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Fluids Mechanisms

MEC 303

* Definition of fluid:

Fluid is a substance that deforms continuously under the application of tangential force, however small it may be

If $\frac{\delta \rho}{\rho} < 5\%$ ⇒ incompressible

for water β_T (~~inco~~ isothermal compressibility) $= 5\times10^{-10}$ m^2/N

i.e it is incompressible

for air $\beta_T = 10^{-5}$ m^2/N ⇒ compressible

* Compressible & Incompressible flow

$$\text{Mach number } (M) = \frac{\text{velocity of flow of any fluid}}{\text{velocity of sound in that fluid}}$$

$$M = \frac{V}{a}$$

$M < 1$ → subsonic flow (domestic flights)

$1 < M < 5$ → supersonic flow (international flights)

$M > 5$ → hypersonic flow (fligher jets)

* Bulk Modulus

$$k = \frac{-dP}{dV/V}$$

$\rho = \frac{m}{V}$ mass is const. $\therefore \rho V = k$

$\therefore \rho dV + V d\rho = 0$ $\therefore \frac{dV}{V} = \frac{-d\rho}{\rho}$

$\therefore K = \dfrac{-dP}{dV/V} = \dfrac{dP}{d\rho/\rho}$ $\rho V = m$ m is const $\therefore \rho dV + V d\rho = 0$ $\therefore \dfrac{-V}{dV} = \dfrac{\rho}{d\rho}$

$\beta = \dfrac{1}{K}$ $a = \sqrt{K/\rho}$

$= \sqrt{\left(\dfrac{dP}{d\rho}\right)_{S=0}}$

β - compressibility
K - Bulk modulus a - velocity of sound in medium

Pressure

Stagnation = Static + Dynamic
P_0 P_S P_D

Stagnation :

When a fluid is brought to rest isoentropically the condition or state is called stagnation state. The properties at that state are called stagnat^n prop. ex. stagnation temp., stag. pressure

$$\boxed{P_0 = P_S + P_D}$$

imp Stagnation pressure is always lost in direct^n of flow.
(As air goes on expanding on heating, pressure will go on decreasing (i.e lost))

Static pressure is measured by instrument having relative velocity 0 wrt fluid.

$$a = \sqrt{\frac{k}{\rho}} = \sqrt{\left(\frac{dP}{d\rho}\right)_{s=0}} = \sqrt{\frac{\frac{1}{2}\rho v^2}{d\rho}}$$

$$a^2 = \frac{1}{2}\frac{\rho v^2}{d\rho}$$

$$\frac{v^2}{a^2} = 2\frac{d\rho}{\rho}$$ for incompressible fluid, $\frac{d\rho}{\rho} = 5/100$

$$M^2 = 2 \cdot \frac{5}{100} = \frac{1}{10}$$

$M \leq 0.3$ ... flow is incompressible

* Continuum

Continuous ~~flow~~ distribution of matter, i.e. no vacant place betn the molecules. If the Knudsen number,

$$K_n = \frac{\lambda}{L} < 0.01$$

the continuum ~~fleh~~ is assumed to be valid where λ is mean free path & L is characteristic length

Any phisysical prop at a point is defined as an average value of that property over a smallest possible volume region δv^* around that point
Smallest volume region δv^* is defined as that vol. when no. of molecules entering into & no. of molecules leaving out actually matter.

$$\vec{\vec{\tau}} = \begin{vmatrix} \tau_{xy} & \tau_{yz} & \tau_{zx} \\ \tau_{yz} & \tau_{zx} & \tau_{xy} \\ \tau_{zx} & \tau_{xy} & \tau_{yz} \end{vmatrix} \quad \begin{vmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{vmatrix}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ i & j & k \end{array} \qquad \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ i & j & k \end{array}$$

Stress is a tensor quantity having 2 directions.

τ_{xx}

↙ plane ↘ direct^n

* Reynold's Transport Theorem (RTT)

$$\frac{dN}{dt} = \frac{\partial}{\partial t}\iiint_{CV} \eta \rho \, d\underbrace{V}_{\text{vol.}} + \iint_{CS} \eta \rho \underbrace{\vec{V}}_{\text{velocity}} \cdot d\vec{A} \qquad \eta = \frac{\text{Extensive prop}}{\text{mass}}$$

CV = Control Vol. CS = Control Surface

first term: rate of change of mass stored in CV

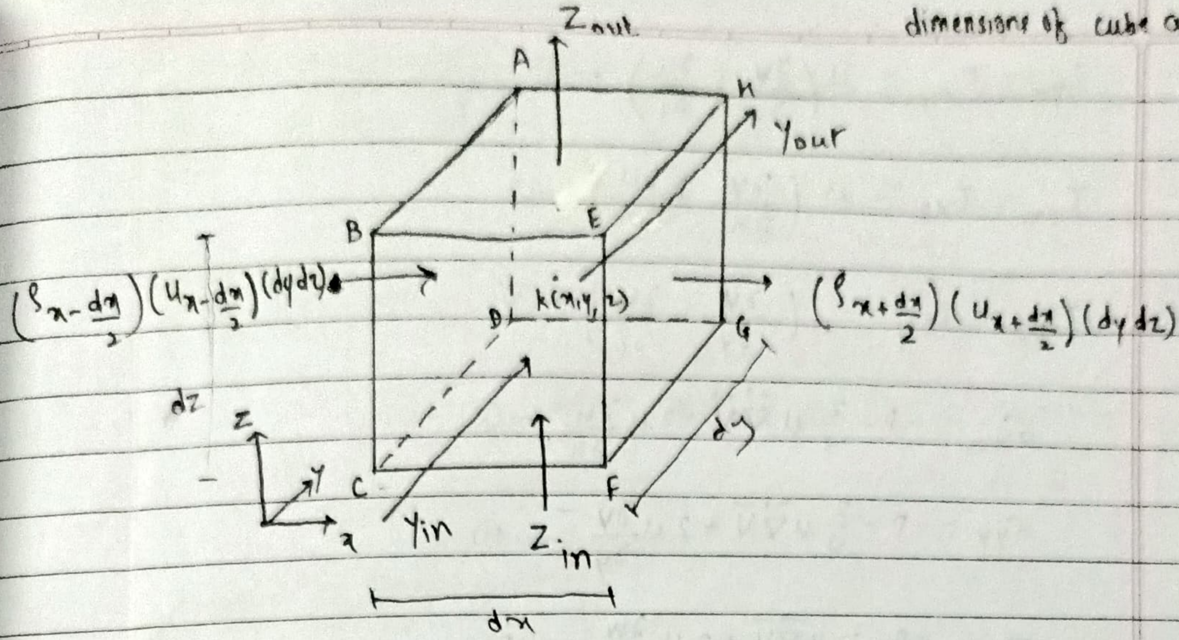
second term: Net outflow of mass across CS

Use: Convert control mass eq^n into Control Vol^n eq^n

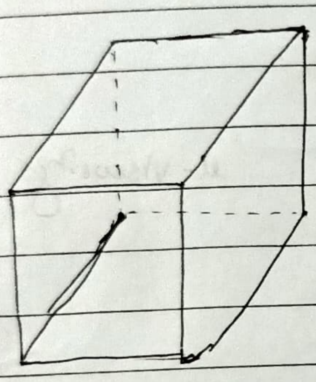
control mass - closed system

control vol - open system

dimensions of cube are dx, dy, ...



* Momentum eq^n



$$dF = \frac{D\vec{V}}{Dt}\,dm \quad \text{—(1)}$$

$$dF = (dF_{sx} + dF_{bx})\hat{i} + (dF_{sy} + dF_{by})\hat{j} + (dF_{sz} + dF_{bz})\hat{k}$$

$$dF_{sx} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) d\forall$$

$$dF_{sy} = \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x}\right) d\forall$$

$$dF_{sz} = \left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) d\forall$$

$$dF_{bx} = \rho\, d\forall\, g_x \qquad dF_{by} = \rho\, d\forall\, g_y \qquad dF_{bz} = \rho\, d\forall\, g_z$$

from (1),

$$\cancel{\rho d\forall}\,\frac{DU}{Dt} = \cancel{\rho}g_x + \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) \quad \text{—(2)}$$

$$\cancel{\rho d\forall}\,\frac{DV}{Dt} = \cancel{\rho}g_y + \frac{1}{\rho}\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right) \quad \text{—(3)}$$

$$\cancel{\rho d\forall}\,\frac{DW}{Dt} = \cancel{\rho}g_z + \frac{1}{\rho}\left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) \quad \text{—(4)}$$

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$$\tau_{yx} = \tau_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \quad -⑤$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad -⑥$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \quad -⑦$$

$$\sigma_{xx} = -p - \frac{2}{3}\mu\vec{\nabla}\vec{V} + 2\mu\frac{\partial u}{\partial x} \quad -⑧$$

$$\sigma_{yy} = -p - \frac{2}{3}\mu\vec{\nabla}\vec{V} + 2\mu\frac{\partial v}{\partial y} \quad -⑨$$

$$\sigma_{zz} = -p - \frac{2}{3}\mu\vec{\nabla}\vec{V} + 2\mu\frac{\partial w}{\partial z} \quad -⑩$$

hydrostatic / Thermodynamic pressure — vol. dilation — lineardilation

μ - viscosity.

$$\cancel{\frac{DV}{Dt} = \vec{g} - \frac{\vec{\nabla}\vec{p}}{\rho} +}$$

Navier Stokes equation

$$\frac{DV}{Dt} = \vec{g} - \frac{\vec{\nabla}\vec{p}}{\rho} + \frac{\mu}{\rho}\cdot\nabla^2 V$$

continuity eq^n in x,y,z coordinate system
& r,θ,z coordinate system

Momentum eq^n

Euler's eq^n along streamline & normal to the Streamline

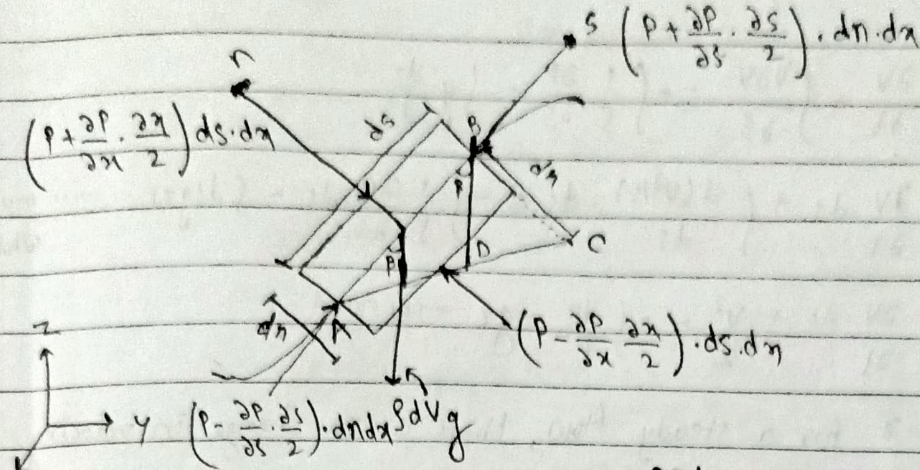
Integral form of energy eq^n.

$$\frac{D\vec{V}}{Dt} = \left(\frac{\partial}{\partial t} + \vec{V}\cdot\vec{\nabla}\right)V = \frac{\partial V}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z}$$

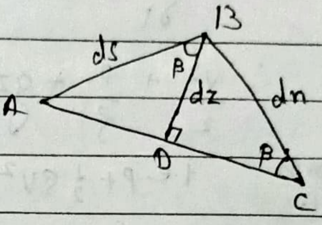
$\frac{Du}{Dt} = \rho g_x$ + put $\sigma_{xx}, \tau_{yx}, \tau_{zx}$ values in ②, ③, ④ from eq^{n}s ⑤ to ⑩

Euler's eqⁿ along streamline & normal to streamline

flow is steady, homogeneous, incompressible, non viscous



BD is perpendicular to AC



$$\cos\beta = \frac{dz}{ds}$$

$$\sin\beta = \frac{dz}{dn}$$

$$\left(P-\frac{\partial P}{\partial s}\cdot\frac{\partial s}{2}\right)dndx - \left(P+\frac{\partial P}{\partial s}\frac{\partial s}{2}\right)dndx - \rho dVg\cos\beta = \rho dV\frac{DV_s}{Dt}$$

component of ρdVg in s directⁿ

mass × acceleⁿ in s directⁿ

~~velocity~~ accelⁿ in s directⁿ is ~~net velocity itself~~ because there is no velocity in n dir.

$V_s = V$

$$-\frac{\partial P}{\partial s}\, ds\cdot dn\cdot dx - \rho dVg\frac{dz}{ds} = \rho dV\frac{DV}{Dt}$$

$ds\cdot dn\cdot dx = dV$

$$\therefore \boxed{\frac{DV}{Dt} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial s} - g\frac{dz}{ds}}$$

integrating,

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_s\frac{\partial V}{\partial s} + V_n\frac{\partial V}{\partial n}$$

$V_n = 0$

V along normal

$$= \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

$\because V_s = V$

multiplying by ds on both sides of Euler's eq^n & integrating both sides,

$$\int \frac{\partial V}{\partial t} + \int \frac{V \partial V}{\partial s} = -\int \frac{1}{\rho}\frac{\partial P}{\partial s} - \int g \frac{dz}{ds}$$

$$\therefore \int \frac{\partial V}{\partial t} ds + \int \frac{d(V^2/2)}{ds} \cdot ds = -\int \frac{1}{\rho}\frac{\partial P}{\partial s} \cdot ds - \int d(gz)$$ ..... multiplied whole by ds

$$\therefore \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} = -\int \frac{dP}{\rho} - gz + \text{constant}$$

for a steady flow, there is no change in velocity

$$\therefore \frac{\partial V}{\partial t} = 0$$

$$\therefore \frac{V^2}{2} + \frac{P}{\rho} + gz = \text{const.}$$

$k = \frac{\text{energy}}{\text{vol.}}$ i.e $P + \frac{1}{2}\rho V^2 + \rho g z = k$ — Bernoulli's eq^n

along normal to streamline,

$$-\frac{\partial P}{\partial n} d\forall - \rho g\, d\forall \sin\beta = \rho a_n d\forall$$

a_n is accel^n along normal

$\because$ it is centrifugal accl^n,

$a_n = -V^2/R$

$$-\frac{\partial P}{\partial n} - \rho g \frac{dz}{dn} = -\frac{\rho V^2}{R}$$

$$\therefore \boxed{\frac{\partial P}{\partial n} + \rho g \frac{dz}{dn} = \frac{\rho V^2}{R}}$$

for real fluid,

$$\underbrace{\frac{P_1}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{V_1^2}{2g}}_{\text{kinetic energy head}} + \underbrace{Z_1}_{\text{potential head}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{\text{energy}}{\text{vol.}} = P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

* Integral form of energy eqn

$$\delta\dot{Q} - \delta\dot{W} = \frac{dE}{dt}_{cm} \qquad \text{cm control mass}$$

we've to convert it into control vol.

∴ we'll use Reynold's Transfer Thm.

$$\frac{dN}{dt} = \frac{\partial}{\partial t}\iiint_{CV} \eta \rho \, d\forall + \iint_{CS} \eta \rho V dA$$

$$\eta = \frac{\text{Extensive prop}}{\text{mass}} = \frac{E}{m} = e \text{ (say)}$$

$$\therefore \delta\dot{Q} - \delta\dot{W} = \frac{\partial}{\partial t}\iiint_{CV} e \rho \, d\forall + \iint_{CS} e \rho V dA$$

$$\delta\dot{W} = \delta\dot{W}_{shaft} + \cancel{\delta\dot{W}_{shear}}^{\,0} + \delta\dot{W}_{pressure} + \cancel{\delta\dot{W}_{other}}^{\,0}$$

$$\delta\dot{W}_{pressure} = \iint_{CS} (p\, d\vec{A}).\vec{V}$$

$$\therefore \delta\dot{Q} - \delta\dot{W}_{shaft} - \iint_{CS} (p\, d\vec{A}).\vec{V} = \frac{\partial}{\partial t}\iiint_{CV} e\rho \, d\forall + \iint_{CS} e \rho V dA$$

$$\therefore \delta\dot{Q} = \frac{\partial}{\partial t}\iiint_{CV} e\rho \, d\forall + \iint_{CS} (p + e\rho)\vec{V} d\vec{A} + \delta\dot{W}_{shaft}$$

~~= 0 it is 0 for streamline flow (steady state flow)~~

$$\therefore \frac{\delta \dot{Q}}{\delta m}\cdot\frac{\delta m}{\delta t} = \cancel{\frac{\partial}{\partial t}\iiint_{CV} e\rho \, d\forall} + \iint_{CS} \left(p\cdot\frac{1}{\rho} + e\right)\rho V \cdot dA + \frac{\delta W}{\delta m}\frac{\delta m}{\delta t}$$

$$q\cdot\dot{m} = \iint_{CS} (p\upsilon + e)\rho V dA + w_{shaft}\,\dot{m}$$

$$q\cdot\dot{m} = \iint_{CS} (p\upsilon + e)\, d\dot{m} + w_{shaft}\,\dot{m}$$

Flow is uniform at a sectn ∴ $(p\upsilon + e)$ is const. $\frac{1}{\rho} = \upsilon$ = specific vol. = vol/mass

$$q = (e + p\upsilon)_2 - (e + p\upsilon)_1 + w_{shaft}$$

now, $e = \frac{E}{m}$ energy per mass

$\therefore$ from Euler's eq^n,

$$e = \frac{E}{m} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

$$= u + \frac{V^2}{2} + gz$$

internal energy per mass

kinetic energy per mass

potential energy per mass

$$\therefore \cancel{q = \left(u + \frac{V^2}{2} + gz\right)_2 - \left(u + \frac{V^2}{2} + gz\right)_1}$$

$$q = \left(u + \frac{V^2}{2} + gz + Pv\right)_2 - \left(u + \frac{V^2}{2} + gz + Pv\right)_1 + w_{shaft}$$

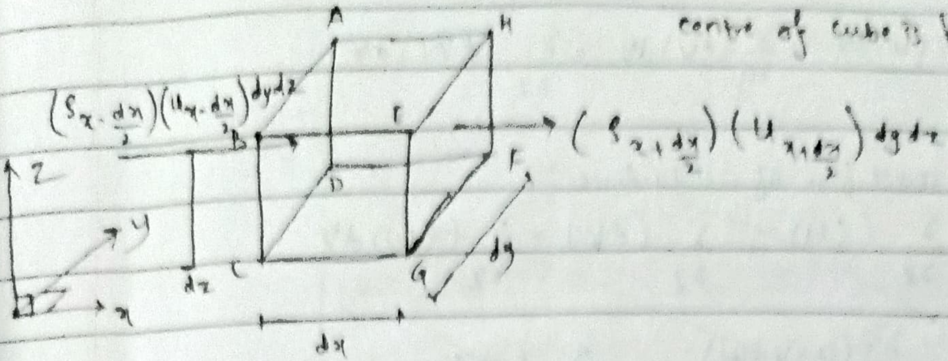
$$\therefore h_1 + \frac{V_1^2}{2} + gz_1 + q_{in} = h_2 + \frac{V_2^2}{2} + gz_2 + w_{shaft}$$

$\because H = U + PV$ $\quad \therefore \frac{H}{m} = \frac{U}{m} + \frac{PV}{m}$

$\therefore h = u + Pv$ $\qquad \frac{V}{m} = v$

* Continuity eq^n in x-y-z co-ordinate system

centre of cube is f(x,y,z)



$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k} \qquad \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$\rho_{(x-\frac{dx}{2})}$ is ~~partial derivative~~ density of flow of ρ ~~wrt~~ at $x - \frac{dx}{2}$

$$\therefore \rho_{x-\frac{dx}{2}} = \rho - \frac{\partial \rho}{\partial x}\cdot\frac{dx}{2} + \cancel{\frac{\partial^2 \rho}{2!\,\partial x^2}\left(\frac{dx}{2}\right)^2 + 0 \cdots\cdots}$$

.... Taylor series

$$f(x+h) = f(x) + \frac{hf'(x)}{1!} + \frac{h^2 f''(x)}{2!} + \cdots$$

$$\rho_{x+\frac{dx}{2}} = \rho + \frac{\partial \rho}{\partial x}\frac{dx}{2}$$

$$u_{x-\frac{dx}{2}} = u - \frac{\partial u}{\partial x}\frac{dx}{2}$$

$$u_{x+\frac{dx}{2}} = u + \frac{\partial u}{\partial x}\frac{\partial x}{2}$$

Net mass flux $= (\text{out})_x - (\text{in})_x$

—— is mass per unit area per unit time

$$\therefore \text{Net mass flux along } x = \left[\left(\rho + \frac{\partial \rho}{\partial x}\frac{\partial x}{2}\right)\left(u + \frac{\partial u}{\partial x}\frac{\partial x}{2}\right) - \left(\rho - \frac{\partial \rho}{\partial x}\frac{dx}{2}\right)\left(u - \frac{\partial u}{\partial x}\cdot\frac{\partial x}{2}\right)\right]dydz$$

$$= \frac{\partial}{\partial x}(\rho u)\,d\forall = \frac{\partial(\rho u)}{\partial x}\,d\forall$$

Mass flux out along y = out$_y$ - in$_y$

$$= \frac{\partial(\rho v)}{\partial y}\cdot d\forall$$

Mass flux is in ~~ret~~^{n} z dir = out$_z$ - in$_z$

$$= \frac{\partial}{\partial z}(\rho w)\,d\forall$$

net mass flux of all surfaces

$$= \frac{\partial}{\partial x}(\rho u)\, d\forall + \frac{\partial}{\partial y}(\rho v)\, d\forall + \frac{\partial}{\partial z}(\rho w)\, d\forall$$

~~= Net mass flux of all surfaces:~~

$$\cancel{= \frac{\partial}{\partial z}(\rho u) + \frac{\partial}{\partial z}(\rho v) + \frac{\partial}{\partial z}(\rho w)\, d\forall}$$

$$\cancel{= \frac{\partial \rho (u+v+w)}{\partial z} = \frac{\partial}{\partial z}(\rho V . d\forall)}$$

~~Time rate of change of mass in control vol.~~

$$\cancel{dV = 0 \quad \therefore V d\rho + \rho dV = 0}$$

$$\cancel{\frac{\partial \rho}{\partial t} + \vec{\nabla} \rho \vec{V}}$$

Rate of change in mass within control vol. $= \frac{\partial (\rho\, d\forall)}{\partial t}$

from conservation of mass law, $\frac{dm}{dt} = 0$

i.e Net mass flow rate out through control surface + rate of change of mass within control volume $= 0$

$$\therefore \left(\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} \right) d\forall = 0$$

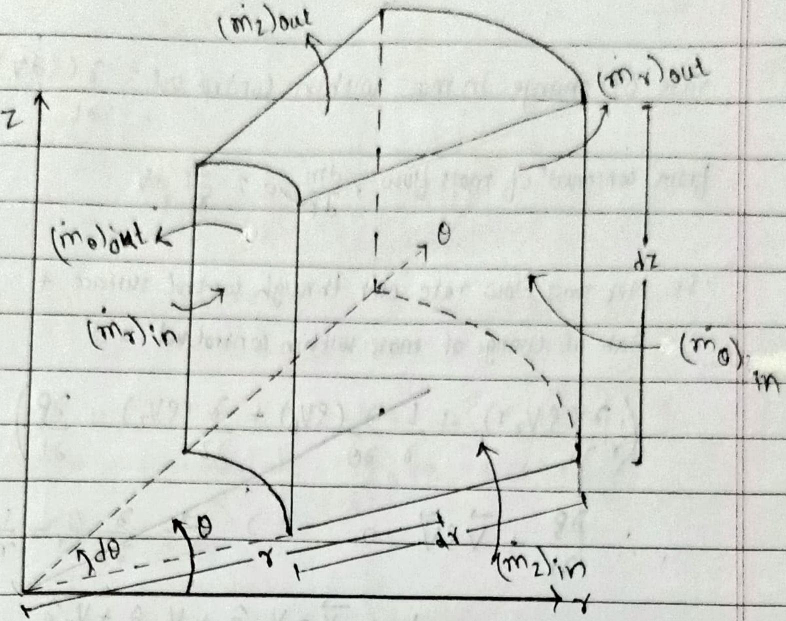
$$\therefore \frac{\partial \rho}{\partial t} + \vec{\nabla} . \rho \vec{V} = 0$$

$$\text{i.e. } \frac{\partial \rho}{\partial t} + \rho \vec{\nabla} V + \vec{V} \vec{\nabla} \rho = 0$$

for incompressible flow, ρ = constant $\Rightarrow \rho \vec{\nabla} V = 0$

for steady flow, $\rho \nabla \vec{V} + V \nabla \rho = 0$

* Continuity eqⁿ in r, θ, z co-ordinate system (cylindrical syst.)



$d\forall = r\,d\theta\,dr\,dz$

$\vec{V} = V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{e}_z$

mass flux is mass per sec

$\dot{m}$ = density × velocity × area

$$(\dot{m}_r)_{in} = \rho_{\left(r-\frac{dr}{2}\right)} V_{r\left(r-\frac{dr}{2}\right)} \left(r-\frac{dr}{2}\right) d\theta\, dz$$

$$(\dot{m}_r)_{out} = \rho_{\left(r+\frac{dr}{2}\right)} V_{r\left(r+\frac{dr}{2}\right)} \left(r+\frac{dr}{2}\right) d\theta\, dz$$

net mass flux through r directⁿ $= (\dot{m}_r)_{out} - (\dot{m}_r)_{in}$

$$= \rho_{\left(r+\frac{dr}{2}\right)} V_{r\left(r+\frac{dr}{2}\right)} \left(r+\frac{dr}{2}\right) d\theta\, dz - \rho_{\left(r-\frac{dr}{2}\right)} V_{r\left(r-\frac{dr}{2}\right)} \left(r-\frac{dr}{2}\right) d\theta\, dz$$

$$= \frac{\partial}{\partial r}(\rho V_r r\, d\theta\, dz\, dr) = \frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r\, r\, d\theta\, dr\, dz) = \frac{1}{r}\frac{\partial (r\rho V_r\, d\forall)}{\partial r}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(\rho V_r r)\, d\forall$$

similarly, mass flux through θ dir $= (\dot{m}_\theta)_{out} - (\dot{m}_\theta)_{in}$

$$= \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta)\, d\forall$$

mass flux through z dir $= \dfrac{\partial}{\partial z}(\rho V_z)\, d\forall$

$$\dot{m} = \frac{dm}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot dA$$

rate of change in mass within control vol $= \frac{\partial(\rho\, d\forall)}{\partial t}$

from conservat^n of mass flow, $\frac{dm}{dt} = 0 \cancel{\neq \frac{\partial\rho}{\partial t} d\forall}$

i.e. Net mass flow rate out through control surface + Rate of change of mass within control vol $= 0$

$$\left(\frac{1}{r}\frac{\partial(\rho V_r r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t}\right) d\forall = 0$$

$$\therefore \frac{\partial \rho}{\partial t} + \vec{\nabla}\rho\vec{V} = 0 \qquad \vec{\nabla} = \frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{e}_\theta + \frac{\partial}{\partial z}\hat{e}_z$$

here, $\vec{V} = V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{e}_z$

Incompressible flow is possible or not?

1] $u = x + 2y \qquad v = x^2 - y^2$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ — continuity eq^n

$LHS = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + 0 + 0 - 2y = \cancel{1 - 2y} \neq 0$
$\neq RHS$

$\therefore$ Not possible

2] $u = x + y, \; v = x - y$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + 0 + 0 - 1 = 0$

Flow is possible

3] find x component for flow to be possible when y component is $v = y^2 - 2x + 2y$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial v}{\partial y} = 2y + 2$

$\therefore \frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y} = -2(y+1)$

$\therefore \partial u = -2(y+1)\,\partial x$

$$\dot{m} = \frac{dm}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot dA$$

rate of change in mass within control vol $= \frac{\partial(\rho \, d\forall)}{\partial t}$

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$$\left(\frac{1}{r}\frac{\partial(\rho V_r r)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) + \frac{\partial \rho}{\partial t}\right) d\forall = 0$$

$$\therefore \frac{\partial \rho}{\partial t} + \vec{\nabla} \rho \vec{V} = 0 \qquad \vec{\nabla} = \frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{e}_\theta + \frac{\partial}{\partial z}\hat{e}_z$$

here, $\vec{V} = V_r \hat{e}_r + V_\theta \hat{e}_\theta + V_z \hat{e}_z$

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LHS $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + 0 + 0 - 2y = 1 - 2y \neq 0$
$\neq$ RHS

$\therefore$ Not possible

2] $u = x + y, \; v = x - y$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + 0 + 0 - 1 = 0$

Flow is possible

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$\therefore \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = -2(y+1)$

$\therefore \partial u = -2(y+1)\,\partial x$

$\therefore u = -2x(y+1) + f(y) \rightarrow$ infinite possible solⁿs

$\vec{V} = u\hat{i} + v\hat{j}$

$= [-2(y+1)x]\hat{i} + [y^2 - 2x + 2y]\hat{j}$

$\vec{V}_{(1,1)} = [-4]\hat{i} + \hat{j}$

$\vec{a} = \frac{D\vec{V}}{Dt} = \left(\frac{\partial}{\partial t}^{0} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(u\hat{i} + v\hat{j})$

for steady flow, $\frac{\partial}{\partial t} = 0$

$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\hat{i} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\hat{j}$

$= \{[-2x(y+1)](-2y-2) + [y^2-2x+2y](-2x)\}\hat{i} +$
$\{[-2x(y+1)](-2) + [y^2-2x+2y](y+2)2\}\hat{j}$

$\vec{a}_{(1,1)}$ = put $x=1, y=1$

4] Incompressible flow is possible or not?

$V_r = u\cos\theta \quad V_\theta = -u\sin\theta$

$\frac{\partial(rV_r)}{\partial r} + \frac{\partial(V_\theta)}{\partial\theta} = 0$

LHS $= u\cos\theta - u\cos\theta = 0$ RHS $= 0$ $\therefore$ possible

5] $V_\theta = -\frac{A\sin\theta}{r^2}$ Find possible r components of velocity how many possible r components are there?

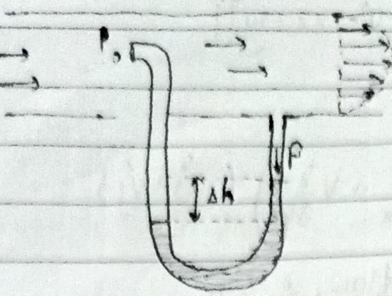
con^n - $\frac{\partial(rV_r)}{\partial r} = \frac{-\partial(V_\theta)}{\partial\theta} \Rightarrow \frac{\partial(rV_r)}{\partial r} = \frac{A\cos\theta}{r^2}$

$\therefore \int \partial(rV_r) = \int \frac{A}{r^2} \cancel{\sin\theta}\cos\theta \, dr + f(\theta)$

$\therefore rV_r = A\cos\theta \cdot \frac{-1}{r} + f(\theta)$

$\therefore V_r = \frac{-A\cos\theta}{r^2} + \frac{f(\theta)}{r}$

* Pitot tube



$$\frac{P_0 - P}{\rho_{Hg}\, g}$$

$$P_0 - P = \frac{1}{2} \rho_f V_f^2$$

$$\Delta h \rho_{Hg}\, g = \frac{1}{2} \rho_f V_f^2$$

$$\therefore \quad V_f = \sqrt{\frac{2 \Delta H \rho_{Hg}\, g}{\rho_f}} = \sqrt{\frac{2 \Delta H S_{Hg}\, g}{S_f}}$$

$$\rho_{Hg} / \rho_{water} = S_{Hg} = \text{specific gravity}$$

1] There's a rise of 30 mm of Hg when a fluid of density 1.23 is used in pitot tube experiment find the velocity of the flow of fluid

$$V_f = \sqrt{\frac{2 \Delta h S_{Hg}\, g}{S_f}} \qquad S_f = \frac{\rho_f}{\rho_{H_2O}} = \frac{1.23}{1000}$$

$$= \sqrt{\frac{2 \times 30 \times 10^{-3} \times 13.6 \times 9.78 \times 10^{3}}{1.23}} , \quad S_{Hg} = 13.6$$

$$= 80.672 \text{ m/s}$$

A pitot tube is an instrument for measuring a flowing fluid's velocity. They are used to calculate airspeed in wind tunnels & aircraft in flight. It consists of 2 hollow tubes that measure pressure at various places within the pipe. One tube gauges the impact or stagnation pressure, other tube measures only the static pressure usually at the pipe wall. These hollow tubes can be mounted separately in a pipe or installed together in one casing as a single device.

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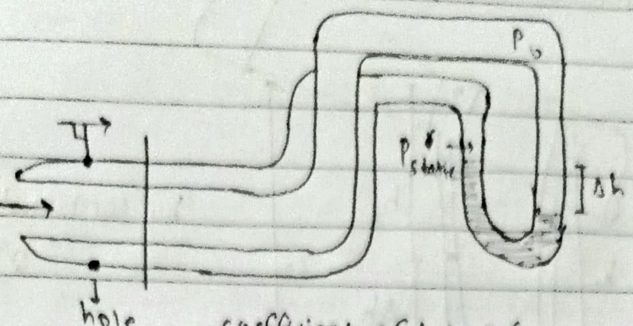
* Pitot Static Tube

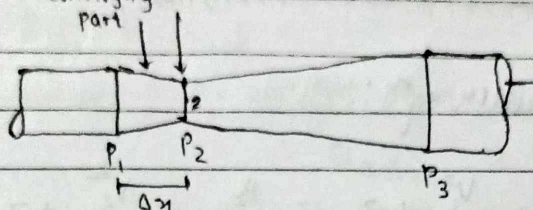


$$\Delta h = \frac{P_0 - P}{\rho_{Hg}\, g}$$

coefficient of tube $= C_V$

$$V_f = C_V \sqrt{2 g \Delta h \left(\frac{\rho_{Hg}}{\rho_f} - 1\right)}$$

Ventury meter



$$Q = AV$$

$P_1 > P_2$

Favourable pressure gradient $= \dfrac{P_1 - P_2}{\Delta x}$

Adverse pressure gradient $= \dfrac{P_3 - P_2}{\Delta x}$ (due to this flow reverse)

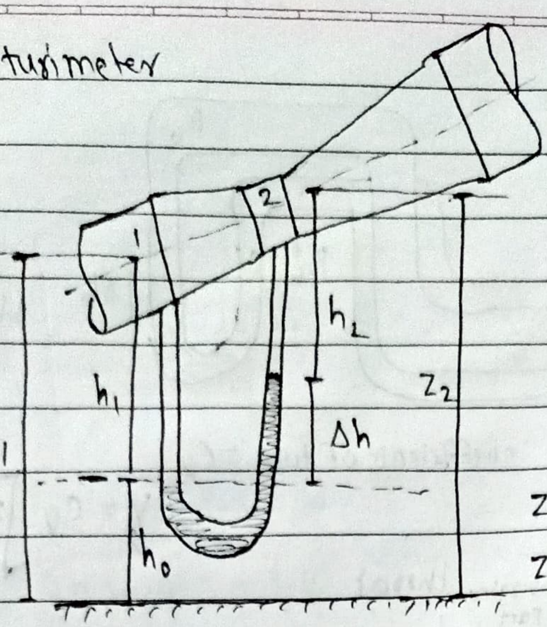
by $A_1V_1 = A_2V_2$, if $A_1 > A_2 \Rightarrow V_1 < V_2$

by Bernaulli's eqn $P_1 > P_2 \Rightarrow V_1 < V_2$

$\therefore A_1 > A_2 \Rightarrow P_1 > P_2$

At the middle point of the cross sectn, velocity is greater but at the vicinity of edges of pipe, velocity is far lesser & hence the fluid particles revert back because they aren't able to overcome adverse pressure gradient.

* Venturimeter



by continuity eq^n,

$$Q = A_1V_1 = A_2V_2 \quad -①$$

$$Z_1 = h_0 + h_1$$

$$Z_2 = h_0 + \Delta h + h_2$$

Bernoulli's eq^n:

$$\frac{P_1}{\rho_f g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_f g} + \frac{V_2^2}{2g} + Z_2 \quad -②$$

$$P_1 + \rho_f g h_1 = P_2 + \rho_f g h_2 + \rho_{Hg} g \Delta h \rightarrow \text{equating pressure at dotted line}$$

$$\therefore P_1 + \rho_f g (Z_1 - h_0) = P_2 + \rho_f g (Z_2 - h_0 - \Delta h) + \rho_{Hg} g \Delta h$$

$$\therefore \frac{P_1}{\rho_f g} + Z_1 - h_0 = \frac{P_2}{\rho_f g} + Z_2 - h_0 - \Delta h + \Delta h \frac{\rho_{Hg}}{\rho_f}$$

$$\therefore \left(\frac{P_1}{\rho_f g} + Z_1\right) - \left(\frac{P_2}{\rho_f g} + Z_2\right) = \Delta h \left(\frac{\rho_{Hg}}{\rho_f} - 1\right) \quad -③$$

$$\frac{\rho_{Hg}/\rho_{H_2O}}{\rho_f/\rho_{H_2O}} = \frac{S_{Hg}}{S_f}$$

$$\rho_{H_2O} = 1000\ kg/m^3$$

from eq^n ②, & from eq ③,

$$\left(\frac{P_1}{\rho_f g} + Z_1\right) - \left(\frac{P_2}{\rho_f g} + Z_2\right) = \frac{V_2^2 - V_1^2}{2g}$$

i.e.

$$\boxed{\begin{aligned} 2g\left[\left(\frac{P_1}{\rho_f g} + Z_1\right) - \left(\frac{P_2}{\rho_f g} + Z_2\right)\right] &= V_2^2 - V_1^2 = \frac{A_1^2 V_1^2}{A_2^2} - V_1^2 \\ &= A_1^2 V_1^2 \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = Q^2 \left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right) = 2g\left(\frac{S_{Hg}}{S_f} - 1\right)\Delta h \end{aligned}}$$

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∵ we've considered Bernaulli's eqⁿ & continuity for ideal fluid, there is no frictⁿ resistance considered

∴ Q can be taken as Q_{th} (theoretical)

$$\boxed{Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\Delta h\left(\frac{S_{Hg}}{S_f} - 1\right)}}$$

$Q_{actual} = C_d Q_{th}$ $\qquad$ C_d - coefficient of discharge

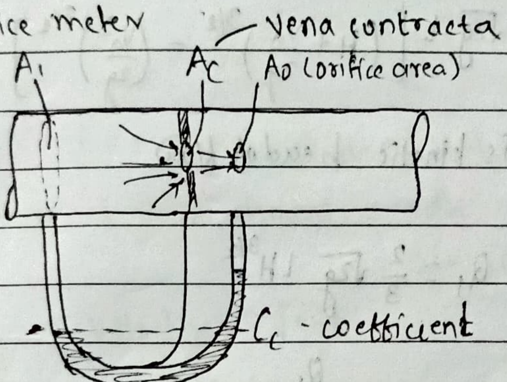
$C_d < 1$

$Q_{th} > Q_{actual}$

for venturimeter, $C_d = 0.95 - 0.99$

∴ $Q_{actual} \approx Q_{th}$

* Orifice meter



$A_O > A_C$

orifice is drill hole

$$\boxed{A_C = A_O C_c}$$

$$A_c V_c = Q_{th} = \frac{A_1 A_c}{\sqrt{A_1^2 - A_c^2}} \sqrt{2g\left(\frac{S_{Hg}}{S_f} - 1\right)\Delta h} \quad \text{—④}$$

Q_{actual} = Area$_{actual}$ Velocity$_{actual}$ $\qquad$ velocity$_{actual}$ = $C_v V_c$

C_v is some constant

i.e. $Q_{ac} = A_c C_v V_c$

$= A_o C_c C_v V_c$

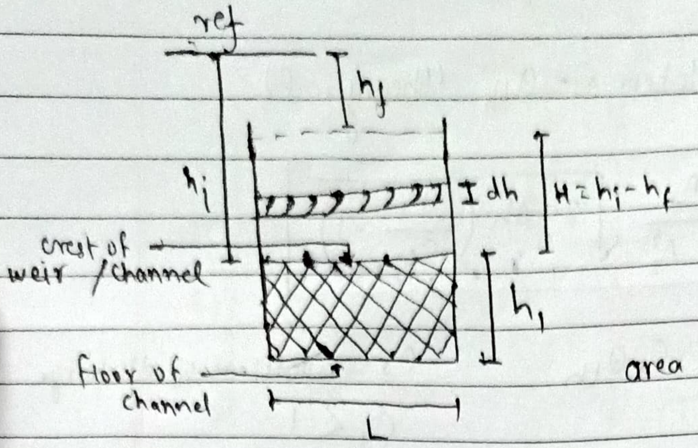
$= A_o C_d V_c$ $\qquad$ ∴ $C_c C_v = C_d$

$$= A_o C_d \times \frac{A_1}{\sqrt{A_1^2 - A_c^2}} \sqrt{2g\Delta h\left(\frac{S_{Hg}}{S_f} - 1\right)} \quad \text{.... from ④}$$

$$\boxed{Q_{ac} = k A_o \sqrt{2g\Delta h\left(\frac{S_{Hg}}{S_f} - 1\right)}}$$

k = flow constant

* Cross Sectⁿ of a channel



velocity $= \sqrt{2gh}$

area of fluid element $= L\,dh$

$$\int_0^Q dQ = \int_0^H L\sqrt{2gh}\,dh = \frac{2}{3}\sqrt{2g}\,LH^{3/2} \quad \ldots \text{ when } V_a = 0$$

Velocity of approach $= V_a$

$$\therefore Q = \frac{2}{3}\sqrt{2g}\,L\left[\left(H + \frac{V_a^2}{2g}\right)^{3/2} - \left(\frac{V_a^2}{2g}\right)^{3/2}\right]$$

$V_a^2/2g$ is kinetic head of flow

for $V_{a_1} = 0$, $\quad Q_1 = \frac{2}{3}\sqrt{2g}\,LH^{3/2}$

for $V_{a_2} \neq 0$, $\quad V_{a_2} = \dfrac{Q_1}{(H+h_1)L}$

$$Q_2 = \frac{2}{3}\sqrt{2g}\,L\left[\left(H + \frac{V_{a_2}^2}{2g}\right)^{3/2} - \left(\frac{V_{a_2}^2}{2g}\right)^{3/2}\right]$$

if $|Q_2 - Q_1| < 10^{-3} \Rightarrow Q_1 = Q$ & $V_{a_2} = V_a$

if not,

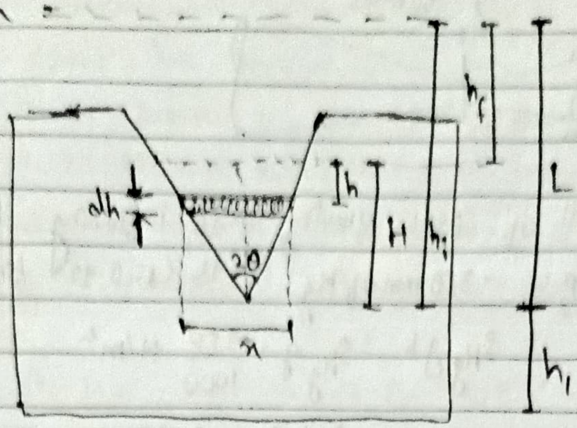
$$V_{a_3} = \frac{Q_2}{(H+h_1)L}$$

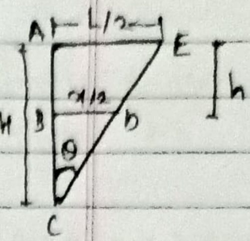
$$Q_3 = \frac{2}{3}\sqrt{2g}\,L\left[\left(H + \frac{V_{a_3}^2}{2g}\right)^{3/2} - \left(\frac{V_{a_3}^2}{2g}\right)^{3/2}\right]$$

if $|Q_3 - Q_2| < 10^{-3} \Rightarrow Q_3 = Q$ & $V_{a_3} = V_a$

If not check for V_{a_4} & so on

V-notch -





$\Delta ACE \sim \Delta BCD$

$$\frac{L/2}{x/2} = \frac{H}{H-h} \qquad \therefore x = \frac{L(H-h)}{H}$$

$$\tan\theta = \frac{L}{2H} \qquad \therefore L = 2H\tan\theta$$

$$\therefore x = \frac{2H\tan\theta\,(H-h)}{H}$$

$$x = 2\tan\theta\,(H-h)$$

area of fluid element $= x\,dh$

$= 2\tan\theta\,(H-h)\,dh$

velocity $= \sqrt{2gh}$

$$dQ = \int_0^Q 2\tan\theta\,(H-h)\sqrt{2gh}\,dh$$

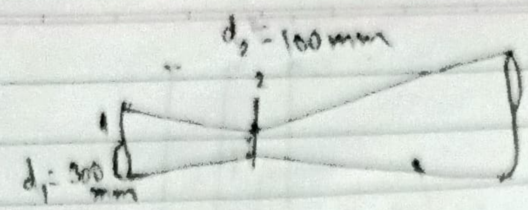
$$Q = \frac{8}{15}\sqrt{2g}\tan\theta\,H^{5/2} \quad \text{when } V_a = 0$$

when $V_a \neq 0$,

$$Q = \int_{V_a^2/2g}^{H+V_a^2/2g} 2\tan\theta\,(H-h)\sqrt{2gh}\,dh$$

$$Q = \frac{8}{15}\sqrt{2g}\tan\theta\left[\left(H+\frac{V_a^2}{2g}\right)^{5/2} - \left(\frac{V_a^2}{2g}\right)^{5/2}\right]$$

1)



Oil of specific gravity 0.88 is flowing $P_1 = 130 \times 10^3$ N/m²

is $C_d = 0.92$ find actual discharge

$P_2 = -350$ mm of Hg

$= \rho_{Hg}\, g h = \rho_{Hg}\, g \times \frac{-350}{1000}$ N/m²

$$Q_{actual} = C_d Q_{th} = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left[\left(\frac{P_1}{\rho_{oil} g} + z_1\right) - \left(\frac{P_2}{\rho_{oil} g} + z_2\right)\right]}$$

$\because$ horizontal, $z_1 = z_2$ $z_1 - z_2 = 0$

$S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$ $\therefore \rho_{oil} = 0.88 \times 1000$

$= 880$

2) In a pitot tube, specific gravity of fluid is 1.25. Flow of water is 1.417 m/s. Find rise in mercury level

$P_0 = P_{static} + P_{dynamic}$

$P_0 - P_{st} = \frac{1}{2} \rho_w V_w^2 = \frac{1}{2} \times 1000 \times (1.417)^2$

$P_{dynamic} = \rho_f g h$

$\therefore h = \frac{500 \times (1.417)^2}{\rho_f \times 9.78}$

$S_f = \frac{\rho_f}{\rho_w}$ $\rho_f = S_f \times \rho_w$

$= 1.25 \times 1000$

$h = \frac{500 \times (1.417)^2}{1.25 \times 1000 \times 9.78} = 81.87$ mm

3) A vertical venturimeter having specific gravity 0.8 & actual flow rate 40 lit/sec has coefficient of discharge 0.96. It has inlet diameter 150 mm & ~~outlet~~ throat diameter 75 mm. The pressure correct^n at the throat is 150 mm above that at the inlet.
Find pressure difference bet^n inlet & throat & difference in level of mercury in a vertical U tube manometer connected bet^n these points.

$Q_{ac} = 40$ lit/sec $\quad S_f = 0.8 \quad \rho_f = 0.8 \times 1000 = 800$

$= 40 \times 10^{-3}$ m^3/sec $\quad C_d = 0.96 \quad d_1 = 150$ mm

$Z_2 - Z_1 = 150$ mm $\quad d_2 = 75$ mm

$$Q_{ac} = C_d Q_{th}$$

$$\frac{Q_{ac}}{C_d} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \left[2g\left(\frac{P_1 - P_2}{\rho_f g} + Z_1 - Z_2\right)\right]^{1/2}$$

put all values to get $P_1 - P_2$

$P_1 - P_2 = \rho_{Hg} g \delta h$ we'll get δh from here

4) loss of head from the entrance to the throat of a 254 mm, 127 mm diameter venturimeter is 1/6 times the throat velocity head. If the mercury in the differential gauge attached to the meter deflects 101.6 mm. What is the flow of water through venturimeter?

$\because$ horizontal venturimeter $\therefore Z_1 - Z_2 = 0$

$C_d = \frac{H - h_f}{H}$ where total head $= H = \frac{P_1 - P_2}{\rho_f g}$

h_f = loss of head $= \frac{V^2}{2g}$

$\delta h = 101.6$

$h_f = \frac{1}{6} \frac{V_2^2}{2g} = \frac{V_2^2}{12g}$ $\quad S_{Hg} = 13.6 \quad S_w = \rho_{water}/\rho_{water} = 1$

$2g\left[\frac{P_1 - P_2}{\rho_f g} + \cancel{Z_1 - Z_2}\right] \cancel{= 2g\delta h\left(\frac{S_{Hg}}{S_f} - 1\right)}$ or $P_1 - P_2 = \rho_{Hg} g \Delta h$

~~ওখান থেকে $P_1 - P_2$ এর value পাবো~~

PTO এখান $P_1 - P_2$ নিবো

5] x component of velocity in a steady incompressible flow field in xy plane is u =

5] for steady, incompressible, laminar flow of thickness h,

find continuity & Navier-Stokes eq^n

Velocity profile

Shear stress eq^n

NS eq^n is

$$\rho \frac{DV}{Dt} = \rho g - \nabla P + \mu \nabla^2 V$$

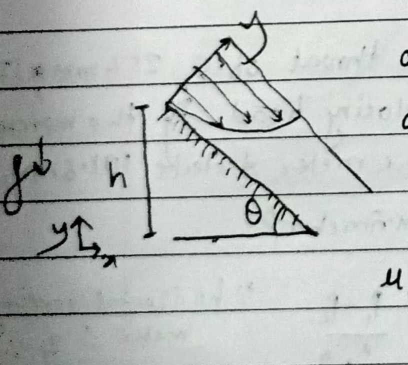
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

LHS = 0

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

LHS = 0

all terms in z component will be 0 $\because$ it is 2D flow



at $y = 0$, $u = v = 0$

at $y = h$, $du/dy = 0$ also $dv/dy = 0 \Rightarrow v = 0$

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho g \sin\theta = 0$$

$$\therefore \frac{\partial u}{\partial y} + \frac{\rho g \sin\theta}{\mu} y = C_1$$

$$u + \frac{y^2 \rho g \sin\theta}{2\mu} = C_1 y + C_2$$

$C_2 = 0$ at $y = h$, $du/dy = 0$

$\therefore C_1 = \rho g h \sin\theta / \mu$

$$\boxed{u = \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right)}$$

$$\boxed{\tau_{yz} = \mu \frac{du}{dy} = \rho g \sin\theta (h - y)}$$

Shear stress τ_{yz} is +ve when shear force acts on -ve y face ~~phase~~ in -ve y & x direct^n

Similarly when shearforce acts on +ve y face & +ve x direct^n, it is -ve

$$Q = \int_0^h u\,dA = \int_0^h u b\,dy = \frac{\rho g \sin\theta\, b h^3}{3\mu}$$

Imp

$$\text{Avg velocity} = V = \frac{Q}{A} = \frac{Q}{bh} = \frac{\rho g \sin\theta\, h^2}{3\mu}$$

4) $$\frac{P_1}{\rho_f g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_f g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$\because h_f$ is loss of head, it is added there.

$$Z_1 - Z_2 = 0$$

$$\frac{P_1 - P_2}{\rho_f g} = \frac{V_2^2}{2g} + \frac{V_2^2}{12g} - \frac{V_1^2}{2g}$$

$A_1 V_1 = A_2 V_2$ $\quad d_1^2 V_1 = d_2^2 V_2$ $\quad V_1$ & V_2 मध्ये rel^n मिळेल

हे put करता V_2 & V_1 मिळेल

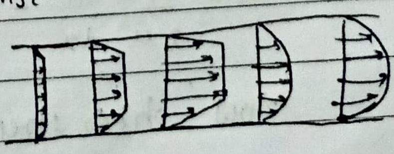
for μ, $1\,kg/ms = 1$ poise

* Flow through parallel plates

Assumptions:

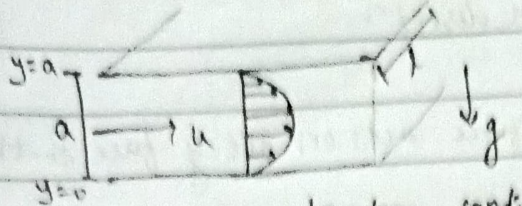
i. Steady flow $\partial/\partial t = 0$

ii. Incompressible flow ρ = const.

iii. fully developed flow →  i.e. $\frac{\partial u}{\partial x} = 0$

iv. It is 2 dimensional flow, $\frac{\partial u}{\partial z} = 0$

A. Both plates are fixed



l is cross sectional length

applying boundary conditions,

at $y=0$, $u=0$, $V=0$

at $y=a$, $u=0$, $V=0$

$$\text{iii) } \frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad \therefore V = \text{const} = 0 \quad \text{—(1)}$$

writing N-S eqn,

$$\rho \frac{Du}{Dt} = \rho g_x - \frac{dP}{dx} + \mu \nabla^2 u$$

$$\therefore \rho\left[\cancel{\frac{du}{dt}} + \cancel{u\frac{du}{dx}} + \cancel{V}\frac{du}{dy} + \cancel{w\frac{du}{dz}}\right] = \cancel{\rho g_x} - \frac{dP}{dx} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\therefore \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dP}{dx}$$

$$\therefore \frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dP}{dx}y + C_1$$

$$u = \frac{y^2}{2\mu}\frac{dP}{dx} + C_1 y + C_2$$

at $y=0$, $u=0$ $\quad \therefore C_2 = 0$

$y=a$, $u=0$ $\quad \therefore C_1 = \frac{-a}{2\mu}\frac{dP}{dx}$

$$\therefore \frac{du}{dy} = \frac{y}{\mu}\frac{dP}{dx} - \frac{a}{2\mu}\frac{dP}{dx} = \frac{1}{\mu}\frac{dP}{dx}\left[y - \frac{a}{2}\right]$$

$$u = \frac{y^2}{2\mu}\frac{dP}{dx} - \frac{ay}{2\mu}\frac{dP}{dx} = \frac{y}{2\mu}\frac{dP}{dx}\left[y-a\right]$$

now shear stress $\boxed{\tau_{yx} = \mu\frac{du}{dy} = \frac{dP}{dx}\left[y - \frac{a}{2}\right]}$

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Page

vol. flow rate per unit length $= \frac{Q}{l}$

$= \frac{1}{l}\int V.dA$

$dA = l\,dy$ y varies from 0 to a

$$\therefore \frac{Q}{l} = \frac{1}{l}\int_0^a V\,l\,dy = \frac{1}{l}\,l\int_0^a u\,dy = \int_0^a \frac{1}{2\mu}\frac{dP}{dx}(y^2 - ay)$$

$$= \frac{1}{2\mu}\frac{dP}{dx}\left[\frac{y^3}{3} - \frac{ay^2}{2}\right]_0^a = \frac{1}{2\mu}\frac{dP}{dx}\left[\frac{-a^3}{6}\right]$$

$$\boxed{\frac{Q}{l} = \frac{-a^3}{12\mu}\frac{dP}{dx}}$$

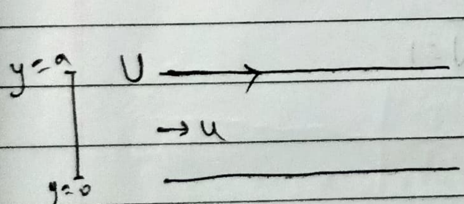
$$\boxed{\text{average velocity } (\bar{V}) = \frac{-a^2}{12\mu}\frac{dP}{dx}}$$

$$V_{max} = u \text{ @ } y = a/2 = \frac{3}{2}\left(\frac{-a^2}{12\mu}\frac{dP}{dx}\right) = \frac{-a^2}{8\mu}\frac{dP}{dx}$$

do $\frac{du}{dy} = 0$ & find y

$$\boxed{V_{max} = \frac{3}{2}\bar{V}}$$

B. Top plate is moving



all terminologies & eq^n's will be same till

$$u = \frac{y^2}{2\mu}\frac{dP}{dx} + C_1 y + C_2$$

at $y = 0,\ u = 0 \quad \therefore C_2 = 0$

$y = a,\ u = U \quad \therefore C_1 = \frac{-a}{2\mu}\frac{dP}{dx} + \frac{U}{a}$

$$\therefore \frac{du}{dy} = \frac{y}{\mu}\cdot\frac{dP}{dx} - \frac{a}{2\mu}\frac{dP}{dx} + \frac{U}{a}$$

$$u = \frac{y^2}{2\mu}\cdot\frac{dP}{dx} - \frac{ay}{2\mu}\frac{dP}{dx} + \frac{Uy}{a}$$

Shear stress $\tau_{yx} = \mu \frac{du}{dy}$

$$\boxed{\tau_{yx} = \frac{\mu U}{a} + \frac{dP}{dx}\left(y - \frac{a}{2}\right)}$$

flow rate per unit length $= \frac{Q}{l} = \frac{1}{l}\int_0^a u\,l\,dy$

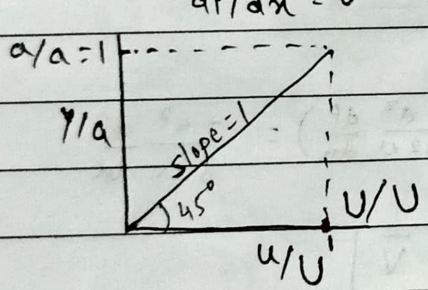
$$= \frac{1}{2\mu}\frac{dP}{dx}\left[\frac{y^3}{3} - \frac{ay^2}{2}\right]_0^a + \frac{U}{a}\left[y^2/2\right]_0^a$$

$$\boxed{Q/l = \frac{Ua}{2} - \frac{a^3}{12\mu}\frac{dP}{dx}}$$

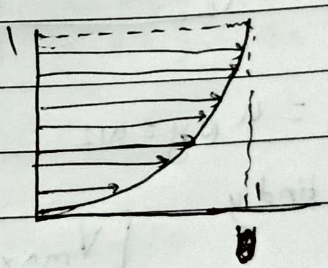
$V_{max} = u \; @ \; y = \frac{a}{2} - \frac{U/a}{\frac{1}{\mu}\frac{dP}{dx}}$

$$\boxed{V_{avg} = \bar{V} = \frac{U}{2} - \frac{a^2}{12\mu}\frac{dP}{dx}}$$

$dP/dx = 0$

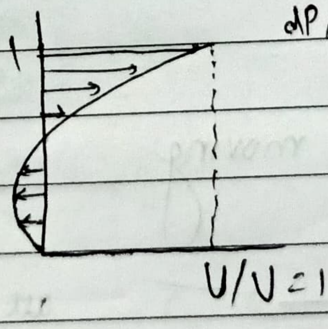


$dP/dx < 0$ favourable con^n

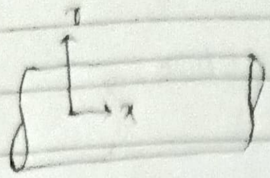


$dP/dx > 0$ adverse con^n

graphs are drawn according to eq^n of u



* Flow through pipe in r-θ-x system



Assumpt^{n}s

i. Steady flow

ii. Incompressible flow

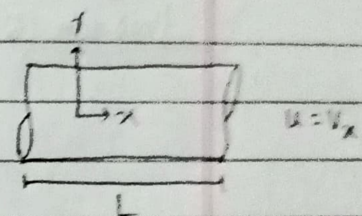
iii. fully developed flow $\quad \partial u / \partial x = 0$

iv. 2D flow in r, x direct^n i.e $\frac{\partial}{\partial \theta} = 0$

applying boundary con^n,

at $r = 0$, $u =$ finite

$r = R$, $u = 0$ $\quad V_r = 0$



$$\frac{1}{r}\frac{\partial (r V_r)}{\partial r} + \cancel{\frac{1}{r}\frac{\partial (V_\theta)}{\partial \theta}}^{0}_{iv} + \cancel{\frac{\partial (V_x)}{\partial x}}^{0}_{iii} = 0$$

$\therefore \; r V_r = \text{const} \quad$ i.e $V_r = \text{const} = 0 \quad$ — ①

$$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0}_{i} + \cancel{\frac{u \partial u}{\partial x}}^{0}_{iii} + \cancel{\frac{V_r \partial u}{\partial r}}^{0} + \cancel{\frac{V_\theta \partial u}{r \partial \theta}}^{0}_{iv}\right] = \cancel{\rho g_x}^{0} - \frac{\partial p}{\partial x} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}}^{0}_{iv} + \cancel{\frac{\partial^2 u}{\partial x^2}}^{0}_{iii}\right]$$

$$\therefore \; \frac{\mu}{r}\frac{\partial}{\partial r}\left(\frac{r \partial u}{\partial r}\right) = \frac{\partial p}{\partial x}$$

$$\therefore \; \int \frac{d}{dr}\left(r\frac{du}{dr}\right) = \int \frac{r}{\mu}\frac{\partial p}{\partial x} + C_1$$

$$r\frac{du}{dr} = \frac{r^2}{2\mu}\frac{\partial p}{\partial x} + C_1$$

$$\therefore \; \frac{du}{dr} = \frac{r\,dp}{2\mu\,dx} + \frac{C_1}{r}$$

$$\therefore \quad u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + C_1 \ln r + C_2$$

boundary con^n at $r=0$ u = finite

this gives $C_1 = 0$

at $r=R$, $u=0$

$$C_2 = \frac{-R^2}{4\mu}\frac{\partial p}{\partial x}$$

$$\therefore \quad u = \frac{-R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right]$$

shear stress = $\boxed{\tau_{yx} = \cancel{\mu \frac{du}{dy}} \; \mu \frac{du}{dr} = \frac{r}{2}\frac{\partial p}{\partial x}}$

$$\text{flow rate } (Q) = \int \vec{V}\, dA = \int_0^R u \, 2\pi r \, dr$$

$$= \int_0^R \frac{-R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-(r/R)^2\right] \cdot 2\pi r \, dr$$

$$\partial p / \partial x = -\Delta P / L$$

$$Q = \frac{-\pi R^2}{2\mu}\frac{\partial p}{\partial x}\int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= \frac{-\pi R^2}{2\mu}\frac{\partial p}{\partial x}\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \frac{\pi}{8\mu L}\Delta P R^4$$

$$\boxed{Q = \frac{\pi R^4 \Delta P}{8\mu L}}$$

$$V_{avg} = \frac{Q}{A} = \frac{Q}{\pi R^2} \Rightarrow \boxed{V_{avg} = \frac{\Delta P R^2}{8\mu L}}$$

V_{max} ~~occurs at $r=0$~~

$$\boxed{V_{max} = \frac{R^2 \Delta P}{4\mu L}}$$

$ds \geq \frac{\partial q}{T}$

ds is written because entropy is point/state funⁿ

∂q ——— " ——— q is path funⁿ

for adiabatic process, $\partial q = 0$

$\therefore ds \geq 0$

if process is irreversible, constantly entropy is being produced $\Rightarrow ds > 0$

if process is reversible, entropy isn't produced

$\therefore ds = 0 \Rightarrow S = 0$

i.e. adiabatic reversible process is isentropic one

$C_p - C_v = R$ R is characteristic gas constant $= 287$ J/kgK

$R = \frac{R_u}{M}$ R_u is universal ——— " ——— $= 8.314$ J/mol K

$P\forall = mRT$ $\therefore P = \rho RT$

$P\forall = nR_uT$ $P\upsilon = RT$ } eqⁿ of state

imp

$\Downarrow$

$mR = nR_u$ $\frac{R_u}{R} = \frac{m}{n} = \frac{m}{m/M} = M$

Assumption

i. Fluid obeys continuum (there is no vacant space betⁿ fluid particles)

ii. ——— " ——— energy of states

iii. Calorically perfect - that means betⁿ temp range 50 k to 600k, value of C_p & C_v don't change

iv Thermally perfect

u & h are funⁿs of P & T but they vary according to P only at higher pressure values.

$\therefore$ u & h are considered as values of T only

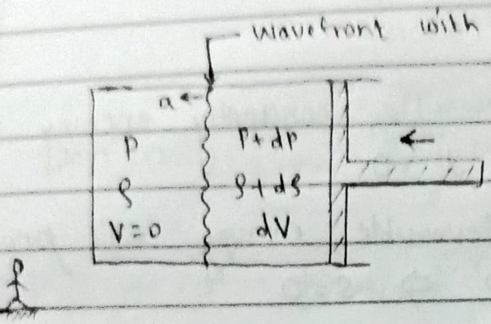
$u = u(T)$

$h = h(T)$

$$\int_1^2 dh = \int_1^2 C_p\,dT \quad \therefore h_2 - h_1 = C_p(T_2 - T_1)$$

$$\int_1^2 du = \int_1^2 C_v\,dT \quad \therefore u_2 - u_1 = C_v(T_2 - T_1)$$

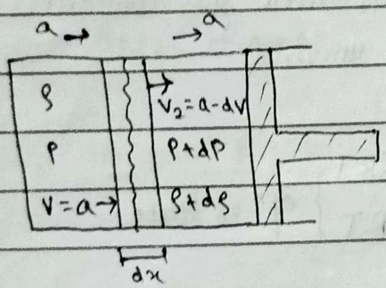
* Sonic velocity of disturbance of infinitely small amplitude through an elastic medium



∵ Observer standing outside is seeing it, it'll change according to time

∴ It is unsteady state problem

To make it consider as a steady state problem, we'll consider observer ~~standing on the wavefront~~



This becomes steady state problem

cross sect^n area = A

applying continuity eq^n,

$$\rho A V = \text{const.} \quad \therefore \rho A a = (\rho + d\rho) A (a - dv)$$

$$\therefore \rho a = \rho a - \rho dV + a\,d\rho + d\rho\,dv$$

$$\therefore dV = \frac{a\,d\rho}{\rho} \quad \text{—①}$$

applying momentum eq^n,

$$F_{net} = \frac{d}{dt}(mV) = \dot{m}V = \dot{m}_2 V_{out} - \dot{m}_1 V_{in}$$

$$= \dot{m}(V_{out} - V_{in}) = \dot{m}(V_2 - V_1) \quad \ldots \dot{m}_2 =$$

$$(P - P - dP)A = \rho A a(a - dv - a)$$

$$-dP = \rho a(-dv)$$

$$dP = \rho a \cdot \frac{a\,d\rho}{\rho} \quad \ldots \text{from ①}$$

$$dP = a^2 d\rho \quad \therefore a = \sqrt{\frac{dP}{d\rho}}$$

i.e $a^2 = \left(\frac{\partial P}{\partial \rho}\right)_{entropy = const}$

for adiabatic process, $PV^\gamma = c$

$P v^\gamma = c$ $\qquad \frac{V}{m} = v = \frac{1}{\rho}$

$\frac{P}{\rho^\gamma} = c$

$\therefore P = c\rho^\gamma$

$\therefore a^2 = \frac{\partial (c\rho^\gamma)}{\partial \rho}$

$\therefore a^2 = c\gamma\rho^{\gamma-1} = \frac{c\rho^\gamma \gamma}{\rho}$

$\therefore \boxed{a^2 = \frac{P\gamma}{\rho}}$

now, $P = \rho R_c T$ $\qquad R_c$ is chara. gas const.

$\therefore a^2 = \gamma RT$ $\qquad \boxed{a = \sqrt{\gamma R_c T}}$

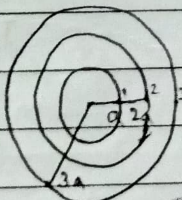
$\boxed{a = \sqrt{K_B/\rho}}$

Mach no. $= M_a = \frac{\text{velocity of medium}}{\text{velocity of sound in that medium}} = \frac{V}{a}$

Type ①: Source is stationary.

i.e $V = 0$ $\qquad V = M_a a$

$0 = 0 \times a$ $\qquad M_a = 0$



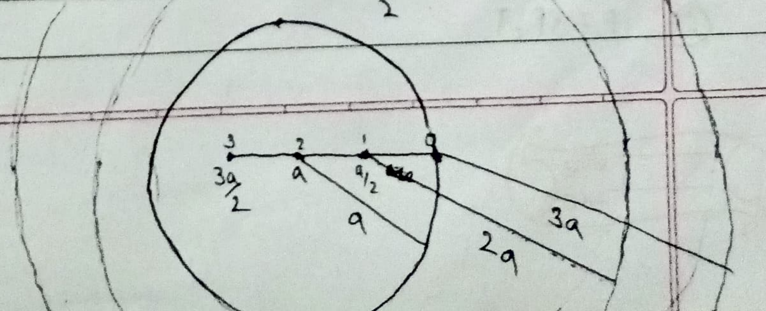
in 1 sec, it'll travel a distance a

" 2 " 2a "

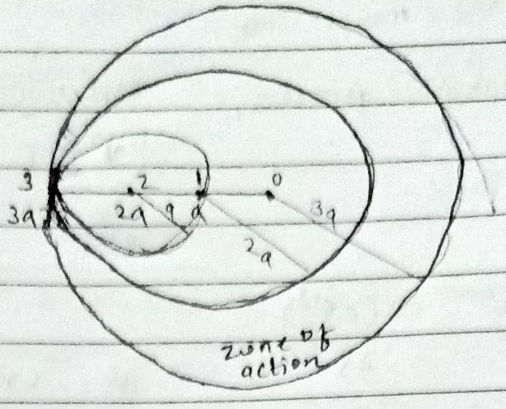
" 3 " 3a "

Type ②: Source is moving with V

case 1: $V < a$ $\qquad$ ex. $V = \frac{a}{2}$ $\qquad \therefore M_a = 0.5$



case 2 : $V = a$ $\quad M_a = 1$

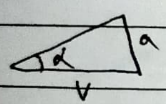


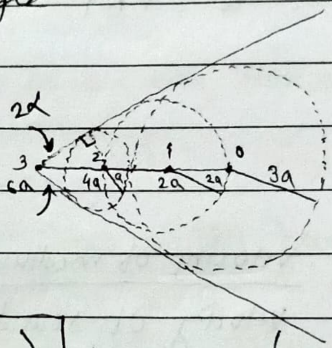
case 3 : $V > a$ $\quad$ ~~Ma =~~

ex. $V = 2a$ $\quad M_a = 2$

α is called Mach angle

$$\sin\alpha = \frac{a}{V} = \frac{1}{M_a}$$





$$\therefore \boxed{\alpha = \sin^{-1}(1/M_a)}$$

Mach cone

Imp

① $$\frac{V_1^2}{2} + h_1 + gz_1 + q_{in} = \frac{V_2^2}{2} + h_2 + gz_2 + W_{shaft}$$

② $$\frac{T_0}{T_s} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

T_0 - stagnation temp

T_s - static temp

③ $$\frac{P_0}{P_s} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma - 1}$$

④ $$\frac{\rho_0}{\rho_s} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma - 1}$$

⑤ $$a = \sqrt{\gamma R T}$$

⑥ $$P = \rho R_c T$$

1]

$\dot{m} = 0.15\,kg/s$

find Area of cross sectⁿ, $\Delta h, \Delta u, \Delta s, M_1$, heat lost

$P_1 = 188\ kPa$ $\quad$ $P_2 = 213\ kPa$

$T_1 = 440\ K$ $\quad$ $T_2 = 351\ K$

$V_1 = 210\ m/s$ $\quad$ $M_2 = 1.337$

$R =$ $\quad$ $\dot{m} = \rho_1 V_1 A = \dfrac{P_1}{R_c T_1} V_1 A$

$0.15 = \dfrac{188}{287 \times 440} \times 210 \times A$ $\quad$ $\therefore A = \cancel{12374}\ m^2$ 0.4797

$C_p = 1004$ $\quad$ $C_v = C_p - R_c$

$= 1004 - 287 = 717$

$\Delta h = h_2 - h_1 = C_p(T_2 - T_1)$

$\Delta u = u_2 - u_1 = C_v(T_2 - T_1)$

$\Delta s = C_p \ln \dfrac{T_2}{T_1} - R \ln \dfrac{P_2}{P_1}$

$a_1 = \sqrt{\gamma R_c T_1} = 420.466\ m/s$

$M_1 = \dfrac{V_1}{a_1} = \dfrac{210}{420.466} = 0.499$

$\dfrac{T_{0_1}}{T_s} = \left(1 + \dfrac{\gamma - 1}{2} M_1^2\right)$ $\quad$ $T_s = T_1$ here

$\therefore T_{0_1} = 440\left(1 + 0.2(0.499)^2\right) = 461.912\ K$

$T_{0_2} = 351\left(1 + 0.2(1.337)^2\right) = 476.487\ K$

$q_{in} = h_{0_2} - h_{0_1} = C_p(T_{0_2} - T_{0_1}) = 1004(14.5753)$

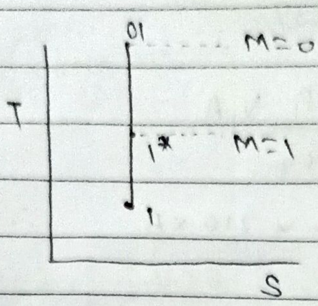
$= 14633.64\ J$

2]

Stagnation state: When fluid is brought to rest isentropically

fluid brough to rest means $V=0$ means $M=0$

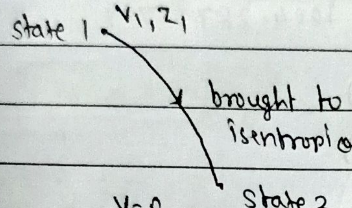
Every static state has its own stagnation state & isentropic star state



1^* is isentropic star state

01 is stagnation state wrt state

1 is static state



for 1 & 2,

$$h_0 = h_1 + \frac{V_1^2}{2} + \cancel{gZ_1}$$

considering it is equipotential surface, Z_1 is neglected

$$h_0 = h_1 + \frac{V_1^2}{2}$$

$$M = \frac{V}{a}$$

$$\therefore V = M\sqrt{\gamma R_c T}$$

$$h_0 = h_1 + \frac{V_1^2}{2} = h_1 + \frac{M^2 \gamma R_c T_1}{2}$$

$$C_p T_0 = C_p T_1 + \frac{M^2 \gamma R_c T_1}{2}$$

$$T_0 = T_1 + \frac{M^2 \gamma R_c T_1}{2} \times \frac{\gamma - 1}{\gamma R_c} \quad \cdots \quad C_p = \frac{\gamma R_c}{\gamma - 1}$$

$$T_0 = T_1 + \frac{M^2 T_1 (\gamma - 1)}{2}$$

$$\boxed{T_0 = T_1\left(1 + \frac{\gamma - 1}{2} M^2\right)}$$

$$T_0 = \frac{h_0}{C_p}, \quad T_1 = \frac{h_1}{C_p}$$

$$\therefore \boxed{h_0 = h_1\left(1 + \frac{\gamma - 1}{2} M^2\right)}$$

For adiabatic process,

$$PV^\gamma = c \Rightarrow PT^{\gamma/1-\gamma} = c$$

$$\therefore \cancel{\frac{P_0}{P_1}} \quad P_0 T_0^{\gamma/1-\gamma} = P_1 T_1^{\gamma/1-\gamma}$$

$$\therefore \frac{P_0}{P_1} = \frac{T_1^{\gamma/1-\gamma}}{T_0^{\gamma/1-\gamma}} = \left(\frac{T_0}{T_1}\right)^{\gamma/\gamma-1}$$

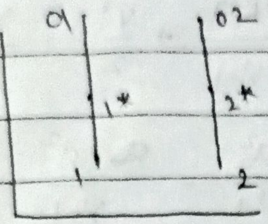
$$\therefore \boxed{p_0 = P_1 \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma - 1}} \qquad \boxed{\rho_0 = \rho \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma - 1}}$$

by energy eq^n for adiabatic process,

$$h_{01} + \cancel{q_{in}} = h_{02} + \cancel{w_s}$$

$$C_p T_{01} = C_p T_{02}$$

$$T_{01} = T_{02}$$



$$\Delta S = C_p \ln \frac{T_2}{T_1} - R_c \ln \frac{P_2}{P_1}$$

substituting static by stagnation terms,

Imp

$$\therefore \boxed{\Delta S = C_p \ln \frac{T_{02}}{T_{01}} - R_c \ln \frac{P_{02}}{P_{01}}}$$

for adiabatic, $T_{02} = T_{01}$

$$\therefore \Delta S = -R_c \ln \frac{P_{02}}{P_{01}}$$

$$\therefore P_{02} = P_{01} e^{-\Delta S / R_c}$$

$$\therefore \boxed{\frac{P_{02}}{P_{01}} = e^{-\Delta S / R_c}}$$ which is less than 1

$P_{02} < P_{01}$

Imp Stagnation pressure is <u>always</u> lost in the direct^n of flow

Isentropic

$$\Delta S = 0$$

$$\frac{P_{02}}{P_{01}} = 1 \qquad \therefore P_{02} = P_{01}$$

in eq^n $h_0 = h + \frac{V^2}{2}$

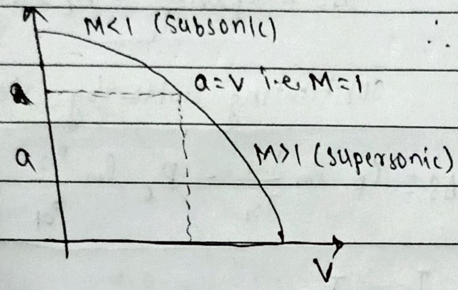
if $h = 0$ V will be V_{max}

$\therefore \frac{V_{max}^2}{2} = h_0$

$$\boxed{V_{max} = \sqrt{2h_0} \qquad V_{max} = \sqrt{2C_pT_0}}$$

$h_0 = h + \frac{V^2}{2}$

$C_pT_0 = C_pT + \frac{V^2}{2}$ $\therefore \frac{\gamma R_c T}{\gamma - 1} + \frac{V^2}{2} = \frac{\gamma R_c T_0}{\gamma - 1}$



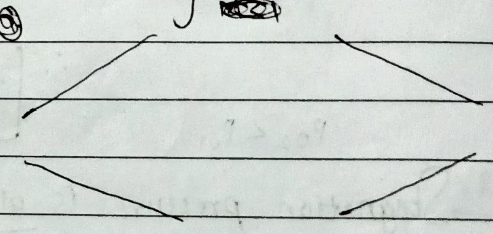
$$\therefore \boxed{\frac{a_0^2}{\gamma - 1} = \frac{a^2}{\gamma - 1} + \frac{V^2}{2}}$$ ... $a = \sqrt{\gamma R_c T}$

... Prandtl velocity ellipse

* Variable flow

with no frict^n & no heat transfer

$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$



case 1: for $M < 1$

when $\frac{dA}{A} > 0 \Rightarrow \frac{dV}{V} < 0$ subsonic diffuse/ deaccelerated flow

when $\frac{dA}{A} < 0 \Rightarrow \frac{dV}{V} > 0$ supersonic diffuse/ accelerated flow

case 2: for $M > 1$

when $\frac{dA}{A} < 0 \Rightarrow \frac{dV}{V} < 0$ subsonic diffuse/ deaccelerated flow

when $\frac{dA}{A} > 0 \Rightarrow \frac{dV}{V} > 0$ supersonic diffuse/ accelerated flow

Date
Page

for adiabatic process

$P = \rho R T$

$\rho_1 A_1 V_1 = \rho_2 V_2 A_2$ ... continuity eq^n

$$\frac{A_1}{A_2} = \frac{\rho_2 V_2}{\rho_1 V_1} = \frac{\frac{P_2}{R_c T_2}}{\frac{P_1}{R_c T_1}} \cdot \frac{M_2\sqrt{\gamma R_c T_2}}{M_1\sqrt{\gamma R_c T_1}} = \frac{P_2 T_1^{1/2} M_2}{P_1 T_2^{1/2} M_1}$$

$$\therefore \boxed{\frac{A_2}{A_1} = \frac{P_1 M_1}{P_2 M_2}\left(\frac{T_2}{T_1}\right)^{1/2}}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_2}{P_{01}/P_1} \times \frac{P_2}{P_1} = \frac{P_2}{P_1} \times \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1} = e^{-\Delta S/R_c}$$

putting these value in above eq^n,

$$\boxed{\frac{A_2}{A_1} = \frac{M_1}{M_2} e^{\Delta S/R_c}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

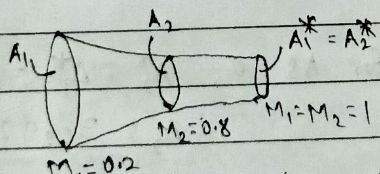
when M_1 becomes 1 for state 1, A_1 becomes A_1^*

& whem M_2 —— " —— 2, A_2 becomes A_2^*

$M_1 = M_2 = 1$

$$\therefore \boxed{\frac{A_2^*}{A_1^*} = e^{\Delta S/R_c}} \quad \text{but} \quad \frac{P_{02}}{P_{01}} = e^{-\Delta S/R_c}$$

$$\therefore \boxed{\frac{A_2^*}{A_1^*} = e^{\Delta S/R_c} = \frac{P_{01}}{P_{02}}} \Rightarrow \boxed{P_{02}A_2^* = P_{01}A_1^*}$$



isenstropic process

for isentropic process, $\Delta S = 0$

$\therefore A_2^* = A_1^*$

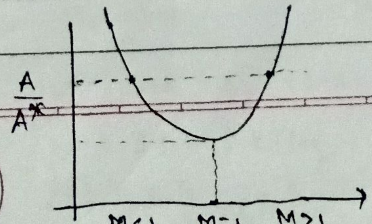
$P_{02} = P_{01}$

in the isentropic flow, let $A_2 = A$ & $A_1 = A^*$

$\therefore M_2 = M$ & $M_1 = M = 1$

$\Delta S = 0$

$$\therefore \boxed{\frac{A}{A^*} = \frac{1}{M} e^{0/R_c}\left[\frac{1+\frac{\gamma-1}{2}M}{\gamma+1}\right]^{\gamma+1/2(\gamma-1)}}$$

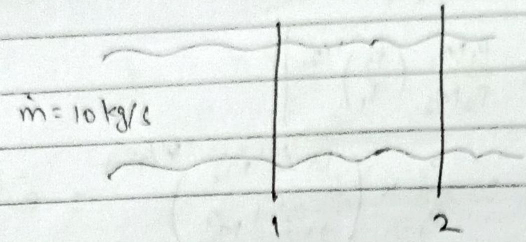


M has 2 values corresponding to given value of A/A^*

$\frac{\gamma+1}{2(\gamma-1)} = 3$ $\therefore$ RHS is a quadratic in M

$\therefore$ M will have two values for given value of A/A^*

1] air at rate of 10 kg/s is flowing in adiabatic duct. At one sect^n the pressure is 2×10^5 Pa. Temp & area are 650°C & 50 cm^2. At a downstream sect^n $M = 1.2$. Sketch general shape of duct. find A_2 if flow is isentropic. find A_2 if entropy change is 42 J/kg K



$$\dot m = \rho_1 A_1 V_1 = \frac{P_1 A_1 V_1}{R_c T_1} \qquad A_1 = 50\ cm^2 = 50\times10^{-4}$$

$$10 = \frac{2\times10^5 \times 50\times10^{-4} \times V_1}{R_c \times 923}$$

$$M_1 = V_1/a_1 = V_1 / \sqrt{\gamma R_c T_1}$$

this gives $M_1 = 4.3$

from gas table,

for $M_1 = 4.3$, $\frac{A_1}{A_1^*} = 13.9549$

for $M_2 = 1.2$, $\frac{A_2}{A_2^*} = 1.03044$

$$\frac{A_1}{A_1^*} = \frac{50\times10^{-4}}{A_1^*} = 13.9549 \qquad A_1^* = 3.58\times10^{-4}\ m^2$$

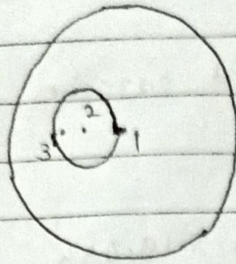
$$\frac{A_2^*}{A_1^*} = e^{\Delta S/R_c} \qquad \text{for } \Delta S = 0,\ A_2^* = A_1^* = 3.58\times10^{-4}\ m^2$$

$$\therefore A_2 = 3.6920\times10^{-4}\ m^2$$

for $\Delta S = 42$ J/kg K, $A_2^* = 4.14\times10^{-4}\ m^2$

$$A_2 = 4.2703\times10^{-4}\ m^2$$

9] A small sound source moves with velocity V to the left in a st line from point 1 in air at temp 293K while generating sound waves as shown. The points 1, 2, 3 are pos^{n}s of the source at various times. The circles represent the sound waves generated at diff times. find speed of source, V



radius of inner circle = 0.8 m

—— outer —— = 1.7 m

$a = \sqrt{\gamma R_c T} = \sqrt{1.4 \times 287 \times 293} = 343.1142$ m/s

1 to 2 $\rightarrow t_1$ 2 to 3 $\rightarrow t_2$

$\therefore \; t_1 + t_2 = \frac{1.7}{a} = 0.004954$ sec

$t_2 = \frac{0.8}{a} = 0.00233158$ sec

$t_1 = 0.002623$

$V = \frac{0.8}{t_1} = 304.99$ m/s

it goes from 1–2 with velocity V in time t_1

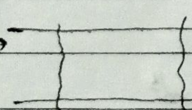
it goes from 2–3 with velocity V in time t_2

at 1, circle is drawn of radius equal to dist travelled by it with speed a during time $t_1 + t_2$

at 2, circle is drawn of radius equal to dist travelled by it with speed a during time t_2

1] Air flows through a long duct of const area at 0.15 kg/s. A short sect^n of duct is cooled by liq nitrogen. The absolute pressure, temp & velocity entering the cooled sect^n are 188 kPa, 440K, 210 m/s resp. At the outlet absolute pressure, temp, mach no. are 213 kPa, 351K, 1.337 resp. Find cross sectional area & changes in enthalpy, entropy, internal energy, flow Mach no. at inlet & amount of heat loss for this system.

$\dot{m} = 0.15$ kg/s $\rightarrow$



$\dot{m} = 0.15 = \rho V_1 A_e = \frac{P_1 V_1 A_e}{R_c T_1} \Rightarrow A = 0.4797\ m^2$

$M_1 = V_1 / a_1 = \frac{210}{\sqrt{\gamma R_c T_1}} \Rightarrow M_1 = 0.499$

$P_1 = 188$ kPa, $T_1 = 440$K $P_2 = 213$ kPa, $T_2 = 351$K

$V_1 = 210$ m/s $M_2 = 1.337$

$M_2 = V_2 / a_2$ $V_2 = 1.337 \sqrt{\gamma R_c T_2} = 502.99$ m/s

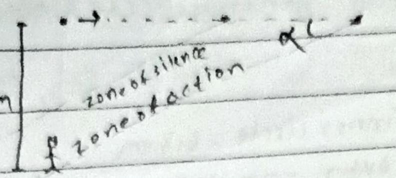
$\Delta h = h_2 - h_1 = C_p \Delta T = 1005$ J/kgK $\times 89$K $= 89.445$ kJ/kg

$\Delta u = u_2 - u_1 = C_v \Delta T = 718$ J/kgK $\times 89$K $= 63.902$ kJ/kg

$\Delta S = C_p \ln \frac{T_2}{T_1} + R_c \ln \frac{P_1}{P_2}$

$= 1005 \ln \frac{351}{440} + 287 \ln \frac{188}{213} = -262.950$ kJ/kgK

2] Aeroplane is flying 2km above ground with $M = 1.5$ when temp $= 25°C$ what is the speed of the plane & how long after passing directly above the ground observer, is the sound of the ~~observer~~ aeroplane heard by the ground observer?



$\sin^{-1}(1/M) = \alpha$

$\sin^{-1}(1/1.5) = 41.81031$

$\tan\alpha = \frac{2}{d}$

$d = 2.236$ km

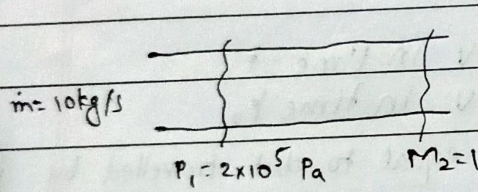
$V = Ma$

$= M\sqrt{\gamma R_c T}$

$= 1.5\sqrt{1.4 \times 287 \times 298} = 519.044$

$t = \frac{d}{V} = \frac{2236}{519.044} = 4.3079$ sec

3] Air flows at the rate 10kg/s in adiabatic channel. One sect^n where area is 50 cm^2, the pressure is 2×10^5 N/m^2 & the temp is 650°C, $M_2 = 1.2$ find A_2 if flow is isentropic



$A_1 = 50 \times 10^{-4}$ m^2

$T_1 = 923$ k

$\dot{m} = \rho_1 A_1 V_1 = \frac{P_1 A_1 V_1}{R_c T_1}$

$V_1 = 2649.01$ m/s

$2649.01 = M_1\sqrt{\gamma R_c T_1}$

$M_1 = 4.349$

$$\frac{P_2}{P_1}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1} = e^{-\Delta S/R_c} \qquad \Delta S = 0$$

$P_2 = P_1 \times 98.666$

$= 197.33 \times 10^5$ Pa

$$\frac{A_2}{A_1} = \frac{M_1}{M_2} e^{\Delta S/R_c}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma+1/2(\gamma-1)}$$

$A_2 = 3.539 \times 10^1$ cm^2

4] Air is flowing at foll. con^n pressure $= 1.6\times10^5$ Pa, $T = 500°C$ $V = 160$ m/s

find P_0 & T_0

$M = 0.287$

$\sqrt{\gamma R_c T}\, M = 160$

$$\frac{P_0}{P} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} \qquad P_0 = 1.6941\times10^5 \text{ Pa}$$

$$\frac{T_0}{T} = 1+\frac{\gamma-1}{2}M^2 \qquad T_0 = 785.7342 \text{ K}$$

* Mass flow rate

$$\frac{\dot m}{A} = \frac{\rho A V}{A} = \frac{\rho^* A^* V^*}{A} = \frac{A^*}{A}\rho^* V^* \quad \text{—①}$$

$$V^* = M a^* = a^* \quad \text{for star con}^n,\ M=1$$

$$= \sqrt{\gamma R_c T^*} \quad \text{—②}$$

$$\rho^* = \frac{P^*}{R_c T^*} \quad \text{—③}$$

$$\therefore \frac{\dot m}{A} = \frac{A^*}{A}\cdot\frac{P^*}{R_c T^*}\sqrt{\gamma R_c T^*} = \frac{A^*}{A}\cdot\frac{P^*}{\sqrt{T^*}}\sqrt{\frac{\gamma}{R_c}} \quad \text{— ④} \quad \text{... from ①,②,③}$$

$$\frac{P_0}{P^*} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \quad \text{.... } M=1$$

$$\frac{T_0}{T^*} = 1+\frac{\gamma-1}{2}M^2 = \frac{\gamma+1}{2} \quad \text{.... } M=1$$

$$\therefore \frac{\dot m}{A} = \frac{A^*}{A}\cdot\frac{P_0}{\left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1}}\cdot\left(\frac{\gamma+1}{2T_0}\right)^{1/2}\sqrt{\frac{\gamma}{R_c}} = \frac{A^*}{A}\frac{P_0}{\sqrt{T_0}}\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\left(\frac{\gamma+1}{2}\right)^{1/2}\sqrt{\frac{\gamma}{R_c}}$$

$$= \frac{A^*}{A}\frac{P_0}{\sqrt{T_0}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}-\frac{1}{2}}\sqrt{\frac{\gamma}{R_c}}$$

$$\frac{\dot m}{A} = \sqrt{\frac{\gamma}{R_c}}\frac{A^* P_0}{A\sqrt{T_0}}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/2(\gamma-1)}$$

$$\therefore \boxed{\frac{\dot m_{max}}{A^*} = \sqrt{\frac{\gamma}{R_c}}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/2(\gamma-1)}\frac{P_0}{\sqrt{T_0}} = 0.0404\frac{P_0}{\sqrt{T_0}}}$$

(when nozzle is choked) ~~for~~ air

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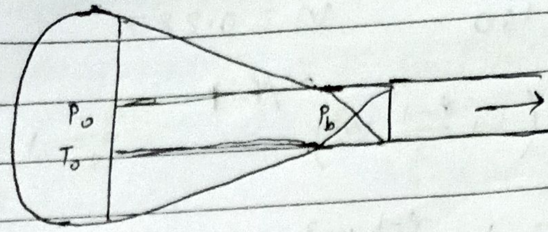
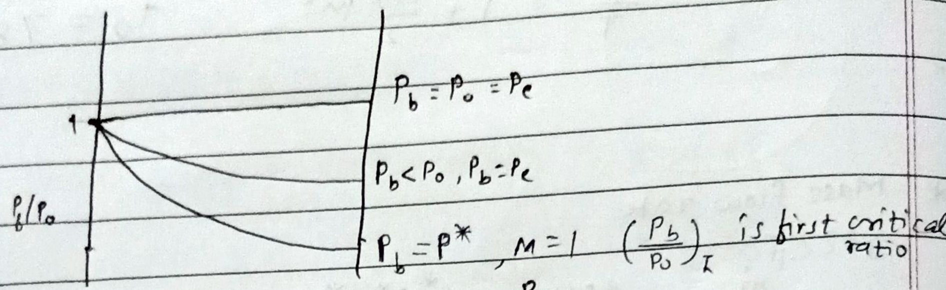
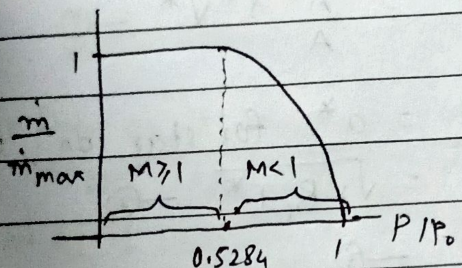
* Nozzle operⁿ

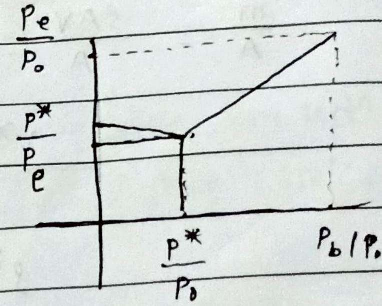


P_e - exit plane pressure

P_b - back pressure or receiver pressure

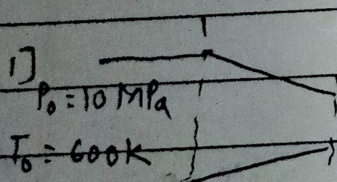






from here onwards,

$\dot m_x = \dot m_{choked}$

1] 

$T_o = 600$k

$A = 10 m^2$

if a) $P_{receiver} = 8.02$ MPa

b) $P_{rec} = 5.284$ MPa

c) $P_{rec} = 4$ MPa

find $\dot m$, velocity at exit point.

a) $\frac{P_{rec}}{P_o} = \frac{8.02}{10} = 0.802 > 0.5284$ .... nozzle not choked

b) $\frac{P_{rec}}{P_o} = \frac{5.284}{10} = 0.584 = 0.5824$ ... nozzle is choked

c) $\frac{P_{res}}{P_o} = \frac{4}{10} = 0.4 < 0.5284$ ... nozzle is choked

a) $\frac{P_{rec}}{P_0} = 0.802$ $\therefore$ from gas table $M_2 = 0.57$

$\therefore \frac{T_2}{T_{0}} = \cancel{0.92}\ 0.939$ $\qquad T_0 = 600k$

$\therefore T_2 = 563k$

$a_2 = \sqrt{\gamma R_c T_2} = 476 m/s$

$V_2 = M_2 a_2 = 271.32 m/s$

$\dot{m} = \rho A V$

$= \frac{P_2}{R_c T_2} A_2 V_2 = 1.35 \times 10^6 kg/s$

b) $\frac{P_{rec}}{P_0} = 0.5824$ $\qquad$ has $M_2 = 1$

$\frac{T_2}{T_0} = 0.833$ $\qquad T_0 = 600K$

$T_2 = 499.8K$

$a_2 = \sqrt{\gamma R_c T_2} = 448.129\ m/s$

$V_2 = M_2 a_2 = 448.129\ m/s$

c) $\frac{P_{rec}}{P_0} = 0.4$ $\qquad$ has $M_2 = 1.22$

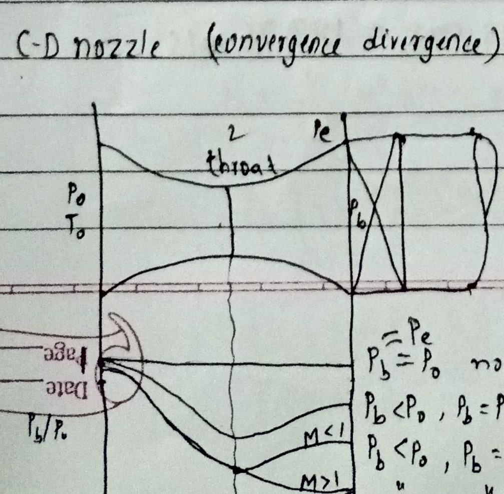
$\frac{T_2}{T_0} = 0.77061$ $\qquad T_0 = 600k$

$T_2 = 462.366$

$a_2 = \sqrt{\gamma R_c T_2} = 431.0205\ m/s$

$V_2 = M_2 a_2 = 525.845\ m/s$

* C-D nozzle (convergence divergence)



$P_b = P_0 = P_e$ no flow

$P_b < P_0$, $P_b = P_e$ but $P_{th} \neq P^*$

$P_b < P_0$, $P_b = P_e$ & $P_{th} = P^*$ $\quad \left(\frac{P_b}{P_0}\right)_I$ is first critical pressure ratio

" " $\quad$ 4 M>1

$(P_b/P_0)_{III}$ is third critical pressure ratio

1] A CD nozzle is designed to operate with an exit Mach no. $M = 2.25$. If it is fed by a large chamber of air at 15 MPa & 600 K & exhausts into the room at 14.7 MPa. Assuming the losses to be negligible, find velocity in the nozzle throat.

→ for $M_3 = 2.25$, $\left(\frac{A_3}{A_3^*}\right)_{designed} = 2.09644$

$M = 0.29 \quad \left(\frac{P}{P_0}\right)_I = 0.94329$

$M = 2.25 \quad \left(\frac{P}{P_0}\right)_{III} = 0.08648$

Given $P_b = 14.7$ MPa $\quad P_0 = 15$ MPa $\quad T_0 = 600$ K

$$\frac{P_b}{P_0} = \frac{14.7}{15} = 0.98 \Rightarrow M = 0.17 \Rightarrow \left(\frac{A_3}{A_3^*}\right)_{operating} = 3.46351$$

by chain rule,

$$\left(\frac{A_2}{A_2^*}\right)_{operating} = \frac{A_2}{A_3} \times \underbrace{\frac{A_3}{A_3^*}}_{operating} \times \frac{A_3^*}{A_2^*} \quad \text{(designed: } \frac{A_2}{A_3}\text{)}$$

∵ isentropic, $A_3^* = A_2^*$

Since pipe is not changing, throat area is not changing

$\therefore A_2^* = A_3^* = A_2$

$$\therefore \frac{A_3}{A_3^*} = \frac{A_3}{A_2} = 2.09644$$

$$\therefore \frac{A_2}{A_2^*} = \frac{1}{2.09644} \times 3.46351 = 1.65212$$

M_2 corresponding to above ratio is $M_2 = 0.38$

$\frac{T_2}{T_0}$ @ $M_2 = 0.38$ is 0.97193

$$T_2 = 0.97193 \times 600 = 582 \text{ K}$$

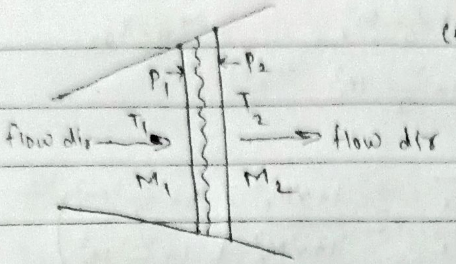
$$a_2 = \sqrt{\gamma R_c T_2} = \sqrt{1.4 \times 287 \times 582} = 483.578 \text{ m/s}$$

$V_2 = M_2 a_2$

$= 0.38 \times 483.578 = 183.76$ m/s

Normal shock:

It is an abrupt discontinuity in 1D supersonic ($M>1$) flow where the wavefront created by compression process is perpendicular to the direct^n of flow. It is characterized by abrupt changes in flow properties such as pressure, temp, density, velocity, entropy across the shock wave.



conditions - $M_1 > 1$

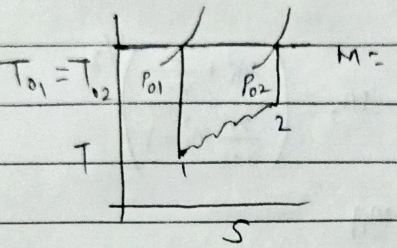
$h_{o_1} = h_{o_2}$ ..... adiabatic

$T_{o_1} = T_{o_2}$

$A_1 = A_2$

$P_{o_1} \neq P_{o_2}$

$M_2 < 1$



$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow \rho_1 V_1 = \rho_2 V_2$$

energy eq^n: $h_{o_1} + \cancel{q_{in}}^{0} = h_{o_2} + \cancel{w_{shaft}}^{0}$

$$q_{in} = w_{shaft} = 0$$

$$h_{o_1} = h_{o_2}$$

$$\therefore h_1 + \frac{V_1^2}{2} + \cancel{gz_1} = h_2 + \frac{V_2^2}{2} + \cancel{gz_2}$$

$$\therefore h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

momentum eq^n: $\Sigma F_x = \dot{m}(V_{out} - V_{in})$

$$P_1 A_1 - P_2 A_2 = \dot{m}(V_2 - V_1) = \rho_1 A_1 V_1 (V_2 - V_1)$$

$$\therefore P_1 - P_2 = \rho_1 V_1 (V_2 - V_1) \quad \ldots A_1 = A_2$$

$$P_1 - P_2 = \rho_1 V_1 V_2 - \rho_1 V_1^2$$

$$P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \quad \ldots \rho_1 V_1 = \rho_2 V_2$$

$$\therefore P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$\therefore P_1 + \frac{P_1}{R_c T_1} M_1^2 a_1^2 = P_2 + \frac{P_2}{R_c T_2} M_2^2 a_2^2$$

$$\therefore P_1 \left[1 + \frac{M_1^2 \gamma R_c T_1}{R_c T_1}\right] = P_2 \left[1 + \frac{M_2^2 \gamma R_c T_2}{R_c T_2}\right] \quad \ldots a = \sqrt{\gamma R_c T}$$

$$\therefore P_1 [1 + \gamma M_1^2] = P_2 [1 + \gamma M_2^2]$$

$$\therefore \boxed{\frac{P_1}{P_2} = \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2}} \quad \text{—(1)}$$

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now, $\rho_1 V_1 = \rho_2 V_2$

$$\therefore \frac{P_1 M_1 \sqrt{\gamma R_c T_1}}{R_c T_1} = \frac{P_2 M_2 \sqrt{\gamma R_c T_2}}{R_c T_2}$$

$$\therefore \frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_2}}$$

$$\therefore \boxed{\frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_2}}} \quad \text{--- (2)}$$

$$\frac{T_2}{T_1} = \frac{T_2 / T_{02}}{T_1 / T_{01}} = \frac{1+\frac{\gamma-1}{2} M_2^2}{1+\frac{\gamma-1}{2} M_1^2} \quad \text{--- (3)} \quad \ldots\ldots T_{01} = T_{02}$$

from (1), (2), (3),

$$\frac{M_1}{M_2} = \frac{P_2}{P_1}\sqrt{\frac{T_1}{T_2}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\left(\frac{1+\frac{\gamma-1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_2^2}\right)^{1/2}$$

on solving,

$$M_2 = \left(\frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1}\right)^{1/2}$$

thus, $\frac{P_2}{P_1} = f_1(\gamma, M_1)$

$$\frac{T_2}{T_1} = f_2(\gamma, M_1)$$

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_2}{P_{01}/P_1} \times \frac{P_2}{P_1} = f_3(\gamma, M_1)$$

1] A C-D nozzle receives air from a tank at 100 MPa & 600 K. The pressure is 28 MPa immediately preceding the shock that is located in the diverging sect^n. M at exit is 0.5. flow rate = 10 kg/sec. find

a) Throat area . b) area at which shock is located

c) outlet pressure req. to operate nozzle in the manner described above

~~d) outlet area~~

~~e) Design mach no.~~

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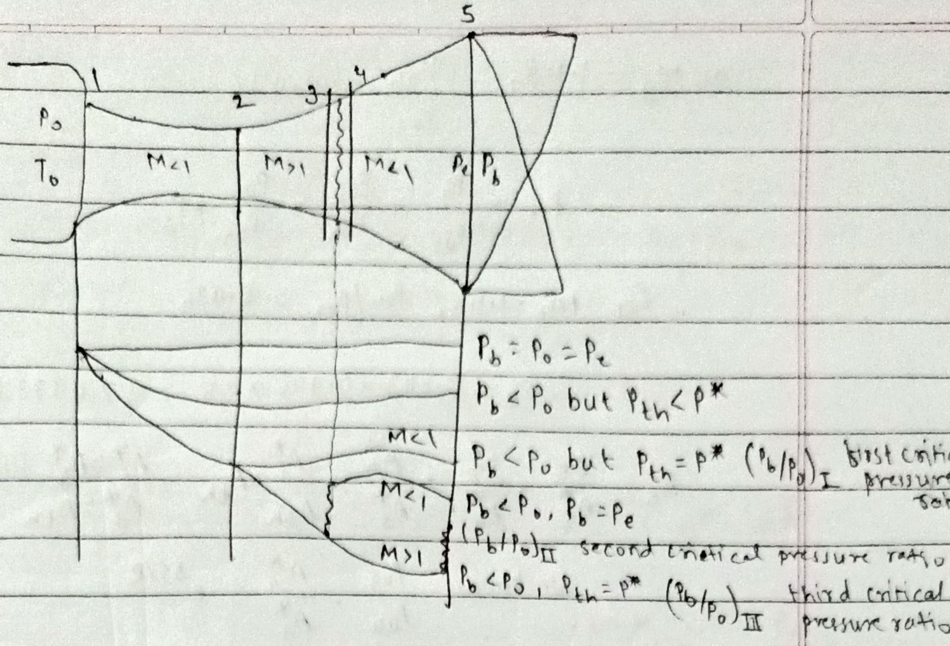


If normal shock is present at exit plane of CD nozzle the ratio of backward pressure to P_0 is 2nd critical pressure ratio

1) If shock is present in CD nozzle, M at throat is 1

$M_2 = 1$ $\quad P_{01} = 100\,MPa$ $\quad T_{01} = 600\,k$ $\quad P_3 = 28\,MPa$ $\quad M_5 = 0.5$

$\Downarrow$ $\quad \dot{m} = 10\,kg/s$

$\dot{m}$ is maximum

$$\therefore \frac{\dot{m}_{max}}{A^*} = 0.0404 \frac{P_0}{\sqrt{T_0}}$$

$$\frac{10}{A^*} = 0.0404 \times \frac{100}{\sqrt{600}} \qquad A^* = A_{th} = 60.631\ m^2$$

as we've assumed flow uptill shock is isentropic,

$$P_{01} = P_{02} = P_{03}$$

$$\therefore \frac{P_3}{P_{03}} = \frac{28}{100} = 0.28 \qquad M_3 = 1.48, \ \frac{T_3}{T_3^*} = 0.6953$$

from gas table for $\frac{P_3}{P_{03}} = 0.28$, $\frac{A_3}{A_3^*} = 1.16$

$\because$ isentropic, $A_3^* = A_2^*$ at throat $M = 1$ $\therefore A_2^* = A_2 = A_{th}$

$$\therefore \frac{A_3}{A_{th}} = 1.16 \qquad \therefore A_3 = 70.332\ m^2$$

$M_5 = 0.5$ $\therefore \frac{P_5}{P_{05}} = 0.843$ ... gas table

$P_{04} = P_{05}$ .... isentropic

as $M_3 = 1.48$, $\frac{P_{04}}{P_{03}} = 0.936$

$$P_5 = \frac{P_5}{P_{05}} \times \frac{P_{05}}{P_{04}} \times \frac{P_{04}}{P_{03}} \times P_3$$

for $M_3 = 1.48$, $P_{04}/P_{03} = 0.936$

$$\therefore P_5 = 0.843 \times 0.936 \times 28 = 22.0933 \text{ MPa}$$

$$A_5 = \frac{A_5}{A_5^*} \times \frac{A_5^*}{A_4^*} \times \frac{A_4^*}{A_3^*} \times \frac{A_3^*}{A_{th}} \times A_{th} \quad \ldots A_5^* = A_4^* \text{ Isentropic}, \; A_3^* = A_{th}$$

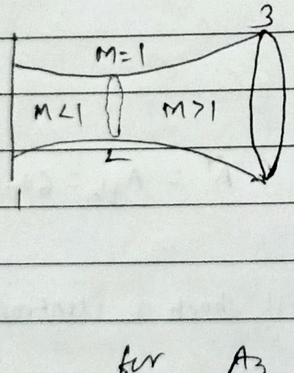
we know, $\frac{P_{03}}{P_{04}} = \frac{A_4^*}{A_3^*} = e^{\Delta S/R}$

$M_5 = 0.5 \quad \therefore \frac{A_5}{A_5^*} = 1.3398$

$$A_5 = 1.3398 \times \frac{P_{03}}{P_{04}} \times A_{th}$$

$$= 1.3398 \times \frac{1}{0.936} \times 60.631 = 86.788 \text{ m}^2$$

Design mach no. means no shock is present



$A_3 = A_5$

$A_2^* = A_3^* = A_2 = A_{th}$

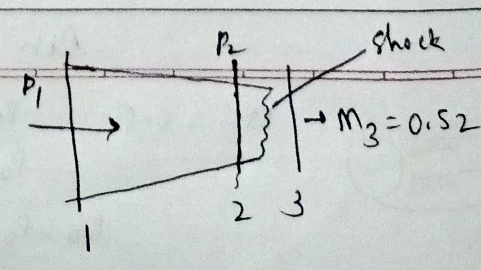
$$\frac{A_3}{A_3^*} = \frac{A_5}{A_{th}} = \frac{86.788}{60.631} = 1.4131$$

for $\frac{A_3}{A_3^*} = 1.4131$ — $M_3 < 1$: $M_3 = 0.455$; $M_3 > 1$: $M_3 = 1.79$

neglecting 0.455, $M_3 = 1.79$

2] Air flows in the system. $M_3 = 0.52$. Considering P_1 & P_2, it is also known that one of these pressures is twice the other

find M_1, A_1/A_2



$M_3 = 0.52$ from Normal shock table, $M_2 = 2.42$

Supersonic diffuser

for $M_2 = 2.42$, $P_2/P_{02} = 0.0663$ $\quad P_{01} = P_{02}$

$$\frac{P_1}{P_{01}} = \frac{P_1}{P_2} \times \frac{P_2}{P_{02}} \times \frac{P_{02}}{P_{01}} = \frac{1}{2} \times 0.0663 = 0.03315$$

$$\frac{P_1}{P_{01}} = 0.03315 \Rightarrow M_1 = 2.87$$

$$\frac{A_1}{A_2} = \frac{A_1}{A_1^*} \times \frac{A_1^*}{A_2^*} \times \frac{A_2^*}{A_2} \quad \ldots\ A_1^* = A_2^*$$

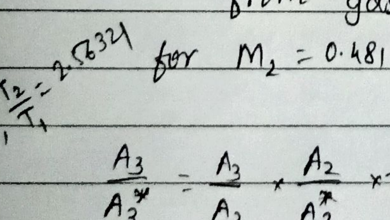
$$= 3.47131 \times \frac{1}{2.44787} \quad \ldots \text{for } M_1 = 2.87,\ \frac{A_1}{A_1^*} = 3.47131$$

for $M_2 = 2.42$, $\frac{A_2}{A_2^*} = 2.44787$

$$= 1.5283$$

3] ~~CD nozzle is designed to produce M = 2.5 with air.~~ A shock stands at inlet to the syst. $M_1 = 2.9$ fluid is Nitrogen
$A_2 = 0.25\ m^2$ $\quad A_3 = 0.2\ m^2$ find M_3 & T_3/T_1

$M_1 = 2.9$



$M_3 > M_2$

from gas table for $M_1 = 2.9$, $M_2 = 0.481$

for $M_2 = 0.481$, $\frac{A_2}{A_2^*} = 1.38$ $\quad \frac{T_{02}}{T_2} = 0.95595$

for $M_1 = 2.9$, $\frac{T_2}{T_1} = 2.56321$

$$\frac{A_3}{A_3^*} = \frac{A_3}{A_2} \times \frac{A_2}{A_2^*} \times \frac{A_2^*}{A_3^*} \quad A_2^* = A_3^*$$

$$= \frac{0.2}{0.25} \times 1.38 = 1.104$$

for $\frac{A_3}{A_3^*} = 1.104$, $M_3 = 1.38$ $\therefore \frac{T_3}{T_{03}} = 0.72418$

$$\frac{T_3}{T_1} = \frac{T_3}{T_{03}} \times \cancel{\frac{T_{03}}{T_{02}}} \times \frac{T_{02}}{T_2} \times \frac{T_2}{T_1} \quad T_{03} = T_{02}$$

$$= 0.7241 \times 0.95595 \times 2.56321$$

$$\approx 2.451$$