

Note on the Rochester Institute of Technology Paper by James Mnatzaganian, Ernest Fokoué and Dhireesha Kudithipudi

The Paper

Jeff Hawkin’s analysis of the biology of the neocortex with the aim of understanding and creating algorithmic models of the brain’s information processing has motivated a number of theories under the general umbrella of Hierarchical Temporal Memory (HTM). An open source software project called NuPIC a realization of many aspects of the HTM theory and a tool of ongoing research. Among these research projects are several attempts to capture the workings of HTM in hardware designs. A widely perceived obstacle to formal specification of HTM, that could be the basis of an optimized electronic circuit, is the lack of mathematical models of the HTM algorithms.

Parameter	Description
n	Number of patterns (samples)
p	Number of inputs (features) in a pattern
m	Number of columns
q	Number of proximal synapses per column
ϕ_+	Permanence increment amount
ϕ_-	Permanence decrement amount
ϕ_δ	Window of permanence initialization
ρ_d	Proximal dendrite segment activation threshold
ρ_s	Proximal synapse activation threshold
ρ_c	Desired column activity level
κ_a	Minimum activity level scaling factor
κ_b	Permanence boosting scaling factor
β_0	Maximum boost
τ	Duty cycle period

TABLE I: User-defined parameters for the SP

17 The following parameters were used to obtain these results: $m = 936$, $q = 353$, $\rho_d = 14$, $\phi_\delta = 0.0105$, $\rho_c = 182$, $\phi_+ = 0.0355$, $\phi_- = 0.0024$, $\beta_0 = 18$, and $\tau = 164$.

18 The following parameters were kept constant: $\rho = 0.5$, 30 training s epochs, and synapses were trimmed if their permanence value ever reached or fell below 10^{-4} .

19 The following parameters were used to obtain these results: $m = 861$, $q = 425$, $\rho_d = 32$, $\phi_\delta = 0.0929$, $\rho_c = 147$, $\phi_+ = 0.0078$, $\phi_- = 0.0576$, $\beta_0 = 11$, and $\tau = 35$.

20 The following parameters were used to obtain these results: $m = 784$, $q = 392$, $\rho_d = 2$, $\phi_\delta = 0.5$, $\rho_c = 182$, $\phi_+ = 0.01$, $\phi_- = 0.02$, $\beta_0 = 3$, $\tau = 8$, and one training epoch. The number of columns was set to be equal to the number of inputs to allow for a 1:1 reconstruction of the SDRs.

In a recently published paper, titled "A Mathematical Formalization of Hierarchical Temporal Memory Cortical Learning Algorithms Spatial Pooler", the authors, James Mnatzaganian, Ernest Fokoué, and Dhireesha Kudithipudi of the Rochester Institute of Technology (RIT), present a mathematical model of the Spatial Pooler component of HTM. It is a very interesting paper and worthwhile reading for anyone seriously interested in experimentation with the Spatial Pooler. It is especially interesting to those experimenting with the NuPIC software, since the paper refers to NuPIC for clarification of the theory at several points in the discussion.

The RIT authors create their own description of the Spatial Pooler algorithm and label their parameters differently. I'm quoting from the paper.

Table I describes the SP parameters as named in the paper, and the numbered entries below them are footnotes 17, 18, and 19 from page 9, and footnote 20 from page 10 of the paper, describing the parameters used for processing images from the famous NIST handwritten digit database. The RIT researchers' implementation of the HTM SP parametrization follows closely that of the Numenta NuPIC code except for the RIT parameter q , the number of proximal synapses per column.

The Parameters

NuPIC defines the `potentialRadius` param from which can be derived the number of inputs available to the proximal dendrite of a column, and the `potentialPct` param giving the fraction of the potential inputs that are actually connected. From these values the equivalent of RIT's q parameter can be derived from the formula:

Quoting from the NuPIC SpatialPooler code comments:

At initialization time we choose

$$((2 * \text{potentialRadius} + 1) \wedge \# \text{inputDimensions}) * \text{potentialPct}$$

input bits to comprise the column's potential pool.

The size of column's potential pool is the number of proximal synapses per column. The examples discussed in the RIT paper are 2-dimensional data from the famous NIST handwritten digit database. The relationship between the RIT q and NuPIC's parameters is:

$$q = \text{potentialPct} (1 + 2 \text{potentialRadius})^2$$

Solving for the radius:

$$\text{potentialRadius} = \frac{\sqrt{(\text{potentialPct} q) - \text{potentialPct}}}{2 \text{potentialPct}}$$

The NuPIC example is 1-dimensional data, so:

$$q = \text{potentialPct} (2 \text{potentialRadius} + 1)$$

therefore:

$$\text{potentialRadius} = \frac{q - \text{potentialPct}}{2 \text{potentialPct}}$$

The potentialPct default of 0.5 is mentioned several times in the Numenta documentation and nothing else seems to be used. Simplifying, then, for 1-dimensional data the potentialRadius is essentially equal to q .

The NIST handwritten digits data is 28 pixel square 2-dimensional data. Note 17 refers to $q=353$ and note 19 to $q=425$. Assuming potentialPct=0.5, potentialRadius is about 12.8 and 14.1 respectively, so the potential field is essentially the entire image excluding the border. This is a bit surprising considering statements in the paper questioning the value of global versus local inhibition.

The Citation

J. Mnatzaganian, E. Fokoué, and D. Kudithipudi, "A Mathematical Formalization of Hierarchical Temporal Memory Cortical Learning Algorithm's Spatial Pooler," arXiv preprint arXiv:1601.06116, 2016.