# ANALISI 1 — LEZIONE 119

Note Title 23/05/2025

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|---|---|---|---|---|---|---|---|---|
| VF | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| MC | 1. | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

$\boxed{\text{TEST}}$ 1

$\boxed{\text{VF1}}$ Se $a_n > 0$ per ogni $n \in \mathbb{N}$ e $\sqrt{n}a_n \to 0$, allora di sicuro $\sum a_n$ converge F

$\boxed{\text{VF2}}$ $\sin(x^3) = x^3 + o(x^6)$ per $x \to 0$ V

$\boxed{\text{VF3}}$ $\forall M \in \mathbb{R}\ \exists K \in \mathbb{R}$ tale che $x^3 \le M$ per ogni $x \le K$ V

$\boxed{\text{VF4}}$ L'equazione differenziale $u''' = 7u$ è lineare V

1 $\sqrt{n}\, a_n \to 0$ $\qquad \dfrac{a_n}{\frac{1}{\sqrt{n}}} \to 0 \quad \rightsquigarrow \quad \underset{\text{BOH}}{a_n} < \underset{\text{Diverge}}{\dfrac{1}{\sqrt{n}}}$ defin.

$a_n = \frac{1}{n} \rightsquigarrow \sqrt{n}\cdot a_n \to 0$ e $\sum a_n$ diverge

$a_n = \frac{1}{n^2} \rightsquigarrow \sqrt{n}\cdot a_n \to 0$ e $\sum a_n$ converge

$\sqrt{n}\, a_n \to 14 \implies \sum a_n$ diverge V

$\dfrac{a_n}{\frac{1}{\sqrt{n}}} \to 14 \begin{matrix} \neq 0 \\ \neq +\infty \end{matrix} \rightsquigarrow \sum a_n$ si comporta come $\sum \frac{1}{\sqrt{n}}$

quindi diverge

2 $\sin(x^3) = x^3 + o(x^8)$

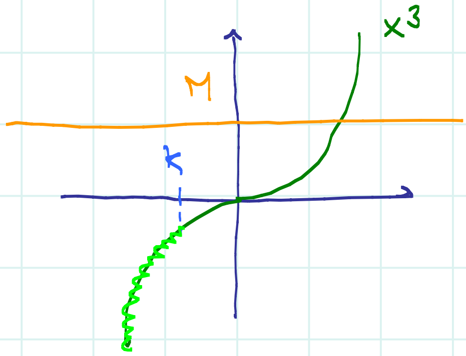
$\sin t = t - \frac{1}{6}t^3 + \dots$

$\sin x^3 = x^3 \underbrace{- \frac{1}{6}x^9}_{\text{viene assorbito da } x^8} + \dots$

$\sin(x^3) = x^3 + o(x^2)$ VERA MA BUFFA

③ $\forall M \in \mathbb{R} \quad \exists k \in \mathbb{R}$ t.c. $x^3 \leq M \quad \forall x \leq k$

Sinonimo di $\lim_{x \to -\infty} x^3 = -\infty$




VF5 La funzione $f(x) = \sin(x^3)$ è pari — F (è dispari)

VF6 Si ha che $2^n \geq n^{3000}$ definitivamente — V

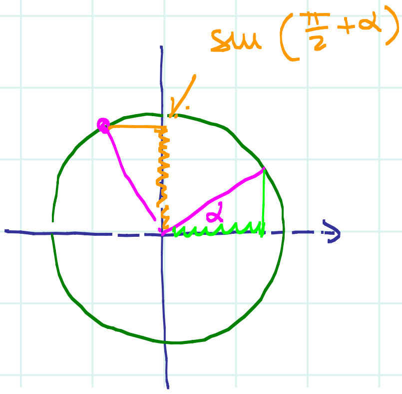
VF7 Per ogni $\alpha \in \mathbb{R}$ si ha che $\sin\left(\frac{\pi}{2} + \alpha\right) = \sin\alpha$ — F $\quad \sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$

VF8 Esiste $\min\{\arctan(x^2) : x \in \mathbb{R}\}$ — V

⑤ $f(-x) = \sin((-x)^3) = \sin(-x^3) = -\sin(x^3) = -f(x)$

⑥ Esponenziale batte potenza $\quad \underbrace{\boxed{\frac{2^n}{n^{3000}}}}_{+\infty} \geq 1$ definitiv.

⑦




⑧

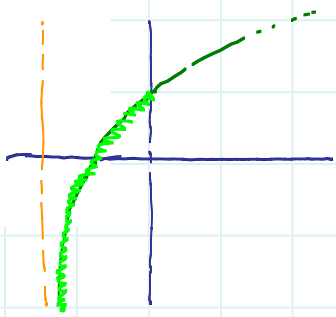

MC1 Se $f(x) = x\sin x$, allora $f''(x) \geq 0$ se e solo se ... $\quad f'(x) = \sin x + x\cos x$

(A) $x\sin x \leq 0$ (B) $\cos x \geq x\sin x$ (C) $\cos x \geq 0$ (D) $x\sin x \geq 0$

~~(E)~~ $2\cos x \geq x\sin x$ $\quad f''(x) = 2\cos x - x\sin x \geq 0$

MC2 $\sup\{\log(3+x) : x < 0\} = \ldots$

(A) $-2$ (B) Non esiste (C) $\log 3$ (D) $-3$ (E) $3$

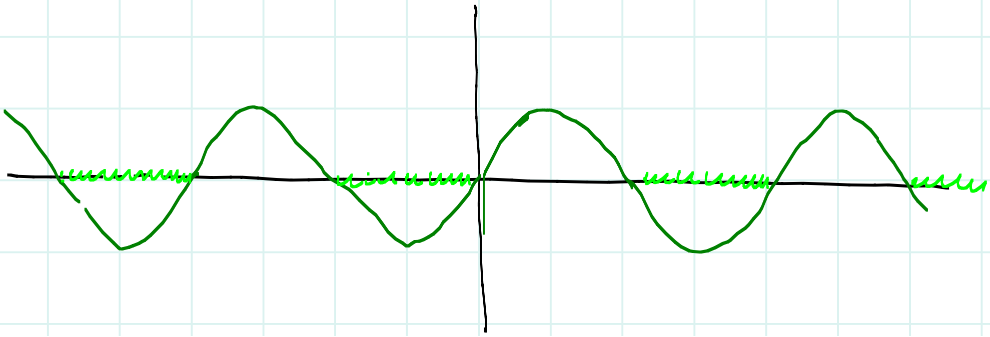


$\boxed{\text{MC4}}$ Consideriamo l'insieme $A = \{x \in \mathbb{R} : \sin x \le 0\}$.

Determinare quale delle seguenti affermazioni sull'insieme $A$ è *vera*.

(A) $A$ è limitato superiormente (B) esiste $\min A$ (C) $A$ ammette minoranti

(D) $2\pi \in A$ (E) $\sup A \in \mathbb{R}$



$\boxed{\text{MC6}}$ Stabilire quali delle seguenti funzioni sono *limitate inferiormente* su tutto $\mathbb{R}$:

$$f(x) = x^5 - \sin(x^4), \qquad g(x) = x^2 - \log(1 + x^{20}), \qquad h(x) = x^7 + \cos(x^8).$$

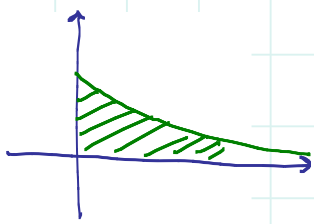
come la 1a

(A) Nessuna $\boxed{\text{(B) Solo } g}$ (C) Solo $f$ (D) Solo $h$ (E) Solo $g$ e $h$

NO LIM INF

$\boxed{\text{MC7}}$ $\displaystyle\int_0^{+\infty} \frac{1}{(x+6)^2}\,dx = \ldots$

$$\left[-\frac{1}{x+6}\right]_0^{+\infty}$$



(A) $\frac{1}{6}$ (B) $\frac{3}{6^3}$ (C) $+\infty$ (D) $-\frac{1}{6}$ (E) $-\frac{3}{6^3}$

$\boxed{\text{MC8}}$ La serie $\displaystyle\sum_{n=1}^{\infty} \frac{n^\alpha - 2000}{\sqrt{n^8+3}}$ converge se e solo se ...

$$\frac{n^\alpha}{n^4} = \frac{1}{n^{4-\alpha}}$$

$4 - \alpha > 1$

$\alpha < 3$

(A) $\alpha \le 7$ $\boxed{\text{(B) } \alpha < 3}$ (C) $\alpha < 7$ (D) $\alpha < 8$ (E) $\alpha < 4$