Simulation Analysis of Linear Quadratic Regulator Control of Gait

Conference Paper · August 2016

2 authors:

Raviraj Nataraj
Stevens Institute of Technology

Antonie J van den Bogert
Cleveland State University

SIMULATION ANALYSIS OF LINEAR QUADRATIC REGULATOR CONTROL OF GAIT

[1,2] Raviraj Nataraj, PhD and [1,2] Antonie J. van den Bogert, PhD

[1] Parker Hannifin Human Motion Control Laboratory, Cleveland, OH, USA
[2] Department of Mechanical Engineering, Cleveland State University, Cleveland, OH, USA
email: r.nataraj@csuohio.edu, web: http://hmc.csuohio.edu/

INTRODUCTION

The linear quadratic regulator (LQR) is a classical optimal control approach to stabilize dynamics of a linear system. Utilizing the LQR to regulate the non-linear biomechanics of gait can facilitate:

- Greater understanding of natural gait control
- Design of control systems for powered assistive devices such as exoskeletons

Exoskeletons to restore gait function have successfully utilized proportional-derivative (PD) control of the hips and knees **[1]**. But controller parameters are not optimized for minimal energy cost and do not use entire body (full-state) feedback. The ***objective*** of this simulation study was to investigate the potential of utilizing time-varying LQR full-state feedback control. Controller performance was evaluated by the maintenance of desired kinematics with minimal effort of a sagittal-plane model walking against external perturbations.

METHODS

The computer model of bipedal gait **[2]** under feedback control included a trunk and bilateral segments for the thigh, shank, and foot. These segments are connected by purely rotational joints representing hips, knees, and ankles. A total of 18 states (9 position, 9 velocity) were defined for the system: 2-D trunk position, trunk tilt, and bilateral joint angles of the hip, knee, and ankle. The model was actuated by torques at the six joints. Trajectory optimization **[2]** was used to find joint torques for walking at 1.3 m/s, while minimizing the integrated squared torque and tracking joint kinematics and ground reaction forces of normative gait **[3]**.

The non-linear gait dynamics, $\dot{x} = f(x,u)$, were linearized about the optimized state and control trajectories (x_o(t), u_o(t)). Surrogate variables y and v represent the state and control deviations from these desired trajectories. After temporal discretization, a linear time-varying (LTV) system was obtained:

$$y_{i+1} = Ay_i + Bv_i$$

A periodic Riccati equation solver **[4]** was used to find the optimal periodic time-varying feedback controller $v = -K(t)y$ that minimized the objective function:

$$J = \sum_{k=0}^{\infty}(y_k^T Q y_k + v_k^T R v_k)$$

The matrices Q, R weigh respective contributions of tracking and effort. LQR controllers were created with Q to R ratio (Q/R) from 10^{-3} to 10^{4} for dimensionless states and controls.

The control system was tested by forward dynamic simulations that included random horizontal force perturbations applied to the hip over 10 gait cycles. The type 1 perturbation was forward-directed and grew in magnitude (10N/sec) to test controller performance stability, indicated by greater time-to-fall. The type 2 perturbation was bi-directional with bounded maximum magnitude (5N) to test controller performance efficiency, indicated by lower root-mean-square (RMS) of the sum of closed-loop joint torque magnitudes. Time-invariant PD controllers with similar gain magnitudes as the LQR controllers were also tested against perturbations for comparison.

RESULTS AND DISCUSSION

Composite results comparing various LQR and PD controllers for time-to-fall and RMS of closed-loop torque are shown in **Fig 1**. Q/R near 1 produced the best average time-to-fall. But RMS of closed-loop torque generally increased with higher Q/R, which corresponded to higher gain magnitudes. Across all LQR and PD controllers, those with relatively high performance in both time-to-fall and closed-loop torque output were those employing LQR feedback.

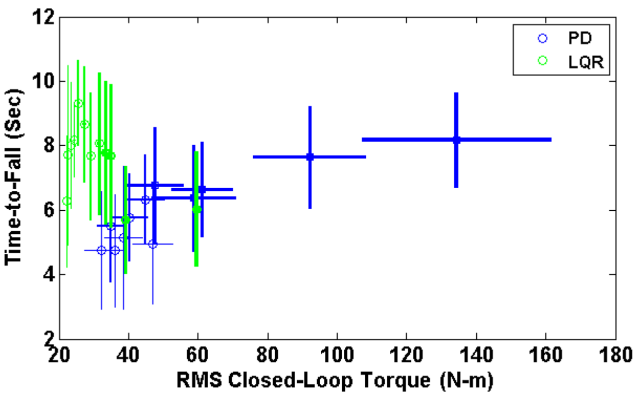

Figure 1: Performance of LQR controllers compared to similar gain-magnitude PD controllers. Thicker lines denote higher gain magnitudes.

The time-varying gain profiles produced for LQR control have smooth yet distinctive features across the gait cycle and can assume positive or negative values. This indicates that sophisticated dynamic coupling must be considered for optimal control of walking. These types of gain profiles may be best implemented explicitly according to phase **[5]** rather than time for real-world applications. Since time-to-fall performance was not proportional to Q/R as was RMS of closed-loop torques, there may be other trade-off factors between tracking and effort with actual non-linear performance than just specification of Q/R for linear control design. The additional feedback states utilized with full-state LQR compared to PD include those denoting trunk position and trunk angle. These states have notable contributions to the generation of closed-loop torques during optimal walking control (**Table 1**).

A limitation of linearization is that the LTV controller is designed for infinitesimally small perturbations. While the controller worked for finite perturbation magnitudes, it failed badly and ungracefully when perturbations became too large. This indicates the need for additional nonlinear control features to handle larger perturbations. This may involve combining the exceptional efficiency of LQR with the robustness of other methods.

CONCLUSIONS

LQR control has the potential to perform similarly to conventional PD controllers with significantly reduced energy cost. Future work will include investigation of linear optimal control performance in the presence of errors in state feedback and with an exoskeleton walking model that includes arm support and limited joint torque. Such work should guide the development of novel sensor-based feedback control systems to address clinical problems such as improved gait restoration following spinal cord injury.

REFERENCES

1. Farris RJ, et al. *IEEE Trans Neural Syst Rehabil*, **22**, 482-490, 2014.
2. Ackermann M, et al., *J Biomech*, **43**, 1055-1060, 2010.
3. Winter DA. *Biomechanics and Motor Control of Human Movement*, Wiley & Sons, Ltd., 1990.
4. Hench JJ, et al., *IEEE Automatic Control*, **39**, 1197-1210, 1994.
5. Gregg RD, et al. *PLoS ONE*, 2014, **9**, e89163, 2014.

ACKNOWLEDGEMENTS

We acknowledge the support of the Parker Hannifin Endowment for Human Motion and Control and NSF grant 1344954.

Table 1: RMS of mean closed-loop torque contributions summed at each joint from feedbacks associated with each group of states for LQR controllers tested under type 2 perturbations.

State Feedback Group	RMS hip torque (N-m)	RMS knee torque (N-m)	RMS ankle torque (N-m)	% all torque contributions
Leg Joint Angles	33.6	31.9	30.4	46.4
Trunk Position	19.8	22.3	27.3	33.6
Trunk Angle	15.4	14.0	12.0	20.1