ai引论 课后练习-3 机器学习

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题目一

平方损失函数

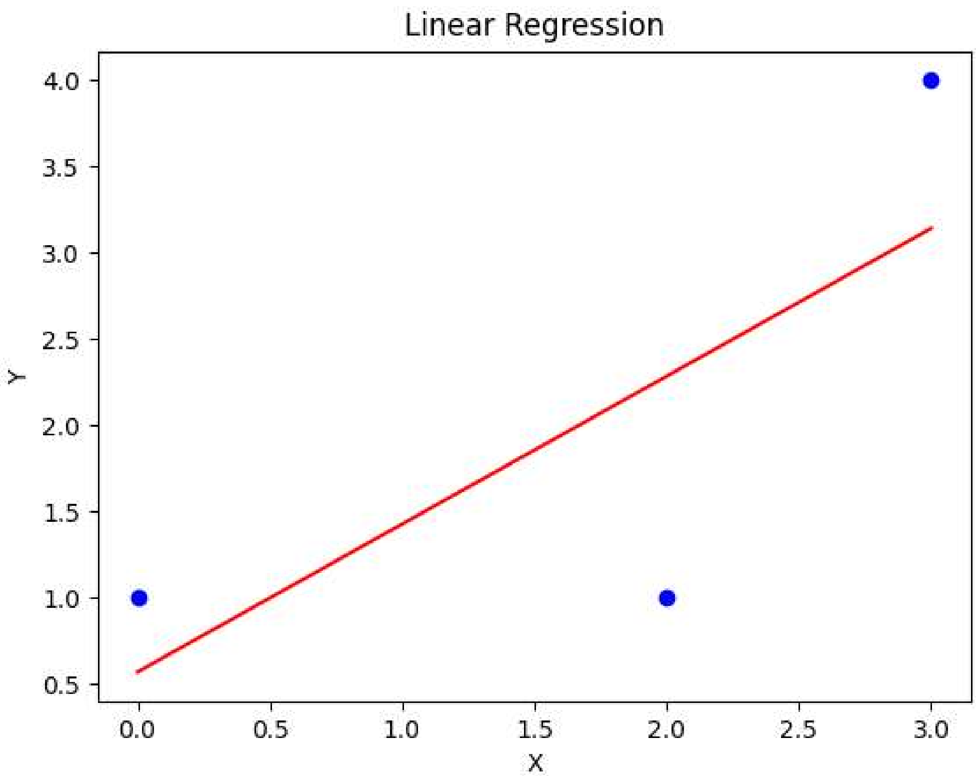
$$Loss = \frac{1}{3}\sum_{i=1}^{3}(wx_i + b - y_i)^2$$

$$\begin{aligned}\frac{\partial L}{\partial w} &= 2(w\bar{x^2} + b\bar{x} - \bar{xy}) = 0 \\ \frac{\partial L}{\partial b} &= 2(w\bar{x} + b - \bar{y}) = 0\end{aligned}$$

代入数据：

$$\begin{aligned}\Rightarrow\ w &= \frac{6}{7} \\ b &= \frac{4}{7}\end{aligned}$$

可以作图：



题目2

1)

仍有正类概率为线性模型f(x)叠加sigmoid函数的映射

$$p(y=1|x;\theta)=\sigma(f(x))$$

负类则为补值:

$$p(y=0|x;\theta)=1-\sigma(f(x))=\sigma(-f(x))$$

2)

以题目给的方式将两者表示为同一形式

$$p(y=y_i|x)=(\sigma(f(x_i)))^{y_i}(\sigma(-f(x_i)))^{1-y_i}$$

做最大对数似然:

$$\begin{aligned}&\max_{w,b}\sum_{i=1}^{n}log[(\sigma(f(x_i)))^{y_i}(\sigma(-f(x_i)))^{1-y_i}]\\=&\max_{w,b}\sum_{i=1}^{n}y_ilog[\sigma(f(x))]+(1-y_i)log[\sigma(-f(x_i))]\\=&\max_{w,b}-\sum_{i=1}^{n}y_ilog[1+e^{-(w^Tx_i+b)}]+(1-y_i)log[1+e^{+(w^Tx_i+b)}]\end{aligned}$$

写成最小化平均损失函数的形式:

$$\min_{w,b}\frac{1}{n}\sum_{i=1}^{n}y_ilog[1+e^{-(w^Tx_i+b)}]+(1-y_i)log[1+e^{+(w^Tx_i+b)}]$$

题目3

1)

先计算属性A的熵:

$$H(D_A)=-\frac{6}{9}\log_2\frac{6}{9}-\frac{3}{9}\log_2\frac{3}{9}=0.9183$$

再计算属性A的增益：
D的熵：

$$H(D) = -\frac{6}{9}\log_2\frac{6}{9} - \frac{3}{9}\log_2\frac{3}{9} = 0.9183$$

分类后的熵：

$$H(D_A) = \frac{3}{9}(0) + \frac{6}{9}(-\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6}) = 0.6667$$

gain ratio A:

$$g_R(D,A) = \frac{H(D) - H(D_A)}{H(D_A)} = 0.274$$

2)

```
               A=1/0
             /       \
            /         \
       C=1/0        [-1,-1,-1]
   [-1,-1,-1,1,1,1]
          /    \
         /      \
    B=1/0      [1]
  [-1,-1,-1,1,1]
       /   \
      /     \
  [-1,-1]  [-1,1,1]
```

因此 $x_* = [1,1,1]$对应标签y=-1

题目4

1)

$$
\begin{aligned}
a_i &= \frac{e^{z_i}}{\sum_k e^{z_k}} \\
\frac{\partial L}{\partial z_i} &= \sum_j \frac{\partial L}{\partial a_j}\frac{\partial a_j}{\partial z_i} \\
if\ i \neq j: \\
\frac{\partial a_j}{\partial z_i} &= -\frac{e^{z_j+z_i}}{(\sum_k e^{z_k})^2} \\
if\ i = j: \\
\frac{\partial a_j}{\partial z_i} &= -\frac{(\sum_{k\neq i} e^{z_k})e^{z_i}}{(\sum_k e^{z_k})^2}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \frac{\partial L}{\partial z_i} &= \frac{e^{z_i}}{(\sum_k e^{z_k})^2}(\frac{\partial L}{\partial a_i}\sum_{j\neq i} e^{z_j} - \sum_{j\neq i}\frac{\partial L}{\partial a_j}e^{z_j}) \\
&= a_i(\frac{\partial L}{\partial a_i}\sum_{j\neq i} a_j - \sum_{j\neq i}\frac{\partial L}{\partial a_j}a_j) \\
&= a_i\sum_{j\neq i}(\frac{\partial L}{\partial a_i} - \frac{\partial L}{\partial a_j})a_j
\end{aligned}
$$

2)

$$
\begin{aligned}
a_i &= ln\frac{e^{z_i}}{\sum_k e^{z_k}} \\
\frac{\partial L}{\partial z_i} &= \sum_j \frac{\partial L}{\partial a_j}\frac{\partial a_j}{\partial z_i} \\
if\ i \neq j: \\
\frac{\partial a_j}{\partial z_i} &= -\frac{e^{z_j}}{\sum_k e^{z_k}} \\
if\ i = j: \\
\frac{\partial a_j}{\partial z_i} &= -\frac{(\sum_{k\neq i} e^{z_k})}{\sum_k e^{z_k}}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \frac{\partial L}{\partial z_i} &= \frac{1}{\sum_k e^{z_k}}(\frac{\partial L}{\partial a_i}\sum_{j\neq i} e^{z_j} - \sum_{j\neq i}\frac{\partial L}{\partial a_j}e^{z_j}) \\
&= \frac{\partial L}{\partial a_i}\sum_{j\neq i} a_j - \sum_{j\neq i}\frac{\partial L}{\partial a_j}a_j \\
&= \sum_{j\neq i}(\frac{\partial L}{\partial a_i} - \frac{\partial L}{\partial a_j})a_j
\end{aligned}
$$