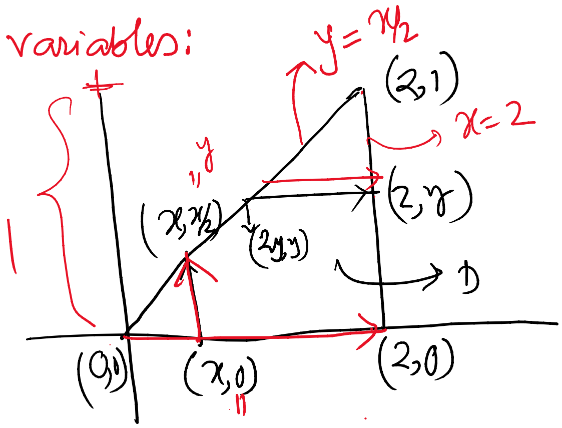
Change of variables:

06 January 2022 08:06

Recap:



$$\iint_D (xy^2)\,dx\,dy$$

$$= \int_0^2 \int_0^{x/2} xy^2\,dy\,dx$$

$$= \int_0^1 \int_{x=2y}^{x=2} xy^2\,dx\,dy$$

$$= 4/15$$

$\boxed{\text{MCQ}}$

LKD

$\boxed{\text{C-D}}$ $\boxed{\text{Question will be same for all}}$

No negative marking

9 → 15 marks

11-01-22

9 to 9:30 AM

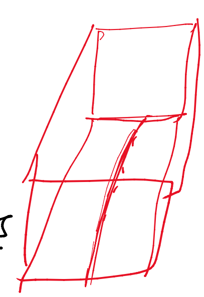
Question: Let $f: D \to \mathbb{R}^+$, $D \subseteq \mathbb{R}^2$, then $\iint_D f\,dx\,dy$ will always exist. (T|F) F

Existence Th. Suppose f is bounded on B and cont. except for discont. along a finite number of graphs of cont. functions

$\Downarrow$

f is integrable

i.e. $\iint_B f\,dx\,dy$ exists



$$\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx$$

Let f is bounded on $[a,b]$ and the set of points of discontinuities D is a zero set.

$\Downarrow$ f is integrable

Q. $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ using double integration

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \rightarrow \int_{-\infty}^{0} e^{-x^2}\,dx + \int_{0}^{\infty} e^{-x^2}\,dx$$

$\int_{0}^{\infty} e^{-x^2}\,dx \rightarrow$ Gamma function

Cylindrical Co-ordinate:

(r, θ, z) of a point

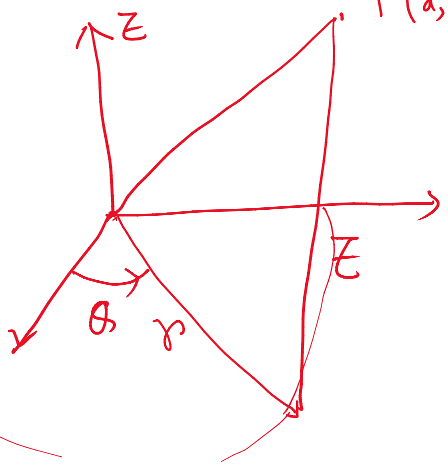
(x, y, z)

$\downarrow$

(r, θ, z): $x = r\cos\theta,\ y = r\sin\theta,\ z = z,\quad r \geq 0,\ 0 \leq \theta \leq 2\pi$

$P(x, y, z)$

$$\boxed{x^2 + y^2 = r^2}$$

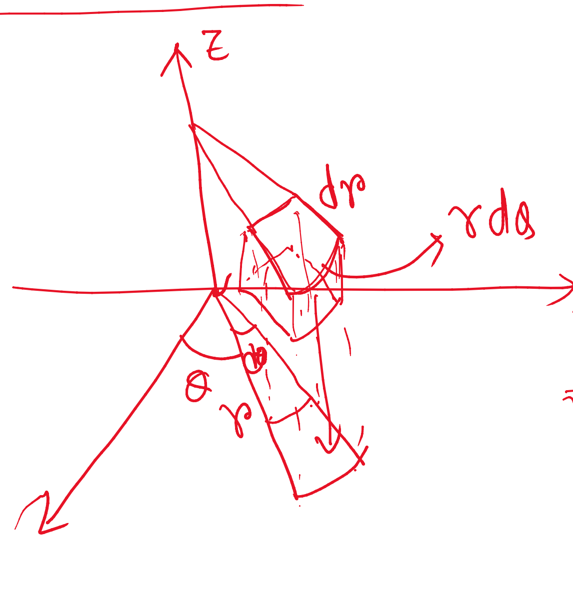


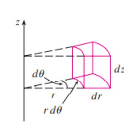
(x, y, z)

$\downarrow$

(r, θ, z)

Infinitesimal volume element:





$$\boxed{r\,dr\,d\theta}$$

x

Triple integration: If W is a region in space,

$$\iiint_W f(x,y,z)\,dx\,dy\,dz = \iiint_{W^*} f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$$

Where W^* is the corresponding region in (r,θ,z) space.

Ex. $\iiint (z^2x^2+z^2y^2)\,dx\,dy\,dz$

$W=\{(x,y,z): x^2+y^2\le 1,\ -1\le z\le 1\}$

Using cylindrical transformation,

$W^*=\{(r,\theta,z): 0\le r\le 1,\ 0\le\theta\le 2\pi,\ -1\le z\le 1\}$

$$= \int_{-1}^{1}\int_0^{2\pi}\int_0^1 z^2 r^2 \boxed{r\,dr\,d\theta\,dz}$$

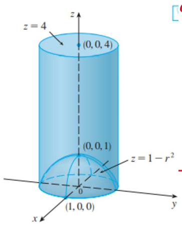
$$= \int_{-1}^{1}\int_0^{2\pi} z^2\cdot \frac{r^4}{4}\Big|_{r=0}^{1} d\theta\,dz = \int_{-1}^{1}\int_0^{2\pi} z^2/4\,d\theta\,dz$$

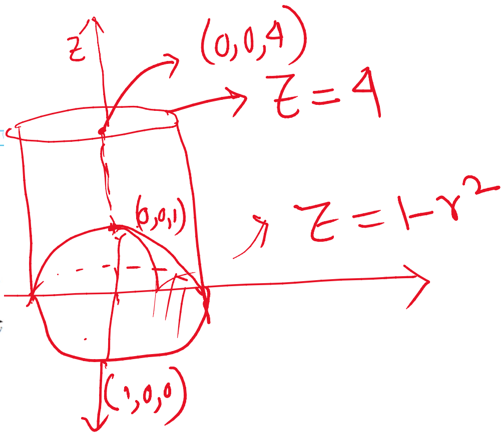
$$= 2\pi\cdot\frac{1}{12}\cdot\left(z^3\Big|_{-1}^{1}\right)$$

$$= \pi/3.$$

Ex. A solid E lies within the cylinder $x^2+y^2=1$, below the plane $z=4$, and above the paraboloid $z=1-x^2-y^2$. The density at any pt

$z = 1 - x^2 - y^2$. The density at any pt is proportional to its distance from the axis of the cylinder. Find the mass of E.





In cylindrical system

$$E = \{(r,\theta,z) : 0 \le \theta \le 2\pi,\ 0 \le r \le 1,\ \underline{1-r^2} \le z \le 4\}$$

Since the density at (x,y,z) is proportional to the distance from the z-axis, so the density function

$$f(x,y,z) = k\sqrt{x^2+y^2} = kr, \quad \underline{k>0} \text{ const.}$$

So, mass of E

$$= \iiint_E k\sqrt{x^2+y^2}\,dV$$

$$= \int_0^{2\pi}\int_0^1\int_{1-r^2}^{4} kr\cdot r\,dz\,dr\,d\theta$$

(do it!) $$= \frac{12\pi k}{5}$$

Ex. Evaluate $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$

$\sqrt{4-x^2}$ $\qquad$ $\sqrt{x^2+y^2}$

Using cylindrical transformation, we get $E = \{(x,y,z) : -2 \le x \le 2,\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2},\ \sqrt{x^2+y^2} \le z \le 2\}$

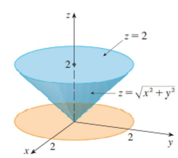
$$E^* = \{(r,\theta,z) : 0 \le \theta \le 2\pi,\ 0 \le r \le 2,\ r \le z \le 2\}$$

$$= \int_0^{2\pi}\int_0^{2}\int_r^{2} r^2 \cdot r \, dz \, dr \, d\theta$$

Check it

$$= \frac{16\pi}{5}$$

$$\boxed{r^2 = x^2+y^2}$$



Figure

I

$\boxed{\text{Spherical co-ordinate}} \longrightarrow$ Next class