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The second law analysis in fundamental convective heat transfer problems

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Abstract

Second law characteristics of heat transfer and fluid flow due to forced convection of steady-laminar flow of incompressible fluid inside channel with circular cross-section and channel made of two parallel plates is analyzed. Different problems are discussed with their entropy generation profiles and heat transfer irreversibility characteristics. In each case, analytical expression for entropy generation number (N_S) and Bejan number (Be) are derived in dimensionless form using velocity and temperature profiles.

Keywords: Bejan number; Entropy generation number; Irreversibility distribution ratio

1. Introduction

The foundation of knowledge of entropy production goes back to Clausius and Kelvin's studies on the irreversible aspects of the Second Law of Thermodynamics. Since then the theories based on these foundations have rapidly developed. However, the entropy production resulting from temperature differences has remained untreated by classical thermodynamics, which motivates many researchers to conduct analyses of fundamental and applied engineering problems based on Second Law (of thermodynamics).

Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Different sources are responsible for generation of entropy like heat transfer across finite temperature gradient, characteristic of convective heat transfer, viscous effect etc. Bejan [1,2] focused on the different reasons behind entropy generation in applied thermal engineering. Generation of entropy destroys available work of a system. Therefore, it makes good engineering sense to focus on irreversibility [2] of heat transfer and fluid flow processes and try to understand the function of entropy generation mechanism. Second law analysis [1,2] focusing on entropy generation and its minimization has been playing a dominant role in recent times to understand the irreversibility in applied engineering and transport processes.

Fluid flow and heat transfer characteristics inside a channel with circular cross-section and a channel made of two parallel plates at different boundary conditions is one of the fundamental researches in engineering [3,4]. Analyses of simpler systems are often useful to understand some important features of complex pattern forming processes in various fields of science and technology. These types of geometry appear in many engineering applications as single unit or as a combination. Bejan [5] presented simplified analytical expression for entropy generation rate in a circular duct with constant heat flux at the wall. This analysis is then extended by calculating optimum Reynolds number [2] as a function of Prandtl number and duty parameter. Further extension is done by Sahin [6] who introduced the second law analysis of viscous fluid in a circular duct at isothermal boundary condition. In the more recent paper, Sahin [7] presented the effect of variable viscosity on entropy generation rate for the constant heat flux boundary condition for circular duct. For non-circular duct, Narusawa [8] gives a theoretical and numerical analysis of second law for flow and heat transfer inside a rectangular duct. Sahin [9] compared the entropy generation rate inside ducts with different geometric shapes (circular, triangular, square etc.) and calculated the optimum duct shape subjected to isothermal boundary condition for laminar flow. Nag and Kumar [10] presented the second law optimization techniques for convective heat transfer through duct at constant heat flux boundary condition. For other flow configuration, like periodic flow, Luiz

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Nomenclature

A	cross-sectional area	m^2
Be	Bejan number, $=1/(1+\Phi)$	
Br	Brinkman number, $=Ec\,Pr$	
C_1	constant	
C_2	constant	
C_P	specific heat	$J\cdot kg^{-1}\cdot{}^\circ C^{-1}$
Ec	Eckert number, $=u_m^2/C_p\Delta T$	
k	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
u	axial velocity	$m\cdot s^{-1}$
U	dimensionless velocity	
v	radial velocity	$m\cdot s^{-1}$
x	axial distance	m
X	simensionless axial distance	
y	direction normal to the axis	m
Y	dimensionless normal direction	
L	half width of the channel	m
m	modified fluid index, $=1+1/n$	
n	fluid index	
N_C	entropy generation number, conduction, $=(\partial\Theta/\partial X)^2/Pe$	
N_F	entropy generation number; fluid friction, $=Br(\partial U/\partial Y)^2/\Omega$	
N_R	entropy generation number; radial, $=(\partial\Theta/\partial R)^2$	
N_S	entropy generation number; total	
N_Y	entropy generation number; normal to the axis, $=(\partial\Theta/\partial Y)^2$	
P	pressure	Pa
Pe	Peclet number, $=(u_m L/\alpha)$	
Pr	Prandtl number, $=(\mu C_p/k)$	
q	constant heat flux at wall	$W\cdot m^{-2}$
R	radial distance	m
r_0	radius of the pipe	m
R	dimensionless radial distance	
Re	Reynolds number, $=(\rho u_m L/\mu)$	
S_G	entropy generation rate	$W\cdot m^{-3}\cdot K^{-1}$
$S_{G,C}$	characteristic entropy transfer rate	
T	temperature	
T_0	reference temperature	

Greek symbols

α	thermal diffusivity	$m^2\cdot s^{-1}$
Γ_n	constants where $n=1,2,3,\ldots$	
λ	angular velocity ratio, $=(\omega_0/\omega_i)$	
μ	dynamic viscosity	Pa·s
ρ	density of the fluid	$kg\cdot m^{-3}$
τ	shear stress	Pa
Φ	irreversibility ratio, $=(N_F/(N_C+N_Y))$	
Π	radius ratio, $=(r_i/r_0)$	
Θ	dimensionless temperature	
Ω	dimensionless temperature difference, $=(\Delta T/T_0)$	
ω	angular velocity	$rad\cdot s^{-1}$

Subscripts and superscripts

av	average
i	based on inner radius
m	maximum
o	based on outer radius
w	value at wall
θ	peripheral component

and Bejan [11] presented the second law analysis of flow and heat transfer through a single tube and bundle of tubes. For other geometry, the second law analysis as well as entropy generation profiles are available in the references by Drost and Zaworski [12] and Bejan [2,5].

The main objective of this article is to analyze the mechanism of entropy generation in basic configurations encountered in convective heat transfer. In this article we seek to identify the origin of entropy production and its distribution through fluid flows most commonly found in convective heat transfer situations. Eight different types of problems are selected in the present study and analyzed. These are forced convection inside a channel with

(a) one fixed plate and one moving plate,
(b) two fixed plates,
(c) circular cross-section for Newtonian fluid,
(d) circular annulus,
(e) axially moving concentric cylinder,
(f) rotating concentric cylinder,
(g) circular cross-section for non-Newtonian fluid,
(h) non-Newtonian fluid through a channel with two parallel plates.

2. Entropy generation in convection heat transfer

Consider the local rate of entropy production inside a fluid engaged in convective heat transfer without internal heat generation. Fig. 1 shows a two-dimensional infinitesimal fluid element ($dx\times dy$). Considering the element as an open thermodynamic system subjected to mass fluxes, energy transfer and entropy transfer interactions through a fixed control surface. If the fluid is Newtonian and incompressible, and if it obeys the Fourier law of heat conduction, the volumetric rate of entropy generation in Cartesian coordinates is [5]:

$$S_G=\frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\mu}{T_0}\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right\}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right] \quad (1)$$

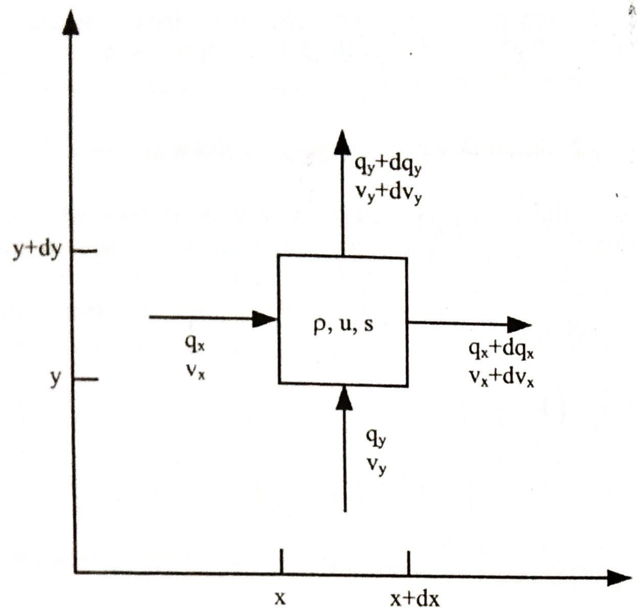

Fig. 1. Entropy generation analysis for an infinitesimal element $dx \times dy$ in convective heat transfer.

The above form of entropy generation shows that the irreversibility is due to two effects, conductive (k) and viscous (μ). Entropy generation rate (S_G) is positive and finite as long as temperature and velocity gradients are present in the medium.

In many fundamental convective heat transfer problems, velocity and temperature distributions are simplified assuming that the flow is hydrodynamically developed ($\partial V/\partial x = 0$) and thermally developing ($\partial T/\partial x \neq 0$) or developed ($\partial T/\partial x = 0$). For example, see references by Bejan [13], Burmeister [14], Shah and London [4] and White [3]. In such situations, Eq. (1) can be reduced into the following form:

$$S_G = \frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T_0}\left[\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2}$$

According to Bejan [1], entropy generation number (N_S) is the dimensionless from of S_G, which is, by definition, equal to the ratio of actual entropy generation rate (S_G) to the characteristic entropy transfer rate ($S_{G,C}$). According to Bejan [1,5], the characteristic entropy transfer rate is equal to:

$$S_{G,C} = \left[\frac{q^2}{kT_0^2}\right] = \left[\frac{k(\Delta T)^2}{L^2 T_0^2}\right] \tag{3}$$

In the above equation, q is the heat flux, T_0 is the absolute reference temperature, ΔT is the reference temperature difference and L is the characteristic length that depends on geometry of the channel and problem type. The first square bracketed term is used for isoflux boundary condition and the second square bracketed term is used for isothermal boundary condition. The velocity u is scaled with reference velocity u_0, distance y is scaled with L, axial distance x is scaled with $L^2 u_0/\alpha$ and dimensionless temperature Θ can be expressed as $(T - T_0)/\Delta T$. After putting into dimensionless form, Eq. (2) can be expressed as:

$$\begin{aligned} N_S &= \frac{1}{Pe^2}\left[\frac{\partial \Theta}{\partial X}\right]^2 + \left[\frac{\partial \Theta}{\partial Y}\right]^2 + \frac{Br}{\Omega}\left[\frac{\partial U}{\partial Y}\right]^2 \\ &= N_C + N_Y + N_F \end{aligned} \tag{4}$$

In the above equation, *Pe* is the Peclet number, which determines the relative importance between convection and diffusion. *Br* is the Brinkman number, which determines the relative importance between dissipation effects and fluid conduction effects (see White [3] for details). Ω is the dimensionless temperature difference, which is equal to $\Delta T/T_0$. On the right-hand side of Eq. (4), the first term (N_C) represents the entropy generation by heat transfer due to axial conduction, second term (N_Y) accounts for entropy generation due to heat transfer in normal direction to the axis and the last term (N_F) is the fluid friction contribution to entropy generation.

3. Fluid friction versus heat transfer irreversibility

Entropy is generated in a process or system due to the presence of irreversibility [1,2]. In convection problem, both fluid friction and heat transfer have contributions to the rate of entropy generation. Expression of entropy generation number (N_S) is good for generating spatial entropy profile, but fails to give any idea which of the fluid friction or heat transfer dominates. According to Bejan [1], the irreversibility distribution ratio (Φ) takes care of the above problem and Φ is equal to the ratio of entropy generation due to fluid friction (N_F) to heat transfer ($N_C + N_Y$). Heat transfer irreversibility dominates over fluid friction irreversibility for $0 \leqslant \Phi < 1$ and fluid friction dominates when $\Phi > 1$. For $\Phi = 1$, both the heat transfer and fluid friction have the same contribution for generating entropy. In many engineering designs and optimization problems [15], contribution of heat transfer entropy generation ($N_C + N_R$) on overall entropy generation rate (N_S) is needed. As an alternative irreversibility distribution parameter, Paoletti et al. [16] define Bejan number (*Be*) which is the ratio of entropy generation due to heat transfer to the total entropy generation. Mathematically Bejan number is:

$$Be = \frac{N_C + N_Y}{N_S} = \frac{1}{1+\Phi} \tag{5}$$

Bejan number ranges from 0 to 1. Accordingly, $Be = 1$ is the limit at which the heat transfer irreversibility dominates, $Be = 0$ is the opposite limit at which the irreversibility is dominated by fluid friction effects, and $Be = 1/2$ is the case in which the heat transfer and fluid friction entropy generation rates are equal.

In the following subsections, the expressions of entropy generation number (N_S), irreversibility distribution ratio (Φ) and Bejan number (*Be*) are determined for a series of important fundamental convective heat transfer configurations.

3.1. Steady flow between a fixed and a moving plate

We start with the simplest example of steady flow between a fixed and a moving plate commonly named as Couette flow. According to Burmeister [14], the velocity and temperature distribution for such flow, provided that the walls are at different temperatures, is:

$$\frac{u}{u_w} = \frac{y}{L} \tag{6}$$

$$\frac{T - T_0}{T_1 - T_0} = \frac{y}{L} + \frac{Ec\,Pr}{2}\frac{y}{L}\left(1 - \frac{y}{L}\right) \tag{7}$$

With proper scaling (see previous section), Eqs. (6) and (7) are put into their dimensionless forms:

$$U = Y \tag{8}$$

$$\Theta = Y + \frac{Br}{2}Y(1 - Y) \tag{9}$$

Using Eqs. (4), (8) and (9), the expression for dimensionless entropy generation number (N_S) for Couette flow with isothermal boundary condition becomes:

$$N_S = \left[1 + \frac{Br}{2} - Br\,Y\right]^2 + \left[\frac{Br}{\Omega}\right] \tag{10}$$

The second square bracketed term on the right-hand side is the fluid friction contribution to entropy generation, which is constant and independent of Y for a particular value of group parameter (Br/Ω). From Eqs. (5) and (10), the Bejan number (Be) for this particular problem becomes:

$$Be = \frac{[2 + Br - 2\,Br\,Y]^2}{[2 + Br - 2\,Br\,Y] + Br/4\Omega} \tag{11}$$

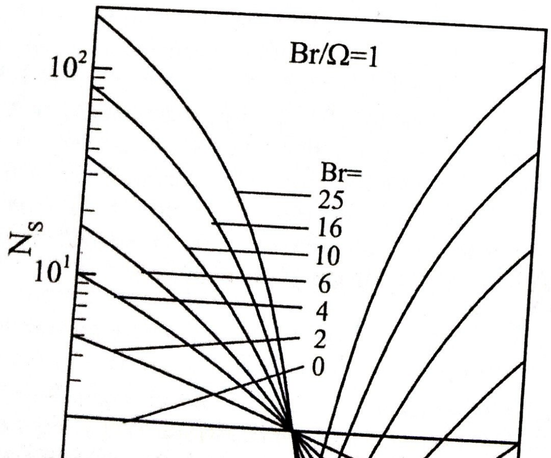

Fig. 2. Entropy generation number as a function of Y at different Brinkman numbers.

Entropy generation number (N_S) is plotted as a function of Y in Fig. 2 at different Brinkman number ranges 0–25 and constant group parameter (= 1).

3.2. Steady flow between two fixed plates

The second example is fluid flow between two fixed parallel plates (Poiseuille flow) with differentially heated isothermal boundary condition. The expressions for the velocity and temperature distribution, according to Burmeister [14], are:

$$u = \left(1 - \frac{y^2}{L^2}\right)u_m \tag{12}$$

$$T = T_0 + \frac{T_1 - T_0}{2}\left(1 + \frac{y}{L}\right) + \frac{\mu}{3k}\left(1 - \frac{y^4}{L^4}\right)u_m^2 \tag{13}$$

With proper scaling, the dimensionless forms of Eqs. (12) and (13) are:

$$U = (1 - Y^2) \tag{14}$$

$$\Theta = \frac{1}{2}(1 + Y) + \frac{Br}{3}(1 - Y^4) \tag{15}$$

Using Eqs. (4), (14) and (15), the expression for dimensionless entropy generation number (N_S) for Poiseuille flow with isothermal boundary condition becomes:

$$N_S = \left[\frac{1}{2} - \frac{4}{3}Br\,Y^3\right]^2 + \left[4\frac{Br}{\Omega}Y^2\right] \tag{16}$$

Bejan number (Be) can derived from Eqs. (5) and (16) and which is equal to:

$$Be = \frac{[3 - 8\,Br\,Y^3]^2}{[3 - 8\,Br\,Y^3]^2 + 144[Br\,Y^2/\Omega]} \tag{17}$$

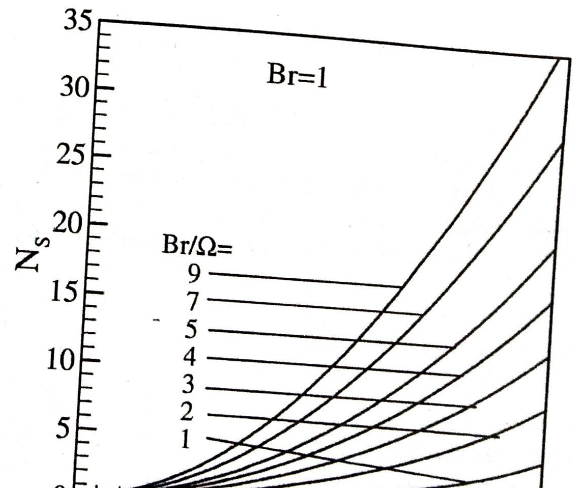

Fig. 3. Entropy generation number as a function of Y at different group parameters.

Entropy generation number (N_S) is plotted as a function of Y in Fig. 3 at different group parameter ranges 1–9 and constant Brinkman number (= 1).

3.3. Convection in a round tube

The third example is fluid flow through a circular pipe (Poiseuille flow) with the constant heat flux boundary condition. Fully developed laminar flow is considered here. The expressions for velocity and temperature distribution, according to Bird et al. [17], are:

$$u = \left(1 - \frac{r^2}{r_0^2}\right) u_m \tag{18}$$

$$T = \frac{q r_0}{k}\left[-4\frac{x\alpha}{R^2 u_m} - \frac{r^2}{r_0^2} + \frac{1}{4}\frac{r^4}{r_0^4}\right] \tag{19}$$

With proper scaling, the dimensionless form of Eqs. (18) and (19) are:

$$U = (1 - R^2) \tag{20}$$

$$\Theta = \left[-4X - R^2 + \frac{R^4}{4}\right] \tag{21}$$

Eq. (4) is derived for Cartesian coordinate. But the problem in the present example is in cylindrical coordinate. A simple substitution of Y with R in Eq. (4) will give the general expression for N_S in cylindrical coordinate. Now using Eqs. (20), (21) and modified form of Eq. (4) in cylindrical coordinate, the expression for dimensionless entropy generation number (N_S) for Poiseuille flow in a round tube with isothermal boundary condition becomes:

$$N_S = \frac{16}{Pe^2} + (R^3 - 2R)^2 + 4\frac{Br}{\Omega}R^2 \tag{22}$$

The first term in the right-hand side of Eq. (22) is related to axial conduction and which is inversely proportional to square of Peclet number. Except for liquid metals ($Pr \ll 1$) and for creeping flow (low Reynolds number), magnitude of Peclet number (Pe) is usually higher in value (negligible Pe^{-2}). For the case where $Pe \gg 4$, the dominance of first term on entropy generation is negligible [5]. Using Eqs. (5) and (22), the expression for Bejan number (Be) for this problem becomes:

$$Be = \frac{16/Pe^2 + (R^3 - 2R)^2}{16/Pe^2 + (R^3 - 2R)^2 + 4Br\,R^2/\Omega} \tag{23}$$

Entropy generation number (N_S) is plotted as a function of R in Fig. 4 at different group parameter ranges 0–1 and for negligible effect of Peclet number.

3.4. Convection in a circular annulus

The next problem is the convection heat transfer inside a circular annular space with isoflux boundary condition. The inner and outer radii of the annulus are r_i and r_0. The velocity profile inside the annular space, according to Kays [18], is:

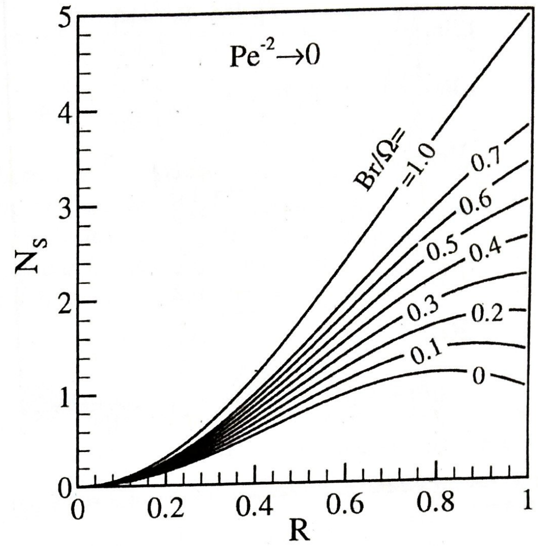

Fig. 4. Entropy generation number as a function of R at different group parameters.

$$u = \frac{2}{C_1}\left[1 - \frac{r^2}{r_0^2} + C_2 \ln\left(\frac{r}{r_0}\right)\right] u_{\mathrm{av}} \tag{24}$$

In the above equation, C_1 and C_2 are two constants and can be defined as a function of Π ($= r_i/r_0$) by using the following relations:

$$C_1 = \frac{1 + \ln(\Pi) + \Pi^2[\ln(\Pi) - 1]}{\ln(\Pi)} \tag{25}$$

$$C_2 = \frac{\Pi^2 - 1}{\ln(\Pi)} \tag{26}$$

With proper scaling, the dimensionless form of Eq. (24) is:

$$U = \frac{2}{C_1}[1 - R^2 + C_2 \ln(R)] \tag{27}$$

For this example, energy equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_{\mathrm{av}}}{aC_1}\left[1 - \frac{r^2}{r_0^2} + C_2 \ln\left(\frac{r}{r_0}\right)\right]\frac{\partial T}{\partial x} \tag{28}$$

To get an expression for temperature distribution, energy equation (Eq. (28)) should be solved with appropriate boundary condition. A separation of variables solution (see Arpaci and Larsen [19]) is assumed in the following form:

$$\Theta(r, x) = \Theta_1(r)\Theta_2(x) + \Theta_1(x) + \Theta_2(r) \tag{29}$$

The first term in the right-hand side of Eq. (29) is significant for decaying initial transition and entrance effect, the second term is significant for axial temperature rise due to accumulated wall heat flux and the third term is significant for radial temperature variation to let wall heat flux into fluid.

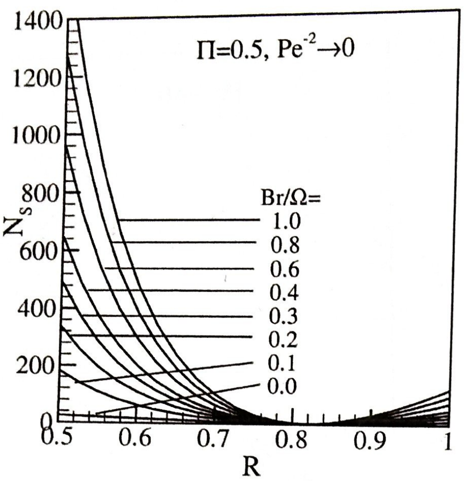

Fig. 5. Entropy generation number as a function of R at different group parameters.

Neglecting entrance effect and assuming that the system already passed the decaying initial transition. Then the first term at the right-hand side of Eq. (29) will disappear. Combination of Eqs. (28) and (29) leaves two separated ordinary differential equations and then the final solution becomes:

$$\Theta = \Gamma + \frac{4X}{C_2 - 1} + \frac{R^2}{C_2 - 1}[1 + C_2 \ln(R) - C_2] - \frac{R^4}{4(C_2 - 1)} \tag{30}$$

In Eq. (30), Γ is a constant of integration. In this particular problem, x is scaled with $2u_m r_0^2/C_1\alpha$. Using Eqs. (27), (30) and modified form of Eq. (4) in cylindrical coordinate, the expression for dimensionless entropy generation number (N_S) for Poiseuille flow in a round tube annulus with the isoflux boundary condition becomes:

$$N_S = \frac{16}{Pe^2(C_2 - 1)^2} + \left[\frac{2R(1 + C_2\ln(R) - C_2)}{C_2 - 1} + \frac{C_2 R - R^3}{C_2 - 1}\right]^2 + \frac{4Br[2R - C_2/R^2]^2}{C_1^2 \Omega} \tag{31}$$

As a special case, when $\Pi = 0$, the problem of annular circular channel reduces to a simple circular channel of radius r_0. In that case $C_1 = 1$ and $C_2 = 0$. In such case, Eq. (31) reduces into Eq. (22). For the circular annulus, Bejan number is:

$$Be = \left\{\frac{16}{Pe^2(C_2 - 1)^2} + \left[\frac{2R(1 + C_2\ln(R) - C_2)}{C_2 - 1} + \frac{C_2 R - R^3}{C_2 - 1}\right]^2\right\} \times \left\{\frac{16}{Pe^2(C_2 - 1)^2} + \left[\frac{2R(1 + C_2\ln(R) - C_2)}{C_2 - 1} + \frac{C_2 R - R^3}{C_2 - 1}\right]^2 + \frac{4Br[2R - C_2/R]^2}{C_1^2 \Omega}\right\}^{-1} \tag{32}$$

Entropy generation number (N_S) is plotted as a function of R in Fig. 5 at different group parameter ranges 0–1 and constant radius ratio ($= 0.5$) assuming the effect of Peclet number is negligible.

3.5. Axially moving concentric cylinder

The next example is the Couette flow inside two concentric circular cylinders. Consider two long concentric cylinders with inner radius r_i and outer radius r_0. For the present problem, the outer cylinder is considered fixed ($u = 0$) with the adiabatic boundary condition and the inner cylinder is considered moving with velocity u_0 with the isoflux boundary condition. According to White [3], velocity distribution inside the annular space can be expressed by the following equation:

$$u = \frac{\ln(r/r_0)}{\ln(r_1/r_0)} u_0 \tag{33}$$

With proper scaling, the dimensionless form of above equation is:

$$U = \frac{\ln(R)}{\ln(\Pi)} \tag{34}$$

To get an expression for temperature distribution, energy equation (Eq. (35)) should be solved with appropriate boundary condition. It is difficult to get a direct analytical solution of Eq. (35). Applying a separation of variable method, as described in Section 3.4, the expression for dimensionless temperature distribution with the isoflux boundary condition at the inner cylinder and adiabatic boundary condition at the outer cylinder of the annulus is presented in Eq. (36).

$$\frac{\partial \Theta}{\partial X} = \frac{1}{R[\ln(R) - \ln(\Pi)]} \frac{\partial}{\partial R}\left(R \frac{\partial \Theta}{\partial R}\right) \tag{35}$$

$$\Theta = \Gamma + \frac{4X + \Pi^2 \ln(R) + R^2[\ln(R/\Pi) - 1]}{\Pi^2 + 2\ln(\Pi) + 1} \tag{36}$$

In above equation, Γ is a constant of integration, Π is radius ratio ($= r_i/r_0$). Distance x is made dimensionless with $r_0^2 u_0/\alpha$. Using Eqs. (34), (36) and modified form of Eq. (4) in cylindrical coordinate, the expression for dimensionless entropy generation number (N_S) for axially moving concentric cylinder with adiabatic outer and isoflux inner wall becomes:

$$N_S = \frac{16}{[\Pi^2 - 2\ln(\Pi) + 1]^2 Pe^2} + \frac{[\Pi^2/R + R\{2\ln(R/\Pi) - 1\}]^2}{[\Pi^2 - 2\ln(\Pi) + 1]^2} + \frac{Br}{2\Omega R^2 \ln(\Pi)} \tag{37}$$

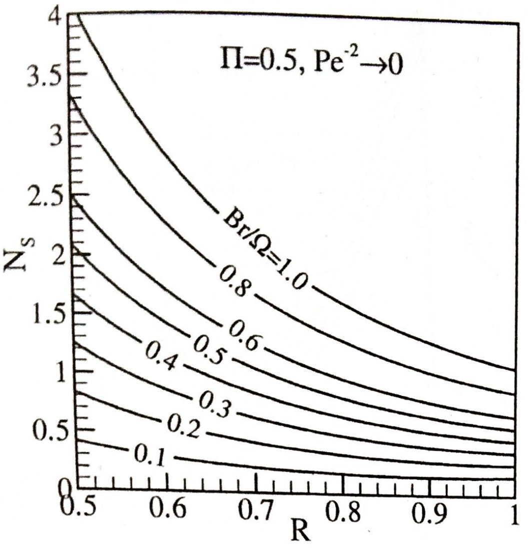

Fig. 6. Entropy generation number as a function of R at different group parameters.

For the present problem expression of Bejan number is:

$$\begin{aligned} Be = {} & \{[16/C_3^2 Pe^2] + [\Pi^2/R + R\{2\ln(R/\Pi) - 1\}]^2/C_3^2\} \\ & \times \{[16/C_3^2 Pe^2] \\ & + [\Pi^2/R + R\{2\ln(R/\Pi) - 1\}]^2/C_3^2 \\ & + Br/[2\Omega R^2 \ln(\Pi)]\}^{-1} \end{aligned} \tag{38}$$

where the constant C_3 is equal to $\Pi^2 - 2\ln(\Pi) + 1$.

Entropy generation number (N_S) is plotted as a function of R in Fig. 6 at different group parameter ranges 0.1–1.0 and constant radius ratio (= 0.5) assuming the effect of Peclet number is negligible.

3.6. Rotating concentric cylinders

Consider the steady flow maintained between two concentric cylinders by steady angular velocity of both cylinders. Let the inner cylinder has radius r_i, angular velocity ω_i and temperature T_i, while the outer cylinder has r_0, ω_0 and T_0. The geometry is such that the only nonzero velocity component is u_θ and the variables u_θ and T must be functions only of radius r. According to White [3], the velocity and temperature profiles can be expressed as:

$$u_\theta = r_i\omega_i \frac{r_0/r - r/r_0}{r_0/r_i - r_i/r_0} + r_0\omega_0 \frac{r/r_i - r_i/r}{r_0/r_i - r_i/r_0} \tag{39}$$

$$\begin{aligned} & \frac{T - T_0}{T_i - T_0} \\ & = Pr\, E \frac{r_0^4(1 - \omega_0/\omega_i)^2}{r_0^4 - r_i^4}\left(1 - \frac{r_i^2}{r^2}\right) \\ & \times \left[1 - \frac{\ln(r/r_i)}{\ln(r_0/r_i)}\right] + \frac{\ln(r/r_i)}{\ln(r_0/r_i)} \end{aligned} \tag{40}$$

With proper scaling, the dimensionless forms of the above equations are:

$$U = \frac{\Pi}{1 - \Pi^2}\left[\frac{\Pi}{\lambda}\left(\frac{1 - R^2}{R}\right) + \left(\frac{R^2 - \Pi^2}{\Pi R}\right)\right] \tag{41}$$

$$\Theta = Br \frac{(1 - \lambda)^2}{1 - \Pi^4}\left(1 - \frac{\Pi^2}{R^2}\right)\frac{\ln(R)}{\ln(\Pi)} + \left[1 - \frac{\ln(R)}{\ln(\Pi)}\right] \tag{42}$$

where λ is the velocity ratio (ω_0/ω_i) and Π is the radius ratio (r_i/r_0). In Eq. (41), peripheral velocity u_θ is made dimensionless with $\omega_0 r_0$. R is equal to r/r_0. For this particular problem, the non-dimensional general expression for entropy generation number, according to Bejan [2], is:

$$N_S = \left(\frac{\partial\Theta}{\partial R}\right)^2 + \frac{Br}{\Omega}\left[R\frac{\partial}{\partial R}\left(\frac{U}{R}\right)\right]^2 \tag{43}$$

Putting Eqs. (41) and (42) into Eq. (43), the entropy generation number (N_S) for rotating concentric cylinder with isothermal boundary condition becomes:

$$\begin{aligned} N_S = {} & \left[2Br\,\Pi^2\Gamma_3\frac{\ln(R)}{R^3} + Br\,\Gamma_3\frac{1}{R}\left(1 - \frac{\Pi^2}{R^2}\right) - \frac{\Gamma_4}{R}\right]^2 \\ & + \frac{4Br}{\Omega}\left[\frac{\Gamma_1 - \Gamma_2\Pi}{R^3}\right]^2 R^2 \end{aligned} \tag{44}$$

Constants Γ_1, Γ_2, Γ_3, Γ_4 of above equation are defined in Eq. (45).

$$\begin{aligned} \Gamma_1 &= \frac{\Pi^2}{\lambda(1 - \Pi^2)}, & \Gamma_2 &= \frac{1}{1 - \Pi^2} \\ \Gamma_3 &= \frac{(1 - \lambda)^2}{(1 - \Pi^4)\ln(\Pi)}, & \Gamma_4 &= \frac{1}{\ln(\Pi)} \end{aligned} \tag{45}$$

For the present problem with boundary condition, the expression for Bejan number is:

$$\begin{aligned} Be = {} & [2Br\,\Pi^2\Gamma_3\{\ln(R)/R^2\} + Br\,\Gamma_3(1 - \Pi^2/R^2) - \Gamma_4]^2 \\ & \times \{[2Br\,\Pi^2\Gamma_3\{\ln(R)/R^2\} \\ & + Br\,\Gamma_3(1 - \Pi^2/R^2) - \Gamma_4]^2 \\ & + \frac{4Br}{\Omega}[\Gamma_1 - \Gamma_2\Pi]^2\}^{-1} \end{aligned} \tag{46}$$

Entropy generation number (N_S) is plotted as a function of R in Fig. 7 at different group parameter ranges 0.0–1.0 and constant radius ratio (= 0.5), velocity ratio (0.5) and Brinkman number (= 1.0).

3.7. Non-Newtonian fluid flow in a circular tube

In many practical situations non-Newtonian fluid appears. Both modeling and analysis involve complexity for non-Newtonian fluid. Second law analysis extends this complexity more. In this example, the governing equations in cylindrical coordinate are simplified and solved analytically for a circular tube with constant heat flux at the wall for a non-Newtonian power law fluid. According to the rheological classification of fluids [20], shear stress (τ) of a non-Newtonian fluid can be expressed as $\mu(du/dr)^n$. This expression for the shear stress covers a wide range of fluids. For Pseudoplastic fluids like fine particle suspensions $n < 1$, for Dilatant fluids like ultrafine irregular particle suspensions

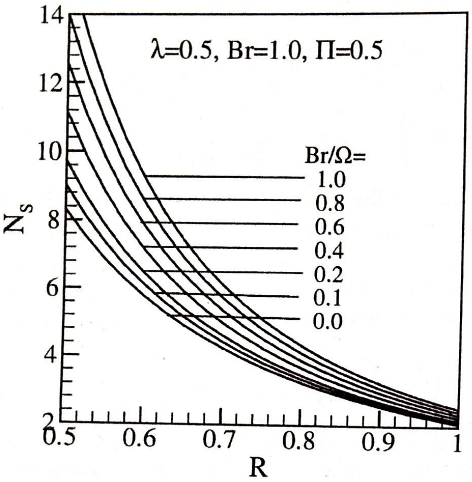

Fig. 7. Entropy generation number as a function of R at different group parameters.

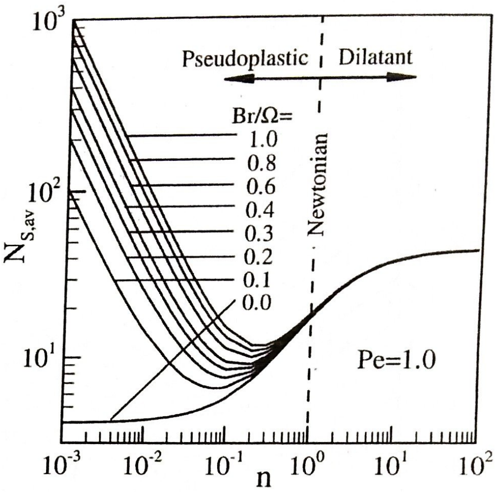

Fig. 8. Average entropy generation as a function of n at different group parameters.

$n > 1$ and for Newtonian fluid like air $n = 1$. Governing momentum equation and its solution for velocity distribution are given in the following equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial u}{\partial r}\right)^n\right] = \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right) \tag{47}$$

$$U = \left(1 - R^{(n+1)/n}\right) \tag{48}$$

In Eq. (48), velocity u is made dimensionless with maximum velocity u_m. Putting $(n+1)/n = m$, the dimensionless form of energy equation is:

$$\frac{\partial \Theta}{\partial X} = \frac{1}{R - R^{m+1}}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta}{\partial R}\right) \tag{49}$$

Using separation of variable method, the solution of Eq. (49) with constant heat flux boundary condition gives the expression for dimensionless temperature distribution:

$$\Theta = \Gamma + \frac{2(m+2)}{m}X + \frac{m+2}{2m}R^2 - \frac{2}{m(m+2)}R^{m+2} \tag{50}$$

where Γ is a constant of integration. Using Eqs. (4), (48) and (50), entropy generation number (N_S) for present example is:

$$N_S = \left[\frac{4(m+2)^2}{m^2}\frac{1}{Pe^2}\right] + \left[\frac{m+2}{m}R - \frac{2}{m}R^{m+1}\right]^2 + \left[\frac{m^2 Br}{\Omega}R^{2m-2}\right] \tag{51}$$

The entropy generation profile expressed by Eq. (51) is valid for wide range of fluid index m $(= 1 + 1/n)$. Starting from flat velocity profile (slug flow) at $n = 0$ $(m = \infty)$ to linear profile at $n = \infty$ $(m = 1)$. For the special case of Newtonian fluid ($n = 1$ or $m = 2$), Eq. (51) reduces to the form given in Eq. (22) which is similar to the solution of Bejan [5]. The expression for Bejan number for the present problem is:

$$\begin{aligned} Be = {} & \left\{\left[4(m+2)^2/Pe^2\right] + \left[(m+2)R - 2R^{m+1}\right]^2\right\} \\ & \times \left\{\left[4(m+2)^2/Pe^2\right] + \left[(m+2)R - 2R^{m+1}\right]^2 \right. \\ & \left. + \left[m^4 Br\, R^{2m-2}/\Omega\right]\right\}^{-1} \end{aligned} \tag{52}$$

Depending on the exponent m or n, Eq. (51) is valid for a wide variety of non-Newtonian fluids. It is really an exhaustive job to focus on the local entropy generation characteristics for each type of fluid. From the expression of N_S, volumetric average entropy generation rate is calculated using Eq. (53).

$$N_{S,\mathrm{av}} = \frac{1}{\forall}\int_{\forall} N_S \,\mathrm{d}\forall \tag{53}$$

Fig. 8 shows the distribution of average entropy generation rate as a function of fluid index n at different group parameters ranges 0–1. Newtonian fluid is indicated by dashed line where $n = 1$.

3.8. Non-Newtonian fluid flow through a channel with two parallel plates

The next example is the flow of non-Newtonian fluid inside two parallel plates with finite gap between them. Heat flux for both walls are same and kept constant. Considering fully developed flow and the power law model [20] for non-Newtonian fluid, the momentum equation in Cartesian coordinate reduces to the form given in Eq. (54). With no slip boundary condition, the solution of Eq. (54) is given in Eq. (55).

$$\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^n\right] = \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right) \tag{54}$$

$$u = \left[1 - \frac{y^m}{L^m}\right]u_m \tag{55}$$

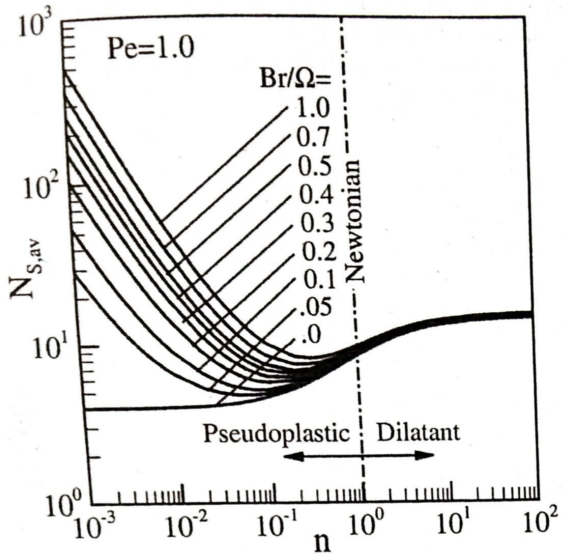

Fig. 9. Average entropy generation as a function of n at different group parameters.

where L is the half width of the channel and u_m is the maximum velocity. Modified exponent m is equal to $(n+1)/n$. With proper scaling, the dimensionless form of the energy equation and its solution are given in Eqs. (56) and (57).

$$\frac{\partial\Theta}{\partial X} = \frac{1}{1-Y^m}\frac{\partial^2\Theta}{\partial Y^2} \tag{56}$$

$$\Theta = \Gamma + \frac{m+1}{m}X + \frac{m+1}{2m}Y^2 - \frac{1}{m(m+2)}Y^{m+2} \tag{57}$$

In the above expressions, y and x are scaled with L and $L^2 u_m/\alpha$, respectively. Γ is a constant of integration. Using Eqs. (4), (55) and (57), the dimensionless entropy generation number for the present problem is:

$$N_S = \left[\frac{(m+1)^2}{m^2}\frac{16}{Pe^2}\right] + \left[\frac{m+1}{m}Y - \frac{1}{m}Y^{m+1}\right]^2 + \left[m^2\frac{Br}{\Omega}Y^{2m-2}\right] \tag{58}$$

The dimensionless Bejan number (Be) for this example is:

$$Be = \{[16(m+1)^2/Pe^2] + [(m+1)Y - Y^{m+1}]^2\} \times \{[16(m+1)^2/Pe^2] + [(m+1)Y - Y^{m+1}]^2 + [(m^4 Br Y^{2m-2})/\Omega]\}^{-1} \tag{59}$$

In Fig. 9, average entropy generation rate is plotted as a function of fluid index n at different group parameters ranges 0–1.

4. Conclusions

The second law of thermodynamics is applied to forced convection inside channel with circular cross-section and channel made of two parallel plates with finite gap between them. Different examples are considered here for which, at least, simplified or approximate analytical expressions for temperature and velocity distributions are available. Both isoflux and isothermal boundary conditions are considered. Non-Newtonian fluid flow is also considered in two examples. General expression for entropy generation number and Bejan number are derived analytically for each case. Each of the expression has three main parts. The first part is related to axial conduction and which is inversely proportional to Peclet number. The second part is related to heat transfer normal to the axis, which is proportional to the distance normal to the axis. The third part is related to fluid friction, which is proportional to a group parameter defined by the product of Brinkman number and inverse of dimensionless temperature difference.

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