Integral Calculus

$\int_a^b f(x)\,dx$

$f(x)$ → integrand

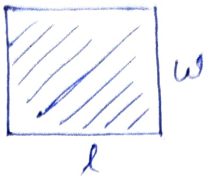
proper integral

i) $f(x)$ is continuous

ii) $f(x) \geq 0 \ \forall\, x \in [a,b]$, $a, b \in \mathbb{R}$, $a < b < \infty$

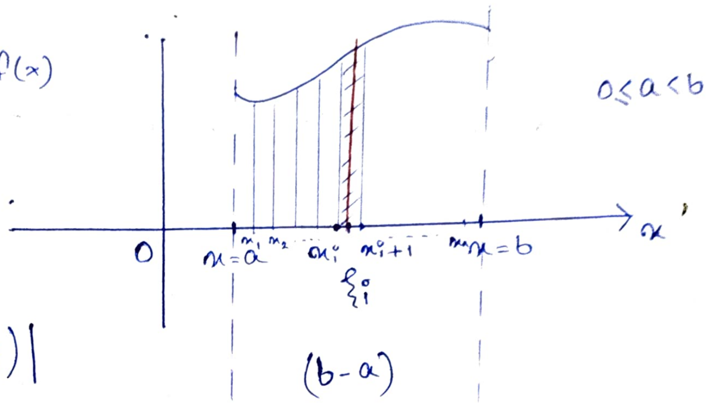
Area = ?

A = ?



area = lw

$\int_a^b f(x)\,dx$ $\quad y = f(x)$



$0 \leq a < b$

$$\sum_{i=0}^{n-1} \left| (x_{i+1} - x_i)\, f(\xi_i) \right|$$

$(b-a)$

$f(\xi_i)$

$f(x) \geq 0$
$x \in [a,b]$

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} (x_{i+1} - x_i)\, f(\xi_i)$$

$$= \int_a^b f(x)\,dx$$

Improper Integral:

depend on limit (a & b) type I

- $b = \infty$
- $a = -\infty$
- $a = -\infty$, $b = \infty$

depend on continuity $f(x)$ type II

- a
- b
- a & b

discontinuous

$\int_0^1 \frac{dx}{1-x}$ $\quad f(x) = \frac{1}{1-x}$ $\quad [0,1]$ is not continuous at $x = 1$

$= \infty$ at $x = 1$

$\int_0^\infty x\,dx = ?$

Type 1

$$\int_a^{\infty} f(x)\,dx = \lim_{B\to\infty}\int_a^{B} f(x)$$

if limit exists and finite

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{A\to-\infty}\int_A^{b} f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx$$

Type - 2

① $\int_a^b f(x)\,dx \Rightarrow f(x)$ is not continuous at $x = b$

$$= \lim_{\epsilon\to 0}\int_a^{b-\epsilon} f(x)\,dx$$

if the limit exists then we can integrate it. and get a finite value.

② $\int_a^b f(x)\,dx$

in upper part we need to substraction. in lower part we need to addition.

$$\int_{a+\delta}^{b} f(x)\,dx$$

③ $\int_a^b f(x)\,dx$

$f(x)$ is not continuous at $c \in (a,b)$

$$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_0^{\infty} \frac{dx}{1+x^2} = \lim_{B\to\infty}\int_0^{B}\frac{dx}{1+x^2}$$

$$= \lim_{B\to\infty}\left[\tan^{-1}x\right]_0^B$$

$$= \lim_{B\to\infty}\left[\tan^{-1}B - \cancel{\tan^{-1}0}^{\,0}\right]$$

$$= \lim_{B \to \infty} \tan^{-1} B$$

$$= \pi/2$$

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{\infty} \frac{dx}{1+x^2}$$

$$= \pi/2 + \pi/2$$

$$= \pi$$

$$\int_0^1 \frac{dx}{1-x} = \lim_{\epsilon \to 0} \int_0^{1-\epsilon} \frac{dx}{1-x}$$

$$= \lim_{\epsilon \to 0} \Big[-\log|(1-x)| \Big]_0^{1-\epsilon}$$

$$= \lim_{\epsilon \to 0} \Big[-\log \epsilon + 0 \Big]$$

$$= \lim_{\epsilon \to 0} \log 1/\epsilon$$

$$= \infty$$