

Mathematics in Rubik's Cube

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In 1947, Hungarian Ernő Rubik invented the Magic Cube, and the name was transmuted to Rubik's Cube in 1980 by Ideal Toy Company. As we know, Rubik's Cube is a brilliant hand exercise more popular in society, there is a growing number of players who broke world records, intercontinental records, and national records. The purpose of this research paper is to present the unconscious discovery from a scattered $3 \times 3 \times 3$ ¹ Rubik's Cube and to explain the puzzle toy to people who are unfamiliar with Rubik's Cube. Assuming that there is a relationship between mathematics and Rubik's Cube. And so, on the other hand, what mathematics is inside Rubik's Cube? A gigantic number in the result, and through the calculation of permutation² on scrambling a $3 \times 3 \times 3$ Rubik's Cube, there are 43252003274489856000 ways. In this paper, the study will focus on the mathematics discovery of the $3 \times 3 \times 3$ Rubik's Cube.

To explore the mathematics inside Rubik's Cube, cubers³ have to first know whether mathematics is linked to the cube. In their study, Koehne et al. (2017, 541) argue that Rubik's Cube permeates different mathematics areas and is a hand exercise in mathematics. The relationship between mathematics and Rubik's Cube has been built because Rubik's Cube touches with mathematics. Furthermore, a science teacher Mr. Brain Rohrig (2010, 54) utilized Rubik's Cube as his teaching material to make his STEM classes students solve problems. These

¹ $3 \times 3 \times 3$ refers a Rubik's Cube of length of 3 *width of 3 *height of 3.

² Permutation refers a calculation without repetitive, and adverse with combination that with repetitive.

³ Cuber refers a Rubik's Cube player.

studies illustrate that Rubik's Cube not only excited mathematical scholars but even scholars also who are outside of mathematics result in a positive status. Their study result reveals that the Rubik's Cube is related to mathematics made my next assumption more convincing.

Mathematicians, statisticians, and cubers usually discover the trick in Rubik's Cube that if some settled cubes are twisted manually, it will be unsolvable. Since mathematicians, statisticians, and cubers know the different definitions of permutation and combination in mathematics. In this case, the selected objects are not repetitive, and novices will apply the calculated method of permutation to resolve how many scramble ways in $3 \times 3 \times 3$ in Rubik's Cube. Linkletter (2020, 110) points out a trap that every novice may make when cubers and novices scramble a Rubik's Cube by twisting the cubes in a manner that renders it unsolvable. His study relates to many cubers not even only novices because when cubers reassemble Rubik's Cube one by one, a cube in the corner is orientated oddly. Through a mass of calculations with permutation in central pieces, edge cubes and corner cubes, an astronomical number is exhibited in this amazing universe.

Many novices commenced learning $3 \times 3 \times 3$ Rubik's Cube by memorizing the formulas that their mathematics teacher and friends gave them. There is a fact that when beginners stop practicing or have fewer chances to practice than usual. Their brain will be blank and have no formulas in their mind and need to start learning a $3 \times 3 \times 3$ Rubik's Cube as a novice again. To be an unobstructed Rubik's Cube player playing $3 \times 3 \times 3$ Rubik's Cube, cubers usually watch videos and read articles to learn new methods to restore Rubik's Cube. The use of Rubik's Cube teaching and using by STEM teachers has become very widespread. Based on the data online, in high school student's education life, many high school mathematics teachers play Rubik's Cube.

There is a relationship between mathematics and Rubik's Cube again. Six faces each with nine cubes... "Rubik's Cube with numbers!" If the numbers are linked to mathematics based on the hypothesis, what is one of the mathematics numbers inside the Rubik's Cube? The discovery in mathematics is more distinctive in Rubik's Cube.

There are many types of Rubik's Cube, for instance, $2 \times 2 \times 2$, $4 \times 4 \times 4$, and $5 \times 5 \times 5$, but $3 \times 3 \times 3$ is the basic cube. This basic cube novices usually play in is a pattern with a $3 \times 3 \times 3$ cube and is colored in one different color on each face. The hypothesis is that the relationship between mathematics and Rubik's Cube is true because pattern recognition and generalizing results permeate different areas of mathematics, such as algebra, geometry, counting, and probability (Koehne et al. 2017, 541). By this means, any $n \times n \times n$ ⁴ cubes and magic dodecahedrons can be restored by calculating mathematics and without rote memorization. What cubers are having is not only hand memory and formulas in their pate but also a mighty mathematics system. The mathematics calculations in Rubik's Cube help cubers breezily restore a Rubik's Cube without memorization by rote.

Koehne et al (2017, 541) emphasize that restoring cubes is to provide students with hands-on experience in a collaborative setting and to connect different areas of mathematics. Obviously, most mathematics teachers encourage their students to engage in Rubik's Cube associations or clubs because there is a relationship between mathematics and Rubik's Cube. Nevertheless, some cubers published their cubing videos on YouTube recently, they got a lot of attention but the

⁴ $n \times n \times n$ denotes that the Rubik's Cube usually refers to the shape of a cube in which each side is n units long, so there are n^3 small units in total. This type of cube is also known as an n -order cube or n -order Rubik's Cube.

reflection from the videos was mediocre. Haters' comments flew around, they underestimated what was behind the Rubik's Cube, and misunderstood that cubing was a childish game for geeks. In other words, while the cube's literal insides may be made of plastic, its real guts are nothing but numbers (Linkletter, 2020, 108). But the stereotype still exists, and many people believe that restoring a Rubik's Cube is only memorizing the formulas with speed nowadays and cubers will become outstanding players if they perform at their fastest speed.

Despite the rumors circulating, the number of people engaging in the Rubik's Cube is on trend. The primary reason is the puzzle mathematics inside of Rubik's Cube was attractive in the past and is an ongoing and long-term process. On the contrary, in a 2009 public newspaper from *The New York Times* written by Jennifer Lee, she emphasizes that “the company – Seven Towns took the relatively cryptic problem-solving guides and made them more student-friendly by adding colorful illustrations and simplifying the instructions” (Lee, “Learning Math from the Rubik’s Cube.”). The invention of Rubik’s Cube itself has already made people curious about restoring and now Seven Towns has easy-to-understand and advanced guides. This innovation also caused an increasing number of field scholars to begin their journey in Rubik’s Cube. The expansion of the Rubik’s Cube in the world exerts a positive influence and the data shows that children have a high percentage of solving the puzzle by following the instructions in the guide (Lee, “Learning Math from the Rubik’s Cube.”). There is an increased number of cubers, and one of the contributing people for spreading Rubik’s Cube culture are teachers, Mr. Brian Rohrig (2010, 54) was no exception. He was a 9th-grade science teacher, and he approached his principal and explained how Rubik’s Cube helps students solve problems in his Physical Science, Biology, Chemistry, and Physics classes to his students.

However, the public also states that “Teachers are attracted to the puzzle-solving lesson because it helps with geometry, algebra, direction-following, memorization, and perseverance.” (Lee, “Learning Math from the Rubik’s Cube.”). In other words, the Rubik’s Cube is always used in STEM and is being used widely in various classes and mathematics fields. The restoring of the Rubik’s Cube helps students enhance their learning and the class’s efficiency. In the durative trend of engaging in cubing, every novice may have their excitement inside of the Rubik's Cube when they first start playing – statistics within mathematics. Such as the foundation mathematics, usually, starts calculating the ways of permutating a $3 \times 3 \times 3$ Rubik’s Cube in the scramble.

It is normal for cubers to drop a Rubik's Cube on the floor when practicing, and usually, all the small cubes are scattered except for the six central pieces. Cubers may discover that there is a core to support them and observe that the scattered cubes and the central pieces have different amounts of sides on small cubes or pieces. Only one color on the center pieces' surface, two colors on the edge cubes' surface where the twenty cubes are in the middle of the edges, and three colors on the corner cubes. With these twenty small cubes and six central pieces in a $3 \times 3 \times 3$ Rubik’s Cube. How many ways can be permuted or combined with scrambling? In this case, order matters are considered and permutation is applied, r is the many objects, selected from n many in total, so the combination formula $C_{n,r} = n! / [(n-r)!r!]$ will not be used in this situation.

To calculate the permutation, either formula $P_{n,r} = n!/(n-r)!$ or R language code: `perm(n,r)` can resolve this problem. Because the central pieces only can be placed in one position and there is no orientation, $P(\text{central pieces}) = \text{position} * \text{orientation} = 1$. Each edge can be placed in one particular position, so $n=12$ and $k=12$. By plugging the numbers into the formula $P_{n,r} = n!/(n-r)!$ results that $P_{12,12} = 12!/(12-12)! = 12!$. The edge cubes can be placed in twelve positions and two orientations, $P(\text{edge cubes}) = \text{position} * \text{orientation} = 12! * 2^{12}$. The corner cubes can be placed in eight positions and three orientations on eight edges, $P(\text{corner cubes}) = \text{position} * \text{orientation} = 8! * 3^8$. After getting permutations from central pieces, edge cubes, and corner cubes, the total permutation is $P(\text{total}) = P(\text{central pieces}) * P(\text{edge cubes}) * P(\text{corner cubes}) = 1 * (12! * 2^{12}) * (8! * 3^8)$.

The calculation has not ended. Why?

If cubers reassembled all the scattered cubes into an integrated Rubik's Cube. There is a very tricky corner cube, it never works no matter how cubers attempt to restore it without twisting the cube. This is a common circumstance when novices want to scramble a solved cube, there is a trap that has caught many novices that they keep the cube intact and scramble it up manually (Linkletter, 2020, 110). Many of the novices got caught. The total permutation $P(\text{total}) = 1 * (12! * 2^{12}) * (8! * 3^8)$ is not the final answer because twelve small cubes cannot be transformed into one another, so only one in twelve chance that the Rubik's Cube can be restored. There are three factors to get twelve which are $3 * 2 * 2$. Twisting small corners, each corner cube has three orientations that can go and three is one of the factors. By flipping any number of edge cubes can

be shimmed into a single edge and there will be two possibilities (Linkletter, 2020, 110). The final factor comes from the edge cubes and corner cubes, the situation is that the players twist two corner cubes while flipping two edge cubes. Briefly, out of these circumstances, there are $[1 * (12! * 2^{12}) * (8! * 3^8)] / 3 * 2 * 2 = 4.3252003274489856 * 10^{19} = 43252003274489856000$ ways to scramble a 3*3*3 Rubik's Cube.

The Rubik's Cube has been widely applied in various fields of society and teaching. Teachers use Rubik's Cube to teach students different skills, the most primitive of Rubik's Cube's applications are mathematics, and always accompanied, but a part of people describe this hand puzzle game to be a "featureless baby toy" for nerds or geeks. However, the stereotype does not stop the progress of novices from becoming professional cubers and influencing promoters. Besides, as eras continued to evolve and the progress of mankind, Seven Towns adopted colorful compositions and easy-to-understand content to make guides for restoring Rubik's Cube. But in the process of learning to restore the Rubik's Cube, cubers and novices may trap themselves in an unsolvable situation because they are not familiar with the permutations of the cube. Nevertheless, through the intellections and directions of mathematics and statistics to calculate the permutation, there will be some scattered pieces of the Rubik's cube that need to be divided into different situations to calculate. The gigantic permutation of numbers attracted a lot of novices to explore the mathematics of Rubik's Cube.

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