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Feedback Stabilization and Tracking of Constrained Robots

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Abstract—**Mathematical models for constrained robot dynamics, incorporating the effects of constraint forces required to maintain satisfaction of the constraints, are used to develop explicit conditions for stabilization and tracking using feedback. The control structure allows feedback of generalized robot displacements, velocities, and the constraint forces. Global conditions for tracking based on a modified computed torque controller and local conditions for feedback stabilization using a linear controller are presented. The framework is also used to investigate the closed-loop properties if there are force disturbances, dynamics in the force feedback loops, or uncertainty in the constraint functions.**

I. Introduction and Motivation

CURRENT industrial robots are characterized by a wide diversity of physical design configurations. However, the basic task capabilities of most current robots are quite limited. The most common tasks involve so-called "pick and place" operations, which characterize the vast majority of current robot applications. If robot technology is to have a more wide-ranging impact on industrial practice, it is essential that the task capabilities of robots be substantially expanded.

A robot can be viewed as a physical mechanism for performing work; the mechanism is often defined as a connection of articulated links constructed so that the end of the last link (which may include a gripper holding an object or a tool holder containing a tool) is the location at which the work is performed. It is natural to focus on the so-called end effector of the robot and to define particular robot tasks in terms of desired motions of the end effector; this is the common view in dealing with "pick and place" operations. However, there are many industrial tasks (most of which cannot be automated using current robots) that are defined in a fundamentally different way. In particular, there are numerous tasks which cannot be defined solely in terms of motion of the end effector. Of specific interest in this paper are tasks which are characterized by physical contact between the end effector and a constraint surface. A long list of such tasks can be given, including scribing, writing, deburring, grinding, and others [1]–[7].

There have been numerous research publications which have dealt with such applications. Although the primary focus of such research has not often been on the role of constraints in defining the tasks, research on compliant control [8]–[11] and force feedback control [12]–[19] are closely related. Several formal control design approaches have been proposed and there have been descriptions of related robot experiments. However, there has been no theoretical framework established which can serve as a basis for the study of robot performance of constrained tasks.

It is the premise of this paper that there is a need for a carefully developed theoretical framework for investigation into application of robots to such tasks. Furthermore, it is our premise that such a theoretical framework should explicitly incorporate the effects of the forces of constraint. Note that the constraint forces are not exogenous but are implicitly defined as the forces required to maintain satisfaction of the constraints. Specifically, we present a dynamic model of a robot, incorporating constraint effects; the form of the model follows from classical results in dynamics [20]–[21] and has recently been recognized as the proper theoretical model for constrained robot problems [22]–[28].

This model is used in this paper to develop theoretical conditions for closed-loop stabilization and tracking using a class of feedback controllers. The constraints are assumed to be nonlinear and the development is based on a coordinate transformation for which the constraints are expressed in a simple form. Global tracking conditions are developed for the case of nonlinear dynamics using a modification of the computed torque method. Local stabilization conditions are also developed using a linear controller. The results are also presented for the case of linear constraints, for simplicity. Such stabilization conditions are new; they serve to provide a theoretical basis for the use of force feedback in constrained systems such as has been suggested in [16]. The constrained model studied in this paper can also form the basis for a rigorous analysis of the closed-loop properties that arise from the use of other control approaches described in the literature [8]–[19].

We further investigate the properties of the closed loop using the proposed controller structure; we show that "high gain displacement feedback loops" reduce the steady-state displacement regulation error for constant force disturbances and uncertainty in the constraint function. We also show that "high gain force feedback loops" reduce the steady-state constraint force regulation error for constant force disturbances and uncertainty in the constraint function. Further, closed-loop stabilization is shown not to be affected by a certain class of dynamics in the constraint force feedback loops.

II. Formulation of Constrained Dynamic Equations

Our development is based on a Lagrangian formulation of robot dynamics, in a coordinate system convenient for characterizing the robot motion. Let $q \in R^n$ denote the vector of generalized displacements, in robot coordinates. If $q: R^1 \to R^n$ is differentiable, then $\dot{q}$ denotes its time derivative; $q \in R^n$ is viewed as a column vector so that its transpose q' is a row vector. We assume the existence of a symmetric, positive definite matrix valued inertia function $M: R^n \to R^{n \times n}$, and a scalar valued potential function $V: R^n \to R^1$, such that the equations of motion of the robot are defined in terms of the Lagrangian function $L(q, \dot{q}) = 0.5\, \dot{q}' M(q)\, \dot{q} - V(q)$ as

$$\frac{d \partial L}{dt \partial \dot{q}}(q, \dot{q}) - \frac{\partial L}{\partial q}(q, \dot{q}) = f + u$$

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where $f \in R^n$ denotes the vector of generalized constraint forces in robot coordinates, and $u \in R^n$ denotes the vector of other generalized force inputs, which includes the control input, disturbance inputs, and any dissipative effects, in robot coordinates. Thus, the equations of motion for the constrained robot, in robot coordinates, are given by

$$M(q)\ddot{q}+F(q, \dot{q})=f+u \tag{1}$$

where

$$F(q, \dot{q})=\frac{d}{dt}[M(q)]\dot{q}-0.5\frac{\partial \dot{q}M(q)\dot{q}}{\partial q}+\frac{\partial V(q)}{\partial q}.$$

Let $p \in R^n$ denote the generalized position vector of the robot end effector, in coordinates in which constraints on the end effector are defined. These (environmental) constraints are such that the generalized position vector of the robot end effector is assumed to satisfy the algebraic equation $\theta(p) = 0$ where the constraint function $\theta: R^n \to R^m$. We assume that the generalized position vector of the robot end effector, in constraint coordinates, can be expressed in terms of the generalized displacement in robot coordinates according to the algebraic equation $p = H(q)$, where the mapping $H: R^n \to R^n$ is invertible. Our subsequent development does not assume that the inverse kinematic relations be expressible in closed form. Thus, the constraint function defined by $\phi(q) = \theta(H(q))$, in robot coordinates, satisfies the constraint equation

$$\phi(q)=0. \tag{2}$$

Since the constraints are holonomic it follows, as shown in [20], [21], that the generalized constraint forces, in robot coordinates, are given by the relation

$$f=J'(q)\lambda \tag{3}$$

where $\lambda \in R^m$ is a vector of generalized multipliers associated with the constraints, and the Jacobian matrix $J(q) = \partial\phi(q)/\partial q$. The constraint force on the end effector, i.e., the contact force between the end effector and the constraint surface, can be expressed, in the constraint coordinates, in terms of the constraint multiplier vector λ.

Note that if $\phi(q) = 0$ is identically satisfied, then also $J(q)\dot{q} = 0$. Thus, the constraint manifold S in R^{2n} defined by $S = \{(q, \dot{q}): \phi(q) = 0, J(q)\dot{q} = 0\}$ is fundamental in the development. In particular, if conditions given in [25]–[26] are satisfied, then there is a unique solution of (1)–(3), denoted by $q(t)$ satisfying $(q(t), \dot{q}(t)) \in S$, for each $(q(0), \dot{q}(0)) \in S$. That is, S is an invariant manifold. Thus, the constraints on the robot can be viewed as restricting the dynamics to the manifold S only rather than to the space R^{2n}; in other words the model described by (1)–(3) is singular on R^{2n} as described in [25]. It is this feature that is critical in defining the constrained robot dynamics; this is also the fundamental source of difficulty in analysis and control of the robot dynamics.

We again emphasize the important role of the constraints in the constrained dynamics, especially with relation to the stabilization problem. In particular, it is easy to see that a closed-loop system that is asymptotically stable if constraints are ignored may, in fact, not be asymptotically stable if the constraints are imposed. A simple theoretical example has been presented in [29] which demonstrates this possibility. In addition, robot experiments have also indicated that a closed loop may be destabilized when a hard constraint (infinitely stiff environment) is imposed on the end effector [30].

III. Transformation of Constrained Dynamic Equations

In order to carry out our subsequent development, we assume that the constraint function satisfies the following.

Assumption 1: There is an open set $\Theta \subset R^{n-m}$ and a function $\Omega: \Theta \lessgtr R^m$ such that

$$\phi(\Omega(q_2), q_2)=0 \qquad \text{for all } q_2 \in \Theta.$$

Suppose a constant vector $\bar{q} \in R^n$ satisfies $\phi(q) = 0$; Assumption 1 holds in some neighborhood of $\bar{q}$ if rank $J(\bar{q}) = m$, according to the implicit function theorem, although a reordering of the variables may be required. Let Assumption 1 hold with $\Theta = R^{n-m}$. A transformation is now made so that the constraint equations are written in a simple form. First define the vector partition $q' = (q_1', q_2')$, where $q_1 \in R^m$, $q_2 \in R^{n-m}$. Define the nonlinear transformation $X: R^n \to R^n$ by

$$x=X(q)=\begin{vmatrix} q_1-\Omega(q_2) \\ q_2 \end{vmatrix}$$

which is differentiable and has a differentiable inverse transformation $Q: R^n \to R^n$ given by

$$q=Q(x)=\begin{vmatrix} x_1+\Omega(x_2) \\ x_2 \end{vmatrix}$$

where the vector partition $x' = (x_1', x_2')$, $x_1 \in R^m$, $x_2 \in R^{n-m}$, is used. We also define the Jacobian matrix of the inverse transformation

$$T(x)=\frac{\partial Q}{\partial x}(x)=\begin{vmatrix} I_m & \frac{\partial \Omega}{\partial x_2}(x_2) \\ 0 & I_{n-m} \end{vmatrix}$$

which is necessarily nonsingular. By abuse of notation we often write $T(x_2)$ or $T(q)$ in place of $T(x)$. The differential equations (1) can be expressed in terms of the variables x as

$$T'(x)M(Q(x))T(x)\ddot{x}+T'(x)\{F(Q(x), T(x)\dot{x}) + M(Q(x))\dot{T}(x)\dot{x}\}=T'(x)u+T'(x)f.$$

For simplicity, introduce the function definitions $\bar{M}(x) = T'(x)M(Q(x))T(x)$ and $\bar{F}(x, \dot{x}) = T'(x)\{F(Q(x), T(x)\dot{x}) + M(Q(x))\dot{T}(x)\dot{x}\}$ and introduce the partitioning of the identity matrix $I_n = [E_1' \mid E_2']$ where E_1 is an $m \times n$ matrix and E_2 is an $(n - m) \times n$ matrix. These equations can be written in a so-called reduced form as

$$E_1\bar{M}(x_2)E_2'\ddot{x}_2+E_1\bar{F}(x_2, \dot{x}_2)=E_1T'(x_2)u+E_1T'(x_2)f \tag{4}$$

$$E_2\bar{M}(x_2)E_2'\ddot{x}_2+E_2\bar{F}(x_2, \dot{x}_2)=E_2T'(x_2)u \tag{5}$$

where we note that in (5), $E_2T'(x_2)f = 0$ follows from (3) and Assumption 1. The notation $F(x_2, \dot{x}_2)$ denotes $F(x, \dot{x})$ evaluated at $x' = (0, x_2')$, $\dot{x} = (0, \dot{x}_2')$, etc. In the transformed coordinates the constraint equation is

$$x_1=0 \tag{6}$$

and the constraint force satisfies

$$f=J'(x_2)\lambda. \tag{7}$$

This reduced form has a useful interpretation that forms the basis for our subsequent development. *The ordinary differential equation (5) characterizes the motion of the robot on the constraint manifold; equation (4) can be viewed as an algebraic equation for the constraint force expressed in terms of the motion on the constraint manifold. It is this structure that is crucial for our approach.*

An important special case occurs if the constraints are linear; it can then be shown that the above development can be carried out explicitly in terms of a singular value decomposition. We make the following assumption.

Assumption 2: Assume that $\phi(q) = Jq$ where J is a constant $m \times n$ matrix with rank of J being m.

Then it follows that there is a singular value decomposition of J such that $J = U\Sigma V'$ where U and V are $m \times m$ and $n \times n$ orthogonal matrices, respectively, and $\Sigma = [D \vdots 0]$, $D = \text{diag}\,[d_1, \cdots, d_m]$ with $d_i > 0$, $i = 1, \cdots, m$ being the singular values of J.

Now the previous development holds with $Q(x) = Vx$, $X(q) = V'q$, and with $T(x) = V$. Our development is considerably simplified in this case; consequently specific results are subsequently presented for the case of linear constraints.

IV. Global Tracking Using Nonlinear Feedback

In this section, we consider a general tracking problem for constrained robots. For simplicity, we formulate the tracking problem in terms of robot coordinates. The desired motion and desired constraint forces, in robot coordinates, are defined by vector functions $q_d: R^1 \to R^n$, $f_d: R^1 \to R^n$. For consistency with the imposed constraints, it is necessary that $\phi(q_d) = 0$ and $f_d = J'(q_d)\lambda_d$, identically, for some multiplier function $\lambda_d: R^1 \to R^m$. Note that λ_d also defines the desired contact force on the end effector in the constraint coordinates.

Our objective is to determine a nonlinear feedback controller to solve the following tracking problem. A feedback control u, depending on the tracking functions q_d, $\dot{q}_d$, $\ddot{q}_d$, f_d, and feedback of the generalized displacement q, the generalized velocity $\dot{q}$, and the generalized constraint force f, is to be selected so that for all $(q(0), \dot{q}(0)) \in S$ it follows that the closed-loop responses satisfy

$$q(t) \to q_d(t) \quad \text{as } t \to \infty$$

$$f(t) \to f_d(t) \quad \text{as } t \to \infty.$$

We consider a version of the computed torque controller [12], [31], modified to accommodate the presence of the constraints, feedback of the constrained motion and the constraint forces, and the simultaneous motion and force tracking objectives. As indicated previously, our development is based on application of the computed torque controller concept to the reduced equations (4)–(7). The controller is chosen so that

$$\begin{aligned} E_1 T'(x_2)u = {} & E_1 \bar{M}(x_2) E_2' \ddot{x}_{2d} - E_1 T'(x_2) J'(x_2)\lambda_d + E_1 \bar{F}(x_2, \dot{x}_2) \\ & + E_1 \bar{M}(x_2) E_2' [G_v(\dot{x}_{2d} - \dot{x}_2) + G_d(x_{2d} - x_2)] \\ & + E_1 E_1' G_f E_1 T'(x_2) J'(x_2)(\lambda - \lambda_d) \end{aligned}$$

$$\begin{aligned} E_2 T'(x_2)u = {} & E_2 \bar{M}(x_2) E_2' \ddot{x}_{2d} - E_2 T'(x_2) J'(x_2)\lambda_d + E_2 \bar{F}(x_2, \dot{x}_2) \\ & + E_2 \bar{M}(x_2) E_2' [G_v(\dot{x}_{2d} - \dot{x}_2) + G_d(x_{2d} - x_2)] \\ & + E_2 E_1' G_f E_1 T'(x_2) J'(x_2)(\lambda - \lambda_d) \end{aligned}$$

where G_v and G_d are $(n-m) \times (n-m)$ constant feedback gain matrices and G_f is an $m \times m$ constant feedback gain matrix. The controller can be expressed in terms of the variables in robot coordinates as

$$\begin{aligned} u = {} & M(q)T(q)T(q_d)^{-1}\ddot{q}_d - J'(q)\lambda_d + F(q, \dot{q}) \\ & + M(q)\{\dot{T}(q)T(q)^{-1}\dot{q} - T(q)T(q_d)^{-1}\dot{T}(q_d)^{-1}\dot{q}_d\} \\ & + M(q)\{T(q)E_2' G_v E_2 [T(q_d)^{-1}\dot{q}_d - T(q)^{-1}\dot{q}] \\ & + T(q)E_2' G_d E_2 [X(q_d) - X(q)]\} \\ & + T'(q)^{-1} E_1' G_f E_1 T'(q) J'(q)(\lambda - \lambda_d). \end{aligned} \tag{8}$$

The controller can also be expressed in terms of the constraint force and the desired constraint force as

$$\begin{aligned} u = {} & M(q)T(q)T(q_d)^{-1}\ddot{q}_d - J'(q)[J(q_d)J'(q_d)]^{-1}J(q_d)f_d \\ & + F(q, \dot{q}) + M(q)\{\dot{T}(q)T(q)^{-1}\dot{q} - T(q)T(q_d)^{-1}\dot{T}(q_d) \\ & \cdot T(q_d)^{-1}\dot{q}_d\} + M(q)\{T(q)E_2' G_v E_2 [T(q_d)^{-1}\dot{q}_d - T(q)^{-1}\dot{q}] \\ & + T(q)E_2' G_d E_2 [X(q_d) - X(q)]\} + T'(q)^{-1}E_1' G_f E_1 T'(q) \\ & \cdot \{f - J'(q)[J(q_d)J'(q_d)]^{-1}J(q_d)f_d\}. \end{aligned} \tag{9}$$

Using the relations $E_2 T'(x_2) J'(x_2) = 0$, $E_1 E_1' = I_m$, and $E_2 E_1' = 0$, the closed-loop equations, in reduced form, can be shown to be given by the linear equations

$$E_1 \bar{M}(x_2) E_2' \{\ddot{e}_2 + G_v \dot{e}_2 + G_d e_2\} = (I_m + G_f) E_1 T'(x_2) J'(x_2)(\lambda - \lambda_d) \tag{10}$$

$$E_2 \bar{M}(x_2) E_2' \{\ddot{e}_2 + G_v \dot{e}_2 + G_d e_2\} = 0 \tag{11}$$

$$e_1 = 0 \tag{12}$$

where $e_2 = x_2 - E_2 X(q_d)$, $e_1 = x_1$.

Conditions on the gain matrices so that the tracking problem is solved are readily obtained from these equations, since (11) is an ordinary differential equation which characterizes the motion on the constraint manifold. The matrix gains G_v and G_d can be selected so that, according to (11), $e_2 \to 0$, and hence $q \to q_d$ as $t \to \infty$. Then from (10) it follows that $\lambda \to \lambda_d$, and hence $f \to f_d$ as $t \to \infty$.

Using this framework, the following result is obtained.

Theorem 1: Suppose that Assumption 1 is satisfied with $\Theta = R^{n-m}$. The closed-loop system defined by the plant equations (1)–(3) and the controller (9) is globally asymptotically stable in the sense that

$$q(t) \to q_d(t) \quad \text{as } t \to \infty$$

$$f(t) \to f_d(t) \quad \text{as } t \to \infty$$

for any $(q(0), \dot{q}(0)) \in S$ if G_v and G_d are symmetric and positive definite and G_f is symmetric and nonnegative definite.

We now consider the simpler case where the constraints are linear. In such case the feedback controller (9) can be written as

$$\begin{aligned} u = {} & M(q)\ddot{q}_d - f_d + F(q, \dot{q}) + M(q)[VE_2' G_v E_2 V'(\dot{q}_d - \dot{q}) \\ & + VE_2' G_d E_2 V'(q_d - q)] + VE_1' G_f E_1 V'(f - f_d) \end{aligned} \tag{13}$$

where G_v and G_d are $(n-m) \times (n-m)$ constant feedback gain matrices and G_f is an $m \times m$ constant feedback gain matrix. Conditions so that the closed-loop system solves the tracking problem in this case are as follows.

Corollary 2: Suppose that Assumption 2 is satisfied. The closed-loop system defined by (1)–(3) and (13) is globally asymptotically stable in the sense that

$$q(t) \to q_d(t) \quad \text{as } t \to \infty$$

$$f(t) \to f_d(t) \quad \text{as } t \to \infty$$

for any $(q(0), \dot{q}(0)) \in S$ if G_v and G_d are symmetric and positive definite and G_f is symmetric and nonnegative definite.

It is important to note that the dependence of the controller on the constraint force in expressions (9) and (13) is crucial; in particular, the second term in the control expression in (13), namely the desired tracking constraint force $-f_d$, cannot be replaced by the feedback constraint force $-f$, because the closed-loop system in such case is ill-posed. Such an incorrect approach has, in fact, been proposed in the literature. One proper form for

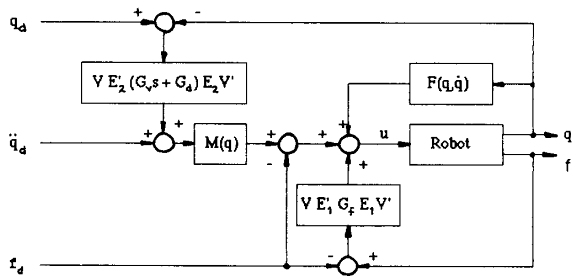

Fig. 1. Closed loop with modified computed torque controller.

introducing feedback of the constraint force is through the controller expressions given by (9) or (13).

Note that the closed loop can be stabilized even if $G_f = 0$; feedback of the constraint force is not required for stabilization. But, as we subsequently indicate, there are potential robustness advantages in using feedback of the constraint force.

The control relations (9) and (13) suggest that feedback of the generalized displacement q, the generalized velocity $\dot{q}$, and the generalized constraint force f, in robot coordinates, is required; but (9) and (13) could be modified to depend on feedback of displacement and velocity of the end effector and feedback of the contact force on the end effector, in the constraint coordinates. Of course, the control expressions (9) and (13) could also be modified to depend on feedback of the motion on the constraint manifold, e.g., through feedback of q_2 and $\dot{q}_2$.

A schematic diagram of the closed-loop system, indicating the control structure (13), is shown in Fig. 1, for the case where the constraints are linear. The viewpoint of the controller as a computed torque controller, modified to conform to the constraints, and depending on feedback of q, $\dot{q}$, and f is clarified from that figure. It should be noted that the control structure (13) is a generalization of the hybrid control architecture presented by Raibert and Craig in [16]; the specific motion and force selection matrices introduced in [16] correspond to the specific case that $\phi(q) = q_1$ in our notation. In addition, we have presented conditions guaranteeing that the closed loop, subject to the imposed constraints, is asymptotically stable, for the general case.

V. Local Stabilization Using Linear Feedback

In this section, we consider a regulation problem where desired *constant* regulation vectors $q_d \in R^n$, $f_d \in R^n$ are given. For consistency with the imposed constraints, it is necessary that $\phi(q_d) = 0$ and $f_d = J'(q_d)\lambda_d$, identically, for some constant multiplier vector λ_d. Our objective is to determine a linear feedback controller to solve the following regulation problem. A feedback control u, depending on the constant regulation vectors q_d and f_d, and feedback of the generalized displacement q, the generalized velocity $\dot{q}$, and the generalized constraint force f, is to be selected so that there is a neighborhood N of $(q_d, 0)$ in R^{2n} such that for all $(q(0), \dot{q}(0)) \in S \cap N$ it follows that the closed-loop responses satisfy

$$q(t) \to q_d \quad \text{as } t \to \infty$$

$$f(t) \to f_d \quad \text{as } t \to \infty.$$

We now consider a linear controller, with feedback of the constrained motion and the constraint forces, that accommodates the presence of the constraints and the simultaneous motion and force regulation objectives. As indicated previously, our development is based on application of linear control concepts to the reduced equations (4)–(7). The controller is chosen to be a linear feedback function of the indicated form and to satisfy the following:

$$\begin{aligned}
&E_1 T'(x_{2d})[u - F(q_d, 0) + f_d] \\
&\quad = E_1 E_2' G_v(\dot{x}_{2d} - \dot{x}_2) + E_1 E_2' G_d(x_{2d} - x_2) \\
&\qquad + E_1 E_1' G_f E_1 T'(x_{2d}) J'(x_{2d})(\lambda - \lambda_d) \\
&E_2 T'(x_{2d})[u - F(q_d, 0) + f_d] \\
&\quad = E_2 E_2' G_v(\dot{x}_{2d} - \dot{x}_2) + E_2 E_2' G_d(x_{2d} - x_2) \\
&\qquad + E_2 E_1' G_f E_1 T'(x_{2d}) J'(x_{2d})(\lambda - \lambda_d)
\end{aligned}$$

where G_v and G_d are $(n - m) \times (n - m)$ constant feedback gain matrices and G_f is an $m \times m$ constant feedback gain matrix. This controller can be expressed in terms of the variables in robot coordinates as

$$\begin{aligned}
u = &F(q_d, 0) - f_d + T'(q_d)^{-1} E_2' G_v E_2 T(q_d)^{-1}(\dot{q}_d - \dot{q}) \\
&+ T'(q_d)^{-1} E_2' G_d E_2 T(q_d)^{-1}(q_d - q) \\
&+ T'(q_d)^{-1} E_1' G_f E_1 T'(q_d) J'(q_d)(\lambda - \lambda_d).
\end{aligned} \tag{14}$$

The controller can also be expressed in terms of the difference of the constraint force and the desired constraint force; the difference $(\lambda - \lambda_d)$ can be expressed in terms of $(q_d - q)$ and $(f - f_d)$ using (3), to first order. The result can be substituted into (14) to obtain

$$\begin{aligned}
u = &F(q_d, 0) - f_d + T'(q_d)^{-1} E_2' G_v E_2 T(q_d)^{-1}(\dot{q}_d - \dot{q}) \\
&+ T'(q_d)^{-1} E_2' G_d E_2 T(q_d)^{-1}(q_d - q) \\
&+ T'(q_d)^{-1} E_1' G_f E_1 T'(q_d)[(f - f_d) \\
&- \left.\frac{\partial (J'(q)\lambda_d)}{\partial q}\right|_{q = q_d} T(q_d) E_2' E_2 T(q_d)^{-1}(q - q_d)].
\end{aligned} \tag{15}$$

Using the relations $E_1 E_2' = 0$, $E_1 E_1' = I_m$, $E_2 E_2' = I_{n-m}$, and $E_2 E_1' = 0$, the linearized closed-loop equations, in reduced form, can be shown to be given by

$$E_1 \bar{M}(x_2) E_2' \ddot{e}_2 + E_1 \bar{K}(x_{2d}) E_2' e_2 = (I_m + G_f) E_1 T'(x_{2d}) J'(x_{2d})(\lambda - \lambda_d) \tag{16}$$

$$E_2 \bar{M}(x_{2d}) E_2' \ddot{e}_2 + G_v \dot{e}_2 + [G_d + E_2 \bar{K}(x_{2d}) E_2'] e_2 = 0 \tag{17}$$

$$e_1 = 0 \tag{18}$$

where

$$\bar{K}(x_{2d}) = \frac{\partial}{\partial x}\{T'(x)[F(Q(x), T(x)\dot{x}) - F(Q(0, x_{2d}), 0) + J'(x_{2d})\lambda_d - J'(x)\lambda_d]\}|_{x = (0, x_{2d}), \dot{x} = (0, 0)}.$$

Conditions on the gain matrices so that the regulation problem is solved are readily obtained using the ordinary differential equation (17) which characterizes the linearized motion on the constraint manifold. The matrix gains G_v and G_d can be selected so that, according to (17), $e_2 \to 0$, and hence $q \to q_d$ as $t \to \infty$ hold locally. Then from (16) it follows that $\lambda \to \lambda_d$, and hence $f \to f_d$ as $t \to \infty$ hold locally.

The following result is obtained.

Theorem 3: Suppose that Assumption 1 is satisfied in some neighborhood of q_d. The closed-loop system defined by the plant equations (1)–(3) and controller (15) is locally asymptotically stable in the sense that there is a neighborhood N of $(q_d, 0)$ such that

$$q(t) \to q_d \quad \text{as } t \to \infty$$

$$f(t) \to f_d \quad \text{as } t \to \infty$$

for any $(q(0), \dot{q}(0)) \in S \cap N$ if G_v and G_d are symmetric and positive definite such that all $2(n - m)$ zeros of

$$\det \{E_2 T'(q_d)[M(q_d)s^2 + K(q_d)] T(q_d) E_2' + G_v s + G_d\}$$

have negative real parts and G_f is symmetric and nonnegative definite where

$$K(q_d) = T'(q_d)^{-1} \frac{\partial}{\partial q} \{T'(q)[F(q, \dot{q}) - F(q_d, 0) + J'(q_d)\lambda_d - J'(q)\lambda_d]\}|_{q=q_d, \dot{q}=0}.$$

We now consider the simpler case where the constraints are linear. In such case the linear feedback controller (15) can be written as

$$u = F(q_d, 0) - f_d - VE_2' G_v E_2 V' \dot{q} + VE_2' G_d E_2 V'(q_d - q) + VE_1' G_f E_1 V'(f - f_d) \quad (19)$$

where G_v and G_d are $(n - m) \times (n - m)$ constant feedback gain matrices and G_f is an $m \times m$ constant feedback gain matrix. Conditions so that the closed-loop system solves the regulation problem are as follows.

Corollary 4: Suppose that Assumption 2 is satisfied. The closed-loop system defined by the plant equations (1)–(3) and controller (19) is locally asymptotically stable in the sense that there is a neighborhood N of $(q_d, 0)$ such that

$$q(t) \to q_d \quad \text{as } t \to \infty$$

$$f(t) \to f_d \quad \text{as } t \to \infty$$

for any $(q(0), \dot{q}(0)) \in S \cap N$ if G_v and G_d are symmetric and positive definite such that all $2(n - m)$ zeros of

$$\det \{E_2 V'[M(q_d)s^2 + K(q_d)] VE_2' + G_v s + G_d\}$$

have negative real parts and G_f is symmetric and nonnegative definite and

$$K(q_d) = \left.\frac{\partial F(q, \dot{q})}{\partial q}\right|_{q=q_d, \dot{q}=0}.$$

Again we mention that the control relations (15) and (19) have been written in terms of feedback of the motion and constraint forces in robot coordinates, but they could be modified to depend on the motion of the end effector and the contact force on the end effector in constraint coordinates.

A schematic diagram of the closed-loop system, indicating the control structure (19), is shown in Fig. 2, for the case where the constraints are linear. The controller consists of a constant bias input plus linear terms proportional to the generalized displacement error, the generalized velocity error, and the generalized constraint force error, all in robot coordinates.

VI. Consequences of Model Imperfections

We have developed conditions which indicate how feedback controllers can be developed so that the closed-loop is asymptotically stable. In this case stability is defined as robustness to changes in the initial data that are consistent with satisfaction of the imposed constraints. As has been demonstrated, feedback can be used to improve the closed-loop properties with respect to such uncertainties.

But feedback can be expected to play a further important role in possibly reducing the effects of disturbances and model imperfections. In this section we briefly consider the implications of using feedback in the case that there are external force disturbances, additional dynamics in the constraint force feedback loops, and uncertainties in the constraints. To avoid unnecessary complica-

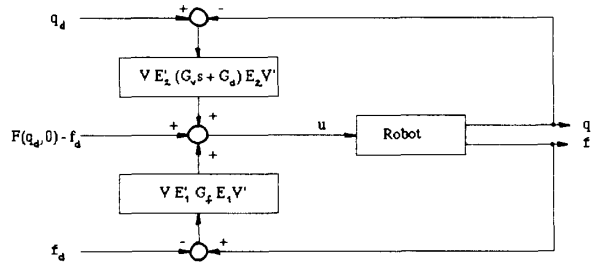

Fig. 2. Closed loop with linear controller.

tions, our developments are based on the linearized equations, assuming that the constraints are linear, i.e., that Assumption 2 holds. Thus, our conclusions are approximations only valid near an equilibrium. Nevertheless, the qualitative features of our conclusions are of substantial importance as they suggest the general implications of the feedback control structure studied. Our conclusions about the effects of model uncertainties are limited; but the indicated linearized equations could form the basis for a more detailed study.

Effects of Force Disturbances

Suppose there is an external force disturbance so that the constrained system is described by

$$M(q)\ddot{q} + F(q, \dot{q}) = u + f + \tau$$

$$Jq = 0$$

$$f = J'\lambda$$

where τ represents the n-vector force disturbance. Recall that constant vectors q_d and f_d are assumed to be consistent with the constraints and they define a constrained equilibrium, corresponding to $\tau = 0$. Assume that the controller is given by (19). Then, following the development indicated previously, the linearized equations, in the reduced form, for the closed loop can be shown to be given by

$$E_1 \bar{M}(x_{2d}) E_2' \ddot{e}_2 + E_1 \bar{K}(x_{2d}) E_2' e_2 = (I_m + G_f) E_1 V'(f - f_d) + E_1 V' \tau$$

$$E_2 \bar{M}(x_{2d}) E_2' \ddot{e}_2 + G_v \dot{e}_2 + \{G_d + E_2 \bar{K}(x_{2d}) E_2'\} e_2 = E_2 V' \tau$$

$$e_1 = 0.$$

The effects of the force disturbance on the regulation accuracy, at least locally, are characterized by these equations.

Suppose that the feedback gain matrices satisfy the conditions of Corollary 4. If τ is a constant force disturbance, then there are steady-state position errors and constraint force errors such that, at least locally, the closed-loop responses satisfy

$$q - q_d \to VE_2'[G_d + E_2 V' K(q_d) VE_2']^{-1} E_2 V' \tau \quad \text{as } t \to \infty$$

$$f - f_d \to J(E_1 V' J')^{-1} [I_m + G_f]^{-1} \{E_1 V' K(q_d) VE_2' [G_d + E_2 V' K(q_d) VE_2']^{-1} E_2 - E_1\} V' \tau \quad \text{as } t \to \infty.$$

Thus, the steady-state displacement error does not depend on the force feedback gain matrix G_f and is inversely proportional to the displacement feedback gain matrix G_d. The steady-state constraint force error does depend on the force feedback gain matrix G_f and is inversely proportional to it. Thus, "high gain" in the displacement feedback loops results in improved steady-state displacement accuracy for additive force disturbances. And "high gain'' in the force feedback loops results in improved steady-state constraint force accuracy for additive force disturbances.

Effects of Dynamics in Force Feedback Loops

Suppose that there are dynamics in the force feedback loops such as might be due to force sensor dynamics. We make a simple assumption about the nature of these dynamics so results are easily obtained; we do not examine the effects of dynamics in the displacement and velocity feedback loops although that might also be of importance.

Recall that q_d and f_d are assumed to be consistent with the constraints and they define a constrained equilibrium if there are no sensor dynamics. Assume that the controller is given by

$$u=F(q_d, 0)-f_d-VE_2'G_vE_2V'\dot{q}+VE_2G_dE_2V'(q_d-q)+VE_1'G_fz$$

$$\dot{z}=-\mu z+\mu E_1V'(f-f_d)$$

where z represents the m vector state of the force feedback loops. This assumes first-order feedback dynamics as a consequence of measurement of the vector component of the constraint force, $E_1V'f$, normal to the constraint surface. For simplicity, we take μ as a positive scalar.

Then, following the development indicated previously, the linearized equations, in the reduced form, for the closed loop given can be shown to be given by

$$E_1\bar{M}(x_{2d})E_2'\ddot{e}_2+E_1\bar{K}(x_{2d})E_2'e_2=G_fz+E_1V'(f-f_d)$$

$$E_2\bar{M}(x_{2d})E_2'\ddot{e}_2+G_v\dot{e}_2+\{G_d+E_2\bar{K}(x_{2d})E_2'\}e_2=0$$

$$\dot{z}=-\mu z+\mu E_1V'(f-f_d)$$

$$e_1=0.$$

The effects of the dynamics in the force feedback loops on the regulation accuracy, at least locally, are characterized by the above equations.

Suppose that the feedback gain matrices satisfy the conditions of Corollary 4. It is easy to show that if $\mu > 0$, the closed-loop equations are locally asymptotically stable in the sense that for initial data near the equilibrium

$$q(t)\to q_d \quad \text{as } t\to\infty$$

$$f(t)\to f_d \quad \text{as } t\to\infty.$$

That is, the dynamics in the force feedback loops do not destabilize the closed-loop system as long as the feedback dynamics are of the assumed simple form and the gains satisfy the conditions of Corollary 4. Note that the assumption that the force feedback gain G_f is nonnegative definite is critical here.

Effects of Constraint Uncertainties

Suppose that there are uncertainties in the constraint function; it is of interest to determine the effect of such uncertainties on the regulation accuracy of the closed loop. Although there are various assumptions that could be made, for simplicity we assume that the constraint is linear and given by

$$Jq=\Delta$$

where Δ represents the constant m vector of constraint uncertainty. Recall that constant regulation vectors q_d, f_d are assumed to be consistent with the constraints and they define a constrained equilibrium corresponding to $\Delta = 0$. Assume that the controller is given by (19). The linearized equations, in the reduced form, for the closed loop can be shown to be given by

$$E_1\bar{M}(x_{2d})E_2'\ddot{e}_2+E_1\bar{K}(x_{2d})E_2'e_2=(I_m+G_f)E_1V'(f-f_d)-E_1K(x_{2d})E_1'(E_1V'J')^{-1}\Delta$$

$$E_2\bar{M}(x_{2d})E_2'\ddot{e}_2+G_v\dot{e}_2+\{G_d+E_2\bar{K}(x_{2d})E_2'\}e_2=-E_2\bar{K}(x_{2d})E_1'(E_1V'J')^{-1}\Delta$$

$$e_1=(E_1V'J')^{-1}\Delta.$$

The effects of the constraint uncertainty on the regulation accuracy, at least locally, are characterized by these equations.

Suppose that the feedback gain matrices satisfy the conditions of Corollary 4. Then there are steady-state position errors and constraint force errors such that, at least locally, the closed-loop responses satisfy

$$q-q_d\to -VE_2'[G_d+E_2V'K(q_d)VE_2']^{-1}E_2V'K(q_d)VE_1'(E_1V'J')^{-1}\Delta \quad \text{as } t\to\infty$$

$$f-f_d\to -J(E_1V'J')^{-1}[I_m+G_f]^{-1}\{E_1V'K(q_d)VE_2'[G_d+E_2V'K(q_d)VE_2']^{-1}E_2V'K(q_d)VE_1'-E_1V'K(q_d)VE_1'\}(E_1V'J')^{-1}\Delta \quad \text{as } t\to\infty.$$

Again we see that the steady-state displacement errors do not depend on the force feedback gain matrix G_f and are inversely proportional to the displacement feedback gain matrix G_d. The steady-state constraint force errors do depend on the force feedback gain matrix G_f and are inversely proportional to it. Thus, ''high gain'' in the displacement feedback loops results in improved steady-state displacement accuracy for uncertainty in the constraint function; and ''high gain'' in the force feedback loops results in improved steady-state constraint force accuracy for uncertainty in the constraint function.

VII. An Example

Consider a simple example of a planar Cartesian manipulator constrained so that the end effector follows an elliptic arc as shown in Fig. 3. We take the equations of motion to be given by

$$\ddot{q}_1=u_1+f_1$$

$$\ddot{q}_2=u_2+f_2$$

with scalar constraint equation given by

$$4(q_1)^2+(q_2)^2-1=0.$$

Thus, the forces of constraint are

$$f_1=8q_1\lambda$$

$$f_2=2q_2\lambda$$

where λ is the constraint multiplier.

The control objective is to choose a feedback controller so that the closed loop is stable and the motion and constraint forces are regulated about the constant values q_{1d}, q_{2d}, f_{1d}, f_{2d}, assumed to be consistent with the imposed constraints. Our approach is based on the developments in Sections III–V.

As in the previous notation, let $q' = (q_1, q_2)$ and let $x' = (x_1, x_2)$ be defined by

$$x=X(q)=\begin{vmatrix} q_1-0.5[1-(q_2)^2]^{1/2} \\ q_2 \end{vmatrix}.$$

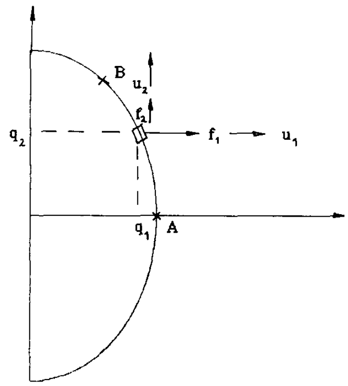

Fig. 3. Cartesian manipulator constrained to ellipse.

Thus

$$T(x)=\begin{vmatrix}1 & -0.5q_2[1-(q_2)^2]^{1/2}\\ 0 & 1\end{vmatrix}.$$

The open-loop nonlinear equations, in the reduced form, are

$$-0.5x_2(1-x_2^2)^{-1/2}\ddot{x}_2-0.5(1-x_2^2)^{-3/2}\dot{x}_2^2=u_1+f_1$$

$$[1+0.25(1-x_2^2)^{-1}x_2^2]\ddot{x}_2+0.25(1-x_2^2)^{-2}x_2\dot{x}_2^2 = -0.5(1-x_2^2)^{-1/2}x_2u_1+u_2$$

$$x_1=0.$$

Results are now presented for two different desired regulation objectives. *We first consider the regulation objective defined by* $q_d'=(0.5,\ 0)$ *and* $f_d'=(1,\ 0)$, *corresponding to stable regulation about point A in Fig. 3 with desired normal contact force of 1.*

Corresponding to the development in Section IV, the modified computed torque controller can be written in the complicated form

$$u_1=-2q_1-0.25(q_1\dot{q}_2-q_2\dot{q}_1)\dot{q}_2q_1^{-2}+(0.25q_2q_1^{-1})(g_v\dot{q}_2+g_dq_2)+g_f(f_1-2q_1)$$

$$u_2=-0.5q_2-g_v\dot{q}_2-g_dq_2+(0.25q_2q_1^{-1})g_f(f_1-2q_1).$$

The closed-loop equations, in reduced form, are

$$(-0.25q_2q_1^{-1})[\ddot{e}_2+g_v\dot{e}_2+g_de_2]=(1+g_f)(f_1-2q_1)$$

$$\ddot{e}_2+g_v\dot{e}_2+g_de_2=0$$

$$e_1=0.$$

Thus, if $g_v>0$, $g_d>0$, $g_f>0$, it follows that $e_2\to 0$ as $t\to\infty$, and therefore

$$q_1\to 0.5,\ q_2\to 0 \qquad \text{as } t\to\infty,$$

$$f_1\to 1,\ f_2\to 0 \qquad \text{as } t\to\infty.$$

Also, a linear feedback controller can be developed using the procedure indicated in Section V, leading to the linear controller

$$u_1=-1+g_f(f_1-1)$$

$$u_2=-g_v\dot{q}_2-g_dq_2.$$

The resulting linearized equations, in the reduced form, for the closed loop are given by

$$0=(1+g_f)(f_1-1)$$

$$\ddot{e}_2+g_v\dot{e}_2+(g_d-0.5)e_2=0$$

$$e_1=0.$$

Thus, if $g_v>0$, $g_d>0.5$, $g_f>0$, it follows that, locally, $e_2\to 0$ as $t\to\infty$, and therefore

$$q_1\to 0.5,\ q_2\to 0 \qquad \text{as } t\to\infty$$

$$f_1\to 1,\ f_2\to 0 \qquad \text{as } t\to\infty.$$

We next consider the regulation objective defined by $q_d'=(0.25,\ 0.866)$ *and* $f_d'=(0,\ 0)$, *corresponding to stable regulation about point B in Fig. 3 with zero desired normal contact force.*

Corresponding to the development in Section IV, the modified computed torque controller can be written in the complicated form

$$u_1=-0.25(q_1\dot{q}_2-q_2\dot{q}_1)\dot{q}_2q_1^{-2}+(0.25q_2q_1^{-1})[g_v\dot{q}_2+g_d(q_2-0.866)]+g_ff_1$$

$$u_2=-g_v\dot{q}_2-g_d(q_2-0.866)+(0.25q_2q_1^{-1})g_ff_1.$$

The closed-loop equations, in reduced form, are

$$(-0.25q_2q_1^{-1})[\ddot{e}_2+g_v\dot{e}_2+g_de_2]=(1+g_f)f_1$$

$$\ddot{e}_2+g_v\dot{e}_2+g_de_2=0$$

$$e_1=0.$$

Thus, if $g_v>0$, $g_d>0$, $g_f>0$, it follows that $e_2\to 0$ as $t\to\infty$, and therefore

$$q_1\to 0.25,\ q_2\to 0.866 \qquad \text{as } t\to\infty$$

$$f_1\to 0,\ f_2\to 0 \qquad \text{as } t\to\infty.$$

Also, a linear feedback controller can be developed using the procedure indicated in Section V, leading to the linear controller

$$u_1=g_ff_1$$

$$u_2=-g_v\dot{q}_2-g_d(q_2-0.866)+0.866g_ff_1.$$

The resulting linearized equations, in the reduced form, for the closed loop are given by

$$-0.866\ddot{e}_2=(1+g_f)f_1$$

$$1.75\ddot{e}_2+g_v\dot{e}_2+g_de_2=0$$

$$e_1=0.$$

Thus, if $g_v>0$, $g_d>0$, $g_f>0$, it follows that, locally, $e_2\to 0$ as $t\to\infty$, and therefore

$$q_1\to 0.25,\ q_2\to 0.866 \qquad \text{as } t\to\infty$$

$$f_1\to 0,\ f_2\to 0 \qquad \text{as } t\to\infty.$$

VIII. Conclusions

Conditions for stabilization of a closed-loop constrained robot have been developed using mathematical models which explicitly include the constraint functions. Global stabilization conditions have been developed using a nonlinear controller which is based on a modification of the computed torque method. Local stabilization conditions have also been developed using a linear controller with a specified feedback structure.

We have also investigated the properties of closed-loop systems using the proposed controller structure; we have shown that "high gain displacement feedback loops" reduce the steady-state displacement regulation errors for constant force disturbances and uncertainty in the constraint function. We also have shown that "high gain constraint force feedback loops" reduce the steady-state constraint force regulation errors for constant force disturbances and uncertainty in the constraint function. Further, closed-loop stabilization has been shown not to be affected by a certain class of dynamics in the constraint force feedback loops.

The suggested approach to investigation of applications of robots to tasks defined by constraints is new. We have shown that, although the underlying mathematical issues are complicated, a formal mathematical approach is tractable. Our emphasis here has been on feedback stabilization. But, as indicated in [23], [25] we believe that this approach provides a theoretical basis for the investigation of a variety of problems that involve the use of force feedback in constrained robot systems.

References

[1] H. Asada and N. Goldfine, "Optimal compliance design for grinding robot tool holders," in *Proc. IEEE Conf. Robotics Automation*, St. Louis, MO, 1985, pp. 316–322.

[2] ——, "Compliance analysis and tool holder design for grinding with robots," in *Proc. Amer. Contr. Conf.*, Boston, MA, 1985, pp. 65–68.

[3] L. Gustafsson, "Deburring with industrial robots," presented at the SME Deburring and Surface Conditioning Conf., 1983.

[4] N. Hogan and S. R. Moore, "Part referenced manipulation—A strategy applied to robotic drilling," in *Proc. ASME Winter Annual Meet.*, Boston, MA, 1983, pp. 183–191.

[5] G. Plant and G. Hirzinger, "Controlling a robot's motion speed by a force-torque sensor for deburring problems," in *Proc. IFIC/IFAP Symp. Inform. Contr. Problems in Manufacturing Technol.*, 1982, pp. 97–102.

[6] T. M. Stepien, *et al.*, "Control of tool/workpiece contact force with application to robotic deburring," in *Proc. IEEE Conf. Robotics Automation*, St. Louis, MO, 1985.

[7] G. P. Starr, "Edge following with a Puma 560 manipulator using VAL-II," in *Proc. IEEE Conf. Robotics Automation*, San Francisco, CA, 1986, pp. 379–383.

[8] N. Hogan, "Impedance control: An approach to manipulation: Part I—Theory; Part II—Implementation; Part III—Applications," *ASME J. Dynam. Syst., Measurement Contr.*, vol. 107, 1985.

[9] H. Kazerooni, *et al.*, "Robust compliant motion for manipulators; Part I: The fundamental concepts of compliant motion; Part II: Design methods," in *Proc. IEEE J. Robotics Automation*, vol. RA-2, pp. 83–92, June 1986.

[10] R. P. Paul and B. Shimano, "Compliance and control," in *Proc. Joint Automat. Contr. Conf.*, 1976, pp. 694–699.

[11] J. K. Salisbury, "Active stiffness control of a manipulator in Cartesian coordinates," in *Proc. IEEE Conf. Decision Contr.*, Albuquerque, NM, 1980, pp. 95–100.

[12] J. J. Craig, *Introduction to Robotics: Mechanics and Control.* Reading, MA: Addison-Wesley, 1986.

[13] G. Hirzinger, "Force feedback problems in robotics," presented at the IASTED Symp. on Modelling, Identification and Contr., Davos, 1982.

[14] ——, "Direct digital robot control using a force torque sensor," in *Proc. IFAC Symp. Real Time Digital Contr. Appl.*, 1983, pp. 243–255.

[15] M. T. Mason, "Compliance and force control for computer-controlled manipulators," *IEEE Trans. Syst., Man., Cybern.*, vol. SMC-11, 1981.

[16] M. H. Raibert and J. J. Craig, "Hybrid position/force control of manipulators," *ASME J. Dynam. Syst., Measurement Contr.*, vol. 102, pp. 126–133, 1981.

[17] H. West and H. Asada, "A method for the design of hybrid position/force controllers for manipulators constrained by contact with the environment," in *Proc. IEEE Conf. Robotics Automation*, St. Louis, MO, 1985.

[18] D. Whitney, "Force feedback control of manipulator fine motions," *ASME J. Dynam. Syst., Measurement Contr.*, pp. 91–97, 1971.

[19] ——, "Historical perspective and state of the art in robot force control," in *Proc. IEEE Conf. Robotics Automation*, St. Louis, MO, 1985, pp. 262–268.

[20] V. I. Arnold, *Mathematical Methods of Classical Mechanics.* New York: Springer-Verlag, 1978.

[21] D. T. Greenwood, *Principles of Dynamics.* Englewood Cliffs, NJ: Prentice-Hall, 1965.

[22] H. Hemami and B. F. Wyman, "Modelling and control of constrained dynamic systems with application to biped locomotion in the frontal plane," *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 526–535, 1979.

[23] H. P. Huang, "Constrained manipulators and contact force control of contour following problems," Ph.D. dissertation, Dep. Elect. Eng. Comput. Sci., Univ. Michigan, Ann Arbor, July 1986.

[24] R. Kankaanranta and H. N. Koivo, "A model for constrained motion of a serial link manipulator," in *Proc. IEEE Conf. Robotics Automation*, San Francisco, CA, 1986.

[25] N. H. McClamroch, "Singular systems of differential equations as dynamic models for constrained robot systems," in *Proc. IEEE Conf. Robotics Automation*, San Francisco, CA, 1986.

[26] N. H. McClamroch and H. P. Huang, "Dynamics of a closed chain manipulator," in *Proc. Amer. Contr. Conf.*, Boston, MA, 1985.

[27] T. Yoshikawa, "Dynamic hybrid position/force control of robot manipulators-description of hand constraints and calculation of joint driving forces," in *Proc. IEEE Conf. Robotics Automation*, San Francisco, CA, 1986, pp. 1393–1398.

[28] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *J. Dynam. Syst., Measurement Contr.*, vol. 102, pp. 119–125, 1981.

[29] N. H. McClamroch and D. Wang, "Feedback stabilization and tracking of constrained robots," Center for Research in Integrated Manufacturing, Robot Syst. Division, Univ. Michigan, RSD-25, Oct. 1986.

[30] S. D. Eppinger and W. P. Seering, "Introduction to dynamic models for robot force control," *IEEE Contr. Syst. Mag.*, vol. 7, no. 2, pp. 48–52, Apr. 1987.

[31] J. Y. S. Luh, "Conventional controller design for industrial robots—A tutorial," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-11, pp. 298–316, 1983.

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