25 - 10 - 2021

Indirect Methods.

1. Transformation Techniques.

If the constraints $g_j(\underset{\sim}{X})$ have simple forms as an explicit fn. of x_i

1. If lower & upper bounds on x_i are given by

$$l_i \leq x_i \leq u_i$$

$$\rightarrow x_i = l_i + (u_i - l_i) \sin^2 y_i$$

$y_i \rightarrow$ new design variable which can take any real value.

- **Penalty Methods:**

These penalty methods transform the constrained optimization problem into a sequential unconstrained optimization problem.

$$\boxed{\begin{array}{c}\text{Find } \underset{\sim}{X} \text{ which minimizes } f(\underset{\sim}{X}) \\ \text{subject to } g_j(\underset{\sim}{X}) \leq 0, \ j = 1, 2, \cdots m\end{array}}$$

$$\downarrow$$

$$\phi_k = \phi(\underset{\sim}{X}, r_k) = f(\underset{\sim}{X}) + r_k \sum_{j=1}^{m} G_j[g_j(\underset{\sim}{X})]$$

G_j is some function of $g_j(\underset{\sim}{X})$ & r_k is a positive constant referred to as penalty parameter.

- Interior penalty Methods
- Exterior penalty Methods.

$$G_j = -\frac{1}{g_j(\underset{\sim}{X})}$$
$$= \log\left[-g_j(\underset{\sim}{X})\right]$$

} Interior.

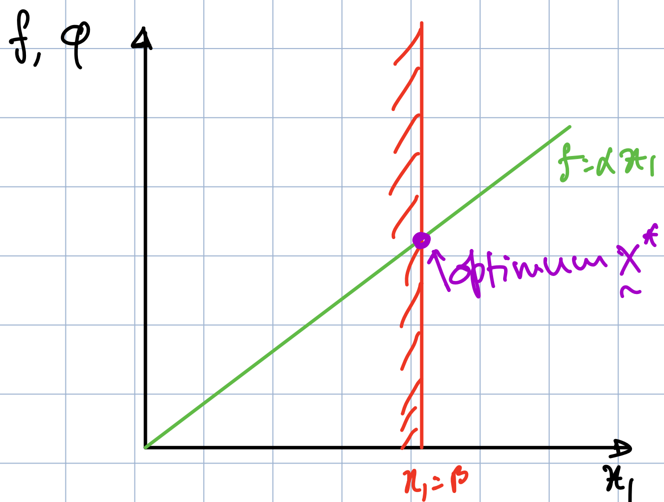
$$G_j = \text{Max}\left[0,\ g_j(\underset{\sim}{X})\right]$$
$$r = \left(\text{Max}\left\{0, g_j(\underset{\sim}{x})\right\}\right)^2$$

} Exterior penalty

Find $\underset{\sim}{X} = x_1$ which minimizes $f(\underset{\sim}{X}) = \alpha x_1$

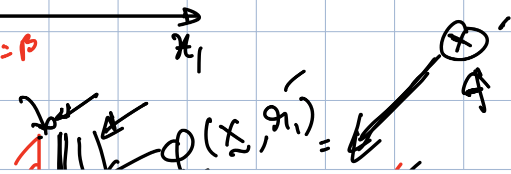
subject to $g_1(\underset{\sim}{X}) = \beta - x_1 \leq 0$

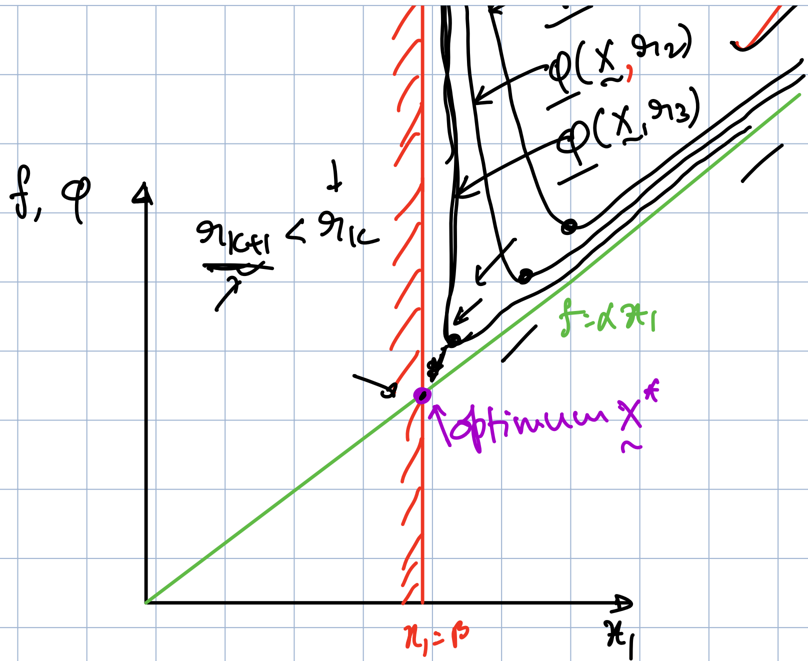
$$G_j^{\text{interior}} = -\frac{1}{\beta - x_1} \qquad G_j^{\text{ext}} = \left\{\max\left(0, (\beta - x_1)\right)\right\}^2$$



$$\varphi_1 = \varphi(\underset{\sim}{X}, r_1)$$
$$\varphi_2 = \varphi(\underset{\sim}{X}, r_2)$$
$$\varphi_3 = \varphi(\underset{\sim}{X}, r_3)$$

$$\boxed{\varphi_k = \alpha x_1 - \frac{r_k}{\beta - x_1}}$$

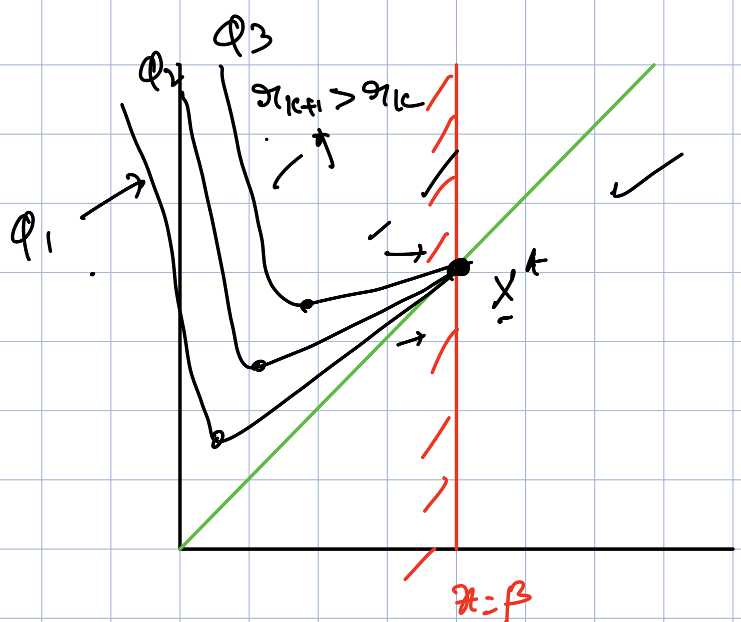




$\varphi_k = \varphi(\underset{\sim}{X}, r_k)$

as $k \to \infty$

$\varphi_k^* \to f^*$



- Automobile companies to adopt to BS 7 by 2025. ←

(39)

Interior penalty function Method.

$$\phi(\underset{\sim}{X}, r_k) = f(\underset{\sim}{X}) - r_k \sum_{j=1}^{m} \frac{1}{g_j(\underset{\sim}{X})}$$

If $\underset{\sim}{X}$ is in the interior of the feasible, $g_j(\underset{\sim}{X}) < 0$

$\therefore \phi(\underset{\sim}{X}, r_k) > f(\underset{\sim}{X})$

If any constraint is critically satisfied.

$g_j(\underset{\sim}{X}) = 0 \Rightarrow \phi \to \infty$

Algorithm:

1. Start with an initial feasible point $\underset{\sim}{X}_1$ that satisfies all the constraints.
   choose $r_1 > 0$ & set $k=1$
2. Minimize $\phi(\underset{\sim}{X}, r_k)$ by using any UCM methods. $\to \underset{\sim}{X}_k^*$
3. Test if X_k^* is the optimum solution for the original problem, i.e. Min $f(\underset{\sim}{X})$
   If yes, $\longrightarrow$ terminate
   If no, $\longrightarrow$ go to step 4
4. Find the value of the next penalty parameter r_{k+1} as $r_{k+1} = c\, r_k$ $c < 1$
5. Set $k = k+1$, $\underset{\sim}{X}_1 = X_k^*$ & go to step 2.

Issues:

1. The starting feasible point $\underset{\sim}{X}_1$ may not be readily available
2. A suitable value for the initial penalty parameter r_1 has to be found.
3. A proper value for c has to be found.
4. Suitable convergence criterion has to be decided.
5. It's important to normalize the constraints wherever possible.

2. Choosing $\underline{r_1}$

$$\boxed{r_1 \approx 0.1 \text{ to } 1.0 \; \frac{f(\underset{\sim}{X}_1)}{-\sum_{j=1}^{m} \frac{1}{g_j(\underset{\sim}{X}_1)}}}$$

3. $r_{k+1} = \underline{c}\, r_k \qquad \underline{\underline{c < 1}}$

$c = 0.1, \; 0.2, \text{ or } 0.5$

$g_j(\underset{\sim}{x}) \leq 0$

$g_j(\underset{\sim}{x}) < 0$

interior of the feasible region

4.

141

4. Convergence

$$\left| \frac{f(\underset{\sim}{x}_k^*) - f(x_{k-1}^*)}{f(x_k^*)} \right| \leq \varepsilon_1$$

$$\left| \underset{\sim}{x}_k^* - \underset{\sim}{x}_{k-1}^* \right| \leq \varepsilon_2$$

$$\max \left(\underset{\sim}{x}_k^* - \underset{\sim}{x}_{k-1}^* \right) \leq \varepsilon_3$$

5. Normalization:

$$g_1(\underset{\sim}{x}) = \delta(\underset{\sim}{x}) \leq \delta_{max} \quad 10^{-4}$$

$$g_2(\underset{\sim}{x}) : \sigma(\underset{\sim}{x}) \leq \sigma_{max} \rightarrow 10^6$$

$$g_1(\underset{\sim}{x}) : \delta(\underset{\sim}{x}) - \delta_{max} \leq 0$$

$$g_2(\underset{\sim}{x}) : \sigma(\underset{\sim}{x}) - \sigma_{max} \leq 0$$

$$\boxed{\begin{aligned} \frac{g_1}{\delta_{max}} &: \frac{\delta(x)}{\delta_{max}} - 1 \leq 0 \\ \frac{g_2}{\sigma_{max}} &: \frac{\sigma(\underset{\sim}{x})}{\sigma_{max}} - 1 \leq 0 \end{aligned}}$$