Def: Thus Binomial distribution_ is $b(r; n,p) = {}^nC_r\, q^{n-r}p^r$

A r.v, X is said to follow Binomial distribution_if it assumes only non negative values whose prob. mass function is given by

$$P(X=x) = f(x) = \begin{cases} {}^nC_x\, q^{n-x}p^x, & x=0,1,2,...,n; \quad q=1-p \\ 0, & \text{otherwise} \end{cases}$$

It is denoted by b(x;n,p).

Corresponding Cummulative Distribution is

$B(x;n,p)=\sum_{t=x} b(t;n,p)$, x=0,1,2,………..,n

EX.1

In how many cases should we expect to get 6 heads and 4 tails if 10 coins are simultaneously tossed 1000 times?

Solution: Here n=10, p=1/2, q=1/2, r=no of heads =6

Since 10 coins are tossed 1000 times

Let us denote the no. of representations=N=1000

Therefore, let occurrence of head denote the successes and occurrence of tail denote the failure.

In general we know that out of the n coins ‘r’ heads can be obtained in

${}^nC_r\, q^{n-r}p^r$ ways.

And since it repeats N times therefore no of cases to get r heads=N ${}^nC_r\, q^{n-r}p^r$

Here we have to get 6 heads and 4 tails.

Therefore total no of cases= 1000 x ${}^{10}C_6\ (1/2)^6(1/2)^4$

$=205.07812 \approx 205.$

Problem 2: The prob. that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease what is the prob. that (a) at least 10 survive? (b) from 3 to 8 survive (c) exactly 5 survive.

Sol: Let X be the no. of people that survive.

(a) $P(X\geq10)=1-P(X<10)=1-\sum_{x=0}^{9} b(x;15,0.4)$

$=1-\sum_{x=0}^{9}\binom{15}{x}(.4)^x\,(.6)^{15-x}$ $=1-.9662=0.0338$ (Ans.)

(b) $P(3\leq X\leq8)=\sum_{x=3}^{8} b(x;15{,}0.4)=\sum_{x=0}^{8} b(x;15{,}0.4)-\sum_{x=0}^{2} b(x;15{,}0.4)$

$=.9050-.0271=0.8779$ (Ans.)

(c) $P(X=5)=b(5;15,0.4)$

$=\sum_{x=0}^{5} b(x;15{,}0.4)-\sum_{x=0}^{4} b(x;15{,}0.4)=0.1859$ (Ans.)

Also find the mean and variance of the binomial r.v.

Here n=15, p=0.4. Therefore Mean=n.p=15.(0.4)=6.0,

Variance (σ^2)=n.p.q=15.(0.4).(0.6)=3.6.

Problem 3: Some body claims that 80% of all industrial accidents can be prevented by paying strict attention to safety regulations. Assuming that the clam is true, what are the probs. that

(a) fewer than 16 of 20 industrial accidents can be prevented by paying strict attention to safety regulation?

(b) 12 of 15 industrial ………………….regulation?

Sol: (a) For this part we have n=20 (sample)

Prob. of preventing accidents = p=.80

Let X be the no. of accidents prevented.

Reqd. to find the prob. that x<16, i.e. P(X<16) = $\sum_{k=0}^{x} b(k;n,p) = B(x;n,p)$, x=0,1,2,….15

Therefore in this case B(x;n,p), x = 1,2,……..15

=B(15; 20, .8) = 0.3704

(b) Here P(X = 12) = b(12; 15,0.8) = B(12; 15, 0.8) – B(11; 15, 0.8) = 0.6020 - 0.3518 = 0.2502 (Ans.)

Mean and standard deviation of Binomial distribution:

For binomial distribution we know that

f(x) = b(x; n,p) = ${}^{n}C_{x}\, q^{n-x}p^{x}$, x=0,1,2,...,n; where p is the prob. of success and q is the prob. of failure.

(a) Since by definition of mean or mathematical expectation

$\mu = \sum_{x=0}^{n} xf(x)$, for discrete r.v., therefore for binomial distribution

$$\mu = \sum_{x=0}^{n} x\, n_{C_x} p^x q^{n-x}$$

$$\mu = \sum_{x=1}^{n} x\, n_{C_x} p^x q^{n-x} \ldots\ldots.(A)$$

[Since the term with x=0 vanishes.]

From (A),

$$\mu = \sum_{x=1}^{n} x \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

$$\mu = \sum_{x=1}^{n} \frac{n(n-1)!}{(x-1)!(n-x)!} p\, p^{x-1} q^{n-x}$$

$$\mu = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!}\; p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np \sum_{x=1}^{n} n-1_{C_{x-1}}\; p^{x-1} q^{(n-1)-(x-1)}$$

$= np \sum_{X=0}^{N} N_{C_X}\; p^X q^{N-X}$, (taking x-1=X & n-1=N)

$$= \text{np}\sum_{X=0}^{N} b(X; N, p)$$

$$\mu = np.1 = np, \qquad [since\ \Sigma f(x) = 1]$$

Variance $\quad \sigma^2 = \sum_{x=0}^{n} (x-\mu)^2 f(x)$

$$= \sum_{x=0}^{n} (x^2 - 2\mu x + \mu^2) f(x)$$

$$= \sum_{x=0}^{n} (x^2) f(x) - 2\mu \sum_{x=0}^{n} x\, f(x) + \mu^2 \sum_{x=0}^{n} f(x)$$

$$= \sum_{x=0}^{n} x^2 f(x) - \mu^2 \text{ , } where\ \mu = np$$

$$\sigma^2 = \sum_{x=0}^{n} x^2 n_{C_x} p^x q^{n-x} - \mu^2$$

$$\sigma^2 = \sum_{x=0}^{n} x^2\, \frac{n(n-1)(n-2)!}{x!\,(n-x)!} p^x q^{n-x} - \mu^2$$

$$\sigma^2 = \sum_{x=0}^{n} \{x(x-1) + x\} \quad p^x q^{n-x} \frac{n!}{x!\,(n-x)!} - \mu^2$$

$$\sigma^2 = \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,(n-2-(x-2))!} p^x q^{n-x} + \mu - \mu^2$$

$$= n(n-1)p^2 \sum_{x=2}^{n} n-2_{C_{x-2}} p^{x-2}\, q^{n-2-(x-2)} + \mu - \mu^2$$

$$= n(n-1)p^2 + \mu - \mu^2$$

$$= np(1-p), \quad (since\ \mu = np)$$

$$= npq.$$

Therefore standard deviation is +ve square root of *npq*.

Thus mean of binomial distribution is $\mu = np$ and standard deviation of this distribution is $\sqrt{npq}$.