Exercise 2: Crops

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This data comes from a sample of farms from three counties in Iowa. We want to know how the factors of the county and whether the farmer is related to the landlord of the farmland is related to the total crop yield of the farms.

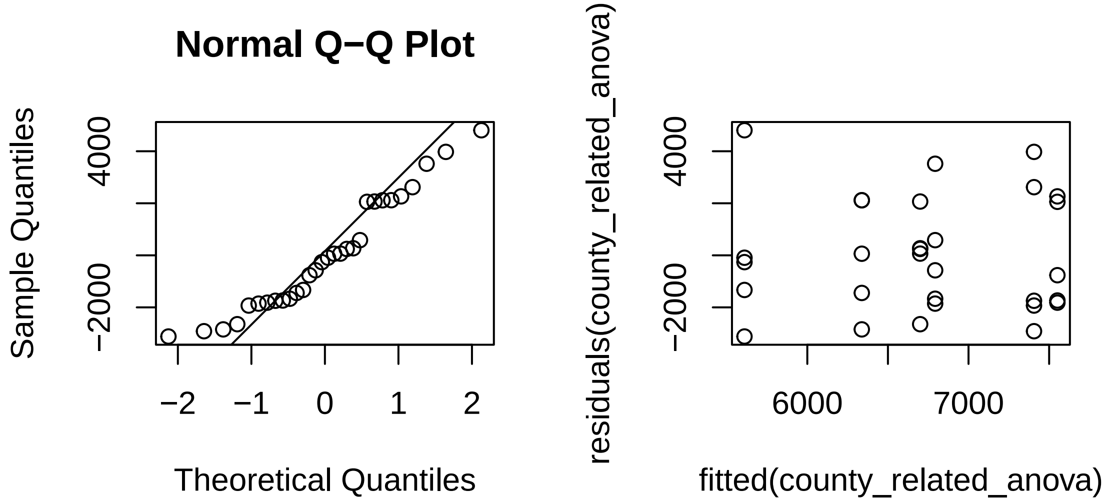
Part A

Firstly, we perform a two-way ANOVA on the Count, Related and Crops columns, which gives

```
anova(county_related_anova)
```

```
## Analysis of Variance Table
##
## Response: Crops
##                Df    Sum Sq Mean Sq F value Pr(>F)
## Related         1   2378957 2378957  0.4113 0.5274
## County          2   8841441 4420721  0.7644 0.4766
## Related:County  2   1497573  748786  0.1295 0.8792
## Residuals      24 138805865 5783578
```

The p-value here is given by the `Pr(>F)` column. For the Related and County factors separately, the p-value is not below 0.05, which means that a linear relation between these factors and the crop yield cannot be conclusively established. This is also true for the interaction between County and Related. As the two-way ANOVA assumes the data is normally distributed, we have to asses the normality of the data.



The left Q-Q plot of the residuals of the ANOVA places the point in roughly a straight line, which implies that this data is normally distributed.

The right plot shows how the spread of the residuals is roughly equal for all values. This implies that the underlying data is normally distributed.

```
summary(county_related_anova)
```

```
## 
## Call:
## lm(formula = Crops ~ Related * County, data = crops_frame)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -3120.4 -1744.7  -176.9  2064.2  4806.6 
## 
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)    
## (Intercept)         6700.0     1075.5   6.230 1.94e-06 ***
## Related1            -362.0     1521.0  -0.238    0.814    
## County2               93.0     1521.0   0.061    0.952    
## County3              851.2     1521.0   0.560    0.581    
## Related1:County2    -820.6     2151.0  -0.381    0.706    
## Related1:County3     217.0     2151.0   0.101    0.920    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 2405 on 24 degrees of freedom
## Multiple R-squared:  0.08393,    Adjusted R-squared:  -0.1069 
## F-statistic: 0.4398 on 5 and 24 DF,  p-value: 0.8163
```

This summary shows that the average farm in county 1 of which the farmer is not related to the landlord is 6700. The average for a farmer in county 3 would then be

```
6700.0 + 851.0
```

```
## [1] 7551
```

This seems plausible.

Part B

```
size_anova <- lm(Crops ~ Size, data = crops_frame)
ancova_county_lm <- lm(Crops ~ Size + County, data = crops_frame)
ancova_related_lm <- lm(Crops ~ Size + Related, data = crops_frame)
```

Now we want to take the size of the farm into account.

```
drop1(ancova_county_lm, test = "F")
```

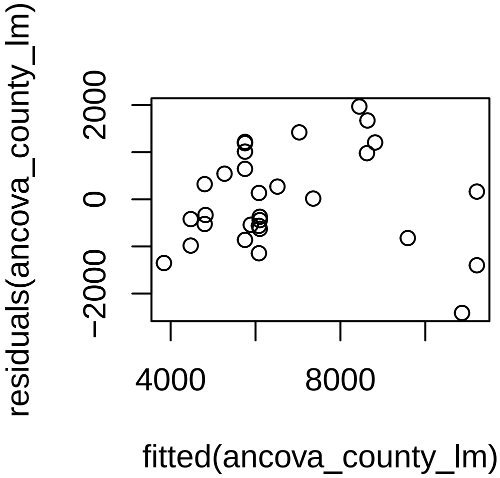
```
## Single term deletions
## 
## Model:
## Crops ~ Size + County
##        Df Sum of Sq       RSS    AIC F value    Pr(>F)    
## <none>                 31187313 423.63                      
## Size    1 111495081 142682394 467.25 92.9504 4.513e-10 ***
## County  2    767179  31954491 420.36  0.3198    0.7291    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

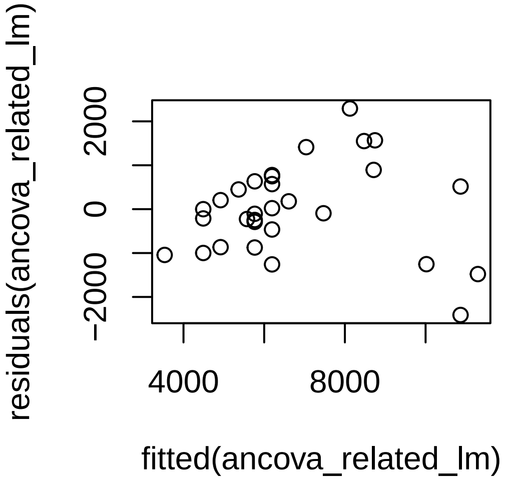
This ANCOVA shows that the size of the farm is strongly correlated with the yield of a farm, which can be seen in the low p-value of $4.5 \cdot 10^{-10}$. The county does no appear to have a significant effect here, as the p-value is to high. The data for the size and related status looks similar:

```r
drop1(ancova_related_lm, test = "F")
```

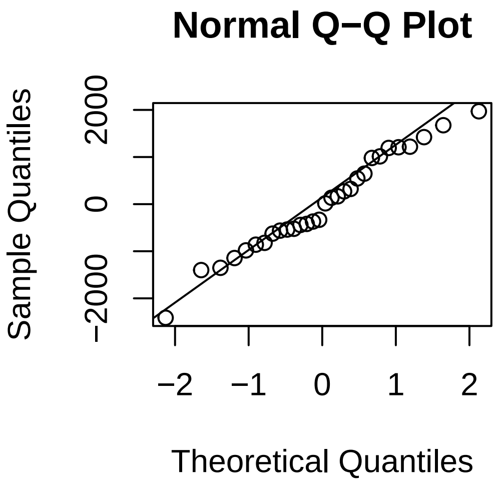
```
## Single term deletions
##
## Model:
## Crops ~ Size + Related
##         Df Sum of Sq       RSS    AIC  F value    Pr(>F)
## <none>                30573906 421.03
## Size     1 118570972 149144879 466.58 104.7107 8.646e-11 ***
## Related  1   1380585  31954491 420.36   1.2192    0.2793
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

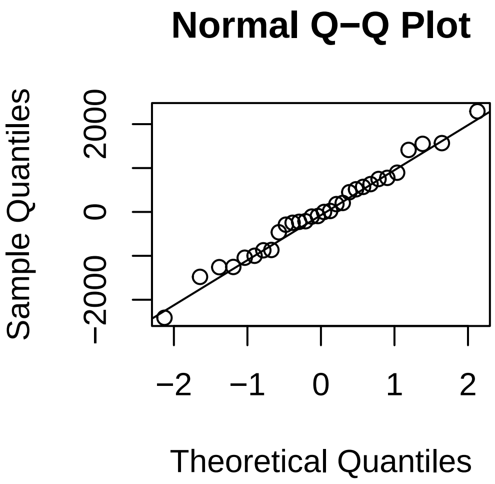
While the correlation between crop yield and farm size is significant, the related status does not appear to have an effect. These results lead us to conclude that the effect of county and relation on the influence of size is negligible. However, as the p-value for related is the smallest, this would be the most appropriate model for this dataset.





As ANCOVA assumes that the underlying data is normally distributed, we will have to check for normality. Both of the above residual plots show no obvious correlation in the variance of the residuals, which implies a normal distribution.





The Q-Q plots above show the points roughly in a straight line. This implies that the underlying data is normally distributed.

Part C

```
inter_relation <- lm(Crops ~ Size * Related, data=crops_frame)
anova(inter_relation)
```

```
## Analysis of Variance Table
##
## Response: Crops
##              Df    Sum Sq   Mean Sq  F value    Pr(>F)
## Size          1 119569344 119569344 105.5276 1.207e-10 ***
## Related       1   1380585   1380585   1.2185    0.2798
## Size:Related  1   1114273   1114273   0.9834    0.3305
## Residuals    26  29459633   1133063
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(inter_relation)
```

```
##
## Call:
## lm(formula = Crops ~ Size * Related, data = crops_frame)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2770.58  -743.18   -45.58   610.48  2156.14
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    1774.978    940.164   1.888   0.0702 .
## Size             28.211      4.841   5.828 3.84e-06 ***
## Related1      -1583.657   1227.310  -1.290   0.2083
## Size:Related1     6.274      6.327   0.992   0.3305
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1064 on 26 degrees of freedom
## Multiple R-squared:  0.8056, Adjusted R-squared:  0.7831
## F-statistic: 35.91 on 3 and 26 DF,  p-value: 2.148e-09
```

In this model, the county factor has no effect on the crop yield. The model can be described as

$$Y_{ik} = \mu + \alpha_i + \beta_i X_{ik} + e_{ik}$$

where μ is the average of the total population, α_i is the effects of all the related factor and β_i is the effect of the interaction between size and related. From the summary above, we can find that

- The average $\mu \approx 1775$
- The factor related $\alpha_i \approx -1584$ if the tenant and landlord are related
- The interaction $\beta_i \approx 6.274$
- The relation between plot size and crop yield $X_{ik} = \gamma \cdot x_{ik}$ where
  - x_{ik} is the size of the plot and
  - $\gamma \approx 28.21$

However, it is important to note that the effect of related is not statistically significant. This may indicate that a simpler model may be sufficient to describe the crop yields of these farms.

Part D

Combining the results from the previous two sections, we can create a numerical model for the crop yield of a farm. For a farm of county 2 of size 165 with relation to the landlord,

$$Y_{ik} = 1775 + -1584 + 6.274 \cdot 28.21 \cdot 165 =$$

```
## [1] 29394.27
```

Which does not appear to be a likely result. The error variance for this value is

```
## [1] 1242.032
```

A more probable result could be obtained by ignoring the effects of relation:

```r
1775 + 28.21 * 165
```

```
## [1] 6429.65
```