Mechanical Springs

Spring

- is an elastic machine component which deflects under the action of the load & returns to its original shape when the load is removed.

Functions & Typical Applications

Functions	Typical applications
Used to absorb shocks & vibration (Cushioning, absorbing, or controlling of energy due to shock and vibration)	Vehicle suspension springs Railway buffer springs Vibration mounts for machine
Used to apply force & control motion	Cam & follower mechanism Engine valve mechanism Force required to engage the clutch & brakes
Used to store energy	Circuit breakers & starters, clocks, toys etc.
Used to measure the force	Weighing balances

Functions & Typical Applications

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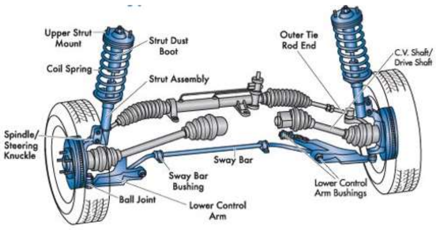
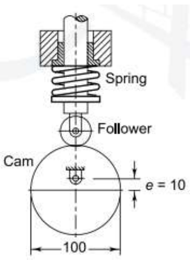
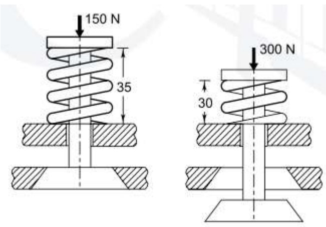
Used to absorb shocks & vibration (Cushioning, absorbing, or controlling of energy due to shock and vibration)	Vehicle suspension springs Railway buffer springs Vibration mounts for machine

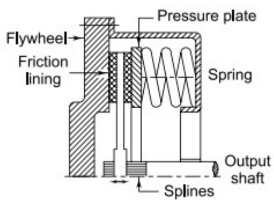


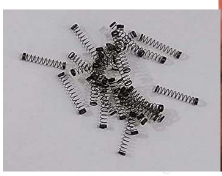
Functions	Typical applications
Used to apply force & control motion	Cam & follower mechanism Engine valve mechanism Force required to engage the clutch & brakes





(a) Closed position (b) Open position





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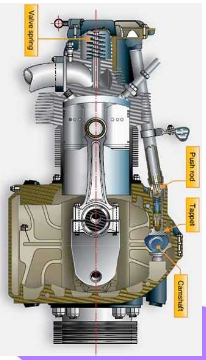
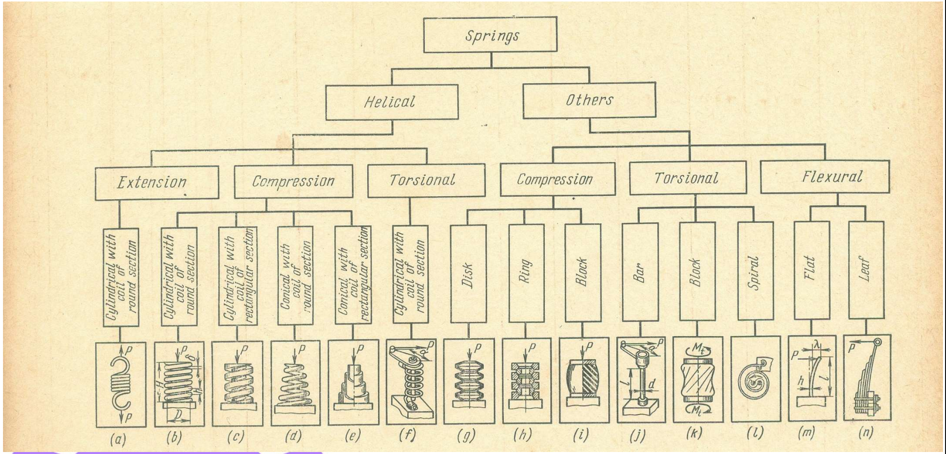


Types of Springs or Spring Configuration

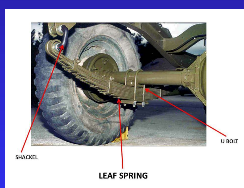
- According to their shape or physical configuration

Types of Springs or Spring Configuration



Types of Springs or Spring Configuration

- Types of Springs
  - Coil or helical
    - Compression
    - Extension
    - Torsion
      - Closely-coiled
      - Open coiled
  - Flat or Beam spring & Leaf Spring
  - Spring washer, Belleville Spring

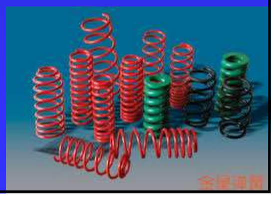


Helical Springs

- Compression
- Torsion
- Extension







Closely-coiled helical spring vs. Open-coiled helical spring

Closely-coiled helical spring	Open-coiled helical spring
When the spring wire is coiled so close that the plane containing each coil is almost at right angles to the axis of the helix. Helix angle is very small.	**When the spring wire is coiled in such a way, that there is large gap between adjacent coils. Helix angle is large**
Helix angle < 10°	Helix angle > 10°
More popular & extensively used	

Spring Materials

Factors are considered for selecting of the materials of the spring wire

- The load acting on the spring
- The range of stress through which the spring operates
- The expected fatigue life
- The environmental conditions in which the spring will operate such as temperature & corrosive atmosphere
- The severity of deformation encountered while making the spring
- The limitation on mass & volume of spring.

Steel wire for spring	% C	Remarks	CI
Cold drawn (or hard drawn) spring steel	0.85-0.95%		1
Oil-tempered & hardened spring steel	0.55-0.75%		1.5
Music wire (hard drawn spring steel)	0.80-0.95%		3.5
Oil-tempered & hardened spring steel (alloyed)	0.55-0.75%		
Alloy steel: Chromium-vanadium Chromium-silicon			4.0
Stainless steel			8.5

Spring Materials

Table 13-1 Common Spring Wire Materials

Source: Reference 2

ASTM #	Material	SAE #	Description
A227	Cold-drawn wire ("hard-drawn")	1066	Least expensive general-purpose spring wire. Suitable for static loading but not good for fatigue or impact. Temperature range 0°C to 120°C (250°F).
A228	Music wire	1085	Toughest, most widely used material for small coil springs. Highest tensile and fatigue strength of all spring wire. Temperature range 0°C to 120°C (250°F).
A229	Oil-tempered wire	1065	General-purpose spring steel. Less expensive and available in larger sizes than music wire. Suitable for static loading but not good for fatigue or impact. Temperature range 0°C to 180°C (350°F).
A230	Oil-tempered wire	1070	Valve-spring quality—suitable for fatigue loading.
A232	Chrome vanadium	6150	Most popular alloy spring steel. Valve-spring quality—suitable for fatigue loading. Also good for shock and impact loads. For temperatures to 220°C (425°F). Available annealed or pretempered.
A313 (302)	Stainless steel	30302	Suitable for fatigue applications.
A401	Chrome silicon	9254	Valve-spring quality—suitable for fatigue loading. Second highest strength to music wire and has higher temperature resistance to 220°C (425°F).
B134, #260	Spring brass	CA-260	Low strength—good corrosion resistance.
B159	Phosphor bronze	CA-510	Higher strength than brass—better fatigue resistance—good corrosion resistance. Cannot be heat treated or bent along the grain.
B197	Beryllium copper	CA-172	Higher strength than brass—better fatigue resistance—good corrosion resistance. Can be heat treated and bent along the grain.
-	Inconel X-750	-	Corrosion resistance.

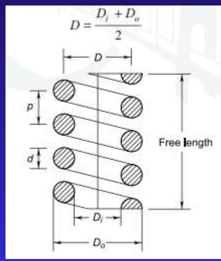
Table 13-2

Preferred Wire Diameters

U.S. (in)		SI (mm)
0.004		0.10
0.005		0.12
0.006		0.16
0.008		0.20
0.010		0.25
0.012		0.30
0.014		0.35
0.016		0.40
0.018	A229	0.45
0.020		0.50
0.022		0.55
0.024		0.60
0.026	A227	0.65
0.028		0.70
0.030		0.80
0.035		0.90
0.038		1.00
0.042		1.10
0.045		
0.048		1.20
0.051		
0.055		1.40
0.059	A401	
0.063		1.60
0.067		
0.072		1.80
0.076		
0.081		2.00
0.085		2.20
0.092		
0.098		2.50
0.105		
0.112		2.80
0.125		3.00
0.135		3.50
0.148		
0.162		4.00
0.177		4.50
0.192		5.00
0.207		5.50
0.225		6.00
0.250		6.50
0.281		7.00
0.312	A228	8.00
0.343		9.00
0.362		
0.375		
0.406		10.0
0.437		11.0
0.469		12.0
0.500	A232	13.0
0.531		14.0
0.562		15.0
0.625		16.0

Terminology of Helical Springs

d = wire diameter of spring (mm)
D_i = inside diameter of spring coil (mm)
D_o = outside diameter of spring coil (mm)
D = mean coil diameter (mm)



Spring Index (C)

- Ratio of mean coil diameter to wire diameter

$$C = \frac{D}{d}$$

$$3 \le C \le 12$$

Preferred Range:

$$6 \le C \le 10$$

In the design of helical springs, the designer should use good judgement in selecting the value of the spring index (C).

Terminology of Helical Springs

The spring index indicates the relative sharpness of the curvature of the coil

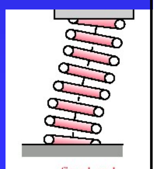
$$C = \frac{D}{d}$$

When C < 3

- A low spring index means high sharpness of curvature.
- When the spring index is low (C < 3), the actual stresses in the wire are excessive due to curvature effect.
- Such a spring is difficult to manufacture & special care in coiling is required to avoid cracking in wires.

When C > 12

- When the spring index is high (C > 12), it results in large variation in the coil diameter. Such spring is prone to buckling.



Terminology of Helical Springs

Terms related to helical compression spring

Solid length

Axial length of the helical compression spring when it is so compressed that adjacent coils touch each other.

Solid length $= N_t\, d$
where,
N_t = total number of coils

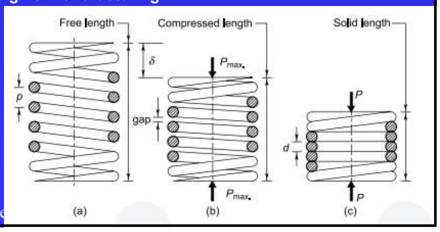
Compressed Length

Axial length of the spring when it is subjected to max. compressive force/max. deflection (δ). In this situation, there should be some gap or clearance between the adjacent coils to prevent clashing of the coils.

Total gap $= (N_t - 1) \times$ Gap between adjacent coils

Free length -is an important dimension in spring design & manufacturing

Axial length of the unloaded helical compression spring (i.e., in free condition prior to assembly.



$$\text{free length} = \text{compressed length} + \delta$$
$$= \text{solid length} + \text{total axial gap} + \delta$$

Terminology of Helical Springs

Terms related to helical compression spring

Pitch of the coil

Axial distance between adjacent coils in uncompressed state of spring.

$$p = \frac{\text{free length}}{(N_t - 1)}$$

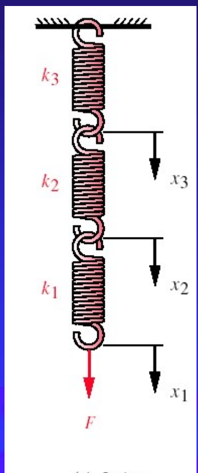
Stiffness of the spring ***or*** **Spring constant or Rate of spring or Gradient of spring**

Is defined as force required to produce unit deflection.

$$k = \frac{P}{\delta}$$

where,
k = stiffness of the spring (N/mm)
P = axial spring force (N)
δ = axial deflection of the spring corresponding to the force P (mm)

Springs in Series & in parallel



(a) Series

$F_i = k_i . x_i$

$F = k_3 . x_3$

$F = k_2 . x_2$

$F = k_1 . x_1$

$$\frac{1}{k_{series}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

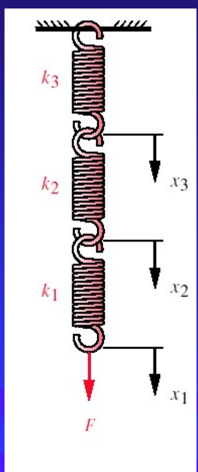
$$k_{parallel} = k_1 + k_2 + k_3$$

$x = x_1 + x_2 + x_3$

$F/k_s = F/k_1 + F/k_2 + F/k_3$

$1/k_s = 1/k_1 + 1/k_2 + 1/k_3$

Springs in Series & in parallel



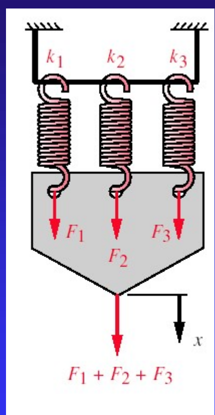
(a) Series

$$F = ky$$

$$k = F / y$$

$$\frac{1}{k_{series}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

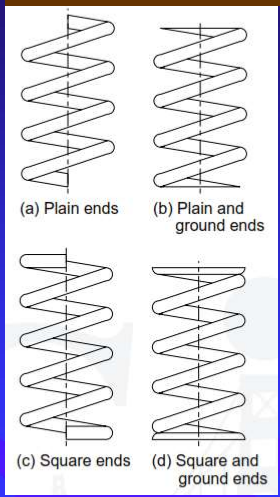
$$k_{parallel} = k_1 + k_2 + k_3$$



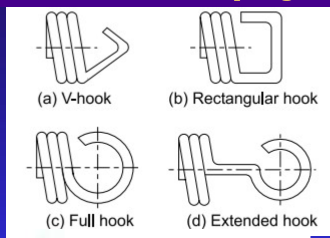
(b) Parallel

End Style or End Design of Helical Springs

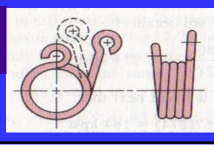
End Style or End Design of Helical Compression Springs



End Style or End Design of Helical Extension Springs



End Style or End Design of Helical torsion Springs



Spring design – end treatment

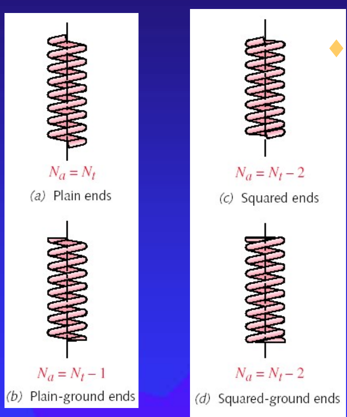


FIGURE 13-9
Four Styles of End-Coil Treatments for Helical Compression Springs

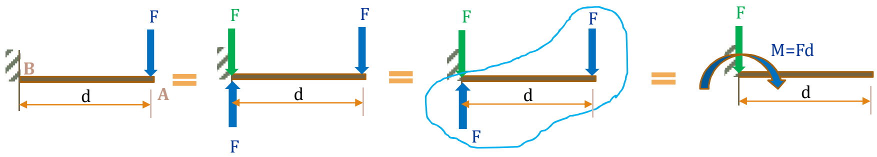
- End details affect active coils
  - Plain ends
  - Squared ends
  - Squared
  - Ground

Design Analysis of Closely coiled Helical Springs

Load-Stress Equation

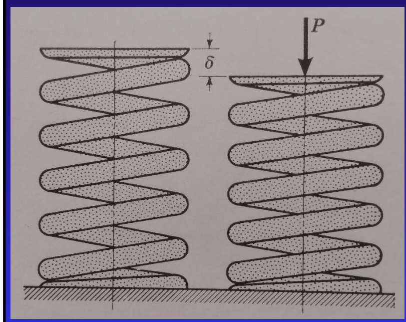
Load-Deflection Equation

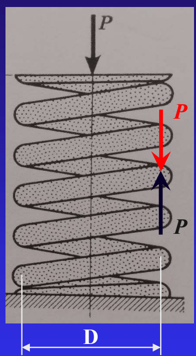
Force-Couple Systems

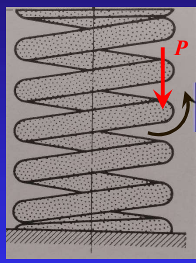


Design Analysis of Closely coiled Helical Springs

Load-Stress Equation





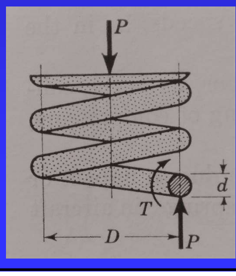


$M = P.\frac{D}{2}$

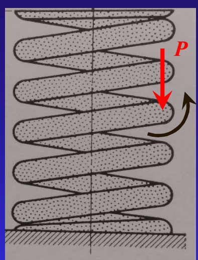
The axial force P can be considered as a direct force P acting on the spring at the mean coil radius together with a couple M=P.D/2 whose moment vector is perpendicular to the axis of the spring.

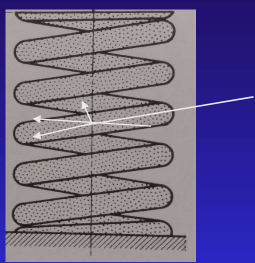
The moment M=P.D/2 is resolved into twisting moment T & Bending Moment M_b.

$T = \frac{P.D}{2} cos\lambda \quad M_b = \frac{P.D}{2} sin\lambda$



FBD





$M = P.\frac{D}{2}$

Front view

Side view

$T = \frac{P.D}{2} cos\lambda \quad M_b = \frac{P.D}{2} sin\lambda$

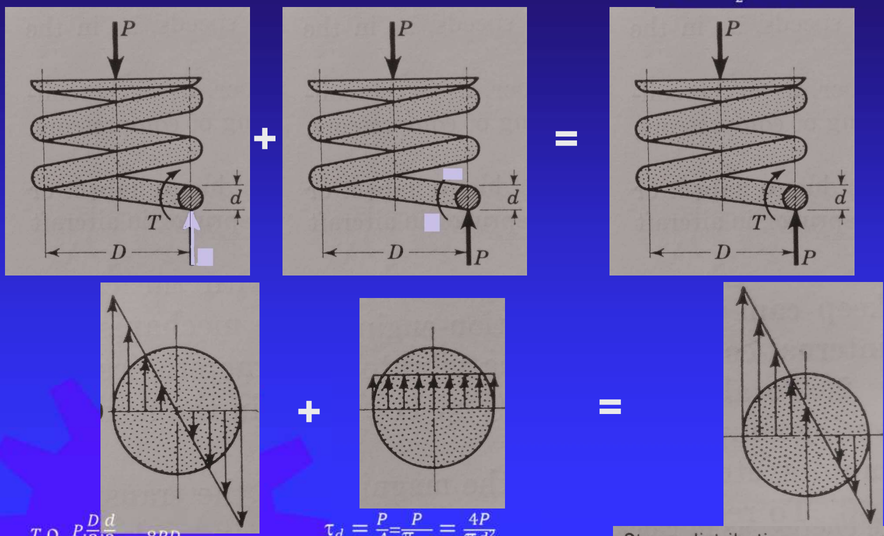
Design Analysis of Closely coiled Helical Springs

Load-Stress Equation

When λ (angle of helix) is very small, cosλ≅1, sinλ≅0

$T = \frac{P.D}{2}cos\lambda = \frac{P.D}{2}$

$M_b = \frac{P.D}{2}sin\lambda = 0$



$\tau_s = \frac{T.\rho}{J} = \frac{P.\frac{D}{2}.\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{8PD}{\pi d^3}$

$\tau_d = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{4P}{\pi d^2}$

Stress distribution across wire (neglecting effect of curvature)

Design Analysis of Closely coiled Helical Springs

Load-Stress Equation

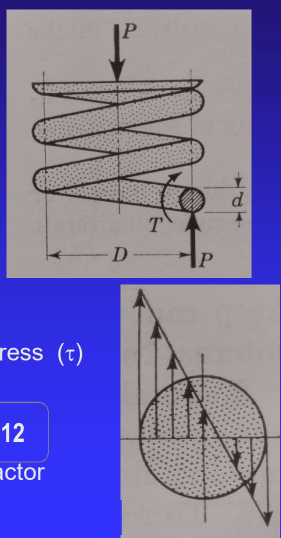
The free-body diagram (FBD) shows that there will be two components of stress on any cross-section of spring wire.

(i) Torsional shear stress (τ_s) due to T

$$\tau_s = \frac{T.\rho}{J} = \frac{P.\frac{D}{2}.\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{8PD}{\pi d^3}$$

(ii) Direct shear stress (τ_d) due to P

$$\tau_d = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{4P}{\pi d^2}$$



These two stresses add directly & maximum shear stress (τ) occurs at the inner fiber of the wire cross-section

$$\tau = \tau_s + \tau_d = \frac{8PD}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{8PD}{\pi d^3}\left(1+\frac{0.5}{C}\right)$$

$3 \leq C \leq 12$

$\tau = K_s \frac{8PD}{\pi d^3}$ Where Ks=shear stress correction factor

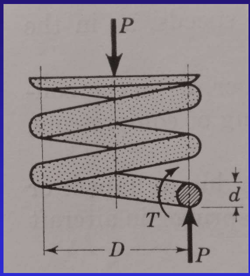
$$K_s = \left(1+\frac{0.5}{C}\right)$$

It is evident that for spring of small C, the effect of direct shear is appreciable.

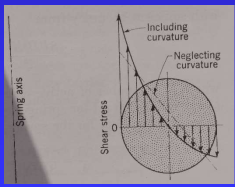
Design Analysis of Closely coiled Helical Springs

Load-Stress Equation

The effect of curvature of the wire as it forms the coil should be considered also.
There is an increase in the intensity of torsional shear stress in inner fiber because of the curvature of the coil.



In order to include both the effect of direct shear and effect of wire curvature on torsional shear stress, a stress factor has been determined by A.M. Wahl (1929) which may be used to determine the maximum shear stress in the inner fiber of the wire cross-section. This stress factor is also known as Wahl Factor (K_w).



$\tau = K_w \frac{8PD}{\pi d^3}$ Where K_w=Wahl factor

$3 \le C \le 12$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

27

Design Analysis of Closely coiled Helical Springs

Load-Deflection Equation

Deflection equation in closely coiled helical springs

Assumption Assuming that direct shear contributes negligibly to the deflection

Torsion formula

$$\frac{T}{J} = \frac{G\theta}{L} \quad \text{where} \quad T = \frac{PD}{2} \qquad L = \pi D N$$

$$\theta = \frac{TL}{GJ} \qquad J = \frac{\pi d^4}{32}$$

Strain energy due to torsional load $U = \frac{1}{2}T\theta \Rightarrow U = \frac{T^2 L}{2GJ} \Rightarrow U = \frac{4P^2D^3N}{Gd^4}$

The deflection due to torsional loading

Castigliano's Theorem $\delta = \frac{\partial U}{\partial P} \Rightarrow \delta = \frac{8PD^3N}{Gd^4}$

Stiffness of the spring $k = \frac{P}{\delta}$

$$k = \frac{G d^4}{8D^3N}$$

28

Design of Closely coiled Helical Springs

Objectives

- Spring should possess sufficient strength to withstand the external load
- Spring should have the required load-deflection characteristics
- Spring should not buckle under the external load

Design parameters

- Wire diameter (d)
- Mean coil diameter (D) *or* Spring Index (C)
- Number of active coils (N)
- Angle of Helix
- Pitch of the coil
- Stiffness of the spring

Practical Limitation

- Space limitations for outside & inside diameter of the coil

Basic Design parameters

- Wire diameter (d)
- Mean coil diameter (D) *or* Spring Index (C)
- Number of active coils (N)

Load-Stress Eq.

Load-Deflection Eq.

Design of Closely coiled Helical Springs

Step 1 Specify functions of the spring

Functions	Typical applications
Used to absorb shocks & vibration	Vehicle suspension springs Railway buffer springs Vibration mounts for machine
Used to apply force & control motion	Cam & follower mechanism Engine valve mechanism Force required to engage the clutch & brakes

Step 2 Load Analysis

- Estimate the maximum spring force (P) &
- The required deflection (δ) or required stiffness (k).

Free Body Diagram of forces

Design under Static Load

Step 3 Material Selection

From Data Book:
Mechanical properties
S_{yt}, S_{ut}

Steel wire for spring
Cold drawn (or hard drawn) spring steel
Oil-tempered & hardened spring steel
Music wire (hard drawn spring steel)
Oil-tempered & hardened spring steel (alloyed)
Alloy steel, Stainless steel

Step 4 Design Analysis

Choose suitable Factor of Safety

- ✓ The factor of safety in the design of springs is usually 1.5 or less.

Reasons for assuming relatively low FOS

- ✓ In most of the applications, springs operate will well defined deflections. Therefore, the forces acting on the spring & corresponding stresses can be precisely calculated. It is not necessary to take higher FOS to account for uncertainty in external forces acting on springs.
- ✓ In case of helical compression springs, an overload will simply close up the gaps between coils without a dangerous increase in deflection & stresses.
- ✓ In case of helical extension springs, usually overload stops are provided to prevent excessive deflection & stresses.

Allowable Tensile stress & Allowable Shear Stress of the spring material

Step 4 Design Analysis

Choose suitable value for spring Index (C)

Preferred Range:

$6 \leq C \leq 10$

Calculate Wahl factor

Calculate wire diameter

Load-Stress Eq.

Calculate mean coil diameter

Calculate the No. of active coils

Load-Deflection Eq.

Calculate the No. of coils

Decide the end style or design of the spring

Calculate the actual deflection

Calculate the solid, free length of the spring

Calculate the pitch of the coils

Calculate the stiffness of the spring

Step 4 Design Analysis

A helical compression spring that is too long compared to the mean coil diameter, acts as a flexible column and may buckle at a comparatively low axial force. The spring should be preferably designed as buckle-proof. Compression springs, which cannot be designed buckle-proof, must be guided in a sleeve or over an arbor. The thumb rules for provision of guide are as follows:

$$\frac{\text{free length}}{\text{mean coil diameter}} \leq 2.6 \quad \text{[Guide not necessary]}$$

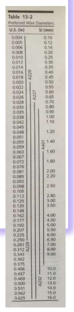
$$\frac{\text{free length}}{\text{mean coil diameter}} > 2.6 \quad \text{[Guide required]}$$

However, provision of guide results in friction between the spring and the guide and this may damage the spring in the long run.

Problem based on Design Analysis

Ex # 1 *A helical compression spring, made of circular wire, is subjected to an axial force, which varies from 2.5 kN to 3.5 kN. Over this range of force, the deflection of the spring should be approximately 5 mm. The spring index can be taken as 5. The spring has square and ground ends. The spring is made of patented and cold-drawn steel wire with ultimate tensile strength of 1050 N/mm² and modulus of rigidity of 81370 N/mm². The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length of the spring;
(vi) free length of the spring;
(vii) required spring rate; and
(viii) actual spring rate



Solution

Given data $P = 2.5$ to 3.5 kN

$\delta = 5$ mm $\quad C = 5$

Material: Cold drawn steel

$S_{ut} = 1050$ N/mm² $\quad G = 81\,370$ N/mm²

The permissible shear stress for the spring is given by,

$\tau = 0.5\, S_{ut} = 0.5\,(1050) = 525$ N/mm²

Step 1: Calculation of wire diameter

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(5)-1}{4(5)-4} + \frac{0.615}{5} = 1.3105$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 525 = (1.3105)\left\{\frac{8(3500)(5)}{\pi d^2}\right\}$$

$\therefore \quad d = 10.55$ or 11 mm

Step 2: Calculation of mean coil diameter

$D = C\,d = 5(11) = 55$ mm

Step 3: Calculation of no. of coils

$$\delta = \frac{8PD^3N}{Gd^4} \quad \text{or} \quad 5 = \frac{8(3500-2500)(55)^3 N}{(81\,370)(11)^4}$$

$N = 4.48$ or 5 coils

For square and ground ends, the number of inactive coils is 2. Therefore,

$N_t = N + 2 = 5 + 2 = 7$ coils

Step 4: Calculation of solid length & Free length of the spring

solid length of spring $= N_t d = 7(11) = 77$ mm

The actual deflection of the spring under the maximum force of 3.5 kN is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(3500)(55)^3(5)}{(81370)(11)^4} = 19.55 \text{ mm}$$

It is assumed that there will be a gap of 0.5 mm between the consecutive coils when the spring is subjected to the maximum force of 3.5 kN. The total number of coils is 7. Therefore, total axial gap will be $(7-1)\times 0.5 = 3$ mm.

Free length = Solid length + Total axial gap + δ

$= 77 + 3 + 19.55$
$= 99.55$ or 100 mm

Step 5: Calculation of spring rate

Required spring rate or stiffness

$$k = \frac{P_1 - P_2}{\delta} = \frac{3500-2500}{5} = 200 \text{ N/mm}$$

Actual spring rate or stiffness

$$k = \frac{Gd^4}{8D^3N} = \frac{(81370)(11)^4}{8(55)^3(5)} = 179.01 \text{ N/mm}$$

Ex # 2 *A railway wagon moving at a velocity of 1.5 m/s is brought to rest by a bumper consisting of two helical springs arranged in parallel. The mass of the wagon is 1500 kg. The springs are compressed by 150 mm in bringing the wagon to rest. The spring index can be taken as 6. The springs are made of oil-hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength. Design the spring and calculate:*

- *(i) wire diameter;*
- *(ii) mean coil diameter;*
- *(iii) number of active coils;*
- *(iv) total number of coils;*
- *(v) solid length;*
- *(vi) free length;*
- *(vii) pitch of the coil;*
- *(viii) required spring rate; and*
- *(ix) actual spring rate.*

Specify functions of the springs

The kinetic energy of the moving wagon is absorbed by the two springs

Determine Loads acting on element

Given $m = 1500$ kg $v = 1.5$ m/s $\delta = 150$ mm
$C = 6$ $S_{ut} = 1250$ N/mm² $G = 81\ 370$ N/mm²

The kinetic energy of the moving wagon

$$KE = \frac{1}{2} m v^2 = \frac{1}{2}(1500)(1.5)^2 = 1687.5 \text{ J or N-m}$$

$$= (1687.5 \times 10^3) \text{ N-mm}$$

Suppose P is the maximum force acting on each spring and causing it to compress by 150 mm.

strain energy absorbed by two springs

$$E = 2\left[\frac{1}{2} P\delta\right] = 2\left[\frac{1}{2} P\ (150)\right] = (150P) \text{ N-mm}$$

Determine Loads acting on element

The strain energy absorbed by the two springs is equal to the kinetic energy of the wagon. Therefore,

$$(150\ P) = 1687.5 \times 10^3$$

$$P = 11\,250 \text{ N}$$

Calculation of wire diameter

The permissible shear stress for the spring wire is given by, $\tau = 0.5\ (1250) = 625 \text{ N/mm}^2$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$

$$= 1.2525$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 625 = (1.2525)\left\{\frac{8(11\,250)(6)}{\pi d^2}\right\}$$

$$\boxed{d = 18.56 \text{ or } 20 \text{ mm}}$$

Calculation of mean coil diameter

Mean coil diameter

$$D = Cd = 6\ (20) = 120 \text{ mm}$$

Calculation of no. of coils

$$\delta = \frac{8\,PD^3\,N}{Gd^4} \quad \text{or} \quad 150 = \frac{8(11\,250)(120)^3\ N}{(81\,370)(20)^4}$$

$$N = 12.56 \text{ or } 13 \text{ coils}$$

Step 4: Calculation of solid length & Free length of the spring

It is assumed that the springs have square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 13 + 2 = 15 \text{ coils}$$

$$\text{Solid length} = N_t d = 15(20) = 300 \text{ mm}$$

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(11\,250)(120)^3(13)}{(81370)(20)^4} = 155.29 \text{ mm}$$

It is assumed that there will be a gap of 2 mm between adjacent coils when the spring is subjected to the maximum force of 11 250 N. Since the total number of coils is 15, the total axial gap will be $(15 - 1) \times 2 = 28$ mm.

Free length = solid length + total axial gap + δ
= 300 + 28 + 155.29

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t - 1)} = \frac{485}{(15 - 1)} = 34.64 \text{ mm}$$

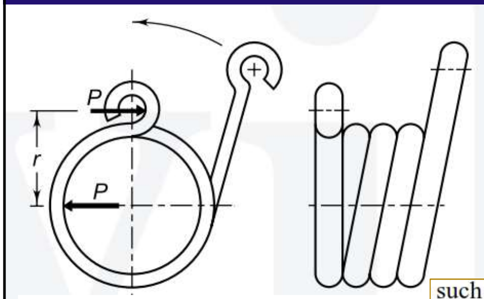
Required spring rate or stiffness

$$k = \frac{P}{\delta} = \frac{11\,250}{150} = 75 \text{ N/mm}$$

Actual spring rate or stiffness

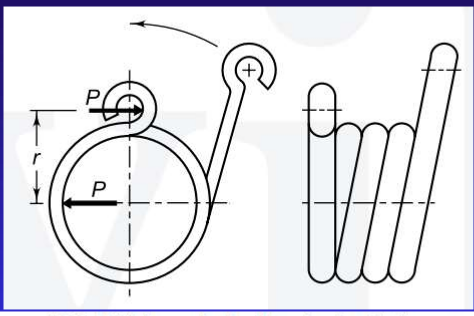
$$k = \frac{Gd^4}{8D^3N} = \frac{(81370)(20)^4}{8(120)^3(13)} = 72.44 \text{ N/mm}$$

Design Analysis of Helical Torsion Springs



such a way that the spring is loaded by a torque about the axis of the coils. The helical torsion spring resists the bending moment ($P \times r$), which tends to wind up the spring. The primary stresses in this spring are flexural in contrast with torsional shear stresses in compression or extension springs. The term 'torsion spring' is somewhat misleading, because the wire of the spring is subjected to bending stresses. Each individual section of the torsion spring is, in effect, a portion of a curved beam. Using the curved beam theory, the bending stresses are given by

Design Analysis of Helical Torsion Springs



$$M_b = Pr$$

$$\sigma_b = K\left(\frac{M_b y}{I}\right)$$

$$\sigma_b = K\left(\frac{32 M_b}{\pi d^3}\right)$$

$$I = \left(\frac{\pi d^4}{64}\right)$$

$$y = \left(\frac{d}{2}\right)$$

AM Wahl analytically derived the expressions for the stress concentration factor K. They are given by,

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)}$$

$$K_o = \frac{4C^2 + C - 1}{4C(C+1)}$$

where K_i and K_o are stress concentration factors at the inner and outer fibres of the coil respectively.

Critical frequency of Helical compression Spring

Surge in Spring

When the natural frequency of vibrations of the spring coincides with the frequency of external periodic force, which acts on it, resonance occurs. In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called Surge of spring. Surge is found in valve springs, which are subjected to periodic force.

It is not a problem in other applications where the external load is steady

Surge in springs is avoided by the following methods:

- **The spring is designed in such a way that the natural frequency of the spring is 15 to 20 times the frequency of excitation of the external force. This prevents the resonance condition to occur.**
- **The spring is provided with friction dampers on central coils. This prevents propagation of surge wave.**

Shot Peening

In a large number of applications, the external force acting on the spring fluctuates with respect to time resulting in fatigue failure. Due to poor surface finish of the spring wire, the fatigue crack usually begins with some surface irregularity and propagates due to tensile stresses. It has been observed that propagation of fatigue crack is always due to tensile stresses.

In order to reduce the chances of crack propagation, a layer of residual compressive stress is induced in the surface of the spring wire. One of the methods of creating such a layer is shot peening. In this process, small steel balls are impinged on the wire surface with high velocities either by an air blast or by centrifugal action. The balls strike against the wire surface and induce residual compressive stresses.

The depth of the layer of the residual compressive stresses depends upon a number of factors, such as

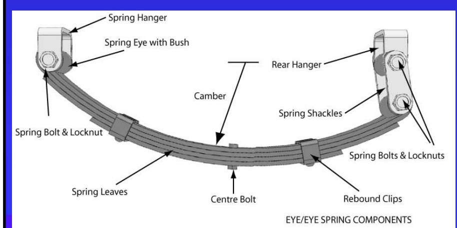
- **size of the balls,**
- **velocity of striking,**
- **Original hardness and ductility of the spring wire.**

These parameters are adjusted in such a manner as to produce the required depth of layer of compressive stress.

Multi-Leaf Spring

Typical Applications

✓ Widely used for the suspension of Automobiles like trucks & railway wagons.

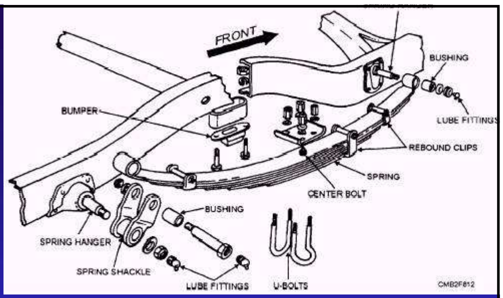


Leaf Spring

spring consists of a series of flat plates, usually of semi-elliptical shape, as shown in Fig. 10.31. The flat plates are called *leaves* of the spring. The leaves have graduated lengths. The leaf at the top has maximum length. The length gradually decreases from the top leaf to the bottom leaf. The longest leaf at the top is called *master leaf*. It is bent at both ends to from the spring eyes. Two bolts are inserted through these eyes to fix the leaf spring to the automobile body. The leaves are held together by means of two U-bolts and a centre clip. Rebound clips are provided to keep the leaves in alignment and prevent lateral shifting of the leaves during operation. At the centre, the leaf spring is supported on the axle. Multi-leaf springs are provided with one or two extra full length leaves in addition to master leaf. The extra full-length leaves are stacked between the master leaf and the graduated length leaves. The extra full-length leaves are provided to support the transverse shear force.

49

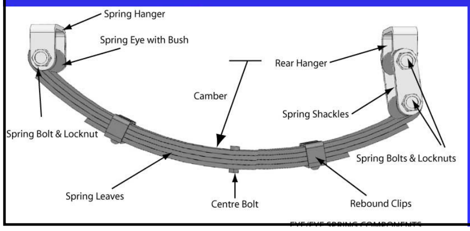
Leaf Spring



Multi-leaf Spring

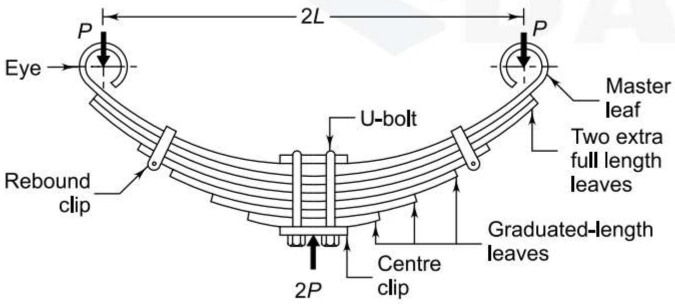
Consisting of

- Master leaf with eyes
- Graduated leaves
- Extra full length leaves
- U-bolt & Centre clip & rebound clip



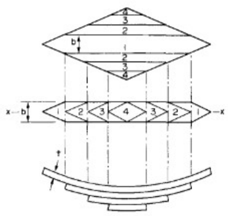
50

Design Analysis of Multi-Leaf Springs

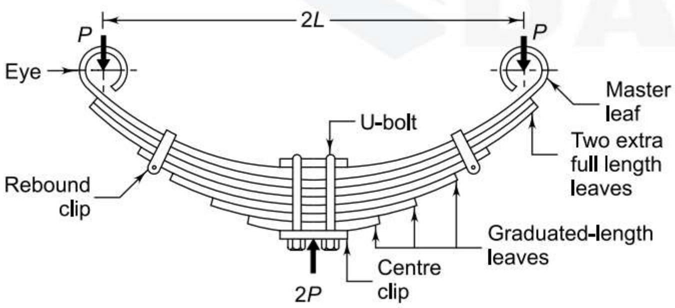


First Group

Second Group



Design Analysis of Multi-Leaf Springs



Notations

n_f = number of extra full-length leaves

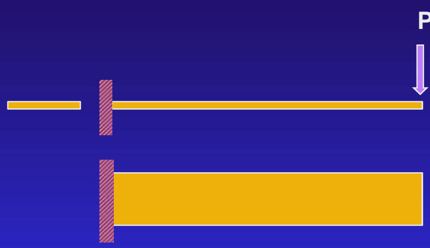
n_g = number of graduated-length leaves including master leaf

n = total number of leaves

b = width of each leaf (mm)

t = thickness of each leaf (mm)

Beam with Constant Width



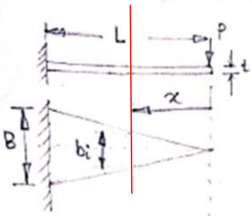
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z}$$

Section Modulus $Z = I/y_{max}$

Beam with Constant Stress or Beam of uniform strength

Analysis of Beam of Uniform Strength in bending (or Uniform-strength Cantilever beams in bending)



STRESS ANALYSIS

The figure shows a cantilever beam having a tapering width.

The moment of a force 'P' at distance x is

$$M = P.x \quad \text{------ (1)}$$

The stress due to bending at a section at distance 'x'

$$\sigma_x = \frac{M.y}{I_x} \quad \text{------ (2)}$$

The c/s at distance 'x' is rectangle of base b_i & height 't'

$$I_x = \frac{b_i t^3}{12}$$

Applying properties of triangle

$$\frac{b_i}{B} = \frac{x}{L} \qquad [b_i = \text{width of rectangular c/s at distance } x]$$

$$\therefore b_i = B.\frac{x}{L}$$

$$y = t/2$$

$$\therefore \sigma_x = \frac{P.x.t/2}{\frac{b_i t^3}{12}}$$

$$\sigma_x = \frac{12.P.x.t/2}{\frac{B.x}{L}.t^3}$$

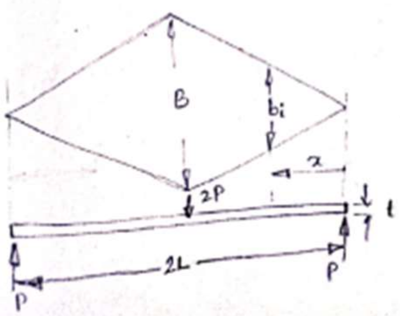
$$\sigma_x = \frac{6\,P.L}{Bt^2} \quad \text{------ (3)}$$

Check:

$$\sigma_b = \frac{P.L.t/2}{Bt^3/12} \quad \text{at } x = L$$

$$\boxed{\sigma_b = \frac{6P.L}{Bt^2}}$$

This eq^n (eq^n 3) shows that the bending stress is constant and not dependent on 'x'. Therefore, it is a beam of uniform strength in bending & this is considered in the design of Leaf Spring.



The fig. shows a simply supported beam having a tapering width

$$M = P.x$$

$$\sigma_x = \frac{M.y}{I_x}$$

$$I_x = \frac{b_i t^3}{12}, \quad y = t/2$$

Apply properties of Δ: $\frac{b_i}{B} = \frac{x}{L} \Rightarrow b_i = B.\frac{x}{L}$

$$\sigma_x = \frac{M.y}{I_x}$$

$$= \frac{P.x.t/2}{b_i t^3/12} = \frac{P.x.t/2}{\frac{B.x}{L}\frac{t^3}{12}}$$

$$= \frac{6PL}{}$$

DEFLECTION ANALYSIS

The deflection of the beam is given by the eqn.

$$\frac{d^2y}{dx^2} = -\frac{M}{EI_x}$$

$$\frac{d^2y}{dx^2} = \frac{P.x}{EI_x} \quad [\text{Where } M = -P.x] \quad \ldots\ldots (4)$$

Here, I_x is variable as width of the rectangular c/s varies.

$$I_x = \frac{b_i t^3}{12}$$

$$I = \frac{Bt^3}{12} = \text{MOI for the max}^m \text{ width c/s}$$

$$b_i = B.\frac{x}{L}$$

$$I_x = \frac{Bt^3}{12}.\frac{x}{L}$$

$$I_x = I.\frac{x}{L}$$

From eqn (4)

$$\frac{d^2y}{dx^2} = \frac{P.x}{E.I_x}$$

$$\frac{d^2y}{dx^2} = \frac{P.x.}{E.I.x/L} = \frac{P.L}{EI}$$

Integrating

$$\frac{dy}{dx} = \frac{P.L}{EI}.x + C_1 \quad \ldots\ldots (5)$$

Since $\frac{dy}{dx} = 0$ at $x = L$

$$\therefore \quad C_1 = -\frac{PL^2}{EI}$$

$$\frac{dy}{dx} = \frac{PL}{EI}x - \frac{PL^2}{EI}$$

Integrating

$$y = \frac{PL}{EI}.\frac{x^2}{2} - \frac{PL^2}{EI}.x + C_2$$

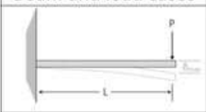
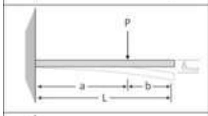
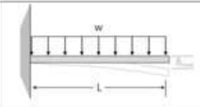
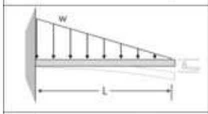
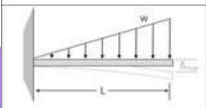
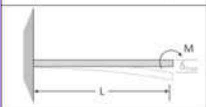
Since $y = 0$ at $x = L$

$$\therefore \quad C_2 = \frac{PL^3}{2EI}$$

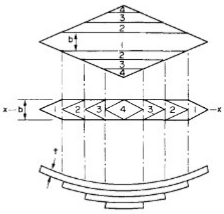
$$\therefore \quad y = \frac{P.L}{EI}.\frac{x^2}{2} - \frac{PL^2}{EI}x + \frac{PL^3}{2EI}.$$

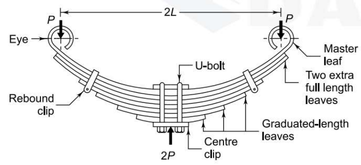
Now, $x = 0$; $\boxed{y = \frac{PL^3}{2EI}}$ ← Deflection at free end or Conc. loaded end.

$$y = \frac{6PL^3}{EBt^3}$$

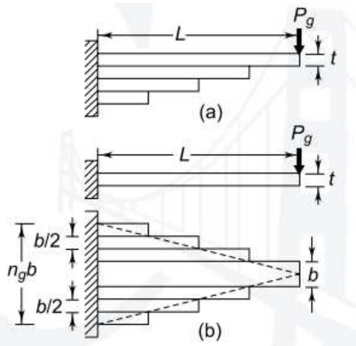
Beam and load cases	Maximum Beam Deflection
	$\delta_{max} = \dfrac{PL^3}{3EI}$
	$\delta_{max} = \dfrac{Pa^2(3L-a)}{6EI}$
	$\delta_{max} = \dfrac{wL^4}{8EI}$
	$\delta_{max} = \dfrac{wL^4}{30EI}$
	$\delta_{max} = \dfrac{11wL^4}{120EI}$
	$\delta_{max} = \dfrac{ML^2}{2EI}$

Design Analysis of Multi-Leaf Springs

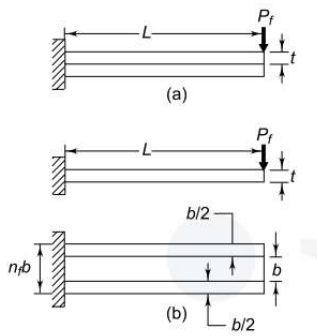


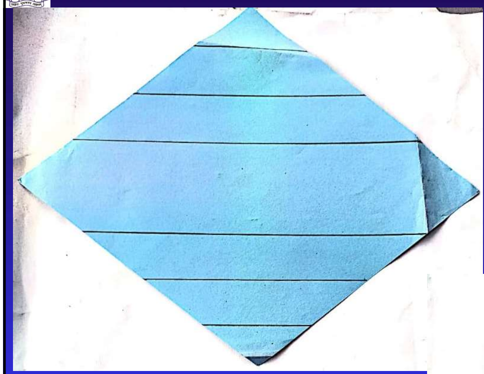


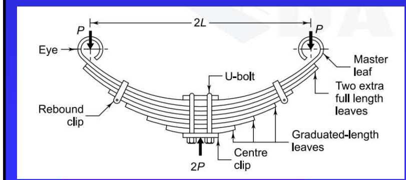
First Group

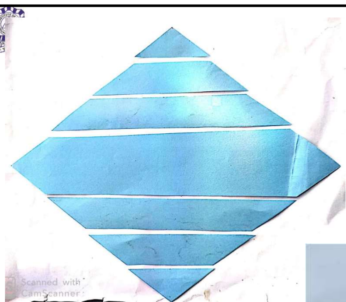


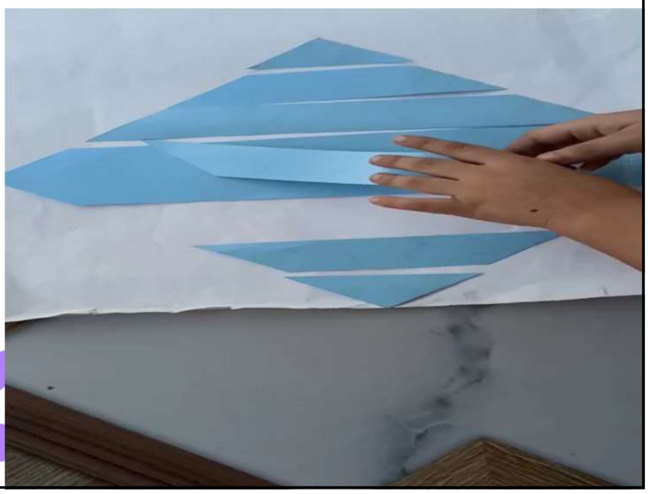
Second Group



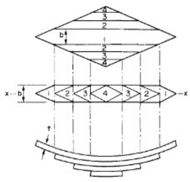


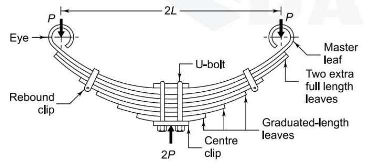

P
2L
P
Eye
Master leaf
U-bolt
Two extra full length leaves
Rebound clip
Graduated-length leaves
Centre clip
2P





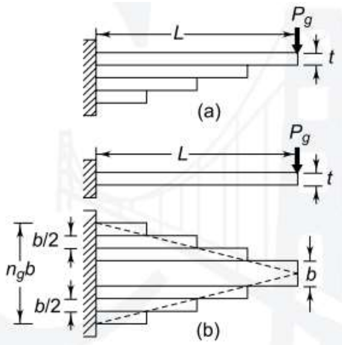
Design Analysis of Multi-Leaf Springs





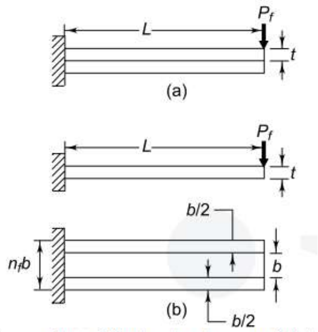
L = length of the cantilever or half the length of semi-elliptic spring (mm)

First Group

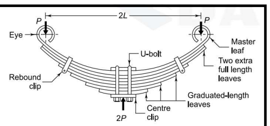


P_g = portion of P taken by the graduated-length leaves (N)

Second Group



P_f = portion of P taken by the extra full-length leaves (N)



For first group.

Stress due to bending in the triangular plate at the support is given by

$$\sigma_g = \frac{M_g y}{I_g} = \frac{P_g . L\,(t/2)}{\frac{1}{12} n_g b . t^3} \quad \text{where} \quad I_g = \frac{Bt^3}{12},\; I_g = \frac{1}{12}(n_g b)t^3 \quad \text{where } B = n_g b.$$

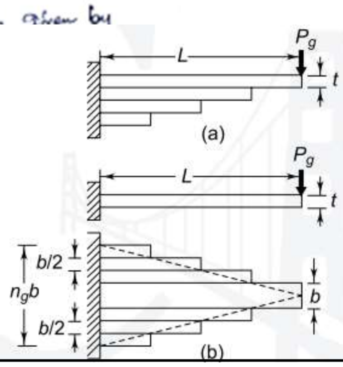
$$P_g + P_f = P$$

$$\sigma_g = \frac{6 P_g L}{n_g b t^2}$$

Deflection at load pt. of the triangular plate is given by

$$\delta_g = \frac{P_g L^3}{2EI}$$

$$\delta_g = \frac{6 P_g L^3}{E(n_g b)t^3}$$



For second group.

Similarly in second group, the extra full-length leaves can be treated as a rectangular plate of thickness 't' and uniform width $(n_f b)$ as shown in fig.

Stress due to bending in the rectangular plate at the support is given as

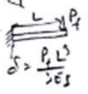
$$\sigma_f = \frac{M_f \cdot y}{I_f} = \frac{P_f \cdot L \cdot t/2}{\frac{1}{12}(n_f b)t^3}$$

$$\sigma_f = \frac{6 P_f L}{n_f b t^2}$$

Deflection at load point of the rectangular plate is given by

$$\delta_f = \frac{P_f L^3}{3EI}$$

$$\delta_f = \frac{4 P_f L^3}{E n_f b t^3}$$



Since the deflection of full length leaves is equal to the deflection of graduated-length leaves

$$\delta_g = \delta_f$$

$$\frac{6 P_g L}{E n_g b t^3} = \frac{4 P_f L}{E n_f b t^3}$$

$$\frac{P_g}{P_f} = \frac{2 n_g}{3 n_f}$$

Moreover, $P_g + P_f = P$

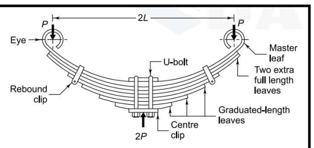
Solving above eqn

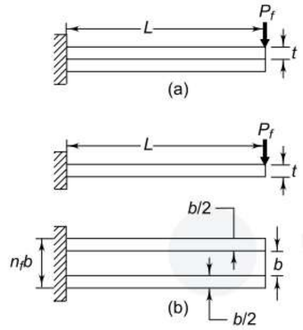
$$P_g = \frac{2 n_g P}{2 n_g + 3 n_f}$$

$$P_f = \frac{3 n_f P}{2 n_g + 3 n_f}$$

$$P_f = \frac{3 n_f}{2 n_g} \cdot P_g$$

$$P_g \left(1 + \frac{3 n_f}{2 n_g}\right) = P \;\therefore\; P_g = \frac{2 n_g P}{3 n_f + 2 n_g}$$





65

$$\sigma_g = \frac{6 P_g L}{n_g b t^2} \qquad \sigma_f = \frac{6 P_f L}{n_f b t^2}$$

Substituting the values of P_g & P_f

$$\left.\begin{aligned} \sigma_g &= \frac{12 PL}{(3n_f + 2n_g) b t^2} \\ \sigma_f &= \frac{18 PL}{(3n_f + 2n_g) b t^2} = 1.5\,\sigma_g \end{aligned}\right\} \text{Load-stress Eq}^n$$

– It is seen from the above eqs. that bending stresses in full-length leaves are 50% more than those in graduated-length leaves.

Deflection at the end of the spring

$$\delta_g = \delta_f = \left.\frac{12 P L^3}{E b t^3 (3n_f + 2n_g)}\right\} \text{Load-deflection Eq}^n.$$

Multi-leaf Springs are designed using (i) load-stress equation & (ii) load-deflection equation.

66

Design Analysis of Multi-Leaf Spring

Problem:

A semi-elliptic multi-leaf spring is used for the suspension of the rear axle of a truck. It consists of two extra full-length leaves and ten graduated-length leaves including the master leaf. The centre-to-centre distance between the spring eyes is 1.2 m. The leaves are made of steel 55Si2Mo90 (Yield strength = 1500 Mpa and E = 207 000 MPa) and the factor of safety is 2.5. The spring is to be designed for a maximum force of 30 kN. For each leaf, the ratio of width to thickness is 5:1. Determine

(i) the cross-section of leaves; and

(ii) the deflection at the end of the spring.

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