example-confidence

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```
In [1]: %load_ext autoreload
        %autoreload 2

In [2]: %pylab inline

Populating the interactive namespace from numpy and matplotlib


In [4]: import networkx as nx

In [18]: from hypotest.setup_hypothgraph import convert_to_hypothgraph
         from hypotest.graph_generation import hypoth_conf
         from hypotest.io import write_dot
```

1 Methodology to assess causal confidence in a hypothesis

In our work we assume that a hypothesis is a directed graph, where nodes are factors, arcs are causal relationships. We focus on causality between two factors f_1 and f_2.

```
In [20]: simple_hypothesis_nodes = [
             ('f1', { 'label': 'f1'}),
             ('f2', { 'label': 'f2'})
         ]
         simple_hypothesis_arcs = [
             ('f1', 'f2', { 'label': 'results in'}),
         ]
         simple_hypothgraph = nx.DiGraph()
         simple_hypothgraph.add_nodes_from(simple_hypothesis_nodes)
         simple_hypothgraph.add_edges_from(simple_hypothesis_arcs)
         simple_hypothgraph = convert_to_hypothgraph.convert_to_hypothgraph(simple_hypothgraph)

In [21]: source, target = 'f1', 'f2'
         conf = hypoth_conf.Hypoth_Conf(source, target, [])
```

1.1 Simplest case of a causal hypothesis

Let's start with a simplest causal hypothesis H_0 where we know that f_1 `results in` f_2.

In [26]:
```
import os
from IPython.display import Image
path_to_figures = './images/example'
```

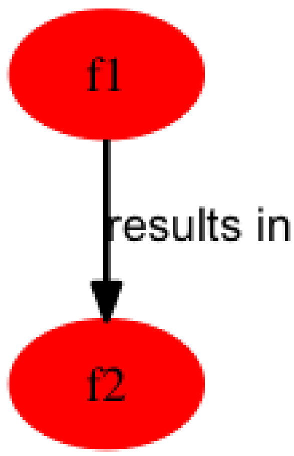
In [23]:
```
simplest_dot = os.path.join(path_to_figures, 'simplest.dot')
simplest_png = os.path.join(path_to_figures, 'simplest.png')

with open(simplest_dot, 'w') as f:
    write_dot.hypothgraph_to_dot(simple_hypothgraph, conf, stream=f)
```

In [24]:
```
!dot -Tpng -o $simplest_png $simplest_dot
```

In [27]:
```
Image(simplest_png)
```

Out[27]:



1.2 Proving a causal hypothesis

In [29]:
```
from hypotest.stats import confidences
```

To prove our hypothesis that f_1 will eventually cause f_2, with the current knowledge, we need to evidence every node which leads us from f_1 to f_2. Let $\pi(f_1, f_2)$ be a set of all possible paths from f_1 to f_2.

In this simplest example there is only one path from f_1 to f_2, the path $\pi_1 = \{f_1, f_2\}$, thus all possible paths $\pi(f_1, f_2) = \{\pi_1\}$.

Thus to prove this hypothesis we need to evidence both f_1 and f_2. In this simple example let us assume that each node if evidenced gives $+1$ contribution to the confidence in the causal hypothesis. If it is not evidenced then it contributes 0 to our confidence in the causal hypothesis.

$$contribution(f_i) = \begin{cases} 1 & \text{node } f_i \text{ is evidenced} \\ 0 & \text{node } f_i \text{ is not evidenced} \end{cases}$$

Our total confidence in the causal hypothesis from f_1 to f_2 is thus

$$Confidence(\pi_1) = contribution(f_1) + contribution(f_2),$$

For any path $\pi_i = \{f_1, f_2, \ldots, f_n\}$, the confidence of this path is

$$Confidence(\pi_i) = \sum_{f_i \in \pi_i} contribution(f_i)$$

Finally, confidence in a hypothesis from factor f_i to factor f_j is the sum of contributions of factors in all paths from f_1 to f_2:

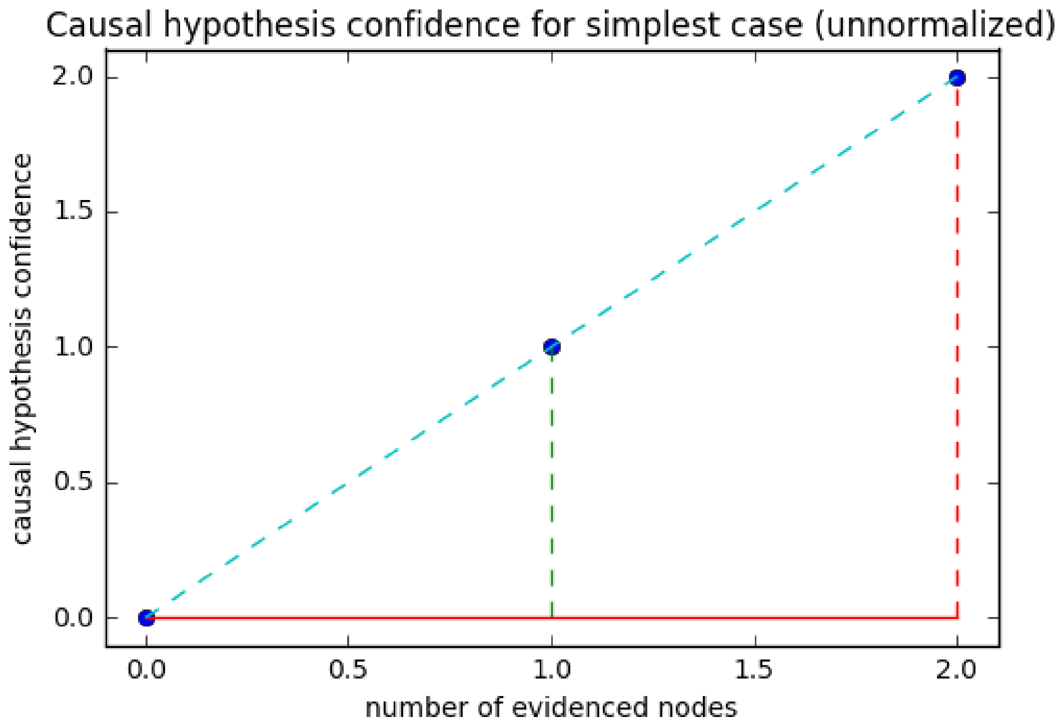
$$Confidence(H_0, f_i, f_j) = \sum_{\pi_i \in \pi(f_i, f_j)} Confidence(\pi_i).$$

Now, suppose we had no evidences for neither of the two facts, what would be our confidence in this causal relation (i.e., $contribution(f_1) = contribution(f_2) = 0$)? 0. By extension, it should be 2 if we evidence all the nodes, which lead from f_1 to f_2. If we had evidenced only one of the two, then our confidence would be 1, assuming that both f_1 and f_2 are equally important for the causal relation `results in`.

```
In [110]: simplest_confidences_unorm = confidences.confidence_spectrum(simple_hypothgraph, sourc
```

```
In [111]: x = range(len(simplest_confidences_unorm))
          y = simplest_confidences_unorm
          simplest_fig = plt.figure('1')
          plt.subplot('111')
          plt.xlim(min(x)-0.1, max(x)+0.1)
          plt.ylim(min(y)-0.1, max(y)+0.1)
          plt.stem(x, y, '--')
          plt.plot(x, y, '--')
          plt.xlabel('number of evidenced nodes')
          plt.ylabel('causal hypothesis confidence')
          plt.legend(loc='upper left')
          plt.title('Causal hypothesis confidence for simplest case (unnormalized)')
```

```
Out[111]: <matplotlib.text.Text at 0x10bd8ea50>
```



Thus the maximum confidence we can achieve is 2, i.e., we could use it to normalize our confidence function, such that its values stay in the $[0, 1]$ range, i.e.,

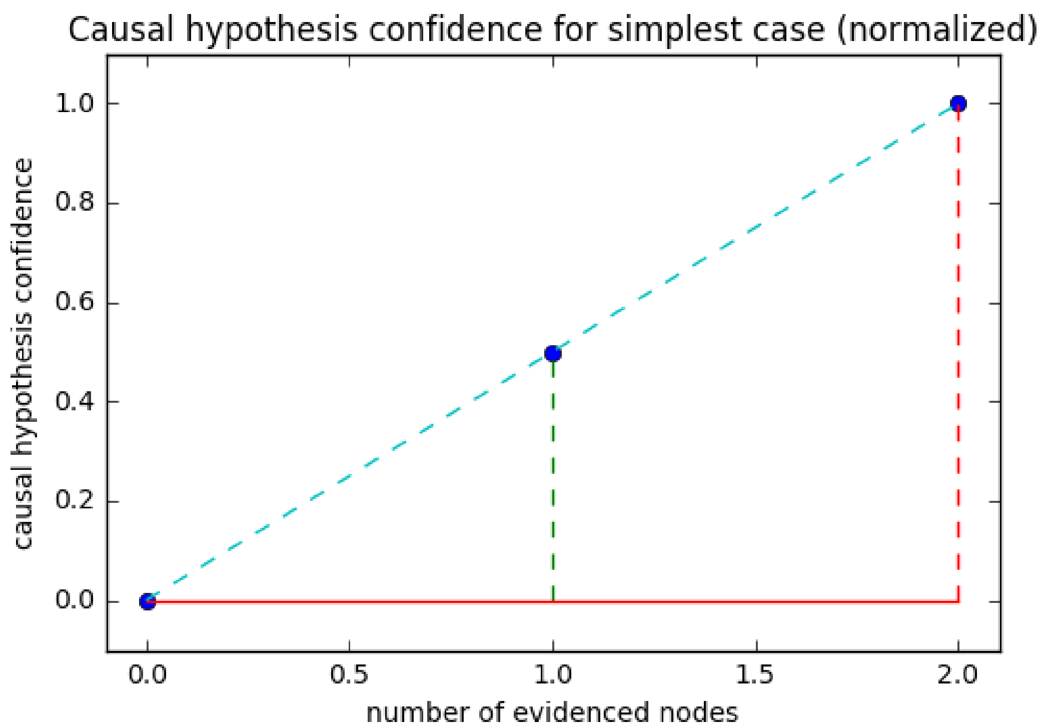
$$Confidence'(f_i, f_j) = \frac{Confidence(f_1, f_j)}{max(Confidence(f_i, f_j))}$$

From now on, we always consider the normalized confidence for other hypothesis graphs.

```
In [34]: simplest_confidences = confidences.confidence_spectrum(simple_hypothgraph, source, targ
```

```
In [112]: x = range(len(simplest_confidences))
          y = simplest_confidences
          simplest_fig = plt.figure('1')
          plt.subplot('111')
          plt.xlim(min(x)-0.1, max(x)+0.1)
          plt.ylim(min(y)-0.1, max(y)+0.1)
          plt.stem(x, y, '--')
          plt.plot(x, y, '--')
          plt.xlabel('number of evidenced nodes')
          plt.ylabel('causal hypothesis confidence')
          plt.legend(loc='upper left')
          plt.title('Causal hypothesis confidence for simplest case (normalized)')
```

```
Out[112]: <matplotlib.text.Text at 0x10c07c890>
```



1.3 More background knowledge on causal shypothesis

Now, let us imagine that our knowledge of the causal hypothesis is expanded to H_n. Namely, we discover that there might be other factors which might have caused f_2, and that are related to f_1. Suppose, additional knowledge is synthesized in the following augmented hypothesis graph.

In [54]:
```
augmented_hypothgraph = simple_hypothgraph.copy()
augmented_nodes = [
    ('f3', { 'label': 'f3'}),
    ('f4', { 'label': 'f4'})
]
augmented_arcs = [
    ('f1', 'f3', { 'label': 'results in' }),
    ('f3', 'f4', { 'label': 'results in'}),
    ('f4', 'f2', { 'label': 'results in'})
]

augmented_hypothgraph.add_nodes_from(augmented_nodes)
augmented_hypothgraph.add_edges_from(augmented_arcs)
```

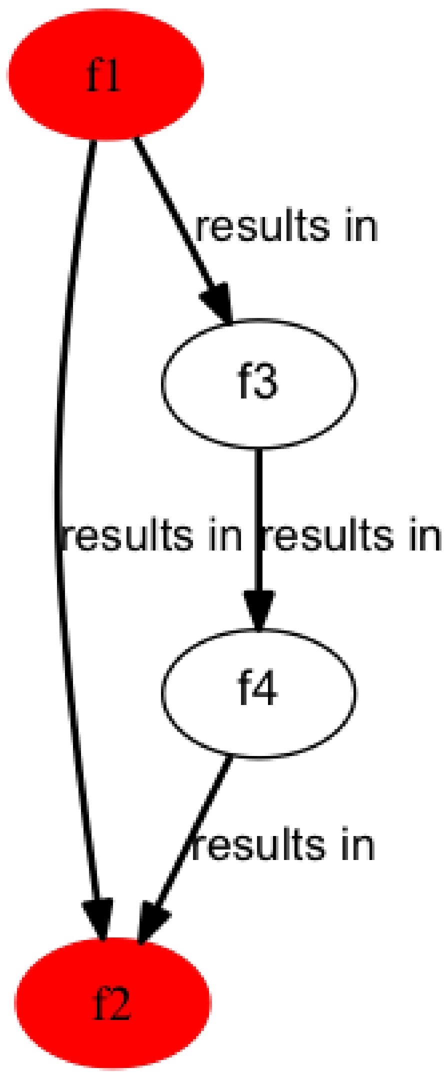
In [55]:
```
augmented_dot = os.path.join(path_to_figures, 'augmented.dot')
augmented_png = os.path.join(path_to_figures, 'augmented.png')
```

```
        with open(augmented_dot, 'w') as f:
            write_dot.hypothgraph_to_dot(augmented_hypothgraph, conf, stream=f)
```

In [56]:
```
!dot -Tpng -o $augmented_png $augmented_dot
```

In [57]:
```
Image(augmented_png)
```

Out[57]:



At this point, our causal hypothesis that f_1 causes f_2 has additional knowledge, namely, we have $f1$ could have caused f_2, by additional causal relations between factors f_3, f_4.

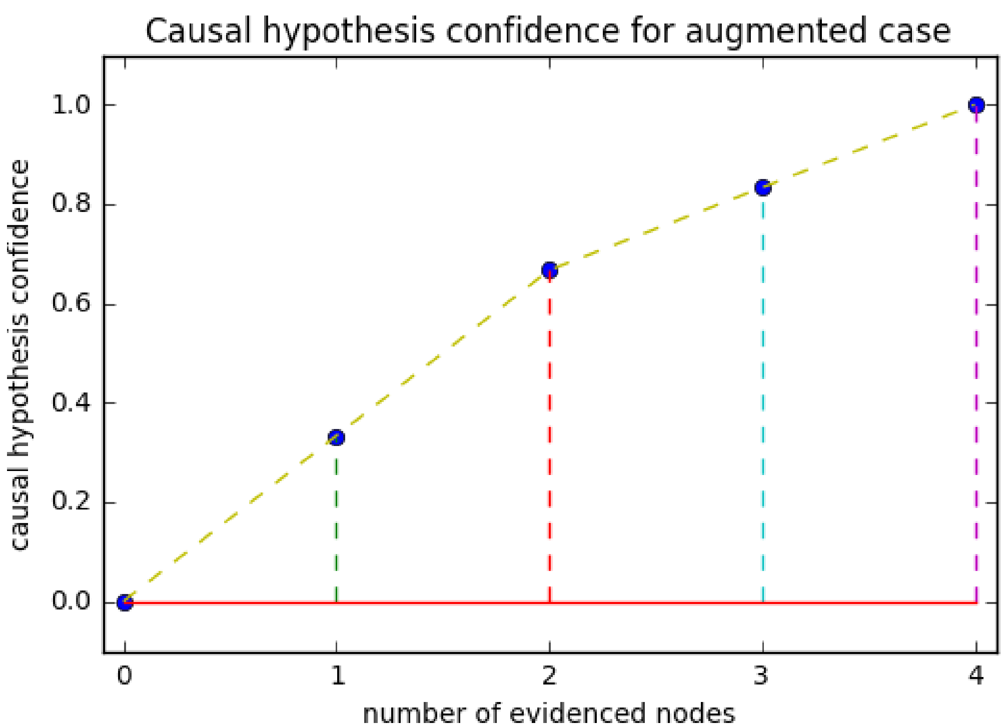
Since, we have to causality paths from f_1 to f_2, to prove it, we need to evidence every factor which lies in one of the paths from f_1 to f_2. Thus, we need to evidence 4 factors, instead of 2 previously.

1.3.1 Augmented hypothesis confidence computation

In [58]:
```
augmented_confidences = confidences.confidence_spectrum(augmented_hypothgraph, source,
```

In [81]:
```
augmented_fig = plt.figure('2')
x = range(len(augmented_confidences))
y = augmented_confidences
plt.subplot('111')
plt.xlim(min(x)-0.1, max(x)+0.1)
plt.ylim(min(y)-0.1, max(y)+0.1)
plt.stem(x, y, '--')
plt.plot(x, y, '--')
plt.xlabel('number of evidenced nodes')
plt.ylabel('causal hypothesis confidence')
plt.legend(loc='upper left')
plt.title('Causal hypothesis confidence for augmented case')
```

Out[81]:
```
<matplotlib.text.Text at 0x10ad44350>
```



1.3.2 Relative confidence

Let us say that H_0 is our knowledge about the causal hypothesis from f_1 to f_2, and the $Confidence(H_0)$ is a function which measures our confidence in this hypothesis as we evidence more nodes. Imagine, there exists a universal hypothesis H_n, in which the hypothesis causality from f_1 to f_2 is augmented with the paths as depicted in Figure 2.

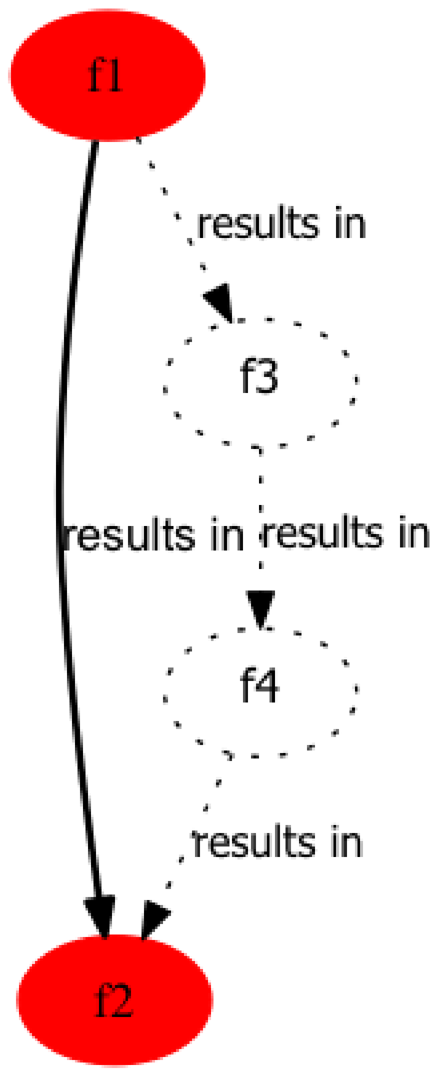
In [88]:
```python
augmented_superimposed_dot = os.path.join(path_to_figures, 'augmented_superimposed.dot'
augmented_superimposed_png = os.path.join(path_to_figures, 'augmented_superimposed.png'

with open(augmented_superimposed_dot, 'w') as f:
    write_dot.big_small_to_dot(augmented_hypothgraph, simple_hypothgraph, conf, stream=
```

In [94]:
```
!dot -Tpng -o $augmented_superimposed_png $augmented_superimposed_dot
```

In [100]:
```python
Image(augmented_superimposed_png)
```

Out[100]:

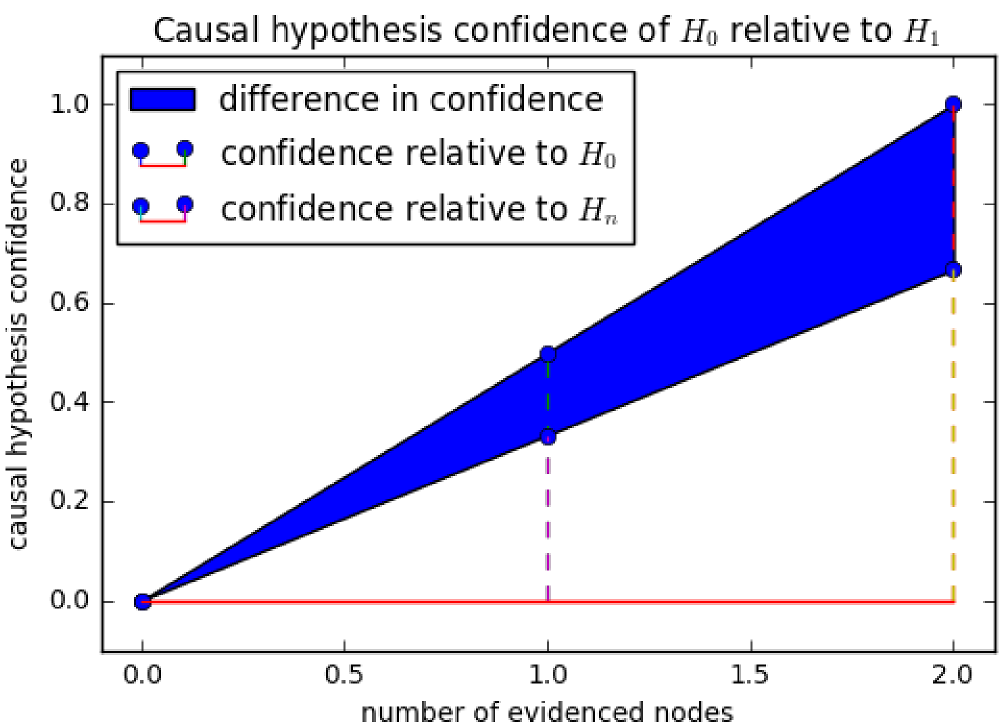


We do not know about the existence of those paths, we evidence nodes which need to be evidenced according to our knowledge H_0, but we want to know how would our confidence function be scaled according to the universal hypothesis H_n.

```
In [103]: relative_simples_to_augmented_confidences = confidences.relative_confidence_spectrum(a
          relative_to_me = relative_simples_to_augmented_confidences['sub_confidence_normalized_
          relative_to_big = relative_simples_to_augmented_confidences['big_confidence_normalized
```

```
In [109]: relative_augmented_to_simplest_fig = plt.figure('3')
          x = range(len(relative_to_me))
          y1 = relative_to_me
          y2 = relative_to_big
          plt.subplot('111')
          plt.xlim(min(x)-0.1, max(x)+0.1)
          plt.ylim(min(y1)-0.1, max(y1)+0.1)
          plt.stem(x, y1, '--', label='confidence relative to $H_0$')
          plt.stem(x, y2, '--', label='confidence relative to $H_n$')
          plt.fill_between(x, y1, y2, label="difference in confidence")
          plt.xlabel('number of evidenced nodes')
          plt.ylabel('causal hypothesis confidence')
          plt.legend(loc='upper left')
          plt.title('Causal hypothesis confidence of $H_0$ relative to $H_1$')
```

```
Out[109]: <matplotlib.text.Text at 0x10bc3c610>
```



1.3.3 Redundant knowledge to hypothesis

Suppose we receive more knowledge that there is a causality relation between f_4 and f_1, how that would change our confidence in the causal hypothesis from f_1 to f_2?

```
In [75]: redundant_hypothgraph = augmented_hypothgraph.copy()
         redundant_arcs = [
             ('f4', 'f1', { 'label': 'results in'} )
         ]

         redundant_hypothgraph.add_edges_from(redundant_arcs)
```

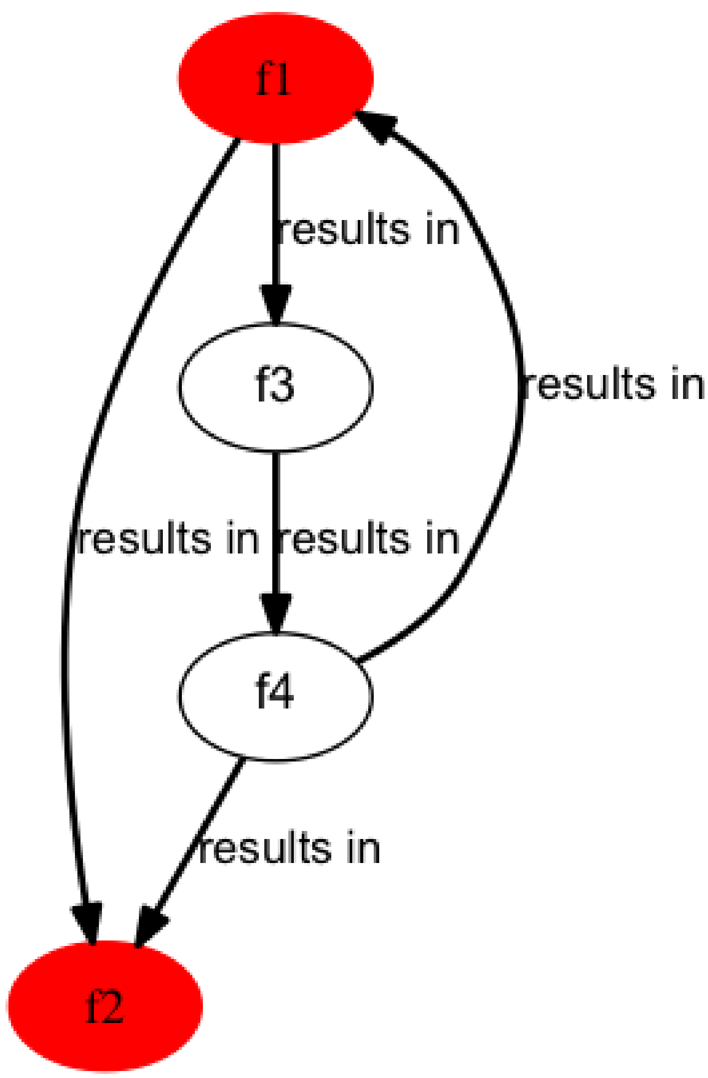
```
In [76]: redundant_dot = os.path.join(path_to_figures, 'redundant.dot')
         redundant_png = os.path.join(path_to_figures, 'redundant.png')

         with open(redundant_dot, 'w') as f:
             write_dot.hypothgraph_to_dot(redundant_hypothgraph, conf, stream=f)
```

```
In [77]: !dot -Tpng -o $redundant_png $redundant_dot
```

```
In [79]: Image(redundant_png)
```

Out[79]:



In [80]:
```
redundant_confidences = confidences.confidence_spectrum(redundant_hypothgraph, source,
```

In [104]:
```
redundant_fig = plt.figure('4')
x = range(len(redundant_confidences))
y = redundant_confidences
plt.subplot('111')
plt.xlim(min(x)-0.1, max(x)+0.1)
plt.ylim(min(y)-0.1, max(y)+0.1)
plt.stem(x, y, '--')
plt.plot(x, y, '--')
plt.xlabel('number of evidenced nodes')
plt.ylabel('causal hypothesis confidence')
plt.legend(loc='upper left')
plt.title('Causal hypothesis confidence for redundant knowledge case')
```

Out[104]:
```
<matplotlib.text.Text at 0x10b3ea5d0>
```