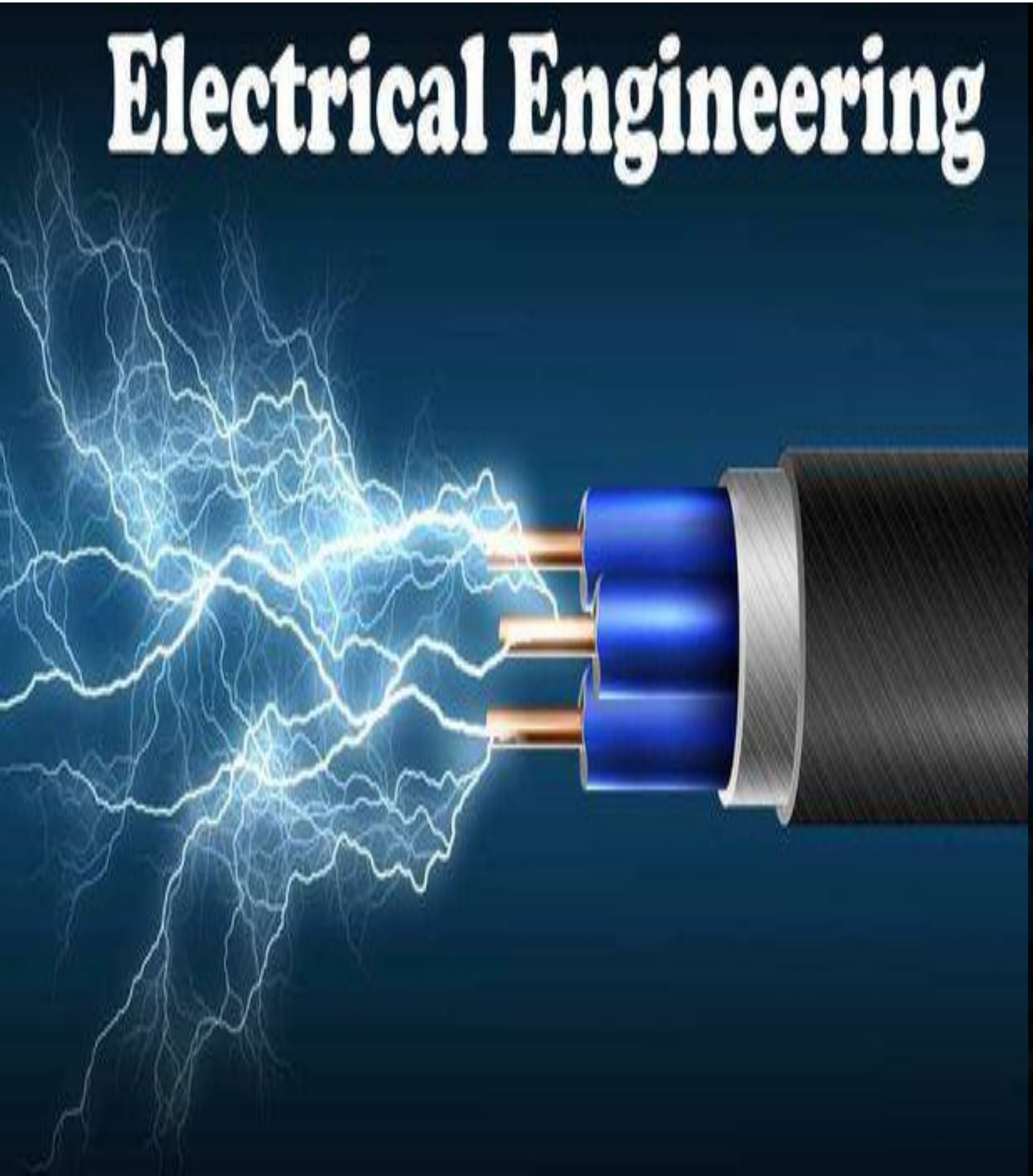
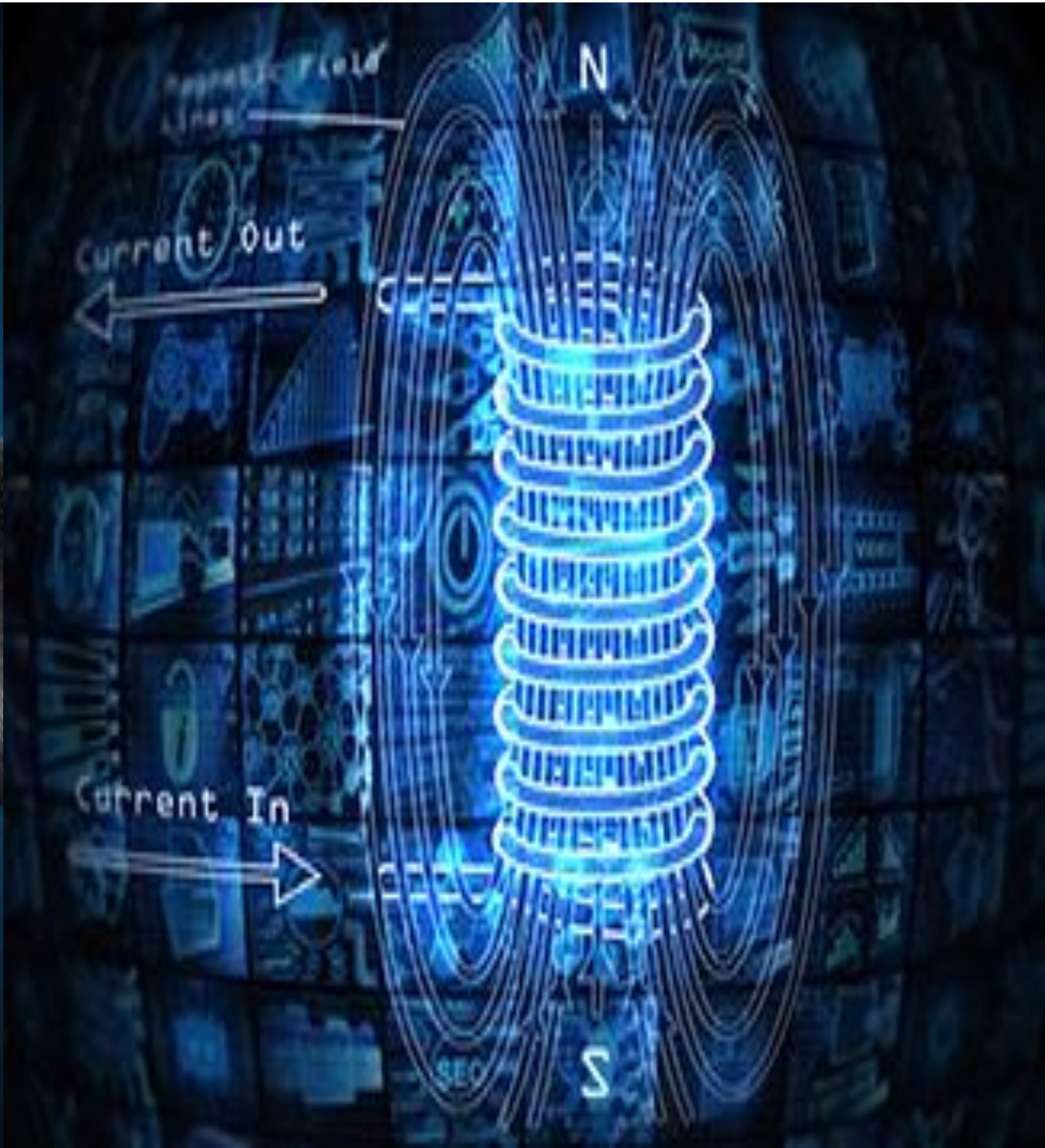
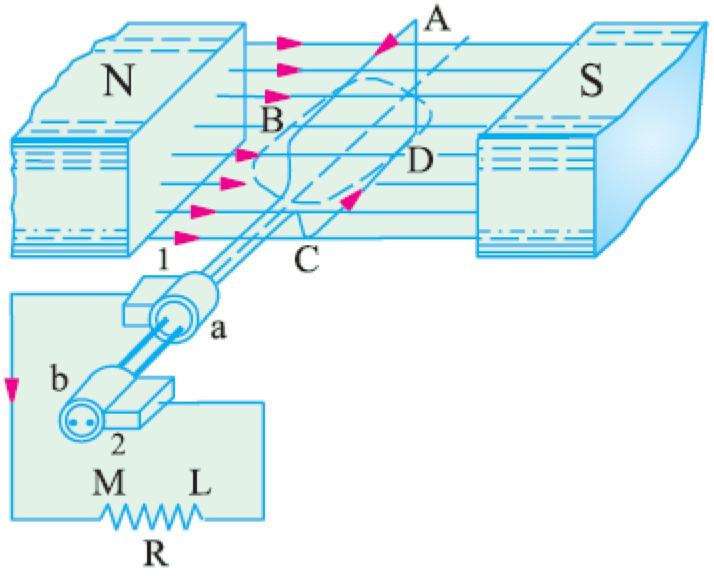

Electrical Engineering

N
Current Out
Current In
S

LECTURE 12

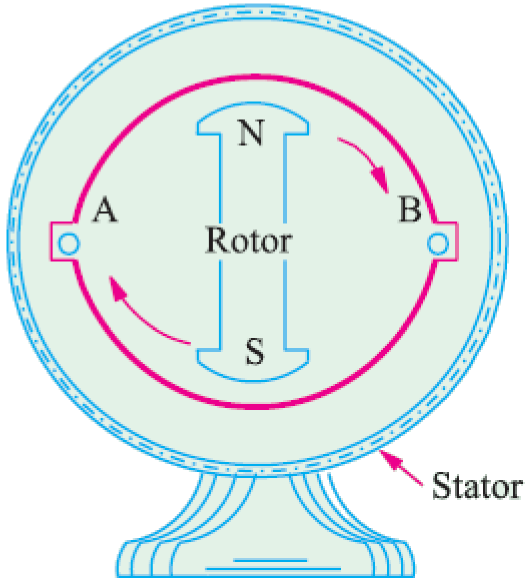
A.C. Fundamentals

Generation of Alternating Voltages and Currents

- Alternating voltage is generated by rotating a coil in a magnetic field, as shown in **Fig. 6.1** (*a*) or by rotating a magnetic field within a stationary coil, as shown in **Fig. 6.1** (*b*).



(a)



(b)

Fig. 6.1

- The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. The rotating-field method is mostly used in practice.

A.C. Fundamentals

➢ Equations of the Alternating Voltages and Currents

- Consider a rectangular coil as shown in **Fig. 6.2** have N turns and rotates with an angular velocity of ω radian/second in a uniform magnetic field.
- Let time be measured from the X-axis. Maximum flux Φ_m is linked with the coil, when its plane coincides with the X-axis.
- In time t seconds, this coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is $\Phi = \Phi_m \cos \omega t$.
- Hence, flux linkages of the coil at any time are $N\,\Phi = N\,\Phi_m \cos \omega\, t$.
- According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil.
- Hence, the value of the induced e.m.f. at this instant (*i.e.* when $\theta = \omega t$) is

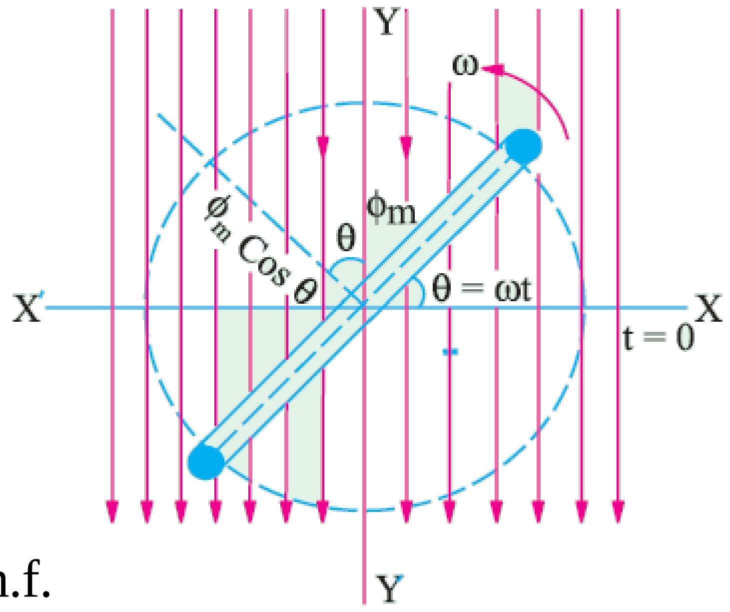


Fig. 6.2

$$e = -\frac{d}{dt}(N\,\Phi)\text{ volt} = -N.\frac{d}{dt}(\Phi_m \cos \omega t)\text{ volt} = -N\Phi_m\,\omega\,(-\sin \omega t)\text{ volt}$$

$$= \omega N\,\Phi_m \sin \omega t \text{ volt} = \omega N\,\Phi_m \sin \theta \text{ volt}$$

A.C. Fundamentals

➢ Equations of the Alternating Voltages and Currents

- When the coil has turned through 90º *i.e.* when θ = 90º, then sin θ = 1, hence e has maximum value, say E_m. Therefore, we get

$$E_m = \omega N \Phi_m = \omega N B_m A = 2\pi f N B_m A \text{ volt}$$

where B = maximum flux density in Wb/m^2 ; A = area of the coil in m^2

f = frequency of rotation of the coil in rev/second

Substituting this value of E_m , we get $e = E_m \sin\theta = E_m \sin\omega t$

- Similarly, the equation of induced alternating current is $i = I_m \sin\omega t$, provided the coil circuit has been closed through a resistive load.

- Since $\omega = 2\pi f$, where f is the frequency of rotation of the coil, the equations of the voltage and current can be written as

$$e = E_m \sin 2\pi f t = E_m \sin\left(\frac{2\pi}{T}\right)t \text{ and } i = I_m \sin 2\pi f t = I_m \sin\left(\frac{2\pi}{T}\right)t$$

where T = time-period of the alternating voltage or current = $1/f$

A.C. Fundamentals

➢ Equations of the Alternating Voltages and Currents

- It is seen that the induced e.m.f. varies as sine function of the time angle ωt. When e.m.f. is plotted against time, a curve known as sinusoidal e.m.f as shown in **Fig. 6.3** is obtained.

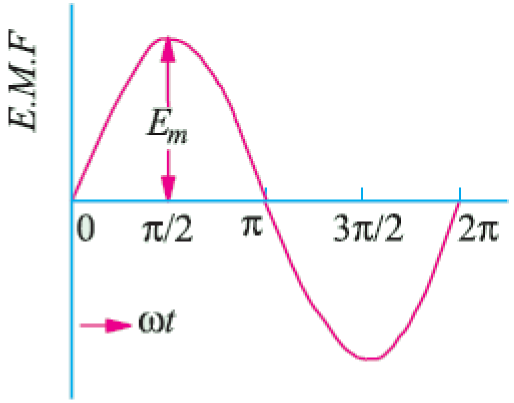


Fig. 6.3

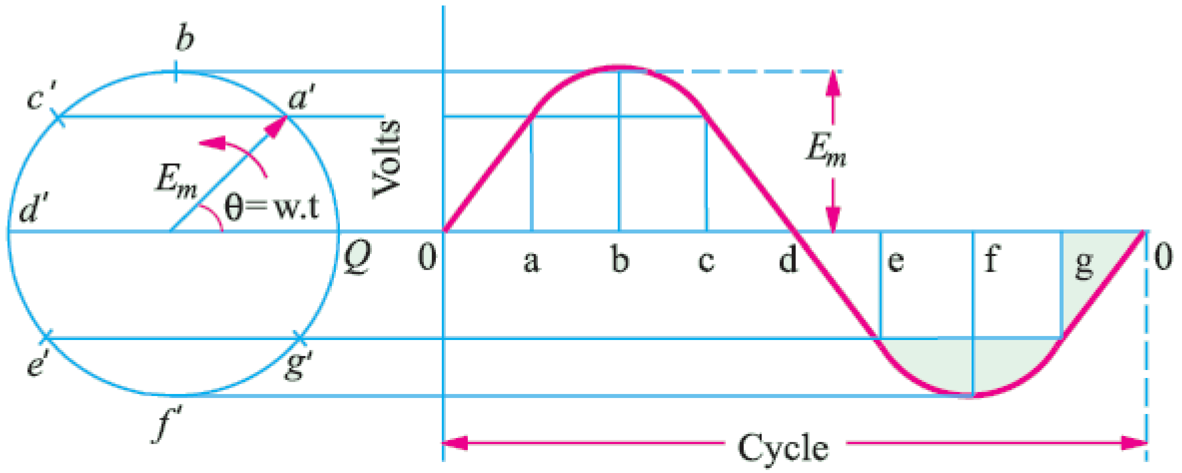


Fig. 6.4

- The sine curve as shown in **Fig. 6.4** is conveniently drawn.
- A vector of length, E_m is drawn. It rotates in the counter-clockwise direction with a velocity of ω radian/second. It makes two loops or one cycle of the generated e.m.f. after one revolution.
- The projection of this vector on *Y*-axis gives the instantaneous value *e* of the induced e.m.f. *i.e.* $E_m \sin \omega t$.

A.C. Fundamentals

- **Equations of the Alternating Voltages and Currents**

  - To construct the curve, lay off along *X*-axis equal angular distance *oa*, *ab*, *bc*, *cd* etc. corresponding to suitable angular displacement of the rotating vector.
  - Now, erect coordinates at the points *a*, *b*, *c* and *d* etc. as shown in **Fig. 6.4** and then project the free ends of the vector E_m at the corresponding positions a', b', c', etc to meet these ordinates.
  - Next draw a curve passing through these intersecting points.

A.C. Fundamentals

- **Simple Waveforms**
  - The shape of the curves are obtained by plotting the instantaneous values of voltage or current as the ordinate against time as a abscissa is called its waveform or wave-shape.
  - Sine wave is the ideal form and is accepted as standard wave to the designer. The waves deviating from the sine wave are termed as distorted waves
  - **Fig. 6.5** shows different alternating waves. An alternating current or voltage is one for which the circuit direction of which reverses at regularly recurring intervals.

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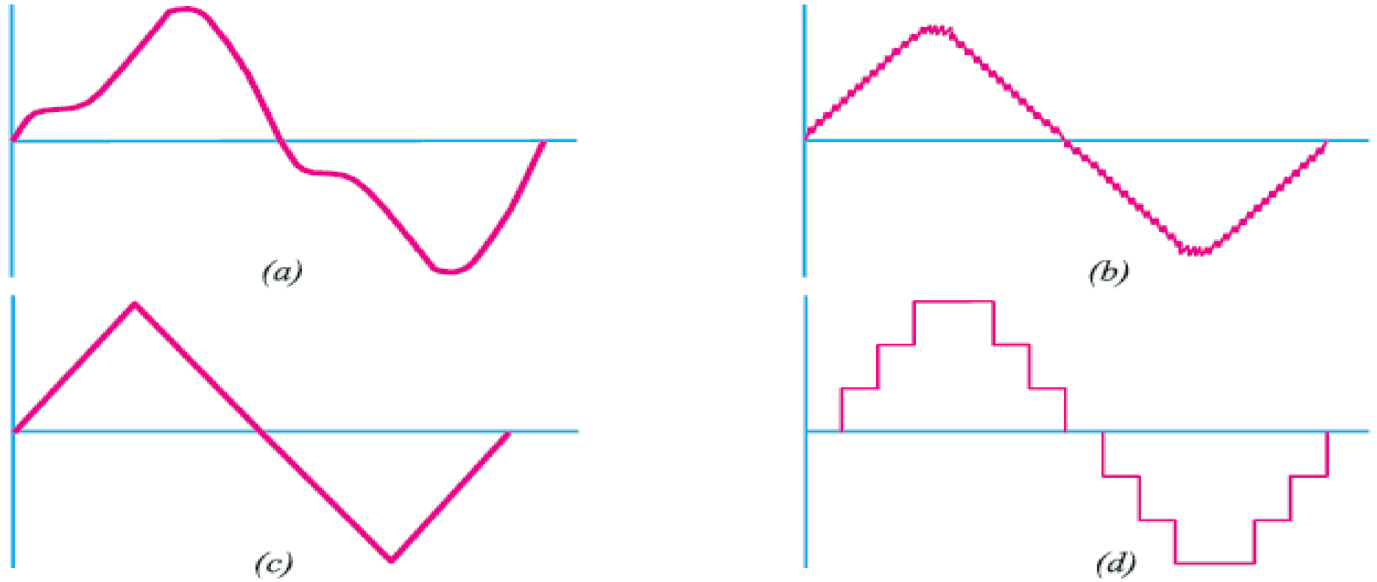


Fig. 6.5

A.C. Fundamentals

- **Complex Waveforms**

  - Complex waves are those which depart from the ideal sinusoidal form.
  - The complex wave having equal positive and negative half cycles can be made up of a number of sine waves with different frequencies.
  - The wave of the lowest frequency is called the fundamental (or first harmonic) wave.
  - The waves of higher frequencies than the lowest one are called harmonics.

    If the fundamental frequency is 50 Hz, then the frequency of the second harmonic is 100 Hz and of the third is 150 Hz and so on.
  - The complex wave may be composed of the fundamental wave and any number of other harmonics.

A.C. Fundamentals

➢ Complex Waveforms

- A complex wave as shown in **Fig. 6.6***(a)* is made up of a combination of fundamental sine wave of frequency 50 Hz and third harmonic of frequency 50 Hz.
- A complex wave as shown in **Fig. 6.6** *(b)* is made up of a combination of fundamental sine wave of frequency 50 Hz, 2nd harmonic of frequency 100 Hz and 3rd harmonic of frequency 150 Hz.

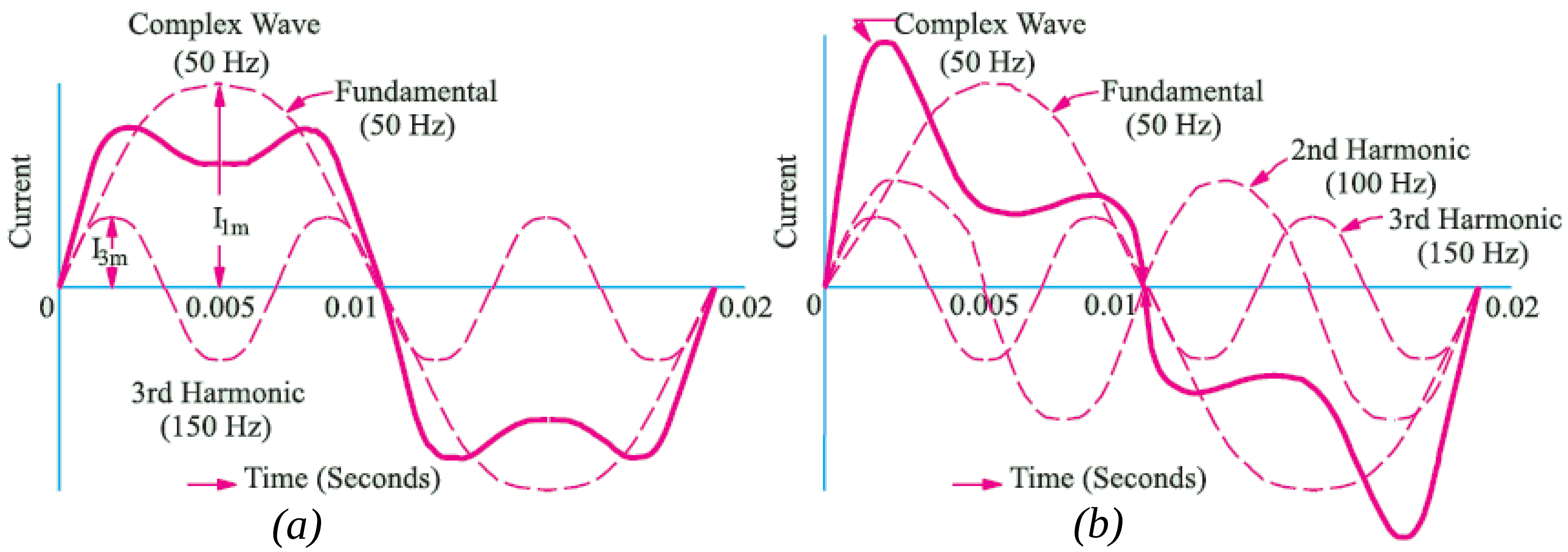
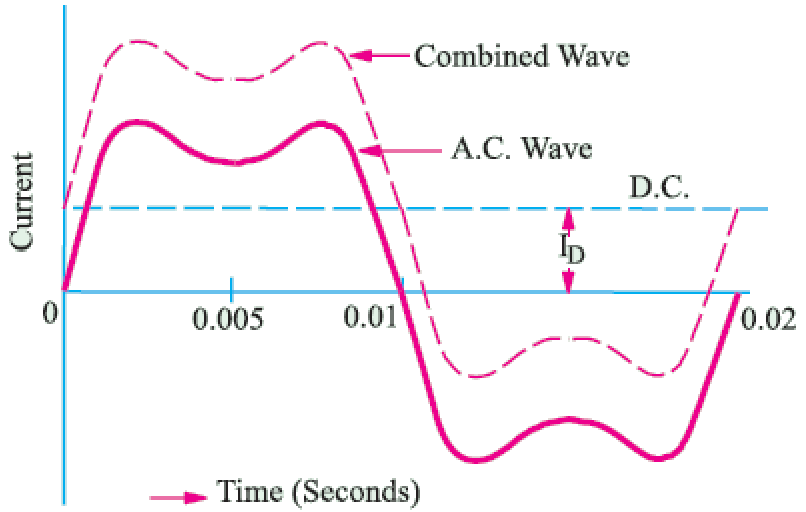


Fig. 6.6

A.C. Fundamentals

- **Complex Waveforms**

A complex wave as shown in **Fig. 6.6(c)** is made up of fundamental, third harmonic and direct current of value $I_{D.}$



(c)

Fig. 6.6

A.C. Fundamentals

- **Definitions**

Cycle: One complete set of positive and negative values of alternating quantity is known as cycle.

A cycle is specified in terms of angular measure. One complete cycle is said to spread over 360º or 2π radians.

Time Period: The time taken by an alternating quantity to complete one cycle is called time period *T*.

For example, a 50-Hz alternating current has a time period of 1/50 second.

Frequency: The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

$$f = PN/120$$

where N = revolutions in r.p.m. and P = number of poles

It may be noted that the frequency is given by the reciprocal of the time period of the alternating quantity.

$$\therefore f = \frac{1}{T}$$

Amplitude: The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

A.C. Fundamentals

- **Definitions**

Phase: The phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference.

In **Fig. 6.7,** the phase of current at point *A* is *T*/4 second, where *T* is time period. It is also expressed in terms of angle, i.e $\pi/2$ radian.

Similarly, the phase of the rotating coil at the instant shown in **Fig. 6**.7 is ωt, called its phase angle.

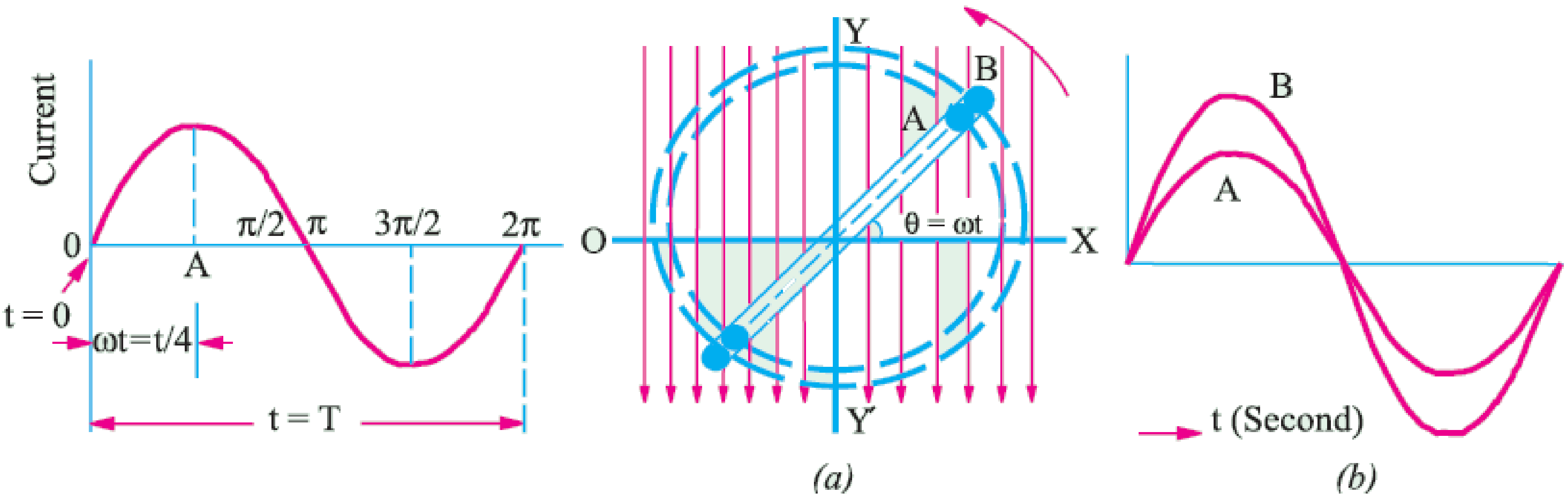
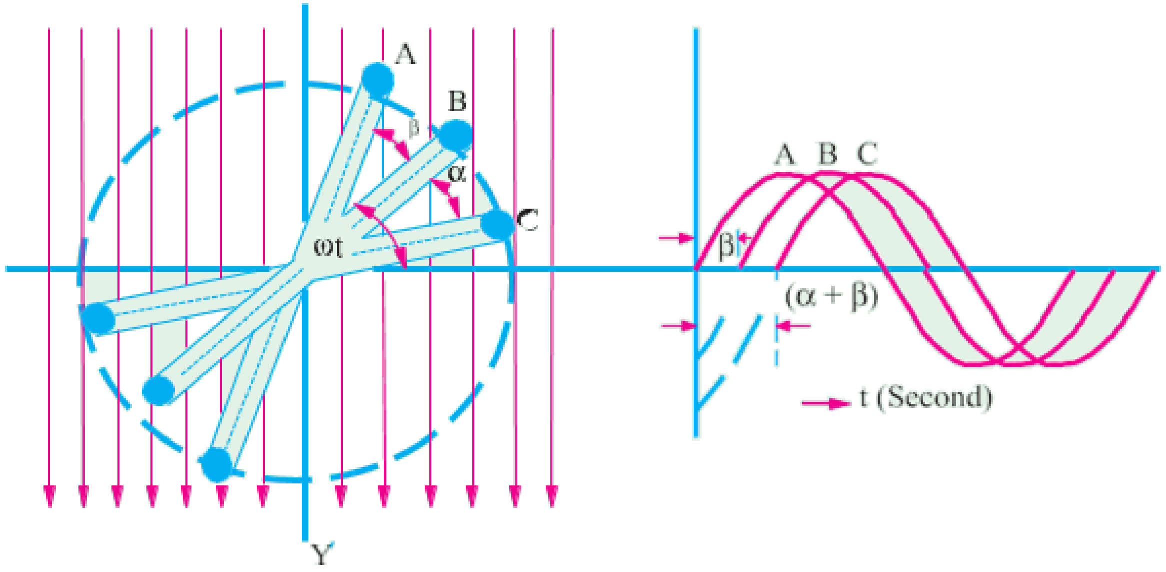


Fig. 6.7

A.C. Fundamentals

- **Definitions**

Phase Difference: There are three similar single-turn coils displaced from each other by angles α and β and rotating in a uniform magnetic field with the same angular velocity as shown in **Fig. 6.8 (a)**



(*a*) (b)

Fig. 6.8

The three sinusoidal waves are shown in **Fig. 6.8** (*b*). It is seen from figure that curves *B* and *C* are displaced from curve *A by* angles β and (α + β) respectively.

It means that phase difference between *A* and *B* is β and between *B* and *C* is α but between *A* and *C* is (α + β).

A.C. Fundamentals

- **Definitions**

Phase Difference:

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity.

For example in **Fig. 6.8** (*b*), *B* lags behind *A* by β and *C* lags behind *A* by (α + β) because they reach their maximum values later.

The three equations for the instantaneous induced e.m.fs. are (Fig. 11.14)

$$e_A = E_m \sin \omega\, t \quad \text{...reference quantity}$$

$$e_B = E_m \sin (\omega t - \beta)$$

$$e_C = E_m \sin [\omega t - (\alpha + \beta)]$$

A.C. Fundamentals

➢ Root-Mean-Square (R.M.S.) Value

- The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.
- A simple experimental arrangement for measuring the equivalent d.c. value of a sinusoidal current is shown in **Fig. 6.9**.
- The two circuits have identical resistances but one is connected to battery and the other to a sinusoidal generator.

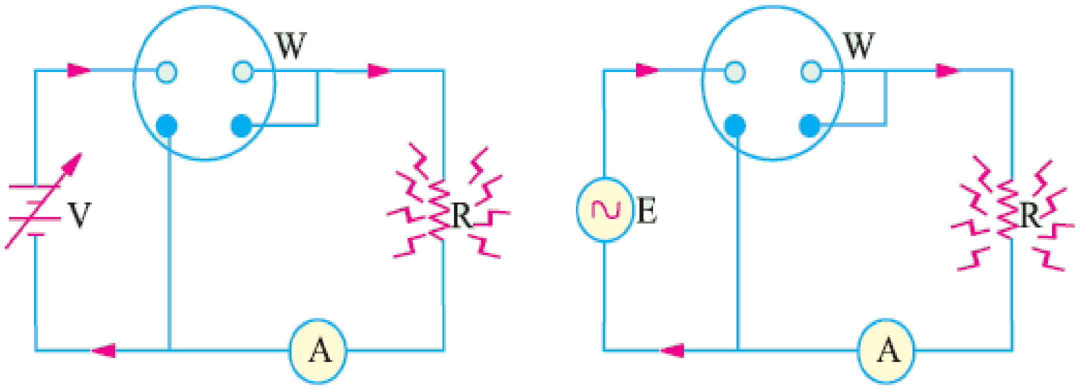


Fig. 6.9

- The wattmeter is used to measure heat power in each circuit.
- The voltage applied to each circuit is so adjusted that heat power production in each circuit is the same.
- The direct current is known as the r.m.s. value of the sinusoidal current.

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**
  - The r.m.s. current is also known as the effective or virtual value of the alternating current.
  - The r.m.s. value of alternating current is computed by the following methods

    (i) mid-ordinate method and

    (ii) analytical method.

Root-Mean-Square (R.M.S.) Value

Mid-ordinate Method

- The positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents are shown in **Fig. 6.10**.

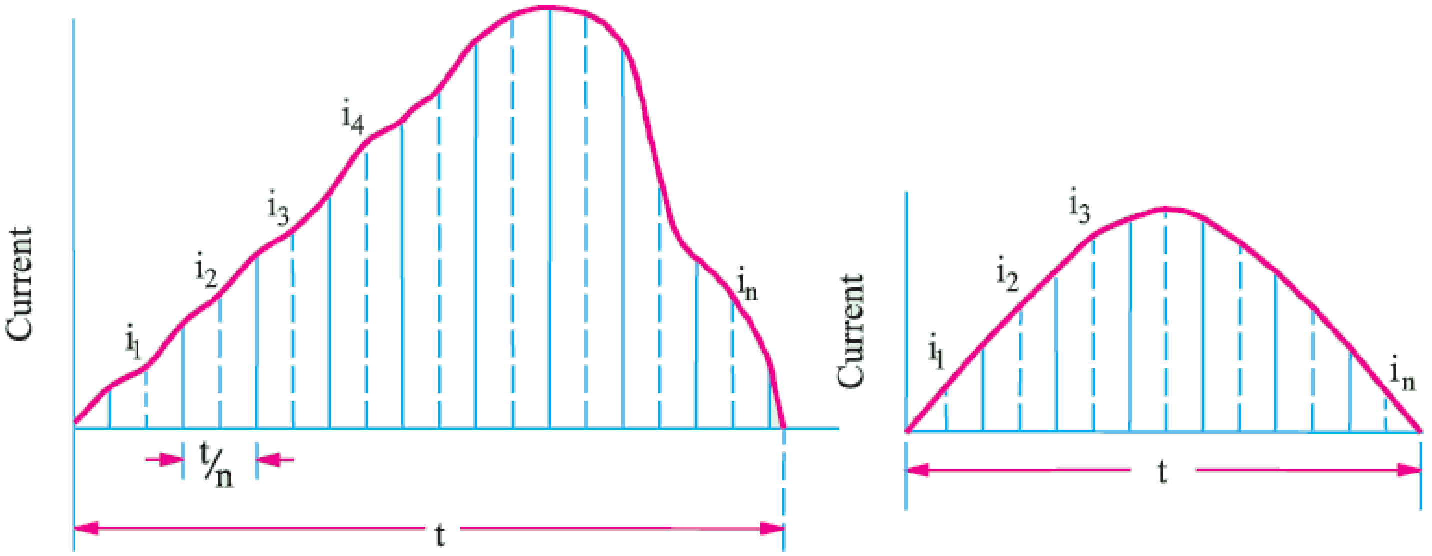


Fig. 6.10

- Divide time base '*t*' into *n* equal intervals of time each of duration *t*/*n* seconds.
- Let the average values of instantaneous currents during these intervals be i_1, i_2, i_3 .... i_n respectively.

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**
  - **Mid-ordinate Method**
    - The alternating current is passed through a circuit of resistance R ohms. Then,

$$\text{Heat produced in 1st interval} = 0.24 \times 10^{-3}\, i_1^2\, Rt/n \text{ kcal} \qquad (\because\ 1/J = 1/4200 = 0.24 \times 10^{-3})$$

$$\text{Heat produced in 2nd interval} = 0.24 \times 10^{-3}\, i_2^2\, Rt/n \text{ kcal}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\text{Heat produced in } n\text{th interval} = 0.24 \times 10^{-3}\, i_n^2\, Rt/n \text{ kcal}$$

$$\text{Total heat produced in } t \text{ seconds is} = 0.24 \times 10^{-3}\, Rt \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right) \text{kcal}$$

- Now, suppose that a direct current of value I produces the same heat through the same resistance during the same time t.
- Heat produced by it is $0.24 \times 10^{-3} I^2 Rt$ kcal. By definition, the two amounts of heat produced should be equal

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**
  - **Mid-ordinate Method**

$$\therefore \quad 0.24 \times 10^{-3} I^2 Rt = 0.24 \times 10^{-3} Rt \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$$

$$\therefore \quad I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \quad \therefore \; I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)}$$

= square root of the mean of the squares of the instantaneous currents

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n} \right)}$$

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**

  - **Analytical Method**

    - The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.
    - The mean of the squares of the instantaneous values of current over one complete cycle is $\int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$
    - The square root of this value is $\sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$
    - Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta\right)} \qquad (\text{put } i = I_m \sin \theta)$$

$$\text{Now, } \cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \; \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**
  - **Analytical Method**

$$\therefore \qquad I = \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\, d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore\ \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

- Hence, for a symmetrical sinusoidal current, r.m.s. value of current = 0.707 × max. value of current
- The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively.
- Average heating effect produced during one cycle is $I^2R = (I_m/\sqrt{2})^2\, R = \frac{1}{2}\, I_m^2\, R$

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**

  - ✓ **R.M.S. Value of a Complex Wave**

    - The R.M.S. value of a complex wave is computed by either the mid-ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known).

    - A current having the equation $i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$ flows through a resistor of R ohm.

    - Then, in the time period T second of the wave, the effect due to each component is as follows :

Fundamental .... $(12/\sqrt{2})^2 \ RT$ watt

3rd harmonic .... $(6/\sqrt{2})^2 \ RT$ watt

5th harmonic .... $(4/\sqrt{2})^2 \ RT$ watt

$\therefore$ Total heating effect $= RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**

  - **R.M.S. Value of a Complex Wave**

    - If I is the r.m.s. value of the complex wave, then equivalent heating effect is I^2RT

$$\therefore \qquad I^2RT = RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$$

$$\therefore \qquad I = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$$

    - There is a direct current of 5 amperes flowing in the circuit, then the r.m.s. value of current is

$$I = \sqrt{(12/\sqrt{2})^2 \quad (6/\sqrt{2})^2 \quad (4/\sqrt{2})^2 \quad 5^2} = 10.93 \text{ A}$$

    - Hence, the r.m.s. value of a complex current wave is equal to the square root of the sum of the squares of the r.m.s. values of its individual components.

A.C. Fundamentals

- **Average Value**

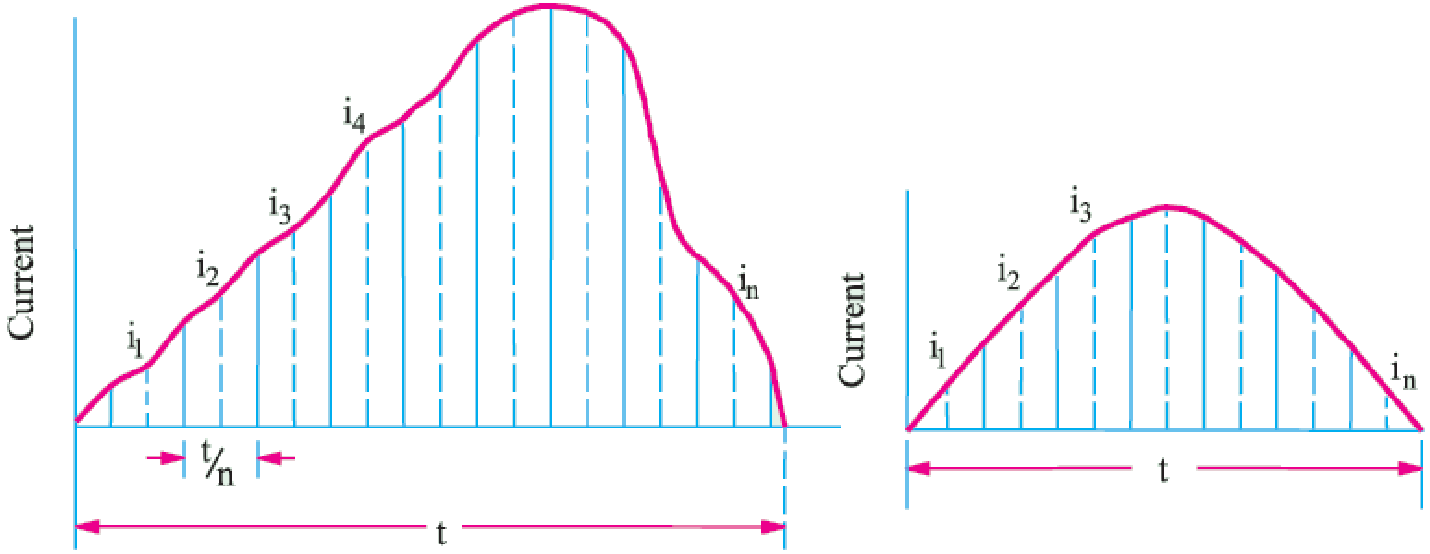
  - The average value I_a of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.
  - The average value of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), over a complete cycle is zero.
  - Hence, the average value of a symmetrical alternating current is obtained by adding or integrating the instantaneous values of current over one half-cycle only.
  - The average value of an unsymmetrical alternating current (i.e. one whose two half-cycles are not similar, like half-wave rectified current) is obtained by adding or integrating the instantaneous values of current over the whole cycle.
  - The average value of alternating current is also computed by the following methods

    (i) mid-ordinate method and

    (ii) analytical method.

A.C. Fundamentals

- **Average Value**
  - **Mid-ordinate Method**
    - The positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents are shown in **Fig. 6.11**.
    - Divide time base '*t*' into *n* equal intervals of time each of duration t/n seconds.
    - Let the average values of instantaneous currents during these intervals be i_1, i_2, i_3 .... i_n respectively.



The average value of the alternating current is

$$I_{av} = \frac{I_1 + I_2 + I_3 \ldots + I_n}{n}$$

Fig. 6.11

A.C. Fundamentals

- **Average Value**

  - **Analytical Method**

    The standard equation of an alternating current is $I = I_m \sin\theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}\left| +1-(-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2}$$

$$= \frac{\text{twice the maximum current}}{\pi}$$

$$\therefore \qquad I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m$$

$\therefore$ average value of current = 0.637 × maximum value

LECTURE 13

A.C. Fundamentals

- **Form Factor**
  - It is defined as the ratio $K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.1$ (for sinusoidal alternating currents only)
  - In the case of sinusoidal alternating voltage also, $K_f = \frac{0.707\, E_m}{0.637\, E_m} = 1.11$
  - The knowledge of form factor is used to find the r.m.s. value from the arithmetic mean value and vice-versa.
- **Crest or Peak or Amplitude Factor**
  - It is defined as the ratio $K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a. c. only)
  - For sinusoidal alternating voltage alsc $K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$
  - This factor is used to find dielectric stress of insulation because it is proportional to the peak value of the applied voltage. The factor is also necessary to measure iron losses, because the iron loss depends on the value of maximum flux.

Root-Mean-Square (R.M.S.) Value

R.M.S. Value of Half-wave (H.W.) Rectified Alternating Current

- Half-wave (H.W.) rectified alternating current is one whose one half-cycle has been suppressed.
- It is shown in **Fig. 6.12** where suppressed half-cycle is shown dotted.

$\therefore$ R.M.S. current of H.W. rectified alternating current

$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\times\pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)}$$

$$\therefore\ I = \frac{I_m}{2} = 0.5 I_m$$

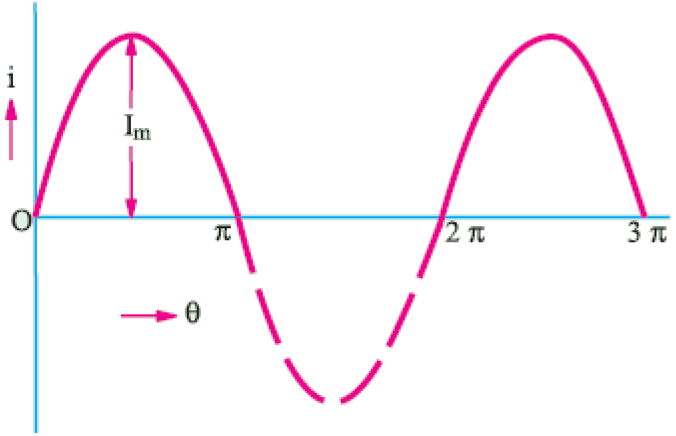


Fig. 6.12

A.C. Fundamentals

- **Root-Mean-Square (R.M.S.) Value**

  - ✓ **Average Value of H.W. Rectified Alternating Current**

    - The integration is carried over from 0 to π for getting average current.

$$\therefore \quad I_{av} = \int_0^{\pi} \frac{i\,d\theta}{2\pi} = \frac{I_m}{2\pi}\int_0^{\pi} \sin\theta\, d\theta \qquad (\because \quad i = I_m \sin\theta)$$

$$= \frac{I_m}{2\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{2\pi}\times 2 = \frac{I_m}{\pi}$$

- **Form Factor of H.W. Rectified Alternating Current**

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

Example – P6.1

Determine the r.m.s. value of the current waveform as shown in Fig.P6.1.

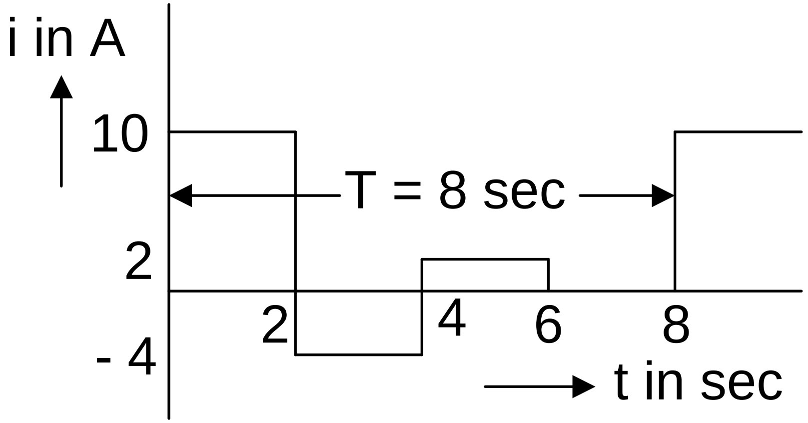


Fig. P6.1

Solution of Example – P6.1

Let the r.m.s. value of current is I

$$\therefore \quad I^2 \times R \times 8 = 10^2 \times R \times 2 \; + \; (-4)^2 \times R \times 2 \; + \; 2^2 \times R \times 2 \; + \; 0$$

or, $8I^2 = 200 + 32 + 8$

$$\therefore \quad I = \sqrt{\frac{240}{8}} = 5.48\ A$$

Example – P6.2

What is the significance of the r.m.s and average values of a wave ? Determine the r.m.s. and average value of the waveform shown in **Fig. P6.2**

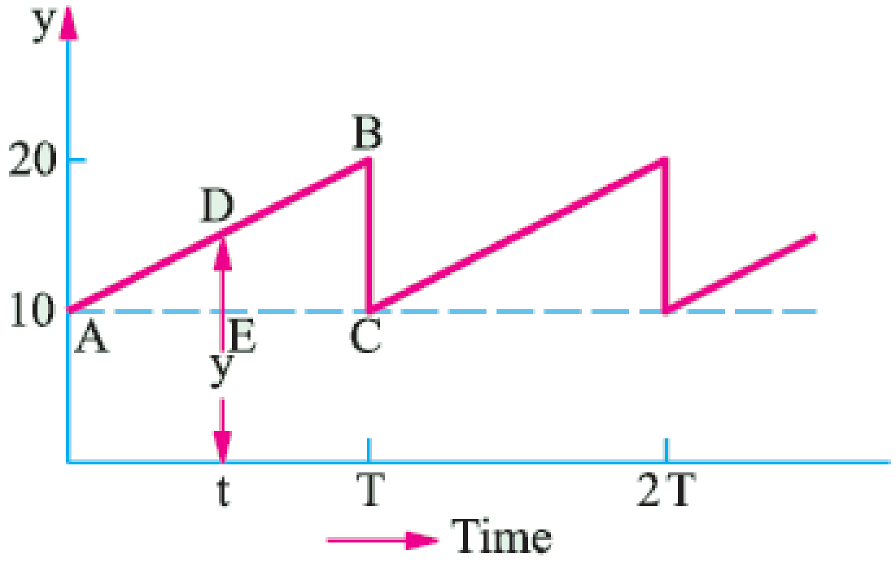


Fig. P6.2

Solution of Example – P6.2

The slope of the curve *AB* is *BC*/*AC* = 20/*T*.

Consider the function *y* at any time *t*.

So, $DE/AE = BC/AC = 10/T.$

or $(y - 10)/t = 10/T$

or $y = 10 + (10/T)t$

This is the equation of the function for one cycle.

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right)dt$$

$$= \frac{1}{T}\int_0^T \left[10.dt + \frac{10}{T}.t.\,dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = 15$$

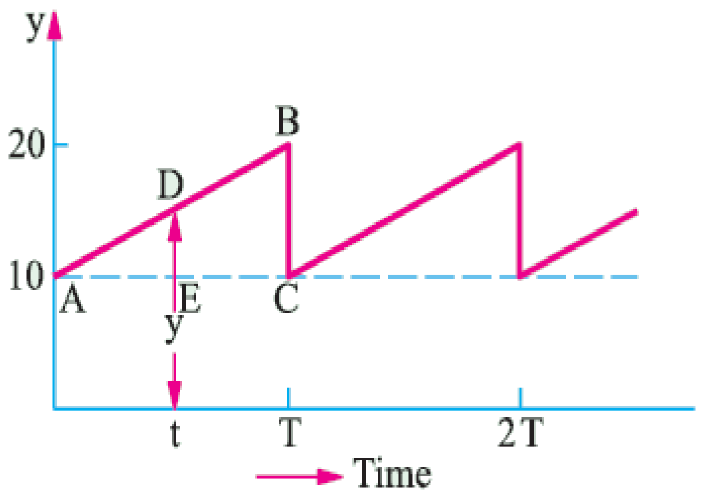


Fig. P6.2

A.C. Fundamentals

Solution of Example – P6.2

Mean square value $= \dfrac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \dfrac{10}{T}t\right)^2 dt$

$$= \frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt = \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

∴ RMS value = 10 7/3 = 15.2

A.C. Fundamentals

- **Representation of Alternating Quantities**

  - The a.c. computations based on the assumption of sinusoidal voltages are cumbersome because instantaneous values of alternating voltage, $e = E_m \sin \omega t$ are to be handled continuously.

  - A conventional vector method is used to employ to represent the sine waves for simplifying the problems in a.c. work.

    - ✓ A vector is a physical quantity which has magnitude as well as direction. Such vector quantities are completely known when the magnitude, direction and the sense in which they act are given.

    - ✓ The vector is graphically represented by straight line.

      The length of the line represents the magnitude of the alternating quantity,

      The inclination of the line with respect to some axis of reference gives the direction of that quantity

      An arrow-head placed at one end indicates the direction in which that quantity acts.

A.C. Fundamentals

Representation of Alternating Quantities

- The alternating voltages and currents are represented by the vectors which are rotating counter-clockwise with the same frequency as that of the alternating quantity as shown in **Fig. 6.13**.

- In **Fig. 6.12**(*a*), *OP* is a vector which represents the maximum value of the alternating current. It's angle with *X* axis gives its phase.

- Let the alternating current be represented by the equation $e = E_m \sin \omega t$.

- The projection of *OP* on *Y*-axis at any instant gives the instantaneous value of that alternating current.

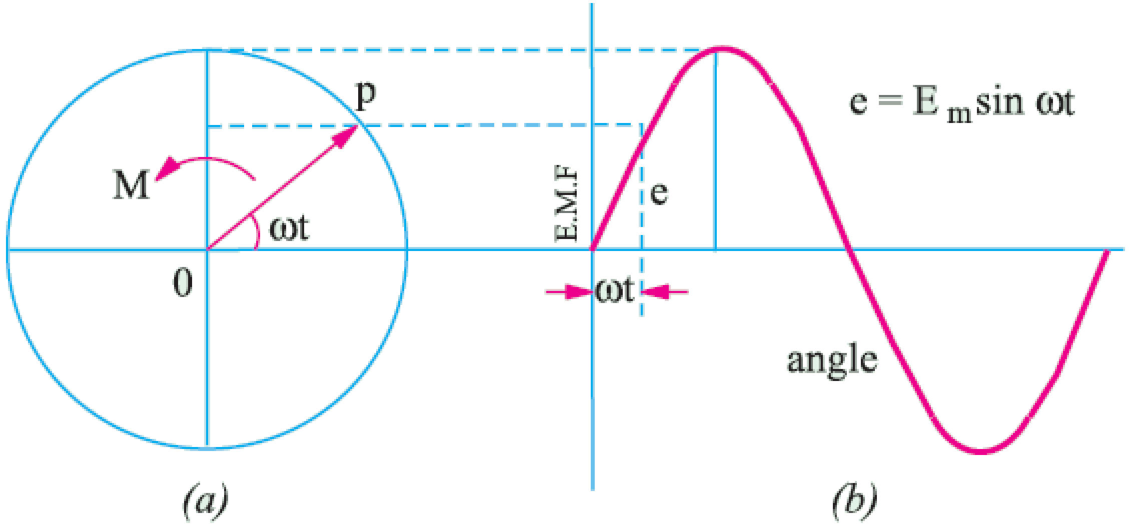


Fig. 6.13

∴ The instantaneous current, $OM = OP \sin \omega t$

or, $e = OP \sin \omega t$

$= E_m \sin \omega t$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$e^{-j\omega t} = \cos \omega t - j \sin \omega t$$

$$Ve^{j\omega t} = V\cos \omega t + j\, V\sin \omega t$$

$$V\sin \omega t = Imag\left(Ve^{j\omega t}\right)$$

A.C. Fundamentals

- **Representation of Alternating Quantities**

  - ✓ The line *OP* is used to represent an alternating voltage if it satisfies the following conditions :

    - The length of the line is equal to the peak value of the sinusoidal alternating voltage to a suitable scale.
    - The line is in the horizontal position at the same instant as the alternating quantity is zero and increasing.
    - The angular velocity of *OP* is to be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

- **Vector Diagram using R.M.S. Valuess**

  It is a very common practice to draw vector diagrams using r.m.s. values of alternating quantities but the projection of the rotating vector on the *Y*-axis does not give the instantaneous value of the alternating quantity.

A.C. Fundamentals

- **Representation of Alternating Quantities**
  - **Vector Diagrams of Sine Waves of Same Frequency**
    - The sinusoidal voltage e and current i of the same frequency are rotating in counter-clock-wise direction are shown in **Fig. 6.14.**
    - The current wave is passed upward through zero at the instant $t = 0$.
    - At the same time, the voltage wave is already advanced an angle α from its zero value.
    - Hence, their equations is written as

$$i = I_m \sin \omega t$$
$$e = E_m \sin (\omega t + \alpha)$$

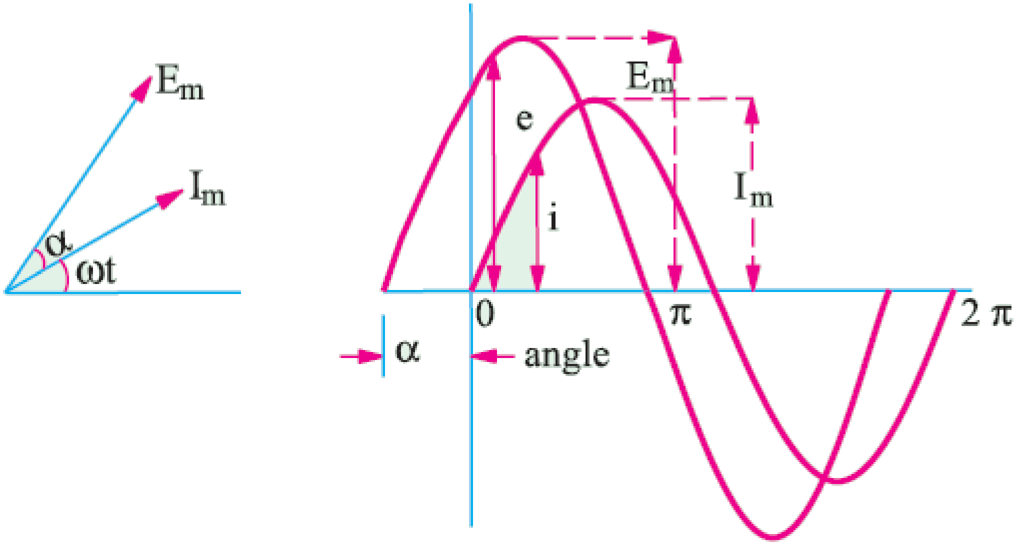


Fig. 6.14

- Sine wave of different frequencies cannot be represented on the same vector diagram.
- The phase angles between them will be continuously changing due to difference in speed of different vectors.

A.C. Fundamentals

➢ A.C. Through Resistance, Inductance and Capacitance

The equation and the phase of the alternating current produced in the circuit containing either resistance or inductance or capacitance are to be find out when alternating voltage, $e = E_m \sin \omega t$ is applied.

✓ A.C. Through Pure Ohmic Resistance

Let the applied voltage, $v = V_m \sin \theta = V_m \sin \omega t$ is applied to the resistive circuit as shown in **Fig. 6.15.**

Where, R = ohmic resistance and i = instantaneous current

So, the applied voltage has to supply ohmic voltage drop only.

Hence, for equilibrium $v = iR$

Putting the value of 'v', we get

$$V_m \sin \omega t = iR \qquad \therefore \; i = \frac{V_m}{R} \sin \omega t$$

Current 'i' is maximum when $\sin \omega t$ is unity $\therefore I_m = V_m/R$

So, $i = I_m \sin \omega t$

Hence, the vectors V_R and I are in phase as shown in **Fig. 6.14**.

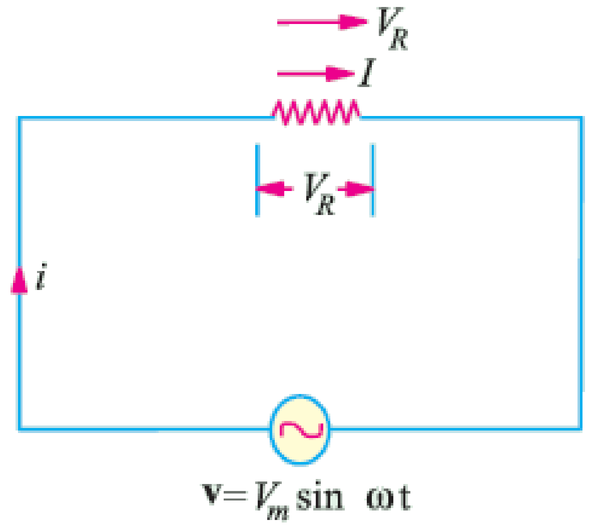


Fig. 6.15

A.C. Fundamentals

✓ A.C. Through Pure Ohmic Resistance

The alternating voltage and current in the resistive circuit are in phase with each other as shown in **Fig. 6.16**.

❑ Power.

Instantaneous power,

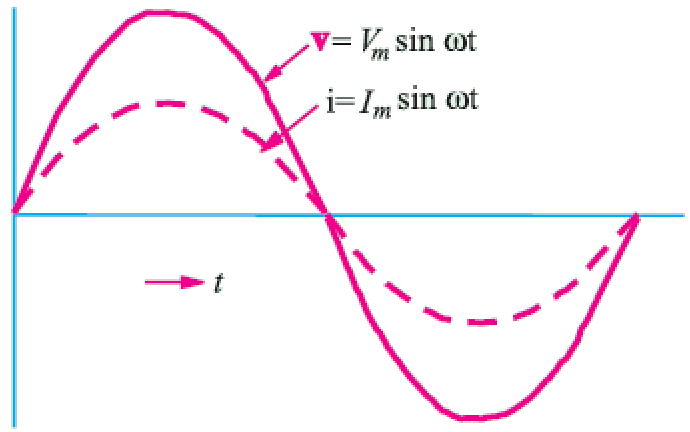


Fig. 6.16

$$p = vi = V_m I_m \sin^2 \omega t$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\,\omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\,\omega t$ of frequency double that of voltage and current waves.

For a complete cycle, the average value of $\frac{V_m I_m}{2}\cos 2\,\omega t$ is zero.

✓ A.C. Through Pure Ohmic Resistance

❑ Power.

Hence, power for the whole cycle is $P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

or $P = V \times I$ watt

where V = r.m.s. value of applied voltage.
I = r.m.s. value of the current.

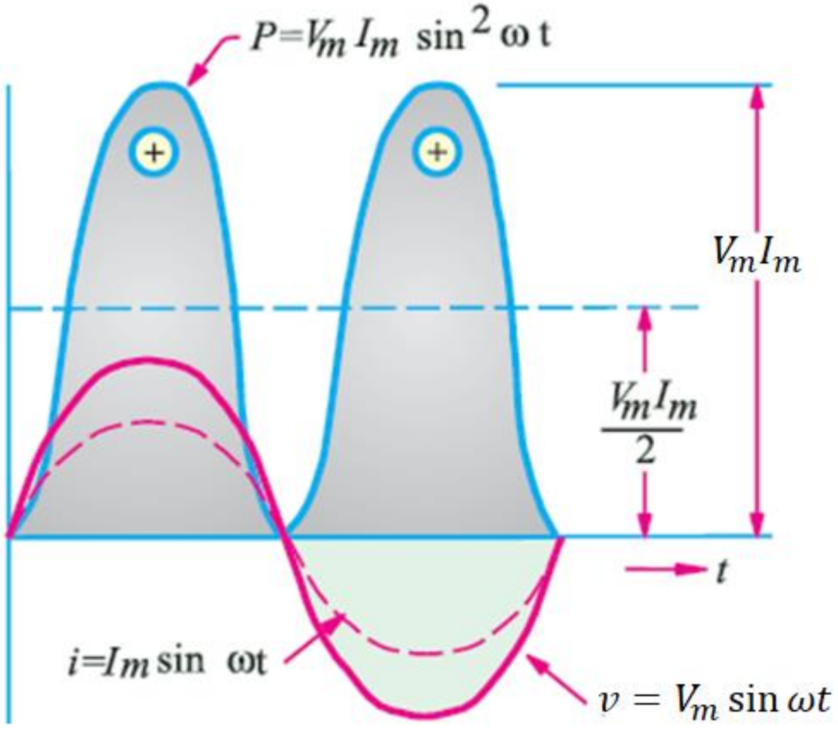


Fig. 6.17

It is seen from **Fig. 6.17** that no part of the power cycle becomes negative at any time.

So, power is never zero in a purely resistive circuit because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

A.C. Fundamentals

✓ A.C. Through Pure Inductance

Let the applied voltage, $v = V_m \sin\theta = V_m \sin\omega t$ is applied to the inductive circuit as shown in **Fig. 6.18.**

When an alternating voltage is applied to a purely inductive coil, a back e.m.f. is produced due to the self-inductance of the coil.

The back e.m.f., at every step, opposes the rise or fall of current through the coil.

So, the applied voltage is to overcome this self-induced e.m.f. only as there is no ohmic voltage drop.

At every step

$$v = L\frac{di}{dt}$$

Now $$v = V_m \sin\omega t$$

$$\therefore \quad V_m \sin\omega t = L\frac{di}{dt} \quad \therefore \; di = \frac{V_m}{L}\sin\omega t \, dt$$

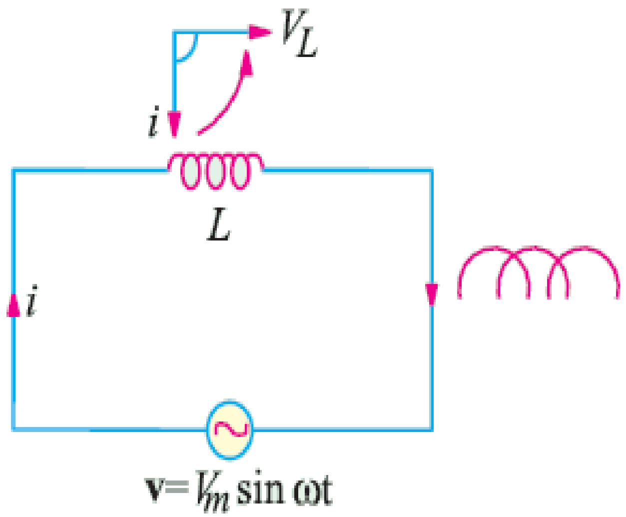


Fig. 6.18

A.C. Fundamentals

- ✓ **A.C. Through Pure Inductance**

Integrating both sides, we get $$\int di = \frac{V_m}{L}\int \sin\omega t\, dt$$

$$\therefore \qquad i = -\frac{V_m}{\omega L}\cos\omega t \qquad \text{(constant of integration = 0)}$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Maximum value of i is $I_m = V_m/X_L$ when $\sin(\omega t - \pi/2)$ is unity

Hence, the equation of the current becomes $i = I_m\sin(\omega t - \pi/2)$

A.C. Fundamentals

- ✓ **A.C. Through Pure Inductance**

The applied voltage is $v = V_m \sin \omega t$

The current flowing in a purely inductive circuit is $i = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) = I_m \sin(\omega t - \pi/2)$

The current lags behind the applied voltage by a quarter cycle or the phase difference between the voltage and current is $\pi/2$ with voltage leading as shown in **Fig. 6.19**.

The vectors V_L and I are graphically depicted in **Fig. 6.18.** It is seen from the figure that the voltage V_L leads the current I by 90^0.

It is seen that $I_m = V_m/\omega L = V_m/X_L$;

Here 'ωL' plays the part of 'resistance'. It is called the inductive reactance X_L of the coil and is given in ohms.

$$\therefore\ X_L = \omega\, L = 2\pi\, f\, L \text{ ohm.}$$

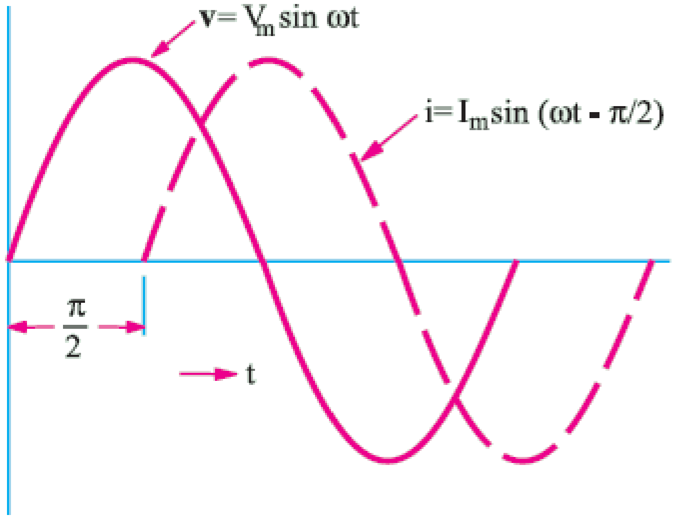


Fig. 6.19.

A.C. Fundamentals

✓ **A.C. Through Pure Inductance**

❑ **Power.**

Instantaneous power $= p = vi = V_m I_m \sin \omega t . \sin(\omega t - \pi/2)$

$$= -V_m I_m \sin \omega t . \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin(2\omega t)$$

Power for whole cycle is $P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin(2\omega t)\, dt = 0$

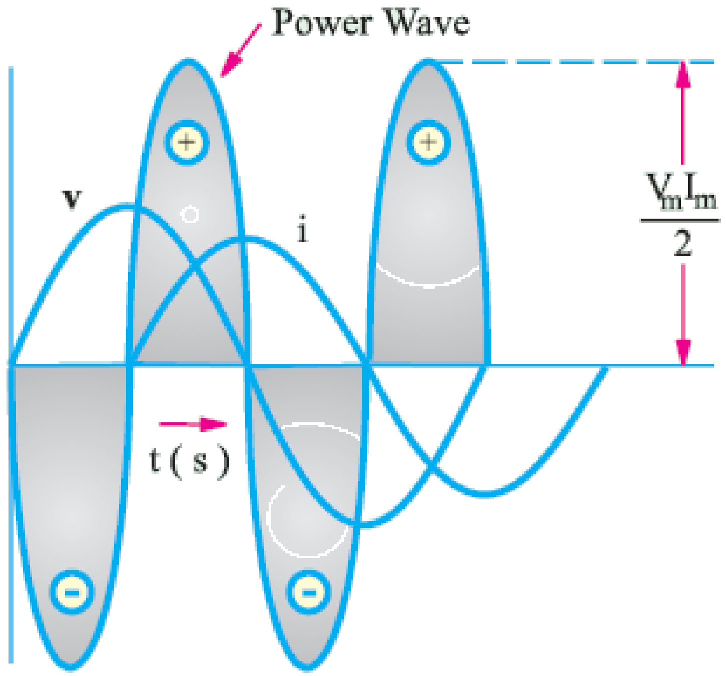


Fig. 6.20.

The power wave is shown in **Fig. 6.20.**

It is also clear from the figure that the average demand of power from the supply for a complete cycle is zero.

It is also seen that power wave is a sine wave of frequency double that of the voltage and current waves.

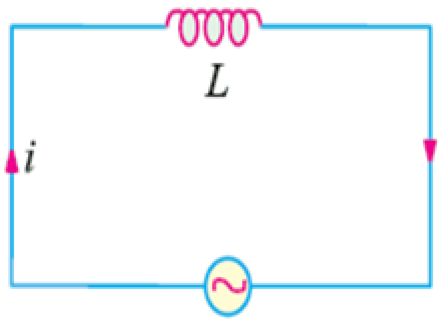
The maximum value of the instantaneous power is $V_m I_m/2$.

A.C. Fundamentals

- **Complex Voltage Applied to Pure Inductance**

The complex voltage in the form of $v = V_{1m} \sin \omega t + V_{3m} \sin 3\omega t + V_{5m} \sin 5\omega t$ is applied to the circuit as shown in **Fig. 6.21.**

The reactances offered for the fundamental voltage wave, $X_1 = \omega L$. for 3rd harmonic, $X_3 = 3\omega L$ and for 5th harmonic, $x_5 = 5\omega L$.



$v = V_{1m} \sin \omega t + V_{3m} \sin 3\omega t + V_{5m} \sin 5\omega t$

Fig. 6.21

Hence, the current would be given by the equation.

$$i = \frac{V_{1m}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{3m}}{3\omega L} \sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{V_{5m}}{5\omega L} \sin\left(5\omega t - \frac{\pi}{2}\right)$$

The harmonics in the current wave are much smaller than in the voltage wave.

For example, the 5th harmonic of the current wave is of only 1/5th of the harmonic in the voltage wave.

It means that the self-inductance of a coil has the effect of 'smoothening' current waveform when the voltage waveform is complex *i.e.* contains harmonics.

A.C. Fundamentals

✓ **A.C. Through Pure Capacitance**

Let the applied voltage, $v = V_m \sin\theta = V_m \sin\omega t$ is applied to the inductive circuit as shown in **Fig. 6.22.**

When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction.

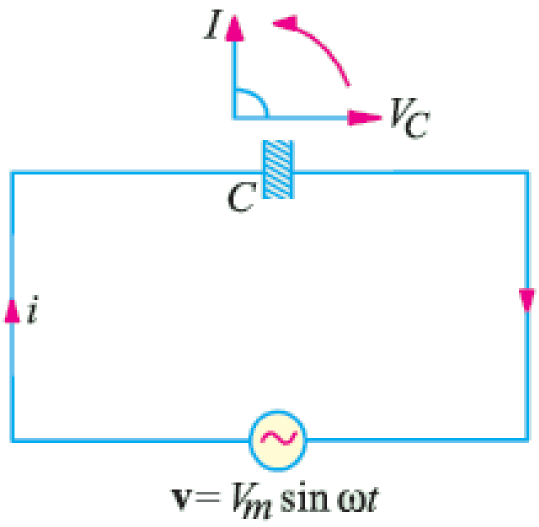


Fig. 6.22

let

v = p.d. developed between plates at any instant

q = Charge on plates at that instant.

Then $q = Cv$ where C is the capacitance

$= C V_m \sin\omega t$

Now, current i is given by the rate of flow of charge

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t) = \omega C V_m \cos\omega t = \frac{V_m}{1/\omega C}\cos\omega t = \frac{V_m}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\because \quad I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$$

A.C. Fundamentals

✓ **A.C. Through Pure Capacitance**

The denominator $X_C = 1/\omega C$ is known as capacitive reactance and is in ohms if C is in farad and ω is in radian/second.

If the applied voltage is $v = V_m \sin \omega t,$ then the current is given by $i = I_m \sin(\omega t + \pi/2)$.

Hence, the current in a pure capacitor leads its voltage by a quarter cycle or phase difference between the voltage and current is $\pi/2$ with the current leading as shown in **Fig. 6.23** .

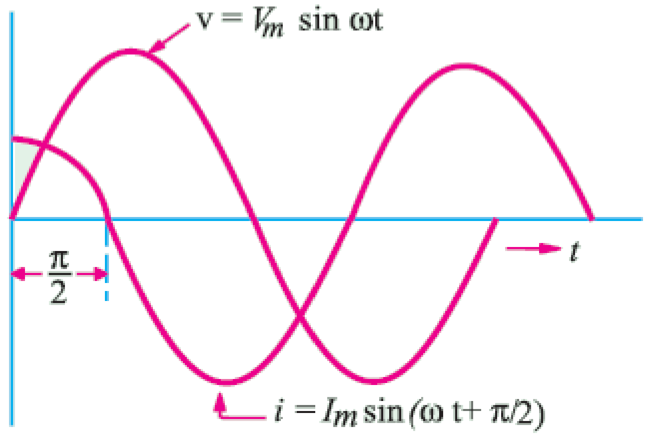


Fig. 6.23

The vectors V_C and I are graphically depicted in **Fig. 6.22.** It is seen from the figure that the voltage V_C lags the current I by 90^0.

A.C. Fundamentals

✓ **A.C. Through Pure Capacitance**

❑ **Power.**

Instantaneous power $= p = vi = V_m I_m \sin \omega t . \sin(\omega t + \pi/2)$

$$= V_m \, I_m \sin \omega t . \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin(2\omega t)$$

Power for whole cycle is $P = -\dfrac{V_m I_m}{2} \displaystyle\int_0^{2\pi} \sin(2\omega t)\, dt = 0$

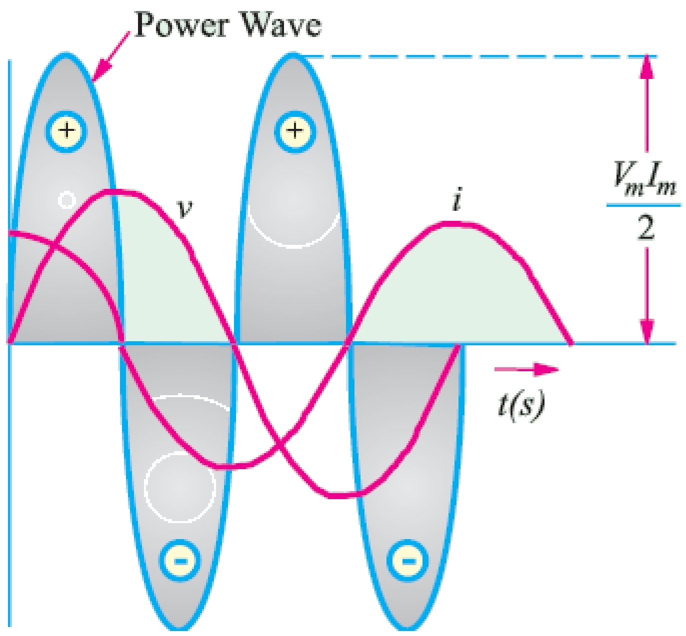


Fig. 6.24

The power is graphically depicted in in **Fig. 6.24**.

The average demand of power from supply is zero in a purely capacitive circuit.

It is also seen that power wave is a sine wave of frequency double that of the voltage and current waves.

The maximum value of the instantaneous power is $V_m I_m/2$.

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

A pure resistance *R* and a pure inductive coil of inductance *L* are connected in series with A.C. Source as shown in **Fig. 6.22**

Let

V = r.m.s. value of the applied voltage and

I = r.m.s. value of the resultant current

So, $V_R = IR$ — Voltage drop across *R* (in phase with *I*),

$V_L = IX_L$ — Voltage drop across coil (ahead of *I* by 90°)

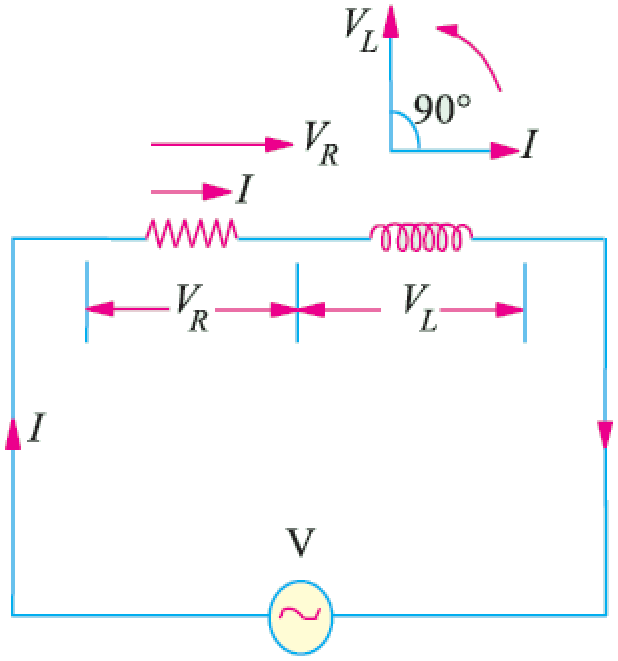


Fig. 6.22

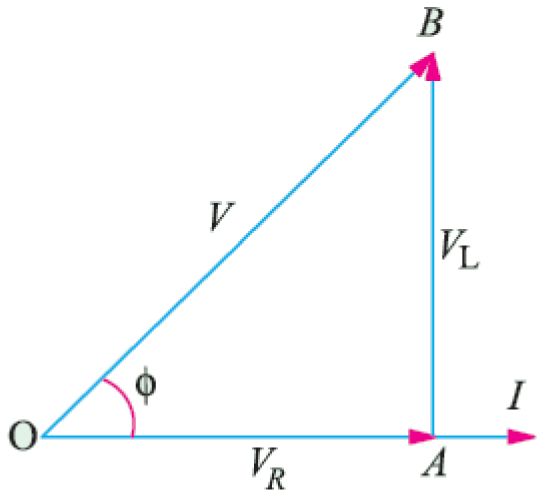


Fig. 6.23

These voltage drops are shown in voltage triangle *OAB* of **Fig. 6.23.**

Vector *OA* represents ohmic drop V_R, *AB* represents inductive drop V_L.

The applied voltage *V* is the vector sum of the two *i.e.* *OB*.

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

$$\therefore V = \sqrt{(V_R^2 + V_L^2)} = \sqrt{[(IR)^2 + (IX_L)^2]} = I\sqrt{(R^2 + X_L^2)}$$

$$\therefore I = \frac{V}{\sqrt{(R^2 + X_L^2)}}$$

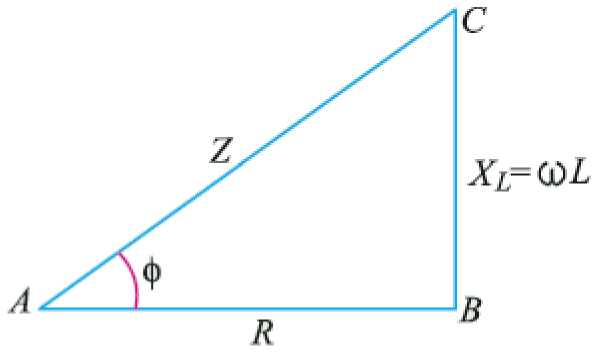


Fig. 6.24

The quantity $\sqrt{(R^2 + X_L^2)}$ is known as the impedance (Z) of the circuit.

The impedance triangle, ABC is shown in **Fig. 6.24** and from the figure, we get

$$Z^2 = R^2 + X_L^2$$

i.e. $(impedance)^2 = (resistance)^2 + (reactance)^2$

From **Fig. 6.23**, it is clear that the applied voltage V leads the current I by an angle $\emptyset$ such that

$$\tan\emptyset = \frac{V_L}{V_R} = \frac{I.X_L}{I.R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}} \qquad \therefore \emptyset = \tan^{-1}\frac{X_L}{R}$$

A.C. Fundamentals

✓ **A.C. Through Resistance and Inductance**

The voltage and current of R-L circuit is shown in **Fig. 6.25**

Hence, if the applied voltage is $v = V_m \sin \omega t$, then the current equation is

$$i = I_m \sin(\omega t - \emptyset) \qquad \text{where } I_m = V_m/Z$$

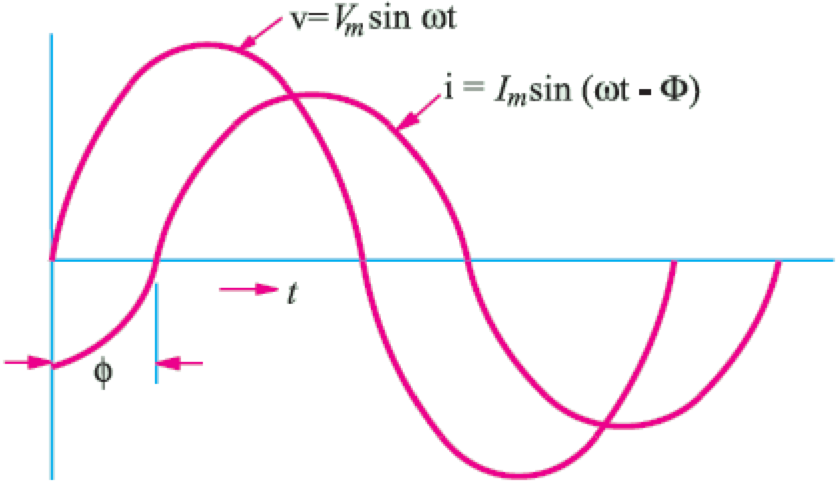


Fig. 6.25

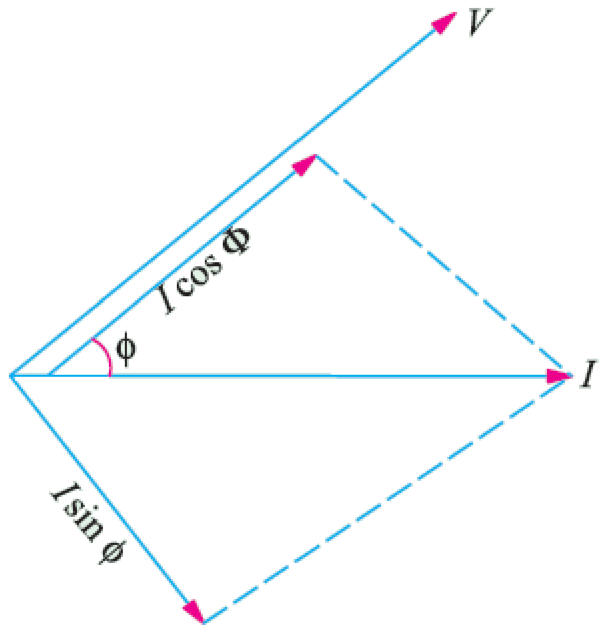


Fig. 6.26

Now, the voltage and current vectors are graphically depected in **Fig. 6.26** where I is resolved into the following two mutually perpendicular components.

$I \cos \emptyset$ along the applied voltage V and

$I \sin \emptyset$ in quadrature (i.e. perpendicular) with V.

A.C. Fundamentals

- ✓ **A.C. Through Resistance and Inductance**

The mean power consumed by the circuit is given by the product of V and the component of the current I which is in phase with V.

So, $P = V \times I \cos \emptyset =$ r.m.s. voltage × r.m.s. current × cos ∅

The term 'cos ∅' is called the power factor of the circuit.

∴ True Power (W) = Volt-Ampere × power factor

$P = VI \cos \emptyset = VI \times (R/Z) = I \times (V/Z) \times R = I^2 R \quad [\because \quad \cos \emptyset = R/Z]$

So, the power is consumed in resistance only because pure inductance does not consume any power.

The graphical representation of the power consumption in R-L circuit is shown in **Fig. 6.27.**

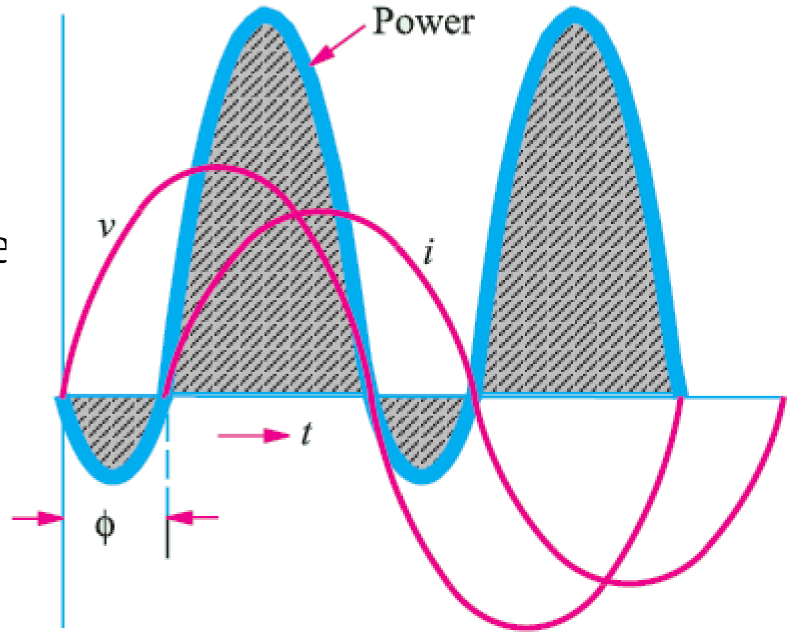


Fig. 6.27

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

The instantaneous power is $p = vi = V_m \sin \omega t \times I_m \sin(\omega t - \emptyset)$

$$= V_m\ I_m \sin \omega t\ \sin(\omega t - \emptyset)$$

$$= \frac{1}{2} V_m I_m [\cos \emptyset - \cos(2\omega t - \emptyset)]$$

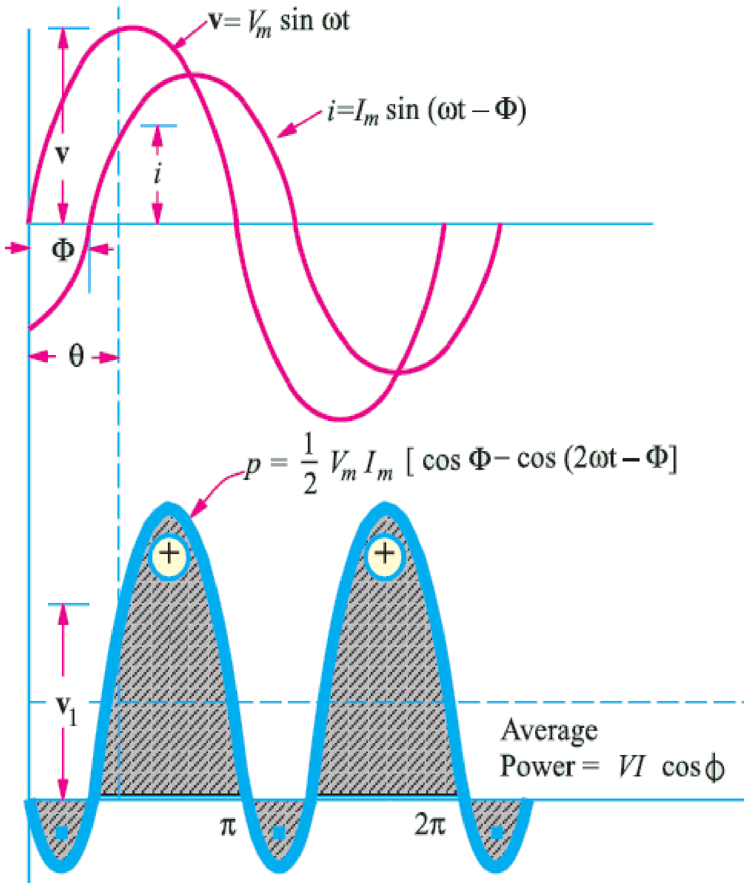


Fig. 6.28

This power consists of following two parts

(i) A constant part $1/2\, V_m I_m \cos \emptyset$ which contributes to real power.

(ii) A pulsating component $1/2\, V_m I_m \cos(2\omega t - \emptyset)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed $= \frac{1}{2} V_m I_m \cos \emptyset = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos \emptyset = VI \cos \emptyset$

where V and I represent the r.m.s. values.

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

Symbolic Notation of Impedance. $Z = R + jX_L$

The numerical value of impedance vector is $\sqrt{(R^2 + X_L^2)}$

The phase angle of it with the reference axis is $\emptyset = \tan^{-1}(X_L/R)$

It may also be expressed in the polar form as $Z = |Z|\angle\emptyset$

(i) Assuming $V = |V|\angle 0^0$; $I = \dfrac{V}{Z} = \dfrac{|V|\angle 0^0}{|Z|\angle\emptyset^0} = \left|\dfrac{V}{Z}\right|\angle - \emptyset^0$

It shows that current vector is lagging behind the voltage vector by $\emptyset^0$. The numerical value of current is V/Z.

(ii) However, it is assumed that $I = |I|\angle 0^0$

Then, $V = IZ = |I|\angle 0^0 \times |Z|\angle\emptyset^0 = |I.Z|\angle\emptyset^0$

It shows that current vector is lagging behind the voltage vector by $\emptyset^0$. The numerical value of current is V/Z.

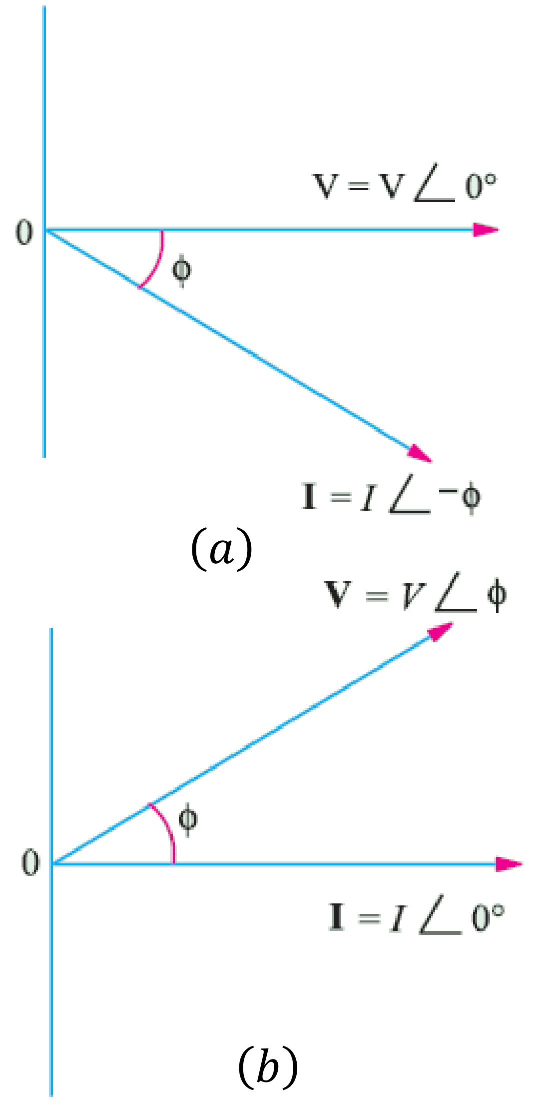


Fig. 6.29

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

Power Factor: It may be defined as

(i) cosine of the angle of lead or lag

(ii) the ratio, $\dfrac{R}{Z} = \dfrac{\text{reristance}}{\text{impedance}}$

(iii) the ratio, $\dfrac{\text{true power}}{\text{apparent power}} = \dfrac{\text{watts}}{\text{volt–amperes}} = \dfrac{W}{VA}$

Active and Reactive Components of Circuit Current *I*

Active component of current is in phase with the applied voltage *V i.e.* *I* cos ϕ. It is known as 'wattful' component.

Reactive component of current is in quadrature with *V i.e.* *I* sin ϕ. It is known as 'wattless' or 'idle' component.

A.C. Fundamentals

✓ A.C. Through Resistance and Inductance

Active and Reactive Components of Circuit Current *I*

The product of volts and amperes in an a.c. circuit gives kilo volt-amperes (kVA). The kVA has two rectangular components as follows:

(i) Active component which is obtained by multiplying kVA by $\cos\emptyset$ ($i.e. kVA\cos\emptyset$)
It is also known as real power and the unit of it is in kW.

(ii) Reactive component which is obtained by multiplying kVA by $\sin\emptyset$ ($i.e.\ kVA\sin\emptyset$)
The unit of it is in kVAr (kilovar).

The relation among kW, $kVAr$ and kVA is $kVA = \sqrt{(kW)^2 + (kVAr)^2}$

where, $kW = kVA\cos\emptyset$ and $kVAr = kVA\sin\emptyset$

The relation among kW, $kVAr$ and kVA are shown in **Fig. 6.30**

It is seen from the figure that kVAr is negative and it indicates lagging kVAr.

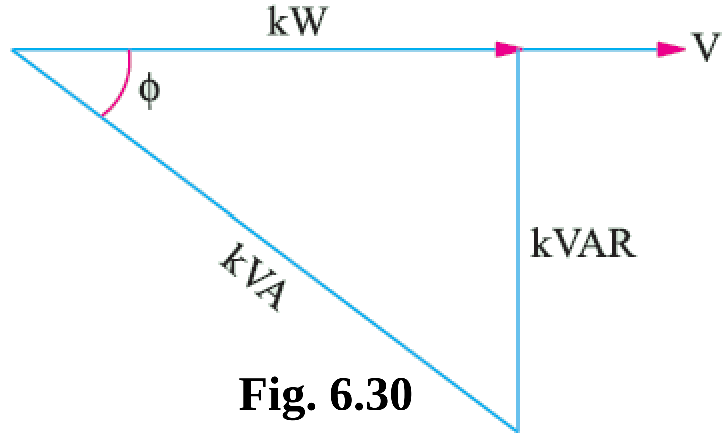


Fig. 6.30

✓ A.C. Through Resistance and Inductance

Active, Reactive and Apparent Power

Let a series $R - L$ circuit draw a current of I when an alternating voltage of r.m.s. value V is applied to it. Suppose that current lags behind the applied voltage by $\emptyset$. The three powers drawn by the circuit are as under and these powers are shown in the power triangle of **Fig. 6.31**.

(i) **apparent power** (S)

It is given by the product of r.m.s. values of applied voltage and circuit current.

$$\therefore \quad S = VI = (IZ).I = I^2Z \quad \text{volt-amperes (VA)}$$

(ii) **active power** (P)

It is the power which is actually dissipated in the circuit resistance.

$$\therefore \quad P = I^2R = VI\cos\emptyset \quad \text{watts}$$

(iii) **reactive power** (Q)

It is the power developed in the inductive reactance of the circuit.

$$\therefore \quad Q = I^2X_L = I^2Z\sin\emptyset = I.(IZ).\sin\emptyset = VI\sin\emptyset \ \text{VAR}$$

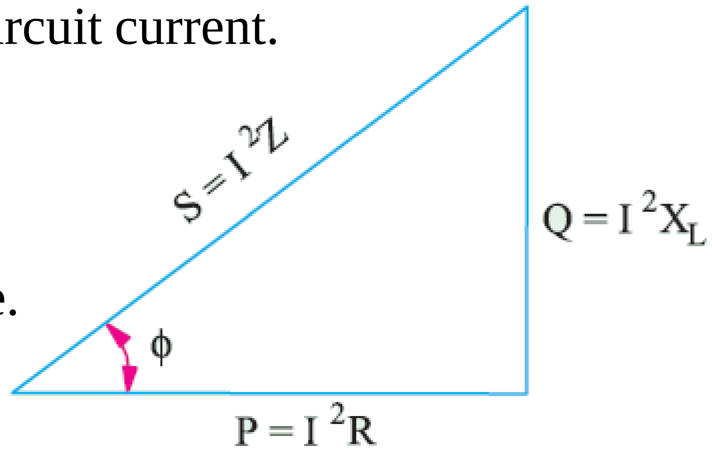


Fig. 6.31

$$S^2 = P^2 + Q^2$$

A.C. Fundamentals

- ✓ **A.C. Through Resistance and Inductance**

❖ Q-factor of a Coil

In practice, the inductor possesses a small resistance in addition to its inductance. The lower the value of this resistor, the better the Q-factor of the inductor. The quality factor or Q-factor of an inductor at operating frequency ω is defined as

$$Q = 2\pi\left[\frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}\right]$$

Consider the sinusoidal voltage $V_m \sin \omega t$ applied to an inductor L having internal resistance R. Let I_m be the peak value of the current in the circuit.

Then, maximum energy stored per cycle $= \dfrac{1}{2} L I_m^2$

Average power dissipated in the inductor per cycle $= \left(\dfrac{I_m}{\sqrt{2}}\right)^2 R$

A.C. Fundamentals

✓ **A.C. Through Resistance and Inductance**

❖ **Q-factor of a Coil**

Hence, energy dissipated in the inductor per cycle = Power×periodic time for one cycle

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R \times T = \left(\frac{I_m}{\sqrt{2}}\right)^2 R \times \frac{1}{f} = \frac{I_m^2 R}{2f}$$

Therefore,

$$Q = 2\pi \frac{\frac{1}{2} L I_m^2}{\frac{I_m^2 R}{2f}} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

$$\therefore \quad Q = \frac{\omega L}{R} = \frac{1}{R/\omega L} = \frac{1}{R/Z} = \frac{1}{\cos \emptyset} = \frac{1}{\text{power factor}}$$

So, the Q-factor of an inductor is defined as the ratio of impedances of the coil to its resistance.

LECTURE 14

A.C. Fundamentals

✓ A.C. Through Resistance and Capacitance

A pure resistance R and a pure capacitor of capacitance C are connected in series with A.C. Source as shown in **Fig. 6.32**

Let

V = r.m.s. value of the applied voltage and

I = r.m.s. value of the resultant current

So, $V_R = IR$ — Voltage drop across R (in phase with I),

$V_C = IX_C$ — Voltage drop across capacitor (lagging I by 90°)

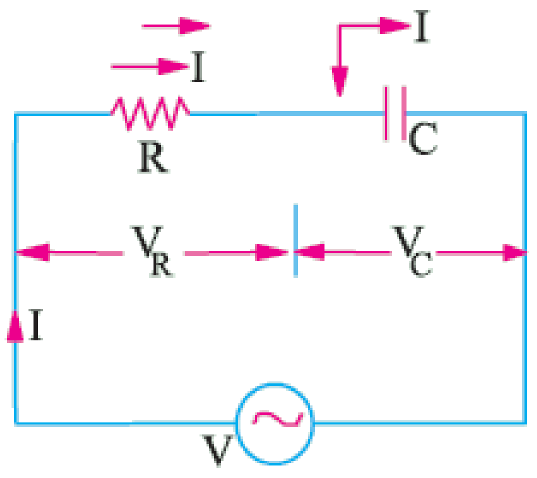


Fig. 6.32

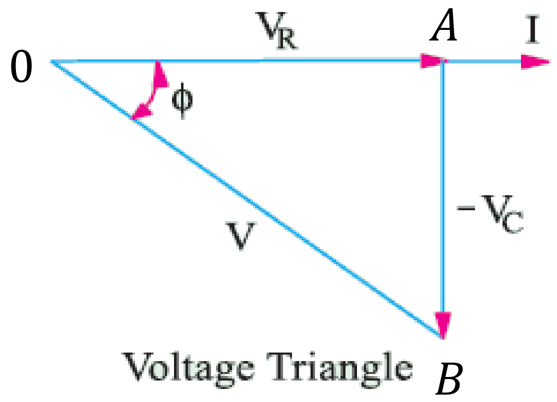


Fig. 6.33

These voltage drops are shown in voltage triangle *OAB* of **Fig. 6.33.**

Vector *OA* represents ohmic drop V_R , *AB* represents capacitive drop V_C.

The applied voltage V is the vector sum of the two *i.e.* *OB.*

A.C. Fundamentals

- ✓ **A.C. Through Resistance and Capacitance**

$$\therefore V = \sqrt{(V_R^2 + V_C^2)} = \sqrt{[(IR)^2 + (IX_C)^2]} = I\sqrt{(R^2 + X_C^2)}$$

$$\therefore I = \frac{V}{\sqrt{(R^2 + X_C^2)}}$$

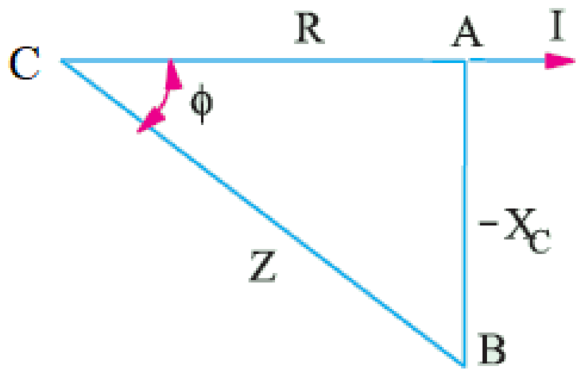


Fig. 6.34

The quantity $\sqrt{(R^2 + X_C^2)}$ is known as the impedance (Z) of the circuit.

The impedance triangle, ABC is shown in **Fig. 6.34** and from the figure, we get

$$Z^2 = R^2 + X_C^2$$

i.e. $(impedance)^2 = (resistance)^2 + (reactance)^2$

From **Fig. 6.33**, it is clear that the applied voltage V lags the current I by an angle $\emptyset$ such that

$$\tan\emptyset = \frac{V_C}{V_R} = \frac{I.X_C}{I.R} = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{\text{reactance}}{\text{resistance}} \qquad \therefore \emptyset = \tan^{-1}\frac{X_C}{R}$$

A.C. Fundamentals

✓ A.C. Through Resistance and Capacitance

Hence, if the applied voltage is $v = V_m \sin \omega t,$ then the current equation is

$$i = I_m \sin(\omega t + \emptyset) \qquad \text{where } I_m = V_m/Z$$

The voltage and current of R-C circuit is shown in **Fig. 6.35**

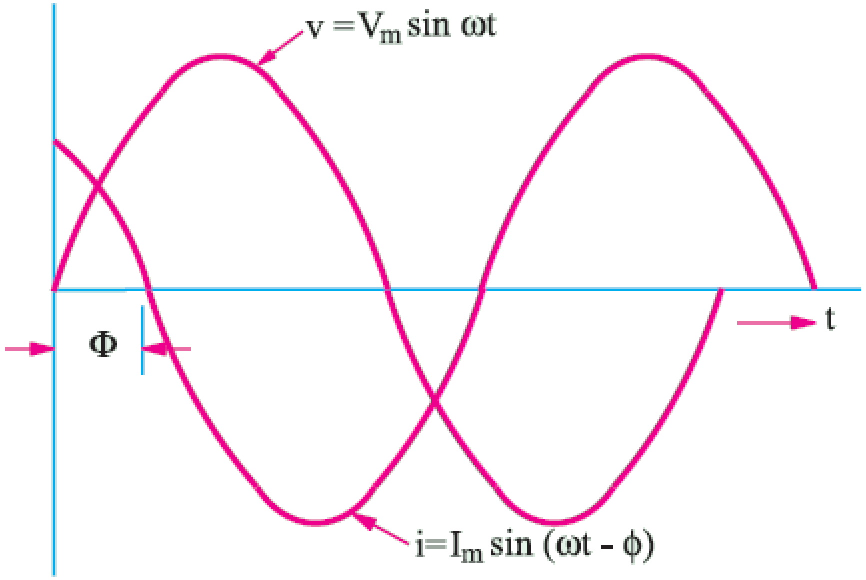


Fig. 6.35

✓ A.C. Through Resistance and Capacitance

❖ Dielectric Loss and Power Factor of a Capacitor

- An ideal capacitor is one in which there are no losses and whose current leads the voltage by 90° as shown in **Fig. 6.36** *(a)*.

- In practice, every capacitor has some dielectric loss and hence it absorbs some power from the circuit. Due to this loss, the phase angle is somewhat less than 90° as shown in **Fig. 6.30** (*b*)

- The phase difference ψ between the ideal and actual phase angles is given by

$\psi = (90 - \phi)$ where ϕ is the actual phase angle.

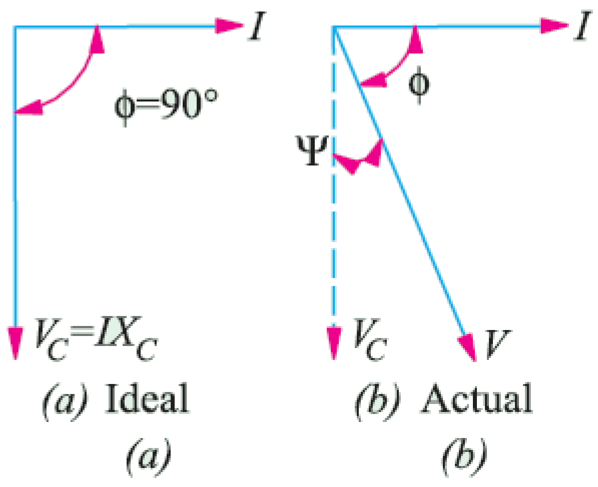


Fig. 6.36

$\therefore$ $\sin\psi = \sin(90 - \phi) = \cos\phi$, where $\cos\phi$ is the power factor of the capacitor.

Since ψ is generally small, $\sin\psi = \psi$ (in radians) $\therefore \tan\psi = \psi = \cos\phi$.

- It is observed that dielectric loss increases with the frequency of the applied voltage. Hence phase difference increases with the frequency *f*.

✓ A.C. Through Resistance and Capacitance

- The dielectric loss appears as heat. So, it is imagined that the capacitor consists of a pure capacitor with an equivalent resistance either in series or in parallel as shown in **Fig. 6.37.**
- These resistances are such that $I^2 R$ loss in them is equal to the dielectric loss in the capacitor.

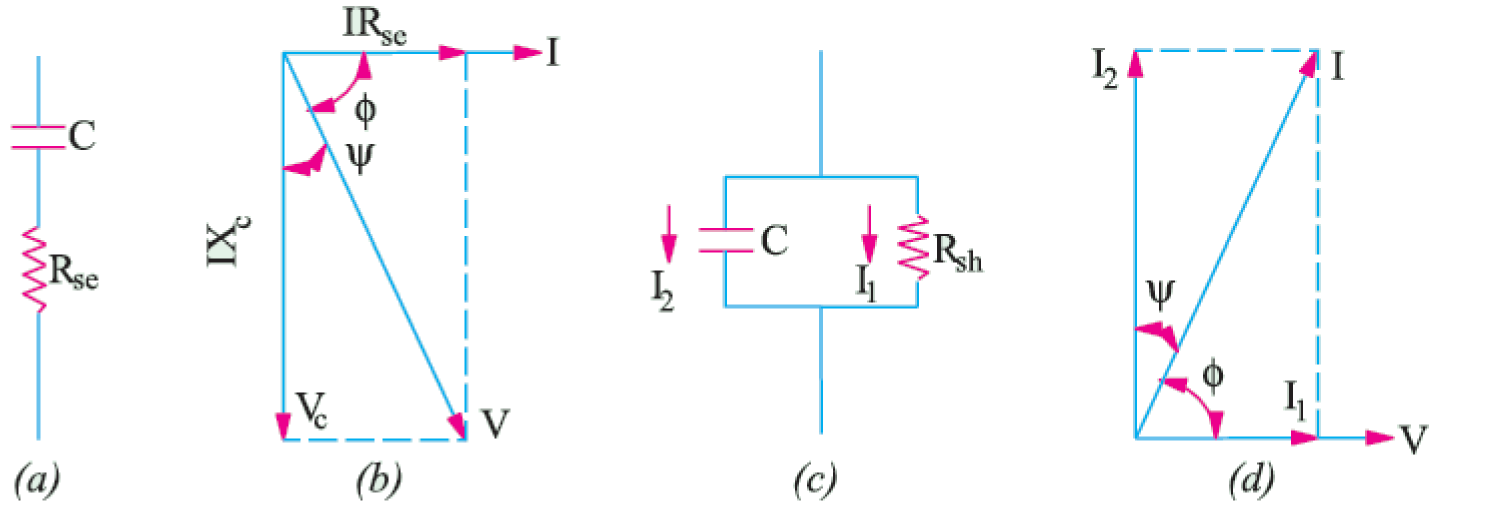


Fig. 6.37

It is seen from **Fig. 6.37** (*b*) that

$$\tan\varphi = \frac{IR_{se}}{I/\omega C} = \omega C R_{se} \qquad \therefore R_{se} = \tan\varphi/\omega C = p.f./\omega C$$

✓ A.C. Through Resistance and Capacitance

Similarly, as seen from **Fig. 6.37** *(d)*,

$$\tan\varphi = \frac{I_1}{I_2} = \frac{V/R_{sh}}{V/X_C} = \frac{X_C}{R_{sh}} = \frac{1}{\omega C R_{sh}}$$

$$\therefore\ R_{sh} = \frac{1}{\omega C.\tan\varphi} = \frac{1}{\omega C \times \text{power factor}} = \frac{1}{\omega C \times \text{p.f.}}$$

The power loss in the shunt resistance is $P_{sh} = V^2/R_{sh} = \omega C V^2 \tan\varphi = \omega C V^2 \times \text{p.f.}$

The power loss in the series resistance is $P_{se} = I^2 R_{se} = (I^2 \times \text{p.f.})/\omega C$

Where p.f. stands for the power factor of the capacitor

A.C. Fundamentals

✓ **A.C. Through Resistance and Capacitance**

❖ **Q-factor of a Capacitor**

In practice, every capacitor C possesses a small resistor R in series with it. The Q-factor or quality factor of a capacitor at the operating frequency ω is defined as the ratio of the reactance of the capacitor to its series Resistor.

Consider a sinusoidal voltage $v = V_m \sin \omega t$ applied to a capacitor C of effective internal resistor R.

Maximum energy stored in the capacitor per cycle $= \frac{1}{2} C V_m^2$

where V_{max} is the maximum value of voltage across the capacitor

But, if $R \ll \omega C$ $\quad \therefore V_{max} = \frac{I_m}{\omega C}$ where I_m is the maximum value of current through C

Therefore, maximum energy stored in the capacitor per cycle $= \frac{1}{2} C V_m^2 = \frac{1}{2} \frac{I_m^2}{\omega^2 C}$

Energy dissipated per cycle $= \frac{I_m^2}{2f}$

A.C. Fundamentals

- ✓ **A.C. Through Resistance and Capacitance**

Therefore $$Q = 2\left(\frac{\frac{I_m^2}{2\omega^2 C}}{\frac{I_m^2 R}{2f}}\right) = \frac{1}{\omega CR}$$

A leaky capacitor is represented by a capacitor C with a high resistance R_{sh} in shunt.

Maximum energy stored in the capacitor $= \frac{1}{2}CV_m^2$ where V_m is the peak value of the applied voltage.

Average power dissipated in R_{sh} $= \frac{\left(V_m/\sqrt{2}\right)^2}{R_{sh}} = \frac{V_m^2}{2R_{sh}}$

Energy dissipated per cycle $= \frac{V_m^2}{2R_{sh}} \times \frac{1}{f}$

Hence, $$Q = 2\pi\left(\frac{\frac{1}{2}CV_m^2}{V_m^2/2R_{sh}f}\right) = \omega CR_{sh}$$

A.C. Fundamentals

✓ A.C. Through Resistance, Inductance and Capacitance

The resistance, inductance and capacitance are joined in series across an a.c. supply of r.m.s. voltage V as shown in **Fig. 6.38.**

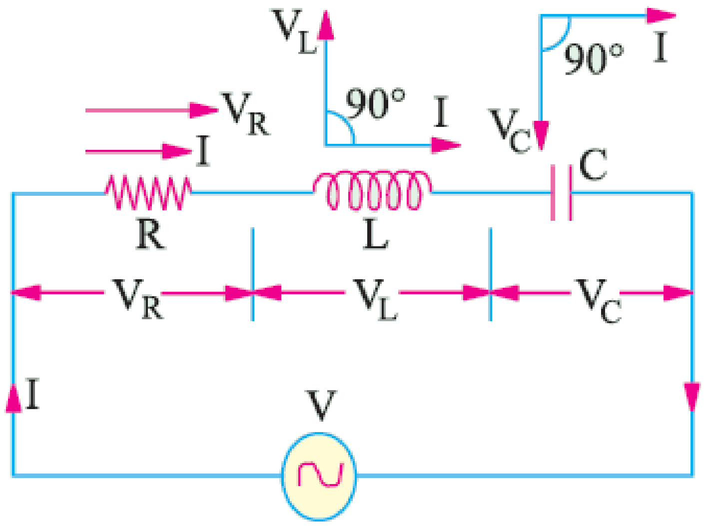


Fig. 6.38

Let

$V_R = IR$ = voltage drop across R —in phase with I

$V_L = I.X_L$ = voltage drop across L —leading I by $\pi/2$

$V_C = I.X_C$ = voltage drop across C —lagging I by $\pi/2$

✓ A.C. Through Resistance, Inductance and Capacitance

- The voltage triangle is shown in **Fig. 6.39**, where *OA* represents V_R, *AB* and *AC* represent the inductive and capacitive drops respectively.

- It will be seen that V_L and V_C are 180° out of phase with each other *i.e.* they are in direct opposition to each other.

- Subtracting *BD* (= *AC*) from *AB*, we get the net reactive drop $AD = I\,(X_L - X_C)$. The applied voltage *V* is represented by *OD* and is the vector sum of *OA* and *AD*

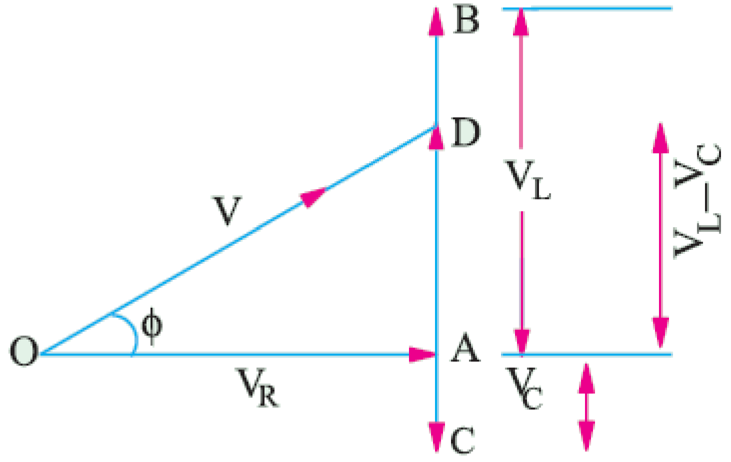


Fig. 6.39

$$\therefore OD = \sqrt{OA^2 + AD^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

or, $$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + X^2}$ is known as the impedance of the circuit

Therefore, $$(impedance)^2 = (resistance)^2 + (net\ reactance)^2$$

A.C. Fundamentals

- ✓ **A.C. Through Resistance, Inductance and Capacitance**

$$\therefore\ Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$$ where X is the net reactance

So, phase angle ϕ is given by $\tan \phi = (X_L - X_C)/R = X/R$ = net reactance/resistance

Power factor, $\cos \emptyset = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{R}{\sqrt{R^2 + X^2}}$

Hence, it is seen that if the equation of the applied voltage is $v = V_m \sin \omega t$, then equation of the resulting current in an R-L-C circuit is given by $i = I_m \sin(\omega t \pm \phi)$

The + ve sign is to be used when current leads *i.e.* $X_C > X_L$.

The –ve sign is to be used when current lags *i.e.* when $X_L > X_C$.

A.C. Fundamentals

✓ **A.C. Through Resistance, Inductance and Capacitance**

In general, the current lags or leads the supply voltage by an angle ϕ such that $\tan\phi = X/R$

So, $Z = R + j\,(X_L - X_C)$

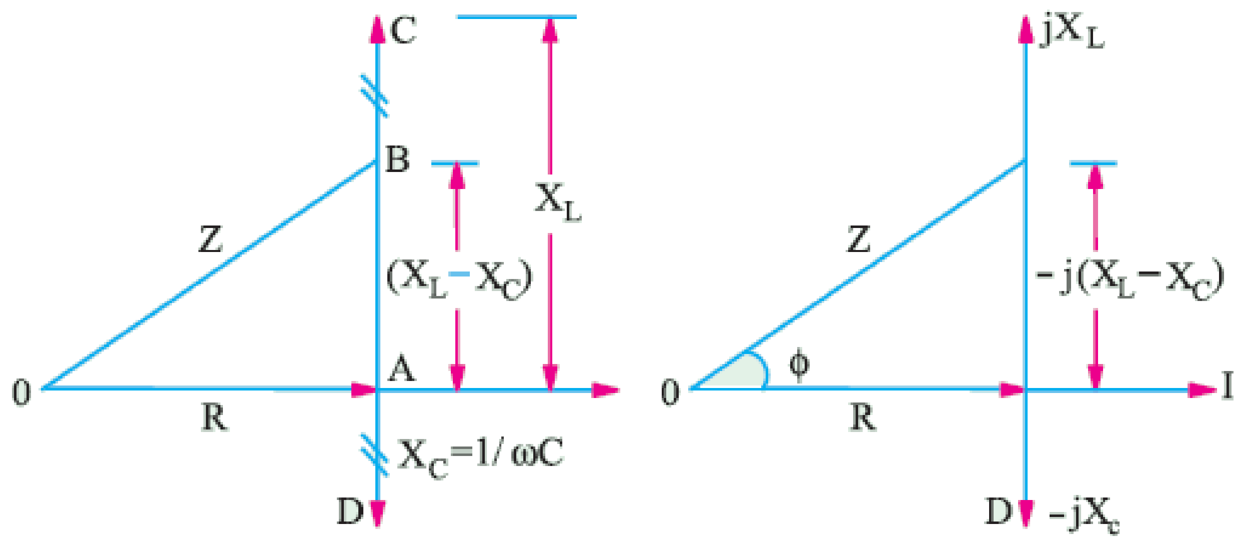
Numerical value of impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Its phase angle is $\Phi = \tan^{-1}[X_L - X_C/R]$

$$Z = Z \angle \tan^{-1}[(X_L - X_C)/R] = Z \angle \tan^{-1}(X/R)$$

If $\mathbf{V} = V \angle 0$, then, $I = V/R$



(a)

(b)

Fig. 6.40

Summary of Results of Series AC Circuits

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2+(\omega L)^2]}$	$0 < \phi < 90°$ lag	$1 >$ p.f. > 0 lag
Resistance and Capacitance	$\sqrt{[R^2+(-1/\omega C)^2]}$	$0 < \phi < 90°$ lead	$1 >$ p.f. > 0 lead
R-L-C	$\sqrt{[R^2+(\omega L \sim 1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

Example – P6.3

A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate (i) impedance, reactance and resistance of the coil (ii) the power absorbed by the coil and (iii) the total power. Draw the vector diagram.

Solution of Example – P6.3

It is seen from the vector diagram of **Fig. P6.1** (*b*)

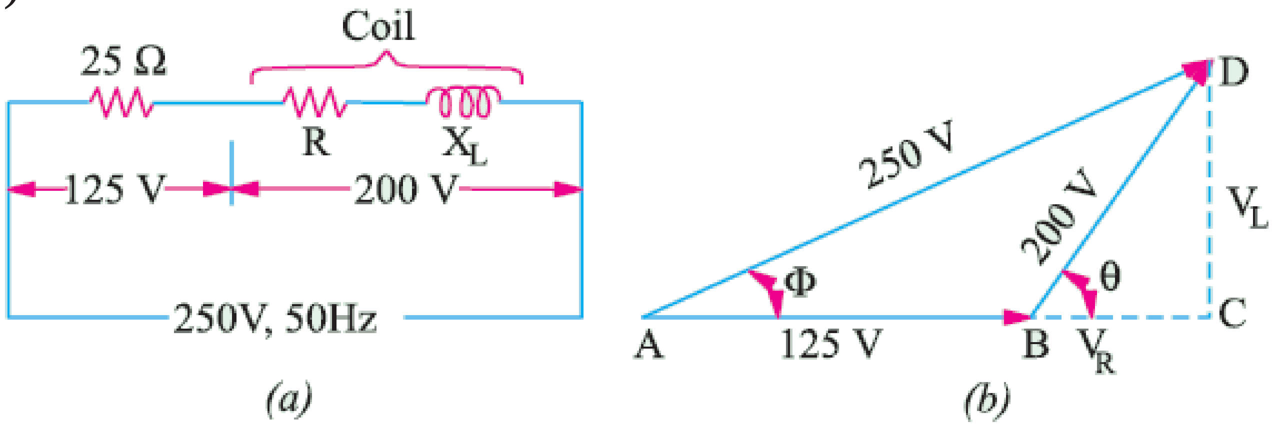


Fig. P6.3

$$BC^2 + CD^2 = 200^2$$

or, $(125 + BC)^2 + CD^2 = 250^2$

or, $(125 + BC)^2 - BC^2 = 250^2 - 200^2$

$\therefore$ $BC = 27.5$ V

$\therefore$ $CD = \sqrt{200^2 - 27.5^2} = 198.1$ V

(i) Coil impedance = 200/5 = 40 Ω

$V_R = IR = BC$ or, $5R = 27.5$ $\therefore$ $R = 27.5/5 = 5.5\ \Omega$

$V_L = I \,.\, X_L = CD = 198.1$ $\therefore$ $X_L = 198.1/5 = 39.62\ \Omega$

Solution of Example – P6.3

(ii) Power absorbed by the coil is = $I^2 R = 5^2 \times 5.5 = 137.5$ W

Also $P = 200 \times 5 \times 27.5/200 = 137.5$ W

(iii) Total power = $VI \cos \phi = 250 \times 5 \times AC/AD$

$= 250 \times 5 \times 152.5/250 = 762.5$ W

The power may also be calculated by using $I^2 R$ formula.

Series resistance = 125/5 = 25 Ω

Total circuit resistance = 25 + 5.5 = 30.5

∴ Total power = $5^2 \times 30.5 = 762.5$ W

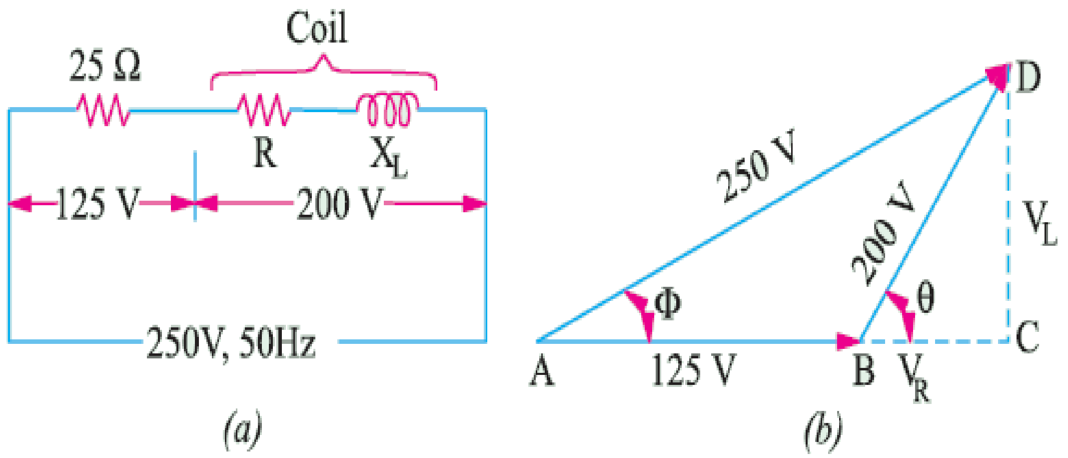


Fig. P6.3

Example – P6.4

Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.

Solution of Example – P6.4

The circuit in **Fig. P6.2** (*a*) and the kVA triangle is shown in **Fig. P6.2** (*b*).

The circuit kVA is given by, $kVA = \sqrt{3^2 + 2^2} = 3.606$

Circuit current = 3,606/240 = 15.03 A $\quad \therefore\ 15.03^2 (R_A + R_B) = 3{,}000$

$\therefore \quad R_A + R_B = 3{,}000/15.03^2 = 13.3\ \Omega$

$\therefore \quad R_B = 13.3 - 5 = 8.3\ \Omega \quad [\because\ R_A = 5\ \Omega]$

Now, impedance of the whole circuit is given by $Z = 240/15.03 = 15.97\ \Omega$

$\therefore \quad X_A + X_B = \sqrt{Z^2 - (R_A + R_B)^2} = \sqrt{(15.97)^2 - (13.3)^2} = 8.84\ \Omega$

Now $\quad X_B = 2\pi \times 50 \times 0.015 = 4.713\ \Omega$

$\therefore \quad X_A = 8.843 - 4.713 = 4.13\ \Omega$ $\quad$ or, $2\pi \times 50 \times L_A = 4.13\ \therefore\ L_A = 0.0132$ H

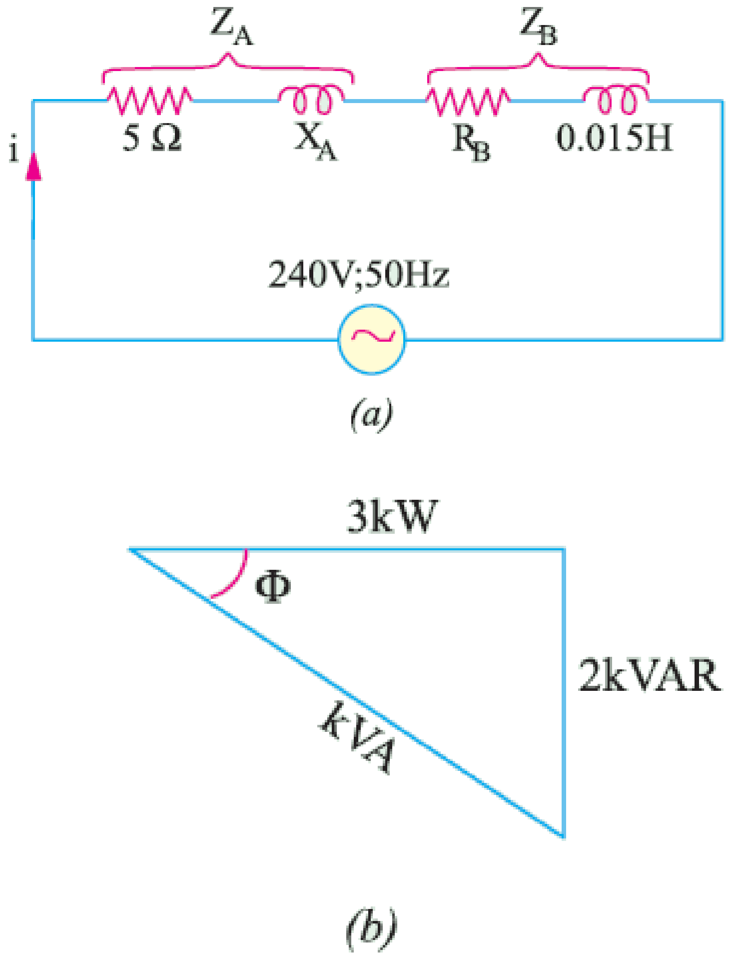


Fig. P6.4

Solution of Example – P6.4

Now $$Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.13^2} = 6.485\ \Omega$$

$$Z_B = \sqrt{R_B^2 + X_B^2} = \sqrt{8.3^2 + 4.713^2} = 9.545\ \Omega$$

∴ P.D. across coil $A = I \cdot Z_A = 15.03 \times 6.485 = 97.5$ V

∴ P.D. across coil $B = I \cdot Z_B = 15.03 \times 9.545 = 143.5$ V

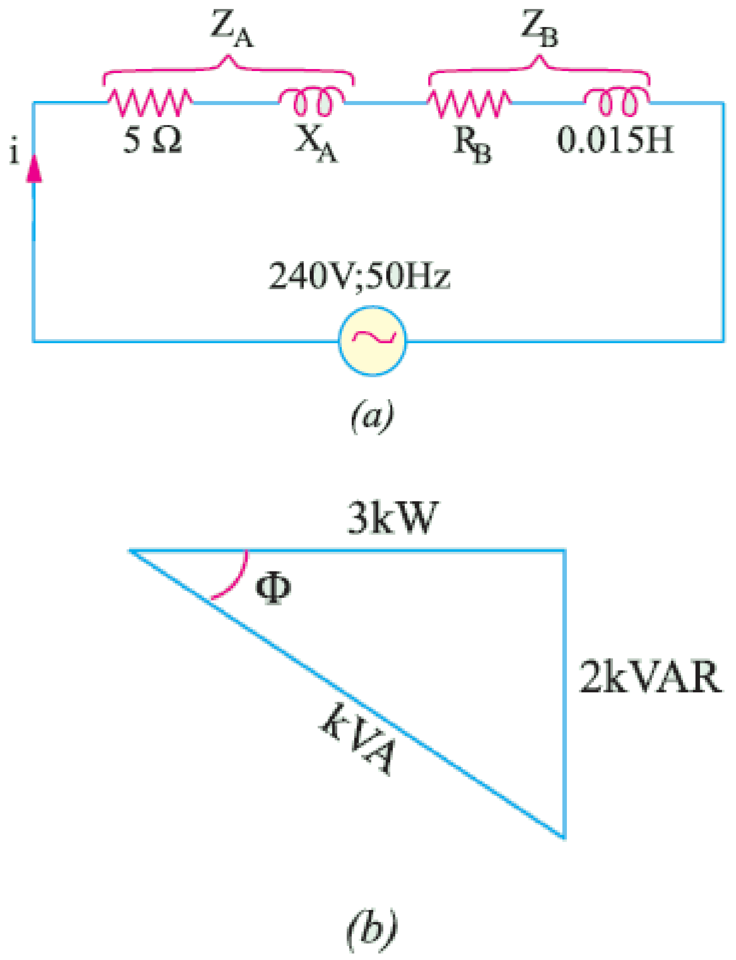


Fig. P6.4

LECTURE 15

A.C. Fundamentals

➢ Resonance in R-L-C Circuits

The resistance, inductance and capacitance are joined in series across an a.c. supply of r.m.s. voltage *V* as shown in **Fig. 6.41.** The phasor diagram of impedance is shown in **Fig. 6.42.**

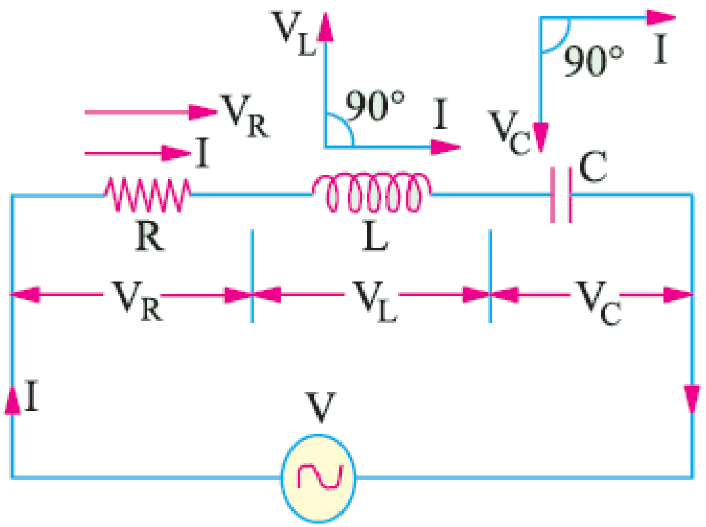


Fig. 6.41

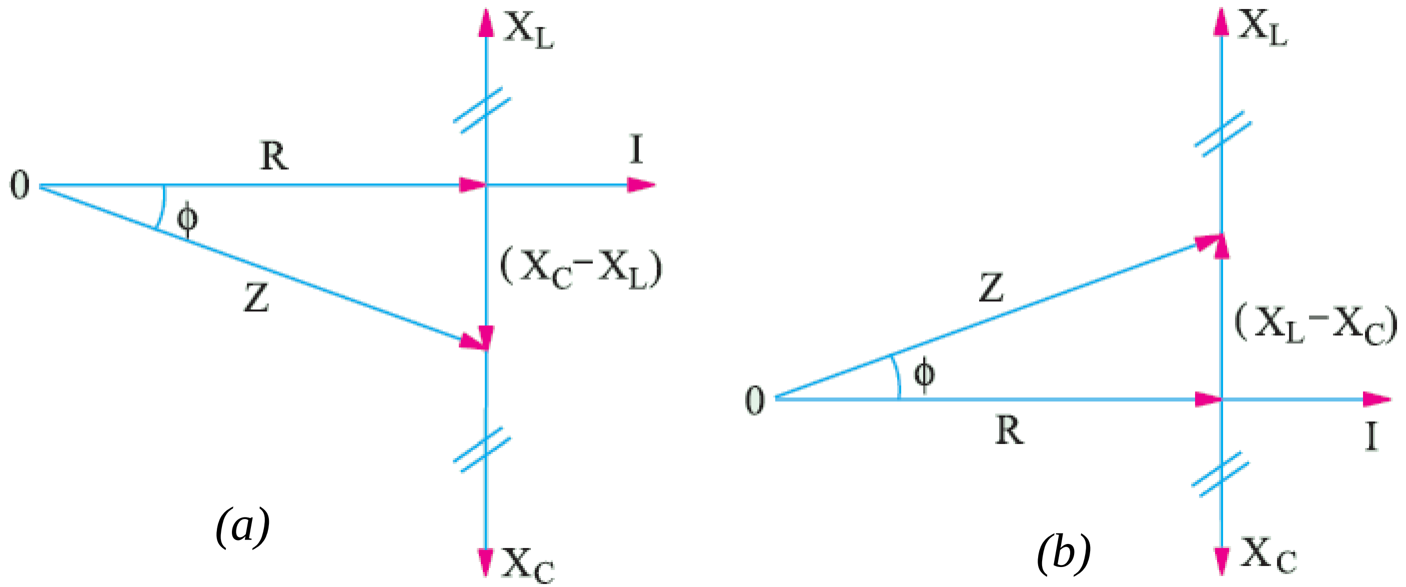


Fig. 6.42

It is seen that the net reactance and impedance in an *R-L-C* circuit is

$$X = X_L \sim X_C \qquad \text{and} \qquad Z = \sqrt{R^2 + X^2}$$

The current either lags or leads the supply voltage by an angle ϕ such that $\tan \phi = X/R$

A.C. Fundamentals

➢ **Resonance in R-L-C Circuits**

- Let the source have constant voltage V but the frequency is varying from zero to infinity.
- There would be a certain frequency of the applied voltage which make X_L equal to X_C in magnitude.
- In that case, $X = 0$ and $Z = R$ as shown in **Fig. 6.43**. Under this condition, the circuit is said to be in electrical resonance.

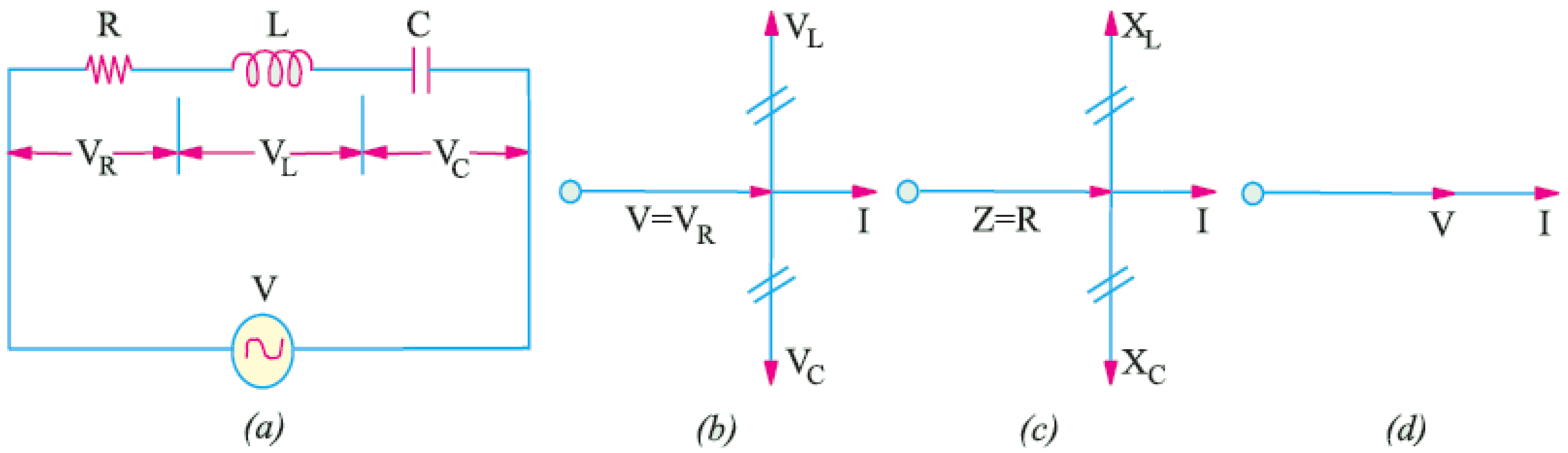


Fig. 6.43

- It is seen from **Fig. 6.43** that $V_L = I.\ X_L$ and $V_C = I.\ X_C$. In the condition of resonance, V_L and V_C are equal in magnitude but opposite in phase.

➢ Resonance in R-L-C Circuits

- So, V_L and V_C cancel each other and the two reactances together act as a short-circuit since no voltage developed across them. Hence, $V = V_R$.
- The phasor diagram for series resonance is shown in **Fig. 6.43** (*d*) as the circuit impedance, $Z = R$. .

❖ Calculation of Resonant Frequency

The frequency at which the net reactance of the series circuit is zero is called the resonant frequency f_0.

$\therefore\ X_L - X_C = 0$ or, $X_L = X_C$ or, $\omega_0 L = 1/\omega_0 C$ or, $\omega_0^2 = \dfrac{1}{LC}$ or, $(2\pi f_0)^2 = \dfrac{1}{LC}$

$$\therefore f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

If L is in henry and C in farad, then f_0 is given in *Hz*.

When a series *R*-*L*-*C* circuit is in resonance, it possesses minimum impedance $Z = R$.

Hence, circuit current is maximum, it being limited by value of *R* alone.

The current $I_0 = V/R$ and is in phase with *V*.

A.C. Fundamentals

- **Resonance in R-L-C Circuits**
  - As the circuit current is maximum, it produces large voltage drops across L and C. These drops are equal and opposite. So, they cancel each other out.
  - So, there is no voltage across the part of a circuit consisting of L an C together (i.e. the circuit act as a short-circuit to the resonance frequency).
  - Hence, a series resonant circuit is sometimes called acceptor circuit and the series resonance is often referred to as voltage resonance.
  - If X_L and X_C are at any frequency f, then the value of the resonant frequency of such a circuit is found by the relation $f_0 = f\sqrt{X_C/X_L}$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

  - **Summary**

    When an *R-L-C* circuit is in resonance

    net reactance of the circuit is zero *i.e.* $(X_L - X_C) = 0$. or $X = 0$.

    circuit impedance is minimum *i.e.* $Z = R$. Consequently, circuit admittance is maximum.

    circuit current is maximum and is given by $I_0 = V/Z_0 = V/R$.

    Power dissipated is maximum i.e. $P_0 = I^2 R = V^2/R$.

    circuit power factor angle $\theta = 0$. Hence, power factor $\cos \theta = 1$.

    although $V_L = V_C$ yet V_{coil} is greater than V_C because of its resistance.

    at resonance, $\omega^2 LC = 1$

    $Q = \tan \theta = \tan 0° = 0$.

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

Graphical Representation of Resonance

An alternating voltage of constant magnitude, but of varying frequency is applied to an R-L-C circuit.

The variations of resistance, inductive reactance X_L and capacitive reactance X_C with frequency are shown in **Fig. 6.44.**

Resistance: It is independent of f. So, it is represented by a straight line.

Inductive Reactance: It is given by $X_L = \omega L = 2\pi f L$. It is proportional to f. So, the graph of it is a straight line passing through the origin.

Capacitive Reactance: It is given by $X_C = 1/\omega C = 1/2\pi f C$. It is inversely proportional to f. The graph of it is a rectangular hyperbola which is drawn in the fourth quadrant because X_C is regarded negative. It is asymptotic to the horizontal axis at high frequencies and to the vertical axis at low frequencies.

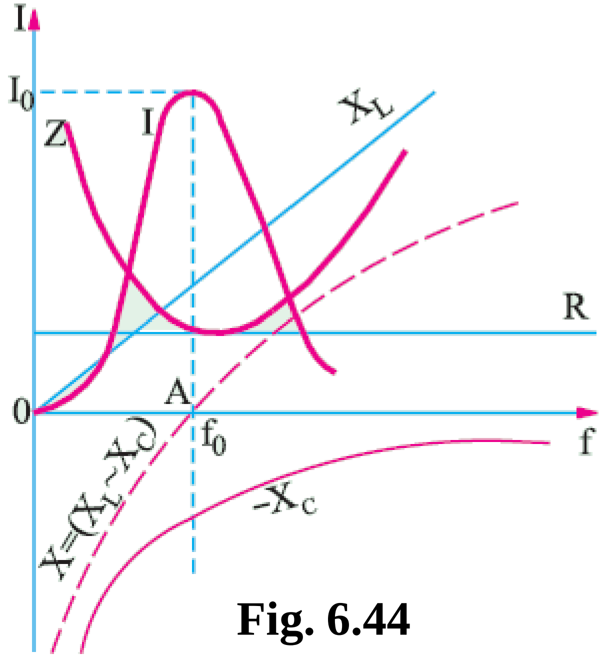


Fig. 6.44

Net Reactance: It is given by $X = X_L{\sim}X_C$. Its graph is a hyperbola (not rectangular) and crosses the X-axis at point A which represents resonant frequency f_0.

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

❖ **Circuit Impedance:** It is given by $Z = \sqrt{R^2 + (X_L \sim X_C)^2} = \sqrt{R^2 + X^2}$

- At low frequencies, Z is large because X_C is large. Since $X_C > X_L$, the net circuit reactance X is capacitive and the p.f. is leading.
- Similarly, at high frequencies, Z is again large (because X_L is large) but is inductive because $X_L > X_C$. Circuit impedance has minimum values at f_0 given by $Z = R$ because $X = 0$ as shown in **Fig. 6.45**.

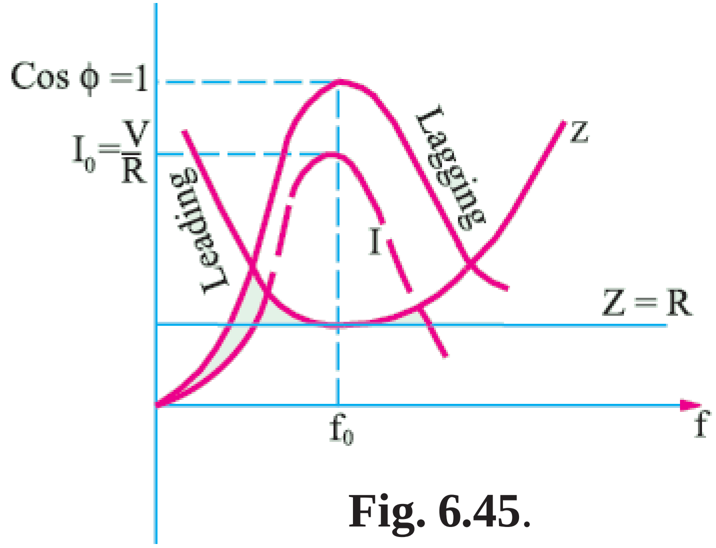


Fig. 6.45.

❖ **Current I_0 :** It is the reciprocal of the circuit impedance.

- When Z is low, I_0 is high and vice versa. As seen, I_0 has low value on both sides of f_0 (because Z is large there) but has maximum value of $I_0 = V/R$ at resonance.
- Hence, maximum power is dissipated by the series circuit under resonant condition.
- At frequencies below and above resonance, current decreases as shown in **Fig. 6.45.**
- Now, $I_0 = V/R$ and $I = V/Z = V/\sqrt{R^2 + X^2}$.
- Hence $I/I_0 = R/Z = R/\sqrt{R^2 + X^2}$ where X is the net circuit reactance at any frequency f.

Resonance in R-L-C Circuits

Power Factor

- It is seen in **Fig. 6.45** that X is capacitive below f_0. Hence, current leads the applied voltage.
- It is also seen in **Fig. 6.45** that X is inductive above f_0. Hence, the current lags the applied voltage.
- The power factor has maximum value of unity at f_0.

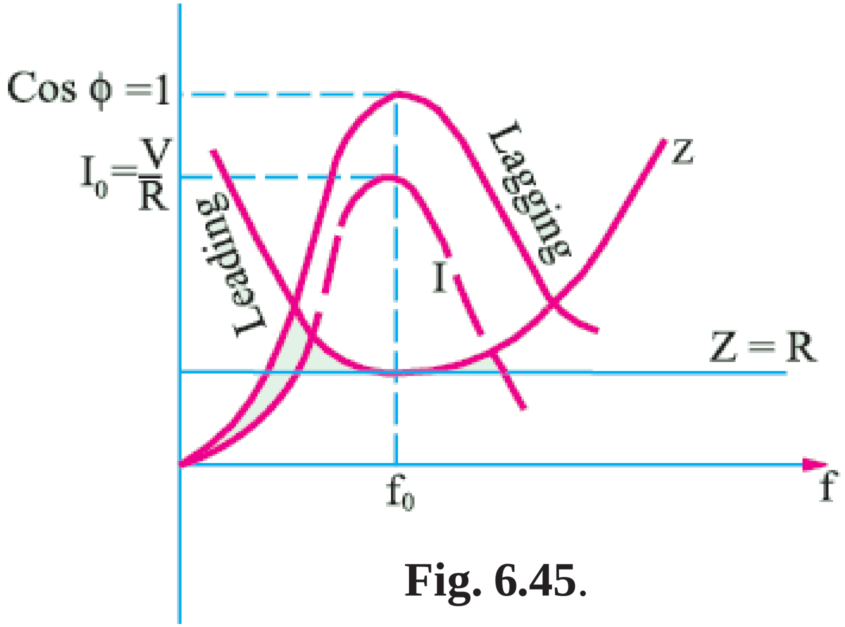


Fig. 6.45.

➢ Resonance in R-L-C Circuits

❖ Resonance Curve

- The curve between circuit current and the frequency of the applied voltage is known as resonance curve.
- The shapes of resonance curve for different values of *R* are shown in **Fig. 6.46**.
- The resonance curve for smaller values of *R* is sharply peaked and this circuit is said to be sharply resonant or highly selective.
- The resonance curve is flat for larger values of *R* and this curve is to have poor selectivity.
- The ability of a resonant circuit to discriminate between one particular frequency and all others is called its selectivity.

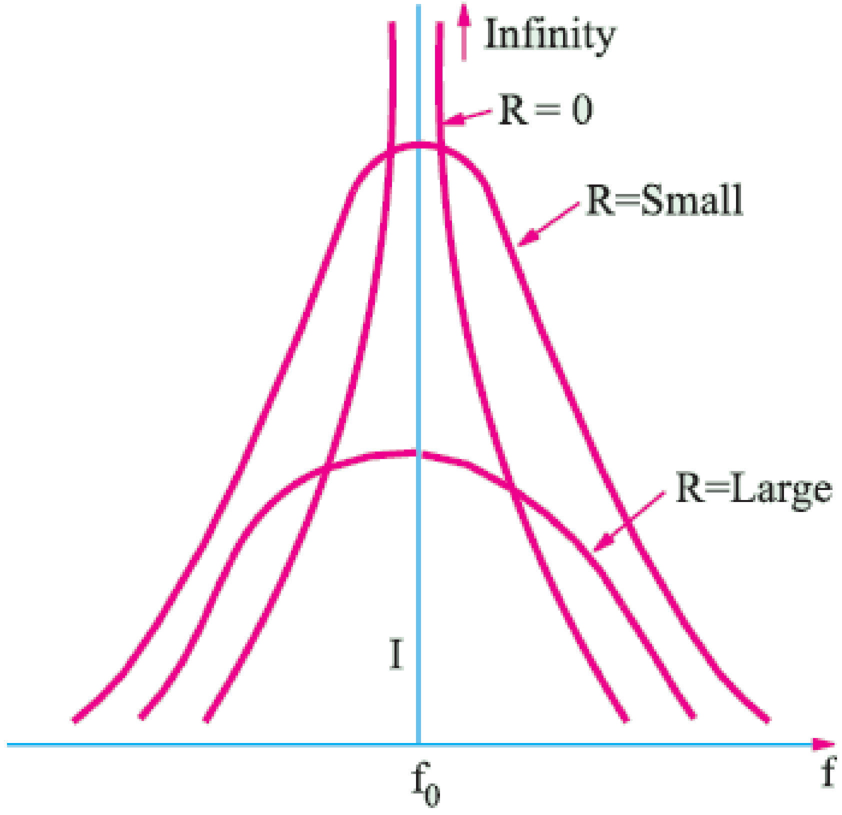


Fig. 6.46

- The selectivities of different resonant circuits are compared in terms of their power bandwidths.

A.C. Fundamentals

➢ Resonance in R-L-C Circuits

❖ Half-Power Bandwidth of a Resonant-Circuit

- The maximum current in an *R-L-C* circuit at resonance is determined by circuit resistance R ($\because X = 0$) but at off-resonance frequencies, the current amplitude depends on Z (where $X \neq 0$).

- The half-wave bandwidth of a circuit is given by the band of frequencies which lies between two points on either side of f_0 where current falls to $I_0/\sqrt{2}$.

- Narrower the bandwidth, higher the selectivity of the circuit and vice versa.

- The half-power bandwidth *AB* as shown in **Fig. 6.47** is given by

$$AB = \Delta f = f_2 - f_1 \qquad \text{or,} \qquad AB = \Delta\omega = \omega_2 - \omega_1$$

where f_1 and f_2 are the corner or edge frequencies.

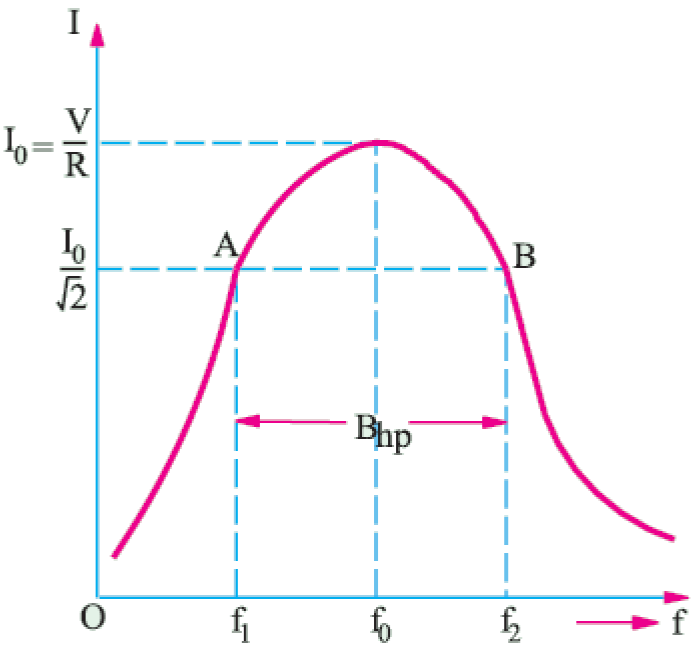


Fig. 6.47

A.C. Fundamentals

➢ Resonance in R-L-C Circuits

❖ Half-Power Bandwidth of a Resonant-Circuit

Maximum power is $P_0 = I^2R$

Power at either of the two points *A* and *B* is $P_1 = P_2 = I^2R$

$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_0^2 R \;= \frac{1}{2} \times \text{ power at resonance}$$

So, the two points *A* and *B* on the resonance curve are known as half-power points and the corresponding value of the bandwidth is called half-power bandwidth B_{hp}.

The decibel power responses at these points, in terms of the maximum power at resonance is

$$10\log_{10} P/P_0 = 10\log_{10}\frac{I_m^2 R/2}{I_m^2 R} = 10\log_{10}\frac{1}{2} = -10\log_{10} 2 = -3\ dB$$

So, it is also called 3*dB* bandwidth.

A.C. Fundamentals

➢ Resonance in R-L-C Circuits

❖ Determination of Upper and Lower Half-power Frequencies

It is seen from **Fig. 6.44** that at lower half-power frequencies, $\omega_1 < \omega_0$ so that $\omega_1 L < 1/\omega_1 C$ and $\phi = -45°$

$$\therefore \quad \frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{or} \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

Putting $\frac{\omega_0}{Q_0} = \frac{R}{L}$ and $\omega_0^2 = \frac{1}{LC}$ We get, $\omega_1^2 + \frac{\omega_0}{Q_0}\omega_1 - \omega_0^2 = 0$

The solution of the equation is $\omega_1 = \omega_0\left[\sqrt{\left(1 + \frac{1}{4Q_0^2}\right)} - \frac{1}{2Q_0}\right]$

Now at the upper half-power frequency, $\omega_2 > \omega_0$ so that $\omega_2 > 1/\omega_2 C$ and $\phi = +45°$

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{or} \quad \omega_2^2 - \frac{\omega_0}{Q_0}\omega_2 - \omega_0^2 = 0$$

A.C. Fundamentals

➢ **Resonance in R-L-C Circuits**

The solution of the equation is $\omega_2 = \omega_0\left[\sqrt{\left(1+\frac{1}{4Q_0^2}\right)}+\frac{1}{2Q_0}\right]$

If $Q_0 > 10$; then the term $1/4Q_0^2$ is negligible as compared to 1.

Hence, $\omega_1 \cong \omega_0\left(1-\frac{1}{2Q_0}\right)$ and $\omega_2 \cong \omega_0\left(1+\frac{1}{2Q_0}\right)$

$$\therefore \quad \omega_2 - \omega_1 = \omega_0/Q_0$$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

  - **Values of Edge Frequencies**

The current at resonance $I_0 = \dfrac{V}{R}$;

The current at any frequency $I = \dfrac{V}{[R^2 + X^2]^{1/2}}$

At points A and B, $I = \dfrac{I_0}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} . \dfrac{V}{R}$

$$\therefore \quad \frac{1}{\sqrt{2}} . \frac{V}{R} = \frac{V}{[R^2 + X^2]^{1/2}}$$

or, $2R^2 = R^2 + X^2$

$\therefore \quad R = X$

So, the net reactance is equal to the resistance

The power factor (p.f.) of the circuit at points A and B is $1/\sqrt{2} = 0.707$. The power factor is leading at point A and lagging at point B.

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

Hence, at point A,

$$R = \frac{1}{\omega_1 C} - \omega_1 L$$

or, $$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

or, $$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(-\frac{1}{LC}\right)}}{2}$$

or, $$\omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

or, $$\omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2} \quad \left[\because \omega_0 = \frac{1}{\sqrt{LC}}\right]$$

or, $$\omega_1 = -\alpha \pm \sqrt{(\alpha)^2 + \omega_0^2} \quad \left[\because \alpha = \frac{R}{2L}\right]$$

$\therefore$ $$\omega_1 = (\omega_0 - \alpha) \text{ rad/s} \quad [\because \omega_0 \gg \alpha]$$

$\therefore$ $$f_1 = f_0 - \frac{R}{4\pi L} \text{ Hz}$$

➢ Resonance in R-L-C Circuits

Hence, at point $B,$

$$R = \omega L - \frac{1}{\omega C}$$

or, $$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

or, $$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(-\frac{R}{L}\right)^2 - 4\left(-\frac{1}{LC}\right)}}{2}$$

or, $$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

or, $$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2} \quad \left[\because \omega_0 = \frac{1}{\sqrt{LC}}\right]$$

or, $$\omega_2 = \alpha \pm \sqrt{(\alpha)^2 + \omega_0^2} \quad \left[\because \alpha = \frac{R}{2L}\right]$$

$\therefore$ $$\omega_2 = (\omega_0 + \alpha) \text{ rad/s} \quad [\because \omega_0 \gg \alpha]$$

$\therefore$ $$f_2 = f_0 + \frac{R}{4\pi L} \text{ Hz}$$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

Half-power bandwidth

$$\therefore\ \Delta\omega = \omega_2 - \omega_1 = 2\alpha = 2 \times \frac{R}{2L} = \frac{R}{L} \quad \text{rad/s}$$

$$\therefore\ \Delta f = f_2 - f_1 = \frac{R}{2\pi L} \quad \text{Hz}$$

$$\therefore\ \omega_1 = \omega_0 - \frac{1}{2}\Delta\omega \quad \text{rad/s} \qquad \text{and} \qquad \omega_2 = \omega_0 + \frac{1}{2}\Delta\omega \quad \text{rad/s}$$

$$\therefore\ \omega_1\,\omega_2 = \omega_0^2 - \frac{1}{4}(\Delta\omega)^2$$

$$\text{or,}\quad \omega_0^2 = \omega_1\omega_2 \qquad \left[\because\ \omega_0^2 \gg \frac{1}{4}(\Delta\omega)^2\right]$$

$$\therefore\ \omega_0 = \sqrt{\omega_1\omega_2} \quad \text{rad/s}$$

$$\therefore\ f_0 = \sqrt{f_1 f_2} \quad \text{Hz}$$

- **Resonance in R-L-C Circuits**

  - ✓ **Q-Factor of a Resonant Series Circuit**

The *Q*-factor of an *R-L-C* series circuit is defined in the following different ways.

(i) It is given by the voltage magnification produced in the circuit at resonance.

It is seen that at resonance current has maximum value, $I_0 = V/R$.

Voltage across coil $V_{L0} = I_0\ X_{L0}$; Voltage across capacitor $V_{C0} = I_0 X_{C0}$

and Supply voltage, $V = I_0 R$

$$\therefore \text{ Voltage magnification } = \frac{V_{L0}}{V} = \frac{I_0 X_{L0}}{I_0 R} = \frac{\text{reactive powere}}{\text{active power}} = \frac{X_{L0}}{R} = \frac{\omega_0 L}{R}$$

$$= \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{\text{reactive powere}}{\text{active power}} = \frac{X_{C0}}{R} = \frac{1}{\omega_0 CR}$$

$\therefore$ Q −factor, $Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{2\pi f_0 L}{R} = \tan \emptyset,$ where $\emptyset$ is the power factor of the coil.

A.C. Fundamentals

➢ Resonance in R-L-C Circuits

✓ Q-Factor of a Resonant Series Circuit

(ii) The Q −factort is also defined as under.

$$\therefore \quad Q\text{ −factor} = 2\pi \times \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}}$$

$$= 2\pi \times \frac{\frac{1}{2}LI_0^2}{I^2RT} == 2\pi \times \frac{\frac{1}{2}L\left(\sqrt{2}I\right)^2}{I^2R(1/f)} = \frac{\omega_0 L}{R}$$

$$\therefore \quad Q\text{ −factor} = 2\pi \times \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}}$$

$$= 2\pi \times \frac{\frac{1}{2}CV_0^2}{I^2RT} == 2\pi \times \frac{\frac{1}{2}C(I_0X_C)^2}{\frac{1}{2}I_0^2R(1/f)} = \frac{1}{\omega_0 CR}$$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

  - **Q-Factor of a Resonant Series Circuit**

(iii) The Q −factort is also defined as under.

$$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Hence, Q-factor can be increased by having a coil of large inductance but of small resistance.

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\omega_0}{R/L} = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \qquad \therefore\ \Delta\omega = \frac{\omega_0}{Q_0}$$

$$\omega_1 = \omega_0 - \frac{R}{2L} = \omega_0 - \frac{1}{2}.\frac{\omega_0}{Q_0} = \omega_0\left(1 - \frac{1}{2Q_0}\right)$$

$$\omega_2 = \omega_0 + \frac{R}{2L} = \omega_0 + \frac{1}{2}.\frac{\omega_0}{Q_0} = \omega_0\left(1 + \frac{1}{2Q_0}\right)$$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

  - **Circuit current at frequencies other than resonant frequencies**

The current at resonance is $I_0 = \dfrac{V}{R}$

The current at other frequencies above resonant frequency is $I = \dfrac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$

This current lags behind the applied voltage by a certain angle $\emptyset$.

$$\therefore \frac{I}{I_0} = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}} \times \frac{R}{V} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega_0 L}{R}\right)^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} = \frac{1}{\sqrt{1 + Q_0^2 \left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2}}$$

A.C. Fundamentals

- **Resonance in R-L-C Circuits**

  - **Relation between resonant power P_0 and off-resonant power P**

The current in the series *RLC* circuit at the resonant frequency f_0 is maximum *i.e.* I_0 .

The maximum power that is dissipated by the circuit at the resonant frequency is P_0 where X_L equals X_C.

Hence, circuit impedance $Z_0 = R$.

$$\therefore P_0 = I_0^2 R = \left(\frac{V}{R}\right)^2 \times R = \frac{V^2}{R}$$

At any other frequency either above or below f_0 the power is

$$P = I^2 R = \left(\frac{V}{Z}\right)^2 \times R = \frac{V^2 R}{R^2 + X^2} = \frac{V^2 R}{R^2 + R^2 Q^2} = \frac{V^2 R}{R^2(1+Q^2)} = \frac{V^2}{R(1+Q^2)} = \frac{P_0}{(1+Q^2)}$$

The circuit power P at any frequency other than f_0 is reduced by a factor of $(1 + Q^2)$, where Q is the tangent of the circuit phase angle.

At resonance, circuit phase angle θ = 0, and $Q = \tan \emptyset = 0$.

Hence, $P = P_0 = V^2/R$.

Example – P6.5

A series RLC circuit has a Q_0 of 5.1 at its resonance frequency of 1000 kHz. Assuming the power dissipation of the circuit is 100 W when drawing a current of 0.8 A, determine the circuit parameters. Also find the bandwidth of the circuit.

Solution of Example – P6.5

The power dissipation of the circuit, $P_R = I^2 R$

or, $100 = (0.8)^2\ R$

$\therefore \quad R = 156\ \Omega$

The quality-factor of the circuit, $Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{2\pi f_0 L}{R}$

or, $5.1 = \dfrac{2\pi \times (1000 \times 10^3)L}{156}$

$\therefore \quad L = 0.126$ mH

The resonant frequency of series RLC circuit, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$\therefore \quad C = \frac{1}{4\pi^2 L f_0} = \frac{1}{4\pi^2 \times (0.126 \times 10^{-3}) \times (1000 \times 10^3)} = 0.201 \text{ nF}$$

The quality-factor of the circuit,

$$Q_0 = \frac{\omega_0}{BW}$$

$$\therefore \quad BW = \frac{\omega_0}{Q_0} \quad \text{rad/s}$$

$$= \frac{f_0}{Q_0} \quad \text{Hz}$$

$$= \frac{1000 \times 10^3}{5.1} \quad \text{Hz}$$

$$= 196 \text{ kHz}$$

A.C. Fundamentals

- **Parallel A.C. Circuits**
  - ✓ **Solving Parallel Circuits**

There are three methods to solve parallel circuits:

(a) Vector or phasor Method

(b) Admittance Method and

(c) Vector Algebra

A.C. Fundamentals

➢ Parallel A.C. Circuits

❖ Vector or Phasor Method

- Consider a circuit as shown in **Fig. 6.48** consisting of two reactors *A* and *B*, joined in parallel across an supply of *V* volts.

- The voltage across two parallel branches *A* and *B* is the same, but the currents through them are different.

For Branch *A*, $Z_1 = \sqrt{R_1^2 + X_L^2}$ $\qquad I_1 = V/Z_1$

$$\cos\emptyset_1 = \frac{R_1}{Z_1} \qquad \therefore\ \emptyset_1 = \cos^{-1}\frac{R_1}{Z_1}$$

Current $\boldsymbol{I_1}$ lags behind the applied voltage by $\emptyset_1$ as shown in **Fig. 6.49**

For Branch *B*, $Z_2 = \sqrt{R_2^2 + X_C^2}$ $\qquad I_2 = V/Z_2$

$$\cos\emptyset_2 = \frac{R_2}{Z_2} \qquad \therefore\ \emptyset_2 = \cos^{-1}\frac{R_2}{Z_2}$$

Current $\boldsymbol{I_2}$ lags behind the applied voltage by $\emptyset_2$ as shown in **Fig. 6.49**

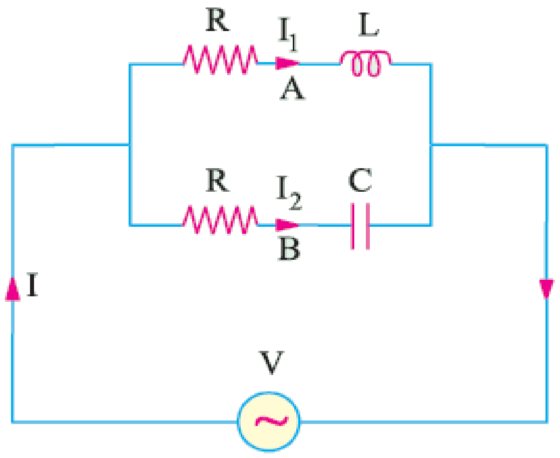


Fig. 6.48

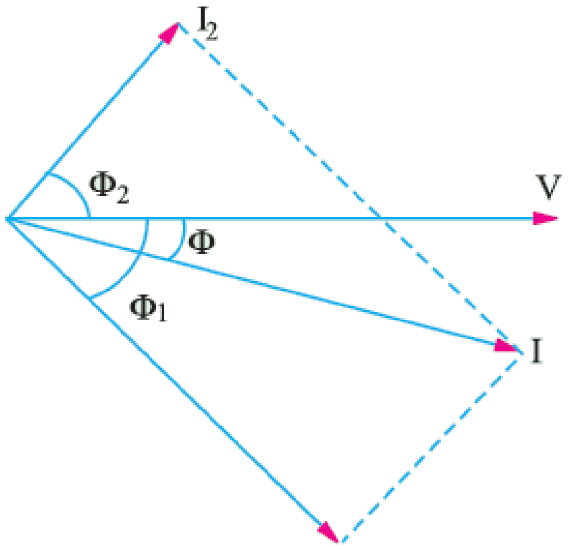


Fig. 6.49

➢ Parallel A.C. Circuits

❖ Resultant Current I

The resultant circuit current I is the vector sum of the branch currents I_1 and I_2 and it is found by

(i) using parallelogram law of vectors as shown in **Fig. 6.49** or

(ii) resolving I_2 into their X- and Y-components (or active and reactive components respectively) and then by combining these components, as shown in **Fig. 6.50.**

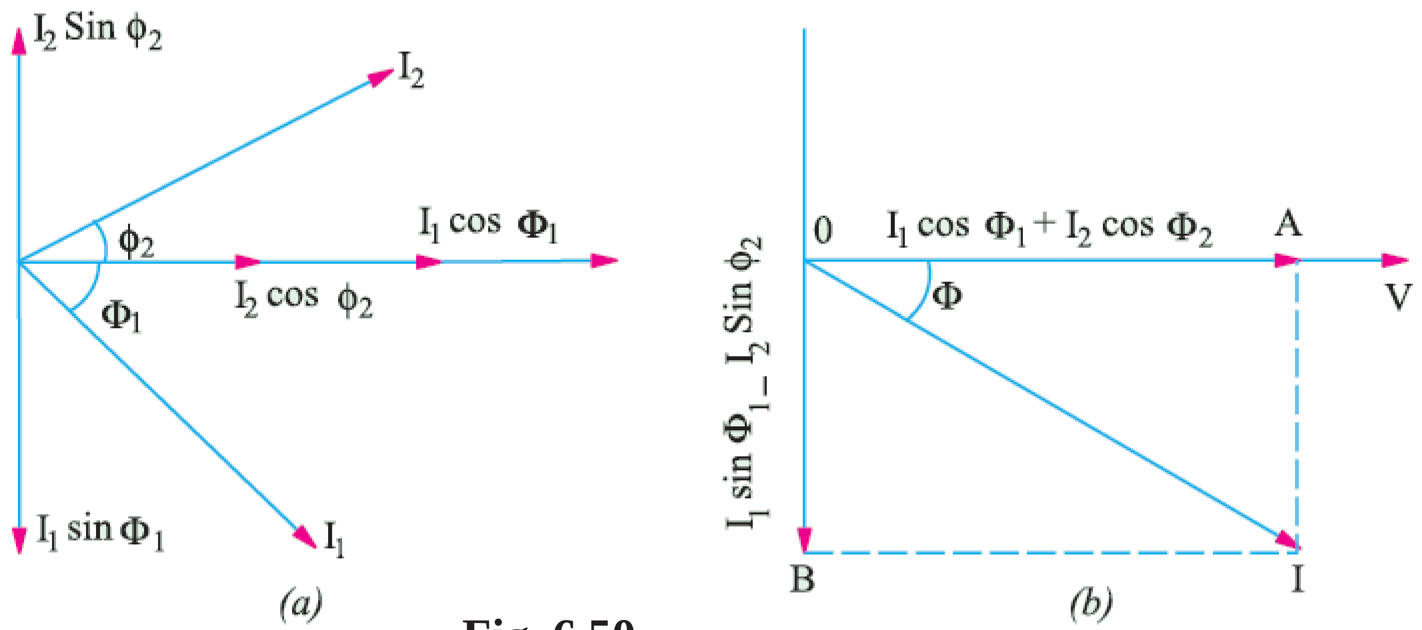


Fig. 6.50.

A.C. Fundamentals

- **Parallel A.C. Circuits**

Sum of the active components of I_1 and I_2 is $(I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2)$

Sum of the reactive components of I_1 and I_2 is $(I_2 \sin\emptyset_2 - I_1 \sin\emptyset_1)$

If I is the resultant current and $\emptyset$ is its phase, then its active and reactive components are equal to the X-and Y-components respectively.

$$I\cos\emptyset = I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2 \qquad \text{and} \qquad I\sin\emptyset = I_2 \sin\emptyset_2 - I_1 \sin\emptyset_1$$

$$\therefore \qquad I = \sqrt{(I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2)^2 + (I_2 \sin\emptyset_2 - I_1 \sin\emptyset_1)^2}$$

$$\text{and} \qquad \tan\emptyset = \frac{I_2 \sin\emptyset_2 - I_1 \sin\emptyset_1}{I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2} = \frac{\text{X−component}}{\text{Y−component}}$$

If $\tan\emptyset$ is positive, then current leads and if $\tan\emptyset$ is negative, then current lags behind the applied voltage V.

Power factor for the whole circuit is given by $\cos\emptyset = \dfrac{I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2}{I} = \dfrac{X - \text{comp}}{I}$

A.C. Fundamentals

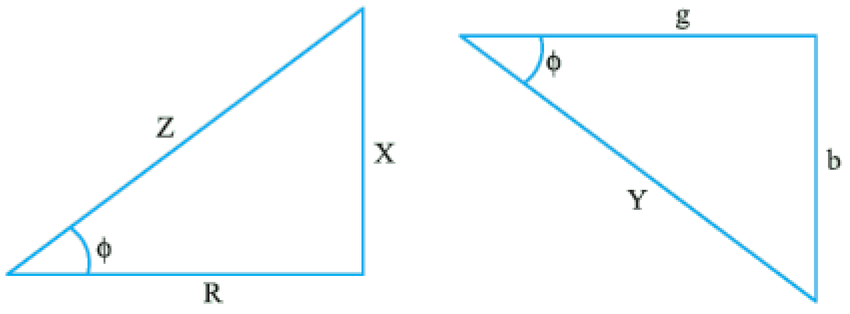
➢ Parallel A.C. Circuits

❖ Admittance Method

Admittance of a circuit is defined as the reciprocal of its impedance. Its symbol is Y.

$$\therefore \quad Y = \frac{1}{Z} = \frac{I}{V} \qquad \text{or,} \qquad Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. } volts}$$

Its unit is Siemens (S). The another unit of it is mho.



(a) (b)

Fig. 6.51

The impedance Z of a circuit as shown in **Fig. 6.51** (a) has two components X and R.

Similarly, The admittance Y of a circuit as shown in **Fig. 6.51** (b) has two components. The X-component is known as conductance and Y-component is known as susceptance.

Conductance, $g = Y\cos\emptyset = \frac{1}{Z}.\frac{R}{Z} = \frac{R}{R^2 + X^2}$ Susceptance, $b = Y\sin\emptyset = \frac{1}{Z}.\frac{X}{Z} = \frac{X}{R^2 + X^2}$

Admittance, $Y = \sqrt{(g^2 + b^2)}$ The unit of g, b and Y are in Siemens.

Capacitive susceptance is positive and inductive susceptance is negative.

A.C. Fundamentals

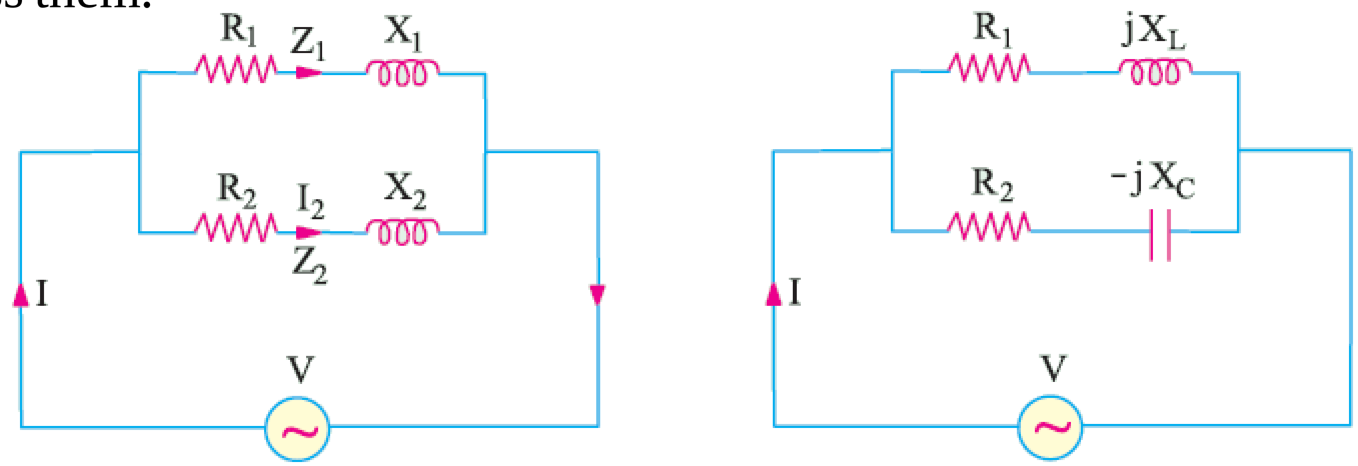
➢ Parallel A.C. Circuits

❖ Complex or Phasor Algebra

Consider a parallel circuit as shown in **Fig. 6.52** (a) consisting of two parallel impedances Z_1 and Z_2. So, they have the same p.d. across them.

Now $\quad I_1 = \dfrac{V}{Z_1}$

and $\quad I_2 = \dfrac{V}{Z_2}$



(a) $\quad$ (b)

Fig. 6.52

Total current $\quad I = I_1 + I_2$

$$= \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

$$= V(Y_1 + Y_2) = VY$$

Where, Y = total admittance = $Y_1 + Y_2$

A.C. Fundamentals

- **Parallel A.C. Circuits**

  - **Complex or Phasor Algebra**

Consider also another parallel circuit as shown in **Fig. 6.52**(b) consisting of two parallel impedances.

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} = \frac{R_1 - jX_L}{R_1^2 + X_L^2} = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

where $g_1 = \dfrac{R_1}{R_1^2 + X_1^2}$ — conductance of upper branch

$b_1 = \dfrac{X_L}{R_1^2 + X_1^2}$ — susceptance of upper branch

Similarly,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{(R_2 + jX_C)}{(R_2 - jX_C)(R_2 + jX_C)} = \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2} = g_2 + jb_2$$

A.C. Fundamentals

➢ Parallel A.C. Circuits

❖ Complex or Phasor Algebra

Total admittance, $Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 + jb_2) = (g_1 + g_2) - j(b_1 - b_2) = G - jB$

$\therefore |Y| = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}$ and $\emptyset = \tan^{-1}\left(\frac{b_1 - b_2}{g_1 + g_2}\right) = \tan^{-1}\frac{B}{G}$

The polar form of admittance is $Y = |Y|\angle\emptyset^0 = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}\ \angle\tan^{-1}\frac{B}{G}$

Total current $I = VY;$ $I_1 = VY_1$ and $I_2 = VY_2$

If $V = |V|\angle 0^0$ and $Y = |Y|\angle\emptyset^0$ Then, $I = VY = |V|\angle 0^0 \times |Y|\angle\emptyset^0 = VY\angle\emptyset^0$

If $V = |V|\angle\alpha$ and $Y = |Y|\angle\beta$ Then, $I = VY = |V|\angle\alpha \times |Y|\angle\beta = VY\angle(\alpha + \beta)$

Example – P6.6

In an electrical circuit as shown in **Fig. P6.6**, two parallel impedances draw currents I_1 and I_2 when $I_1 = 40\angle 20^0$ A, $I_2 = 30\angle -65^0$ A. If the supply voltage be $100\angle 0^0\ V,$ obtain the values of line current and power factor. What is the input power?

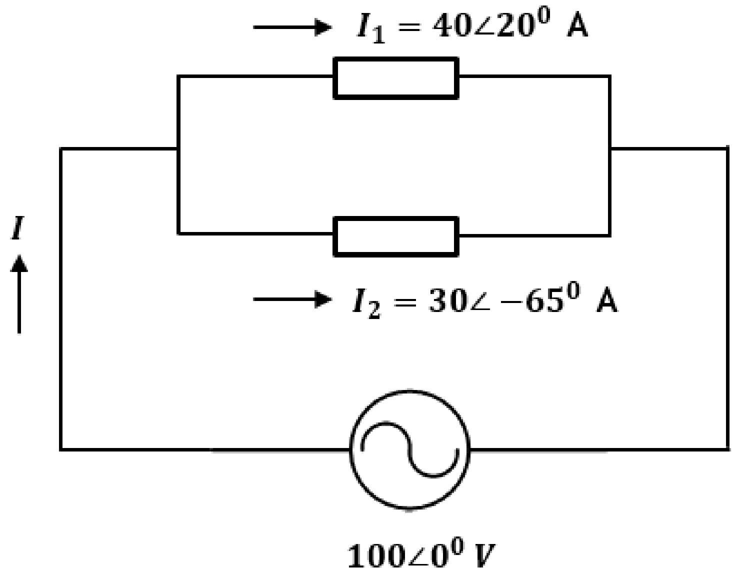


Fig. P6.6

Solution of Example – P6.6

Given

$$I_1 = 40\angle 20^0 \text{ A} \qquad \text{and} \qquad I_2 = 30\angle -65^0 \text{ A}$$

$$\therefore \quad I = I_1 + I_2 = 40\angle 20^0 + 30\angle -65^0 \text{ A}$$

$$= 37.587 + j13.680 + 12.678 - j27.189 \text{ A}$$

$$= 50.265 - j13.569 \text{ A}$$

$$= 52.048\angle -15.043^0$$

$\therefore$ The line current $= 52.048$ A

$\therefore$ The power factor $= \cos 15.043^0 = 0.965$

$\therefore$ Input power $= VI\cos\emptyset = 100 \times 52.048 \times 0.965 = 5.022$ kW

Example – P6.7

Determine the total current, branch currents, active and reactive power from the supply for the circuit as shown in **Fig. P6.7.**

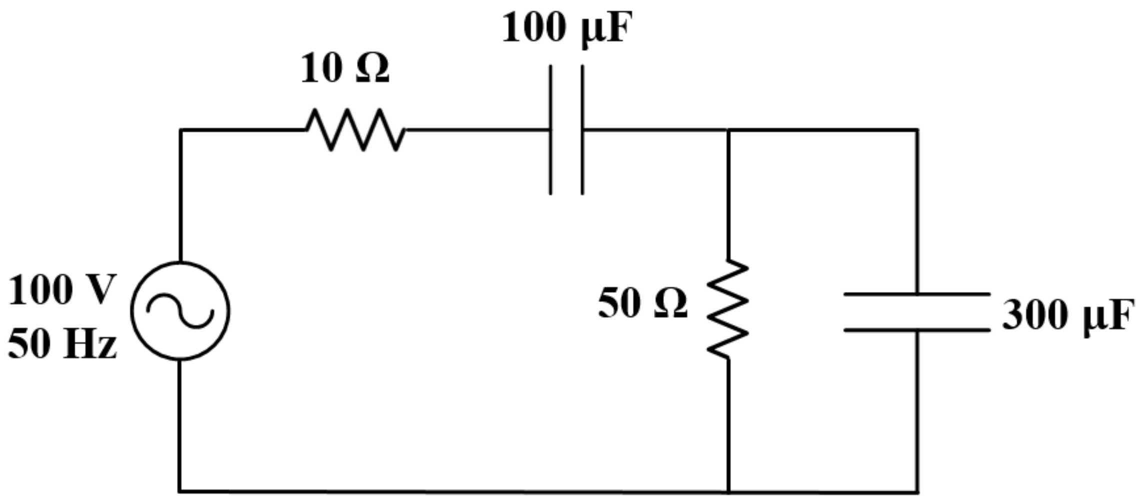


Fig. P6.7

LECTURE 16

Parallel A.C. Circuits

Resonance in Parallel Circuits

A coil is connected in parallel with a capacitor as shown in **Fig. 6.53**.

The circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero.

The frequency at which this happens is known as resonant frequency.

The vector diagram for this circuit is shown in **Fig. 6.54**.

Net reactive or wattless component is

$$I_C - I_L \sin \phi_L = 0$$

or, $$I_L \sin \phi_L = I_C$$

Now, $I_L = \dfrac{V}{Z}$ $\quad \sin \phi_L = \dfrac{X_L}{Z}$ $\quad$ and $\quad I_C = \dfrac{V}{X_C}$

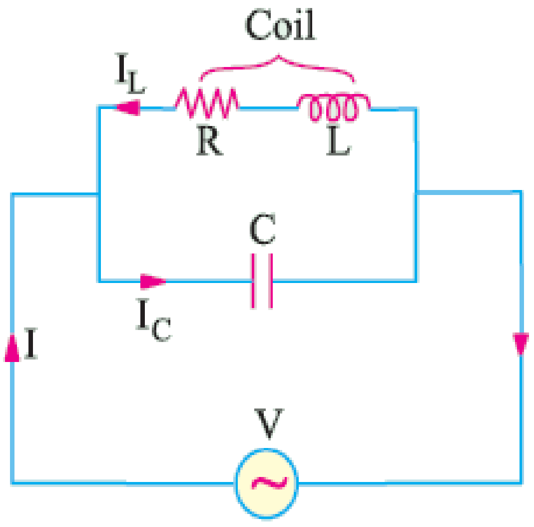


Fig. 6.53

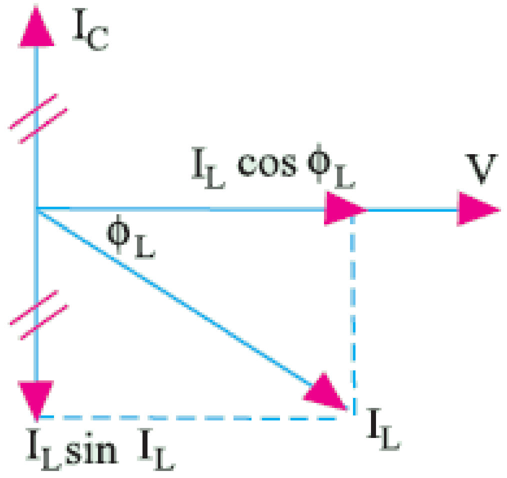


Fig. 6.54

A.C. Fundamentals

➢ Parallel A.C. Circuits

❖ Resonance in Parallel Circuits

Hence, condition for resonance becomes

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C} \qquad \text{or,} \quad X_L \times X_C = Z^2$$

$$\text{or,} \quad \omega L \times \frac{1}{\omega C} = R^2 + X_L^2$$

$$\text{or,} \quad \frac{L}{C} = R^2 + (2\pi f_0 L)^2$$

$$\therefore \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This is the resonant frequency and is given in Hz if R is in ohm, L is in henry and C is in farad.

If R is negligible, then $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

➢ Parallel A.C. Circuits

❖ Current at Resonance

Since, wattles component of the current is zero, the circuit current is

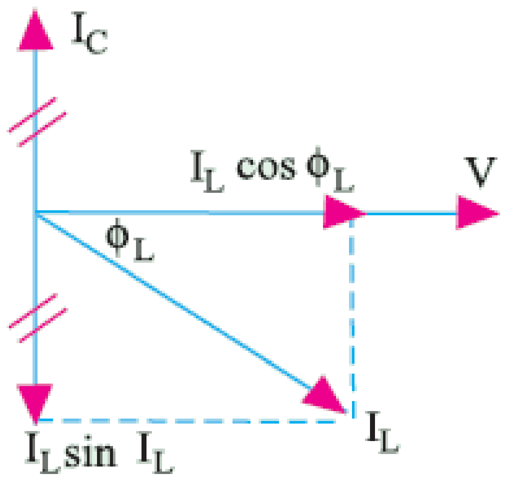


Fig. 6.54

$$I = I_L \cos \emptyset_L = \frac{V}{Z}.\frac{R}{Z} = \frac{VR}{Z^2} = \frac{VR}{L/C} = \frac{V}{L/CR}$$

The ratio L/CR is known as the equivalent or dynamic impedance of the parallel circuit at resonance and the impedance is 'resistive' only.

Since current is minimum at resonance, L/CR represent the maximum impedance of the circuit.

Current at parallel resonance is minimum. So, the circuit (when used in radio work) is sometimes known as rejector circuit because it rejects (or takes minimum current of) that frequency to which it resonates.

This resonance is often referred to as current resonance because the current circulating between the two branches is many times greater than the line current taken from the supply.

➢ Parallel A.C. Circuits

❖ Resonance in Parallel Circuits

The variations of impedance and current with frequency are shown in **Fig. 6.55.**

It is seen from the figure that impedance at resonant frequency is maximum and equals L/CR.

The current at resonance is minimum and equals $V\,/\,(L/CR)$.

The impedance decreases and current increases at off-resonance frequencies.

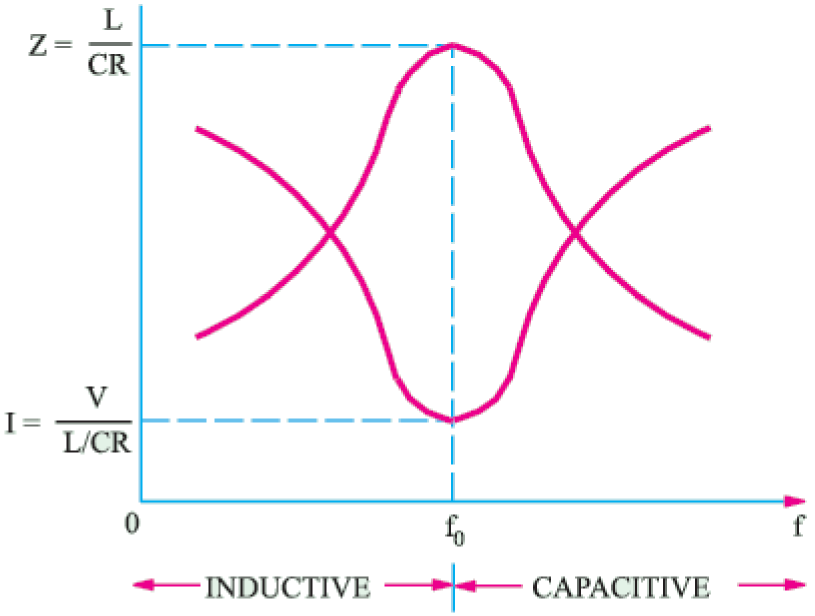


Fig. 6.55.

A.C. Fundamentals

- **Parallel A.C. Circuits**

❖ Resonance in Parallel Circuits

Alternate Method

$$Y_1 = \frac{1}{R + jX_L} = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}; \qquad Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C}$$

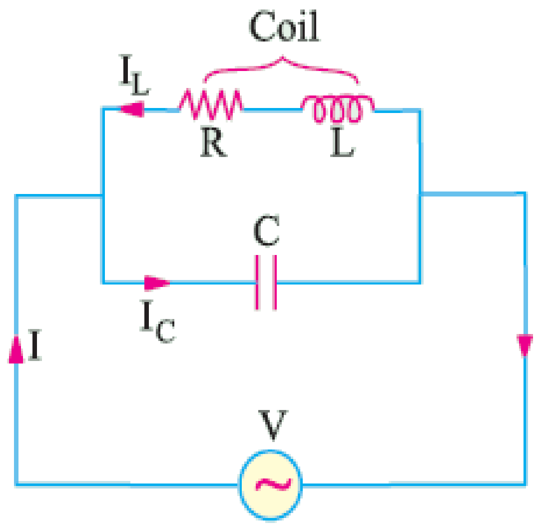


Fig. 6.53

$$Y = \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

Now, circuit would be in resonance when *j*-component of the complex admittance is zero.

$$\therefore \quad \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$$

or, $\quad R^2 + (2\pi f_0 L)^2 = X_L X_C \qquad \therefore \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

A.C. Fundamentals

- **Parallel A.C. Circuits**

❖ Resonance in Parallel Circuits

The relations are in terms of susceptance as under

Inductive susceptance, $B_L = \dfrac{X_L}{R^2 + X_L^2}$ Capacitive susceptance, $B_C = \dfrac{1}{X_C}$

Net susceptance, $B = B_C - B_L$ $\therefore \quad Y = G + j(B_C - B_L) = G + jB$

The parallel circuit is said to be in resonance when $B = 0$.

$$B_C - B_L = 0$$

or, $\dfrac{1}{X_C} = \dfrac{X_L}{R^2 + X_L^2}$ $\therefore \quad f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

or, $R^2 + (2\pi f_0 L)^2 = X_L X_C$

➢ Parallel A.C. Circuits

❖ Graphical Representation of Parallel Resonance

The effect of variation of frequency on the susceptance of parallel circuit of **Fig. 6.56** are shown in **Fig. 6.57.**

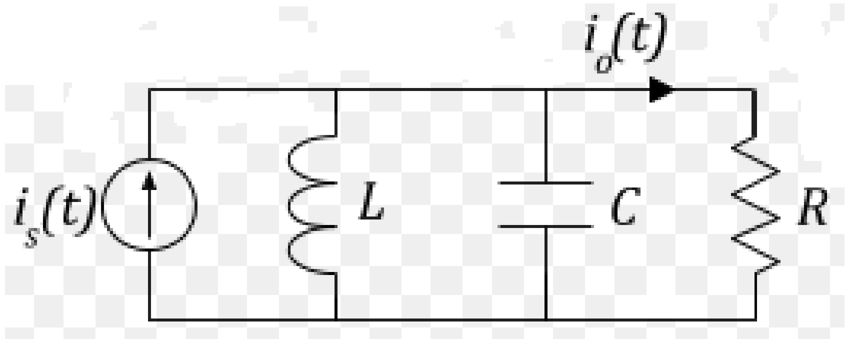


Fig. 6.56

- ✓ **Inductive susceptance:** $b = \frac{1}{jX_L} = -j\frac{1}{X_L} = -j\frac{1}{2\pi f L}$
  - It is inversely proportional to the frequency of the applied voltage.
  - Hence, it is represented by a rectangular hyperbola drawn in the fourth quadrant (∴ it is assumed negative).
- ✓ **Capacitive susceptance** $b = j\frac{1}{X_C} = j\omega C$
  - It increases with increase in the frequency of the applied voltage.
  - Hence, it is represented by a straight line drawn in the first quadrant (it is assumed positive).

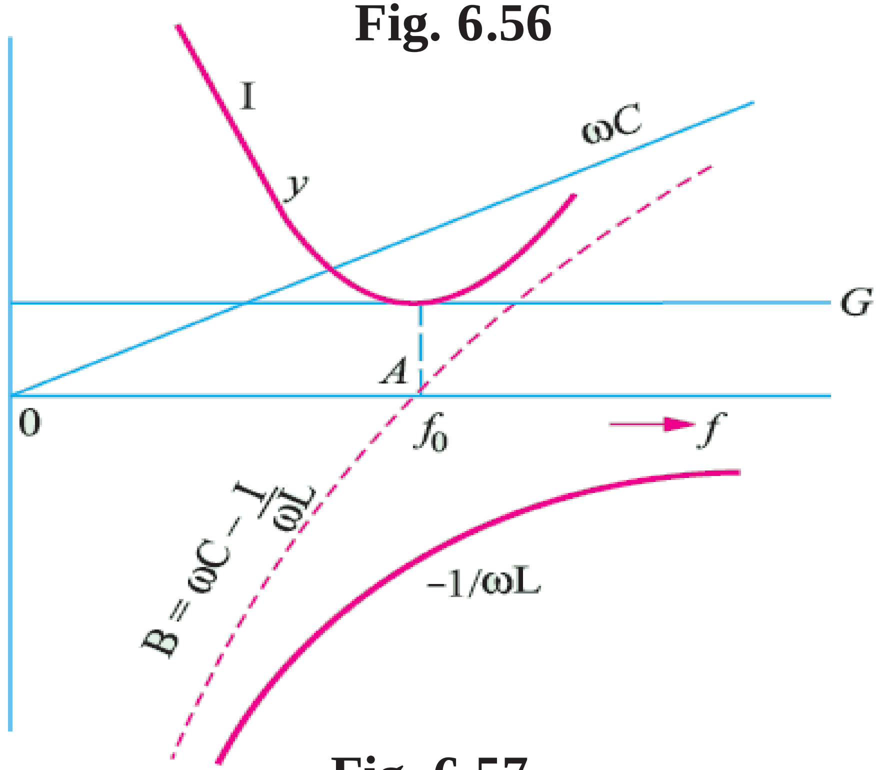


Fig. 6.57

A.C. Fundamentals

➢ Parallel A.C. Circuits

❖ Graphical Representation of Parallel Resonance

Net Susceptance

- It is the difference of the two susceptances as represented by the dotted hyperbola.
- Net susceptance at point A is zero. Hence, admittance is minimum and equal to G. So, the line current is minimum at point A.
- Inductive susceptance pre-dominates below resonant frequency (corresponding to point A). Hence, line current lags behind the applied voltage.
- Capacitive susceptance predominates above the resonant frequency. Hence, line current leads the applied voltage.

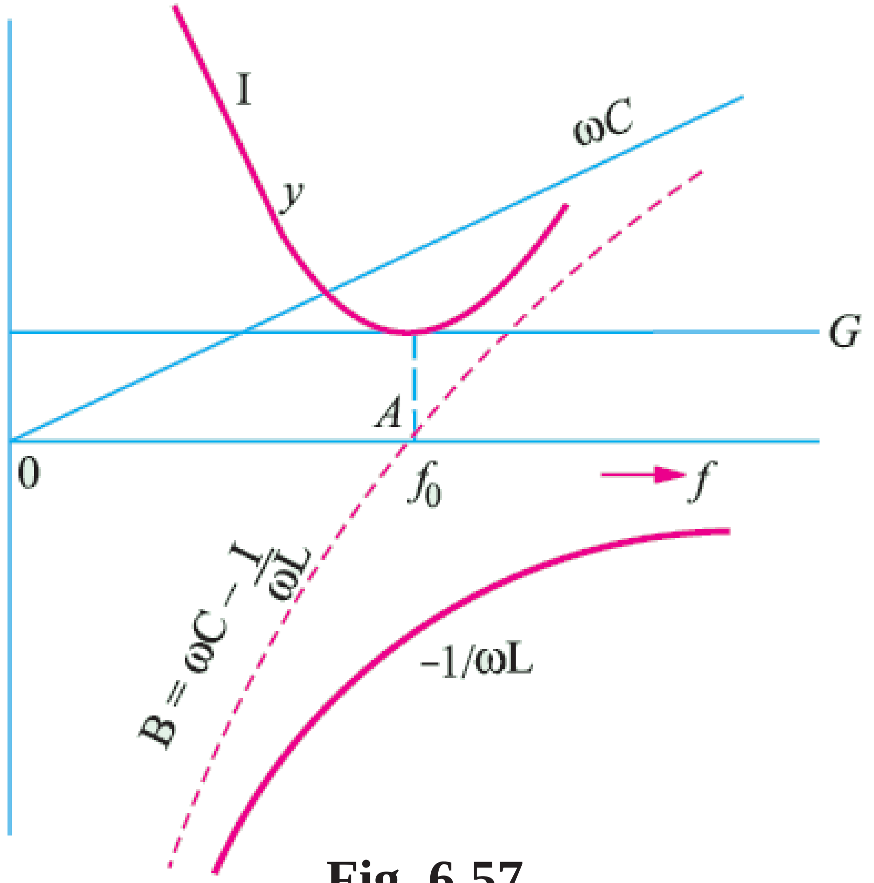


Fig. 6.57

➢ Parallel A.C. Circuits

❖ Bandwidth of a Parallel Resonant Circuit

A parallel circuit comprising of resistance R, inductance L and capacitance C is shown in **Fig. 6.58**

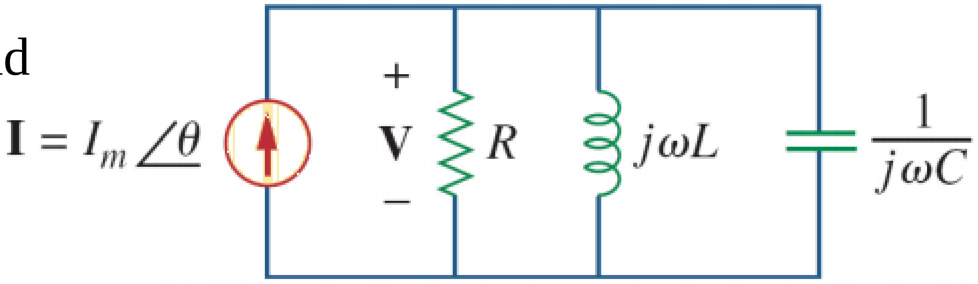


Fig. 6.58

The admittance of the circuit is

$$Y = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = G + jB$$

At resonance, $\omega C - \frac{1}{\omega L} = 0$ $\quad\therefore\ \omega_0 = \frac{1}{\sqrt{LC}}$ rad/s

At resonance, the parallel LC combination acts like an open circuit so that the entire current flows through R.

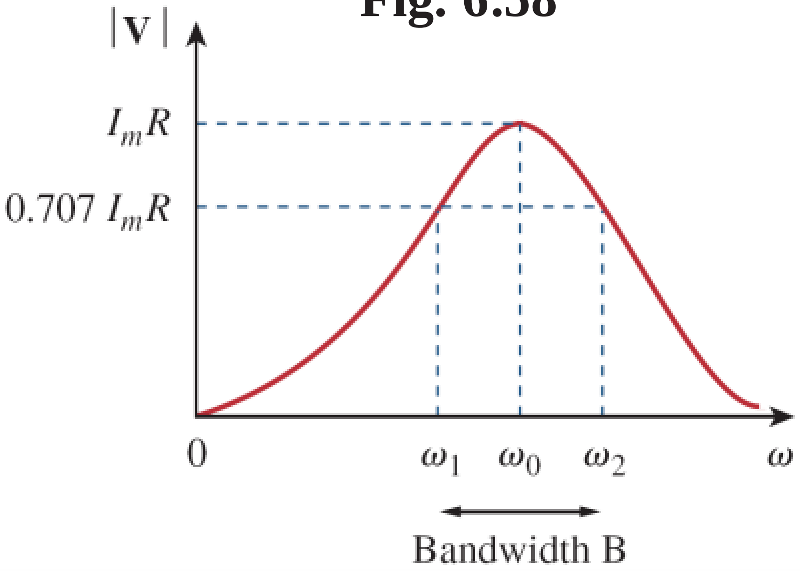


Fig. 6.59

A.C. Fundamentals

➢ Parallel A.C. Circuits

❖ Bandwidth of a Parallel Resonant Circuit

This circuit also has upper and lower half-power frequencies where power dissipation is half of that at resonant frequency like series RLC circuit.

$$\therefore \quad \frac{V^2}{R} = \frac{1}{2}.\frac{V_0^2}{R} \qquad \text{or,} \quad V = \frac{1}{\sqrt{2}}V_0 \qquad \text{or,} \quad \frac{I}{Y} = \frac{1}{\sqrt{2}}.\frac{I}{G} \qquad \text{or,} \quad Y = \sqrt{2}G$$

$$\text{or,} \quad Y^2 = 2G^2 \qquad \text{or,} \quad G^2 + B^2 = 2G^2 \qquad \therefore \quad G = B$$

Hence, $Y = \sqrt{G^2 + B^2} = \sqrt{2}\,G$ and $\emptyset = \tan^{-1}(B/G) = \tan^{-1}(1) = 45^0$

Hence, the net susceptance B equals the conductance G at resonance.

At f_2, $B = B_{C2} - B_{L2} = G$ or, $\omega_2 C - \dfrac{1}{\omega_2 L} = \dfrac{1}{R}$ or, $\omega_2^2 - \dfrac{1}{RC}\omega_2 - \dfrac{1}{LC} = 0$

$$\therefore \quad \omega_2 = \frac{\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 + \frac{4}{LC}}}{2} = \frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad \therefore \quad \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \quad [\because \ \omega_2 > 0\ ]$$

A.C. Fundamentals

- **Parallel A.C. Circuits**

  - **Bandwidth of a Parallel Resonant Circuit**

At f_1, $B = B_{L1} - B_{C1} = G$ or, $\dfrac{1}{\omega_1 L} - \omega_1 C = \dfrac{1}{R}$ or, $\omega_1^2 + \dfrac{1}{RC}\omega_1 - \dfrac{1}{LC} = 0$

$$\therefore \ \omega_1 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 + \frac{4}{LC}}}{2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\therefore \ \omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \quad [\because \ \omega_1 > 0\ ]$$

Half power band−width, $BW_{hp} = \omega_2 - \omega_1 = \dfrac{1}{RC}$

Quality factor, $Q = \dfrac{\omega_0}{BW_{hp}} = \omega_0 RC = \dfrac{R}{\omega_0 L}$

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}$$

$$\omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

For high Q values

$$\omega_1 \approx \omega_0 - \frac{\omega_0}{2Q} \qquad \omega_2 \approx \omega_0 + \frac{\omega_0}{2Q}$$

A.C. Fundamentals

➢ Parallel A.C. Circuits

✓ Q-factor of a Parallel Circuit

It is defined as the ratio of the circulating current through the inductance or capacitance to the line current drawn from the supply or simply, as the current magnification.

The circulating current between capacitor and coil branches is I_C.

Hence Q-factor = I_C/I

Now, $I_C = V/X_C = V/(1/\omega C) = \omega CV$ and $I = V/(L/CR)$

$$\therefore\ Q - \text{factor} = \frac{\omega CV}{V/(L/CR)} = \frac{\omega L}{R} = \tan \emptyset\,,$$ where $\emptyset$ is the power factor angle of the coil.

Now, $f_0 = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{LC}}$ $$\therefore Q = \frac{2\pi f_0 L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is seen that in series circuits, Q-factor gives the voltage magnification, whereas in parallel circuits, it gives the current magnification.

➢ Parallel A.C. Circuits

Following points about parallel resonance should be noted and compared with those about series resonance. At resonance.

1. net susceptance is zero *i.e.* $1/X_C = X_L/Z^2$ or $X_L \times X_C = Z^2$ or $L/C = Z^2$
2. the admittance equals conductance
3. reactive or wattless component of line current is zero.
4. dynamic impedance = L/CR ohm.
5. line current at resonance is minimum and $\dfrac{V}{L/CR}$ but is in phase with the applied voltage
6. power factor of the circuit is unity.

Example – P6.8

A coil of resistance 20 Ω and inductance 200 μH is in parallel with a variable capacitor. This combination is in series with a resistor of 8000 Ω. The voltage of the supply is 200 V at a frequency of 10^6 H . Calculate

(i) the value of C to give resonance

(ii) the Q of the coil

(iii) the current in each branch of the circuit at resonance.

Solution of Example – P6.7

The circuit is shown in **Fig. P6.7**.

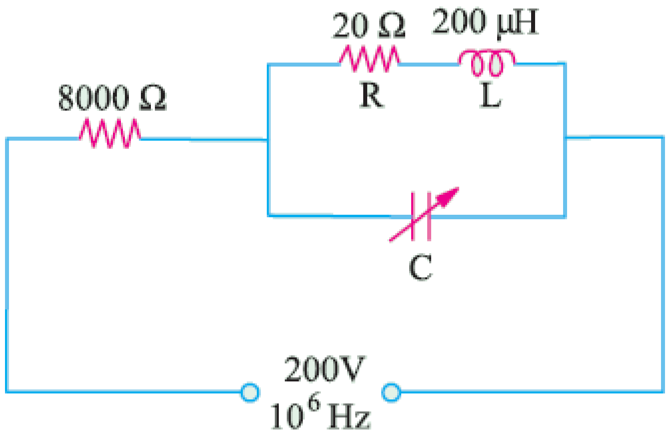


Fig. P6.7.

$X = 2\pi fL = 2\pi \times 10^6 \times 200 \times 10^{-6} = 1256\,\Omega$

Since coil resistance is negligible as compared to its reactance, the resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{or,} \quad 10^6 = \frac{1}{2\pi\sqrt{200 \times 10^{-6} \times C}}$$

(i) $\therefore \quad C = 125\ \mu\mu\text{F}$

(ii) $$Q = \frac{2\pi f L}{R} = \frac{2\pi \times 10^6 \times 200 \times 10^{-4}}{20} = 62.8$$

Solution of Example – P6.8

(iii) Dynamic resistance of the coil is

$$\frac{L}{CR} = \frac{200 \times 10^{-6}}{125 \times 10^{-12} \times 20} = 80{,}000\ \Omega$$

Total equivalent resistance of the tuned circuit is 80,000 + 8,000 = 88,000 Ω

P.D. across tuned circuit = current × dynamic resistance = 2.27 × 10 × 80,000 = 181.6 V

Current through inductive branch $= \dfrac{181.6}{\sqrt{10^2 + 1256^2}} = 0.1445\ \text{A} = 144.5\ \text{mA}$

Current through capacitor branch is

$$\frac{V}{1/\omega C} = \omega CV = 181.6 \times 2\pi \times 10^6 \times 125 \times 10^{-12} = 142.7\ \text{mA}$$

Comparison of Series and Parallel Resonant Circuits

item	*series circuit (R–L–C)*	*parallel circuit (R–L and C)*
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum = V/R	Minimum = $V/(L/CR)$
Effective impedance	R	L/CR
Power factor at resonance	Unity	Unity
Resonant frequency	$1/2\pi\sqrt{(LC)}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}-\frac{R^2}{L^2}\right)}$
It magnifies	Voltage	Current
Magnification is	$\omega L/R$	$\omega L/R$


Thank you