Uncertainty

AIMA Chapter 13

Outline

- Uncertainty
- Probability
- Syntax and Semantics
- Inference

Uncertainty: Motivating Example

Let taxi agent's action A_t = leave for airport t minutes before flight. **Will A_t get me there on time?**

- Sources of uncertainty:
  1. Partial observability (e.g., road state, other drivers' plans, ...)
  2. Noisy sensors (e.g., traffic reports, fuel sensor, ...)
  3. Uncertainty in action outcomes (e.g., flat tire, accident, ...)
  4. Complexity in modeling and predicting traffic (e.g., congestion)
- Logical agent either

  Logical agent can only say Yes or No and nothing more

  1. risks falsehood: "A_{25} will get me there on time", or
  2. reaches weaker conclusion: "A_{25} will get me there on time **if** there's no accident on the bridge **and** it doesn't rain **and** my tires remain intact…"

Dealing with Uncertainty

- Probability – degree of belief
- Summarizing uncertainty using Probability

- Decisions – based on utility (usefulness)
- Rational agent – prefers state with higher utility

Random Variables

Domains

- Boolean: coin is either heads or tails (true or false)
- Discrete: a die can have values $\{1, \ldots, 6\}$

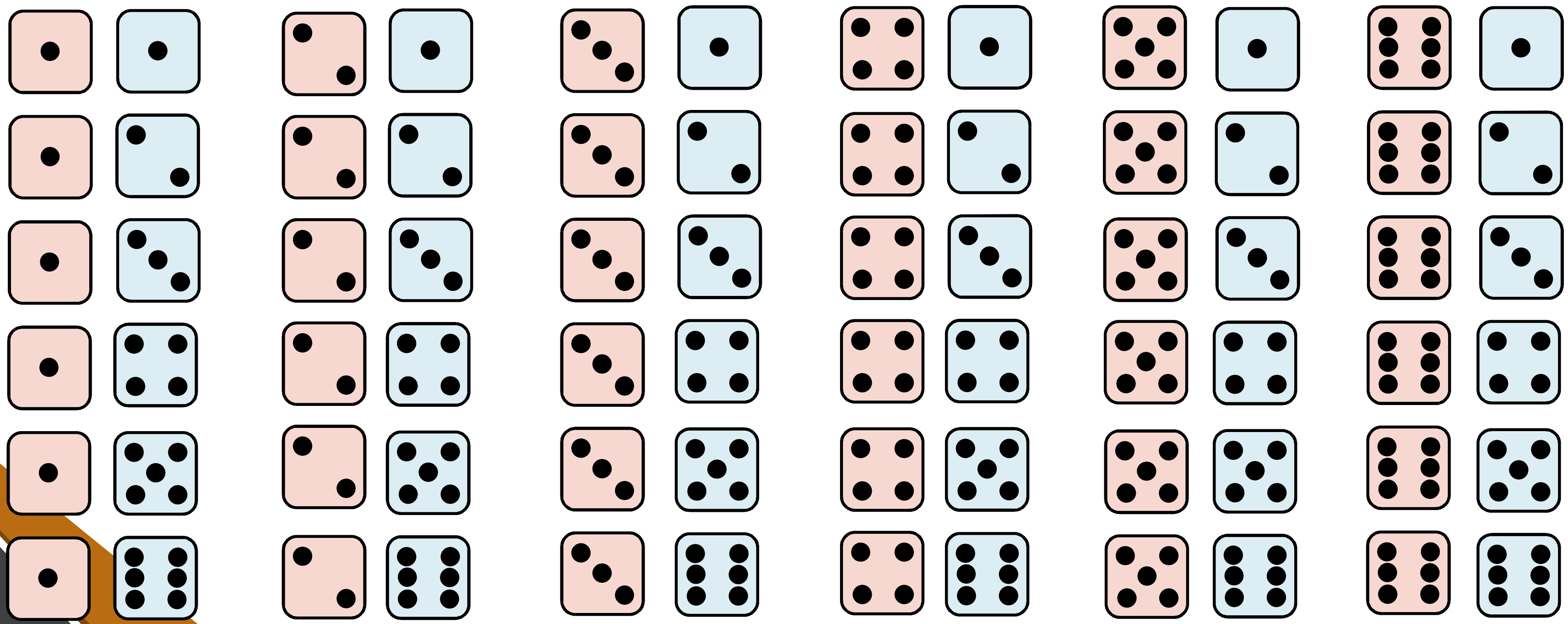
Events: subsets of domains

- $Heads(X)$ the coin flipped to heads
- $Even(X)$ the die has value $\in \{2,4,6\}$

Events

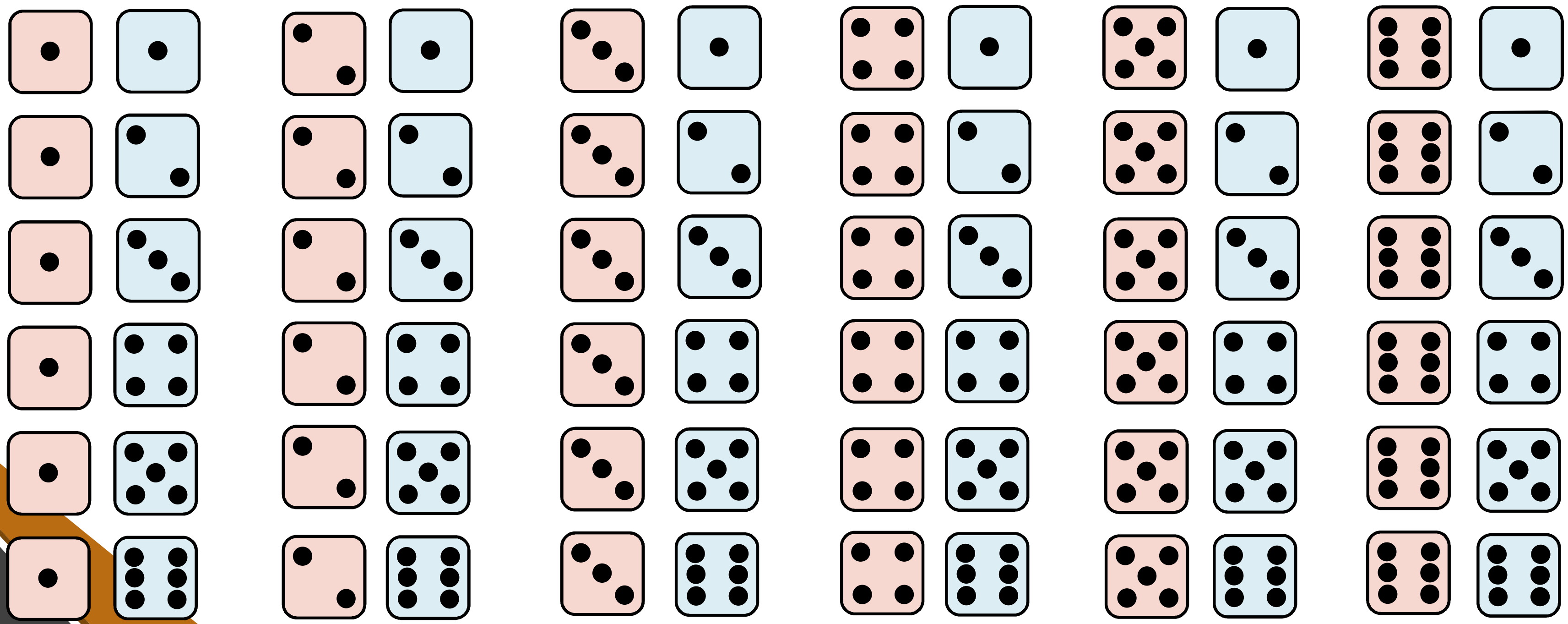
- Given a random variable X, let D_X be its domain.
- **Atomic event (possible world):** an assignment of a value to each random variable; a singleton event
  - We roll two different dice

Each of the 36 combinations is an atomic event



Events

- Red die = RV X_1, blue die = X_2
- Event: $X_1 + X_2 = 8$



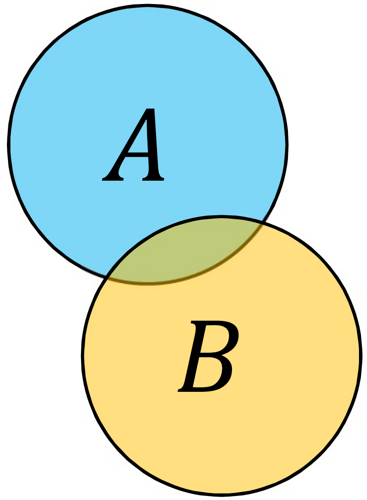
Axioms of Probability

- Let X be a random variable with finite domain D_X.
- A probability distribution over D_X assigns a value $p_X(x) \in [0,1]$ to every $x \in D_X$ s.t.

$$\sum_{x \in D_X} p_X(x) = 1$$

- For any event $A \subseteq D_X$ we have

$$\Pr[X \subseteq A] \equiv \Pr_X[A] = \sum_{x \in A} p_X(x)$$



- In particular

$$\Pr[A] + \Pr[B] = \Pr[A \cap B] + \Pr[A \cup B]$$

Joint Probability

- Given two random variables X and Y, the **joint probability** of an atomic event $(x, y) \in D_X \times D_Y$ is $p_{X,Y}(x, y) = \Pr[X = x \wedge Y = y]$
- In particular $p_X(x) = \sum_{y \in D_Y} p_{X,Y}(x, y)$

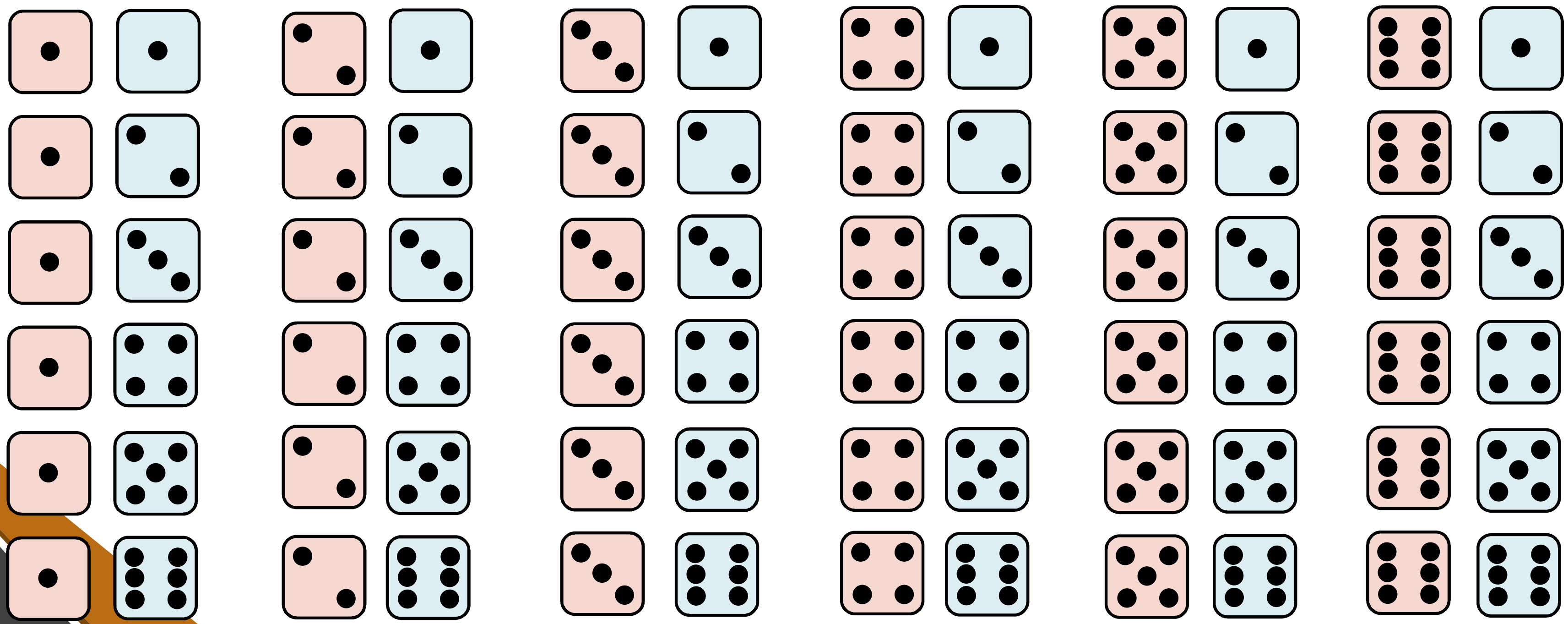
Income (in SGD)	15-24	25-34	35-44	45-54	55-64	65+
$< S\$2500$	0.062	0.051	0.037	0.019	0.015	0.039
$S\$2500 - S\$5000$	0.078	0.068	0.061	0.057	0.031	0.053
$> S\$5000$	0.015	0.051	0.094	0.119	0.111	0.039

$$\Pr[Age = (25-34)] = 0.051 + 0.068 + 0.051 = 0.17$$

Posterior/Conditional Probability

Probability that an event occurs, given that some other event occurs.

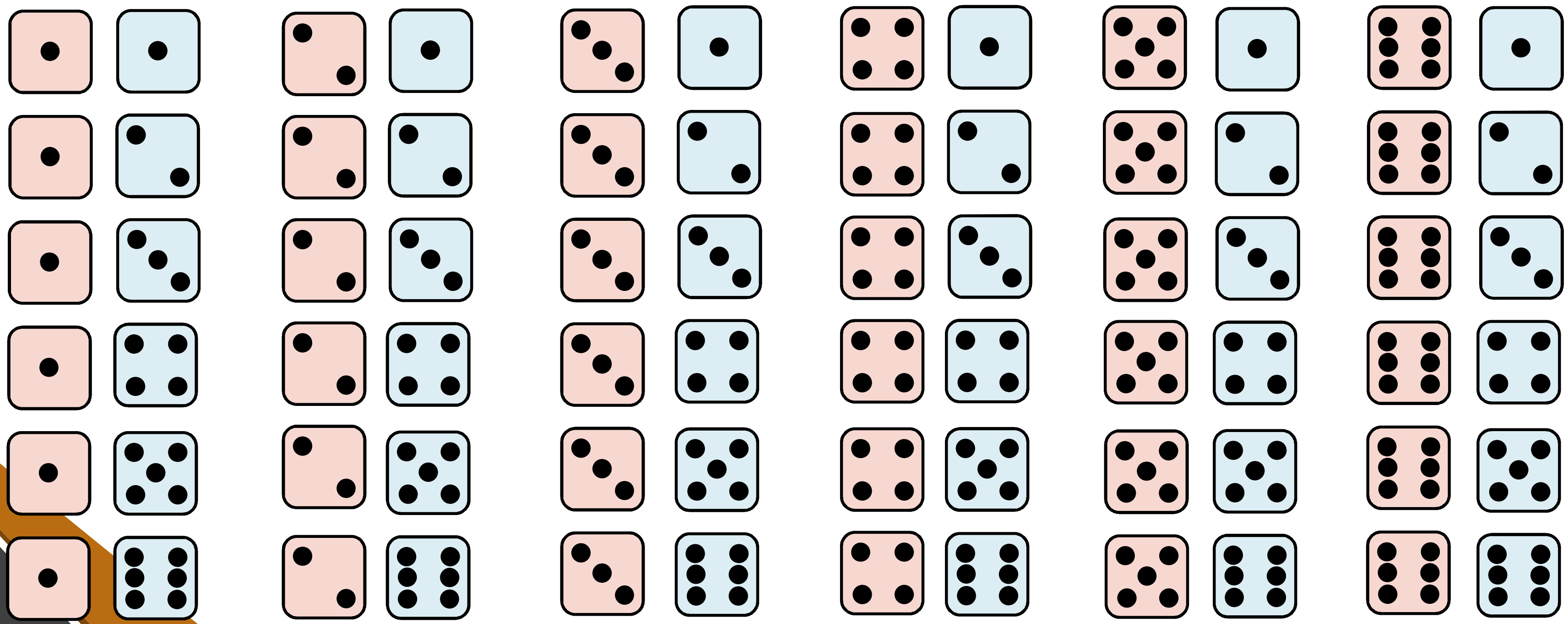
$$\Pr[X_1 = 2]$$



Posterior/Conditional Probability

Probability that an event occurs, given that some other event occurs.

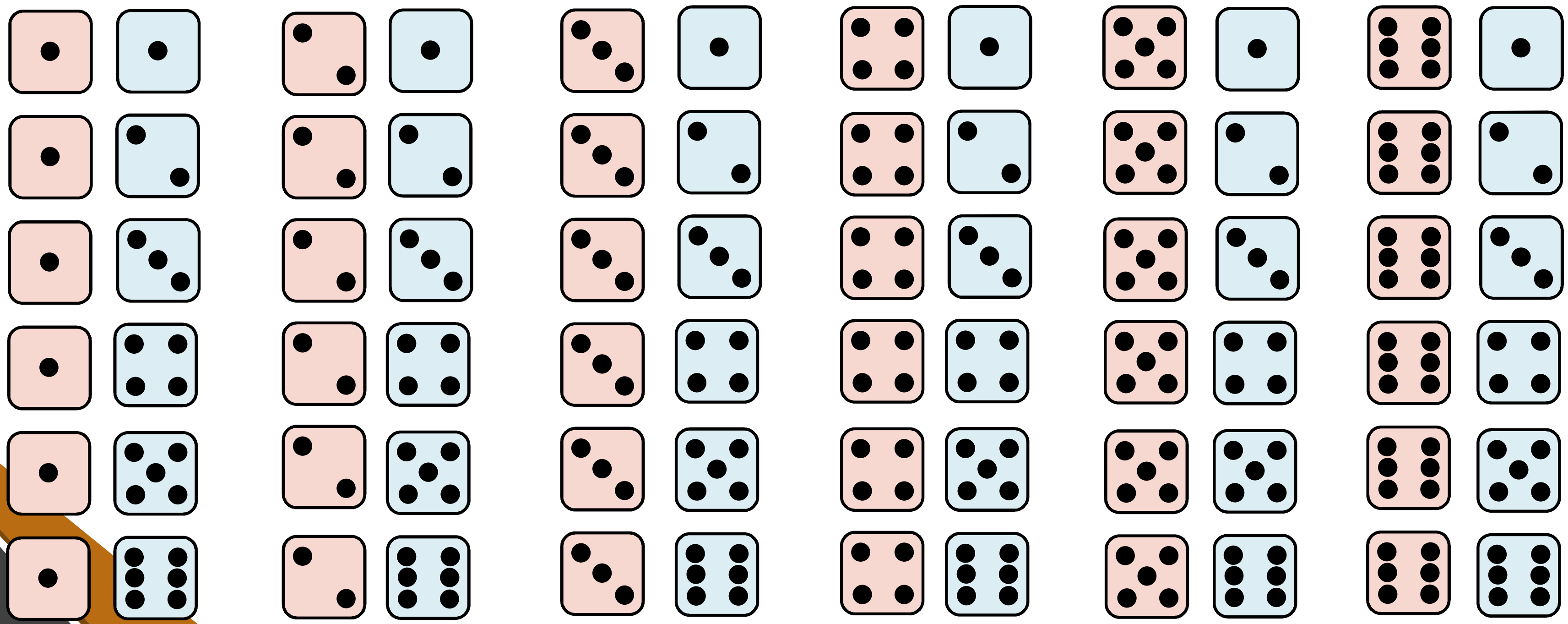
$$\Pr[X_1 = 2 \mid X_1 + X_2 = 8]$$



Posterior/Conditional Probability

Probability that an event occurs, given that some other event occurs.

$$\Pr[\, X_1 + X_2 = 8 \mid X_1 = 2\,]$$



Posterior/Conditional Probability

- $\Pr[A \mid B] = \frac{\Pr[A \wedge B]}{\Pr[B]}$ assuming that $\Pr[B] > 0$

Among all the worlds where B occurs, find the $A-$worlds s.t., $A \wedge B$ occurs

Axio

- Total probability of set of worlds: $p_X(x) \in [0,1]\ \forall x \in D_X$ then, $\sum_{x \in D_X} p_X(x) = 1$
- For any event $A \subseteq D_X$ we have, $\Pr_X[A] = \sum_{x \in A} p_X(x)$
- Disjunction of events: $\Pr[A \vee B] = \Pr[A] + \Pr[B] - \Pr[A \wedge B]$

Independence

A and B are independent if $\Pr[A \wedge B] = \Pr[A] \cdot \Pr[B]$. Equivalent to $\Pr[\, A \mid B \,] = \Pr[A]$.

"Knowing B adds no information about A"

Toothache, cavity and weather

$$\Pr[\, Weather = cloudy \mid toothache \wedge cavity \,] = ?$$

Inference by Enumeration

- Start with the joint probability distribution:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

- For any proposition (event) X, sum the atomic events y where X holds: $\Pr[X] = \sum_{y \in X} \Pr[X = y]$
- $\Pr[\text{toothache}] = 0.108 + 0.016 + 0.012 + 0.064 = 0.2$

Marginalization / summing out

$\Pr[\mathit{toothache}] = \sum_z \Pr[\mathit{toothache}, z]$ where z is all each possible value of other variables.

Inference by Enumeration

- Start with the joint probability distribution:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

- For any proposition (event) X, sum the atomic events y where X holds: $\Pr[X] = \sum_{y \in X} \Pr[X = y]$
- $\Pr[\text{toothache} \lor \text{cavity}] = 0.108 + 0.016 + 0.012 + 0.064 + 0.072 + 0.008 = 0.28$

Inference by Enumeration

- Start with the joint probability distribution:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

- For any proposition (event) X, sum the atomic events y where X holds: $\Pr[X] = \sum_{y \in X} \Pr[X = y]$
- $\Pr[\neg\text{cavity} \mid \text{toothache}] = \dfrac{\Pr[\neg\text{cavity} \wedge \text{toothache}]}{\Pr[\text{toothache}]}$

$$= \frac{0.016 + 0.064}{0.108 + 0.016 + 0.012 + 0.064} = 0.4$$

Inference by Enumeration

- Start with the joint probability distribution:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

- $\Pr[\text{cavity} \mid \text{toothache}] = \dfrac{\Pr[\text{cavity} \wedge \text{toothache}]}{\Pr[\text{toothache}]}$

Normalization

$\Pr[toothache]$ is common. Treat it as a constant α (normalization constant)

The Power of Independence

- We have n random variables $X_1, \ldots, X_n$; domains of size d. How big is their joint distribution table?
- Suppose that $X_1, \ldots, X_n$ are independent: maintain only dn values!
- Independence is good (if you can find it)