CASE STUDIES OF MATH AT WORK: EXPLORING DESIGN-ORIENTED MATHEMATICAL PRACTICES IN SCHOOL AND WORK SETTINGS

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Final Report to the National Science Foundation

NSF Grant RED–9553648
September 30, 1999

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PROJECT SUMMARY

The Math-at-Work Project was a four year investigation of how mathematical practices are organized and develop in schools and workplaces where people design things (Hall, 1995). Math-at-Work was structured to produce detailed case studies of how mathematics is used in design projects, with an explicit focus on how the work of design is supported by quantitative reasoning and modeling. Studies were conducted to document, analyze, and make explicit aspects of mathematical practice that are seldom unpacked in educational research. These include: (a) developing representations of complex systems, (b) coordinating different representations for the purposes of description, prediction, and decision making, and (c) using these representations and surrounding analyses to justify a particular series of design decisions. Case studies of mathematical activity in middle school classrooms and adult workplaces were also used to investigate, as an empirical matter, widespread calls made for "authentic" or "realistic" mathematics education by educational researchers and policy makers.

Research methods used in these studies combined ethnographic observation (writing field notes, collecting working documents on design projects, interviewing study participants) with close analysis of interaction captured in video/audio recordings of work on design projects. A large corpus of field materials collected in this way served as the basis for an analysis of mathematical activity in classrooms and workplaces, including comparisons across these settings. For classroom research purposes, student design projects were anchored around curriculum units developed with NSF support at the Institute for Research on Learning, Palo Alto, California. The units we adopted (Antarctica, Guppies, and Excursions Projects are now available from Voyager Expanded Learning, http://www.iamvoyager.com/) included task memos directing the activities of student groups, worksheets and supporting case material for the contexts of design problems, and software tools that allowed students to model and investigate structures and processes in architectural design, population biology, and cartography.

During the first three years of the Math-at-Work Project (October, 1995 to September, 1998), we pursued the following activities:

- A team of university researchers (Hall and graduate students) recruited middle school mathematics teachers from three urban public schools (sites we called *Pine*, *Atticus*, and *Hill*);

- We also recruited adult workplaces in specific types of design work (architectural design, population biology, and mapping with geographic information systems);
- We worked with project teachers to study adult design practices in these workplaces (sites we called *JC Architects*, the *BugHouse*, and *HCP Consulting*, respectively;
- We selectively adopted and extended project-based curriculum units in content areas that were thematically similar to the workplaces we were studying; and
- We conducted classroom studies of group design projects by students in grades 7 and 8.

During the fourth year of this project (October, 1998 to September, 1999, granted as a no cost extension), we continued studies of mapping for habitat conservation, analyzed a sizable corpus of ethnographic and interactional data collected across schools and workplaces, and wrote additional papers for the educational research community.

RESEARCH FINDINGS AND DISSEMINATION

Results of these studies have been presented at national and international research conferences, provided as case materials for workshops with teachers and researchers in education and technical design, used as teaching materials for graduate seminars at the University of California, Berkeley (doctoral and credential students), and published as articles in the educational research community. This section lists and discusses our current findings and analyses as they relate to classrooms, workplaces, and a comparison of schools and workplaces. The References section at the end of this document lists research reports and presentations coming out of the Math-at-Work Project. There are two appendices: Appendix A describes our study sites and the corpus of data we have collected, as well as giving more detail on the kinds of mathematical environments we constructed in classrooms or found in workplaces. Appendix B includes excerpts from selected papers or presentations coming out of the Math-at-Work Project.

Findings in Math-at-Work Project Classrooms

Our data collection in classrooms provided several types of evidence for changes in the way that students understood important concepts in design domains and used mathematical tools to ask and answer questions about these domains. To support the most fine-grained level of analysis, we filmed and collected documents (physical and electronic) during the daily classroom work of two or three "focus groups" in each classroom. These focus group records allowed us to follow both the intended and emergent structure of design projects in a longitudinal analysis. In doing this, we were generally able to reconstruct (using field notes, student documents, and close analysis of interaction) student contributions to particular understandings as they changed or stabilized over time. We also filmed and collected documents during whole class discussions and group design reviews, and these data allowed us to compare the work of focus groups with other students in the class. Finally, we interviewed student groups after design projects during our first year of classroom research (Antarctica and Guppies Projects), and we conducted "design challenges" with each group before and after the units taught during our second year of classroom research (Excursions and Guppies Projects). These interviews and design challenges allowed us to characterize how groups' approaches to design problems changed over the course of work on a unit, and to identify particular conceptions (intended or emergent) that we followed more closely within focus groups.

Based on our analysis of student design projects in Math-at-Work classrooms, we report three top level findings:

(1) In each of the curriculum units, we found evidence for changes both in what students understood about the content domain of a design project and the mathematical means through which they asked and answered new questions about these domains (Hall & Stevens, 1995; John, Luporini, & Lyons, 1997; Stevens, 1997; Torralba, 1999);

(2) The trajectories of particular student groups (focus groups) had complex histories that reflected the intended and emergent mathematics of the curriculum, changing and distributed forms of quantitative reasoning and modeling, and different understandings of what counted as mathematics (John, Torralba, & Hall, 1999; John, 1999; Stevens, 1999);

(3) Problems with infrastructure were challenging in each of the schools where we worked, though we found effective strategies for repurposing and augmenting that infrastructure to support student inquiry and learning (Hall, 1997; Hall & Torralba, 1997; Stevens, 1999).

MMAP curriculum units were originally developed to focus on broad NCTM Standards (1989) concerning problem solving, communication, reasoning, and connections, as well as central mathematical concepts of proportional reasoning, functions, and algebra. These were considered critical, transitional ideas for middle school learners. We adopted three units for use in Math-at-Work classrooms, and each stressed these core ideas in different ways (see our description of mathematical environments in Appendix A; also see http://www.irl.org/mmap/curricsoft/default.html for a more detailed description of MMAP curriculum units.).

For example, in the Guppies Project, students were asked to model a population of fish over several years and then to design a tank environment that could accommodate this growing population. Students modeled fish populations using constraint networks (e.g., graphically-linked quantities for population size, birth rate, and death rate), and these quantities were defined by their estimates of birth and death rates as percentages or algebraic expressions. Students then examined the dynamic behavior of fish populations by varying model assumptions and looking for changes in quantitative behavior, using linked, time-based tables and graphs. We extended this unit to include the problem of returning fish populations to a "wild" stream environment, then asked students to model and make predictions about a variety of new conditions in this environment (e.g., upstream pollution, harvesting by rice farmers who use fish to control mosquitoes, and the influence of an exotic predator).

In an analysis of a "design challenge" given before and after this unit (i.e., a shortened task in which student teams were asked to model the relation between mice and cat populations in a grain storage facility), we found coarse-grained evidence that five of seven student groups in a grade 8 classroom implemented and explored the behavior of more elaborate population models on the second challenge. Students' models were more elaborate in the sense that they were better able to coordinate quantities involved in population growth (i.e., birth and death rates, carrying capacity, and time-indexed changes in population size), they explicitly linked populations through a relation of predation (i.e., estimating how many mice are killed by a changing population of cats), and they were better able to evaluate and refine model assumptions in light of observed quantitative behavior.

These coarse-grained contrasts were useful, but they told us little about how students arrived at more sophisticated understandings of content domains (e.g., the carrying capacity of a population in some habitat) or learned how to use mathematical tools to ask and answer questions within these domains (e.g., resolving the shape of a nonlinear graph in terms of assumptions made about the level, time basis, and influence of different rates). A second part of our analysis followed these changes back into the daily records of focus groups we had collected as the units were taught. These longitudinal case studies, in turn, revealed a complex history of student contributions and perspectives during group design projects. Three aspects of these analyses are particularly important.

First, and as documented in other research on teaching and learning classroom mathematics (e.g., Hall & Rubin, 1998; Lampert & Blunk, 1998; and O'Conner, 1998), the intended structure of the curriculum, including teaching decisions made on a daily basis, do not determine and may not always anticipate the emergent structure of students' activities or understandings. In a longitudinal case study of a group of seventh graders working on the Antarctica Project (i.e., students are asked to design and analyze a research station), Stevens (1997, 1999) found that students established divisions of labor around design, computer-based drafting, and calculation. On the positive side, students found mathematically-rich design problems that were not intended by curriculum designers or their teacher (e.g., arguments over the scale and fit of live/work space, or constructing a regular geometric shape with computer-based drawing tools). But on the negative side, these emergent activities and topics were difficult for the classroom teacher to detect, and not all students in the group participated equally in their framing or solution.

A second aspect of our longitudinal analyses concerned what students understood about the history of their own work on a design project as a resource for communicating the results of their work to others. Torralba (1999) found that sharp disagreements over what would be an "interesting" modeling problem in the Guppies Project were critical to the local history of a group's work, but these disagreements (and their resolution in very different models) were not considered particularly relevant for presenting "results" outside the group. Intermediate working documents, through which students were able to translate and refine their understanding of alternative models, were included and discussed only after students were told that these were valuable for understanding how their models evolved and what they had learned. In this sense, students' expectations about using mathematics in design domains were often structured around getting "answers" (i.e., a traditional view of mathematical problem solving), even though the design problems they were asked to solve were intended to be open-ended.

A third aspect of these longitudinal studies concerned how students constructed and understood the generality of their modeling activities, either with respect to other design situations or by comparison with the work of other student groups. John (1999; John, Luporini, & Lyon, 1997) analyzed interactional processes of generalizing that (a) changed the scope of modeled phenomena, (b) fit or tailored abstract procedures or definitions to specific circumstances, and (c) refined or edited explanatory accounts for public presentation. Under this kind of analysis, a "general" argument was the outcome of processes distributed over students and different modeling tools (e.g., narrative accounts and graphical displays provide different resources for generalizing), rather than the object of any individual student's thinking, per se. Particularly important (and related to Torralba's analysis, above), were demands on students' accounts of architectural or population models as their work moved from local to public settings for talk inside the classroom (Hall & Rubin, 1998).

Findings in Math-at-Work Project Workplaces

Data collection in workplaces differed in that we were not providing for the intended structure of design projects, so our methods more closely resembled a growing body of ethnographic research on technical and scientific work (Goodwin, 1994; Lynch & Woolgar, 1990; Star, 1995; Suchman, 1995). After a period of field observation, we arranged with workplace participants to make video and audio recordings of naturally-occurring "chunks of work" (a term we used when negotiating access to workplaces) that made up the activities of design projects. This usually took us across distinctly different settings (e.g., from the forest floor, through multiple laboratories, and into meetings over resultant documents in a study of field entomology), where we made film recordings that closely followed work with representational forms as well as the spoken (or written) contributions of different project participants. After we had collected and annotated these recordings, using notes written in the field and copies of working documents, we conducted interviews with major participants using (in most instances) examples of their mathematical activity from the annotated corpus. Finally, we invited workplace participants to review examples of student design projects, then to visit Math-at-Work classrooms to act as design reviewers or judges during student presentations (these were also filmed). These kinds of data records allowed us to inventory the kinds of mathematics used in different work sites, the kinds of representational technologies that supported quantitative reasoning and modeling, and the kinds of interactions that occurred between student and adult designers during design reviews.

Based on our analysis of project activities in adult workplaces, we report five top level findings:

(1) There are site-specific distributions of physical media, recurrent task structures, and linked participation structures that create environments for mathematical activity in adult design projects (Hall, Torralba & John, 1997; Hall & Stevens, 1995; Stevens, 1997, 1999; also see Appendix A for more details);

(2) People inside these environments work with strongly conventionalized quantities that, as complex historical artifacts, reflect agreements about how to model/represent physical or biological processes and structures (Hall, 1998; Stevens & Hall, 1998);

(3) These agreements sometimes break down, either when people need to communicate across specialties, when the purposes or staffing of a design problem change, or when people with different levels of experience/expertise work together (Hall, 1998; Hall, Stevens & Torralba, in press);

(4) Breakdowns and renegotiated agreements about how to model/represent things create what we call "teaching/learning events" that are consequential for individual participants (Hall and Stevens, 1996; Stevens and Hall, 1998);

(5) As people move across specific environments in the workplace, there are tensions between the need to get work done and the need to create and maintain organizational capacity. These tensions, described by Engestrom (1999) as an "invisible battlefield" for organizational development, sometimes result in disruptions to representational infrastructure that change work environments and relations among participants (Hall, Stevens & Torralba, in press).

Three additional aspects of our analyses were important, particularly around the issue of whether mathematics in design projects at work bears a substantive family resemblance to what happens (or could happen) in school classrooms. First, we found that while quantitative reasoning was strongly conventionalized in the workplace (e.g., types of measurement in architectural design or comparing graphs of abundance in the chemical taxonomy of insects), there were many situations in which people needed to reorganize or to extend conventional ways of measuring or calculating to deal with unforeseen and dynamic aspects of their work (Hall, Torralba, & John, 1998; Stevens, 1999). For example, Torralba (1999) analyzed an argument, contributed in a project meeting by a junior entomologist, that juxtaposed possible sampling errors with new findings about termite foraging behavior. The entomologist supported her argument by rearranging conventional tables of field data values and directing the attention of other project members to striking quantitative differences. Similarly, Stevens (1999) analyzed situations in architectural design where the reference frame for conventional measuring schemes needed to be flexibly extended to capture occluded parts of an existing, historic building. These type of findings corroborate observational studies of less routine types of manufacturing work and clinical medicine (Hoyles, Noss, & Pozzi, 1998; Smith, in press; Steen & Forman, 1995), all supporting the argument that flexible quantitative reasoning is an important and wide-ranging form of mathematics in the workplace.

Second, the interactional structure of workplace "teaching and learning events" (Hall & Stevens, 1996) has been elaborated further by Stevens (1999; Stevens & Hall, 1998) as a grounded model of "disciplining" perception. This model consists of events that are recognizable in the sequential organization of design conversation: (1) one participant recognizes a disparity in perspective or understanding, (2) alerts interactional partners to the disparity, and (3) redirects the attention of others with statements like "look at it this way." These directives are usually accompanied by (4) embodied activity that puts representational media into new forms of coordination, and this activity is often (5) combined with justifications for why the alternative view is better or more productive. Finally (6), recipients of the demonstration may reproduce or rehearse this new way of seeing the situation. This model provides a way to follow what happens when intersubjective understandings between co-workers (or students and teachers) break down and are repaired, as an approach to a microgenetic analysis of learning.

Third, our research in workplaces has underscored the importance of understanding mathematical practices from a historical perspective, paying careful attention to how stable representational infrastructures for quantitative reasoning and modeling are established and change over time (Hall, Stevens, & Torralba, in press). In a comparative analysis of design conversations between people from different disciplinary backgrounds (e.g., field entomology, chemistry, and a biostatistics), we found striking differences in how participants from different disciplines oriented to and acted on the same situation. Some of these differences appeared to reflect the orientations of makers versus users of representational devices, but we also found evidence that participants who might be called users (or non-experts) could selectively unpack or open up devices made by others. We interpreted this selective use and mention of others' orientations to shared situations as evidence that specialists can cross over disciplinary divides in the course of disrupting or changing representational infrastructure (e.g., a specialist in statistics was not in a fixed position outside the representational devices of entomology, or vice versa).

Findings from a Comparison of Classrooms and Workplaces

Our comparison of how mathematics is used in classroom and workplace design projects took a "case-oriented" (Ragin & Becker, 1992) approach, in two senses. First, we investigated the empirical basis of "authenticity" in project-based mathematics teaching, asking how mathematics in classroom design projects was organized in ways that resembled (or did not resemble) what goes on in workplaces. Second, we used comparative research to "tinker" or "experiment" with curriculum materials and classroom environments for learning about mathematics through design (Gravenmeijer, 1994; Brown, 1992), reversing the comparison to take activity structures from the workplace into classrooms. Specifically, we took mathematics in design as a concept to be problematized, unpacked, and re-specified (Lynch, 1991) through comparative analysis of specific empirical cases. We simultaneously conducted and used this comparative analysis over cycles of implementation and study in classrooms (i.e., using case materials from workplaces to extend curriculum units, making comparisons across classrooms for the same unit, refining teaching strategies within or across years). In pursuing this approach, we were simultaneously studying and doing design, using the results of our comparative analysis to change the types of "authenticity" that might be possible in middle school mathematics classrooms.

Based on our comparative analysis and attempts to tinker with classroom structure, we report four top level findings:

(1) Quantitative reasoning that supports modeling physical or biological systems was common across the workplaces we studied, and these mathematical practices closely resembled the intended curricular scope of middle school mathematics (Stevens, 1997, 1999; John, Luporini, and Lyons, 1997);

(2) The design conversations we studied in both schools and workplaces shared common features: participants contributed "explanatory sequences" (Ochs, Taylor, Rudolph, & Smith, 1992) when constructing and comparing design alternatives, they challenged each others' accounts of how to proceed, and they edited or redrafted their contributions accordingly. These exchanges, as basic interactional processes that make and use models, involved quantitative reasoning about time, space, agency, and physical processes (Hall, 1999; Hall, Stevens, & Torralba, in press; John, 1999; Ochs, Jacoby, & Gonzales, 1994);

(3) Modeling, as a basic system of activities in design, involved linked processes of looking *at* representational technologies that implement a model and looking *through* these technologies at the modeled world. This distinction (at/through), usually unproblematic in the workplaces we studied, was a daily struggle in classrooms. These kinds of flexible and purposeful quantitative reasoning are just beginning to be explored in research on mathematics education (e.g., Cobb, Yackel, & McClain, 1999; Greeno & Hall, 1997; Nemirovsky, in press; Noss & Hoyles, 1999);

(4) Stable representational infrastructures existed in both classroom and work settings, though they had different histories and reflected different purposes. Drawing from our analysis of design activity in workplaces, we found several ways to repurpose

and augment existing school infrastructure to better support student design projects (Hall, 1997; Hall & Torralba, 1997; Stevens, 1999).

Two further points are important regarding these findings. First, while there were differences of history, scale, and depth of understanding in how students and professional designers used quantities to model things, these did not appear to be differences in kind. In this sense, we propose (and have demonstrated) that it is possible to tinker with project-based mathematics pedagogy in ways that encourage flexible uses of quantitative reasoning and modeling. Our efforts occupy one region in a larger design space that is being explored in classroom-based educational research, and our comparative studies of workplaces are a critical contribution to this effort. One of the most promising of our experiments was the creation of what we called "hybrid interactional spaces" (Hall & Torralba, 1997; Hall, 1997, Stevens, 1999), where middle schoolers and adult design professionals met each other over student's ongoing design projects. We undertook these reviews as an alternative to working with video case materials from our workplace studies (video segments, descriptions of design activities, and documents/artifacts from adult practice). We initially tried to use these video cases as a way of "seeding" adult representational practices within student activities. But while students found case descriptions and images of adult work interesting, they were not clear (and we had difficulty making it clear) how these would be relevant to their concerns as students, either in terms of the intended curriculum or their own emergent interests.

Project reviews, on the other hand, were activities that were familiar both to students (i.e., group or individual presentations of student work) and and to their adult visitors (i.e., either "design crits" for architects, "poster sessions" for entomologists, or map reviews for cartographers). In this sense, the structure of these reviews as occasions for public talk were familiar to both students and their visitors, and this familiarity included shared systems of representation supporting presentation and review. Much was not shared, however, both in the generative use of these technologies to pursue questions or explain things, and in what constituted appropriate criticisms or questions and reasonable answers. So these reviews were "hybrids" by bringing into juxtaposition understandings of design domains and uses of representational technology that could diverge (i.e., students' versus adults' purposes). Hybrid interactional spaces not only juxtaposed different representational practices (i.e., form-function relations, as described by Saxe, 1991), but they also framed classroom discussions in ways that allowed students, their teachers, and ourselves (as active participants in curriculum design) to recognize innovative or unreasonable design arguments.

Second, while we had some success with this approach to design experiments in project-based teaching, there were also serious challenges. For example, local work space in schools was generic, meaning that students had to empty the space for use by others around an hourly schedule that has existed in U.S. public schools since the turn of the century (Tyack and Tobin, 1994). In contrast, local work spaces in our adult study sites were owned by participants or projects, who could keep and use a material history of their work in a single physical location. Not only did students need to learn to work as a team with (for them) novel systems of representation, but they also needed to assemble and then dismantle the local environment of their work on a project every day. This made the coherence of a project (i.e., design rationales, visual and computational resources that support comparison, and a sense of progress) difficult to maintain in the classroom.

Much of our comparative analysis was directed towards identifying good candidates for repurposing or augmentating existing infrastructures to support learning and doing mathematics in middle school classrooms. This required recognizing the stock of existing representational technologies in school sites (they varied across particular sites), deciding how to reorganize activity so that these existing resources could be used for more advantageous purposes, and finding ways to incorporate new resources that fit or extended students' expecations about what should be going on as they learn mathematics.

Many of these supports were simple matters of practical classroom organization. These included dedicating display spaces to students' in-progress work on a design, providing individual and group folders that could be quickly retrieved without teacher involvement at the start of a class session, and offering an overnight printing service to groups when their work depended on shared use or comparison of physical documents (network functions as simple as printing were weak in every school we studied). Other supports were less obvious. These included organizing peer assessments of technical skills and comparisons of designs in-progress, organizing public discussions around central or benchmark concepts in a project (diSessa and Minstrell, 1998; Orsolini & Pontecorvo, 1992), and inscribing the results of these benchmark discussions into public display spaces that were resources for later work by different groups.

RESEARCH IMPLICATIONS AND FUTURE WORK

Ranging across studies conducted in workplaces and middle school classrooms, we found dense empirical support for rich and complexly-layered mathematical activity in design projects. Much of this mathematical activity bore little resemblance to a mathematics pedagogy consisting of problem sets organized around common procedures. Instead, the similarities between these workplaces and classrooms using project-based curriculum were strong around matters of quantitative reasoning and modeling.

In the workplaces we studied , there were well-established divisions of labor that separated people by seniority and educational cohort (e.g., only junior architects make flexible use of CAD tools in *JC Architects*) and by type of specialization (e.g., chemists versus entomologists or statisticians at the *BugHouse*). These led to strikingly different perspectives on design projects, and while these differences sometimes created obstacles in joint work, they also provided both sources of innovation (Hall, Stevens, & Torralba, in press) and opportunities for learning (Hall & Stevens, 1996; John, Torralba, & Hall, 1999; Stevens & Hall, 1998).

Both in principle and as a matter of practical organization, these divisions of labor and specialization were not initially present in the classrooms that we studied, so rich differences in perspective that might create either obstacles or opportunities for learning were attenuated (Stevens, 1999). In principle, students in these classrooms were expected to participate in design projects (or any other curricular activity) on an equal footing, and while they might be asked to play different roles in project work at different times, their teachers (with our help) and curriculum designers intended that all students have equal access to project activities and opportunities to learn mathematics. As a matter of practical organization, these classrooms (like most other public school classrooms) were composed of same-age children with similar educational backgrounds, and their teachers acted as isolated representatives of the disciplines (mathematics, architecture, biology, cartography) under instruction. These differences in organization reflect very different, historically-specific objectives in institutional settings we call school or work.

The dissimilarities, particularly at the level of work organization, point towards a continuing need to restructure curriculum, teaching, and assessment if we want to expand a view of mathematics as purposeful, representational practices among student learners. For example, even the basic idea of a classroom "discussion" (Lemke, 1990), seemingly familiar in any classroom, is difficult to structure so that different ideas are expressed and compared on open-ended problems. This is true whether the intended "realism" (Hall, 1995) of mathematics teaching encourages learners to make warranted arguments about fundamental mathematical ideas (Ball & Lampert, 1999) or to justify or refine assumptions in models used to examine, evaluate, and choose among design alternatives (the project-based mathematics implemented in this study and offered in MMAP curriculum units).

We are still bringing these findings and case materials to research and professional communities with a stake in mathematics education (e.g., an upcoming NSF-sponsored conference on "Mathematics In and Out of School", organized by Jim Greeno and Shelley Goldman, and a conference on "Children's ways with words in science and mathematics: A conversation across disciplines?", organized by Ann Rosebery and co-sponsored by the National Center for Improving Student Learning and Achievement). The corpus of recordings, working documents, and field observations we have collected can support a variety of further uses and analysis. One future direction is to design, disseminate, and study the use of comparative video cases in teacher professional development, expanding what we have done in courses or workshops on a relatively small scale into a broader network of certification programs that provide pre- and in-service training. Another line of future work is to circulate these materials and our existing analysis across a growing community of cognitive and educational researchers who study conceptual change and learning environments using methods that span different levels of analysis (i.e., ongoing interaction, individual trajectories, and the organization of collective activity). Finally, we hope to extend the idea of "hybrid interactional spaces" (i.e., using adult representatives of discipinary communities as reviewers, consultants, or coaches for student projects) as a way to expand the activities that students, teachers, and researchers recognize as mathematics inside classrooms.

Acknowledgements

Beyond generous support from the National Science Foundation, this four year project would not have been possible without the dedicated work and contributions of many people. Chief among these are the Math-at-Work Project teachers, who travelled further across institutional boundaries than any other participants in these studies (i.e., from their classrooms, into our university lab, then into adult worksites, back into their own classrooms, and finally out into the educational research community). Relations of colleageality and friendship developed during this work, but we were also all challenged by the size of these displacements. On most occasions, we learned from these difficulties, improved our analysis of mathematical practices in design projects, and created better environments for student learning. In addition, the generous contributions of workplace participants at *OutBack*, *JC Architects*, the *BugHouse*, and *HCP Consulting* were, in the end, surprising even by comparison with our initial, naive optimism about gaining access to adult design projects.

This project also reflects the generous contributions of project advisors, program officers, academic colleagues, and family members. As project advisors, Jim Kaput, Shelley Goldman, Lise Dworkin, and Lucy Suchman each listened carefully to our plans, picked out problems and prospects that had not occurred to any of us who were buried in the details of the project, and made very thoughtful suggestions about how to move forward. As our work progressed, we got similarly high quality feedback from colleagues Sharon Derry, Yrjo Engestrom, Charles Goodwin, Jim Greeno, Celia Hoyles, Jean Lave, Rich Lehrer, Jay Lemke, Ray McDermott, Ricardo Nemirovsky, Richard Noss, Susan Newman, Chris Ritter, Mike Rose, Michael Roth, Leona Schauble, Jack Smith, Leigh Star, Beth Warren, and Karen Wieckert. There is no unified perspective here, of course, but we thank them all.

Dick Lesh, Barbara Lovitts, and Eamonn Kelly created an ongoing discussion about research methods in classroom research that was critical for proposing and getting started with this comparative research project. Each later provided solid support as program officers in what was the NSF Division of Research on Teaching and Learning. Finally, we have all subjected our families and intimates to endless conversations about mathematics in and out of school, as well as to concerns about how and whether to continue with this kind of research. Thanks to Anne, Eva, Joe, Jordan, Karen, Ed, and Maren in particular.

References

The follow lists include articles, book chapters, working papers, and presentations coming out of the Math-at-Work Project. See Appendix B for summaries or excerpts from selected papers.

Research Articles, Book Chapters, and Working Papers

Greeno, J. G. & Hall, R. P. (1997, January). Practicing representation: learning with and about representational forms. *Phi Delta Kappan*, 361-367. To be reprinted in the *Handbook of Research in Educational Psychology*, Allyn and Bacon, Publishers.

Hall, R. (in press). Video recording as theory. In D. Lesh & A. Kelley (Eds.) *Handbook of Methods for Research in Learning and Teaching Science and Mathematics*. Mahweh, NJ: Lawrence Erlbaum.

Hall, R. (1999). The organization and development of discursive practices for "having a theory". *Discourse Processes, 27*(2), 187-218.

Hall, R. (1998). Following mathematical practices in design-oriented work. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum, Volume 10, Studies in Mathematics Education Series* (pp. 29-47). London: Falmer Press.

Hall, R. (1998). A case study of making and using representations of quantity in architectural design. Math-at-Work Project Working Paper. University of California, Berkeley.

Hall, R. (1995) Exploring design-oriented mathematical practices in school and work settings. *Communications of the ACM*, September, p. 62.

Hall, R. and Goldstein, B. (1999). Category boundaries and their quantitative consequences: Looking at/through representational technologies for mapping habitats and conserving endangered species. Math-at-Work Project Working Paper. University of California, Berkeley.

Hall, R. and Stevens, R. (1996). Teaching/learning events in the workplace: a comparative analysis of their organizational and interactional structure. (pp. 160-165) In *Proceedings of the Eighteenth Annual Conference of the Cognitive Science Society*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Hall, R. and Stevens, R. (1995) Making space: a comparison of mathematical work in school and professional design practices. (pp. 118–145) S.L. Star (Ed.) *The cultures of computing*. London: Basil Blackwell.

Hall, R., Stevens, R. & Torralba, A. (in press). Disrupting representational infrastructure in conversations across disciplines. In Derry, S. & M. A. Gernsbacker (Eds.), *Problems and promises of interdisciplinary collaboration: A cognitive science perspective*. Mahweh, NJ: Lawrence Erlbaum.

Hall, R., Torralba, T., and John, S. (1997). "That's a good comparing" Experimentation, contingency, and discretion among scientists working on the chemical taxonomy of insects. Math-at-Work Project Working Paper. University of California, Berkeley. This paper was also presented in B. Warren and R. Hall (Organizers) *Contingency in scientific work: cases from the classroom and the laboratory*. Symposium at the American Educational Research Association, San Diego, CA.

John, S. (1999). Generalizing in context: A case study of mathematical generalizing in a middle school mathematics classroom. Math-at-Work Project Working Paper, Graduate School of Education, University of California, Berkeley.

John, S., Torralba, T., & Hall, R. (1999). Comparing logics of participation: Working at the boundary between errors and findings in classroom and professional biology. Math-at-Work Project Working Paper. University of California, Berkelely. This paper was also presented M. Roth & R. Hall (Panel Chairs), "Of lizards, ants, and arthropods: Scientists' and middle school students' representations of nature." Panel at the American Educational Research Association Annual Meetings, Montreal, Canada, April, 1999.

Schwartzburg, A. and Stevens, R. (1996) *Making space: a collaborative investigation of embodied knowledge*. Hand-crafted book, exhibited in “Science Imagined”, Berkeley Arts Center, Berkeley, CA. October 27- December 29. Catalog. Hayden Press.

Stevens, R. (1997, December). Using the division of labor concept to compare computer-supported collaborative work and learning across settings. In R. Hall & N. Miyake (Eds.) *Proceedings of CSCL '97: The Second International Conference on Computer Support for Collaborative Learning*. December 10-14, 1997.

University of Toronto/Ontario Institute for Studies in Education (OISE). Mahweh, NJ: Lawrence Erlbaum and Associates. A fuller version of this paper will appear in T. Koschmann, R. Hall, & N. Miyake (Eds.) *CSCL II: Continuing the conversation*. Mahweh, NJ: Lawrence Erlbaum and Associates.

Stevens, R. (1999). *Disciplined perception: Comparing the development of embodied mathematical practices in school and at work*. Dissertation, Graduate School of Education, University of California, Berkeley.

Stevens, R. and Hall, R. (1998). Disciplined perception: learning to see in technoscience. In M. Lampert and M. Blunk (Eds.) *Talking mathematics in school: Studies of teaching and learning (pp. 107-149)*. New York: Cambridge University Press.

Stevens, R. & Hall, R. (1997). Seeing the Tornado: how VideoTraces mediate visitor understandings of (natural?) spectacles in a science museum. *Science Education, 81*(6), 735–747.

Torralba, J. A. (1999). Local translations: Biological representational forms and practices in middle school classrooms and professional laboratories. Math-at-Work Project Working Paper, Graduate School of Education, University of California, Berkeley.

Presentations at Conferences or Workshops

Hall, R. (1998, November). How to follow learning in the coordinated assembly of representational states. T. Koschmann (Chair) Six readings of a single text: a videoanalytic session (pp. 407-409). In S. Poltrock and J. Grudin (Eds.) Proceedings of Computer Supported Cooperative Work (CSCW '98). New York, NY: The Association for Computing Machinery.

Hall, R. (1997, October). Aligning images of design/mathematical practices across schools and workplaces. Paper presented in a panel on "Crossing Boundaries" (S. Newman, Organizer) Annual meetings of the Society for the Social Studies of Science, Tucson, Arizona, October, 1997.

Hall, R. (1996, November). Following mathematical practices in design oriented work. Talk in the Cognition and Instruction Symposium Series, University of Wisconsin-Madison.

Hall, R. (1996, August). Mathematical practices in school and work settings. Invited talk and presentation of case materials in a conference on "Workplace Simulations." Learning Research and Development Center, University of Pittsburgh, Pittsburgh, PA.

Hall, R. (1995). Exploring mathematical practices in design-oriented work. Video case and accompanying documentation. Discussion piece for an AERA session on "Assessing Cases of Algebraic Reasoning." San Francisco, CA.

Hall, R. (1995). Assembling representations across talk, action, and inscription. Talk at the Cognitive Science Colloquium Series, University of California, Berkeley.

Hall, R., John, S., Torralba, T., and Danielson, S. (1998). Cases of conceptual change as the reorganization of representational practices in classrooms. Presentation to Researchers/Developers in the Middle School Mathematics through Applications Project. Institute for Research on Learning, Palo Alto, CA.

Hall, R. and Smith, J. (1997). What can studies of mathematics in the workplace tell us about teaching and learning mathematics in school? Symposium at the American Educational Research Association Annual Meetings, Chicago, IL.

Hall, R. and Stevens, R. (1996, November). Talking across disciplinary boundaries in design-oriented work. Symposium on "Understanding interdisciplinary teamwork: challenges for research and practice." National Institute for Science Education, University of Wisconsin-Madison.

Hall, R. and Stevens, R (1996). The BugHouse: sorting insects across disciplinary boundaries. Talk in a symposium on "Conversations and representational media: accomplishing shared meaning." Annual meetings of the American Educational Research Association, New York City.

Hall, R. and Stevens, R (1996). Assembling mathematical competence in school and work settings. Talk in a symposium on "Development through work: activity, artifacts, and persons." Annual meetings of the American Educational Research Association, New York City.

Hall, R., Stevens, R. and Torralba, T. (1997). Cases of quantitative reasoning and modeling in design-oriented work. Presentation at a Summer Workshop for Teacher/Developers in the Middle School Mathematics through Applications Project. Institute for Research on Learning, Palo Alto, CA.

Hall, R., Stevens, R, and Torralba, A. (1995). Case studies of math at work: exploring design-oriented mathematical practices in school and work settings. Talk given at the Institute for Research on Learning (MMAP) and the Xerox Palo Alto Research Center (WPT).

Hall, R. and Torralba, T. (1997). Bringing images of design-oriented work into middle school mathematics classrooms. Talk in a panel: What can studies of mathematics in the workplace tell us about teaching and learning mathematics in school? Symposium at the American Educational Research Association Annual Meetings, Chicago, IL.

John, S., Luporini, D., and Lyon, L. (1997). Making, interpreting, and animating graphical representations in an architectural design project. Talk in a panel: What can studies of mathematics in the workplace tell us about teaching and learning mathematics in school? Symposium at the American Educational Research Association Annual Meetings, Chicago, IL.

Roth, W. and Hall, R. (1999, April). Of lizards, ants, and arthropods: Scientists' and middle school students' representations of nature. Panel at the American Educational Research Association Annual Meetings, Montreal, Canada.

Stevens, R. (1997). What does STS *have* to say about learning? Paper presented at the Annual Meetings of the Society for the Social Studies of Science, October, 1997, Tucson, AZ.

Stevens, R. (1997) Disciplined perception: Examples of ways that architects' visual practices are mathematical. Talk in a panel: What can studies of mathematics in the workplace tell us about teaching and learning mathematics in school? Symposium at the American Educational Research Association Annual Meetings, Chicago, IL.

Stevens, R. and Hall, R. (1996, October). Seeing a tornado: the organization of (natural?) spectacles in a science museum. Talk in a panel on "Science as Spectacle" (M. Lynch, organizer). Annual meetings of the Society for the Social Studies of Science, Bielefeld, Germany.

Stevens, R. and Hall, R. (1996, October) Disciplined perception: learning to see in technoscience. Talk in a panel on "Ethnographies of Visual Practices" (A. Cambrosio, organizer). Annual meetings of the Society for the Social Studies of Science, Bielefeld, Germany.

Torralba, J. A. (1999a). (a) Survey of audio/video equipment in educational ethnographic work; (b) What are useful strategies for indexing and annotating?; (c) How can you follow teaching and learning outside the classroom?" Presentations at a Workshop on Collecting and Analyzing Video and Audio Records. Center for the Integrated Study of Teaching and Learning (CISTL), University of California at Berkeley. Graduate School of Education. February, 1999.

Torralba, J. A. (1999b). An archeology of biological activity in a grade 8 mathematics classroom. Paper presented at the American Educational Research Association meeting. Montreal, Canada. April, 1999.

Torralba, J. A., John, S. A. (1998b). The interactional work of group work: How students use talk to develop mathematical understanding. Paper presented at the American Educational Research Association (AERA) meeting. San Diego, CA. April 1998.

Related Publications (cited in Project Summary)

Ball, D. L., and Lampert, M. (1999). Multiples of evidence, time, and perspective: Revising the study of teaching and learning. In E. C. Lagemann and L.S. Shulman (Eds.) *Issues in Education Research: Problems and possibilities*. San Francisco: Jossey-Bass Publishers.

Brown, A. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences, 2*(2), 141-178.

Cobb, P., Yackel, E., and McClain, K. (1999). *Symbolizing and communicating: Perspectives on mathematical discourse, tools, and instructional design*. Mahweh, NJ: Lawrence Erlbaum and Associates, Inc.

diSessa, A. A. & Mistrell, J. (1998). Cultivating Conceptual Change with Benchmark Lessons. In J.G. Greeno and S.V. Golman (Eds.), *Thinking practices in mathematics and science* (pp. 155-187). Mahweh, NJ: Lawrence Erlbaum and Associates.

Engestrom, Y. (1999b). Expansive visibilization at work: an activity-theoretical perspective. *Computer Supported Cooperative Work 8*, 63-93.

Goodwin, C. (1994). Professional vision. *American Anthropologist, 96*(3), 606-633.

Gravemeijer, K. E. P. (1994). *Developing realist mathematics education*. Utrecht, The Netherlands: CD-Beta Press.

Hall, R. (1996) Representation as shared activity: Situated cognition and Dewey's cartography of experience. *The Journal of the Learning Sciences 5(3)*, 209-238.

Hall, R. (1995) Realism(s) for learning algebra. In C. B. LaCampagne, W. Blair, and J. Kaput (Eds.), *The Algebra Initiative Colloquium.* (Volume 2, pp. 33–51) Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement.

Hall, R., Knudsen, J., and Greeno, J.G. (1996) A case study of systemic attributes of assessment technologies. *Educational Assessment 3*(4), 315-361.

Hoyles, C, Noss, R. & Pozzi, S. (1998). Mathematizing in practice. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum, Volume 10, Studies in Mathematics Education Series* (pp. 48-62). London: Falmer Press.

Lampert, M. (1985). Mathematics learning in context: "The Voyage of the Mimi". *The Journal of Mathematical Behavior,* 4, 157-168.

Lampert, M. & Blunk, M. (1998), *Talkiing mathematics: Studies of teaching and learning in school* . New York: Cambridge University Press.

Lemke, J. L. (1990). *Talking science : Language, learning, and values.* Norwood, N J: Ablex.

Lynch, M. (1991). Method: measurement – ordinary and scientific measurement as ethnomethodological phenomena. In G. Button (Ed.), *Ethnomethodology and the human sciences* (pp. 77-108). Cambridge: Cambridge University Press.

Lynch, M. & Woolgar, S. (1990). *Representation and scientific practice*. Cambridge, MA: The MIT Press.

National Council of Teachers of Mathematics (NCTM) (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nemirovsky, R. (in press). How one experience becomes part of another. In K. Beach (Ed.), Special issue of *The Journal of the Learning Sciences.*

Noss, R. & Hoyles, C. (1999). Rethinking expertise: A study of proportional reasoning in the workplace. Manuscript.

Ochs, E., Jacoby, S., & Gonzales, P. (1994). Interpretive journeys: How physicists talk and travel through graphic space. *Configurations, 1*, 151-171.

Ochs, E., Taylor, C., Rudolph, D., & Smith, R. (1992). Storytelling as a theory-building activity. *Discourse Processes, 15*, 37-72.

O'Conner, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. In M. Lampert & M. Blunk (Eds.), *Talkiing mathematics: Studies of teaching and learning in school* (pp. 17-55). New York: Cambridge University Press.

Orsolini, M. & Pontecorvo, C. (1992). Children's talk in classroom discussions. *Cognition and Instruction,* 9(2), 113-136.

Ragin, C. & Becker, H. (1992). *What is a case? Exploring the foundations of social inquiry*. Cambridge: Cambridge University Press.

Saxe, G. B. (1991). *Culture and cognitive development: studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum and Associates.

Smith, J. (in press). Tracking the mathematics of automobile production: Are schools failing to prepare students for work? *American Journal of Educational Research*.

Star, S. L. (1995). *Ecologies of knowledge: Work and politics in science and technology* . Albany, NY: State University of New York Press.

Steen, L. A., & Forman, S. L. (1995). Mathematics for work and life. In I. M. Carl (Ed.), *Prospectsfor school mathematics: seventy five years of progress*. Reston, VA: National Council of Teachers of Mathematics.

Suchman, L. (September, 1995) Making work visible (pp. 56-64). In L. Suchman (Ed.) *Representations of work*. Special issue of *Communications of the ACM 38*(9).

Tyack, D. & Tobin, W. (1994). The "grammar" of schooling: Why has it been so hard to change? *American Educational Research Journal, 31*(3), 453-479.