Kinematic Synthesis of Planar Mechanisms

(Mechanisms Synthesis)

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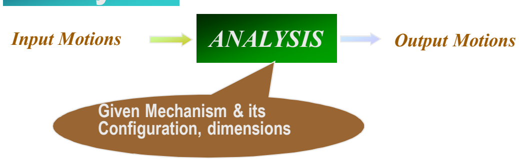
Analysis vs. Synthesis

Analysis



In Kinematic Analysis one is given a mechanism & the task is to determine the various relative motion that can take place in that mechanism.

Synthesis

- decision –making process
- Innovative or creative process
- process of creating new mechanism
- Selecting optimum/best configuration from no. of existing mechanism
- Determination of optimum dimensions of the elements of the mechanism on the basis of analysis

In Kinematic Synthesis one has to be come up with a design of mechanism to generate prescribed motion characteristic.

Kinematics Synthesis of Plane Mechanisms or Linkages

Aim:

Design or creation of a mechanism to obtain a desired set of motion characteristics.

Objective

- design of mechanisms to satisfy certain kinematic specification.

- In other words, motion characteristics are given & the mechanism is to be found

Kinematic Synthesis Problems

- Type Synthesis
- Number Synthesis
- Dimensional Synthesis

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Kinematic Synthesis of Mechanisms

Synthesis Problems

Kinematic Synthesis

- Type Synthesis — Refers to the kind of mechanism selected
- Number Synthesis — Deals with the number of links & nos. of joints that are required to obtain a certain mobility
- Dimensional Synthesis
  - Motion generation /Body Guidance
  - Path generation
  - Function generation

By Dimensional Synthesis, we mean the determination of kinematic dimensions of the individual links of a mechanism to satisfy specified motion characteristics or specified tasks.

Clasiification of Dimensional Synthesis Problems

Depending on the required kinematic characteristics to be satisfied by the designed mechanism or linkage, dimensional synthesis problems can be broadly classified as given below:

Motion generation /Body Guidance

In this general class of synthesis problem, the linkage has to be so designed that a rigid body (i.e., one link of the mechanism, for example the coupler of a 4R linkage) can be guided in a prescribed manner.

The guidance may or may not be coordinated with the input motion

Path generation

If a point on the floating link (i.e. link not connected to the frame , like coupler) of a mechanism has to be guided along a prescribed path, then such a problem is classified as a path- generation problem .

This refers to a problem in which a coupler point is to generate a path having a prescribed shape

The generation of a prescribed path may or may not be coordinated with the input motion

Function generation

In this class of problem, the motion parameters (displacement, velocity, acceleration etc.) of the output & input links are to be coordinated so as to satisfy a prescribed functional relationship.

The output & input motion characteristics have to maintain a specified functional relationship

Steps in Kinematics Synthesis of Plane Mechanisms

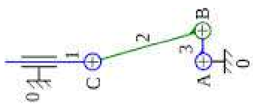
Type Synthesis

↓

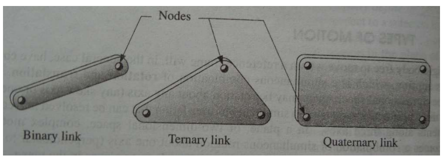
Number Synthesis

↓

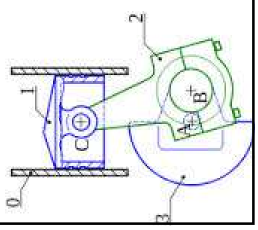
Dimensional Synthesis

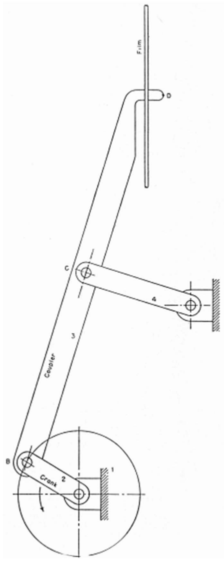


What is Kinematic dimensions?



Node-node distance or joint centre to centre distance etc.





Path generation

Function generation problem

In function generation, rotation (or translation) motion of input and output links must be correlated. The kinematic synthesis task may be to design a linkage to correlate input and output such that as the input moves by 'x', the output moves by $y = f(x)$ for the range $x_0 \le x \le x_{n+1}$

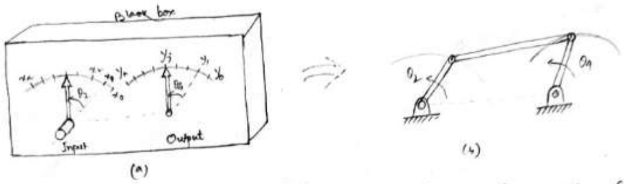
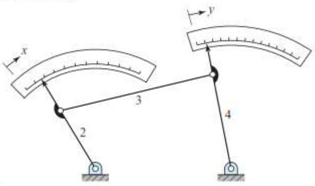


Fig. 1 Function-generator mechanism (a) exterior view, (b) Schematic of the mechanism inside.
(i.e four-bar linkage function generator)



In the case of rotary input and output, the angles of rotation θ_2 & θ_4 are the linear analogs of x and y respectively.

When the input is rotated to a value of the independent parameter 'x', the mechanism in the 'black box' causes the output link to turn to the corresponding value of the dependent variable $y = f(x)$. This may be regarded as a simple case of a mechanical analog computer.

Function generation problem

Fig.2 Shows a Six-link function generator mechanism in which two four-link mechanisms are joined in a Series. The objective in this linkage is to provide a measure of flow rate (i.e y) through the weir where the input is the vertical translation 'x' of the water level.

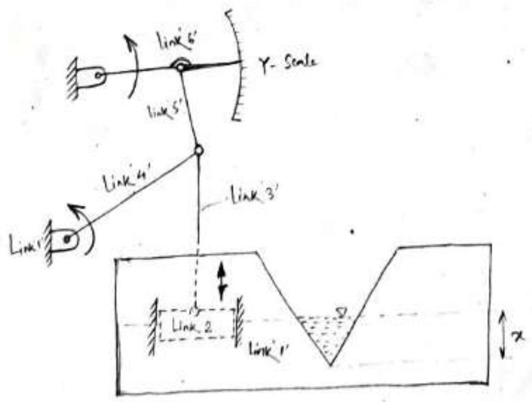


Fig. 2: Flow rate indicator mechanism, $y = f(x)$

Dimensional Synthesis Problems

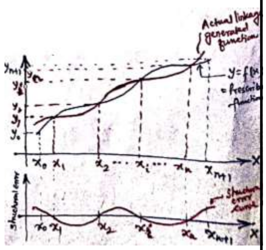
Function generation problem

Synthesis

- **Exact Synthesis** → By exact synthesis, we mean that the generated function by the physical mechanism fits the desired function at all points in the interval
- **Approximate Synthesis** → By approximate synthesis, we mean that the generated function by the physical mechanism fits the desired function at a finite number of points in the interval

Accuracy points / Precision points

The points at which the generated and desired functions agree



Structural Error

It is defined as the theoretical difference between the function generated by the synthesized linkage & the function originally prescribed

Structural error is inherent in approximate synthesis

Chebyshev's Spacing of Accuracy Points

Let y=f(x) be the function desired to be generated in an interval $x_0 \le x \le x_{n+1}$:

Let the mechanism generated function be $F(x, R_1, R_2, \ldots, R_k)$ where $R_1, R_2, \ldots, R_k$ are design parameters

Structural Error

$$E(x)=f(x)-F(x, R_1, R_2, \ldots, R_k)$$

The best choice for the spacing of accuracy points will be that which gives the min. value of E(x) between any two adjacent points:

However, Chebyshev's spacing of accuracy points can always be taken as a first approximation

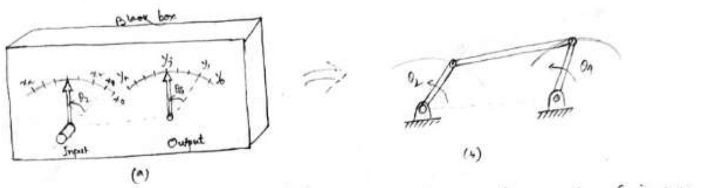
A very good trial for the spacing of these precision positions is called Chebyshev Spacing

Chebyshev's Spacing of Accuracy Points

For 'n' precision positions in the range $x_0 \le x \le x_{n+1}$, the Chebyshev's spacing is

$$x_j = \left(\frac{x_{n+1}+x_0}{2}\right) - \frac{(x_{n+1}-x_0)}{2}\cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } j=1,2,\ldots n.$$

Determine the three accuracy points with Chebyshev's spacing for a 4 bar linkage to generate the function $y=x^{0.8}$ in the interval $1 \le x \le 3$,



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Determine the three accuracy points with Chebyshev's spacing for a 4 bar linkage to generate the function $y=x^{0.8}$ in the interval $1 \le x \le 3$,

Here $n=3$; $x_0 = 1$; $x_{3+1} = x_4 = 3$

$$x_j = \left(\frac{x_{n+1}+x_0}{2}\right) - \left(\frac{x_{n+1}-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } j=1,2,3$$

$$x_j = \left(\frac{x_4+x_0}{2}\right) - \left(\frac{x_4-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{6}\right\}$$

$$x_1 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(2-1)\pi}{2\times 3}\right\} = 2-\cos\pi/6 = 1{\cdot}134$$

$$x_2 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(4-1)\pi}{2\times 3}\right\} = 2-\cos\pi/2 = 2$$

$$x_3 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(6-1)\pi}{6}\right\} = 2-\cos 5\pi/6 = 2{\cdot}866$$

Accuracy pts.

The corresponding values of 'y' to be

$$y_1 = x^{0.8} = (1{\cdot}134)^{0.8} = 1{\cdot}106$$

$$y_2 = (2)^{0.8} = 1{\cdot}741$$

$$y_3 = (2{\cdot}866)^{0.8} = 2{\cdot}322.$$

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Function generation problem

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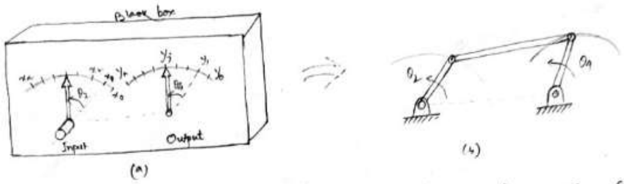

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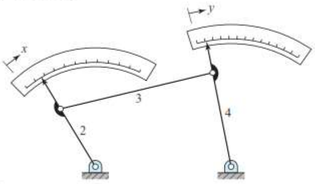

In the case of rotary input and output, the angles of rotation θ_2 & θ_4 are the linear analogs of x and y respectively.

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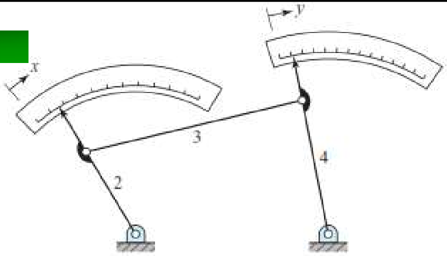
Scale Factor for Input & Output motion

Mechanized variables: θ_2 & θ_4

Functional variables: 'x' & 'y'



The orientation of the driver link (θ_2) represents the independent variable 'x'.
The orientation of the driven link (θ_4) represents the dependent variable 'y'.
The mechanized variables θ_2 & θ_4 are proportional to the functional variables 'x' & 'y'.
The relation betⁿ Δx and $\Delta\theta_2$ & that betⁿ Δy and $\Delta\theta_4$ is usually assumed to be linear.

With the mappings betⁿ function variable Space (x, y) and mechanism joint Space (θ_2, θ_4) known, we can map the three function precision points to corresponding precision joint angles.

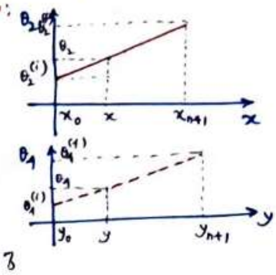
Let $\theta_2^{(i)}$ be the initial value of θ_2 representing x_0

$\theta_4^{(i)}$ be " " " of θ_4 " $y_0 = f(x_0)$

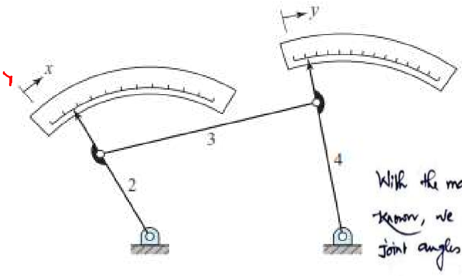
The input & output scale factors m_x & m_y resp. are defined as:

$$\therefore \text{Scale factor } m_x = \frac{\Delta\theta_2}{\Delta x} = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0} = \frac{\theta_2 - \theta_2^{(i)}}{x - x_0}$$

$$m_y = \frac{\Delta\theta_4}{\Delta y} = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0} = \frac{\theta_4 - \theta_4^{(i)}}{y - y_0}$$



The Superscripts 'i' & 'f' denote the initial & final values of θ_2 & θ_4.



$$\theta_2 - \theta_2^{(i)} = m_x(x - x_0) \Rightarrow \theta_2 = \theta_2^{(i)} + m_x(x - x_0) \text{ where } m_x = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0}$$

$$\theta_4 - \theta_4^{(i)} = m_y(y - y_0) \Rightarrow \theta_4 = \theta_4^{(i)} + m_y(y - y_0) \text{ where } m_y = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0}$$

With the mappings betⁿ function variable Space (x, y) and mechanism joint Space (θ_2, θ_4) known, we can map the three function precision points to corresponding precision joint angles.

Dimensional Synthesis for Function generation problem

- Methods
  - Graphical Methods
  - Analytical Methods
    - Bloch's method
    - Freudenstein's Method

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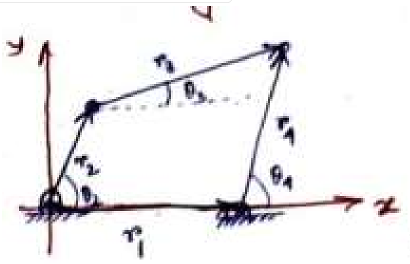
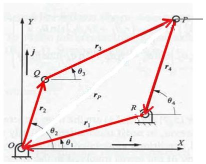
Displacement Analysis of 4R linkage

Freudenstein's Method





Loop Closure Equation in Scalar Form

Two scalar eqns.

$$r_2\cos\theta_2 + r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4 \Rightarrow r_2\cos\theta_2 + r_3\cos\theta_3 = r_1 + r_4\cos\theta_4$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 = r_1\sin\theta_1 + r_4\sin\theta_4 \Rightarrow r_2\sin\theta_2 + r_3\sin\theta_3 = r_4\sin\theta_4$$

as $\theta_1 = 0^\circ$

$$r_3\cos\theta_3 = r_1 + r_4\cos\theta_4 - r_2\cos\theta_2$$

$$r_3\sin\theta_3 = r_4\sin\theta_4 - r_2\sin\theta_2$$

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$r_3 \cos\theta_3 = r_1 + r_4\cos\theta_4 - r_2\cos\theta_2$

$r_3 \sin\theta_3 = r_4\sin\theta_4 - r_2\sin\theta_2$

$$r_3^2 = \left(r_1 + r_4\cos\theta_4 - r_2\cos\theta_2\right)^2 + \left(r_4\sin\theta_4 - r_2\sin\theta_2\right)^2$$

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\cos\theta_2 + 2r_1r_4\cos\theta_4 - 2r_2r_4\cos\theta_2\cos\theta_4 - 2r_2r_4\sin\theta_2\sin\theta_4$$

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\cos\theta_2 + 2r_1r_4\cos\theta_4 - 2r_2r_4\cos(\theta_2 - \theta_4)$$

$$2r_1r_4\cos\theta_4 - 2r_1r_2\cos\theta_2 + (r_1^2 + r_2^2 + r_4^2 - r_3^2) = 2r_2r_4\cos(\theta_2 - \theta_4)$$

$$\frac{r_1}{r_2}\cos\theta_4 - \frac{r_1}{r_4}\cos\theta_2 + \frac{r_1^2 + r_2^2 + r_4^2 - r_3^2}{2r_2r_4} = \cos(\theta_2 - \theta_4)$$

$$K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4) \quad \text{------ (A)} \quad F$$

FREUDENSTEIN'S η^n

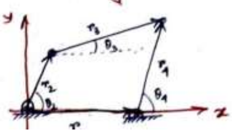
FREUDENSTEIN'S METHOD: Function Generation with Three Accuracy Points.

With three accuracy points, the number of design parameters that can be determined is three.

Example: Four-bar Function Generators with Three Accuracy Points

Loop-closure eq^n or vector loop eq^n

$$\vec{r_2} + \vec{r_3} = \vec{r_1} + \vec{r_4}$$



$$K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4) \quad \text{------ (A) Freudenstein's eq}^n$$

Substituting the three related pairs $(\theta_2^{(1)} \,\&\, \theta_4^{(1)})$, $(\theta_2^{(2)} \,\&\, \theta_4^{(2)})$ and $(\theta_2^{(3)} \,\&\, \theta_4^{(3)})$ successively in eq^n (A), we obtain three linear simultaneous eq^{n}s in K_1, K_2 & K_3.

$$K_1\cos\theta_4^{(1)} - K_2\cos\theta_2^{(1)} + K_3 = \cos(\theta_2^{(1)} - \theta_4^{(1)}) \quad \text{------ (I)}$$

$$K_1\cos\theta_4^{(2)} - K_2\cos\theta_2^{(2)} + K_3 = \cos(\theta_2^{(2)} - \theta_4^{(2)}) \quad \text{------ (II)}$$

$$K_1\cos\theta_4^{(3)} - K_2\cos\theta_2^{(3)} + K_3 = \cos(\theta_2^{(3)} - \theta_4^{(3)}) \quad \text{------ (III)}$$

Solving above linear eq^{n}s, we get the K_1, K_2, & K_3 i.e link length ratios (design parameters)

Manual Method:

(II) − (I) : $K_1[\cos\theta_4^{(2)} - \cos\theta_4^{(1)}] - K_2[\cos\theta_2^{(2)} - \cos\theta_2^{(1)}] = \cos(\theta_2^{(2)} - \theta_4^{(2)}) - \cos(\theta_2^{(1)} - \theta_4^{(1)})$

(III) − (I) : $K_1[\cos\theta_4^{(3)} - \cos\theta_4^{(1)}] - K_2[\cos\theta_2^{(3)} - \cos\theta_2^{(1)}] = \cos(\theta_2^{(3)} - \theta_4^{(3)}) - \cos(\theta_2^{(1)} - \theta_4^{(1)})$.

Solve two eq^n for two unknowns

Example # 2

Determine the lengths of the links of a 4bar linkage to generate $y = \log_{10} x$ in the interval $1 \le x \le 10$. The length of the smallest link is 5 cm. Use three accuracy points with Chebyshev's spacing. Given $\theta_2^{(i)} = 45°$, $\theta_2^{(f)} = 105°$, $\theta_4^{(i)} = 135°$ & $\theta_4^{(f)} = 225°$

$$45° \le \theta_2 \le 105° \quad ; \quad 135° \le \theta_4 \le 225°$$

Given data: $n = 3$, $x_0 = 1$, $x_4 = 10$

$$x_j = \left(\frac{x_{n+1}+x_0}{2}\right) - \left(\frac{x_{n+1}-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{2n}\right\}$$

ie $$x_j = \left(\frac{x_4+x_0}{2}\right) - \left(\frac{x_4-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{6}\right\}$$

$$x_j = \frac{11}{2} - 4.5\cos(2j-1)\pi/6$$

$x_1 = 5.5 - 4.5\cos\pi/6 = 1.6 \Rightarrow y_1 = \log_{10}(1.6) = 0.204$

$x_2 = 5.5 - 4.5\cos\pi/2 = 5.5 \Rightarrow y_2 = \log_{10}(5.5) = 0.741$

$x_3 = 5.5 - 4.5\cos 5\pi/6 = 9.4 \Rightarrow y_3 = \log_{10}(9.4) = 0.974$

$x_0 = 1 \Rightarrow y_0 = \log_{10}(1) = 0$

$x_4 = 10 \Rightarrow y_4 = \log_{10}(10) = 1.$

Scale Factor for Input & Output motion

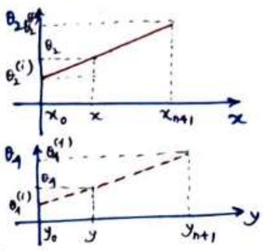
Let $\theta_2^{(i)}$ be the initial value of θ_2 representing x_0

$\theta_4^{(i)}$ be " " " of θ_4 " $y_0 = f(x_0)$

The input & output scale factors m_x & m_y resp. are defined as:

$$\therefore \text{Scale factor } m_x = \frac{\Delta\theta_2}{\Delta x} = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0} = \frac{\theta_2 - \theta_2^{(i)}}{x - x_0}$$

$$m_y = \frac{\Delta\theta_4}{\Delta y} = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0} = \frac{\theta_4 - \theta_4^{(i)}}{y - y_0}$$



The superscripts 'i' & 'f' denote the initial & final values of θ_2 & θ_4.

$$m_x = \frac{105 - 45°}{10 - 1} = \frac{\theta_2 - 45°}{x - 1} \quad ; \quad m_y = \frac{225 - 135}{1 - 0} = \frac{\theta_4 - 135}{y - 0}$$

$$\theta_2 = \left(\frac{60}{9}\right)(x-1) + 45 \quad ; \quad \theta_4 = 90(y-0) + 135$$

$$\theta_2 = \left(\frac{60}{9}\right)(x-1) + 45 \quad ; \quad \theta_4 = 90(y-0) + 135$$

Accuracy Points or Precision Points

Position	x_j	$\theta_2^{(j)}$	y_j	θ_4	$\theta_4^{(j)}$
1	1.6	49°	0.204	~~63.36~~	153.36
2	5.5	75°	0.741	~~111.69~~	201.69
3	9.4	101°	0.974	~~132.66~~	222.66

Freudenstein's eqn

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)$$

$$K_1 \cos 153.36 - K_2 \cos 49° + K_3 = \cos(49 - 153.36)$$

$$K_1 \cos 201.69 - K_2 \cos 75° + K_3 = \cos(75 - 201.69)$$

$$K_1 \cos 222.66 - K_2 \cos 101° + K_3 = \cos(101 - 222.66)$$

Solving

$K_1 = 2.0$; $K_2 = -0.7015$; $K_3 = 1.081$

$$K_1 = 2.0 \; ; \; K_2 = -0.7015 \; ; \; K_3 = 1.081$$

$$K_1 = \frac{r_1}{r_2} = 2 \; ; \; K_2 = \frac{r_1}{r_4} = -0.7015$$

$$r_2 = r_1/2 \; ; \; r_4 = -\frac{r_1}{0.7015}$$

$$K_3 = \frac{r_1^2 + r_2^2 + r_4^2 - r_3^2}{2 r_2 r_4} = 1.081$$

$$\frac{r_1^2 + \frac{r_1^2}{4} + \left(\frac{r_1}{0.7015}\right)^2 - r_3^2}{\cancel{2} r_1/\cancel{2} \cdot \left(-\frac{r_1}{0.7015}\right)} = 1.081$$

$$+\left[\frac{r_1^2 + r_1^2/4 + \left(\frac{r_1}{0.7015}\right)^2 - r_3^2}{r_1^2}\right] = -\frac{1.081}{0.7015}$$

$$1 + \frac{1}{4} + \left(\frac{1}{0.7015}\right)^2 - \left(\frac{r_3}{r_1}\right)^2 = -\frac{1.081}{0.7015}$$

$$\left(\frac{r_3}{r_1}\right)^2 = (2.1962)^2 \quad \therefore \frac{r_3}{r_1} = 2.1962$$

$$\frac{r_1}{r_3} = 0.462$$

$$\left|\frac{r_1}{r_2}\right| = 2 \; ; \; \left|\frac{r_1}{r_4}\right| = +0.7015, \quad \left|\frac{r_1}{r_3}\right| = 0.462$$

from above it is clear that $\quad [r_2 < r_1 < r_4 < r_3]$

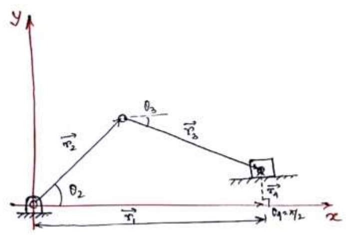
$$r_2 < r_1$$
$$< r_4 \quad \text{as } r_1 < r_4$$
$$< r_3 \quad \text{as } r_1 < r_3$$

$\therefore$ r_2 is the smallest link

$$r_2 = 5 \text{ cm}$$

$$\therefore r_1 = 10 \text{ cm} \; ; \; r_4 = 14.2 \text{ cm}, \; r_3 = 21.85 \text{ cm}.$$

Synthesis of the Slider-Crank Mechanism with three accuracy points



Loop-closure eqn

$$\vec{r_2} + \vec{r_3} = \vec{r_1} + \vec{r_4}$$

Scalar component of the eqn.

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_1 \cos\theta_1 + r_4 \cos\theta_4$$
$$r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_1 \sin\theta_1 + r_4 \sin\theta_4$$

where $\theta_1 = 0°$; $\theta_4 = \pi/2$

r_1 is variable

$$r_3 \cos\theta_3 = r_1 + 0 - r_2 \cos\theta_2$$
$$r_3 \sin\theta_3 = 0 + r_4 - r_2 \sin\theta_2$$

Squaring & adding

$$r_3^2 = (r_1 - r_2\cos\theta_2)^2 + (r_4 - r_2\sin\theta_2)^2$$

$$r_3 \cos\theta_3 = r_1 + 0 - r_2 \cos\theta_2$$
$$r_3 \sin\theta_3 = 0 + r_4 - r_2 \sin\theta_2$$

Squaring & adding

$$r_3^2 = (r_1 - r_2\cos\theta_2)^2 + (r_4 - r_2 \sin\theta_2)^2$$

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1 r_2 \cos\theta_2 - 2r_2 r_4 \sin\theta_2$$

$$2r_1 r_2 \cos\theta_2 + 2r_2 r_4 \sin\theta_2 - (r_2^2 - r_3^2 + r_4^2) = r_1^2$$

$$K_1 S \cos\theta_2 + K_2 \sin\theta_2 - K_3 = S^2, \text{ where } \quad K_1 = 2r_2$$

$$K_2 = 2r_2 r_4$$

$$K_3 = r_2^2 - r_3^2 + r_4^2$$

Substituting the three related pairs

$[\theta_2^{(1)}, s^{(1)}]$, $[\theta_2^{(2)}, s^{(2)}]$ & $[\theta_2^{(3)}, s^{(3)}]$ Variable $r_1 = S$ (Sliding)

$$K_1 S \cos\theta_2 + K_2 \sin\theta_2 - K_3 = S^2$$

Successively in above eq^n, we obtain three linear simultaneous eq. in K_1, K_2 & K_3.

$$\left.\begin{aligned} K_1 s^{(1)} \cos\theta_2^{(1)} + K_2 \sin\theta_2^{(1)} - K_3 &= \{s^{(1)}\}^2 \\ K_1 s^{(2)} \cos\theta_2^{(2)} + K_2 \sin\theta_2^{(2)} - K_3 &= \{s^{(2)}\}^2 \\ K_1 s^{(3)} \cos\theta_2^{(3)} + K_2 \sin\theta_2^{(3)} - K_3 &= \{s^{(3)}\}^2 \end{aligned}\right\} \cdots\cdots \text{(I)}$$

Example #

Design a slider-crank mechanism in which the slider displacement is proportional to the square of the crank ~~rotation~~ angular displacement in the interval $45° \le \theta_2 \le 135°$, The initial and final value of slider ~~displacement~~ position from ref. frame are 10 cm & 3 cm, respectively. Use the three point chebyshev spacing.
The direction of slider motion is parallel to x-axis

Given : $\theta_2^{(i)} = 45°$; $\theta_2^{(f)} = 135°$, $s^{(i)} = 10$ cm , $s^{(f)} = 3$ cm

The slider displacement is proportional to the square of the crank ~~rotation~~ Angular displacement (crank ang. Disp)

Displacement = Change in position = final position − initial position (Difference)

$\therefore \quad s - s^{(i)} = c\left(\theta_2 - \theta_2^{(i)}\right)^2$ where 'c' is a constant of proportionality

Now when $\theta_2 = \theta_2^{(f)} = 135°$ then $s = s^{(f)} = 3$ cm

From eq(1) $s^{(f)} - s^{(i)} = c\left(\theta_2^{(f)} - \theta_2^{(i)}\right)^2$

$$c = \frac{s^{(f)} - s^{(i)}}{\left\{\theta_2^{(f)} - \theta_2^{(i)}\right\}^2} = \frac{-10+3}{\{135-45°\}^2} = -\frac{7}{90^2}$$

$$\therefore \quad s - s^{(i)} = -\frac{7}{90^2}\left(\theta_2 - \theta_2^{(i)}\right)^2$$

i.e. $s - 10 = -7/90^2\,(\theta_2 - 45°)^2$ ------- (2)

The three accuracy points $\theta_2^{(1)}$, $\theta_2^{(2)}$ and $\theta_2^{(3)}$ are determined by chebyshev's spacing

$$\theta_2^{(j)} = \frac{\theta_2^{(f)} + \theta_2^{(i)}}{2} - \left[\frac{\theta_2^{(f)} - \theta_2^{(i)}}{2}\right]\cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } n = 3$$

----- (3)

The three accuracy points $\theta_2^{(1)}$, $\theta_2^{(2)}$ and $\theta_2^{(3)}$ are determined by Chebyshev's spacing

$$\theta_2^{(j)} = \frac{\theta_2^{(f)} + \theta_2^{(i)}}{2} - \left[\frac{\theta_2^{(f)} - \theta_2^{(i)}}{2}\right] \cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } n = 3 \qquad \text{----- (3)}$$

$$\theta_2^{(1)} = \frac{135^\circ + 45^\circ}{2} - \frac{135^\circ - 45^\circ}{2} \cos\frac{\pi}{6}$$

$$\theta_2^{(1)} = 90^\circ - 45^\circ \cos \pi/6 = 51.03^\circ$$

$$\theta_2^{(2)} = 90^\circ - 45^\circ \cos \pi/2 = 90^\circ$$

$$\theta_2^{(3)} = 90^\circ - 45^\circ \cos 5\pi/6 = 128.97^\circ$$

The corresponding values of 's' are obtained by eqn (2)

$$s = 10 - \frac{7}{90^2}(\theta_2 - 45^\circ)^2$$

$$s^{(1)} = 10 - \frac{7}{90^2}(51.03 - 45)^2 = 9.97 \text{ cm}$$

$$s^{(2)} = 10 - \frac{7}{90^2}(90 - 45)^2 = 8.25 \text{ cm}$$

$$s^{(3)} = 10 - \frac{7}{90^2}(128.97 - 45)^2 = 3.91 \text{ cm}$$

Now, substituting the related pairs $(51.03^\circ, 9.97 \text{ cm})$, $(90^\circ, 8.25 \text{ cm})$ and $(128.97^\circ, 3.91 \text{ cm})$ in eqn (1), we get

$$K_1 \cdot 9.97 \cos 51.03^\circ + K_2 \sin 51.03^\circ - K_3 = (9.97)^2 \Rightarrow 6.27K_1 + 0.78K_2 - K_3 = 99.4$$

$$K_1 \cdot 8.25 \cos 90^\circ + K_2 \sin 90^\circ - K_3 = (8.25)^2 \Rightarrow K_2 - K_3 = 68$$

$$K_1 \cdot 3.91 \cos 128.97^\circ + K_2 \sin 128.97^\circ - K_3 = (3.91)^2 \Rightarrow -2.47K_1 + 0.78K_2 - K_3 = 15.3$$

$$6.27K_1 + 0.78(68 + K_3) - K_3 = 99.4 \Rightarrow 6.27K_1 - 0.22K_3 = 46.36$$

$$-2.47K_1 + 0.78(68 + K_3) - K_3 = 15.3 \Rightarrow -2.47K_1 - 0.22K_3 = -37.74$$

$$\therefore K_1 = 9.62, \quad K_2 = 131.1, \quad K_3 = 63.1$$

$$K_1 = 2r_2 = 9.62 \Rightarrow r_2 = 4.81 \text{ cm}$$

$$K_2 = 2r_2 r_4 = 131.1 \Rightarrow r_4 = \frac{131.1}{2 \times 4.81} = 13.62 \text{ cm}$$

$$K_3 = r_2^2 - r_3^2 + r_4^2 = 63.1 \Rightarrow r_3 = 12.06 \text{ cm}$$