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Bipedal Walking Pattern Design Considering the Effect of Double Support Phase

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***Abstract*— In this paper, under the condition of acceleration continuance, the effect of double support phase in bipedal walking is investigated, and a new design approach for bipedal waking pattern based on the synchronization of the motions in sagittal and lateral planes is developed. The analysis of the motions both in the sagittal and lateral planes reveals that the motions in these two planes are tightly coupled together. The motion parameters such as walking speed, phase motion time, and phase stride can be easily adjusted simply by altering the start and finish points of double support phases in the lateral plane. Consequently, a new bipedal walking pattern for adjusting walking speed by controlling the double support phase is developed.**

***Index Terms*— Walking pattern design, biped robot, inverted pendulum, double support phase**

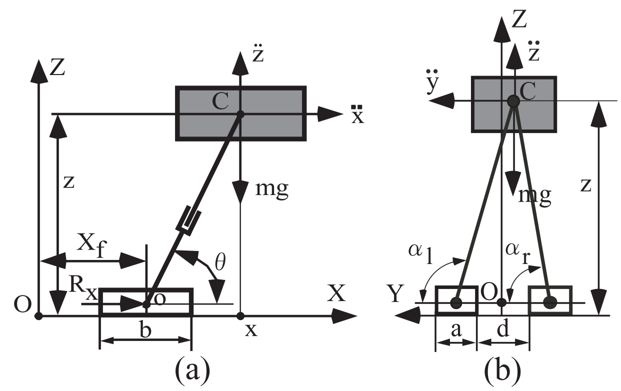

Fig. 1. 3D inverted pendulum model of biped robot. (a) motion in sagittal plane. (b) motion in lateral plane

I. Introduction

Up so far, many bipedal and humanoid robots are successfully developed ([1]-[6]). In these robots, the gait planning and control for bipedal walking are based on ZMP concept ([7]-[9]). The typical approach to implement the dynamic and stable walking is using a 3D inverted pendulum model, in which, the whole robot mass is simply assumed to be concentrated to the robot's CoG (Center of Gravity) and the robot locomotion in the sagittal and lateral planes are supposed to be independent ([10]-[14]).

However, partially because few attentions are paid on the motion in double support phase and its effect to whole bipedal walking, as well as the investigation is not done intensively even for single support phase, up to now, the bipedal walking properties and limitations, such as walking speed, step length, walking cycle time, and so on, have not been completely understood.

This paper mainly discusses the above two problems based on ZMP concept and an inverted pendulum model with the assumption that ZMP is fixed at the center of the robot sole in single support phase. To do this, the relation between the motions in the sagittal and lateral planes is investigated first. By dividing a whole bipedal walking cycle into a double support, a deceleration, and an acceleration phases, and synchronizing the motions in the sagittal and lateral planes, we point out that the motions in these two planes are strongly coupled together, the motion in the lateral plane restricts the motion in the sagittal plane, vice versa. Then, the role of the double support phase in biped walking is discussed. By changing the start and finish points of the double support phases in the lateral plane, the walking parameters such as walking speed, walking cycle, step length and so forth, are consequently changed. Further, an approach for adjusting the instantaneous speed at the end of the acceleration phase is obtained, and a numerical example is given out.

II. Robot Model and Bipedal Walking

A. Robot Model and Assumptions

In this study, a bipedal robot is modeled as a 3D inverted pendulum of which the mass of the robot is supposed to concentrate on the point C, the CoG of the robot, as shown in Fig.1. $OXYZ$ is the world coordinate system, where X, Y axes are respectively the walking and swinging directions. XZ plane and YZ plane are respectively called the *sagittal plane* and *lateral plane*; and $oxyz$ is the local system fixed to the center of the support sole of the robot.

For simplicity, the following *assumptions* are made.

1) The height of the robot's CoG is a constant, i.e., $\ddot{z} = 0$.
2) The origin o of the local system $oxyz$ is always set at the sole center of support foot.
3) The desired ZMP in single support phase (SSP) is also always set at the sole center of the support foot; hence, the desired ZMP in SSP is identical to o.

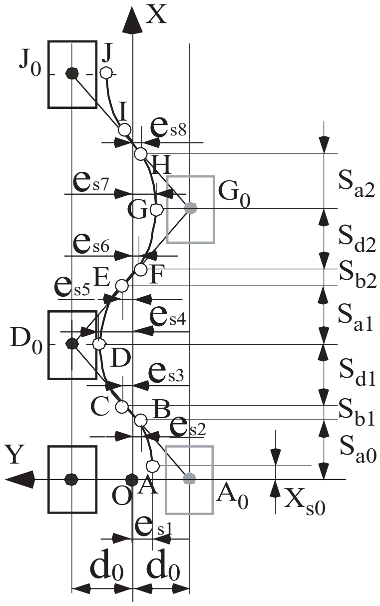

Fig. 2. Trajectories of CoG and ZMP in bipedal walking

4) The equivalent foot length of two feet is b.
5) The robot moves with constant speed in the double support phase.
6) The robot just walks forward, i.e., the distance d_0 in the lateral plane is a constant (Fig. 2).
7) In single support phase, the CoG never passes over the X axis (the midline) of its two feet. This means that the single support phase ends and the double support phase starts before the CoG crosses the X axis.

In SSP (Single Support Phase), ZMP (Zero moment point) is in fact a center of pressure of the support foot sole, or saying, the acting point of the resultant reaction force of the support floor. Therefore, ZMP is always within the inside of the sole of the support foot in SSP, even when the robot is falling down (at this moment, the ZMP is at one of the edges of the support foot). Consequently, the safest case is that the desired ZMP is at the sole center of the support foot in SSP. This paper just discusses the CoG's trajectory of the robot with the precondition of the desired ZMP at the sole center of the support foot in SSP, and doesn't consider the detail control problem.

B. Trajectories of Robot CoG and ZMP

The robot motion, in which its CoG trajectory is shown in Fig.2, is as follows. Before the robot starts to walk, its CoG is at the midpoint O of the two support feet. When the robot starts to walk, the CoG is first shifted from the point O to A. Here the distances between OA in X and Y direction are respectively X_{s0} and e_{s1}. From the point A, the left leg of the robot (here, assume the left leg of the robot first steps out) steps forward while the right leg supports the robot, and the robot moves forward along the curve AB. When the CoG moves to the point B, the left leg lands to the floor and the robot enters into the double support phase. When the CoG reaches to the point C, the right leg lifts off and the robot is in the single support phase again. During the left leg support phase, the CoG moves along the curve CDE. When the CoG reaches to E, the right leg lands to the floor and the robot is in the double support phase again. In this way, the swing leg and the support leg switches each other in turn at the points B, E, H, the robot moves forward in X direction (sagittal plane) while swings in Y direction (lateral plane). The positions of each point from A to H are shown in Fig.2.

As our assumptions indicate, the ZMP is fixed at the sole center of the support foot in the single support phase, such as points A_0, D_0, G_0, and J_0, and the ZMP moves from A_0 to D_0, from D_0 to G_0 in the double support phase as shown in Fig.2.

In the local system $oxyz$, the ZMP can be expressed as

$$x_{ZMP} = x - \frac{\ddot{x}z}{g} \tag{1}$$

$$y_{ZMP} = y - \frac{\ddot{y}z}{g} \tag{2}$$

Let

$$\omega^2 = \frac{g}{z} \tag{3}$$

then the above two equations can be rewritten as

$$\ddot{x} = \frac{g}{z}(x - x_{ZMP}) = \omega^2(x - x_{ZMP}) \tag{4}$$

$$\ddot{y} = \frac{g}{z}(y - y_{ZMP}) = \omega^2(y - y_{ZMP}) \tag{5}$$

Eq.(4) shows that for the motion in the sagittal plane XZ, when the robot is forward-leaning (the CoG of the robot is in front of the support foot, $x - x_{ZMP} \geq 0$), the robot has to accelerate forward. Contrarily, when the robot is backward-leaning (the CoG is at the rear of the support foot, $x - x_{ZMP} \leq 0$), the robot should decelerate. Therefore, with the switching of the support leg and swinging leg in walking, the robot continually accelerates and decelerates.

In a single support phase, the general motion equations (the solutions of Eqs. (4) and (5)) of the CoG of the robot in the two planes are respectively as follows,

$$x(t) = x_{ZMP} + c_1 e^{\omega t} + c_2 e^{-\omega t} \tag{6}$$

$$y(t) = y_{ZMP} + c_3 e^{\omega t} + c_4 e^{-\omega t} \tag{7}$$

where, x_{ZMP} and y_{ZMP} are respectively constants as our assumption; c_1 to c_4 are coefficients determined by initial conditions.

III. Motions in Sagittal and Lateral Planes

In our previous work ([14][15]), a bipedal walking is divided into initial acceleration, double support, deceleration and acceleration phases. Here the motion in each phase is investigated in more detail by combining and synchronizing the motion in the lateral plane. In this paper, the distances $e_{sn}(\mathrm{n} = 0, 1, 2, \cdots)$ from the robot's CoG to X axis in Y direction (See Fig. 2) are supposed positive, i.e., $e_{sn} > 0 \quad (\mathrm{n} = 0, 1, 2, \cdots)$.

A. Initial acceleration phase AB

(Motion time: $0 \leq t \leq t_{a0}$, travelled distance in X and Y direction: $X_{s0} \leq x(t) \leq S_{a0}$, $-e_{s1} \leq y(t) \leq -e_{s2}$)

In this phase, the robot starts walking from the standstill and the robot has to accelerate. According to the motion equations (4) and (5), in order to guarantee the ZMP at the sole center of the support foot, the robot has to lean forward in the XZ (sagittal) plane and lean left in the YZ (lateral) plane, respectively. Thus, at the start point A, the offsets X_{s0} and e_{s1} respectively in X and Y axes are necessary as shown in Fig.2. (If $X_{s0} = 0$, then $x_{ZMP} = x$. It leads $\ddot{x} = 0$. This means the robot cannot move from the rest ($\dot{x} = 0$). If $e_{s1} = 0$, then the robot will have to move to left and pass over the X axis. This violates the assumption (7)).

The above condition implies that the projection of the CoG must be within the support polygon. This is (refer to Fig.1)

$$0 < X_{s0} \leq \frac{b}{2} \tag{8}$$

With the initial conditions $x(0) = X_{s0}$, $\dot{x}(0) = 0$, $y(0) = -e_{s1}$, $\dot{y}(0) = 0$, $x_{ZMP} = 0$, and $y_{ZMP} = -d_0$, the coefficients of Eqs.(6) and (7) are respectively $c_1 = c_2 = X_{s0}/2$, and $c_3 = c_4 = (d_0 - e_{s1})/2$. Therefore the two motion equations are respectively

$$x(t) = \frac{1}{2} X_{s0}(e^{\omega t} + e^{-\omega t}) = X_{s0} \cosh(\omega t) \tag{9}$$

$$\begin{aligned} y(t) &= -d_0 + \frac{1}{2}(d_0 - e_{s1})(e^{\omega t} + e^{-\omega t}) \\ &= -d_0 + (d_0 - e_{s1}) \cosh(\omega t) \end{aligned} \tag{10}$$

In the sagittal plane, the terminal conditions at the point B are $x(t_{xa0}) = S_{a0}$ and $\dot{x}(t_{xa0}) = V_{xah0}$, where, t_{xa0} and V_{xah0} are respectively the motion time and terminal speed in the sagittal plane in this phase. On the other hand, for the motion in the lateral plane, with the terminal conditions $y(t_{ya0}) = -e_{s2}$ and $\dot{y}(t_{ya0}) = V_{yah0}$, similar to the above, t_{ya0} and V_{yah0} are respectively the motion time and the terminal speed of the phase in the lateral plane). Therefore, the motion time t_{xa0} and t_{ya0} can be respectively expressed as

$$e^{2\omega t_{xa0}} = \frac{\omega S_{a0} + V_{xah0}}{\omega S_{a0} - V_{xah0}} \tag{11}$$

$$e^{2\omega t_{ya0}} = \frac{\omega(d_0 - e_{s2}) + V_{yah0}}{\omega(d_0 - e_{s2}) - V_{yah0}} \tag{12}$$

Obviously, the two motions in the two planes should be synchronized, that is, there should be $t_{xa0} = t_{ya0}$. Thus, we can further get

$$\frac{X_{s0}}{S_{a0}} = \frac{d_0 - e_{s1}}{d_0 - e_{s2}} = \rho_{a0} < 1 \tag{13}$$

Meanwhile, the two terminal velocities in the two planes are respectively expressed as

$$V_{xah0} = \omega \sqrt{{S_{a0}}^2 - {X_{s0}}^2} = \omega S_{a0} \sqrt{1 - {\rho_{a0}}^2} \tag{14}$$

$$\begin{aligned} V_{yah0} &= \omega \sqrt{(d_0 - e_{s2})^2 - (d_0 - e_{s1})^2} \\ &= \omega (d_0 - e_{s2}) \sqrt{1 - {\rho_{a0}}^2} \end{aligned} \tag{15}$$

By substituting (13) into (11) or (12), we can get

$$t_{a0} = t_{xa0} = t_{ya0} = \frac{1}{\omega} \ln \frac{1}{\rho_{a0}} (1 + \sqrt{1 - {\rho_{a0}}^2}) \tag{16}$$

Further, by substituting (13) into (14) and (15), we can find the following relation

$$\frac{V_{xah0}}{V_{yah0}} = \frac{S_{a0}}{d_0 - e_{s2}} = \frac{X_{s0}}{d_0 - e_{s1}} \tag{17}$$

Since there should be $e_{s2} \geq 0$ as shown in Fig.2, from (13) there is

$$X_{s0} \leq \frac{d_0 - e_{s1}}{d_0} S_{a0} = (1 - \frac{e_{s1}}{d_0}) S_{a0} \tag{18}$$

If e_{s1} and e_{s2} are determined, then from (8) and (13), S_{a0} should be

$$S_{a0} < \frac{b}{2} / \rho_{a0} \tag{19}$$

Eqs.(13), and (18) show that the motions in the sagittal and lateral planes are tightly coupled together. The determination of the initial and terminal positions in one plane automatically and simultaneously decides the initial and terminal positions in another plane, therefore completely decides the motions in the two planes.

Eqs. (13) and (16) show that the motion time is independent of the moving distance, but determined by the ratio of the distance from the CoG to ZMP at the terminal time to the distance at the initial time in either of two planes. Eq. (17) shows that the ratio of the terminal velocities in the two planes is equal to the ratio of two distances from the CoG to ZMP at the terminal time. Eq. (17) also implies that big S_{a0} and big X_{s0} are helpful to get a big V_{xah0}. Note that in this phase, an offset x_{s0} is necessary, otherwise the robot cannot start walking.

B. Double support phase BC

(Motion time: $0 \leq t \leq t_{b1}$, traveled distance in X and Y direction: $0 \leq x(t) \leq S_{b1}$, $-e_{s2} \leq y(t) \leq e_{s3}$)

Once the CoG of the robot moves over the point B, the robot enters into the double support phase. To avoid the robot having a big shock in walking, the robot should move forward with continuous accelerations A_{xh0}, A_{yh0} and A_{xd10}, A_{yd10} respectively in X and Y directions at the beginning and end of the DS phase. Note that, the beginning of the DS phase is just the end of previous (initial) acceleration phase, and the end of the DS phase is just the beginning of the next deceleration phase it will discussed in the next subsection.

The initial and terminal conditions for CoG's motion in X direction are as follows,

$$x(0) = S_{a0}, \quad \dot{x}(0) = V_{xah0}, \quad \ddot{x}(0) = A_{xah0} \tag{20}$$

$$\dot{x}(t_{b1}) = V_{xbe1}, \quad \ddot{x}(t_{b1}) = A_{xd10} \tag{21}$$

Therefore, a fifth order polynomial function of time t, $x(t) = \sum_{k=0}^{4} a_k t^k$, is employed to represent CoG's trajectory.

For the motion in Y direction, the initial and terminal conditions for CoG's motion are as follows,

$$y(0) = -e_{s2}, \quad \dot{y}(0) = V_{yah0}, \quad \ddot{y}(0) = A_{yah0} \tag{22}$$

$$y(t_{b1}) = e_{s3}, \quad \ddot{y}(t_{b1}) = A_{yd10} \tag{23}$$

Similarly, a fifth order polynomial function $y(t) = \sum_{k=0}^{4} b_k t^k$, is used to represent CoG's trajectory in Y direction.

Here, the motion time t_{b1} and the traveled distance S_{b1} in X direction are unknown. We use the terminal condition

$$\dot{y}(t_{b1}) = V_{ybe1} \tag{24}$$

to determine t_{b1}. Then, S_{b1} can be obtained. Again, the above terminal conditions are determined in the following deceleration phase.

C. *Deceleration phase CD*

(Motion time: $0 \le t \le t_{d1}$, traveled distance in X and Y direction: $-S_{d1} \le x(t) \le 0$, $e_{s3} \le y(t) \le e_{s4}$)

After the robot moves over the point C, its former support leg (here, is the right leg) will lift off and the robot is in the single support. Since the CoG of the robot is at the behind and right of its support foot, the robot must decelerate in both of two directions. This phase CD is called the *deceleration phase*.

With initial conditions $x(0) = -S_{d1}$, $\dot{x}(0) = V_{xbe1}$, $y(0) = e_{s3}$, $\dot{y}(0) = V_{ybe1}$, and $x_{ZMP} = 0$, $y_{ZMP} = d_0$, from Eqs.(6) and (7), the motion equations in the sagittal and lateral planes are respectively expressed as follows,

$$x(t) = \frac{V_{xbe1}}{\omega} \sinh(\omega t) - S_{d1} \cosh(\omega t) \tag{25}$$

$$y(t) = d_0 + \frac{V_{ybe1}}{\omega} \sinh(\omega t) - (d_0 - e_{s3}) \cosh(\omega t) \tag{26}$$

The terminal condition in the sagittal plane is $x(t_{d1}) = 0$, but $\dot{y}(t_{d1}) = 0$ for the lateral plane. The motion times in the two planes are respectively as follows,

$$e^{2\omega t_{xd1}} = \frac{V_{xbe1} + \omega S_{d1}}{V_{xbe1} - \omega S_{d1}} \tag{27}$$

$$e^{2\omega t_{yd1}} = \frac{\omega(d_0 - e_{s3}) + V_{ybe1}}{\omega(d_0 - e_{s3}) - V_{ybe1}} \tag{28}$$

The above two motion times should be the same; therefore there exists the following condition,

$$\frac{\omega S_{d1}}{V_{xbe1}} = \frac{V_{ybe1}}{\omega(d_0 - e_{s3})} \tag{29}$$

The relation between the terminal speed V_{xdl1} (the lowest in this phase) and V_{xbe1} can be expressed as

$$\frac{V_{xdl1}}{V_{xbe1}} = \frac{d_0 - e_{s4}}{d_0 - e_{s3}} = \rho_{d1} < 1 \tag{30}$$

Concretely, V_{xdl1} and S_{d1} are as follows,

$$V_{xdl1} = \sqrt{{V_{xbe1}}^2 - (\omega S_{d1})^2} = \rho_{d1} \cdot V_{xbe1} \tag{31}$$

$$S_{d1} = \frac{1}{\omega} \sqrt{1 - {\rho_{d1}}^2}\, V_{xbe1} \tag{32}$$

The swinging amplitude e_{s4} in this phase is

$$e_{s4} = d_0 - \sqrt{(d_0 - e_{s3})^2 - (V_{ybe1}/\omega)^2} \tag{33}$$

and V_{ybe1} can be expressed as

$$V_{ybe1} = \sqrt{1 - {\rho_{d1}}^2} \cdot \omega (d_0 - e_{s3}) \tag{34}$$

Meanwhile, to guarantee the robot continually moving forward, the terminal speed should be $V_{xdl1} > 0$, that is

$$\omega S_{d1} < V_{xbe1} \tag{35}$$

The above condition implies that

$$e_{s4} > 0 \quad \text{and} \quad 0 < e_{s3} < \frac{d_0}{d_0 - e_{s2}} < 1 \tag{36}$$

The motion time t_{d1} is

$$t_{d1} = t_{xd1} = t_{yd1} = \frac{1}{\omega} \ln \frac{1}{\rho_{d1}} (1 + \sqrt{1 - {\rho_{d1}}^2}) \tag{37}$$

D. *Acceleration phase DE*

(Motion time: $0 \le t \le t_{a1}$, traveled distance in X and Y direction: $0 \le x(t) \le S_{a1}$, $e_{s5} \le y(t) \le e_{s4}$)

In the sagittal plane, once passing over the point D, the CoG of the robot will be in front of its support foot. Thus, the robot must accelerate. On the other hand, the robot reaches its swinging peak at D with 0 speed, and its CoG is in the right of its support foot. Therefore, the robot will accelerate in the lateral plane, too. This phase is called the *acceleration phase*.

With the initial conditions $x(0) = 0$, $\dot{x}(0) = V_{xdl1}$, $y(0) = e_{s4}$, $\dot{y}(0) = 0$, and $x_{ZMP} = 0$, $y_{ZMP} = d_0$, from Eqs.(6) and (7), the motion equations in the sagittal and lateral planes are respectively,

$$x(t) = \frac{V_{xdl1}}{\omega} \sinh(\omega t) \tag{38}$$

$$y(t) = d_0 - (d_0 - e_{s4}) \cosh(\omega t) \tag{39}$$

The terminal conditions are $x(t_{a1}) = S_{a1}$, $y(t_{a1}) = e_{s5}$. The motion times in the two planes can be respectively expressed as follows,

$$e^{2\omega t_{xa1}} = \frac{V_{xah1} + \omega S_{a1}}{V_{xah1} - \omega S_{a1}} \tag{40}$$

$$e^{2\omega t_{ya1}} = \frac{\omega(d_0 - e_{s5}) + V_{yae1}}{\omega(d_0 - e_{s5}) - V_{yae1}} \tag{41}$$

where, V_{xah1} and V_{yae1} are respectively the terminal speeds in the two planes. With the condition $t_{xa1} = t_{ya1}$, we can get the following relationship,

$$\frac{V_{xah1}}{V_{xdl1}} = \frac{d_0 - e_{s5}}{d_0 - e_{s4}} = \rho_{a1} > 1 \tag{42}$$

Further, V_{xah1} and V_{yae1} can be expressed as

$$V_{xah1} = \sqrt{{V_{xdl1}}^2 + (\omega S_{a1})^2} = \rho_{a1} V_{xdl1} \tag{43}$$

$$\begin{aligned} V_{yae1} &= -\omega\sqrt{(d_0 - e_{s5})^2 - (d_0 - e_{s4})^2} \\ &= -\omega(d_0 - e_{s4})\sqrt{{\rho_{a1}}^2 - 1} \end{aligned} \tag{44}$$

These two velocities are the highest in this phase.

From (42), e_{s5} can be expressed as

$$e_{s5} = (1 - \rho_{a1})d_0 + \rho_{a1}e_{s4} > 0 \tag{45}$$

And ρ_{a1} should be

$$1 < \rho < \frac{d_0}{d_0 - e_{s4}} \tag{46}$$

The motion time t_{a1} in this phase is

$$t_{a1} = t_{xa1} = t_{ya1} = \frac{1}{\omega}\ln(\rho_{a1} + \sqrt{{\rho_{a1}}^2 - 1}) \tag{47}$$

Note that $BCDE$ is a whole walking cycle which consists of a double support, a deceleration, and a acceleration phases. After the acceleration phase, the robot will enter into the double support phase and start a new walking cycle again; the motion is repetitive and the equations are also the same.

IV. Adjustment of Walking Velocity

Further, from Eqs. (30), (31), (42), and (43), the following three relations can be derived out,

$$S_{a1} = \sqrt{\frac{(d_0 - e_{s5})^2 - (d_0 - e_{s4})^2}{(d_0 - e_{s3})^2 - (d_0 - e_{s4})^2}} \cdot S_{d1} \tag{48}$$

$${V_{xah1}}^2 - {V_{xbe1}}^2 = \omega^2({S_{a1}}^2 - {S_{d1}}^2) \tag{49}$$

$$\frac{V_{xah1}}{V_{xbe1}} = \frac{d_0 - e_{s5}}{d_0 - e_{s3}} \tag{50}$$

These three equations are used to adjust the terminal speed V_{xah1} of the acceleration phases. Note that e_{s4} is a function of e_{s3} and determined by eq.(33).

With Eqs. (48) to (50), it is easily concluded that

1) $V_{xah1} = V_{xbe1}$ requires $S_{a1} = S_{d1}$ or $e_{s5} = e_{s3}$. The walking cycle is called *isometric speed cycle*.
2) $V_{xah1} > V_{xbe1}$ implies $S_{a1} > S_{d1}$ or $e_{s5} < e_{s3}$. The walking cycle is *accelerated*. In other words, if the start point (ex., point E) of a double support phase is closer to the midline (X axis) of two feet than the finish point (ex., point C) of the previous double support phase, the robot will be accelerated.
3) Contrarily, $V_{xah1} < V_{xbe1}$ means $S_{a1} < S_{d1}$ or $e_{s5} < e_{s3}$. The walking cycle is *decelerated*.

Note that the above three conditions also imply that if the landing point D_0 is just at the midpoint of CE (Fig.2), then the robot will just make an isometric speed walking; if the landing point D_0 is at the rear of the midpoint of CE, the robot will accelerate; contrarily, the robot will decelerate if the point D_0 is in front of the midpoint of CE. Thus, by adjusting the phase stride S_a and S_d, or the positions e_{s2}, e_{s3}, e_{s5}, e_{s6}, and e_{s8} of which the start and finish points of double support phases, the walking speed can be changed.

TABLE I

Motion Parameters for ZMP fixed case

S_{a0}	S_{b1}	S_{d1}	S_{a1}	S_{b2}	S_{d2}	S_{a2}	X_{s0}	d_0	z_0	V_{xah0}	V_{xbe1}
0.35	0.24	0.28	0.39	0.20	0.34	0.19	0.10	0.09	0.50	1.48	1.78
t_{a0}	t_{b1}	t_{d1}	t_{a1}	t_{b2}	t_{d2}	t_{a2}	V_{xdl1}	V_{xah1}	V_{xbe2}	V_{xdl2}	V_{xah2}
0.43	0.14	0.20	0.26	0.09	0.20	0.12	1.24	2.14	2.14	1.50	1.71
e_{s1}	e_{s2}	e_{s3}	e_{s4}	e_{s5}	e_{s6}	e_{s7}	e_{s8}	V_{yah0}	V_{ybe1}	V_{yah1}	V_{ybe2}
0.07	0.02	0.02	0.041	0.006	0.02	0.041	0.034	0.30	0.22	-0.30	-0.22

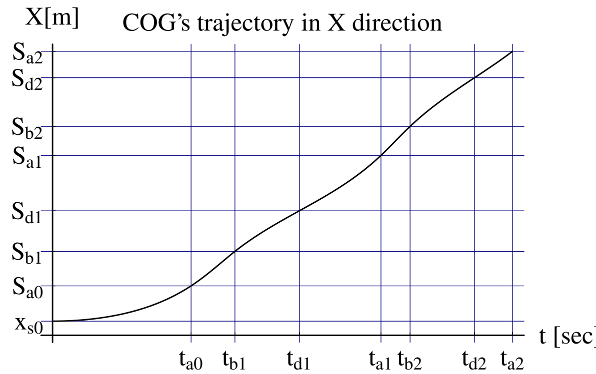

Fig. 3. CoG's trajectory in X direction

V. Numerical Example

Here, we give out a numerical example with the above theoretical results. Parameters used for motion planning are as follows: $z = 0.50$[m], $b = 0.22$[m], the others and results are listed in Table I . For the first walking cycle $BCDE$, in the double support phase BC, we set V_{xbe1}=1.2V_{xah0}, and in deceleration phase CD, set ρ_{d1}=0.7, $e_{s5} < e_{s3}$ (in fact, ρ_{a1}=1.2/ρ_{d1}). This means that the robot is accelerated not only in double support phase BC but also in phase CDE. For the second one $EFGH$, in the double support phase EF, we set V_{xbe2}=V_{xah1}, and in deceleration phase EF, set ρ_{d2}=0.7, $e_{s8} < e_{s6}$ (in fact, ρ_{a2}=0.8/ρ_{d2}). This means that the robot's velocity is kept in double support phase EF but decelerated in phase FGH. The trajectories, velocities, and accelerations in X and Y directions of the robot's CoG, are respectively shown in Fig. 3 to 8.

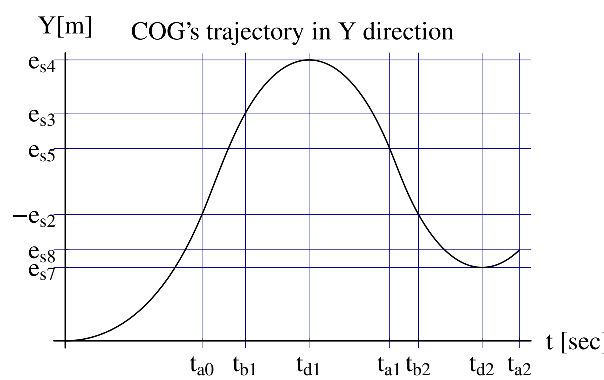

Fig. 4. CoG's trajectory in Y direction

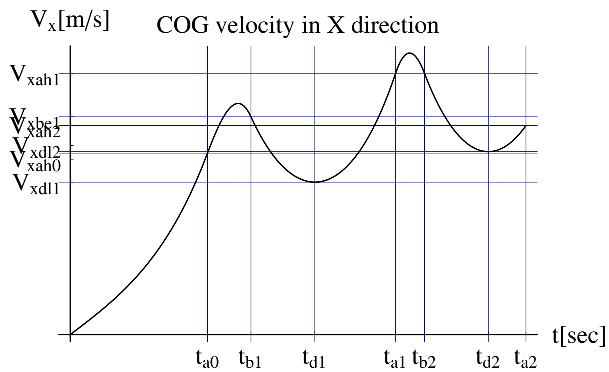

Fig. 5. CoG's speed in X direction

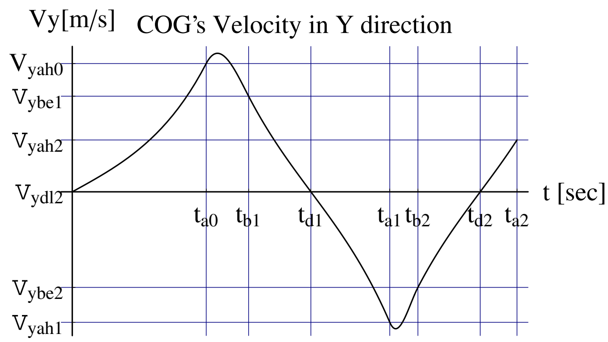

Fig. 6. CoG's speed in Y direction

VI. Conclusion

In this paper, with acceleration continuous condition, a new design approach of bipedal walking pattern based on the synchronization of the motions in the sagittal and lateral planes are presented. With the discussion on the motions in these two planes, the fact is clarified that the motions in the sagittal and lateral planes are tightly coupled together. The motion parameters in the sagittal plane such as walking speed, walking time, and phase stride can be easily adjusted by altering the start and finish points of the double support phase in the lateral plane. Therefore, an approach for adjusting the walking speed by controlling the double support phase is naturally developed. It is expected to apply this theory to a real bipedal robot and extend it to the jumping and running robot.

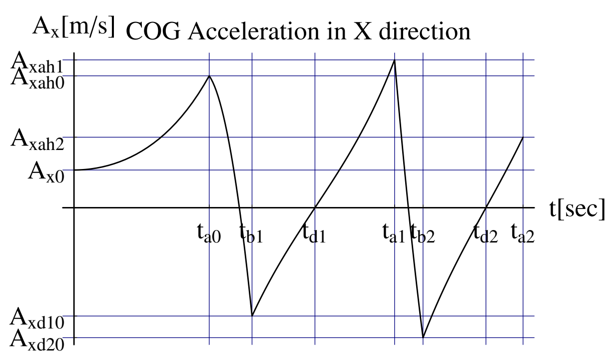

Fig. 7. COG's acceleration in X direction

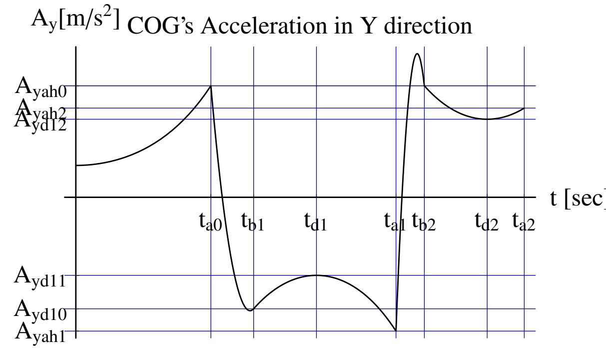

Fig. 8. COG's acceleration in Y direction

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