Convection :- { to bring together / to convert into one place

~~Topics~~

Energy $= ML^2T^{-2}$ (it is a derived quantity)

$m_1 v_1 = m_2 v_2$

mass gain $= m = \dfrac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = m_0\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}$

m = moving mass

m_0 = rest "

$= m_0\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2+\frac{1}{2}\left(\frac{1}{2}+1\right)\cancel{\left(\frac{v}{c}\right)^4}\ldots\right]$

$\cancel{2}(m-m_0)c^2 = \frac{1}{2}mv^2$

$\Rightarrow \Delta m c^2 = 1/2\, mv^2$

Eq. energy gain = K.E.

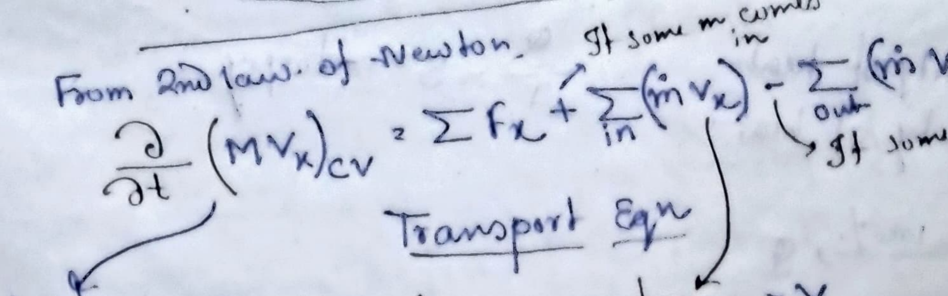
$F \propto \frac{d}{dt}(mv)$

$F = K\frac{d}{dt}(mv) = km\frac{dv}{dt} = kma$

$m=1,\ a=1,\ F=1,\ K=1$

Momentum Theorem:

$\dot{m} = \frac{dm}{dt}$



From 2nd law of Newton

$$\frac{\partial}{\partial t}(MV_x)_{cv} = \sum F_x + \sum_{in}(\dot{m}V_x) - \sum_{out}(\dot{m}V_x)$$

If some m comes in

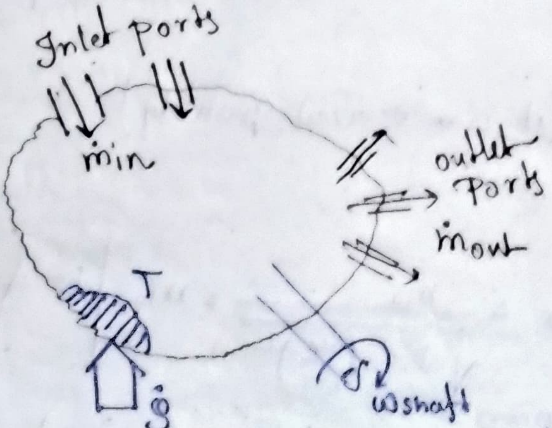
If some m comes out.

Transport Eqn

Rate of change of momentum

$\frac{mV}{m} \rightarrow$ specific momentum $= V$

First Law & Second law of Thermodynamics for open and control system:



$\frac{\dot q}{T}$ (entropy transfer rate)

$$\frac{\partial}{\partial t}(mv_x)_{cv} = \sum_{in}(\dot m v_x) - \sum_{out}(\dot m v_x) + \sum F_x$$

Momentum Conservation.

$$\frac{\partial}{\partial t}(m) = \sum_{in}\dot m - \sum_{out}\dot m \quad \text{(mass conservation)}$$

$$\frac{\partial}{\partial t}(E) = \sum_{in}\dot m e - \sum_{out}\dot m e + \dot q - \dot w_{sh}$$

1st law $\quad h = u + pv$

$e = h + \frac{1}{2}v^2 + gz$

$\underbrace{h + \frac{1}{2}v^2}$ = stagnation enthalpy

(when we heat it will combine with h.)

$h + \frac{1}{2}v^2 + gz$ = methalpy

q, T

q = quantity of heat $\qquad \frac{q}{T} = S$ = entropy

T = quality of heat

$$w \xrightarrow{100\%} q$$

$$q \xrightarrow{<100\%} w$$

$$\frac{\partial E}{\partial t} = \sum_{in}\dot m e - \sum_{out}\dot m e + \dot q - \dot w_{sh}$$

$\frac{\partial S}{\partial t} \geqslant \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \frac{\dot{Q}}{T} - 0$ (100% conservation is possible, so no more can we get)

$\downarrow$ 2nd law

> Irreversible process
= Reversible process.

$$\dot{S}_{gen} = \frac{\partial S}{\partial t} - \frac{\dot{Q}}{T} + \sum_{out} \dot{m}s - \sum_{in} \dot{m}s \to 0$$

Entropy Generation = Degree of Irreversibility of a System.

$\rightarrow$ $E_2 - E_1 = \int_1^2 \delta Q - \int_1^2 \delta W$

$S_2 - S_1 \geqslant \int_1^2 \frac{\delta Q}{T}$

$S_{gen} = (S_2 - S_1) - \int_1^2 \frac{\delta Q}{T} \geqslant 0$

Reynolds Transport Theorem : (RTT)

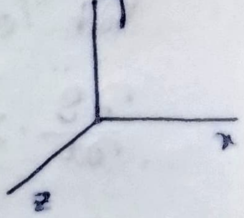
The Problem of fact :-

$\uparrow S = \frac{Q}{T\downarrow} \to$ physical quantity

non

= $\frac{\text{Quantity}}{\text{Quality}}$

$Z = Z(x, y)$



Fluid Flow : P - V - T

Solid mech :- $\sigma - \epsilon - T$ $\quad \sigma = \sigma(\epsilon, T)$

(i) Flow (Action)
(ii) on the verge of flow. (Intention)

$Q \xrightarrow{<100\%} W \to$ Degree of Imperfection

$S_{gen} \to$ degree of Irreversibility

Thermodynamics ~ Heat Transfer

Laws of Thermodynamics

The nature of motive force springer.

(i) Zeroth Law
(ii) First "
(iii) Second "
(iv) Third "
(v) Constructal Law (Adrian Bejan, 1996)
(vi) Law of motive force (2

$$Q \xrightarrow{<100\%} W$$

$$Q_{upgradation} + Q_{degradation} \rightarrow Q_{total}$$

$$W + T\left(\frac{Q}{T}\right) = Q$$

$$W + TS = Q$$

An estimate of lost Available work :- The Gouy-Stodola Thm.

$$\left.\frac{\partial E}{\partial t}\right|_{cv} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz\right) + \dot{Q} - \underbrace{\dot{W}_{sh}}_{\text{shear work}} \quad \text{(i)}$$

$$\frac{\partial S}{\partial t} \geq \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \frac{\dot{Q}}{T_0} \quad \text{(ii)}$$

$$S_{gen} = \frac{\partial S}{\partial t} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s - \frac{\dot{Q}}{T_0} \quad \text{(iii)}$$

Eliminate $\dot{Q}$ both (i) & (ii)

(i) − (ii) × T_0

$$\dot{W}_{sh} \leq \sum_{in} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz - T_0 s\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz - T_0 s\right) - \frac{\partial}{\partial t}\left(E - T_0 S\right) \quad \text{(iv)}$$

$$\left(\dot{W}_{sh}\right)_{max} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz - T_0 S\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}v^2 + gz - T_0 S\right) - \frac{\partial}{\partial t}\left(E - T_0 S\right) \quad \text{(v)}$$

$$\left(\dot{W}_{sh}\right)_{lost} = \underset{\text{(v)}}{\left(\dot{W}_{sh}\right)_{max}} - \underset{\text{(i)}}{\dot{W}_{sh}} \quad \text{(vi)}$$

$$\dot{W}_{lost} = T_0\left(\frac{\partial S}{\partial t} - \frac{\dot{Q}}{T_0} - \sum_{in}\dot{m}s + \sum_{out}\dot{m}s\right) \quad \text{—— (vii)}$$

$$\dot{W}_{lost} = T_0 \cdot \dot{S}_{gen} \quad \text{—— (viii)}$$

$$\boxed{\dot{W}_{lost} \propto \dot{S}_{gen}} \quad \text{—— (ix)}$$

→ Gouy - Stodola Thm.

T/D → $W_{sh}\uparrow$

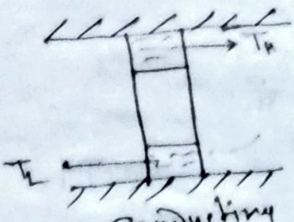
Correction → $W_{sh}\downarrow$

<u>Entropy</u> <u>Generation</u> <u>Minimization</u> :- (EGM)

Finite Time Thermodynamics (FTT)

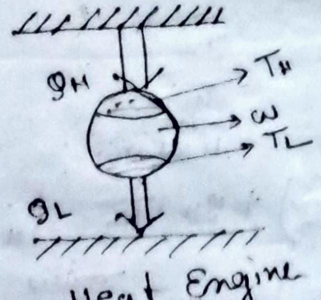
EGM = FTT

<u>On</u> <u>the</u> <u>qualitative</u> <u>Similarities</u> of <u>H.T</u> <u>objectives</u>:-



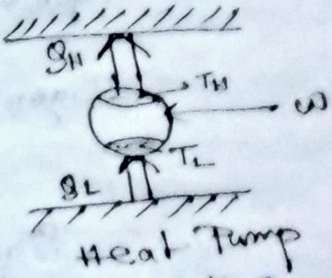
Conducting Layers

Dissipative Structure.



Heat Engine

Dissipative Structure.



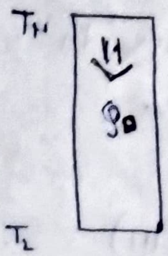
Heat Pump

Dissipative Structure.

$$\Rightarrow \underset{\text{inward}}{\underset{\downarrow}{S}} = \frac{Q \rightarrow \text{Outward}}{T}$$

Entropy Generation Minimization in a General Heat transfer

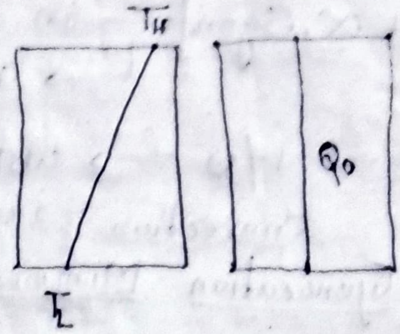
objective :-

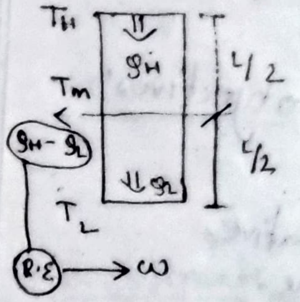


A, L, K

(T_H, T_L)

$T_H > T_L$





$$q_H - q_L = \omega$$

$$q_L \to q_H \Rightarrow \omega \to 0$$

$$q_0 = \frac{KA}{L}(T_H - T_L) \quad \text{---(i)}$$

$$S_{gen,0} = \underbrace{\frac{q_0}{T_L}}_{\text{Entropy out}} - \underbrace{\frac{q_0}{T_H}}_{\text{Entropy in}}$$

$$S_{gen,0} = q_0\left(\frac{1}{T_L} - \frac{1}{T_H}\right) \quad \text{---(ii)}$$

$$S_{gen} = \underbrace{\frac{q_L}{T_L} + \frac{q_H - q_L}{T_m}}_{\text{Heat out}} - \underbrace{\frac{q_H}{T_H}}_{\text{Heat in.}}$$

$$S_{gen} = q_L\left(\frac{1}{T_L} - \frac{1}{T_m}\right) + q_H\left(\frac{1}{T_m} - \frac{1}{T_H}\right) \quad \text{---(iii)}$$

$$q_H = \frac{KA}{L/2}(T_H - T_m) \quad \text{---(iv)}$$

$$q_L = \frac{KA}{L/2}(T_m - T_L) \quad \text{---(v)}$$

Now, from (iii)

$$\therefore S_{gen} = \frac{KA}{L/2}(T_m - T_L)\left(\frac{1}{T_L} - \frac{1}{T_m}\right) + \frac{KA}{L/2}(T_H - T_m)\left(\frac{1}{T_m} - \frac{1}{T_H}\right)$$

$$\therefore S_{gen} = S_{gen}(T_m)$$

for $\min^m S_{gen}$,

$$\frac{d(S_{gen})}{dT_m} = 0$$

We know, that, for $\min^m S_{gen}$, $\max^m$ work

$$T_{m,opt} = \sqrt{T_H T_L} \quad \text{--- (vi)} \rightarrow \text{optimum midpoint temp.}$$

Put Eq^n (vi) in III

$$S_{gen,min} = 4\left(\sqrt[4]{\frac{T_H}{T_L}} - \sqrt[4]{\frac{T_L}{T_H}}\right)^2 \frac{KA}{L} \quad \text{(vii)}$$

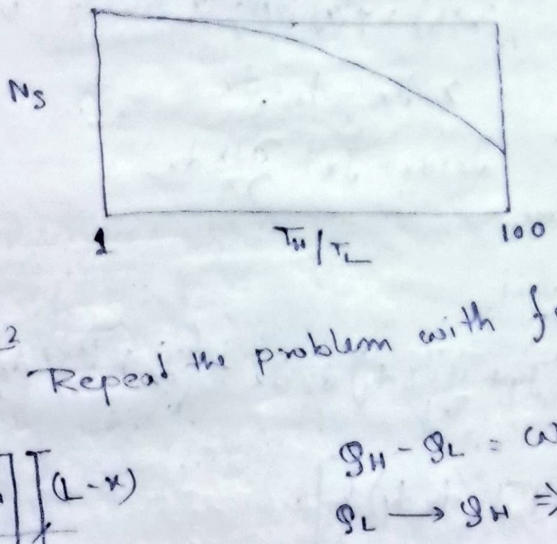
(vii) $\div$ (ii)

(Entropy Generation number)

$$N_S = \frac{S_{gen,min}}{S_{gen,0}} < 1 \left[\frac{\text{Entropy generation in Engg. syst}}{\text{E.G in pre Engineered system}}\right] \quad \text{--- (viii)}$$

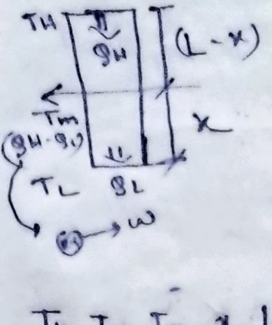
$$N_S = \frac{4\sqrt{\frac{T_H}{T_L}}}{\left(\sqrt{\frac{T_H}{T_L}}+1\right)^2} \;(\leq 1) \quad \text{--- (ix)}$$

H-T-1



(MATLAB)
After Mid Term.
Printed with Name &
Roll. No.

H.T.2
Repeat the problem with following →



$$q_H - q_L = w$$

$$q_L \rightarrow q_H \Rightarrow w \rightarrow 0$$

$$q_0 = \frac{KA}{L}(T_H - T_L)$$

$$S_{gen,0} = \frac{q_0}{T_L} - \frac{q_0}{T_H}$$

$S_{gen,0}$

T_H, T_L, T_m, x, L

D.O.F → reduction of D.O.F

Non dimensional prob →

$\tau_H = \frac{T_H}{T_L}$, $\tau_m = \frac{T_m}{T_L}$, $\xi = \frac{x}{L}$

$T_H = \tau_H \cdot T_L$ $\qquad x = \xi L$

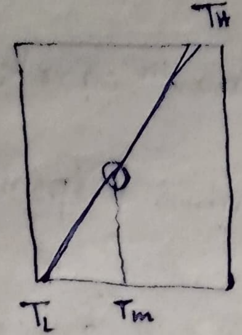
$$\frac{\partial}{\partial x}(S_{gen}) = 0, \quad \frac{\partial}{\partial T_m}(S_{gen}) = 0$$

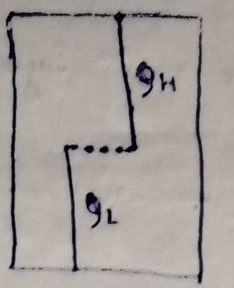
$$\Downarrow$$

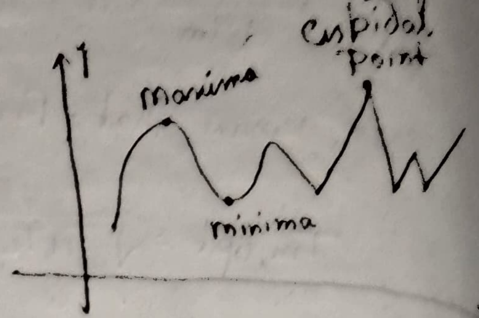
$$\frac{\partial}{\partial \xi}(S_{gen}) = 0 \quad \frac{\partial}{\partial \tau_m}(S_{gen}) = 0$$

Analytically $T_H >> T_L$

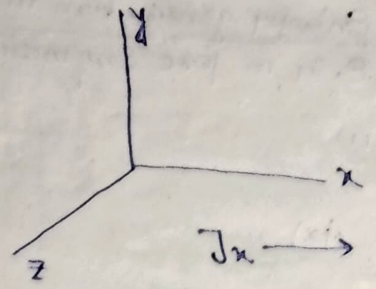
Show Newton Raphson Code with MATLAB.







J (Heat flux) $= J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$



(orthogonal)

$J_x \Rightarrow$ influx.
Efflux

Net efflux
= Efflux − influx

Taylor's Series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \dots$$

$$x = J_x,\ h = dx$$

$$J_{x+dx} = J_x + \frac{\partial J_x}{\partial x}dx$$

Net efflux per unit volume =

$$= (\text{Efflux} - \text{Influx}) / \text{volume}$$

$$= \frac{\left[\left\{\left(J_x + \frac{\partial J_x}{\partial x}dx\right)dy\,dz + \left(J_y + \frac{\partial J_y}{\partial y}dy\right)dz\,dx + \left(J_z + \frac{\partial J_z}{\partial z}dz\right)dx\,dy - \left(J_x\,dy\,dz + J_y\,dx\,dz + J_z\,dx\,dy\right)\right\}\right]}{dx\,dy\,dz}$$

$$= \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}$$

$$= \left[\hat{i}\frac{\partial}{\partial x}(\ ) + \hat{j}\frac{\partial}{\partial y}(\ ) + \hat{k}\frac{\partial}{\partial z}(\ )\right] \cdot \left[\hat{i}J_x + \hat{j}J_y + \hat{k}J_z\right]$$

$$= \nabla \cdot \hat{J} = \underbrace{\text{div}\,\hat{J}}_{\text{Faster motion}}$$

$q = -kA \frac{dT}{dx} \rightarrow$ in x dirn

$q = -kA \, \text{grad}\, T \rightarrow$ in 3D $\longrightarrow$ Slower motion

$\text{grad}\, \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$

The generalised Differential eqn for a conservation principle

Change = Transfer ... (i)

Change > Transfer ... (ii)

From (i) & (ii)

Change ≥ Transfer ... (iii)

Change = Transfer + ∈ ... (iv)

∈ ≥ 0

→ In nature, SLOWER motion originates first.

Rate + Faster motion (convective) = Slower motion (Diffusion) + Source

ϕ = Flux = Intensive

ρ = local density

$\rho\phi$ = extensive = Total flux for CV

Rate = R = $\frac{\partial}{\partial t}(\rho\phi)$

Convection = div $(\rho \phi \hat{u})$ → velocity of fluid → faster

D = Diffusion = div $(\Gamma_\phi \, \text{grad}\, \phi)$ → slower

coefficient of Diffusion

Source = S_ϕ

R = S

C = D

R + C = D + S

$$\boxed{\frac{\partial}{\partial t}(\rho\phi) + \text{div}(\rho\phi\hat{u}) = \text{div}(\Gamma_\phi \, \text{grad}\, \phi) + S_\phi} \quad \text{CFD}$$

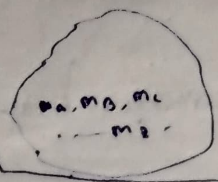
2nd order, Unsteady eqn — Convection — Conduction — Radiation

Conservation of ϕ eqn

Ex → 1

Conservation of a chemical species :-

Mass fraction, $m_L = \dfrac{m_L}{m_A + m_B + \cdots m_Z}$



Step I :- $\phi \equiv m_L$

J_L = chemical flux = $-\Gamma_L \, \text{grad} \, m_L$ $\quad \therefore J_L \propto \text{grad} \, m_L$

$= |\Gamma_L| \, |\text{grad} \, m_L|$

Fick's Law of Diffusion.

$S_\phi = R_L \rightarrow$ No. of new species (created, destroyed, nor create nor destroy).

$A + B = C + D, \quad S_\phi = R_L = 0$

$A + B = C + D + E, \quad S_\phi = R_L = 1$

$A + B = + C \quad ; \quad S_\phi = R_L = -1$

$$\frac{\partial}{\partial t}(\rho m_L) + \text{div}(\rho m_L \hat{u}) = \text{div}(\Gamma_L \, \text{grad} \, m_L) \mp R_L$$

Ex - 2

Conservation of mass :

$m_L = +1$

$R_L = 0$

$$\therefore \frac{\partial}{\partial t}(\rho) + \text{div}(\rho u) = 0 \quad (\text{Continuity Eq}^n)$$

Procedure :

(i) Meaning of ϕ

(ii) Law of Diffusion.

(iii) Nature of source term.

Eq (iii)

Conservation of Momentum :→ (Navier - Stokes Eqn).-

$$R + C = D + S$$

$$\frac{\partial}{\partial t}(\rho\varphi) + \mathrm{div}(\rho\varphi\hat{u}) = \mathrm{div}(\Gamma_\varphi\,\mathrm{grad}\,\varphi) + S_\varphi \quad \text{--(i)}$$

$$\frac{\text{Momentum}}{\text{mass}} = \frac{mv}{m} = v \quad (\text{Intensive Property})$$

(I) $\frac{mu}{m} = \varphi = u$

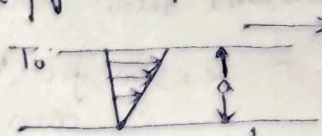
(II) Law of momentum Diffusion

$$J_u = -\mu\,\mathrm{grad}\,u = \mu\,|\mathrm{grad}\,u|$$

Newton's Law of viscosity

$$S_u = -\frac{\partial p}{\partial x} + B_x + V_x$$

Body force, surface forces



(Couette flow)
(Shear driven flow)

[when the fluid velocity is very high, then every thing can't be accomodate in diffusion term, so, one extra term will be happend, it is called surface forces.]

Hagen Pouiselliez.
(Pr. driven flow)

From (i)

$$\frac{\partial}{\partial t}(\rho u) + \mathrm{div}(\rho u\hat{u}) = \mathrm{div}(\mu\,\mathrm{grad}\,u) - \frac{\partial p}{\partial x} + B_x + V_x \rightarrow \text{Navier Stokes Eqn.}$$

(iv)

Conservation of Energy :→

$$\frac{\partial}{\partial t}(\rho\varphi) + \mathrm{div}(\rho\varphi\hat{u}) = \mathrm{div}(\Gamma_\varphi\,\mathrm{grad}\,\varphi) + S_\varphi$$

(I) $\varphi = h = u + pv$ → flow energy.
internal energy

$h = C_p T$

$h \propto T$

(II) $J_h = -K\,\mathrm{grad}\,T = K\,|\mathrm{grad}\,T|$

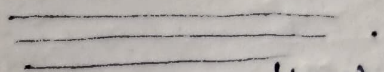
Fourier's law of conduction

(iii) $S_\phi = S_h$ it can 0 when it is no convection. then it is totally conductive term.

$$\frac{\partial}{\partial t}(\rho h) + \text{div}(\rho h \hat{u}) = \text{div}(k \text{ grad } T) + S_h$$

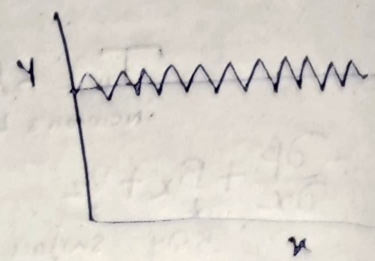
Ex→5 Turbulence :→

(Turbulent flow)



(Laminar flow)

we assume that, here,



(I) $\phi = k$ = specific K.E in conserved

$$= \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$$

Given that,

Conservation of turbulent kinetic energy

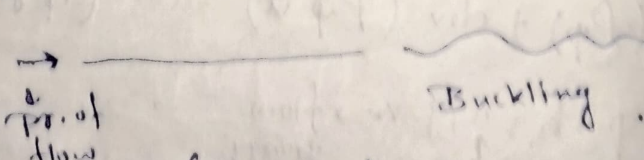
then,

(i) $\phi = k$

(ii) $J_k = -\Gamma_k \text{ grad } k$

(iii) $S_k = G - \rho\epsilon$, kinematic rate of dissipitiation of turbulent K.E — Dynamic

Source — rate of generation of turbulence K.E



$$\frac{\partial}{\partial t}(\rho\phi) + \text{div}(\rho\phi\hat{u}) = \text{div}(\Gamma_\phi \text{ grad } \phi) + S_\phi$$

$$\boxed{\frac{\partial}{\partial t}(\rho k) + \text{div}(\rho k \hat{u}) = \text{div}(\Gamma_k \text{ grad } k) + G - \rho\epsilon}$$

[K-E model of turbulence]

(f) Elemental volume