Esercizio 1 $\lim_{x\to+\infty} \left(\sqrt{x+3} - \sqrt{x-25}\right)$ $[+\infty-\infty]$

Idea: $\sqrt{A}-\sqrt{B} = (\sqrt{A}-\sqrt{B})\frac{\sqrt{A}+\sqrt{B}}{\sqrt{A}+\sqrt{B}} = \frac{A-B}{\sqrt{A}+\sqrt{B}}$

$$\left(\sqrt{x+3}-\sqrt{x-25}\right)\frac{\sqrt{\dots}+\sqrt{\dots}}{\sqrt{\dots}+\sqrt{\dots}} = \frac{\cancel{x}+3-\cancel{x}+25}{\sqrt{x+3}+\sqrt{x-25}} \to 0 \quad \left[\frac{28}{+\infty}\right]$$

Esercizio 2 $\lim_{x\to-\infty} \sqrt{x^2+x-\cos x} + x$ $[+\infty-\infty]$

Achtung! Non pensare nemmeno di fare $\sqrt{\dots}+x \sim \sqrt{x^2}+x$
$= x+x = 2x \to -\infty$

L'errore è che $\sqrt{x^2} = x$ va bene per x positivi, in generale
$\sqrt{x^2} = |x|$

Poniamo $y=-x$ così diventa lim per $y\to+\infty$:

$$\lim_{y\to+\infty} \sqrt{y^2-y-\cos y} - y$$ Razionalizzando

cos è pari

$$\sqrt{\dots}-y \;\frac{\sqrt{\dots}+y}{\sqrt{\dots}+y} = \frac{\cancel{y^2}-y-\cos y-\cancel{y^2}}{\sqrt{y^2-y-\cos y}+y} \to -\frac{1}{2}$$

Nell' ultimo passaggio ho raccolto y sopra e sotto

$$\frac{\cancel{y}\left(-1-\boxed{\frac{\cos y}{y}}\right)}{\cancel{y}\left(\sqrt{1-\underset{\downarrow\\0}{\frac{1}{y}}-\underset{\downarrow\\0}{\frac{\cos y}{y^2}}}+1\right)} \to \frac{-1}{2}$$

$\frac{\cos y}{y} \to 0$

$$-\frac{1}{y} \le \frac{\cos y}{y} \le \frac{1}{y} \quad \dots \qquad -\frac{1}{y^2} \le -\frac{\cos y}{y^2} \le \frac{1}{y^2} \quad \dots$$

Esempio 3 $\sqrt[3]{n^2+n} - \sqrt[3]{n^2+3}$

Prima: $A^2-B^2 = (A+B)(A-B)$

Ora: $A^3-B^3 = (A-B)(A^2+AB+B^2)$

Uso questa relazione con $A = \sqrt[3]{n^2+n}$ e $B = \sqrt[3]{n^2+3}$

Quindi devo moltiplicare e dividere per A^2+AB+B^2

$$\sqrt[3]{\ldots} - \sqrt[3]{\ldots} \quad \frac{\sqrt[3]{(n^2+n)^2} + \sqrt[3]{(n^2+n)(n^2+3)} + \sqrt[3]{(n^2+3)^2}}{\ldots}$$

$$= \frac{\cancel{n^2}+n-\cancel{n^2}-3}{\sqrt[3]{(n^2+n)^2}+\sqrt[3]{(\ )(\ )}+\sqrt[3]{(n^2+3)^2}} \quad \left[\frac{+\infty}{+\infty}\right] \to 0$$

$$\sim \frac{n}{n^{4/3}}$$

Esempio 4 $\sqrt{n+3} - \sqrt[3]{n^2+5} \to -\infty$

Brutalmente è: $\sqrt{n} - \sqrt[3]{n^2} = n^{\frac{1}{2}} - n^{\frac{2}{3}} \to -\infty$ perché $\frac{2}{3} > \frac{1}{2}$

Esempio 5

$\frac{e^{2x}-\cos(3x)}{\log(1+\tan x)}$	2

$\boxed{x \to 0}$

Successione	Li
$n^2 \cos\frac{1}{n^2}$	$+\infty$
$n^2 \tan^2\frac{1}{n}$	1
$n(\log(n+1)-\log n)$	1
$n(\sqrt[n]{7}-\sqrt[n]{5})$	$\log\left(\frac{7}{5}\right)$

$$\frac{e^{2x}-\cos(3x)}{\log(1+\tan x)} = \underbrace{\boxed{\frac{\tan x}{\log(1+\tan x)}}}_{\downarrow\ 1} \underbrace{\boxed{\frac{x}{\tan x}}}_{\downarrow\ 1}\left[\underbrace{\boxed{\frac{e^{2x}-1}{x}}}_{\downarrow\ 2} - \underbrace{\boxed{\frac{1-\cos(3x)}{x^2}}}_{\downarrow\ \frac{9}{2}} \cdot \underbrace{\boxed{x}}_{\downarrow\ 0}\right]$$

$y = \tan x$

$$\underbrace{n^2}_{+\infty}\ \underbrace{\cos\frac{1}{n^2}}_{1} \longrightarrow +\infty \qquad n^2\tan^2\frac{1}{n} = \frac{\tan^2\frac{1}{n}}{\frac{1}{n^2}} = \left[\frac{\tan\frac{1}{n}}{\frac{1}{n}}\right]^2 \longrightarrow 1$$

$x = \frac{1}{n}$

$$n\left(\log(n+1) - \log n\right) \qquad [+\infty\cdot(+\infty-\infty)]$$

$$= n\log\left(\frac{n+1}{n}\right) = n\log\left(1+\frac{1}{n}\right) = \log\left[\left(1+\frac{1}{n}\right)^n\right] \to \underbrace{\log e}_{=1}$$

$$\frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \frac{\log(1+x)}{x} \ldots$$

$$n\left(\sqrt[n]{7} - \sqrt[n]{5}\right) = n\left(\sqrt[n]{7}-1\right) - n\left(\sqrt[n]{5}-1\right)$$

$$\to \log 7 - \log 5 = \log\frac{7}{5} \qquad \text{(vedi lez. 25)}$$

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$$\left(n+\sqrt{n}+\sqrt[3]{n}\right)^{1/\log n}$$

Brutale: $\sim n^{\frac{1}{\log n}} = e^{\frac{1}{\log n}\cdot\log n} = e$:)

$$\boxed{n^{\frac{1}{\log n}}} \boxed{\left(1+\frac{1}{\sqrt{n}}+\frac{1}{n^{2/3}}\right)^{\frac{1}{\log n}}}$$

$\boxed{n^{\frac{1}{\log n}}} = e$

$\left(1+\frac{1}{\sqrt{n}}+\frac{1}{n^{2/3}}\right)^{\frac{1}{\log n}} \to 1^0 = 1$

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$\lim_{x\to 0}\dfrac{\sqrt{1+\sin x}-\cos x}{\tan(3x)}$	$\frac{1}{6}$	$\lim_{x\to 0}\dfrac{\sqrt{\cos x}-1}{\sqrt[3]{x^2+1}-e^x}$	0

$$\frac{\sqrt{1+\sin x}-\cos x}{\tan(3x)}\cdot\frac{\sqrt{1+\sin x}+\cos x}{\sqrt{1+\sin x}+\cos x} = \frac{1+\sin x-\cos^2 x}{\tan(3x)}\ \boxed{\frac{1}{\sqrt{\ldots}+\cos x}}$$

$\boxed{\frac{1}{\sqrt{\ldots}+\cos x}} \to \frac{1}{2}$

$$\frac{1+\sin x-\cos^2 x}{\tan(3x)} = \boxed{\frac{3x}{\tan(3x)}}\left[\boxed{\frac{\sin x}{3x}} + \frac{1-\cos^2 x}{3x}\right]$$

$\boxed{\frac{3x}{\tan(3x)}} \to 1 \qquad \boxed{\frac{\sin x}{3x}} \to \frac{1}{3}$

$$\frac{1-\cos^2 x}{3x} = \boxed{\frac{\sin^2 x}{3x^2}}\ \boxed{x}$$

$\boxed{\frac{\sin^2 x}{3x^2}} \to \frac{1}{3} \qquad \boxed{x} \to 0$

$$\frac{\sqrt{\cos x}-1}{\sqrt[3]{x^2+1}-e^x}\cdot\frac{\sqrt{\cos x}+1}{\sqrt{\cos x}+1}=\boxed{\frac{\cos x-1}{x^2}}\frac{x^2}{\sqrt[3]{x^2+1}-e^x}\boxed{\frac{1}{\sqrt{\cos x}+1}}$$

$$\frac{\cos x-1}{x^2}\to-\frac{1}{2}\qquad\frac{1}{\sqrt{\cos x}+1}\to\frac{1}{2}$$

Dico che il centrale tende a 0

$$\frac{x^2}{\sqrt[3]{x^2+1}-e^x}\cdot\frac{\sqrt[3]{(x^2+1)^2}+e^x\sqrt[3]{x^2+1}+e^{2x}}{\dots}$$

$$=\frac{\underset{\to 0}{x}\left[\sqrt[3]{(x^2+1)^2}+e^x\sqrt[3]{x^2+1}+e^{2x}\right]^{\nearrow 3}}{\underbrace{\boxed{\frac{x^2+1-e^{3x}}{x}}}_{\to -1}}$$

[e^{3x} corretto dopo video]

$$\frac{x^2+1-e^{3x}}{x}=x-\frac{e^{3x}-1}{x}\longrightarrow-3$$

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