R_3: B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

Inference

- Given a knowledge base, infer something non-obvious about the world.
- Mimic logical human reasoning
- After exploring 3 squares, we have some understanding of the Wumpus world
- Inference ⇨ Deriving knowledge out of percepts

Given KB and α, we want to know if $KB \vdash \alpha$

Truth Table for Inference

Is α_1 true whenever KB is true?

		,1	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
		se	*false*	*false*	*false*	*false*	*false*	*true*
alse	*false*	*false*	*false*	*false*	*false*	*true*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*false*

$R_1: \neg P_{1,1}$

$R_4: \neg B_{1,1}$

$R_5: B_{2,1}$

$\alpha_1 = \neg P_{1,2}$

Does KB entail α_1?

Can we infer that [1,2] is safe from pits?

Inference by Truth-Table Enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, { })
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
      if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
      else return true // when KB is false, always return true
   else do
      P ← FIRST(symbols)
      rest ← REST(symbols)
      return (TT-CHECK-ALL(KB, α, rest, model ∪ {P = true})
              and
              TT-CHECK-ALL(KB, α, rest, model ∪ {P = false }))
```

Check all possible truth assignments

Validity and Satisfiability

A sentence is valid if it is true in all models,

e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $\left(A \wedge (A \Rightarrow B)\right) \Rightarrow B$

Validity is connected to entailment via the Deduction Theorem:

$\boxed{KB \vDash \alpha \text{ iff } (KB \Rightarrow \alpha) \text{ is valid}}$

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to entailment via the following:

$KB \vDash \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

Proof Methods

Applying inference rules (aka theorem proving)

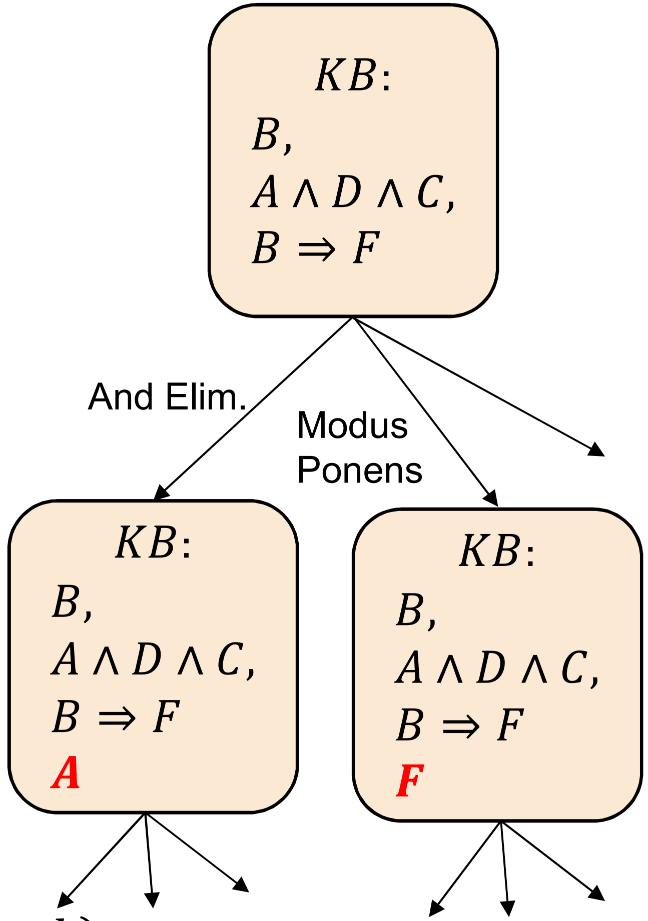
- Generation of new sentences from old
- Proof = sequential application of inference rules
- Inference rules are actions in a search algorithm
- Proof can be more efficient: ignores irrelevant propositions
- Transformation of sentences into a normal form

Model checking

- Truth table enumeration (time complexity exponential in n)
- Improved backtracking, e.g. DPLL
- local search in model space (sound but incomplete), e.g. min-conflicts-like hill-climbing algorithm

Applying Inference Rules

- Equivalent to a search problem
  - States: KBs (initial state is initial KB)
  - Actions: Inference rules
  - Transition: add sentence to current KB
  - Goal: KB contains sentence to prove
- Examples of inference rules
  - And-Elimination (A.E.): $a \wedge b \vDash a$
  - Modus Ponens (M.P.): $a \wedge (a \Rightarrow b) \vDash b$
  - Logical Equivalences: $(a \vee b) \vDash \neg(\neg a \wedge \neg b)$

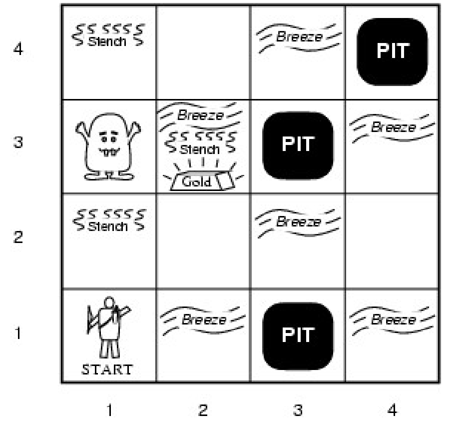


Logical equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Example: No pit in [1, 2]/ $\neg P\_(1,2)$

- KB:
  - $R_1: \neg P_{1,1}$
  - $R_2: B_{1,1} \Leftrightarrow (\mathrm{P}_{1,2} \lor P_{2,1})$
  - $R_3: B_{2,1} \Leftrightarrow (P_{1,1,} \lor P_{2,2} \lor P_{3,1})$
  - $R_4: \neg B_{1,1}$



Resolution for Conjunctive Normal Form (CNF)

- conjunction of "disjunctions of literals" (clauses)
- E.g., $(x_1 \vee \neg x_2) \wedge (x_2 \vee x_3 \vee \neg x_4)$
- Resolution: if a literal x appears in C_1 and its negation $\neg x$ appears in C_2, it can be deleted:

$$\frac{(x_1 \vee \cdots \vee x_m \vee x) \wedge (y_1 \vee \cdots \vee y_k \vee \neg x)}{(x_1 \vee \cdots \vee x_m \vee y_1 \vee \cdots \vee y_k)}$$

(delete duplicate variables as necessary)

- Resolution is **sound** and **complete** for propositional logic

Conversion to CNF: the Rules

1. Convert $\alpha \Leftrightarrow \beta$ to $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$
2. Convert $\alpha \Rightarrow \beta$ to $\neg\alpha \vee \beta$
3. Move $\neg$ inwards using De Morgan and double negation
    1. Convert $\neg(\alpha \vee \beta)$ to $\neg\alpha \wedge \neg\beta$
    2. Convert $\neg(\alpha \wedge \beta)$ to $\neg\alpha \vee \neg\beta$
    3. Convert $\neg(\neg\alpha)$ to α
4. Convert $\big(\alpha \vee (\beta \wedge \gamma)\big)$ to $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

Resolution Algorithm

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
  α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\}$
  loop do
    for each pair of clauses C_i, C_j **in** $clauses$ **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if $resolvents$ contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false*
    $clauses \leftarrow clauses \cup new$

Resolution closure

What does an empty clause imply??

Proof by contradiction: show that $KB \wedge \neg\alpha$ is unsatisfiable

Resolution theorem:

If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains the empty clause.

Why is Resolution for CNF Sound and Complete?

Resolution Example

- $KB = \left(B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})\right) \wedge (\neg B_{1,1})$
- $\alpha = \neg P_{1,2}$

Negate the premise via proof by contradiction

Resolution Example

- $KB = \left(B_{1,1} \Leftrightarrow \left(P_{1,2} \vee P_{2,1}\right)\right) \wedge \left(\neg B_{1,1}\right)$
- $\alpha \; = \neg P_{1,2}$

Forward and Backward Chaining

- Horn Form (restricted)
  - KB = conjunction of Horn clauses
  - Horn clause = definite clause or goal clause
    - Definite clause : $\bigwedge_j \alpha_j \Rightarrow \beta$
    - Goal clause : $\bigwedge_j \alpha_j \Rightarrow False$
  - e.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- Inference with Horn clauses: forward chaining or backward chaining algorithms. Easy to interpret, run in linear time
- Inference is Modus Ponens (for Horn Form): sound for Horn KB

$$\frac{\alpha_1, \ldots, \alpha_k; \bigwedge_j \alpha_j \Rightarrow \beta}{\beta}$$

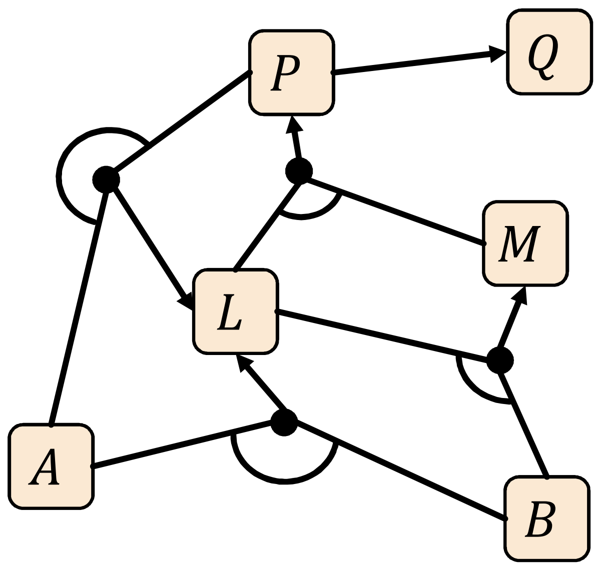
Forward Chaining (FC)

- Idea: Fire any rule whose premise is satisfied in the KB, add its conclusion to the KB, repeat until query is found

KB of horn clauses

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

AND-OR graph



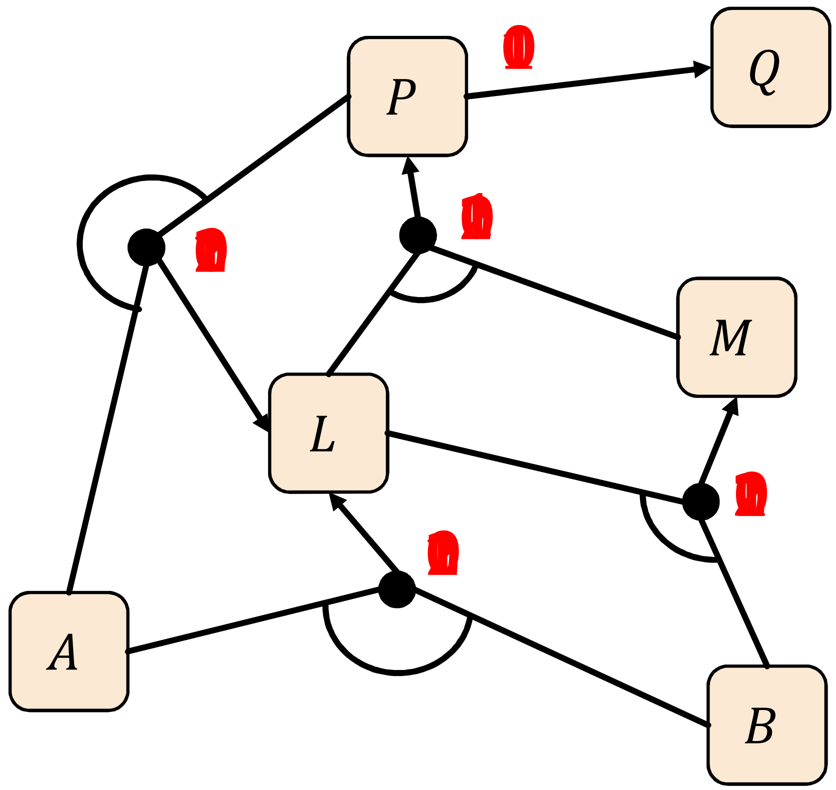
Forward Chaining (FC) Algorithm

- For every rule c, let $\text{count}(c)$ be the number of symbols in c's premise.
- For every symbol s, let $\text{inferred}(s)$ be initially $False$
- Let agenda be a queue of symbols (initially containing all symbols known to be true.
- While $\text{agenda} \neq \emptyset$:
  - pop a symbol p from $agenda$; if it is q we're done
  - Set $inferred(p) = True$
  - For each clause $c \in KB$ such that p is in the premise of c, decrement $count(c)$. If $count(c) = 0$, add c's conclusion to $agenda$.

Forward chaining is sound and complete for Horn KB

Forward Chaining Example

Iteration 1: $[A, B]$
Iteration 2: $[B]$
Iteration 3: $[] \Rightarrow [L]$
Iteration 4: $[] \Rightarrow [M]$
Iteration 5: $[] \Rightarrow [P]$
Iteration 6: $[] \Rightarrow [L, Q]$
Iteration 7: $[Q]$
Iteration 8: $[]$



Proof of Completeness

FC derives every atomic sentence entailed by Horn KB

1. Suppose FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model m that assigns true/false to symbols based on the inferred table
3. Every clause in the original KB is true in m

   $$\alpha_1 \wedge \cdots \wedge \alpha_k \Rightarrow \beta$$

4. Hence, m is a model of KB
5. If $KB \vDash q$, then q is true in every model of KB, including m.

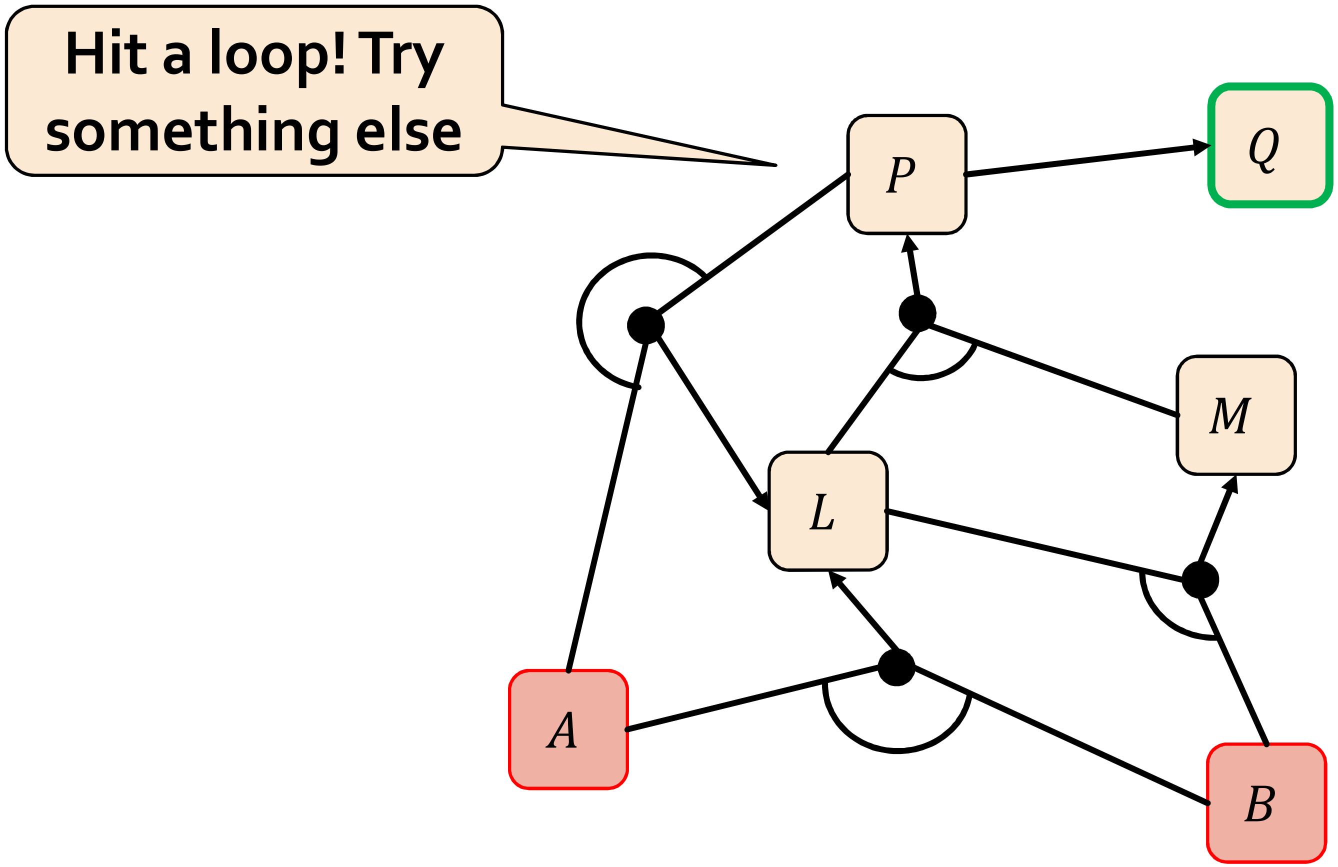
Backward Chaining (BC)

Backtracking depth-first search algorithm

Idea: work backwards from the query q

- To prove q by BC,
  - check if q is known already, or
  - prove by BC the premise of some rule concluding q
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  - has already been proven true, or
  - has already failed

Backward Chaining Example



Forward vs. Backward Chaining

FC = data-driven reasoning

- e.g., object recognition, routine decisions
- May do a lot of work that is irrelevant to the goal

BC = goal-driven reasoning

- e.g., Where are my keys? How do I get into Google?
- Complexity of BC can be sublinear in $|KB|$.

Proof Methods

Applying inference rules (aka theorem proving)

- Generation of new sentences from old
- Proof = sequential application of inference rules
- Inference rules are actions in a search algorithm
- Proof can be more efficient: ignores irrelevant propositions
- Transformation of sentences into a normal form

Model checking

- Truth table enumeration (time complexity exponential in n)
- Improved backtracking, e.g. DPLL
- local search in model space (sound but incomplete), e.g. min-conflicts-like hill-climbing algorithm

Efficient Propositional Model Checking

Two families of efficient algorithms for propositional model checking:

- Complete backtracking search algorithms
  - DPLL algorithm (Davis, Putnam, Logemann, Loveland)
- Incomplete local search algorithms
  - WALKSAT algorithm

These algorithms test a sentence for satisfiability; used for inference.

Recall: Satisfiability is connected to entailment via

$KB \vDash \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

DPLL Algorithm

How are DPLL and CSP related?

Determine if a given CNF formula $\phi = C_1 \wedge \cdots \wedge C_m$ is satisfiable

Improvements over truth table enumeration:

1. Early termination

   (a) A clause is true iff any literal in it is true.

   (b) The formula ϕ is false if any clause is false.

2. Pure symbol heuristic

   Least constraining value

   Pure symbol: always appears with the same "sign" in all clauses.

   e.g., in $(A \vee \neg B) \wedge (\neg B \vee \neg C) \wedge (C \vee A)$, A and B are pure; C is impure.

   Make a pure symbol's literal true: Doing this can never make a clause false.

   Ignore clauses that are already true in the model constructed so far.

3. Unit clause heuristic

   Most constrained variable

   Unit clause: only one literal in the clause.

   The only literal in a unit clause must be true.

DPLL Algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, { })

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols* – P, *model* ∪ {P=*value*})
  value ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  P is non-null **then return** DPLL(*clauses*, *symbols* – P, *model* ∪ {P=*value*})
  ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, *model* ∪ {P=*true*}) **or**
      DPLL(*clauses*, *rest*, *model* ∪ {P=*false*}))

Early Termination

Try to apply heuristics

If it doesn't work, brute force.

WALKSAT Algorithm

- Incomplete, local search algorithm
- Evaluation function: minimize the number of unsatisfied clauses
- Balance between greediness and randomness

WALKSAT Algorithm

CNF formula: $\phi = C_1 \wedge \cdots \wedge C_m$

1. Start with a random variable assignment $\ell_1 \ldots \ell_n$, where $\ell_i \in \{True, False\}$
2. If $\vec{\ell}$ satisfies the formula return $\vec{\ell}$.
3. Choose a random unsatisfied clause $C_j \in \phi$
4. With probability p flip the truth value of a random symbol $x_i \in C_j$; else flip a symbol $x_i \in C_j$ that maximizes number of satisfied clauses in ϕ.
5. Repeat steps 2-4 $MaxFlips$ times.

Why is WalkSat incomplete?

How are WALKSAT and local search related?

Inference-Based Agents in the Wumpus World

A wumpus-world agent using propositional logic:

$\neg P_{1,1}$

$\neg W_{1,1}$

$B_{i,j} \Leftrightarrow \left(P_{i,j+1} \lor P_{i,j-1} \lor P_{i+1,j} \lor P_{i-1,j}\right)$

$S_{i,j} \Leftrightarrow \left(W_{i,j+1} \lor W_{i,j-1} \lor W_{i+1,j} \lor W_{i-1,j}\right)$

$W_{1,1} \lor W_{1,2} \lor \cdots \lor W_{4,4}$

$\neg W_{1,1} \lor \neg W_{1,2}$

$\neg W_{1,1} \lor \neg W_{1,3}$

...

Each i, j rule is its own proposition

There is exactly one Wumpus

64 distinct proposition symbols, 155 sentences

Expressiveness Limitation of Propositional Logic

- KB contains "physics" sentences for every single square
- For every time t and every location $[i, j]$,

$$L_{i,j}^{t} \wedge FacingEast^{t} \wedge Forward^{t} \Rightarrow L_{i+1,j}^{t}$$

Rapid proliferation of clauses