LOCAL SEARCH

AIMA Chapter 4.1

Previously…

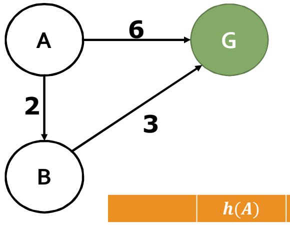
Use Heuristics to Guide Search

- Greedy best-first search
- A^* search

A^* Search Heuristic

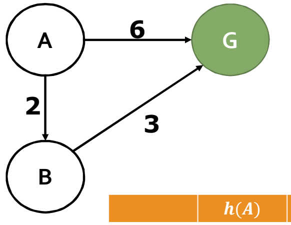
- If $h(n)$ is admissible, then A^* w. Tree-Search is optimal
- If $h(n)$ is consistent, then A^* w. Graph-Search is optimal
- A heuristic that dominates another incurs lower search cost

Consider the search problem



	$h(A)$	$h(B)$	$h(G)$
H-1	4	3	0
H-2	5	2	0

Consider the search problem



Consistent?

$$h(A) - h(B) \leq pathCost$$

Admissible?

$$h(A) \leq pathToGoal^*$$

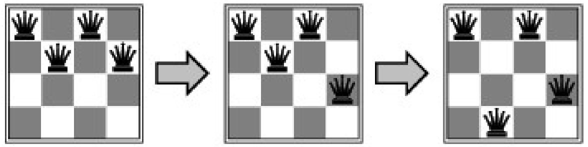
	$h(A)$	$h(B)$	$h(G)$
H-1	4	3	0
H-2	5	2	0

Local Search Algorithms

- The path to goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find final configuration satisfying constraints, e.g., n-queens
- Local search algorithms: maintain single "current best" state and try to improve it
- Advantages:
  - very little/constant memory
  - find reasonable solutions in large state space

Example: n-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



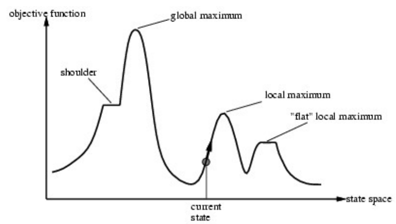
Hill-Climbing Search

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  current ← MAKE-NODE(problem.INITIAL-STATE)
  loop do
      neighbor ← a highest-valued successor of current
      if neighbor.VALUE ≤ current.VALUE then return current.STATE
      current ← neighbor
```

"Like climbing Mt. Everest in thick fog with amnesia"

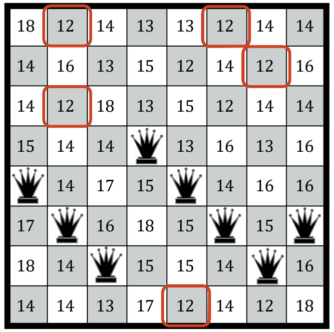
Hill-Climbing Search

- Problem: depending on initial state, can get stuck in local maxima



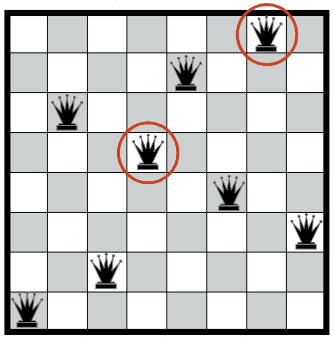
- Non-guaranteed fixes: sideway moves, random restarts

Hill-Climbing Search: 8-Queens



- h = number of pairs of queens that are attacking each other, either directly or indirectly
- $h = 17$ for the above state

Hill-Climbing Search: 8-Queens

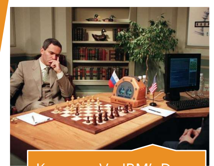


Local Minimum with $h = 1$

Adversarial Search

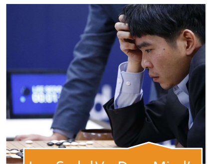
AIMA Chapter 5.1 – 5.5

AI vs. Human Players: the State of the Art



Kasparov Vs. IBM's Deep Blue (1997)

World Poker Champs Vs. Libratus (2017)



Lee Sedol Vs. DeepMind's AlphaGo (2016)

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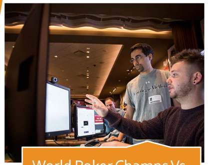

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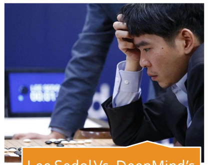

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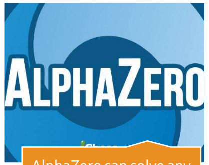

AlphaZero can solve any zero-sum game! (2018)

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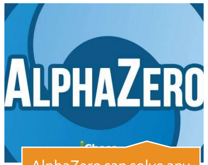


AlphaZero can solve any zero-sum game! (2018)

OpenAI Five

OpenAI 5 vs world champs—DOTA2 (2019)

Deterministic Games in Practice

- Chinook (Checkers, 1994). Precomputed endgame database ⇒ perfect play for all positions with ≤ 8 pieces on the board (total of 444 billion positions).
- Deep Blue (Chess, 1997). Searches 200 million positions/sec + evaluation functions + secret sauce.
- Logistello (Othello, 1997). Human champions refuse to play against AI.
- AlphaGo + Alphazero (Go/everything above, 2016-2017). Learning for evaluation functions + database and efficient search + secret sauce.

Outline

- Adversarial search problems (aka games)
- Optimal (i.e., Minimax) decisions
- α-β pruning
- Imperfect, real-time decisions

Games vs. Search Problems

Utility maximizing opponent

- solution is a strategy specifying a move for every possible opponent response.

Time limit

- unlikely to find goal, must approximate

Let's Play!

- Two players in a **zero-sum game**:
  - Winner gets paid and loser pays.
- Easy to think in terms of a **max player** and **min player**
  - Player 1 wants to maximize value (MAX player)
  - Player 2 wants to minimize value (MIN player)

Game: Problem Formulation

A game is defined by 7 components:

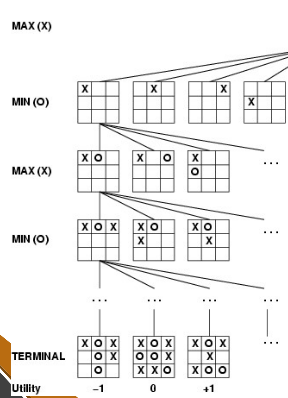
1. Initial state s_0
2. States
3. Players: $PLAYER(s)$ defines which player has the move in state s.
4. Actions : $ACTIONS(s)$ returns set of legal moves in state s
5. Transition model : $RESULT(s, a)$ returns state that results from the move a in state s.

Game: Problem Formulation

A game is defined by 7 components:

6. Terminal test $TERMINAL(s)$ returns true if game is over and false otherwise
   - Terminal states: states where the game has ended
7. Utility function $UTILITY(s, p)$ gives final numeric value for a game that ends in terminal state s for a player p
   - Chess: White wins $+1$; Black wins -1; draw 0.

Game Tree
(2-Player, Deterministic, Turn-Taking)



Key property: **zero-sum** game

- Loosely, it means that there's a loser for every winner
- Total utility score over all agents sum to zero (or constant)
- Aka constant-sum game
- Makes the game adversarial

Non Zero-Sum Games?

Example : Game of NIM

Several piles of sticks are given. We represent the configuration of the piles by a monotone sequence of integers, such as (1,3,5). A player may remove, in one turn, any number of sticks from one pile. Thus, (1,3,5) would become (1,1,3) if the player were to remove 4 sticks from the last pile. The player who takes the last stick loses.

Example : Game of NIM

<table>
<tr><th>Start</th><th>||| (A)</th><th>|||| (B)</th><th>||||| (C)</th></tr>
<tr><td>Bob take 2 from A</td><td>|</td><td>||||</td><td>|||||</td></tr>
<tr><td>Alice take 3 from C</td><td>|</td><td>||||</td><td>||</td></tr>
<tr><td>Bob take 1 from B</td><td>|</td><td>|||</td><td>||</td></tr>
<tr><td>Alice take 1 from B</td><td>|</td><td>||</td><td>||</td></tr>
<tr><td>Bob take 1 from A</td><td>0</td><td>||</td><td>||</td></tr>
<tr><td>Alice take 1 from B</td><td>0</td><td>|</td><td>||</td></tr>
<tr><td>Bob take 2 from C</td><td>0</td><td>|</td><td>0</td></tr>
</table>

Bob wins!!

|| | ||

Represent the NIM game (1,2,2) **as a game tree.**

Represent the NIM game $(1,2,2)$ as a game tree.

Play against what kind of player?
Stupid? Smart?

Player Strategies

A strategy s for player i: what will player i do at every node of the tree that they make a move in?

Need to specify behavior in states that will never be reached!

Winning Strategy

A strategy s_1^* for player 1 is called **winning** if for any strategy s_2 by player 2, the game ends with player 1 as the winner.

A strategy t_1^* for player 1 is called **non-losing** if for any strategy s_2 by player 2, the game ends in either a tie or a win for player 1.

Nash Equilibrium

When players knows the strategies of all opponents: no one wants to change her strategy

Stronger form of Nash equilibrium – subgame perfect Nash equilibrium

Every subgame is a Nash equilibrium

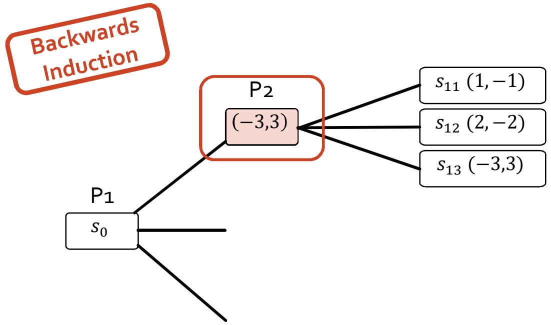
Optimal Strategy at Node - Minimax

$$Minimax(s) = \begin{cases} Utility(s) & \text{if TerminalTest}(s) \\ \max_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s,a)) & \text{if Player}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s,a)) & \text{if Player}(s) = \text{MIN} \end{cases}$$

Intuitively,

- MAX chooses move to maximize the minimum payoff
- MIN chooses move to minimize the maximum payoff

Minimax Play (Subperfect Nash Equilibrium)



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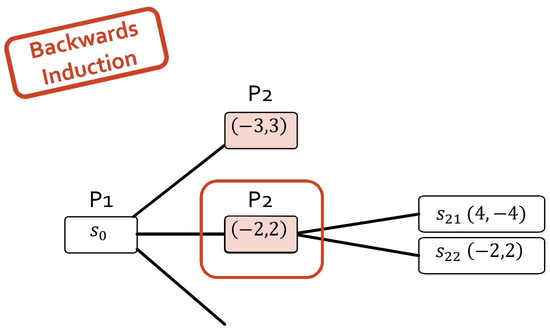

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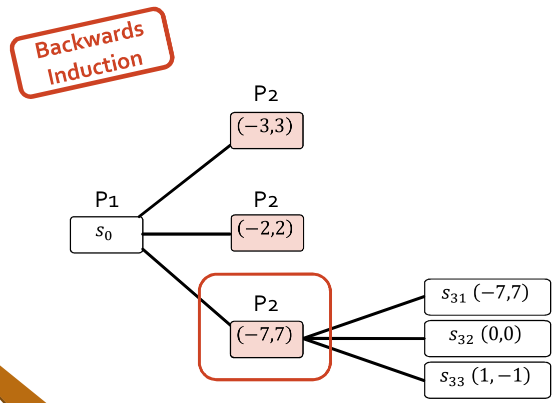
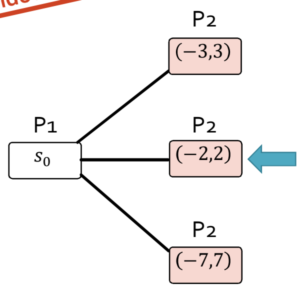


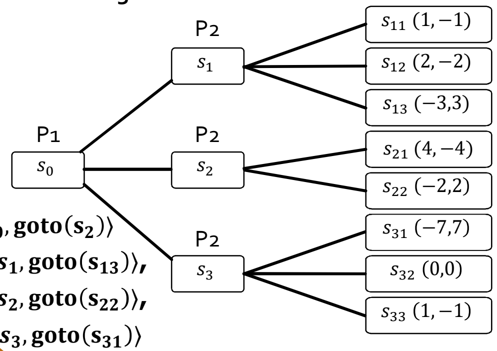
Minimax Play
(Subperfect Nash Equilibrium)

Backwards Induction



Minimax Play
(Subperfect Nash Equilibrium)

What are the optimal strategies in this game?



P1 $\rightarrow \langle s_0, \mathbf{goto}(s_2) \rangle$

P2 $\rightarrow \langle s_1, \mathbf{goto}(s_{13}) \rangle$, $\langle s_2, \mathbf{goto}(s_{22}) \rangle$, $\langle s_3, \mathbf{goto}(s_{31}) \rangle$

Properties of Minimax

Computes a Nash equilibrium of the game

Property	
Complete?	Yes (if game tree is finite)
Optimal	Yes (optimal gameplay)
Time	$\mathcal{O}(b^m)$
Space	Like DFS: $\mathcal{O}(bm)$

Minimax Algorithm

- Runs in time polynomial in tree size
- Returns a sub-perfect Nash equilibrium: the best action at every choice node.

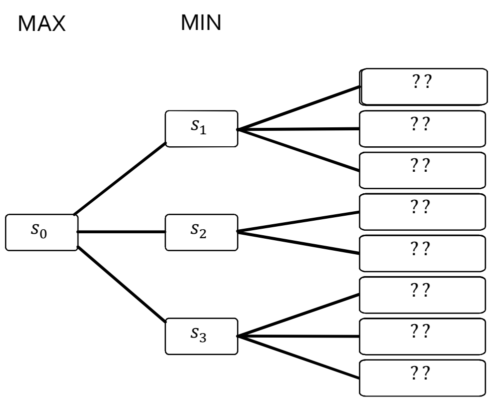
Are we done here?

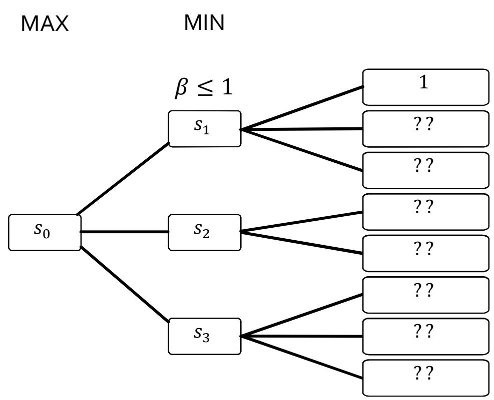
Backwards Induction

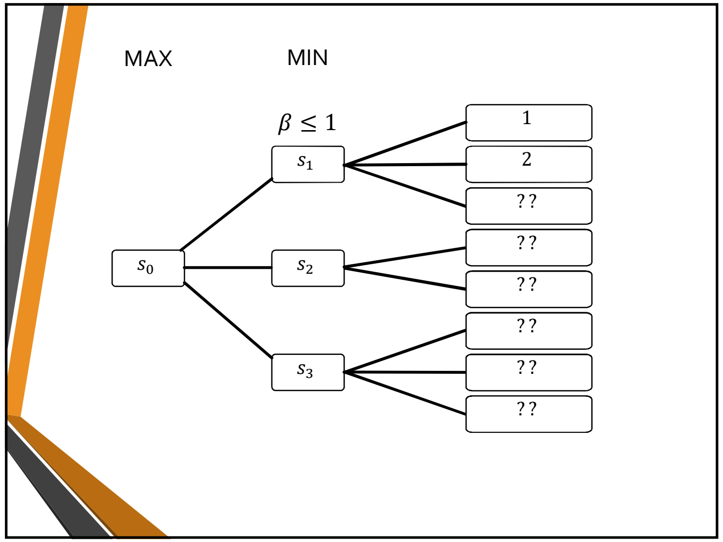
- Game trees are huge: chess has game tree with $\sim 10^{123}$ nodes (planet Earth has $\sim 10^{50}$ atoms)
- Impossible to expand the entire tree

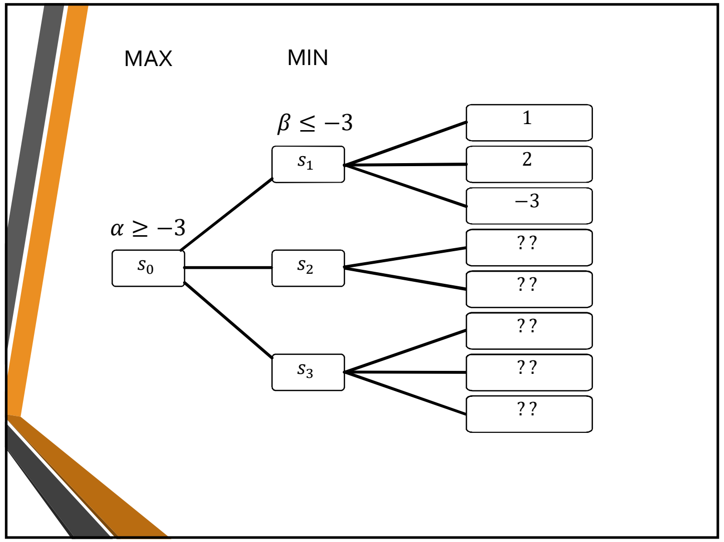
α-β Pruning

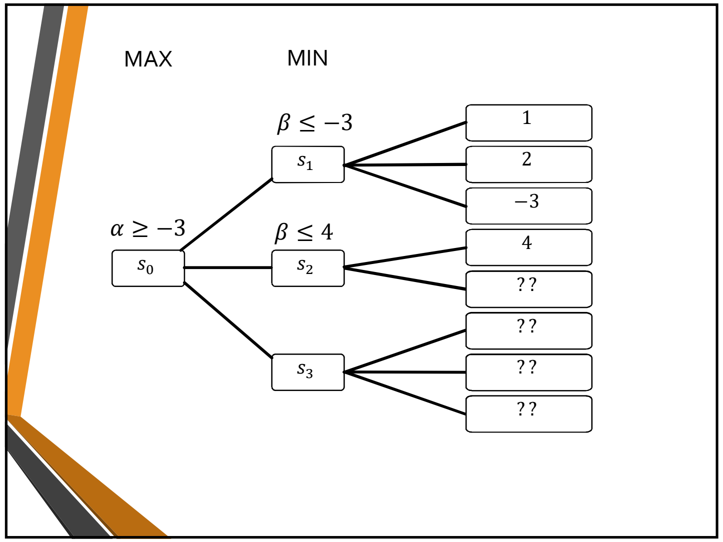
- **Basic idea:** *"If you have an idea that is surely bad, don't take the time to see how truly awful it is."* -- Pat Winston
- Maintain a **lower bound α** and **upper bound β** of the values of, respectively, MAX's and MIN's nodes seen thus far. We can prune subtrees that will never affect minimax decision.

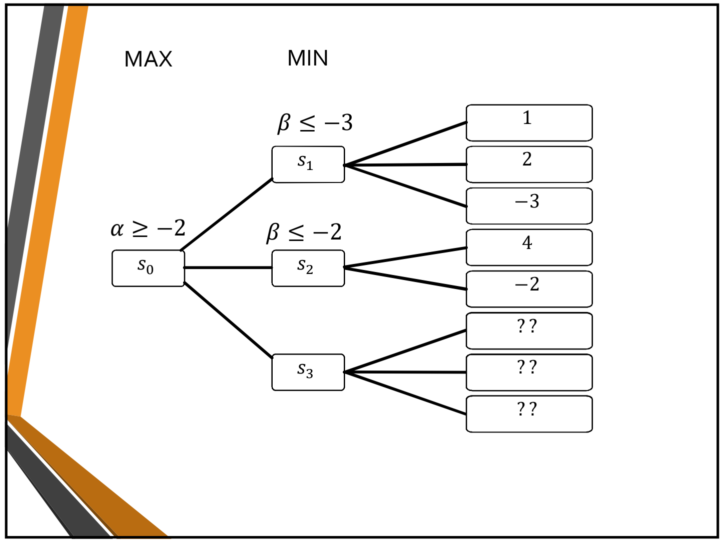





MAX
MIN
$\beta \leq 1$
s_1
s_0
s_2
s_3
1
2
??
??
??
??
??
??


MAX
MIN
$\beta \leq -3$
$\alpha \geq -3$
s_1
s_0
s_2
s_3
1
2
-3
??
??
??
??
??


MAX
MIN
$\beta \leq -3$
$\alpha \geq -3$
$\beta \leq 4$
s_1
s_0
s_2
s_3
1
2
-3
4
??
??
??
??


MAX
MIN
$\beta \leq -3$
$\alpha \geq -2$
$\beta \leq -2$
s_1
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4
-2
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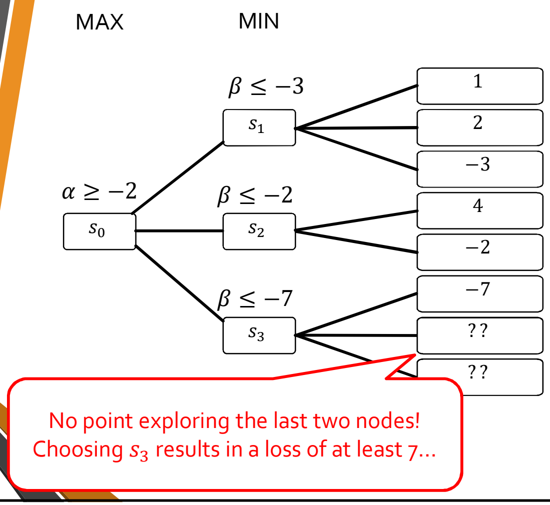


α-β Pruning

- MAX node n: $\alpha(n)$ = highest **observed** value found on path from n; initially $\alpha(n) = -\infty$
- MIN node n: $\beta(n)$ is the lowest observed value found on path from n; initially $\beta(n) = +\infty$
- Pruning:
  - given a MIN node n, stop searching below n if there is some MAX ancestor i of n with $\alpha(i) \ge \beta(n)$
  - given a MAX node n, stop searching below n if there is some MIN ancestor i of n with $\beta(i) \le \alpha(n)$

Analysis of α-β Pruning

- When we prune a branch, it **never** affects final outcome.
- Good move ordering improves effectiveness of pruning
- "Perfect" ordering: time complexity $= O\left(b^{\frac{m}{2}}\right)$
  - → Good pruning strategies allow us to search twice as deep!
  - Chess: simple ordering (checks, then take pieces, then forward moves, then backwards moves) gets you close to best-case result.
  - It makes sense to have good expansion order heuristics.
- Random ordering: time complexity = $O\left(b^{\frac{3m}{4}}\right)$ for $b < 1000$

Summary: α-β Pruning Algorithm

- Initially, $\alpha(n) = -\infty$, $\beta(n) = +\infty$
- $\alpha(n)$ is max along search path containing n
- $\beta(n)$ is min along search path containing n
- If a MIN node has value $v \le \alpha(n)$, no need to explore further.
- If a MAX node has value $v \ge \beta(n)$, no need to explore further.

Time Limit

- **Problem:** very large search space in typical games
- **Solution:** α-β pruning removes large parts of search space
- Unresolved problem: Maximum depth of tree
- Standard solutions:
  - evaluation function = estimated expected utility of state
  - cutoff test: e.g., depth limit

Heuristic Minimax Value

$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \boxed{\text{if TERMINAL-TEST}(s)} \\ \max_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

$$\text{H-MINIMAX}(s,d) = \begin{cases} \text{EVAL}(s) & \boxed{\text{if CUTOFF-TEST}(s,d)} \\ \max_{a \in \text{ACTIONS}(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in \text{ACTIONS}(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

Run minimax until depth d; then start using the evaluation function to choose nodes.

Evaluation Functions

- An evaluation function is a mapping from game states to real values: $f: \mathcal{S} \rightarrow \mathbb{R}$
  - So far:
    $$f(s) = \begin{cases} UTILITY(s) \text{ if } TERMINAL(s) \\ 0 \; otherwise \end{cases}$$
- Should be cheap to compute
- For non-terminal states, must be strongly correlated with actual chances of winning
- Tic-Tac-Toe:
  $f(n) = [\text{\#of open lines for X}] - [\text{\# of open lines for O}]$

Evaluation Functions

- Chess:
  - Alan Turing's evaluation function:
    $f(n) = \frac{w(n)}{b(n)}$ where $w(n)$ is the point value of white's pieces and $b(n)$ is the point value of black's pieces.
  - Modern evaluation functions: weighted sum of position features
    $f(n) = w_1 F_1(n) + w_2 F_2(n) + \cdots + w_k F_k(n)$
  - Example features: piece count, piece placement, controlled squares etc.
  - Deep Blue has about 6000 features.
  - How do we determine weights? Do they change dynamically?

Evaluation Functions

- Suppose that $f(n) = \sum_{j=1}^{k} w_j F_j(n)$
- If for $n, n' \in \mathcal{S}$ we have $F_j(n) = F_j(n')$ for all j, then they're indistinguishable.
- … evaluates all states with same value.

- Let $\mathcal{S}(\vec{q}) = \{n \in \mathcal{S}: \forall j \in [k], F_j(n) = q_j\}$: all states whose feature values are as specified in the vector $\vec{q}$.
- "All states where white has two pawns but black has a bishop."
- Suppose we know that in this case,
  - Black wins 61% of games
  - White wins 15% of games
  - 24% of games end in a draw
- Then expected utility for black is
  $0.61 \times 1 + 0.15 \times (-1) + 0.24 \times 0 = 0.46$
- Evaluation function need not return actual expected values, just maintain relative order of states.

Evaluation alone is not enough!

- Opening move search doesn't result in useful utility estimates
- Applying evaluation functions on end game scenarios may not solve the game
  - Western chess KRK end game
- Use a policy or lookup table (taken from expert knowledge or previous game history)

Cutting Off Search

- Modify minimax or α-β pruning algorithms by replacing
  - TERMINAL-TEST(*state*) with CUTOFF-TEST(*state*, *depth*)
  - UTILITY(*state*) is replaced by EVAL(*state*)
- Can also be combined with iterative deepening

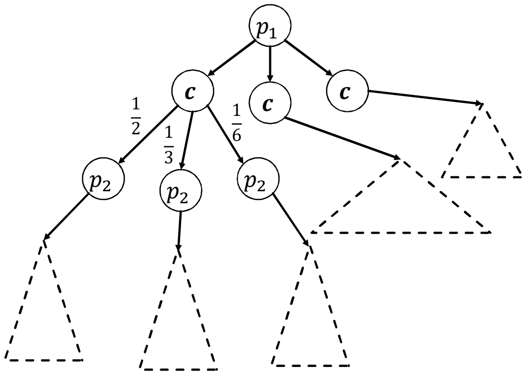
Cutting Off Search

- Why is it useful?
  - For chess, $b^m = 10^6, b = 35 \Rightarrow m = 4$
  - Ply = 4 – Novice chess player
  - Ply = 8 – Typical PC engine
  - Ply = 12~14 – Grandmaster chess (Deep blue)

Stochastic Games

- Many games have randomization:
  - Backgammon
  - Settlers of Catan
  - Poker
- How do we deal with uncertainty?
- Can we still use minimax? Yes, but search space is much bigger

Adding Chance Layers



Calculate the expected value of a state
(MUCH harder than deterministic games)