Non-linear programming - constrained optimization

Find $\underset{\sim}{X}$ that minimizes $f(\underset{\sim}{X})$

subject to

$$g_j(\underset{\sim}{X}) \leq 0 \qquad j = 1, 2, \cdots, m$$

$$h_k(\underset{\sim}{X}) = 0 \qquad k = 1, 2, \cdots, p$$

- Direct
  - → constraints are handled explicitly
  - Random search
  - Heuristic search (complex)
  - objective & constraint approximation
    - sequential linear programming
    - sequential quadratic programming
  - Method of feasible directions
    - Zoutendijk's method
    - Rosen's gradient projection
  - ? Generalized reduced gradient method

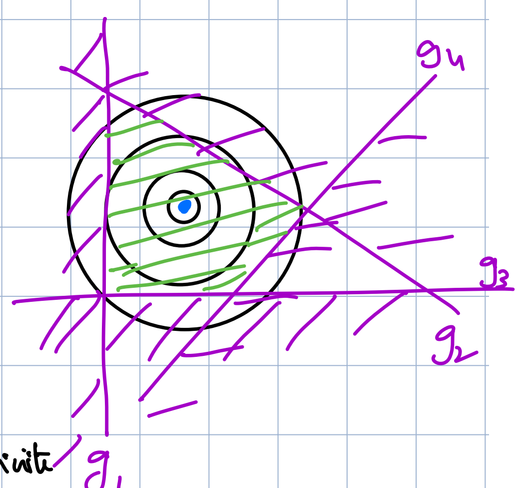
- Indirect
  - series of unconstrained minimization methods
  - Transformation of variables technique
  - sequential UCM
    - Interior penalty
    - Exterior penalty
    - ? Augmented Lagrange Multiplier method.

Key characteristics of a COP

1. The constraints do not have any effect on the optimum.

$$\nabla f \Big|_{\underset{\sim}{X}^*} = 0$$

$$H \Big|_{\underset{\sim}{X}^*} = \left[\frac{\partial^2 f}{\partial x_i \partial x_j}\right]_{\underset{\sim}{X}^*} \rightarrow \text{+ve definite}$$

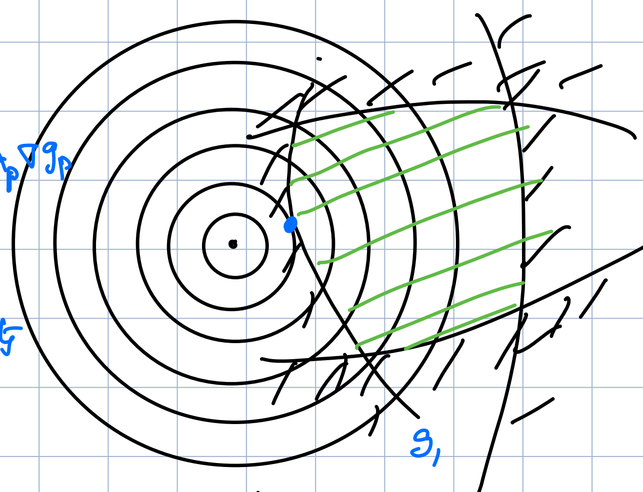


2. The optimum (unique) solution occurs on a constraint boundary

$$-\nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2 + \cdots + \lambda_p \nabla g_p$$
$$= \sum_{j=1}^{p} \lambda_j \nabla g_j$$

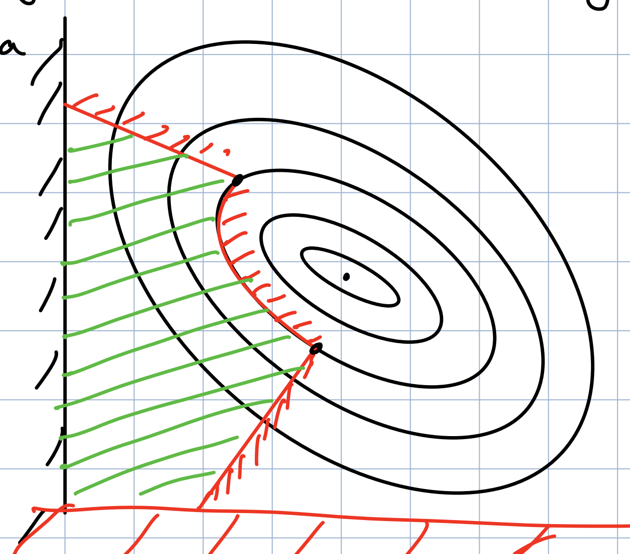
$j = 1, p \in$ active constraints

$$-\nabla f = \lambda_1 \nabla g_1$$



3. If the UCMP has more than one local minima, then the CMP may have multiple minima.

4. Even If the UCMP has only one minima the CMP may have multiple minima.



Direct Methods

1. **Random search Methods.**

- Generate a trial design vector using a random number for each design variable.
- verify whether the constraints are satisfied or not. For equality constraints choose a tolerance.
- If the trial vector satisfies all the constraints, compare that with the previously generated valid solution. $f(\underset{\sim}{X}_{i+1}) \le f(\underset{\sim}{X}_i)$ $\quad \underset{\sim}{X}_{i+1} \leftarrow \underset{\sim}{X}^*$

Minimize

$$F(\underset{\sim}{X}) = f(\underset{\sim}{X}) + a \sum_{j=1}^{m} G_j(\underset{\sim}{X})^2 + b \sum_{k=1}^{p} H_k(\underset{\sim}{X})^2$$

$$[G_j(\underset{\sim}{X})]^2 = \underline{\underline{\max}}\,[0,\ g_j(\underset{\sim}{X})]^2$$

$$g_j(\underset{\sim}{X}) \le 0$$

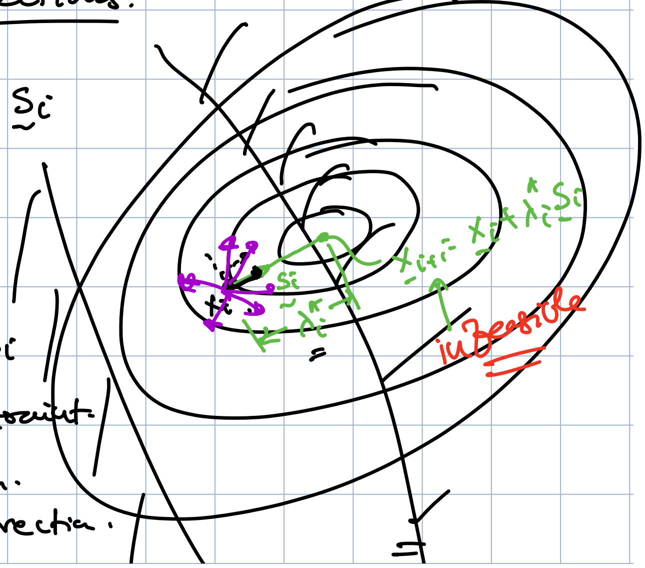
Method of feasible directions:

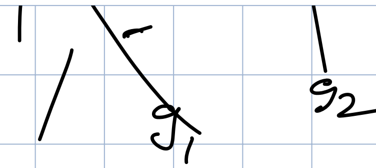
$$\underset{\sim}{X}_{i+1} = \underset{\sim}{X}_i + \lambda_i \underset{\sim}{S}_i$$

The value of λ_i is chosen such that

(1) a small movement along $\underset{\sim}{S}_i$ does not violate any constraint.

(2) the value of the objective fn. can be reduced in that direction.





In otherwords.

→ the direction should be feasible

→ the direction should be usable.

1) → feasible direction

1)&2) → usable feasible direction -

A direction $\underset{\sim}{S}$ is feasible at a point $\underset{\sim}{X}_i$ if

$$\frac{d}{d\lambda} g_j(\underset{\sim}{X}_i + \lambda \underset{\sim}{S}) \Big|_{\lambda=0} = \underset{\sim}{S}^T \nabla g_j(\underset{\sim}{X}_i) \leq 0$$

For a direction $\underset{\sim}{S}$ to be usable feasible direction, it should satisfy

$$\underset{\sim}{S}^T \nabla f(\underset{\sim}{X}_i) < 0$$

$$\underset{\sim}{S}^T \nabla g_j(\underset{\sim}{X}_i) \leq 0$$