Week 2 Physics Notes

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1 Vectors and Kinematics

1.1 Vectors and Scalars

1.1.1 Definitions

- **Scalar Quantity**: represents a physical quantity possessing *only magnitude.*

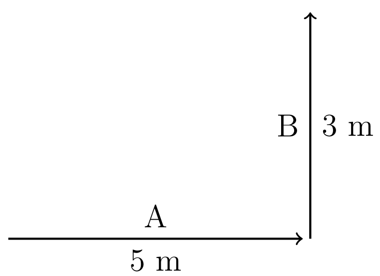
  ex. distance, speed, time, volume, density, energy, mass, power, work, etc.

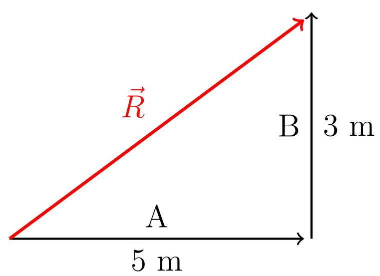
- **Vector Quantity**: a physical quantity with both *magnitude and direction.*

  ex. displacement, velocity, acceleration, force, impulse, momentum, pressure, etc.

- **Comparison Example** (Distance vs. Displacement)
  - **Distance** is a scalar quantity that describes the *total length traveled* by an object.
  - **Displacement** is a vector quantity that describes the *change in position* of an object.

  Take for instance an object traveling 5m, West then going 3m, North.





The distance in this case would be the sum of the lengths, A and B (this is the magnitude of the displacement). This gives:

$$
\begin{aligned}
d &= A + B \\
d &= 5\,\mathrm{m} + 3\,\mathrm{m} \\
d &= 8\,\mathrm{m}
\end{aligned}
$$

On the other hand, the resultant vector, $\vec{R}$, represents the displacement of the object—the difference between its final and original position.

Since $A \perp B$, then:

$$
\begin{aligned}
\vec{R} &= \sqrt{A^2 + B^2} \\
\vec{R} &= \sqrt{(5\,\mathrm{m})^2 + (3\,\mathrm{m})^2} \\
\vec{R} &= \sqrt{34} \approx 5.831\,\mathrm{m}
\end{aligned}
$$

1.1.2 Vectors

A vector is denoted by a magnitude, a unit, and the direction of the vector. Variables may also have an arrow cap to indicate that it is a vector quantity.
ex.

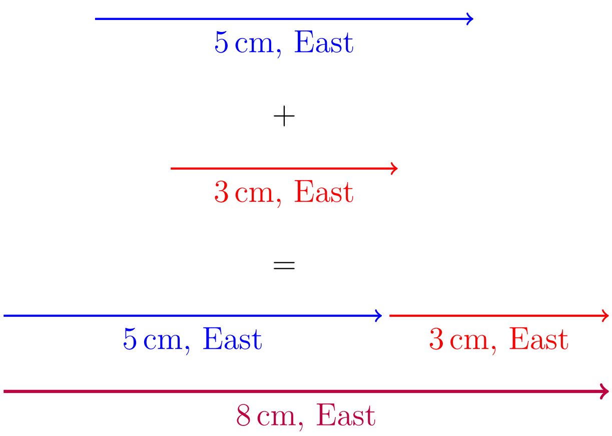
- $25\,\text{km/s}^2$, East
- $3\,\text{m},\ \frac{1}{2}\pi$
- $11.58\,\text{N},\ 45°$ West of South
- $\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2}$

1.1.3 Vector Addition

The addition of vectors is the same as regular addition, but a few considerations must be made. Direct addition, that is:

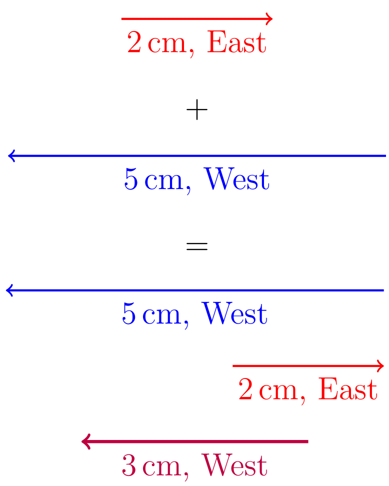
$$\vec{c} = \vec{a} + \vec{b}$$

Is only possible if, and only if, the vectors being summed together are on the same axis. In other words, the vectors can only be summed directly if both of them face the same direction, or are facing opposite directions.



Which is, assuming that west and south are negative, algebraically equivalent to:

$$5\,\text{cm} + 3\,\text{cm} = 8\,\text{cm}, \text{East}$$



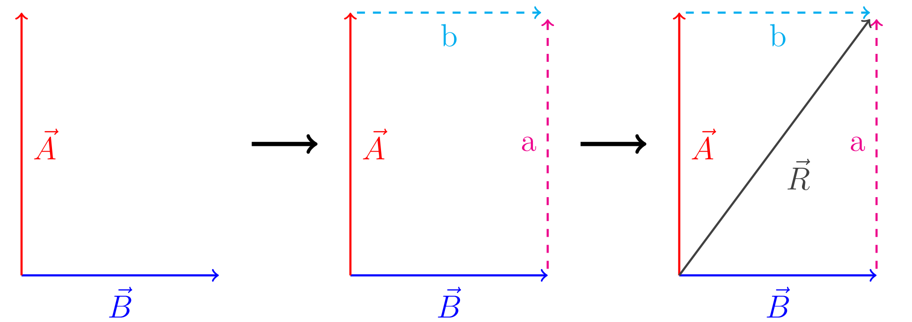
Which is, assuming that west and south are negative, algebraically equivalent to:

$$2\,\text{cm} + (-5\,\text{cm}) = -3\,\text{cm} = 3\,\text{cm}, \text{West}$$

1.1.4 Other Methods of Vector Addition

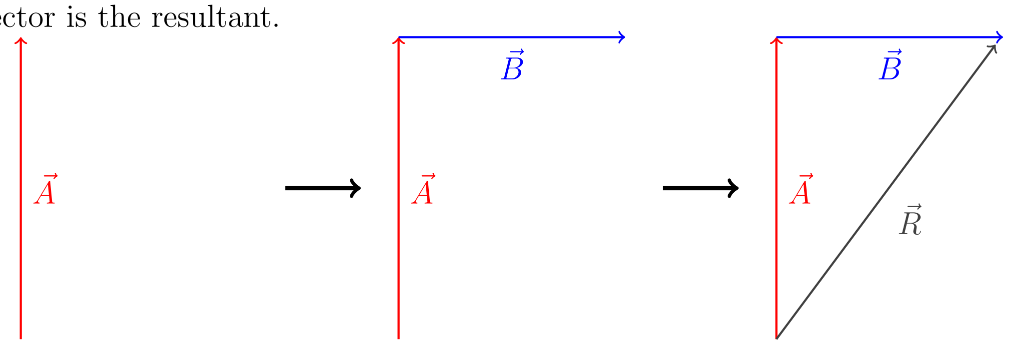
However, if the vectors being summed are not along the same axis, one can utilize various methods.

- Graphical Method
  1. Parallelogram Method
     - Used for adding only 2 vectors.
     - All vectors, including the resultant, are drawn from the origin



Step 1 Draw vectors starting at origin | Step 2 Complete the parallelogram | Step 3 Draw the resultant from the origin to its opposite angle

  2. Head-to-Tail Method
     - Can be used for two or more vectors.
     - The line starting from the tail of the first vector to the head of the last vector is the resultant.



Step 1 Draw the first vector | Step 2 Draw the subsequent vectors starting from the head of the previous vector | Step 3 Draw the resultant from the tail of the first vector to the head of the last vector

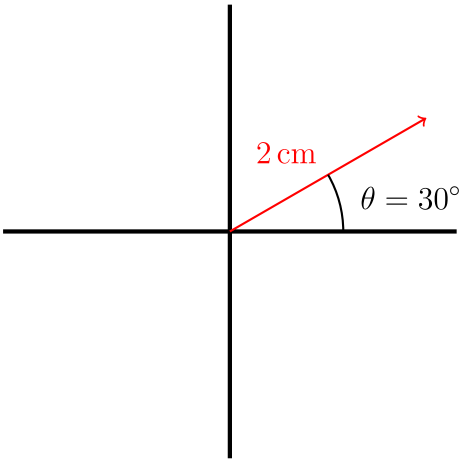
- Algebraic Method
  - The use of vector resolution and addition to find the resultant without the use of graphs.

1.1.5 Vector Resolution

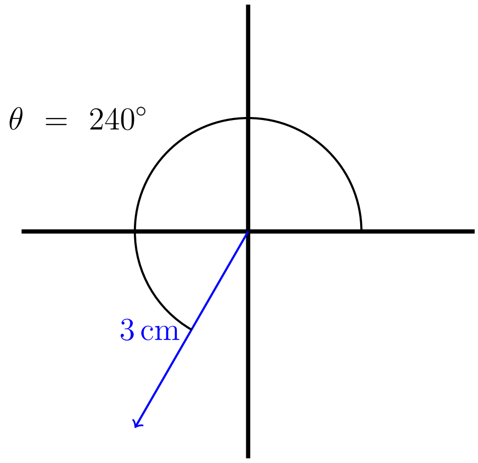
A resultant can also be algebraically determined through breaking down the x and y components of each vector, then summing like components.

Given the following vectors:

Vector A



Vector B



$\vec{A} = 2\,\text{cm}, 30°$

$\vec{B} = 3\,\text{cm}, 240°$

First find the x and y components of each vector:

Vector A

Using the following,

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{\vec{R_A}}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{\vec{R_A}}$$

, we can determine $\vec{A}$'s x and y components as such:

$$\cos 30° = \frac{\vec{A}_x}{2\,\text{cm}}$$
$$\vec{A}_x = \cos 30° \cdot 2\,\text{cm}$$
$$\vec{A}_x = \sqrt{3}$$

$$\sin 30° = \frac{\vec{A}_y}{2\,\text{cm}}$$
$$\vec{A}_y = \sin 30° \cdot 2\,\text{cm}$$
$$\vec{A}_y = 1$$

Vector B

The same formulas,

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{\vec{R_B}}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{\vec{R_B}}$$

, are also used to determine $\vec{B}$'s x and y components:

$$\cos 240° = \frac{\vec{B}_x}{3\,\text{cm}}$$
$$\vec{B}_x = \cos 240° \cdot 3\,\text{cm}$$
$$\vec{B}_x = -\frac{3}{2}$$

$$\sin 240° = \frac{\vec{B}_y}{3\,\text{cm}}$$
$$\vec{B}_y = \sin 240° \cdot 3\,\text{cm}$$
$$\vec{B}_y = -\frac{3\sqrt{3}}{2}$$

Then, sum the like components of the two vectors:

x-components

$$\vec{R_x} = \vec{A}_x + \vec{B}_x$$
$$\vec{R_x} = \sqrt{3} + (-\frac{3}{2})$$
$$\vec{R_x} = \frac{-3 + 2\sqrt{3}}{2}$$
$$\vec{R_x} = \frac{-3 + 2\sqrt{3}}{2} \approx 0.232$$
$$\vec{R_x} = 0.232\,\mathrm{m}, \mathrm{East}$$

y-components

$$\vec{R_y} = \vec{A}_y + \vec{B}_y$$
$$\vec{R_y} = 1 + (-\frac{3\sqrt{3}}{2})$$
$$\vec{R_y} = \frac{2 - 3\sqrt{3}}{2} \approx -1.598$$
$$\vec{R_y} \approx 1.598\,\mathrm{m}, \mathrm{South}$$

Finally, use the Pythagorean theorem to find the resultant, $\vec{R}$:

$$Hypotenuse = Leg_1 + Leg_2$$
$$\vec{R}^2 = \vec{R_x}^2 + \vec{R_y}^2$$
$$\vec{R}^2 = (0.232\,\mathrm{m}, \mathrm{East})^2 + (1.598\,\mathrm{m}, \mathrm{South})^2$$
$$\vec{R}^2 = (0.232)^2 + (-1.598)^2$$
$$\vec{R} = \sqrt{2.607}$$
$$\vec{R} = 1.615\,\mathrm{m}$$

Use the inverse tangent to find the direction:

$$\tan^{-1}\theta = \frac{opp}{adj}$$
$$\tan^{-1}\theta = \frac{\vec{R_y}}{\vec{R_x}}$$
$$\tan^{-1}\theta = \frac{0.232}{-1.598}$$
$$\theta = \tan\frac{0.232}{-1.598}$$
$$\theta = -8.261°$$

Final Answer:

$$\vec{R} = \boxed{1.615\,\text{meter},\ -8.261°}$$

Graphical Solution through Head-to-Tail Method: