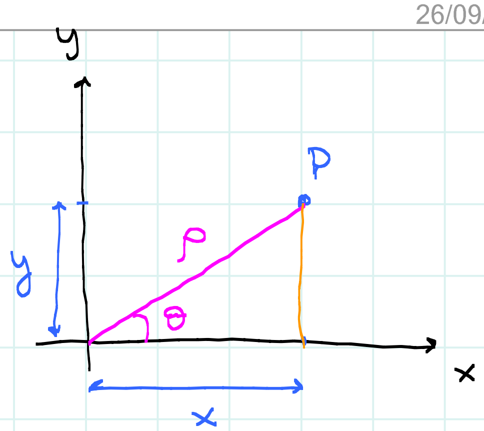
# ALGEBRA LINEARE - LEZIONE 02

Note Title 26/09/2023

## COORDINATE POLARI NEL PIANO




Coord. cartesiane : $(x, y)$

Coord. polari : $\rho, \theta$

**Relazioni**

→ Se conosco $\rho$ e $\theta$, allora $\boxed{x = \rho\cos\theta}$ $\boxed{y = \rho\sin\theta}$

→ Se conosco $x$ e $y$, allora $\boxed{\rho = \sqrt{x^2+y^2}}$

Come trovo $\theta$? $x = \rho\cos\theta$, $y = \rho\sin\theta$ $\rightsquigarrow$ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$ $\rightsquigarrow$ $\tan\theta = \dfrac{y}{x}$

$\boxed{\tan\theta = \dfrac{y}{x}}$ con mille cautele :

- ci sono problemi quando $x=0$ (in questi casi $\theta = \pm\frac{\pi}{2}$)
- ci sono due angoli fra 0 e $2\pi$ che hanno la stessa tangente. Per stabilire quello giusto guardiamo la figura.

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## Coord. polari e prod. scalare nel piano

$\vec{u} = (x_1, y_1)$ $\vec{v} = (x_2, y_2)$

$\langle \vec{u}, \vec{v} \rangle = x_1x_2 + y_1y_2$

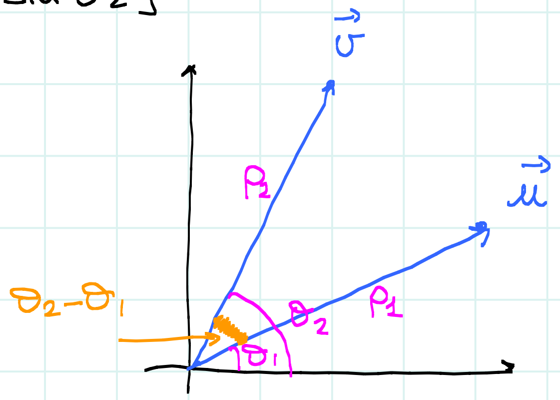
Scriviamo i due vettori in coord. polari:

$\vec{u} = (\rho_1\cos\theta_1, \rho_1\sin\theta_1)$ $\vec{v} = (\rho_2\cos\theta_2, \rho_2\sin\theta_2)$

$$\langle \vec{u}, \vec{v} \rangle = \rho_1 \rho_2 \cos\theta_1 \cos\theta_2 + \rho_1 \rho_2 \sin\theta_1 \sin\theta_2$$

$$= \rho_1 \rho_2 \left[ \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \right]$$

$$= \underset{\|\vec{u}\| \cdot \|\vec{v}\|}{\rho_1 \rho_2} \cos \underbrace{(\theta_1 - \theta_2)}_{\text{angolo compreso}}$$

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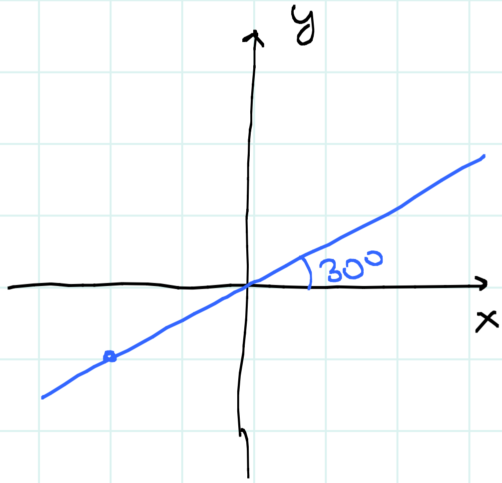
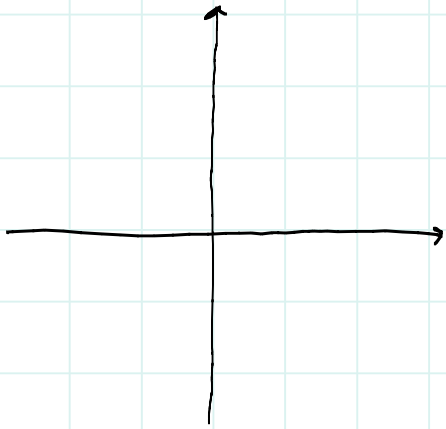
| x | y | ρ | θ | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | | |
| 0 | 1 | 1 | $\frac{\pi}{2}$ | | |
| −1 | 0 | 1 | $\pi$ | | |
| 1 | 1 | $\sqrt{2}$ | $\frac{\pi}{4}$ | | |
| 1 | −1 | $\sqrt{2}$ | $-\frac{\pi}{4}$ | $\frac{7\pi}{4}$ | $\frac{15\pi}{4}$ |
| −4 | 4 | $4\sqrt{2}$ | $\frac{3\pi}{4}$ | $135^\circ$ | |
| 1 | $-\sqrt{3}$ | 2 | $-\frac{\pi}{3}$ | | |
| 0 | 0 | 0 | qualunque | | |
| $-\sqrt{3}$ | −1 | 2 | $210^\circ$ | $\frac{7\pi}{6}$ | |
| 0 | 2 | 2 | $\frac{\pi}{2}$ | | |
| −3 | 0 | 3 | $\pi$ | | |
| −1 | 0 | 1 | $5\pi$ | | |
| 0 | 0 | 0 | $\pi$ | | |
| 0 | 0 | 0 | $\frac{\pi}{4}$ | | |
| $\sqrt{3}$ | 1 | 2 | $\frac{\pi}{6}$ | | |
| 0 | −6 | 6 | $-\frac{\pi}{2}$ | | |
| $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{6}}{2}$ | $\sqrt{2}$ | $-\frac{4\pi}{3}$ | | |




$$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}}$$



$\rho \cos\theta$ $\quad$ $\rho \sin\theta$

$\sqrt{2}\left(-\frac{1}{2}\right)$ $\quad$ $\sqrt{2}\left(-\frac{\sqrt{3}}{2}\right)$

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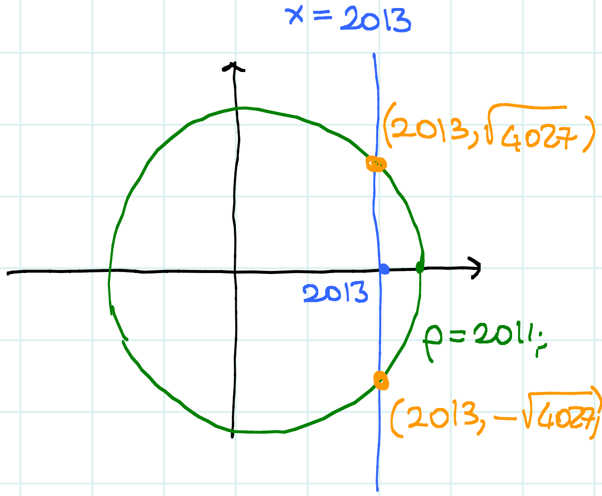
Trovare tutti i p.ti del piano tali che




① $x = 2013 \qquad \rho = 2014$

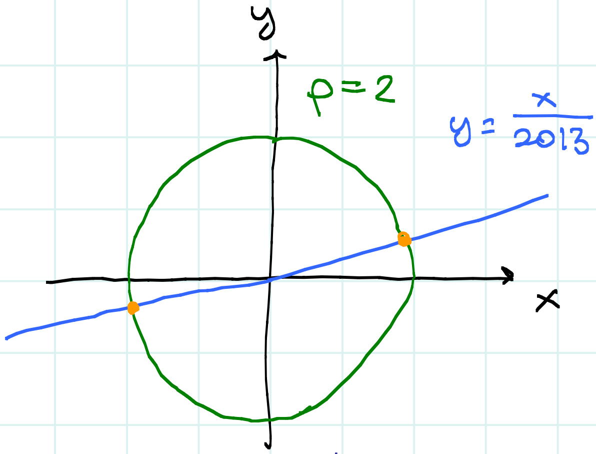
Dalla figura sono due punti

Come li trovo? Risolvo

$$\begin{cases} x = 2013 \\ \sqrt{x^2+y^2} = 2014 \end{cases} \qquad \begin{cases} x = 2013 \\ x^2+y^2 = 2014^2 \end{cases}$$

$$y^2 = 2014^2 - x^2 = 2014^2 - 2013^2 = 4027 \quad \rightsquigarrow \quad y = \pm\sqrt{4027}$$

$$(a^2-b^2) = (a+b)(a-b)$$

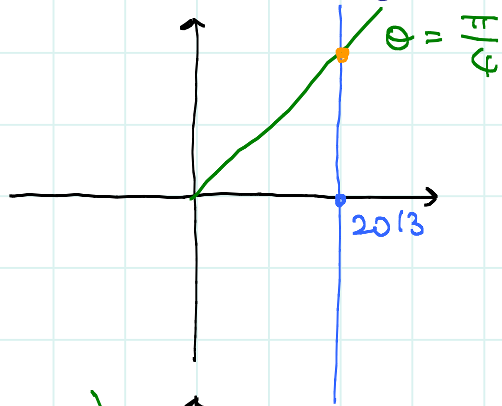



② $x = 2013y \qquad \rho = 2$

$$y = \frac{1}{2013}x$$

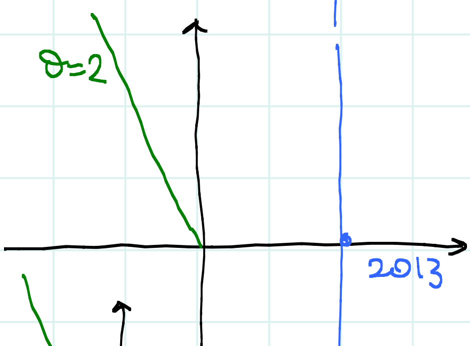
Ho due punti, che trovo risolvendo

$$\begin{cases} x^2+y^2 = 4 \qquad (\rho = 2) \\ x = 2013y \end{cases}$$ $\rightsquigarrow$ sostituisco nella 1ª e risolvo eq. di 2° grado




③ $x = 2013 \qquad \theta = \frac{\pi}{4}$

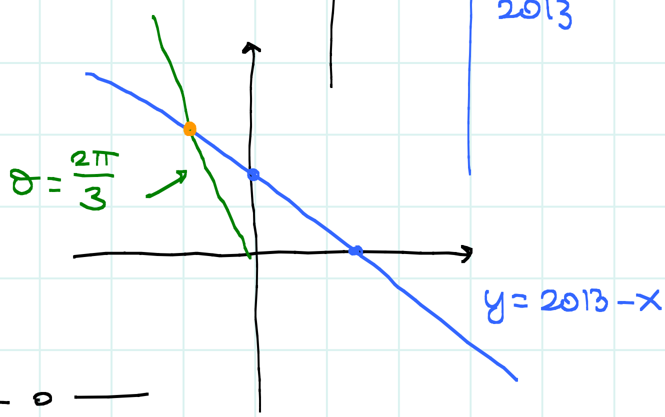
L'unico p.to è $(2013, 2013)$




④ $x = 2013 \qquad \theta = 2$

↑ radianti

Nessuna soluzione




⑤ $x+y = 2013 \qquad \theta = \frac{2\pi}{3}$

$y = 2013 - x$. Una soluzione

che risolve il sistema

$$\begin{cases} x+y = 2013 \\ y = -\sqrt{3}\,x \end{cases} \qquad \frac{y}{x} = \tan\theta = -\sqrt{3}$$

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