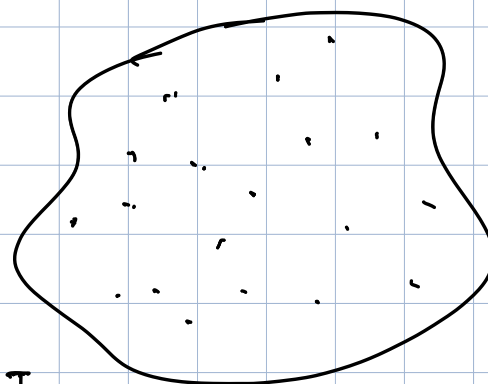
24-Sep-2021

(88)

Direct Search Methods : No gradient is computed

Zeroth - order Methods.

- Random search
  - → Random Jump ✓
  - → Random walk. ✓



Random Jump:

$$\underset{\sim}{X} = \{x_1, x_2, \ldots x_n\}^T$$

1) Establish bounds on the design variables.

$$l_i \leq x_i \leq u_i \qquad i = 1, 2, \cdots n$$

2) generate a set of 'n' random numbers that are uniformly spaced & between 0 & 1

$$\underset{\sim}{r} = \{r_1, r_2, \ldots r_n\}^T \quad 0 \leq r_i \leq 1$$

$$\underset{\sim}{X} = \begin{Bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} l_1 + r_1 (u_1 - l_1) \\ \vdots \\ \vdots \\ l_n + r_n (u_n - l_n) \end{Bmatrix}$$

3) Evaluate $f(\underset{\sim}{X})$ at $\underset{\sim}{X}$

4) Do the same for several random numbers

5) find the smallest $f(X)$ & designate $\underset{\sim}{X}^*$

Random walk method:

$$\boxed{\underset{\sim}{X}_{i+1} = \underset{\sim}{X}_i + \lambda\, \underset{\sim}{u}_i}$$

serach direction

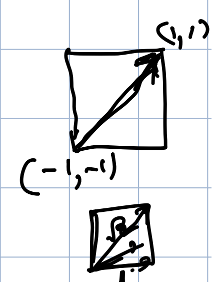
1. Start with an initial guess point $\underset{\sim}{X}_1$, sufficiently large initial step length (λ) a minimum allowable step length ε & maximum number of iterations N.

   $\underset{\sim}{X}_1, \quad \lambda, \quad \varepsilon, \quad N$

2. Find $f_1 = f(\underset{\sim}{X}_1)$
3. Set iteration counte $i = 1$
4. Generate a set of n random numbers $[-1, 1]$ & formulate $\underset{\sim}{u}$

$$\underset{\sim}{u} = \frac{1}{(r_1^2 + r_2^2 + \dots + r_n^2)^{1/2}} \begin{Bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{Bmatrix}$$

To avoid bias towards the diagonals of the unit hypercube,



$R = \|\underset{\sim}{r}\|$

accept $\underset{\sim}{u}$ only if $R \leq 1$ & discard if $R > 1$

5. Compute $\underset{\sim}{X} = \underset{\sim}{X}_1 + \lambda \underset{\sim}{u}$ & $f(\underset{\sim}{X})$
6. Compare $f(\underset{\sim}{X})$ with f_1

7. If $f < f_1$, set $\underset{\sim}{X}_1 = \underset{\sim}{X}$ & $f_1 = f$ & go to step 3.

If $f \geq f_1$ go to next step.

8. If $i \leq N$, $i = i+1$ & go to step 4.

if $i > N$, go to next step

9. Reduce step length $\lambda = \lambda/2$ If $\lambda \leq \varepsilon$ go to next step; If $\lambda > \varepsilon$ go to step 4

10. Stop the procedure. $\underset{\sim}{X}^* = \underset{\sim}{X}_1$ & $f^* = f_1$

Random walk with direction exploitation.

$f(x) = x_1^2 + x_2^2 - 2x_1x_2 - x_1 - x_2$

$\underset{\sim}{X}_{i+1} = \underset{\sim}{X}_i + \lambda u_i$ ← Random walk

$\boxed{\underset{\sim}{X}_{i+1} = \underset{\sim}{X}_i + \lambda_i^* \underset{\sim}{u}_i}$ direction exploitation.

$X_i = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ $\quad \underset{\sim}{X}_{i+1} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \lambda_1 \begin{Bmatrix} 0.5 \\ -0.5 \end{Bmatrix}$

$u_i = \begin{Bmatrix} 4\sqrt{2} \\ -4\sqrt{2} \end{Bmatrix}$ $\quad \underset{\approx}{X_2} = \begin{Bmatrix} 1 + 0.5\lambda_1 \\ 2 - 0.5\lambda_1 \end{Bmatrix}$

$$f(X_1) = 1 + 4 - 2\times2\times1 - 1 - 2$$
$$= 5 - 4 - 1 - 2$$
$$= -2$$

$$f(\underset{\sim}{X}_2) = (1 + 0.5\lambda_1)^2 + (2 - 0.5\lambda_1)^2 - 2(1 + 0.5\lambda_1)(2 - 0.5\lambda_1) - 1 - 0.5\lambda_1 - 2 + 0.5\lambda_1$$

$$\rightarrow f(\lambda_1) =$$

$$\frac{df}{d\lambda_1} = 0 \Rightarrow \lambda_1^*$$

$$\underset{\sim}{X}_2 = \underset{\sim}{X}_1 + \lambda_1^* \underset{\sim}{u}_1$$

If $f(\underset{\sim}{X}_2) < f(\underset{\sim}{X}_1)$

$$\underset{\sim}{X}_3 = X_2 + \lambda_2^* \underset{\sim}{u}_2$$

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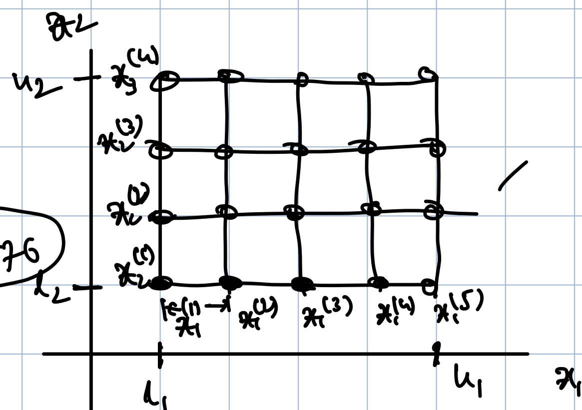
Grid Search Method:

$n = 10$

$p_i = 4$

grid points : $4^{10} = 1048576$



$n = 2$ $\quad 4^2 = 16$

$p_i = 4$

Univariate Method:

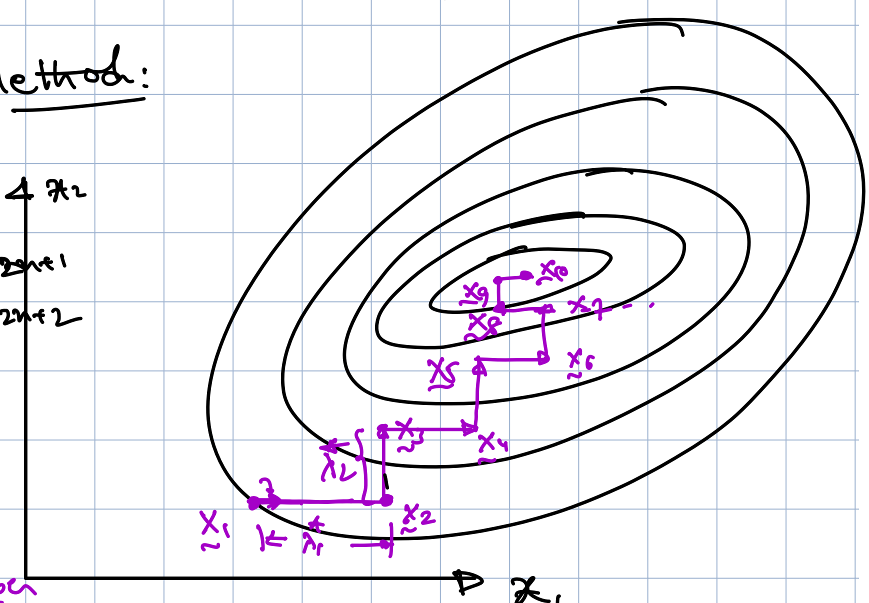
$$\underset{\sim}{X}_{i+1} = \underset{\sim}{X}_i + \lambda_i^* \underset{\sim}{S}_i$$

$$\underset{\sim}{S}_i = \begin{cases} [1 \;\; 0]^T & i = 1, n+1, \cdots 2n+1 \\ \text{or } [0 \;\; 1]^T & = 2, n+2, \cdots 2n+2 \end{cases}$$

$$f_1^+ = f(\underset{\sim}{X}_1 + \varepsilon \underset{\sim}{S}_1)$$

$$f_1^- = f(\underset{\sim}{X}_1 - \varepsilon \underset{\sim}{S}_1)$$

ε a small number



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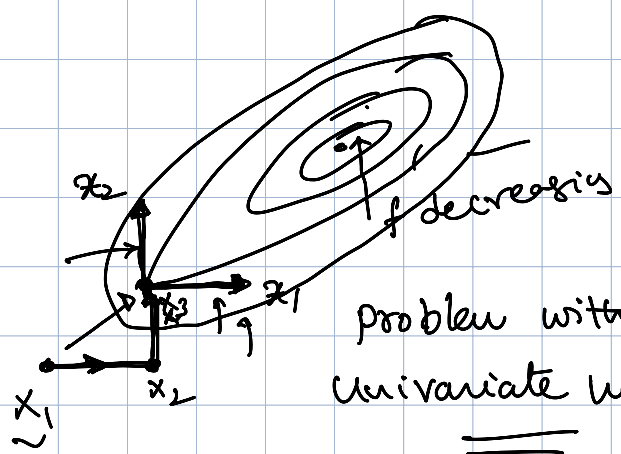
$$\underset{\sim}{S_i} = \begin{cases} (1, 0, \cdots 0) & \text{for } i = 1, n+1, 2n+1 \cdots \\ (0, 1, \cdots 0) & \text{for } i = 2, n+2, 2n+2 \cdots \\ \vdots & \vdots \\ (0, 0, \cdots 1) & \text{for } i = n, 2n, 3n, \cdots \end{cases}$$



problem with univariate method.