Heat And Mass Transfer :- (MEC403)

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→ Heat transfer :- (q)

the heat/thermal energy in transit due to temperature difference

↳ q_x = rate of heat transfer along x direction ⇒ J/s or W

↳ q'_x = rate of heat transfer along x direction per unit length

↳ q''_x = rate of heat transfer along x direction per unit area.

↳ q'''_x = rate of heat transfer along x direction per unit volume.

(q''_x = Heat flux)

→ Conduction :- (Diffusion)

process by which heat transfer occurs through the collisions b/w neighbouring atoms or molecules via a conductor.

Fourier's Law of Conduction :-

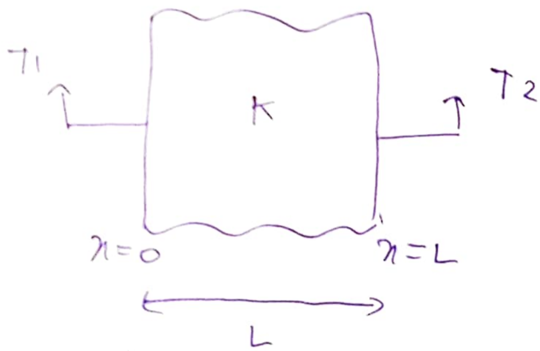
"The time rate of heat transfer through a material is proportional to the negative gradient in the temperature & to the area at 90° to that gradient, through which the heat flows."

i.e. $q_x = -KA\left(\frac{dT}{dx}\right)$ → temp grad.

K → thermal conductivity of material

A → area

($K = [W/mK]$)



conduction occurs through a medium and its dominant mode of heat transfer in solid

→ Convection :- (Advection + Diffusion)

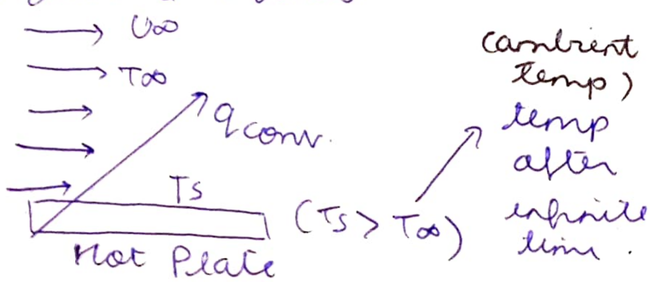
Advection → transport of substance or quantity by bulk motion of fluid.

process by which heat transfer occurs through the bulk movement of molecules within fluids such as gases & liquids.

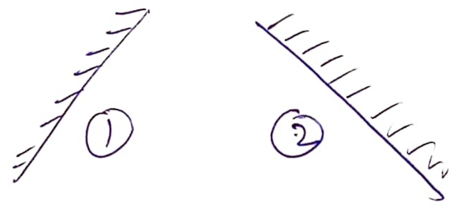
i.e. $q_{conv} = hA(T_s - T_\infty)$

h → convective heat transfer coefficient

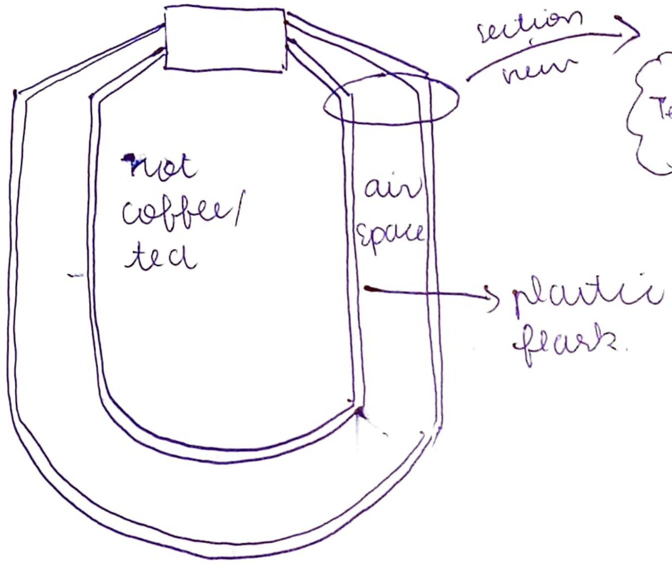
A → area.

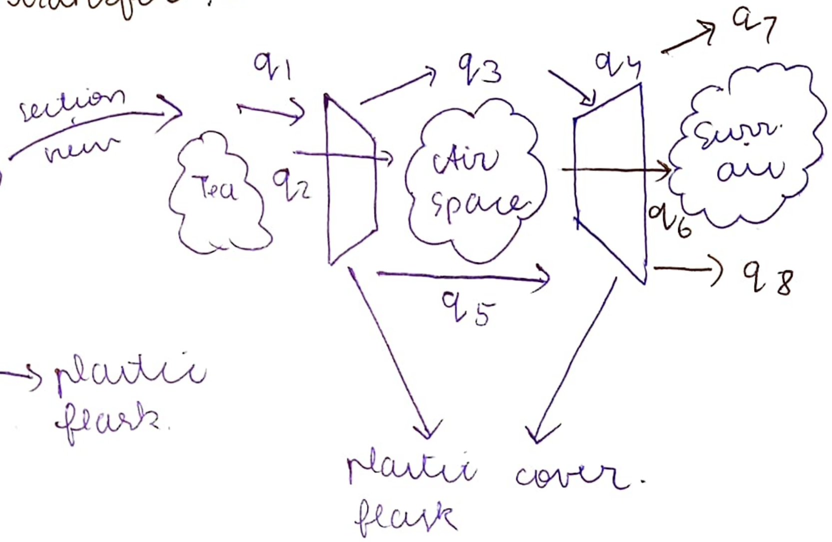


→ Radiation :-
process by which heat transfer occurs through the emission or transmissions of energy in the form of waves or particles through space or through material medium.
i.e. $q_{rad} = \sigma A T^4$
(Stefan - Boltzman Equation)



→ Identify different modes of heat transfer :-





q_1 = free convection from coffee / Tea to plastic flask.
q_2 = conduction through plastic flask
q_3 = free convection from ~~air to cover~~ plastic flask to air.
q_4 = free convection from air to cover
q_5 = Net radiation exchange b/w the outer surface of plastic flask and inner surface of cover.

q_6 = conduction through cover
q_7 = surrounding air
q_8 = Net radiation exchange b/w outer surface of cover & surrounding air.

∴ Net heat loss = $\sum_{i=1}^{8} q_i$ } flask becomes cool over

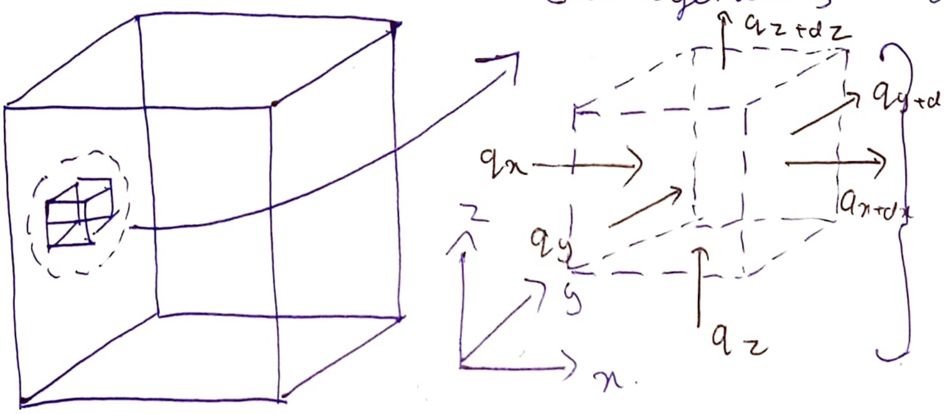
To reduce the heat loss from flask :-

q_5 & q_8 by silver / aluminium lining. ??

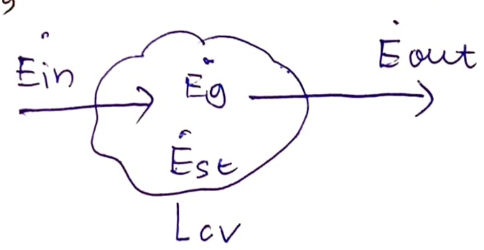
q_3 & q_4 by creating vacuum.

→ 3-D Heat Conduction / Diffusion Equation in Cartesian Coordinate System :-

Homogenous body & no advection.



1st law of thermodynamic



$\dot{E}_g$ = energy generated

$\dot{E}_{st}$ = energy storage ⟶ stored in form of sensible heat

$$\therefore \dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \quad \text{—— (I)}$$

∴ considering infinitesimally small control volume of volume $= dx \cdot dy \cdot dz$

$$\therefore \dot{E}_{in} = q_x + q_y + q_z \quad \text{—— (1)}$$

$$\dot{E}_{out} = q_{x+dx} + q_{y+dy} + q_{z+dz}$$

using Taylor series,

$$q_{i+di} = q_i + \frac{\partial q_i}{\partial i} \cdot di$$

neglecting HOT.

$$\therefore \dot{E}_{out} = \left(q_x + \frac{\partial q_x}{\partial x} dx\right) + \left(q_y + \frac{\partial q_y}{\partial y} dy\right) + \left(q_z + \frac{\partial q_z}{\partial z} dz\right) \quad \text{—— (2)}$$

$$\dot{E}_{out} = E_{in} + \frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy + \frac{\partial q_z}{\partial z} dz$$

$$\because \dot{E}_g = q''' \cdot (dx\, dy\, dz) \quad \text{—— (3)}$$

↘ heat generated per unit volume. (W/m3)

$$\frac{\dot{E}_{st}}{dx\, dy\, dz} = \rho \cdot c_p \frac{\partial T}{\partial t} \quad \text{—— (4)} \qquad \therefore \frac{\partial T}{\partial t} = \frac{\dot{E}_{st}}{\rho\, c_p \cdot dV}$$

Putting ① ② ③ ④ in (I)

$$\therefore (q_x + \cancel{q_y + q_z}) + (q''' \times (dx\,dy\,dz)) - (q_x + \cancel{q_y} + q_z + \frac{\partial q_x}{\partial x}dx + \frac{\partial q_y}{\partial y}dy + \frac{\partial q_z}{\partial z}dz) = (\rho c_p \frac{\partial T}{\partial t} dx\,dy\,dz)$$

$$\therefore \frac{\partial q_x}{\partial x}dx + \frac{\partial q_y}{\partial y}dy + \frac{\partial q_z}{\partial z}dz = \left(\rho C_p \frac{\partial T}{\partial t} - q'''\right) dv \quad \text{— (II)}$$

$$\left.\begin{aligned} \because q_x &= -k\,dy\,dz\frac{\partial T}{\partial x} \\ q_y &= -k\,dz\,dx\frac{\partial T}{\partial y} \\ q_z &= -k\,dx\,dy\frac{\partial T}{\partial z} \end{aligned}\right\} \text{— (III)}$$

Also considering k as not a constant.
(unit of k = W/mK)

putting (III) in (II)

$\therefore$

$$\boxed{\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + q''' = \rho C_p \frac{\partial T}{\partial t}}$$

↳ 3-D heat conduction equation.

considering k as constant and not steady steady state:-

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)k + q''' = \rho C_p \frac{\partial T}{\partial t}$$

considering steady state condition and k not constant:-

$$\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + q''' = 0$$

considering no internal heat generation and k not constant:-

$$\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] = \rho C_p \frac{\partial T}{\partial t}$$

→ Let $\frac{k}{\rho c_p} = \alpha = \frac{\text{thermal conductivity}}{\text{heat capacity}}$ = Thermal diffusivity

Thermal conductivity ↓ larger value ∴ ↑ heat transfer

Heat capacity ↓ ability of the material to store the heat. In solids & liq ∴ $\rho \cdot c_p$ is small (ρ is ↑ & c_p ↓)

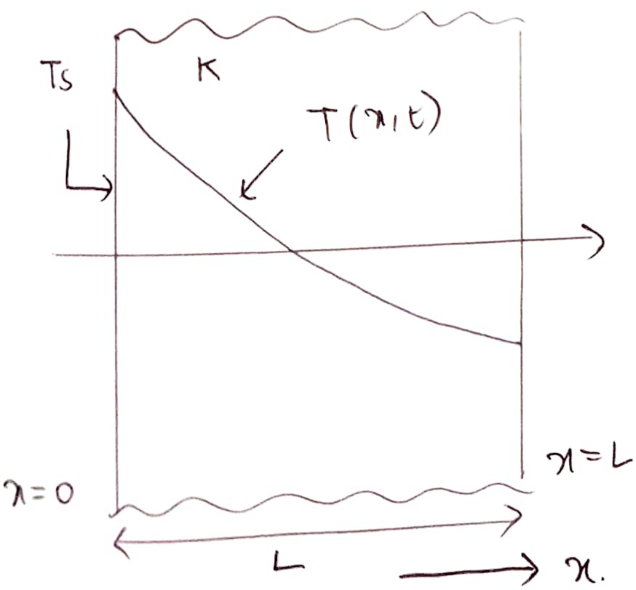
→ α is not always constant :-
Its usually $k = f(T)$ &
i) solid (metal) ∴ $T \uparrow$ ∴ $k \downarrow$
ii) gas (air) ∴ $T \uparrow$ ∴ $k \uparrow$

} occurs b/c in metals, the free e^- starts moving highly & not move according to the req. vibration for transferring the heat. Whereas in gas, the random ↑se in vibration of molecules hit each other more frequently thereby k ↑se

→ Boundary conditions :-

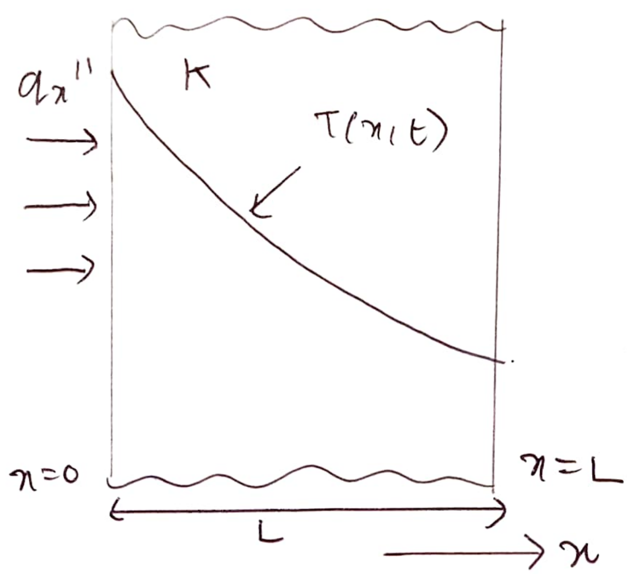
I) for 1-D :-

1) Specific surface temp B.C
(OR)
B.C. of 1st kind
(OR)
Dirichlet B.C.



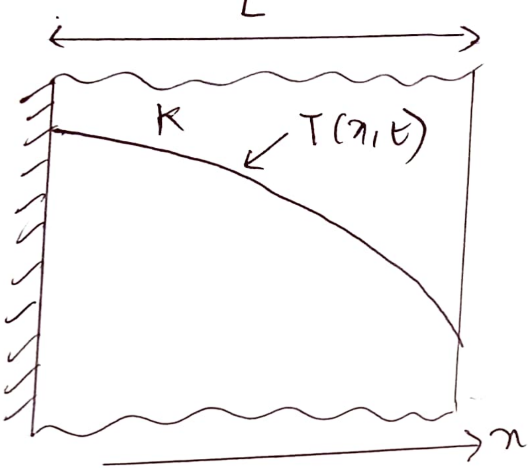
$$\boxed{T(0,t) = T_s}$$

2) Constant heat flux B.C
(OR)
B.C. of 2nd kind
(OR)
Neumann B.C.



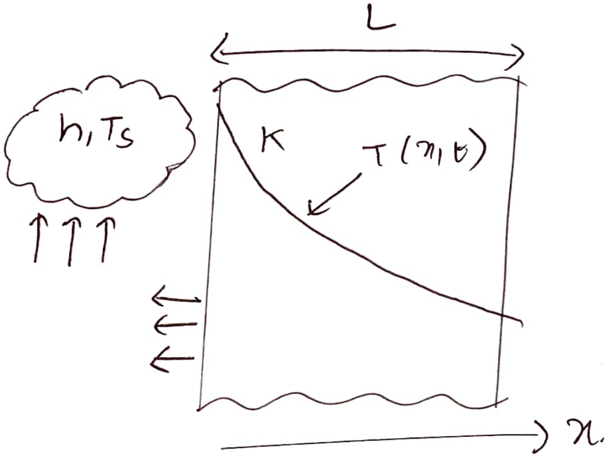
$$\boxed{-k \frac{dT}{dx}\bigg|_{x=0} = q_x''}$$

3) Special case of 2nd B.C
i.e Insulated B.C
i.e. $k \neq 0$.



$$\boxed{\left.\frac{dT}{dx}\right|_{x=0} = 0}$$

4) Convective B.C also called
(OR)
B.C of 3rd kind.



$$-kA\left.\frac{dT}{dx}\right|_{x=0} = hA\left[T(0,t) - T_\infty\right]$$

$$-k\left.\frac{dT}{dx}\right|_{x=0} = h\left[T(0,t) - T_\infty\right]$$

→ Solve the following :-

Q. The temp distribution across a wall 1m thick at a certain instant of time is given as $T(x) = a + bx + cx^2$ where T is in °C & x is in metres while $a = 900°C$, $b = -300°C/m$, $c = -50°C/m^2$. A uniform heat generation $q''' = 1000\ W/m^3$ is present in the wall of area of $10\ m^2$ having the properties $\rho = 1600\ kg/m^3$, $k = 40\ W/m°C$, $C_p = 4\ kJ/kg°C$.

1) Determine the rate of heat transfer entering the wall (meaning $x = 0$) & leaving the wall ($x = 1m$)

2) Determine the rate of change of energy storage in the wall (Hint find $\dot{E}_{st}$)

3) Determine the time rate of Temp change at $x = 0, 0.25, 0.5\ m$

(find $\left.\frac{\partial T}{\partial t}\right|_{x=0,\,0.25,\,0.5\ m}$)

(is $\partial T/\partial t$ always constant as $\dot{E}_{st} = \rho C_p \partial T/\partial t$ & $\dot{E}_{st}$ is constant as it depends only on initial & final qn condition)

8) Sol^n :- 1) $q_x^* = -KA\frac{dT}{dx} = -KA(b+2cx)$

∴ $q_x^*|_{x=0} = -40\times10\times(-300+2\times(-50)\times0) = 120$ kW

$q_x^*|_{x=1} = -40\times10\times(-300+2\times(-50)\times1) = 160$ kW

2) $q_x^*|_{x=0} = \dot{E}_{in} = 120$ kW

$q_x^*|_{x=1} = \dot{E}_{out} = 160$ kW.

$\dot{E}_g = q'''_x \times Vol = 1000\times(10\times1) = 10$ kW.

∴ $\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 120-160+10 = -30$ kW

3) $\frac{\partial T}{\partial t} = \frac{K}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2}\right] + \frac{q'''}{\rho c_p} = \frac{1}{\rho c_p}\left[k(2c)+q'''\right]$ (independent of x)

$= \frac{1\times(40\times2\times(-50)+10^3)}{16\times10^2\times4\times10^3}$

II $= -6.25\times10^{-4} + 1.5625\times10^{-4}$

$= -4.6875\times10^{-4}$ °C/s

Also we can find $\frac{\partial T}{\partial t}$ as:-

$\frac{\partial T}{\partial t} = \frac{\dot{E}_{st}}{\rho c_p dV} = \frac{-30\times10^3}{16\times10^2\times4\times10^3\times(10\times1)} = -4.6875\times10^{-4}$ °C/s

→ $k = f(T)$ where let $k = k_0(1+\alpha T)$

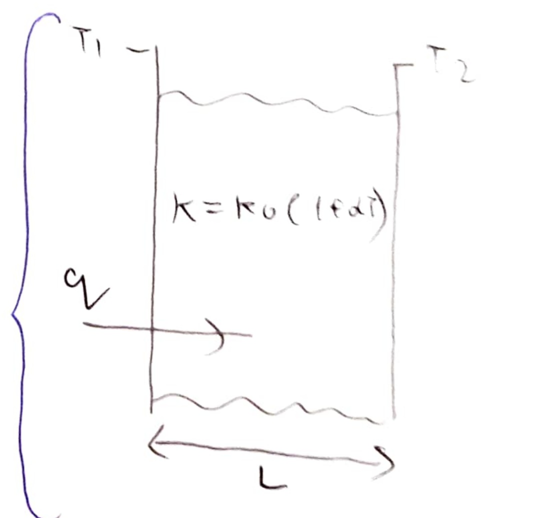
↳ $\alpha > 0 \Rightarrow$ for non-metal

↳ $\alpha < 0 \Rightarrow$ for metal

↳ $\alpha = 0 \Rightarrow k \neq f(T)$

$q = -kA\frac{dT}{dx}$

$q = -k_0(1+\alpha T)A\frac{dT}{dx}$



$$\therefore q\int_0^L dx = -KA\int_{T_1}^{T_2}(1+\alpha T)\,dT$$

$$qL = -K_0A\left[T+\alpha T^2/2\right]_{T_1}^{T_2}$$

$$q = \left(\frac{K_0A}{L}\right)\left[(T_1-T_2)+\left(\frac{\alpha}{2}\right)(T_1^2-T_2^2)\right]$$

$$q = \left(\frac{K_0A}{L}\right)(T_1-T_2)\left[1+\alpha\left(\frac{T_1+T_2}{2}\right)\right]$$

let K_m = mean value of thermal conductivity at mean temp $= K_0\left[1+\alpha\left(\frac{T_1+T_2}{2}\right)\right]$

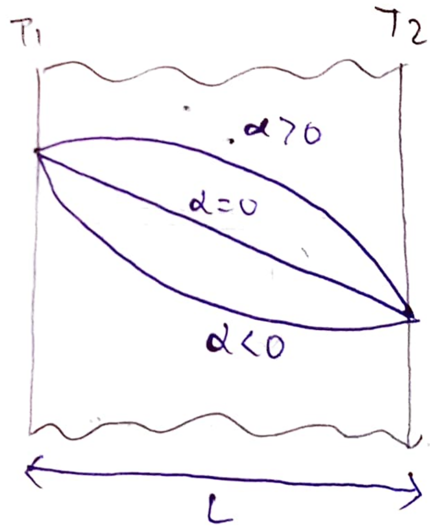
$$\therefore \boxed{q = \left(\frac{K_mA}{L}\right)(T_1-T_2)}$$

→ slope $= \frac{dT}{dx} = \frac{q}{K_0(1+\alpha T)}$ } for $K = K_0(1+\alpha T)$

1) if $\alpha = 0$ ∴ constant line of slope ∴ $K = K_0$

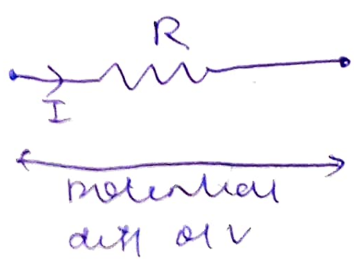
2) if $\alpha > 0$ ∴ slope is +ve along material direction ∴ K ↑ses with α

3) if $\alpha < 0$ ∴ slope is −ve along material direction ∴ K ↓ses with α.



→ Electrical Analogy for Heat transfer Problems:-

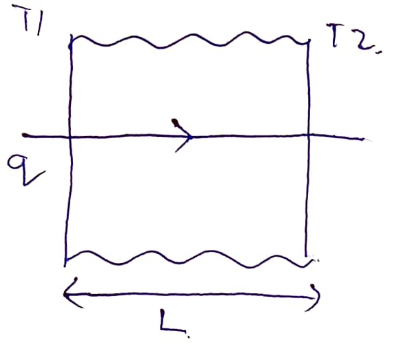
for electrical circuits:-



from ohm's law

$I = V/R$.

For heat transfer :-



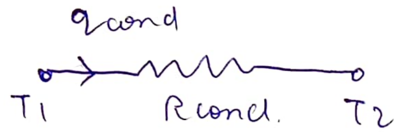
for $T_1 > T_2$ for conduction

∴ treating T_1 & T_2 analogous to voltage

q analogus to current.

$$\therefore q_{cond} = \left(\frac{KA}{L}\right)(T_1 - T_2)$$

$$\therefore q_{cond} = \left(\frac{T_1 - T_2}{L/KA}\right) = \frac{T_1 - T_2}{R_{cond}}$$



where $R_{cond} = L/KA$

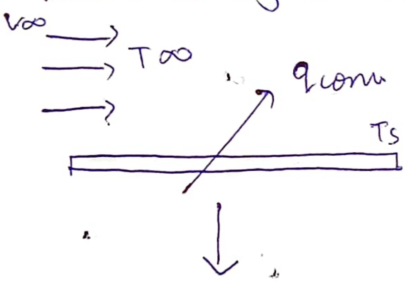
Hence we can replace conduction problem with electrical problem as

$V \sim (T_1 - T_2)$

$I \sim q_{cond}$

$R_{cond} \sim R$.

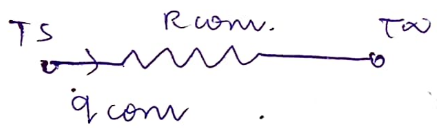
Similarly for convection :-



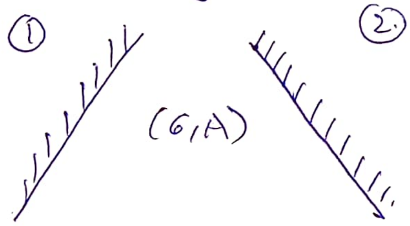
$$q_{conv} = hA(T_s - T_\infty)$$

$$q_{conv} = \frac{T_s - T_\infty}{(1/hA)} = \frac{T_s - T_\infty}{R_{conv}}$$

$$R_{conv} = \frac{1}{hA}.$$



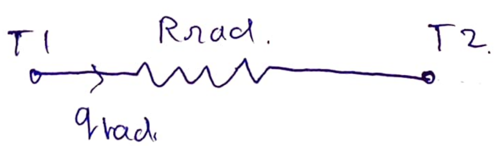
Similarly for Radiation :-



$$q_{rad} = \sigma A (T_1^4 - T_2^4)$$

$$q_{rad} = \sigma A (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$$

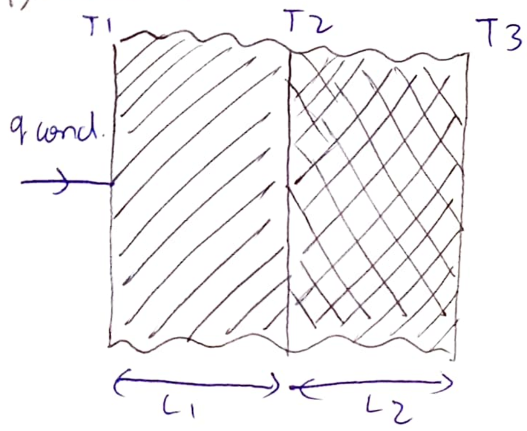
$$q_{rad} = \frac{T_1 - T_2}{\frac{1}{\sigma A (T_1^2 + T_2^2)(T_1 + T_2)}} = \frac{T_1 - T_2}{R_{rad}}$$



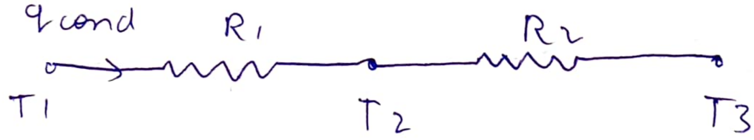
$$R_{rad} = \frac{1}{\sigma A (T_1^2 + T_2^2)(T_1 + T_2)}$$

→ Composite Wall :-

1) Series :-



$T_1 > T_2 > T_3$



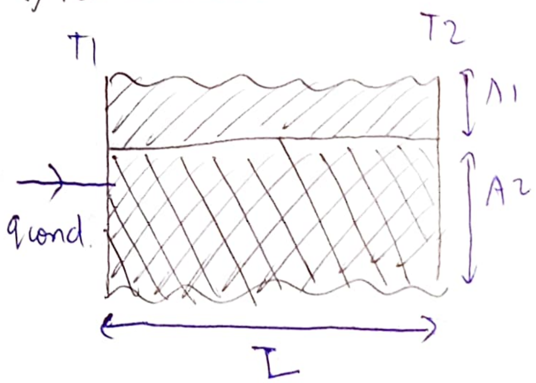
$$q_{cond} = \frac{T_1 - T_3}{R_1 + R_2}$$

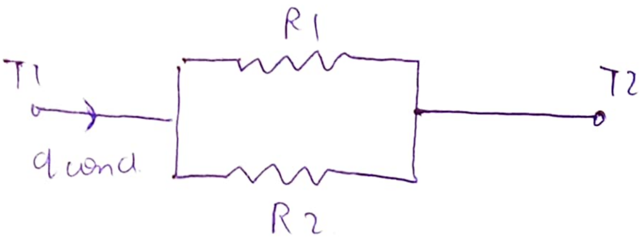
$$\frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

$$R_1 = \frac{L_1}{k_1 A}$$

$$R_2 = \frac{L_2}{k_2 A}$$

2) Parallel :-

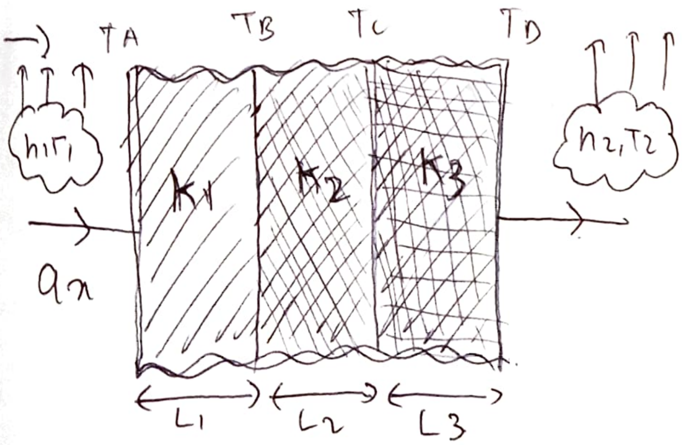


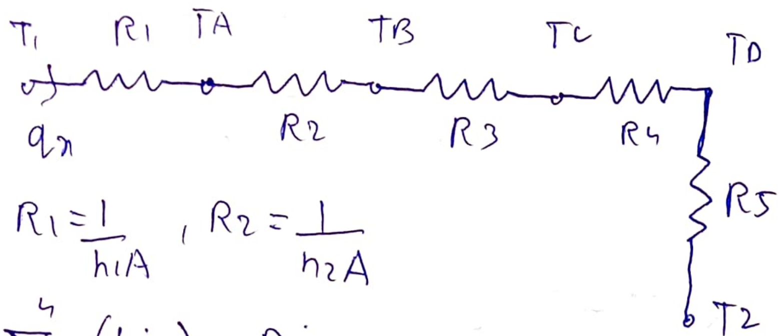


$$q_{cond} = \frac{T_1 - T_2}{1/R_1 + 1/R_2}$$

$$R_1 = \frac{L}{k_1 A_1}$$

$$R_2 = \frac{L}{k_2 A_2}$$





$$R_1 = \frac{1}{h_1 A}, \quad R_2 = \frac{1}{h_2 A}$$

$$\sum_{i=2}^{4} \left(\frac{L_i}{k_i A}\right) = R_i$$

$$q_x = \frac{T_1 - T_2}{\sum_{i=1}^{5} (R_i)} = \frac{T_1 - T_A}{R_1} = \frac{T_A - T_B}{R_2} = \frac{T_1 - T_B}{R_1 + R_2} = \dots$$

Let U = overall heat transfer coefficient (W/m^2K)

$$q_x = UA(T_1 - T_2) = \frac{T_1 - T_2}{1/UA}$$

$$\therefore \frac{1}{UA} = \frac{1}{h_1 A} + \frac{1}{h_2 A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}$$

$$\therefore \frac{1}{U} = \frac{1}{h_1} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_2}$$

→ solve:-

Q) A cold storage room has walls made of 0.23 m of brick on the outside, 0.80 m of plastic foam & finally 1.5 cm of wood on the inside. The outside & inside air temp are 22°C & -2°C respectively. If the inside & outside heat transfer coefficients are 29 & 12 W/m²K and thermal conductivity of brick, foam & wood are 0.98, 0.02 & 0.17 W/mK respectively. Determine.

1) Rate of heat removed by refrigeration if the total area is 90 m²

2) Temp. of inside surface of brick.

Sol^n 1) $R_1 = \frac{1}{h_1 A} = \frac{1}{29 \times 90} = 3.83 \times 10^{-4}$ $\quad R_5 = \frac{1}{h_2 A} = \frac{1}{12 \times 90} = 9.25 \times 10^{-4}$

$$\left.\begin{aligned} R_2 &= 2.60 \times 10^{-3} \\ R_3 &= 0.44 \\ R_4 &= 9.80 \times 10^{-4} \end{aligned}\right\} \quad R_i = \frac{L_i}{K_i A}$$

$$\therefore R_{net} = R_1 + R_2 + R_3 + R_4 + R_5 = 0.444888$$

$$\therefore q_x = \frac{T_1 - T_2}{R_{net}} = \frac{22 - (-2)}{0.444888} = 53.946 \text{ W}$$

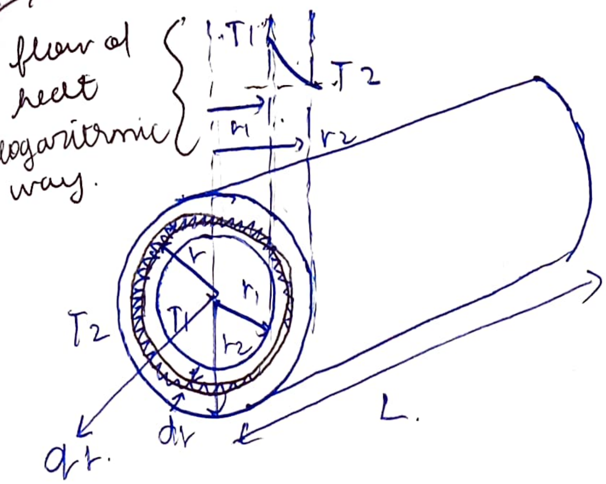
2) $$\frac{T_1 - T_A}{R_1 + R_2} = q_x$$

$$\therefore \frac{22 - T_A}{3.83 \times 10^{-4} + 2.6 \times 10^{-3}} = 53.946$$

$$T_A = 22 - 53.946 \times 2.983 \times 10^{-3}$$

$$T_A = 21.839 \,^{\circ}C$$

→ Heat conduction along hollow cylinder :-



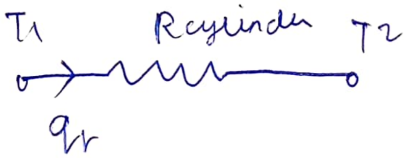
$$q_r = -KA\frac{dT}{dr}$$

$$q_r = -K \times (2\pi r L)\frac{dT}{dr}$$

$$\therefore \int_{T_1}^{T_2} dT = -\frac{q_r}{2\pi K L}\int_{r_1}^{r_2}\frac{dr}{r}$$

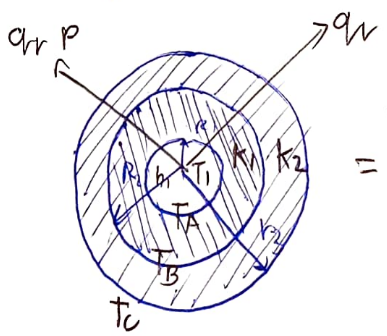
$$\therefore (T_2 - T_1) = -\frac{q_r}{2\pi K L}\left[\ln\left(\frac{r_2}{r_1}\right)\right]$$

$$\therefore q_r = \frac{2\pi K L (T_1 - T_2)}{\ln(r_2/r_1)} = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]} = \frac{T_1 - T_2}{R_{cylinder}}$$



$$R_{cylinder} = \frac{\ln(r_2/r_1)}{2\pi K L}.$$

→ composite cylinder :-



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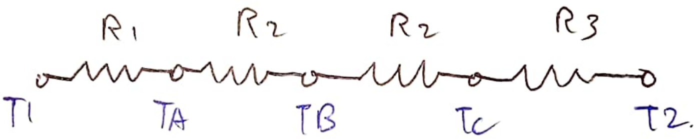


T_1 = inside
T_2 = outside

$$q_r = \frac{T_1 - T_2}{R_1 + R_2 + R_2 + R_4}$$

$R_1 = 1/h_1A_1 = 1/h_1(2\pi r_1 l)$, $R_2 = \frac{\ln(r_2/r_1)}{(2\pi k_1 l)}$, $R_3 = \frac{\ln(r_3/r_2)}{(2\pi k_2 l)}$, $R_4 = 1/h_2A_2 = 1/h_2(2\pi r_3 l)$

$$\therefore q_r = \frac{T_1 - T_2}{\frac{1}{h_1 2\pi r_1 l} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{h_2 2\pi r_3 l}}$$

$$q_r = \frac{2\pi l (T_1 - T_2)}{\frac{1}{h_1 r_1} + \frac{1}{h_2 r_2} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}}$$

Solve:-

Q) Hot air at a temp of 60°C is flowing through a steel pipe of 10cm diameter. The pipe is covered with 2 layers of diff insulating materials of thickness 5cm & 3cm and their corresponding thermal conductivities are 0·23 & 0·37 W/m°K. The inside & outside heat transfer coefficient are 58 & 12 W/m² K. The atmosphere is at 25°C find the rate of heat loss from a 50m pipe length. Neglect the resistance of the steel pipe

Soln

$r_2 - r_1 = 5cm$, $r_3 - r_2 = 3cm$

$r_1 = 5cm$, $r_2 = 10cm$, $r_3 = 13cm.$

$T_1 = 60°C$, $T_2 = 25°C$

$h_1 = 58$, $k_1 = 0·23$, $l = 50m.$

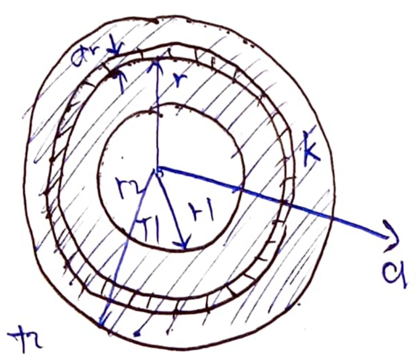
$\therefore h_2 = 12$, $k_2 = 0·37$

$$q_r = \frac{2\pi l(T_1 - T_2)}{\frac{1}{h_1 r_1} + \frac{1}{h_2 r_2} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}}$$

$$q_r = \frac{2\pi \times 50 \times (60 - 25)}{\frac{100}{58\times5} + \frac{100}{12\times10} + \frac{\ln(10/5)}{0·23} + \frac{\ln(13/10)}{0·37}}$$

$q_r = 2243·5658$ W

→ Heat Conduction through hollow sphere:-



$$q_r = -kA(dT/dr)$$

$$q_r = -k(4\pi r^2)dT/dr.$$

$$\therefore \int_{T_1}^{T_2} dT = \left(-\frac{q_r}{4\pi k}\right)\int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$T_2 - T_1 = \frac{q_r}{4\pi k}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$R_{sphere} = \left(\frac{1}{4\pi k}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

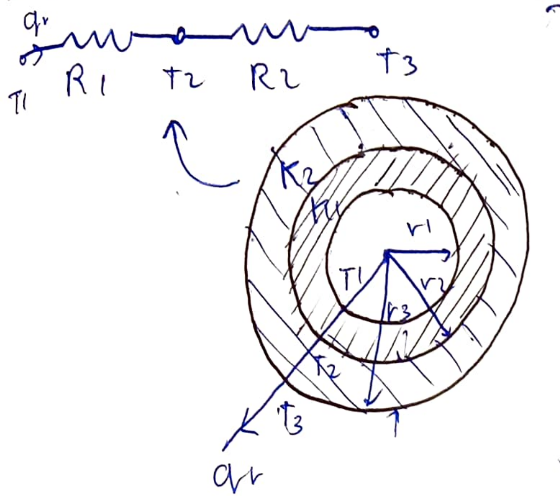
T_1 R_{sphere} T_2

q_r

$$q_r = -\frac{4\pi k(T_2 - T_1).r_1 r_2}{(r_2 - r_1)}$$

$$q_r = \frac{4\pi k(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \frac{T_1 - T_2}{R_{sphere}}$$

→ composite walled hollow sphere:-



$$R_1 = \frac{1}{4\pi k_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$R_2 = \frac{1}{4\pi k_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$q_r = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}.$$

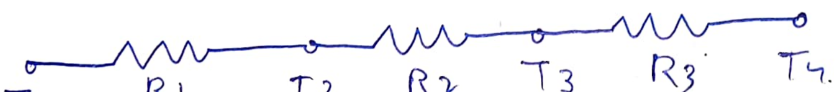
Solve:-

Q) A hollow Aluminium sphere of thermal conductivity 230 W/mK with an electric heater in the centre is used in tests to determine the thermal conductivity of insulating materials. The inner & outer radii of the sphere are 0.15 & 0.18 m respectively & testing is done under steady-state conditions with the inner surface of the Aluminium maintained at 250°C. In a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12 m. The system is in room for which the air temp is 20°C & the convective coefficient at the outer surface of the insulation is 30 W/m²K. If 80 W are dissipated by the heater under steady state condition, what is the thermal conductivity of the insulation?

Sol^n $r_1 = 0.15\,m$ $\quad T_1 = 250°C$ $\quad k_1 = 230\,W/mK$

$r_2 = 0.18\,m$ $\quad T_3 = 20°C$ $\quad h_1 = 30\,W/m^2K$

$r_3 = 0.30\,m.$ $\quad q_r = 80\,W.$



$$R_1 = \frac{1}{4\pi k_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi \times 230} \times \left(\frac{1}{0.15} - \frac{1}{0.18}\right) = 3.84 \times 10^{-4}\ K/W$$

$$R_2 = \frac{1}{4\pi k_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi \times k_2} \times \left(\frac{1}{0.18} - \frac{1}{0.3}\right) = \frac{0.177}{k_2}\ K/W$$

$$R_3 = \frac{1}{h_1 A_1} = \frac{1}{30 \times 4\pi \times 0.3^2} = 0.0295\ K/W$$

$$\therefore q_r = \frac{T_1 - T_4}{R_1 + R_2 + R_3}$$

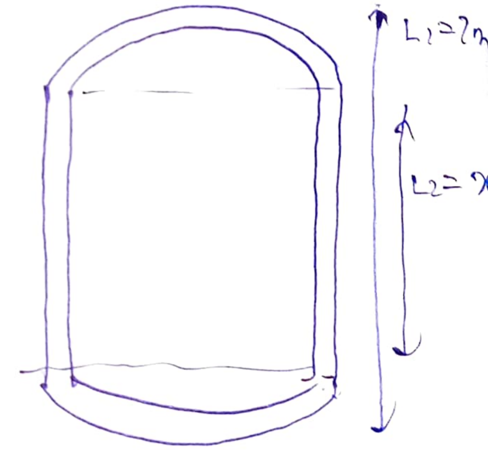
$$80 = \frac{250-20}{3.84\times10^{-4} + 0.0295 + \frac{0.177}{K_2}}$$

$$\frac{0.177}{K_2} = 2.875 - 0.02488$$

$$K_2 = \frac{0.177}{2.845116}$$

$$\boxed{K_2 = 0.06221} \text{ W/mK}$$

** Q) A cylindrical liq O_2 tank has a diameter of 1.4m, 7m long & has spherical ends. The boiling pt of liq. oxygen is $-182°C$ & its latent heat of evaporation is ~~20~~ 214 KJ/kg. The tank is insulated in order to reduce the heat transfer to the tank in such a way that in steady state, the rate of O_2 boil off would not exceed 14 kg/hr. Calculate the thermal conductivity of the insulating material if the 8 cm thick layer of insulation is applied & its outside surface is maintained at 30°C



Sol^n q_r = latent heat of evaporation × boiling rate

$$= 214\times10^3 \times \frac{14}{3600} = 832.22 \text{ W}$$

$x = 7 - 0.14$
$= 5.6$ m.

$r_1 = 0.7$m.
$r_2 = 0.7 + 0.08 = 0.78$ m.

$q_r = q_{cylinder} + q_{sphere}$

$$q_r = \frac{T_1 - T_3}{\left[\frac{\ln(r_2/r_1)}{2\pi K l}\right]} + \frac{T_1 - T_3}{\left[\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]}$$

$$832.22 = \frac{30+182}{\frac{\ln(0.78/0.7)}{2\pi K\times5.6}} + \frac{30+182}{\frac{1}{4\pi K}\left(\frac{1}{0.7} - \frac{1}{0.78}\right)}$$

$$\therefore K = \frac{832.22}{\frac{2\pi\times5.6(212)}{\ln(78/70)} + \frac{4\pi\times0.7\times0.78\times(212)}{(0.78-0.7)}}$$

$$\boxed{K = 9.5532\times10^{-3}} \text{ W/mK}$$

Q) A spherical thin walled metallic container is used to store liq. nitrogen at 77°K. The container has a diameter of 0.5 m & is covered with an evacuated reflective insulation composed of silica powder. The insulation is 25 mm thick & its outer surface is exposed to ambient air at 300°K. The convection coefficient is known to be 20 W/m²K. What is the rate of heat transfer to the liq Nitrogen if thermal conductivity of insulation is 0.0017 W/m°K.

Soln

$$\left.\begin{aligned} T_1 &= 300\,K \\ T_2 &= 77\,K \end{aligned}\right\} \quad \left.\begin{aligned} r_1 &= 0.25\,m \\ r_2 &= 0.275\,m \end{aligned}\right\} \quad \begin{aligned} h &= 20\,W/m^2K \\ k &= 0.0017\,W/mK \end{aligned}$$

$$\therefore q = \frac{T_1 - T_2}{R_1 + R_2} = \frac{T_1 - T_2}{\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{hA}}$$

$$q = \frac{(300 - 77)}{\left[\frac{1}{4\pi \times 0.0017} \times \left(\frac{1}{0.25} - \frac{1}{0.275}\right)\right] + \left[\frac{1}{20 \times 4\pi \times 0.225^2}\right]}$$

$$q = 13.0603\,W.$$