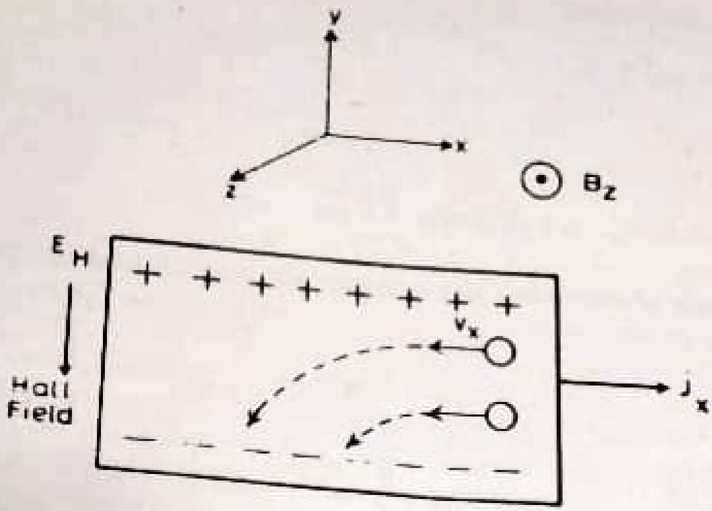

Fig. 6.33 Origin of Hall effect and Hall field.

there. This combination of positive and negative surface charges creates a downward electric field, which is called the Hall field.

The Lorentz force F_L which produces the charge accumulation in the negative y-direction, has the value.

$$F_L = ev_xB_z$$

Now the field created by the surface charges produces a force which opposes this Lorentz force. The accumulation process continues until the Hall force completely cancels the Lorentz force. Thus, in the steady state, $F_H = F_L$.

$$eE_H = ev_xB_z$$

$$E_H = v_xB_z \tag{6.153}$$

The current density, j_x is given by the equation

$$j_x = -nev_x \tag{6.154}$$

Dividing equation (6.153) by equation (6.154),

$$\frac{E_H}{j_x} = -\frac{B_z}{ne}$$

$$E_H = -\left(\frac{1}{ne}\right) j_xB_z$$

The Hall field is thus proportional, both to the current, and to the magnetic field.

The proportionality constant, that is $\frac{E_H}{j_x B_z}$ is known as the Hall constant and is usually usually denoted by R_H, i.e.,

$$\frac{\frac{E_H}{j_x}}{B_z} = -\frac{1}{ne} = R_H \qquad (6.155)$$

Now the *Hall constant* or *Hall coefficient*, R_H is defined as the ratio of the electric field strength produced per unit current density to the transverse magnetic field. It will be noted tht R_H depends on the sign of e and the reader should verify that if E_H is in a certain direction for a flow of negative charges, then it will be in the opposite sense for the same current when it is produced by a flow of positive charges in a reverse direction. In the monovalent metals, R_H is negative, which is consistent with our belief that the current is produced by a flow of negatively charged particles; the magnitude of R_H is then such that there is of the order of one moving charge per atom. In more complicated metals, particularly those in which there is band overlap, R_H can be positive (e.g., in zinc and cadmium), and here it is assumed that most of the conduction occurs by the motion of positive holes. From equation (6.155)

$$R_H = \frac{E_H}{j_x B_z} = \left(\frac{V_y}{y}\right)\left(\frac{yz}{I_x}\right)\frac{1}{B_z} = \frac{V_y Z}{I_x B_z} \qquad (6.155a)$$

$$\text{Unit of } R_H = \frac{\text{volt-m}}{\text{amp-weber}/m^2} = \text{Vm}^3\text{A}^{-1}\text{wb}^{-1}$$

Table 6. E Hall coefficient and mobilities for some metals at 300 K

Metal	R_H ($\text{Vm}^3\text{A}^{-1}\text{wb}^{-1}$) in 10^{-10}	μ ($\text{m}^2\text{V}^{-1}\text{s}^{-1}$)
Silver	−0.84	0.0056
Copper	−0.55	0.0032
Gold	−0.71	0.0030
Sodium	−2.50	0.0052
Aluminium	−0.31	0.0012
Lithium	−1.70	0.0018
Zinc	+0.30	0.0060
Cadmium	+0.60	0.0080

The general expression for current density is

$$j_x = nev_x$$

i.e., electrical conductivity

$$\sigma = \frac{j_x}{E_x} = \frac{(ne)v_x}{E_x}$$

The drift velocity produced for unit electric field is called the *mobility* of charge carriers.

i.e.,

$$\sigma = \frac{j_x}{E_x} = ne\mu_e \quad \text{or} \quad \mu_e = \left(\frac{1}{ne}\right)\sigma$$

$$\mu_e = R_H \sigma \quad \text{or} \quad \mu_e = \frac{E_H}{j_x B_z}\left(\frac{j_x}{E_x}\right)$$

$$\mu_e = \left(\frac{V_y}{B_z y}\right)\left(\frac{x}{V_x}\right) = \left(\frac{V_y}{V_x}\right)\left(\frac{x}{y}\right)\left(\frac{1}{B_z}\right) \tag{6.156}$$

From this, the mobility of electrons may be determined. Measurement of Hall voltage helps one to determine the following:

1. The sign of current-carrying charges can be determined.
2. The number of charge carriers present in unit volume can be calculated from the magnitude of R_H.
3. The mobility of the charge carriers may be obtained directly from the measurement of Hall voltage. The Hall coefficients and mobilities of some selected metals are given in Table 6.E.

Magnetoresistance

The magneto-resistance of a crystal refers to the change of electrical resistance of a crystal when a magnetic field is imposed. This effect is due to the fact that, when the magnetic field is imposed, the paths of the electrons become curved and do not go exactly in the direction of the superimposed electic field. In the case of metals the resistance generally increases. The magneto resistivity, $\frac{\Delta\rho}{\rho_0}$, where ρ_0 is zero field resistivity and $\Delta\rho = \rho(B) - \rho_0$ is found to increase with magnetic field and varies as B^2 in small fields. But in large fields this may vary as B or even tend to saturation or vary as either B^2 or as B.

For a magnetic field applied perpendicular to the electric field, the magneto resistivity is given by

$$\frac{\Delta\rho}{\rho_0} = \frac{\alpha(B/\rho_0)^2}{1 + \gamma(B/\rho)^2} \tag{6.157}$$

where α and γ are constants. For small fields, the magneto resistivity varies as B^2. If $\gamma = 0$, this is directly proportional to B^2, but if $\gamma \neq 0$, it tends to a finite limit. We also notice an isotropy of magneto-resistance. For example, in lead the magneto resistivity attains a saturation value in one direction, while it increases without limit as B^2 in another direction as shown in Fig. 6.34. It can be shown theoretically that magneto-resistance is absent if τ is independent of velocity. However, under certain conditions like dominant lattice scattering, and presence of large B, $\Delta\rho/\rho_0$ can also be shown to saturate. In the case of polycrystalline specimens, $\Delta\rho/\rho_0$

$$\int_0^\infty x^n e^{-ax} dx = \frac{\angle n}{a^{n+1}}$$

Thus
$$(\Delta W) = \frac{e^2E^2}{2m\tau}\frac{2}{(1/\tau)^3}$$

$$(\Delta W) = \frac{e^2E^2\tau^2}{m} \tag{6.151a}$$

If n is the number of electrons present in unit volume, the total energy dissipated per unit volume per second is

$$W = \frac{n}{\tau}(\Delta W)$$

$$W = \frac{n}{\tau}\frac{e^2E^2\tau^2}{m} = \sigma E^2 \tag{6.152}$$

This agrees with the experimental relation.

XXX. EFFECT OF THE MAGNETIC FIELDS

Many interesting effects arise when a metal is subjected to a magnetic field. Amongst them are the *Hall effect* and *magneto resistance*.

Hall effect

The results of the wellknown experiment on *Hall effect* sometimes also seem to contradict completely the classical picture of conduction. If a sample conducting material is placed in a uniform magnetic field and a current is passed along the length of the conductor as shown in Fig. 6.33, a voltage is found to develop at right angles to both the direction of the current flow and that of the magnetic field. This voltage is known as the *Hall voltage*, and its value is found to depend on the magnetic field strength and on the current passed. The mathematics of Hall effect is based on the simple dynamics of charges moving in electromagnetic fields.

Consider a specimen in the form of a rectangular cross-section as shown in Fig. 6.33 carrying a current I_x in the x-direction. If a uniform magnetic field B_z is applied along the z-axis, it is found that an emf develops along the y-axis i.e., in a direction perpendicular to I_x and B_z. This voltage is called Hall voltage.

Let us first consider the situation before the magnetic field is introduced. There is an electric current flowing in the positive x-direction, which means that the conduction electrons are drifting with a velocity v_x in the negative x-direction. When the magnetic field is introduced, the *Lorentz force* F_L causes the electrons to bend downward as shown in Fig. 6.33. As a result, electrons accumulate on the lower surface, producing a net negative charge there. Simultaneously a net positive charge appears on the upper surface, because of the deficiency of electrons