Trajectory Planning For Biped Walk With Non-instantaneous Double Support Phase

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***Abstract*—In this paper we present a technique for finding periodic gaits for five-link planar bipedal robot with point feet. Unlike most works that are limited to the consideration of only underactuated single support phase (SSP) we show that this phase change condition is state dependant and extend our approach to the presence of double support phase (DSP). The concept of virtual holonomic constraints is used with full set of constraints during SSP and during SSP to DSP change.**

***Index Terms*—five-link biped robot, virtual holonomic constraint, trajectory planning, double support phase**

I. Introduction

The control of planar point contact bipeds as underactuated mechanical systems has been widely studied. Trajectory planning of a biped based on a concept of virtual holonomic constraints [1] and subsequent use of hybrid zero dynamics [2] or transversal linearization [3] allows to investigate orbital stability and design control systems. Parametrization of the virtual constraints allows to choose optimal trajectories in terms of energy consumption or any other cost function.

In most earlier works on the biped robot with point feet trajectory consists of a SSP and an instantaneous switch to the next SSP. However in human like locomotion there is a DSP besides a SSP that improves the stability of robot motion. Furthermore, since in DSP the system is overactuated, it is easier to control robot velocity change in speeding up or slowing down. A few papers considered a DSP for a biped robot with point feet to analyze the influence on stability and time optimality [4], [5]. However these works either are not based on virtual constraints concept, which complicates the further task of orbital stabilization or the full set of physical constraints on SSP and SSP to DSP phase change are not considered.

In this paper, we study the SSP trajectory planning task taking into account the full set of mechanical constraints imposed both during SSP and SSP to DSP phase change moment. Necessary conditions for SSP to SSP transitions are derived. The algorithm to find trajectories with DSP is proposed.

II. Problem Statement

Consider five-link planar biped robot shown in Fig.(1). In general it has seven degrees of freedom (DoF), two of them correspond to position of any point on the robot, for example, point $p_1 = [p_1^x;\ p_1^y]$, which describes position of the robot with respect to inertial frame. While the rest five correspond to angular position of each link, $q = [q_1; \ldots; q_5]$. During walking the robot experiences repetitive SSP, impact and double support DSP. During the SSP the robot is completely defined by q as the stance leg is attached to ground, however has only four actuator (there is no torque applied between ground and the stance leg), therefore the system has underactuation of degree one. During DSP both legs of the robot are attached to ground, thus robot has three DoF and four actuators that makes the system over-actuated, because of that wide range of trajectories can be realized in contrast to SSP. Therefore in this paper we limit our discussion to SSP and impact of the swing leg with ground. The main goal of this paper is to lay out optimization procedure that allows to find the trajectory of the biped during SSP, so that after impact the system enters DSP. The underlying challenge is the presence of underactuation and constraints on torques.

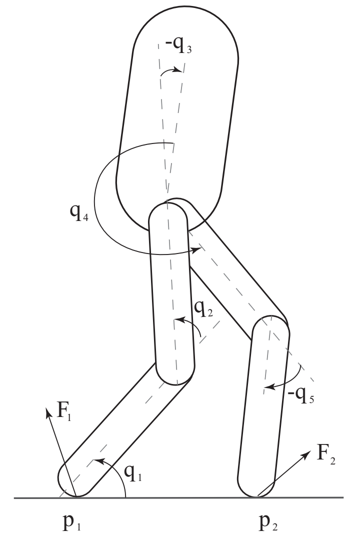

Fig. 1. Schematic drawing of the five-link planar biped robot during the impact

III. Dynamic Model

For dynamic model considered in this paper we have to make the following assumption: the robot is made of five rigid

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links connected by four frictionless and rigid joints. Under this assumption applying Euler-Lagrange method [6] the dynamic model of the robot in SSP is easily obtained

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix} 0_{1\times4} \\ I_{4\times4} \end{bmatrix} \tau, \tag{1}$$

where $D(q)$ is the inertia matrix; $C(q,\dot{q})$ is the Coriolis and Centrifugal matrix; $G(q)$ is the gravity vector; and the matrix on the right hand side, usually denoted as B is input matrix, that has rank four due to underactuation. After SSP when the swing leg contacts the ground an impact occurs that modeled as a contact between two rigid bodies. In the literature, there exist a large number of rigid impact models [7]–[9], and any of them can be used to compute mapping between the generalized velocity just after the impact and generalized velocity and position just before the impact. In this paper we use the model described in [9] that is based on the hypotheses that: the impact is instantaneous; after the impact the swing leg does not slip and stays on the ground; and during the impact the externally applied forces can be represented by impulses.

The derivation of the impact model requires the ground reaction forces at the leg ends, and thus seven- DoF unpinned model of the robot.

$$D_e(q_e)\ddot{q}_e + C_e(q_e,\dot{q}_e)\dot{q}_e + G_e(q_e) = B_e\tau + \delta F_{ext}, \tag{2}$$

By integrating Eq.(2) over the duration of the impact, we obtain

$$D_e(q_{e,int}^+)\dot{q}_{e,int}^+ - D_e(q_e^-)\dot{q}_e^- = \begin{bmatrix} J_{p_1}^T & 0 \\ 0 & J_{p_2}^T \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \tag{3}$$

where J_{p_i} $(i = 1,2)$ is a Jacobian matrix of point p_i; F_i $(i = 1,2)$ is the ground reaction force at point p_i; $\dot{q}_e^-$ is the velocity just before the impact and $\dot{q}_{e,int}^+$ is the velocity just after the impact. Solving together Eq.(3) and velocity constraints at points p_1 and p_2 after the impact, one can obtain $\dot{q}_e^+$, F_i

$$F_1 = \Delta_{F_1}\dot{q}^-, \quad F_2 = \Delta_{F_2}\dot{q}^-, \quad \dot{q}_{int}^+ = \bar{\Delta}_{\dot{q}}\dot{q}^- \tag{4}$$

Combined model of SSP and the impact that is of hybrid nature becomes

$$\begin{cases} D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu, & (q;\dot{q}) \notin \Gamma, \\ q_{int}^+ = q^-, \ \dot{q}_{int}^+ = \bar{\Delta}_{\dot{q}}\dot{q}^- & (q;\dot{q}) \in \Gamma, \end{cases} \tag{5}$$

where the switching surface is

$$\Gamma = \{(q,\dot{q}) \mid p_2^x(q) = 0, \ p_2^y(q) > 0\}. \tag{6}$$

Notice that if only SSP walking scheme is considered then the angular momentum conservation principle can be applied insted of solving (3). In this case the angular momentum respect to impact point p_2 just before and after collision is conserved. However, commonly it is not mentioned that it is not always possible to realize SSP to SSP change and it depends on the robot state at the moment of collision. Namely, the angular momentum respect to point p_2 before collision is linear function of joint angular velocities and equal to $K_{p_2} = K_{p_1} + \overline{p_2p_1} \times Q = H\dot{q}$, where K_{p_1} is the angular momentum respect to point p_1, $Q = MV_{com}$ is the total linear momentum of the robot. Since after collision K_{p_2} keeps its value, SS to SS transition is realizable only when the sign of K_{p_2} value corresponds to clockwise rotation, that is to moving forward condition. Since K_{p_1} value sign corresponds to the desired direction of rotation, then step length is the parameter which influences the condition of switch between phases. In other words, the bigger the step length, the easier to find trajectory corresponding to SSP to DSP case. Similar result shown for simple compass biped with fixed joint angle [10], where for joint angles more than some certain value SSP to SSP transitions are not possible.

In a classical setting trajectory planning for an underactuated mechanical system boils down to finding $(q^*(t),\ \dot{q}^*(t))$ by solving an optimization problem to obtain feed-forward torques, $U_{ff}(t)$ required to keep the system on the trajectory. There are two main problems associated with this approach. First of all, as state space of the system, $x = [q;\ \dot{q}]$ is large, $x \in \mathbf{R}^{10}$, in addition to constraints, optimization will be very time consuming. Secondly, we assume that the walking of the robot is symmetric and cyclic, therefore it is not important to follow trajectory as a function of time but rather to converge to closest point on the cycle. To overcome these problems we have to introduce kinematic relations between generalized coordinates, usually called virtual holonomic constraints (VHC) [1], [11], [12]. Assume that the angle between the stance leg and the ground q_1 is the generator of motion s, then introduce the following relations

$$q = \Phi(s), \ \dot{q} = \Phi'(s)\dot{s}, \ \ddot{q} = \Phi'(s)\ddot{s} + \Phi''(s)\dot{s}^2, \tag{7}$$

where $\phi_i(s)$ can be any function but usually chosen as regular polynomial, Bzier polynomial or trigonometric polynomial of degree n. By substituting (7) into (1) we obtain dynamics written in $(s,\dot{s})$

$$D(s)\Phi'(s)\ddot{s} + \Big(D(s)\Phi''(s) + C(s)\Phi'(s)\Big)\dot{s}^2 + G(s) = Bu \tag{8}$$

In underactuated systems the main challenge is to stabilize passive DoF, whose dynamics for the robot can be obtained by premultiplying both parts of (8) by $B^\perp$ $(B^\perp B = 0)$

$$\alpha(s)\ddot{s} + \beta(s)\dot{s}^2 + \gamma(s) = 0 \tag{9}$$

Resulting dynamics is called reduced dynamics of the system [11], [13] and has set of properties [14] useful for trajectory planning and control design for mechanical systems. Introduction of VHC allows to rewrite dynamics in a form such that new state space of the system, $x_s = [s;\ \dot{s}]$ becomes of dimension two. It greatly reduces search domain of the optimization problem. Moreover, by means of VHC we were able to reparametrize time, therefore solving both problems of "classical" trajectory planning problem for underactuated systems.

IV. Optimization

Initial and final positions of the SSP of the robot can be defined by the following set of parameters: position of the torso with respect to vertical, σ, vertical position of the hip, p^y_{hip}, step length, L_{step}, and a parameter that defines vertical position of the hip, p^x_{hip}, both in the beginning and in the end of SSP. These parameters allow to uniquely define q^+ and q^-.

In order to get rid of step duration and convert second order differential equation (9) into first order differential equation we introduce $Y(s) = \dot{s}^2$. Reduced dynamics written in terms of $Y(s)$ has the form

$$\alpha(s)\dot{Y}(s) + 2\beta(s)Y(s) + 2\gamma(s) = 0. \quad (10)$$

As optimization requires solving reduced dynamics for each set of parameters, solving first order equation instead of second order facilitates to the reduction of time needed to find optimal trajectory. Optimization parameters are initial velocity of the motion generator, $\dot{s}_0$ and $(m+1)$ coefficients of the $\Phi(s)$. Altogether there are $4\times(m+1)+1$ parameters, P, to optimize. As the posed problem is non-convex with many constraints, we divide it into two subproblems: first one is search for feasible points and the second one is minimization of cost function whether it is cost of transportation (COT), min max of the torques or any other chosen cost. To solve these problems we use MATLAB's built in function *fmincon* with *SQP* solver for finding feasible points and *interior point* for minimization of the cost.

To solve subproblem one we set cost function to zero and try to find parameters, initialized randomly, to satisfy the following constraints on:

- initial and final pose of the robot, $A_{eq}P = [0; q^+; q^-]$;
- angular velocity of the stance leg during SSP, $\dot{s} < 0$;
- GRF of the stance leg during SSP, $F^N_1 > 0$, $\frac{|F^T_1|}{|F^N_1|} < \mu$;
- linear velocity of the swing leg at contact, $v^y_s < 0$;
- GRF at points p_i at the impact, $F^N_i > 0$, $\frac{|F^T_i|}{|F^N_i|} > \mu$;
- angular velocities during SSP, $\dot{q}_{min} < \dot{q} < \dot{q}_{max}$;
- torques, $\tau_{min} < \tau < \tau_{max}$.

Special attention is paid to $F^N_i > 0$ constraints which describe the unilateral nature of contact forces. It turned out that this constraint is a bottleneck for finding trajectories with SSP to DSP transition. Finally by increasing the step length we could eliminate this problem. Once desired number of vectors P are found, second subproblem can be solved by initializing optimization problem at feasible points and minimizing cost function.

V. Conclusion

Planar biped robot motion trajectory search algorithm is presented in the paper with extension to double support phase and utilizing virtual constraints approach. Additional attention is paid on considering the contact force restrictions during SSP to DSP change. Future work will concentrate on DSP continuous dynamics, full cyclic motion and its orbital stabilization.

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