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Optimal trajectory stabilization of lower limb exoskeleton involving hybrid contact dynamics

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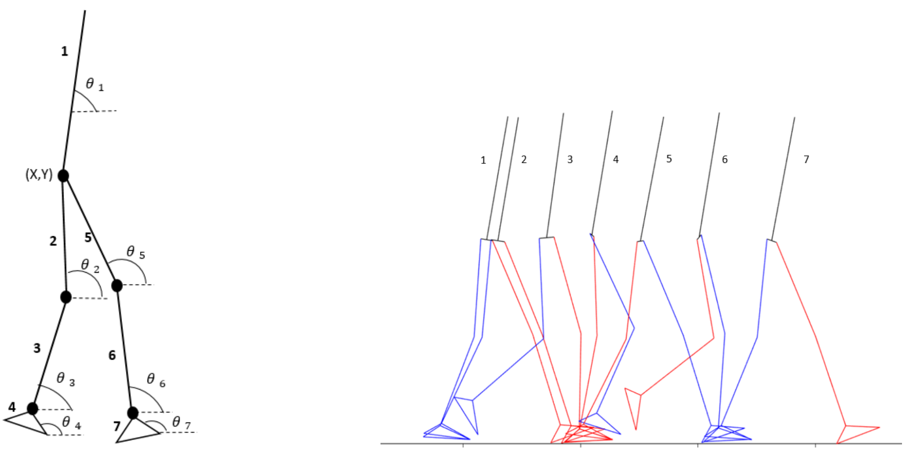
Abstract

The work proposed here discusses the dynamics and control of a lower limb exoskeleton. The primary task of imitating the human gait in the robotic exoskeleton with a model-based feedback control lies in developing its dynamics with high fidelity. However, the inherently complex nature of human gait makes the dynamics of the system hybrid, highly constrained and nonlinear.

We investigate the framework of finding the multi-phase optimal control for the dynamics of a human gait having frictional contact, to track the reference trajectory with minimal deviation subjected to external perturbations. The trajectory stabilization control we have used is the LQ tracking. The formulation of the LQ tracking with contact considered makes it a constrained LQ tracking problem, increasing the complexity as the widely used method of dynamic programming stands invalid. In the upcoming part of the abstract, we explain how we obtain the contact formulation, the methodology used to solve and optimizing the design matrix for better tracking.

The model developed has a kinematic topology of a 7-link Biped and nine degrees of freedom of the hip joint translation, torso orientation, right hip angle, right knee angle, right ankle angle, left hip angle, left knee angle, and left ankle angle as shown in Figure 1a). All joints are frictionless and revolute, and gait is over a straight horizontal terrain. During walking, the kinematics of the gait changes from open chain in Single support phase (SSP) to closed chain in Double support phase (DSP) as shown in Figure 1b), the Matlab simulation of human gait for one cycle, the red limb indicates the front leg and the blue limb indicates the rear leg. The contact established between the foot and the ground is subjected to changes based on the gait phase, so we distinguish the contact events from the start till the end of a gait cycle to obtain a predefined contact sequence as tabulated in Table 1. A clear illustration of hybrid contact events can also be noticed in Figure 1b). The gait starts with an initial heel contact on the front leg and flat foot contact for the rear leg, and on the transition to the next phase, the contact changes to flat foot contact on the front leg and toe-off in the rear leg, in a similar manner the contact sequence continues to achieve the walking gait. The Equation (1) to Equation (3) gives the rigid body equation



(a) (b)

Figure 1: a) Seven link Biped. b) Walking gait simulation.

Table 1: Contact events in gait cycle

Numbering	Phases	Rear leg	Front leg
-	Initial start	Flat foot	Heel contact
1	DSP 1 sub-phase 1	Flat foot	Flat foot
2	DSP 1 sub-phase 2	Toe contact	Flat foot
3	R-SSP sub-phase 1	Mid swing	Flat foot
4	R-SSP sub-phase 2	Heel contact	Toe contact
5	DSP 2 sub-phase 1	Flat foot	Toe contact
6	L-SSP sub-phase 1	Flat foot	Mid swing
7	L-SSP sub-phase 2	Flat foot	Heel contact

of motion subjected to an external contact involving friction [3]. Considering the contact constraints implicitly applied, the constrained dynamics is obtained in Equation (4), here $\beta(\mathbf{q},\dot{\mathbf{q}})$ is the contact Hessian and $\mathbf{C_T}(\mathbf{q})$ represents the contact Jacobian.

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau \tag{1}$$

$$\mathbf{g_n}(\mathbf{q}) = 0 \tag{2}$$

$$\mathbf{q_f}(\mathbf{q}) = 0 \tag{3}$$

$$\begin{bmatrix} \mathbf{M}(\mathbf{q}) & \mathbf{C_T}(\mathbf{q})^T \\ \mathbf{C_T}(\mathbf{q}) & 0 \end{bmatrix} \times \begin{bmatrix} \ddot{\mathbf{q}} \\ -\lambda \end{bmatrix} = \begin{bmatrix} -\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{G}(\mathbf{q}) + \tau \\ \beta(\mathbf{q},\dot{\mathbf{q}}) \end{bmatrix} \qquad \text{for} \lambda \geq 0 \tag{4}$$

The constraints in the Equation (4) will be updated for different phases based on the hybrid contact sequences leading to the switching of the equations for the respective phases[1]. Now, we formulate the optimal control problem using a Linear Quadratic Regulator for tracking the nominal trajectory for the dynamics defined in the Equation (4). The nominal trajectory is represented by $\boldsymbol{x}_t^R$ for $t \in T$, is obtained from the OpenSim Gait 2354 model for different walking speed [2] and modified by a factor α_{us} depending on the user specifics.

$$V_N(\mathbf{x_t},\mathbf{u_t}) = \min_u \frac{1}{2}\sum_{t=0}^{N-1}((\mathbf{x_t} - \mathbf{x_t}^R)^T\mathbf{Q}(\mathbf{x_t} - \mathbf{x_t}^R) + \mathbf{u_t}^T\mathbf{R}\mathbf{u_t}) + \frac{1}{2}(\mathbf{x_N} - \mathbf{x_N}^R)^T\mathbf{P}(\mathbf{x_N} - \mathbf{x_N}^R) \tag{5}$$

The objective function of the trajectory stabilization is stated in the Equation (5), here $\boldsymbol{u} \in \mathbb{R}^3$, $\boldsymbol{x} \in \mathbb{R}^3$ represent the control input and states space at any given time t , $\mathbf{P}$ is the terminal cost matrix and the matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ represent the penalty cost on the trajectory deviation and control input. The objective function is taken as a minimization problem of the weighted sum of the state error, and the control input, as shown in the Equation (5) and is subjected to the constrained dynamics in the discretised state space form. Thus our trajectory stabilization formulation becomes a constrained LQ tracking problem. The constraint arises as in our case, the Equation (5) is not only subjected to the linear dynamics in discrete form but also subjected to a linear equality constraint. The Equation (5) and The Equation (4) is the formulation of the LQ tracking for the hybrid contact dynamics defined.

The formalization we state looses its recursive nature, so we cant solve it with the standard Algebraic Riccati Differential Equation. So, we solve our formalization using the nonlinear equality constrained programming technique and the optimization problem takes the form of a QP. Further, for the problem stated the $\boldsymbol{R}$ matrix with weighted diagonal elements is considered non-uniform to find the best penalty for each element of the control input.

References

[1] Hogan, F.R.; Rodriguez, A.: Feedback Control of the Pusher-Slider System: A Story of Hybrid and Underactuated Contact Dynamics. arXiv preprint arXiv:1611.08268, 2016.

[2] John, C.T.; Seth, A.; Schwartz, M.H.; Delp. S.L.: Contributions of muscles to mediolateral ground reaction force over a range of walking speeds. Journal of Biomechanics, Vol. 45, No. 14, pp. 2438–2443, 2012.

[3] Shabana, A.A.: Computational Dynamics (3rd edition). John Wiley Sons, 2014.