An

Ans_

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Section - B
Semester - 5^{th}
Course - B.Tech CSE (H)
Tutorial sheet

1

Ans 1 (3) O (N + M) time
O(1) space

Ans 2. $T(n) = O(n)$, space $O(1)$

Ans 3.
$T(n) = O(\log_2 n)$, Space $O(1)$

4
```
int sum = 0, i;
for(i=0; i*i <n, i++)
    { sum = i };
    }
```

$= n + (n-1) + (n-4) + (n-9) + \cdots (n-k)$

$= n + (n\, k) - (1^2 + 2^3 + 3^3 + \cdots k^2)$

$= \sqrt{n}$

$i^2 < n$

$i < n$

$T(n) = O(\sqrt{n})$, Space $O(1)$

Ans 5
```
int j = 1, i = 0
while (i<=n)
   { i = i+j;
     j++
   }
```

$0 <= n$ 1

$1 <= n$ 1

$3 <= n$

$(0, 1, 3, 6, 10, 15, 21 \cdots n)$
k terms.

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$$k^{th} \text{ term} = \frac{k*(k+1)}{2}$$

$$n = \frac{k^2+k}{2}$$

$$k^2+k = 2n$$

$$k^2+k-2n = 0$$

$$k = \frac{- \pm \sqrt{1^2+8n}}{2}$$

$$k = \frac{\sqrt{8n+1}}{2} + 1$$

$$k = \frac{\sqrt{8n+1}}{2}$$

$$k = \frac{\sqrt{8n}}{2} = \sqrt{n}.$$

$T(n) = \sqrt{n}$ Space $- O(1)$

Ans 6 Void Recursion (int n) → T(n)

```
{ if (n==1)return;
  recursion (n-1) → T(n-1)
  print(n) → 1
  recursion(n-1) → T(n-1)
}.
```

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(n-1) & n>1 \end{cases}$$

$$T(n) = 2T(n-1)+1 \quad -(1)$$

$$T(n-1) = 2T(n-2)+1$$

$$t(n) = 2(2T(n-2)+1)+1$$

$$T(n) = 4T(n-2)+(1+2)$$

$$T(n-2) = 2(T(n-3))+1$$

Ans7. It's a Binary Search Algorithm.

$T(n) = \log_2 n$

$T(n) = T(\frac{n}{2}) + 1$

by using master's method (can't be solved)

$T(n) = aT(\frac{n}{b}) + f(n)$

so $a=1$

$b=2$

$f(n)=1$

$c = \log_b a = \log_2 1 = 0$

Answer 8. 1. $T(1) = 1$

$T(n) = T(n-1) + 1$ — 1

$T(n) = T(n-2) + (n+(n-1))$ — 2

$T(n) = T(n-3) + (n+(n-1)+(n-2))$ — 3

$T(n) = T(n-k) + (n+(n-1)+(n-2) \dots (n-k)]$

$T(n-k) = T(1)$

$n = k+1$

$k = n-1$

2 → $T(n) = T(1) + (n+(n-1)+(n-2) \dots \dots (n-1+1-n))$

$T(n) = 1 + (n+(n-1)+(n-2)+ \dots 1)$

$T(n) = 1 + \frac{n(n+1)}{2} = \frac{n^2+1}{2} + 1$

$T(n) = \frac{n^2+2}{2}$

$T(n) = O(n^2)$.

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Ans 8 (3) $T(n) = T(n/2) + 1$

$T(n/2) = T(\frac{n}{4}) + 1$

$T(n) = T(\frac{n}{4}) + 2$ — 2

$T(\frac{n}{4}) = T(\frac{n}{8}) + 1$

$T(n) = T(\frac{n}{8}) + 3$ — 3

$T(n) = T(\frac{n}{2^k}) + k$ — 4

$\frac{n}{2^k} = 1$

$2^k = n$

$k = \log_2 n$

$T(n) = T(1) + \log_2 n$

$T(n) = O(\log_2 n)$

Ans 8 (4) $T(n) = 2T(\frac{n}{2}) + 1$

$c = 1$

$n^c = n$

$f(n) = 1$

$n^c > f(n)$

$T(n)^o = O(n)$

Ans 8 (5) $T(n) = 2T(n-1) + 1$

$T(n) = O(2^n)$

Ans 8 (5) $T(n) = T(\sqrt{n}) + n$

Ans 8 (6) $T(n) = 3T(n-1),\ T(0) = 1$

$T(n) = 3(T(n-1)$ — (1)

$T(n-1) = 9\,T(n-2)$

$T(n) = 3^3\,T(n-3)$

⋮

$T(n) = 3^k\,T(n-k)$

$n-k = 0$

$\boxed{k = n}$

$T(n) = 3^n\,T(0)$

$\boxed{T(n) = 3^n}$

$\boxed{Tn = O(3^n)}$

Ans 8 (7)

$T(n) = T(\sqrt{n}) + 1$

$T(\sqrt{n}) = T(n^{1/4}) + 1$

$T(n) = T(n^{1/4}) + 2$

$T(n) = T(n^{1/8}) + 3$

⋮

$T(n) = T(n^{\frac{1}{k}}) + 2^k$

$n^{\frac{1}{k}} = 2$

$n^{\frac{1}{2^k}} = 1$

$\frac{1}{2^k} \log n = 1$

$2^k = \log n$

$\log_2 n = k \quad \log_2(\log n)$

$T(n) = O(\log(\log n))$

Ans 8 (c)

$T(n) = T(\sqrt{n}) + n$

$T(\sqrt{n}) = T(n^{1/4}) + n^{1/2}$

$T(n) = T(n^{1/4}) + n + n^{1/2}$

$T(n) = T(n^{1/8}) + (n + n^{1/2} + n^{1/4})$

$-$ $|$

$T(kn) = T(n^{1/k}) + n + n^{1/2} + n^{1/4} \ldots$ k terms

$n^{\frac{1}{2k}} = 1$

$2^k = \log n$

$k = \log(\log(n))$

$T(n) = 1 + (n + \sqrt{n} + \sqrt{n}\sqrt{n} + \ldots k\text{terms})$

$T(n) = 1 + \left(n \left(\frac{(\sqrt{n})^k - 1}{k-1} \right) \right)$

$T(n) = 1 + n \left[\frac{\sqrt{n}^{\log(\log n)} - 1}{\log \log(n) - 1} \right]$

$T(n) = n \cdot \log\log(n)$ (By neglecting other values)

$T(n) = O(n \cdot \log(\log n))$

Ans 9.

```
int sum = 0, i;
for ( i=0; i <n; i++)
  {
    sum+=i
  }
```

$0, 1, 2, \ldots n$

So $T(n) = O(n)$, Space $= O(1)$,

Ans 10 $O(N^*(N, N-1, \cdots 1))$

$O(N^*(\frac{N+1}{2}))$

$O(N^* N)$

Ans 11. $O(\frac{n}{2} * \log_2 N))$

$O(n \log n)$

12. (a) The best choice will be X for large inputs

13. (4) $O(\log n)$

Ans 14. $T(n) = 7(T(\frac{n}{2})) + (3n^2+2)$

$f(n) = 3n^2+2$

$a = 7$

$b = 2$

$c = \log_b a = \log_2 7 = 2.807$

$n^c = n^{2.8} \approx n^{2.8}$

$f(n) = 3n^2+2$

So $n^c > f(n)$

So $T(n) = \theta(n^{2.8})$

or (c) $\theta(n^{2.8})$
(a). $\theta(n^{2.8})$
(d) $\theta(n^3)$