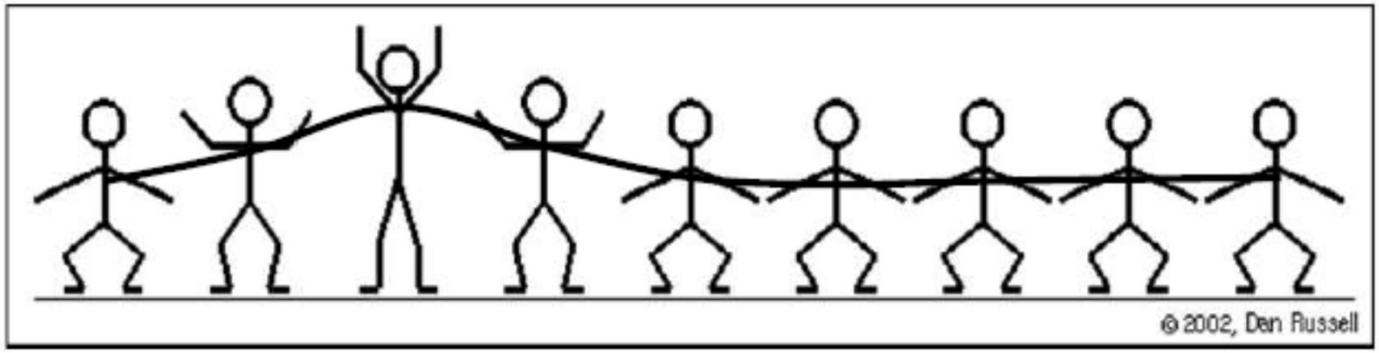
Waves and Wave Properties

What is a wave?

Answer: A wave is a disturbance that carries energy from place to place.



A wave does NOT carry matter with it! It just moves the matter as it goes through it.

Think of a stadium wave: the people are moving up and down, but the wave goes *around* the stadium

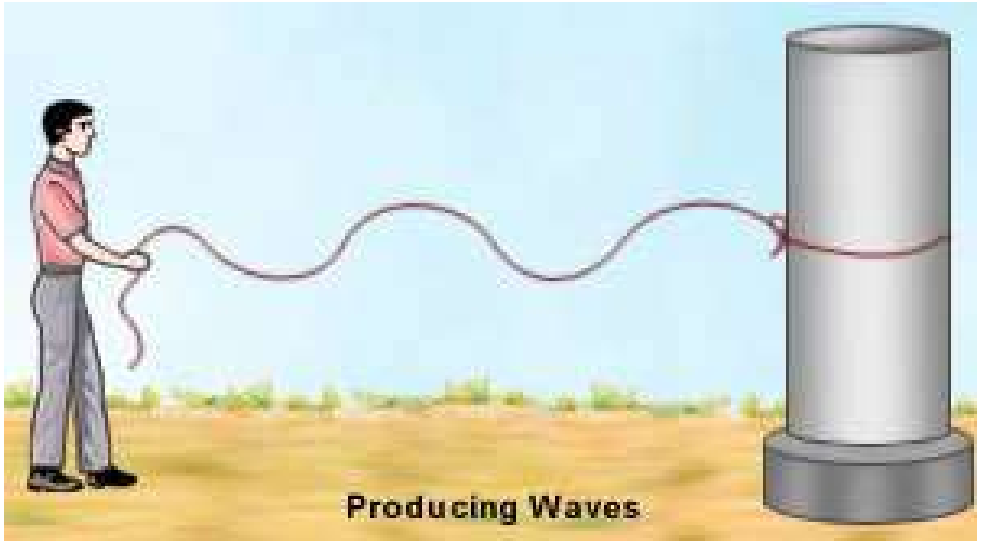
Some waves do not need matter (called a "medium") to be able to move (for example, through space).

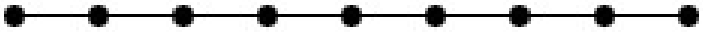
These are called electromagnetic waves (or EM waves).

Some waves MUST have a medium in order to move. These are called mechanical waves.

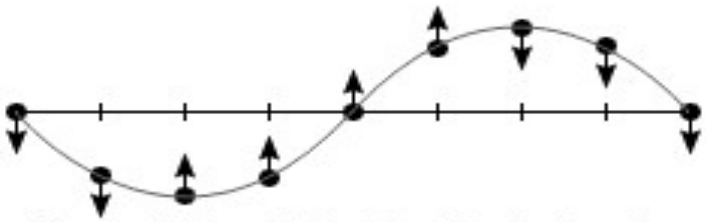
Wave Types

1. Transverse waves: Waves in which the medium moves at right angles to the direction of the wave





Initial Position of Particles

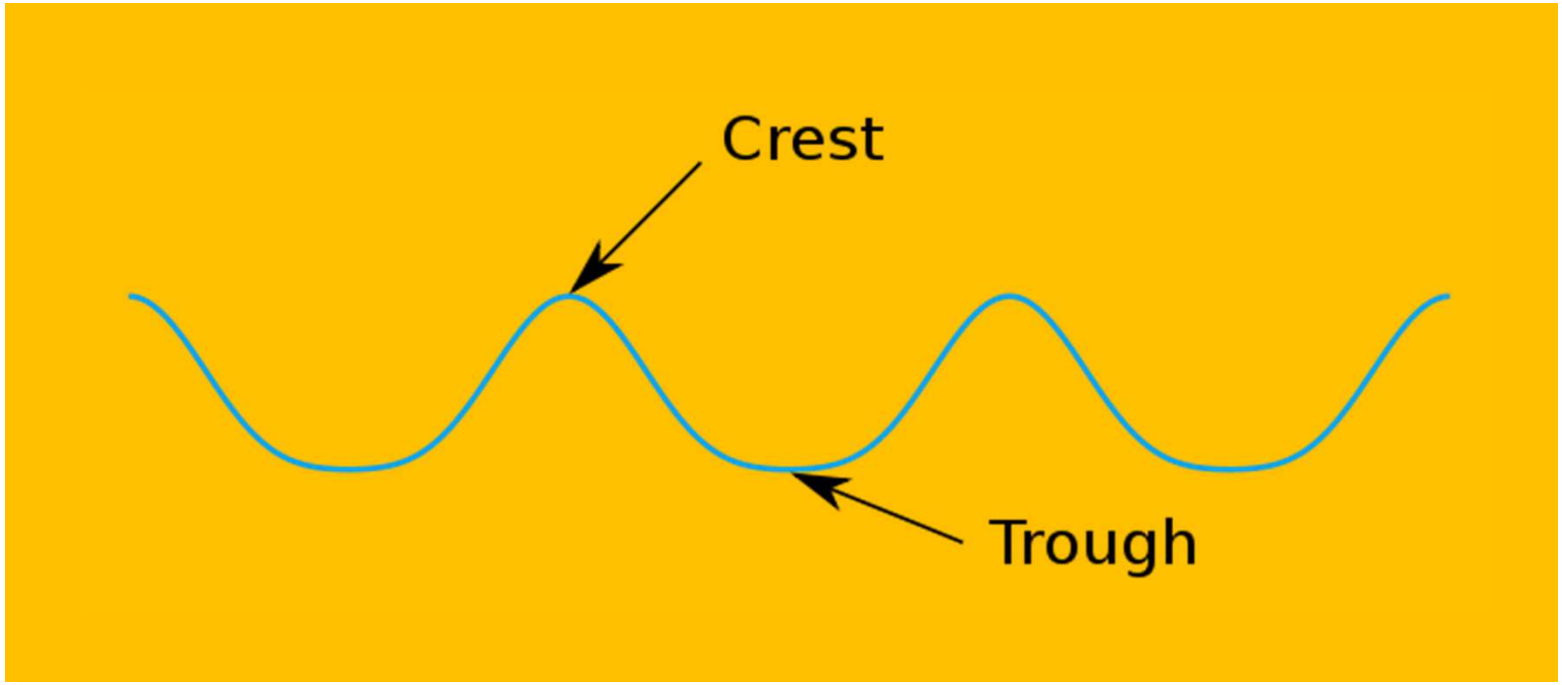


Curve obtained by joining the displaced Positions of the particles

Parts of transverse waves:

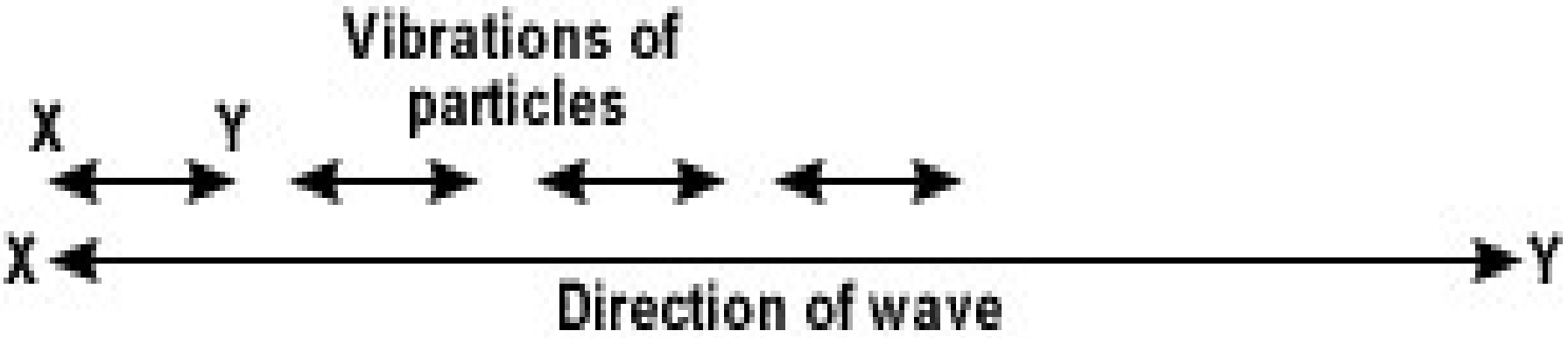
Crest: the highest point of the wave

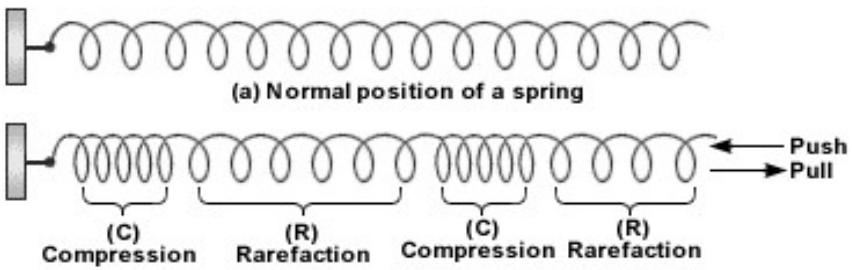
Trough: the lowest point of the wave



2. Longitudinal (or Compressional) waves:

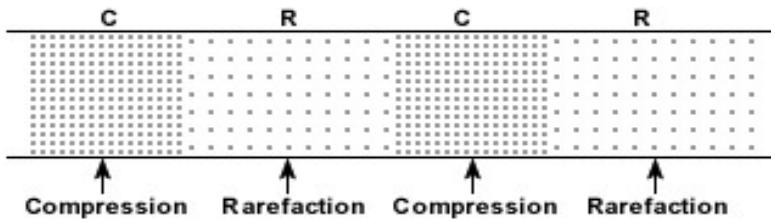
Waves in which the medium moves back and forth in the same direction as the wave





(a) Normal position of a spring

(b) Longitudinal wave in a spring

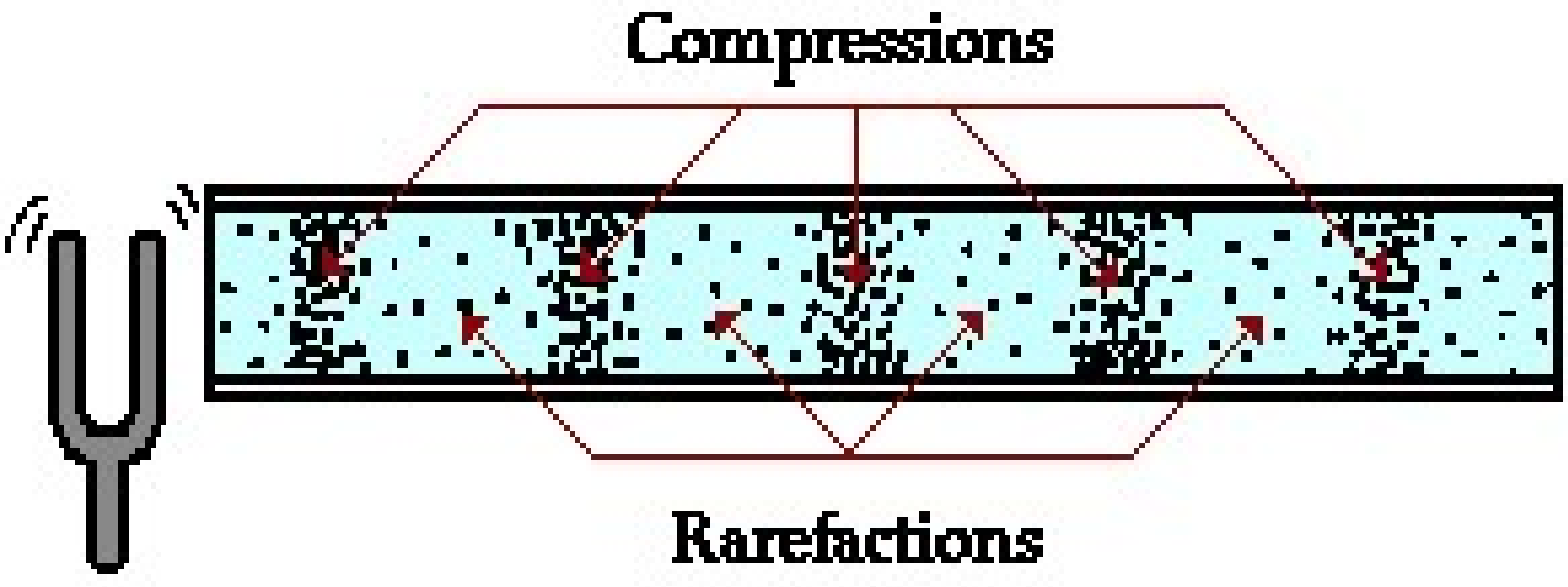


Compressions and rarefactions of a longitudinal wave. Regions marked C are compressions and regions marked R are rarefactions.

Parts of longitudinal waves:

Compression: where the particles are close together

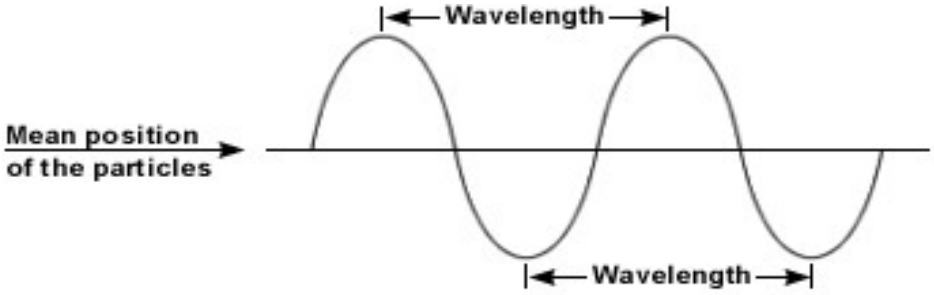
Rarefaction: where the particles are spread apart



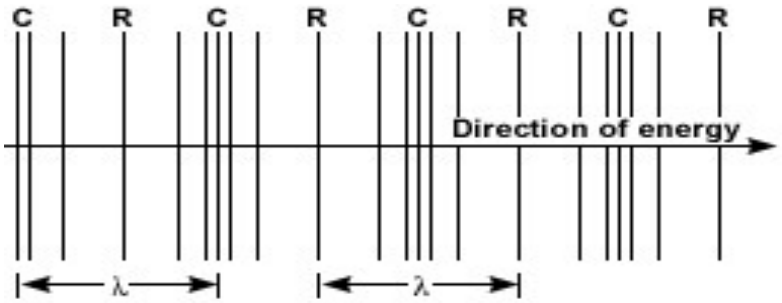
Wave Properties

Wave properties depend on what (type of energy) is making the waves.

1. Wavelength: The distance between one point on a wave and the exact same place on the next wave.



Wavelength of Transverse Waves



Wavelength of Longitudinal Waves

2. Frequency: How many waves go past a point in one second; unit of measurement is hertz (Hz).

The higher the frequency, the more energy in the wave.

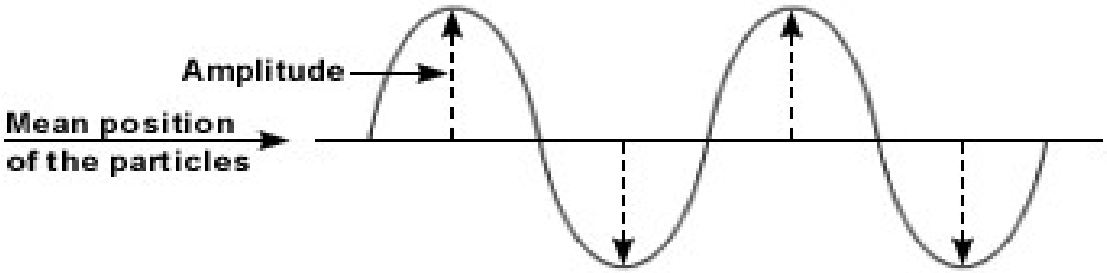
10 waves going past in 1 second = 10 Hz

1,000 waves go past in 1 second = 1,000 Hz

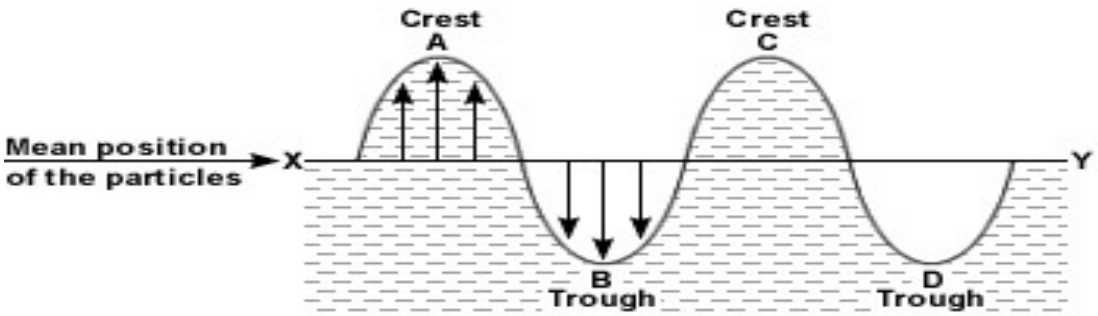
1 million waves going past = 1 million Hz

3. Amplitude: How far the medium moves from rest position (where it is when not moving).

Remember that for transverse waves, the highest point is the crest, and the lowest point is the trough.



Amplitude of a Transverse Wave



Crest and troughs of a transverse wave.
Here, A and C are crests
B and D are troughs

Remember that for compressional waves, the points where the medium is close together are called compressions and the areas where the medium is spread apart are called rarefactions.

The closer together and further apart the particles are, the larger the amplitude.

compression

rarefaction

4. Wave speed: Depends on the medium in which the wave is traveling. It varies in solids, liquids and gases.

A mathematical way to calculate speed:

wave speed = wavelength x frequency

(in meters) (in Hz)

OR

$$v = f \times \lambda$$

Problem: If a wave has a wavelength of 2 m and a frequency of 500 Hz, what is its speed?

The energy of a wave is proportional to the square of its amplitude. Mathematically speaking . . .

$$E = CA^2$$

Where:

E = energy (the capacity to do work)
C = a constant (depends on the medium)
A = amplitude

For example:

If the amplitude is equal to 3 units
(and we assume C = 1 for this case) . . .

$$E = (1)\,(3)^2 = (1)\,(9) = 9 \text{ units}$$

Mathematical Representation of a Traveling Wave

Suppose the shape of a wave is given by at t=0:

$$y(x, t = 0) = A \sin \frac{2\pi}{\lambda} x.$$

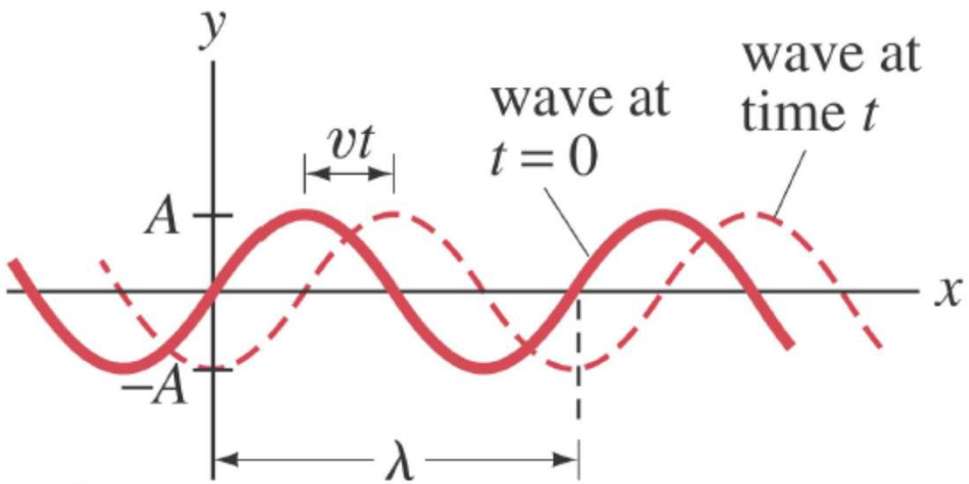
After a time t, the wave crest has traveled a distance vt, so we write:

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

Or: $y(x, t) = A \sin(kx - \omega t),$

with $\omega = 2\pi f\,, \quad k = \dfrac{2\pi}{\lambda}.$

$v = \lambda f$



The Wave Equation

$$y(x,t) = A\sin(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial x} = kA\cos(kx - \omega t) \qquad \frac{\partial y(x,t)}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\sin(kx - \omega t) \qquad \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\boxed{\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}}$$

In three dimensional wave

$$\frac{\partial^2 \psi(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \psi(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \psi(x,y,z,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,y,z,t)}{\partial t^2}$$

$$or,\ \nabla^2\psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \Leftrightarrow \psi(r,t) = A\sin(\vec{k}.\vec{r} - \omega t) \Rightarrow \psi(r,t) = Ae^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

Maxwell's Equation & Electromagnetic wave

The Basic laws of electricity and magnetism are the following

Maxwell's equations are the basic equations of electromagnetism. They involve calculus; here is a summary:

1. Gauss's law relates electric field to charge

2. A law stating there are no magnetic "charges"

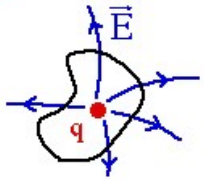
3. A changing electric field produces a magnetic field

4. A magnetic field is produced by an electric current, and also by a changing electric field

Maxwell's Equations

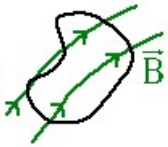
$$\oiint \vec{E} \bullet \hat{n}\, dS = \frac{q}{\varepsilon_o}$$

Gauss's Law

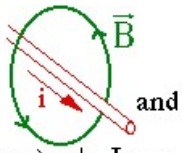


$$\oiint \vec{B} \bullet \hat{n}\, dS = 0$$

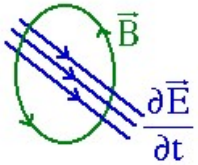
(no monopoles)



$$\oint \vec{B} \bullet d\vec{l} = \mu_o \left(i + \varepsilon_o \frac{d}{dt} \Phi_E \right)$$



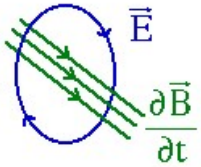
and



Ampère's Law

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \Phi_B$$

Faraday's Law



$$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_o} \qquad \vec{\nabla} \times \vec{B} = \mu_o \left(\vec{j} + \varepsilon_o \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{\nabla} \bullet \vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

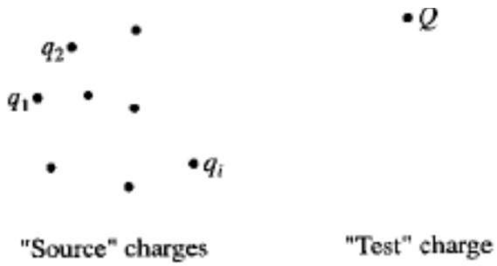
(Differential Forms)

The Electric Field:

What is the force on the test charge Q due to a source charge q?

The law was first discovered in 1785 by French physicist Charles-Augustin de Coulomb

We shall consider the special case of the **electrostatics** in which all the source charges are stationary.



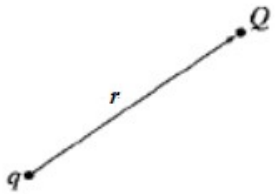
The **principle of superposition** states that the interaction between any two charges is completely unaffected by the presence of others.

Coulomb's Law

$$F = \frac{kqQ}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$$

Where $k = 9\times10^9 \dfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$

and $\varepsilon_0 = 8.86\times10^{-12}\dfrac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$



How does one particle sense the presence of the other?

The electric charge creates an electric field in the space around it. A second charged particle does not interact directly with the first; rather, it responds to whatever field it encounters. In this sense, the field acts as an *intermediary* between the particles.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots = \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{\mathrm{r}_1^{\,2}}\hat{\mathrm{r}}_1 + \frac{q_2}{\mathrm{r}_2^{\,2}}\hat{\mathrm{r}}_2 + \cdots\right) = Q\mathbf{E}$$

$$\text{where } \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\mathrm{r}_1^{\,2}}\hat{\mathrm{r}}_1 + \frac{q_2}{\mathrm{r}_2^{\,2}}\hat{\mathrm{r}}_2 + \cdots\right) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathrm{r}_i^{\,2}}\hat{\mathrm{r}}_i$$

The electric field strength is defined as the force per unit charge placed at that point.

Example

On a clear day there is an electric field of approximately 100 N/C directed vertically down at the earth's surface. Compare the electrical and gravitational forces on an electron.

Solution:

The magnitude of the electrical force is

$Fe=eE=1.6\text{x}10^{-19}\text{x}100=1.6\text{x}10^{-17}$ N. (upward)

The magnitude of the gravitational force is

$Fg=mg=9.11\text{x}10^{-31}\text{x}9.8=8.9\text{x}10^{-30}$ N. (downward)

Continuous Charge Distributions

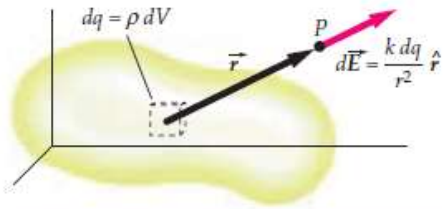


FIGURE An element of charge dq produces a field $d\vec{E} = (k\, dq/r^2)\,\hat{r}$ at point P. The field at P is found by integrating over the entire charge distribution.

In order to find the electric field due to a continuous distribution of charge, one must divide the charge distribution into infinitesimal elements of charge dq which may be considered to be point charges.

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathbf{r}^2}\hat{\mathbf{r}} \quad \Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\mathbf{r}^2}\hat{\mathbf{r}}$$

Thus the electric field of a line charge is

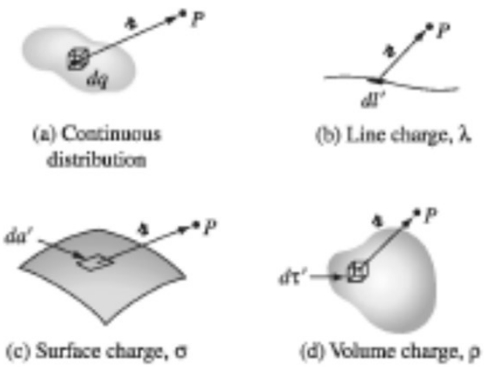
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dl';$$

for a surface charge,

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{S}} \frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, da';$$

and for a volume charge,

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau'.}$$



The Divergence of Electric Field

The electric field can be expressed in the following form

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{\text{all space}} \frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}} = \frac{1}{4\pi\varepsilon_0}\int_{\text{all space}} \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\rho(\mathbf{r}')d\tau'$$

Divergence of the electric field is

$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{\text{all space}} (\nabla\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2})\rho(\mathbf{r}')d\tau'$$

Why the divergence operator doesn't apply on the r' coordinate?

Since $(\nabla\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}) = 4\pi\delta^3(\mathscr{r})$,

$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{\text{all space}} 4\pi\delta^3(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')d\tau' = \frac{1}{\varepsilon_0}\rho(\mathbf{r})$$

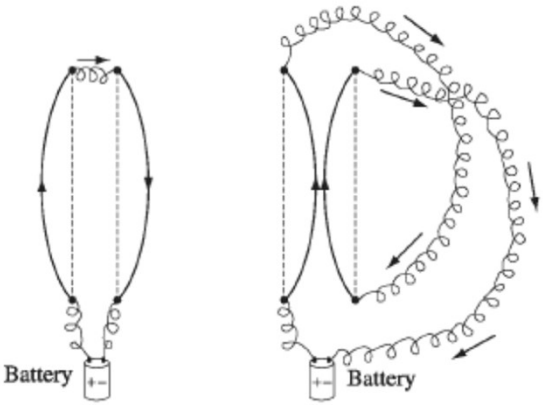
Statue of Johann Carl Friedrich Gauss at his Birthplace, Brunswick, Germany

The Curl of the Electric Field

Gauss's law

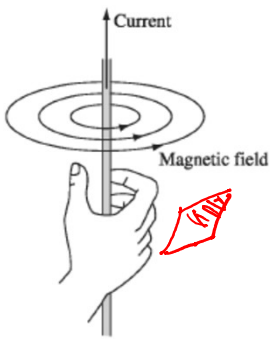
Magnetostatics

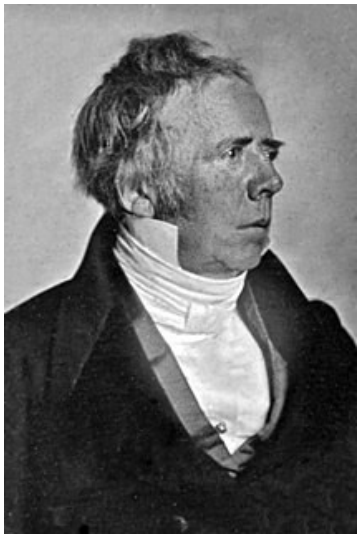
THE LORENTZ FORCE LAW & Magnetic field



(a) Currents in opposite directions repel.

(b) Currents in same directions attract.





This was discovered on 21 April 1820 by Danish physicist Hans Christian Ørsted

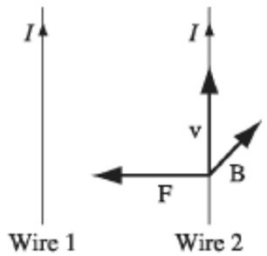
In fact, this combination of directions is just right for a cross product: the magnetic force on a charge Q, moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, is[2]

$$\boxed{\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B}).}$$

Lorentz force law $\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})].$

One implication of the Lorentz force law

$$\boxed{\textbf{Magnetic forces do no work.}}$$



The Biot–Savart law in magnetism is named after Biot and his colleague Félix Savart for their work in 1820

The magnetic field of a steady line current is given by the **Biot-Savart law**:

$$\boxed{\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}'\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.}$$

$$\partial\rho/\partial t = 0$$

$$\nabla\cdot\mathbf{J} = 0.$$

Continuity equation

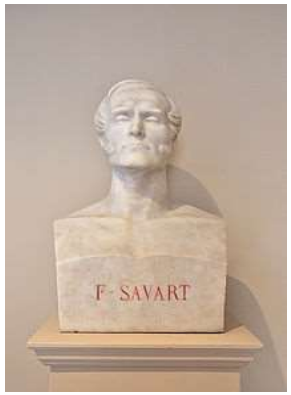
$$\nabla\cdot\mathbf{J} = \partial\rho/\partial t$$

The constant μ_0 is called the **permeability of free space**:

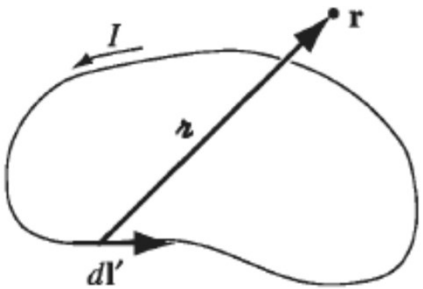
$$\mu_0 = 4\pi\times 10^{-7}\ \mathrm{N/A^2}.$$

These units are such that **B** itself comes out in newtons per ampere-meter (as required by the Lorentz force law), or **teslas** (T):[10]

Jean-Baptiste Biot



Félix Savart



The Bio-Sarvart Law
for the Surface and Volume Current

The **Biot-Sarvart law**: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathbf{r}}}{\mathbf{r}^2}dl' = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{l}'\times\hat{\mathbf{r}}}{\mathbf{r}^2}$

For surface current: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{K}\times\hat{\mathbf{r}}}{\mathbf{r}^2}da'$

For volume current: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}\times\hat{\mathbf{r}}}{\mathbf{r}^2}d\tau'$

For a moving charge: Wrong, why?

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}\times\hat{\mathbf{r}}}{\mathbf{r}^2}d\tau' = \frac{\mu_0}{4\pi}\int \frac{q\mathbf{v}\delta(\mathbf{r}-\mathbf{r}')\times\hat{\mathbf{r}}}{\mathbf{r}^2}d\tau' = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{\mathbf{r}^2}$$

A point charge does not constitute a steady current.

The divergence of B:

$$\nabla\cdot\mathbf{B}(\mathbf{r})=\nabla\cdot\left(\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')\times\hat{\mathbf{r}}}{\mathbf{r}^2}d\tau'\right)=\frac{\mu_0}{4\pi}\int\nabla\cdot\left(\frac{\mathbf{J}(\mathbf{r}')\times\hat{\mathbf{r}}}{\mathbf{r}^2}\right)d\tau'$$

$$\nabla\cdot\left(\frac{\mathbf{J}(\mathbf{r}')\times\hat{\mathbf{r}}}{\mathbf{r}^2}\right)=\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}\cdot(\nabla\times\mathbf{J})-\mathbf{J}\cdot\left(\nabla\times\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}\right)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B})=\mathbf{B}\cdot(\nabla\times\mathbf{A})-\mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}=-\nabla\left(\frac{1}{\mathbf{r}}\right)\quad\text{(Prob. 1.13)}$$

$$\nabla\cdot\left(\frac{\mathbf{J}(\mathbf{r}')\times\hat{\mathbf{r}}}{\mathbf{r}^2}\right)=\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}\cdot\underbrace{(\nabla\times\mathbf{J})}_{0}-\mathbf{J}\cdot\underbrace{\left(\nabla\times\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}\right)}_{0}$$

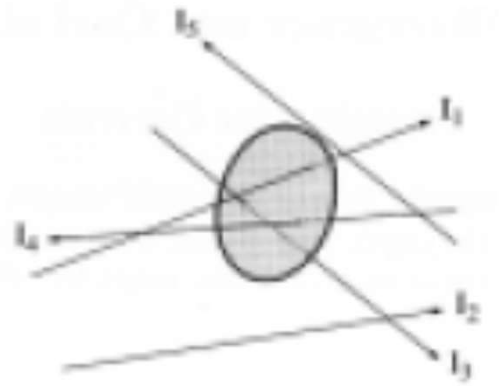
$\therefore\ \underline{\nabla\cdot\mathbf{B}=0}$ The divergence of a magnetic field is zero.

The Differential Form of **B**

Suppose we have a bundle of straight wires. Only wires that pass through the loop contribute $\mu_0 I$.
The line integration then be

$$\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \qquad I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{a}$$

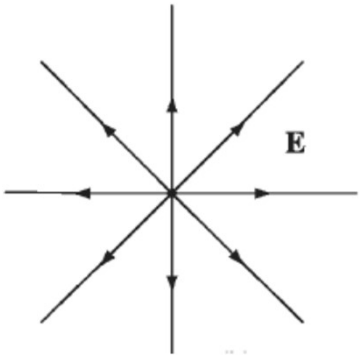
The total current enclosed by the integration loop.

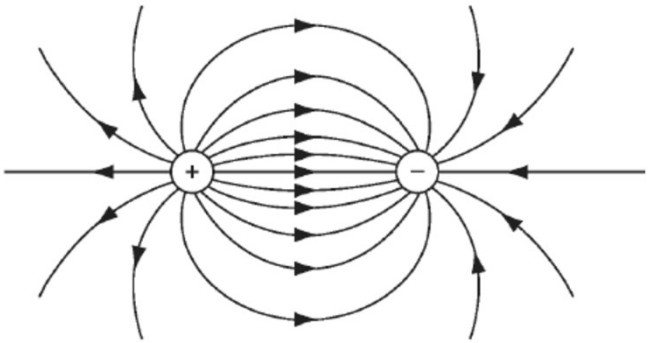


$$\oint \mathbf{B} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{B}) \cdot d\mathbf{a} = \int \mu_0 \mathbf{J} \cdot d\mathbf{a}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

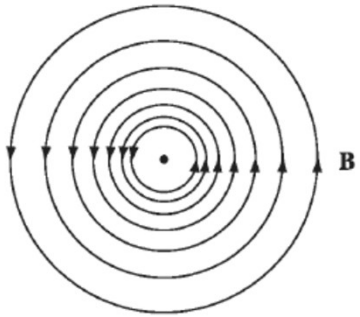
Does this differential equation apply to any shape of the current loop? Yes, to be prove next.



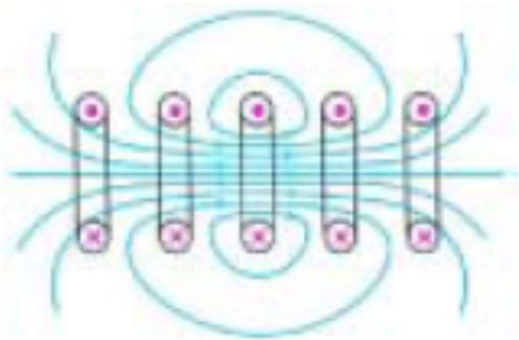


Opposite charges

Field lines for E



Straight wire



Solenoid

Field lines for B

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{Ampere's law in differential form}$$

$$\int (\nabla \times \mathbf{B}) \cdot d\mathbf{a} = \oint_{\text{amperian loop}} \mathbf{B} \times d\mathbf{l} = \int \mu_0 \mathbf{J} \cdot d\mathbf{a} = \mu_0 I_{\text{enc}}$$

$$\oint_{\text{amperian loop}} \mathbf{B} \times d\mathbf{l} = \mu_0 I_{\text{enc}} \quad \text{Ampere's law in integral form}$$

Just as the Biot-Savart law plays a role in magnetostatics that coulomb's law assumed in electrostatics, so Ampere's play the role of Gauss's.

Electrostatics:	Coulomb	→ Gauss,
Magnetostatics:	Biot-Savart	→ Ampere.

ELECTROMAGNETIC INDUCTION

Faraday's Law

In 1831 Michael Faraday reported on a series of experiments, including three that (with some violence to history) can be characterized as follows:

Experiment 1. He pulled a loop of wire to the right through a magnetic field (Fig. 7.21a). A current flowed in the loop.

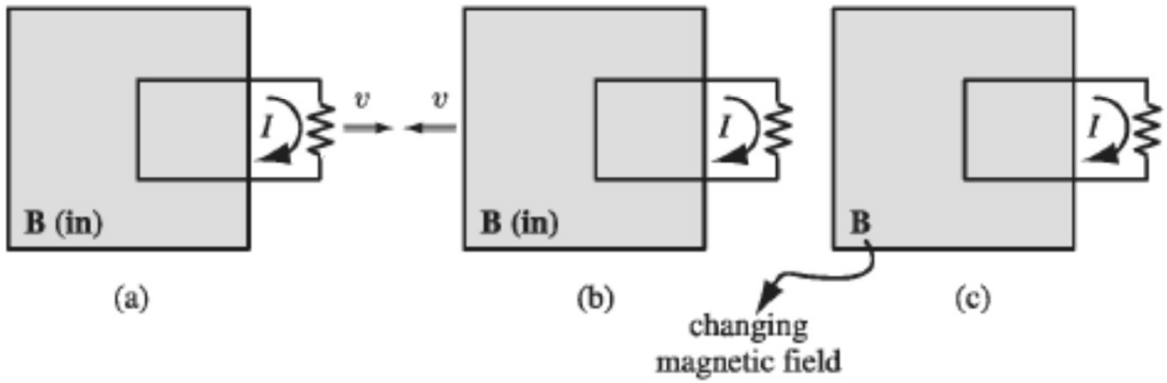
Experiment 2. He moved the *magnet* to the *left*, holding the loop still (Fig. 7.21b). Again, a current flowed in the loop.

Experiment 3. With both the loop and the magnet at rest (Fig. 7.21c), he changed the *strength* of the field (he used an electromagnet, and varied the current in the coil). Once again, current flowed in the loop.

The first experiment, of course, is a straightforward case of motional emf; according to the flux rule:

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

A changing magnetic field induces an electric field.



It is this induced[8] electric field that accounts for the emf in Experiment 2.[9] Indeed, if (as Faraday found empirically) the emf is again equal to the rate of change of the flux,

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt},$$

then **E** is related to the change in **B** by the equation

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}.$$

This is **Faraday's law**, in integral form. We can convert it to differential form by applying Stokes' theorem:

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.}$$

The Equations of Electromagnetism (Before Maxwell …)

Gauss' Law for Electrostatics $\nabla . E = \frac{\rho}{\varepsilon_0}$ or $\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$

Gauss' Law for Magnetism $\nabla . B = 0$ or $\oint \underline{B} \bullet \underline{dA} = 0$

Faraday's Law of Induction

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{ or, } \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

Ampere's Law $\nabla \times B = \mu_0 J$ or, $\oint \underline{B} \bullet \underline{dl} = \mu_0 I$

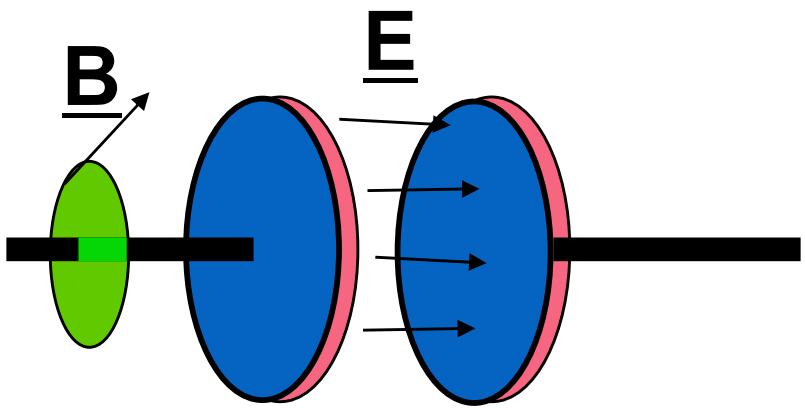
Lorentz force law $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Continuity equation $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

...lets take a look at charge flowing into a capacitor...

...when we derived Ampere's Law
we assumed constant current...

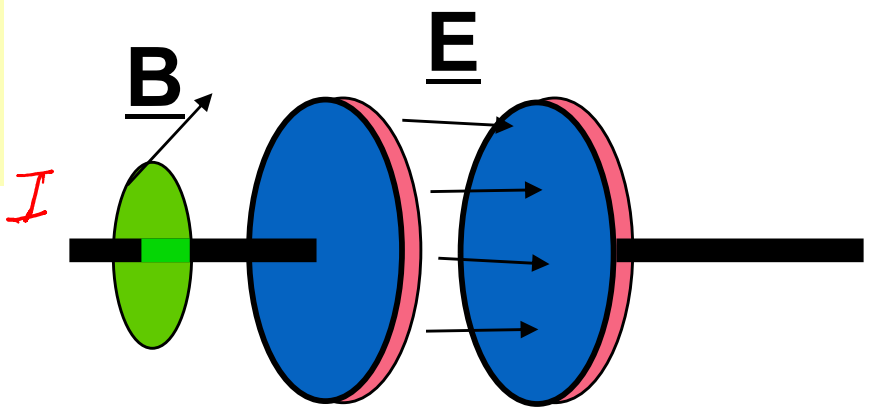
$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 I}$$



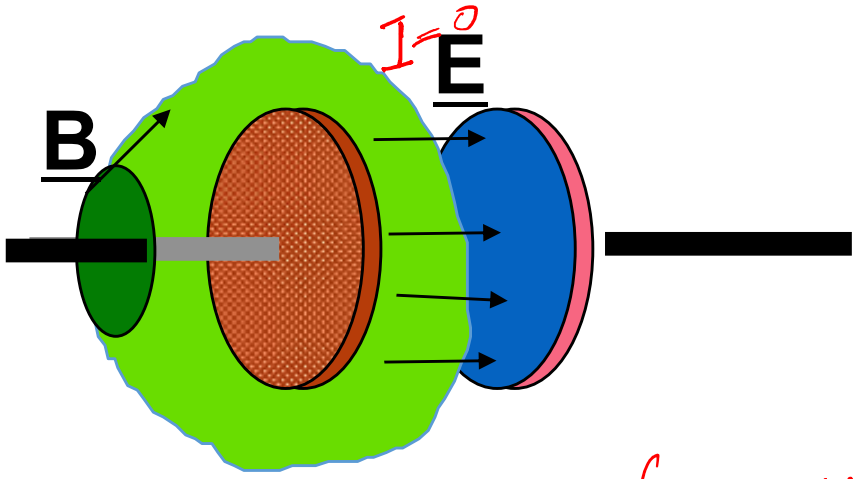
...lets take a look at charge flowing into a capacitor...

...when we derived Ampere's Law we assumed constant current...

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

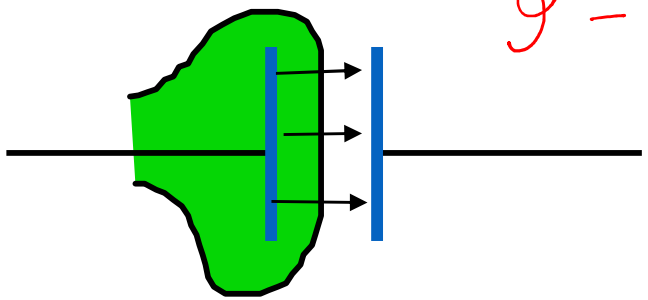


.. if the loop encloses one plate of the capacitor..there is a problem … I = 0



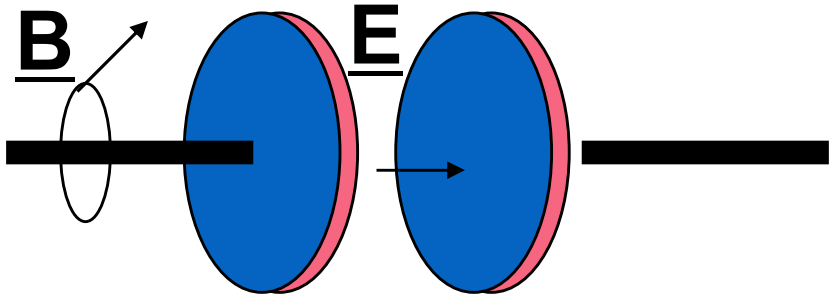
$$\oint \underline{B} \cdot \underline{dl} = 0$$

Side view: (Surface is now like a bag:)



Maxwell solved this problem by realizing that....

Inside the capacitor there must be an induced magnetic field...



How?.

Differential form of Ampere's Law

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

Inconsistency in Ampere's Law

$$\nabla . \nabla \times B = \mu_0 \nabla . J$$

$$-\frac{\partial \rho}{\partial t}$$

$$\neq 0$$

$$0$$

How Maxwell Fixed Ampere's Law

Applying the continuity equation and Gauss's law, the offending term can be rewritten:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial(\varepsilon_0 \nabla \cdot \mathbf{E})}{\partial t} = \nabla \cdot (-\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t})$$

A new current $\mathbf{J}' = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ $\leftarrow$ kills off the extra divergence

$$\nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 (\nabla \cdot \mathbf{J}') = \mu_0 \nabla \cdot (\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}) = 0$$

When E is constant (electrostatic+magnetostatic), we will have $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$.

$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ plays a crucial role in the EM wave propagation.

Electric Analogy of Faraday's Law

Maxwell's term cures the defect in Ampere's law, and moreover, it has a certain aesthetic appeal.

Faraday's law

A changing magnetic field induces a electric field.

A changing electric field induces a magnetic field.

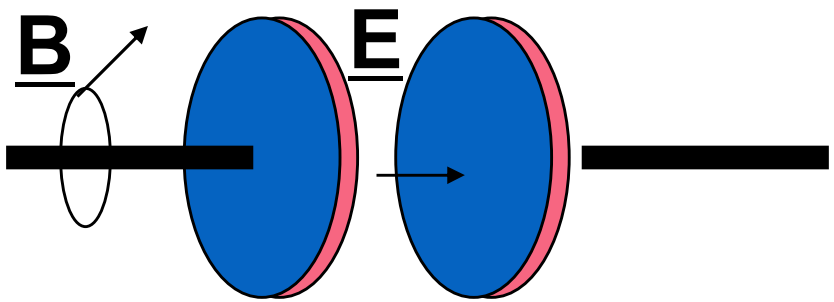
Maxwell called this extra term "the displacement current".

$$\mathbf{J}_d \equiv \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

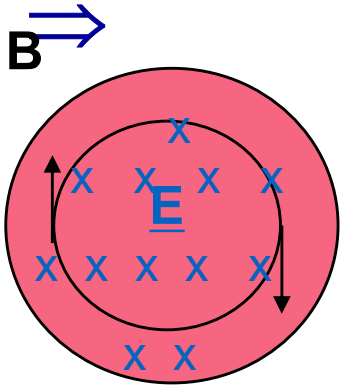
a misleading name, nothing to do with current

Maxwell solved this problem by realizing that....

Inside the capacitor there must be an induced magnetic field...



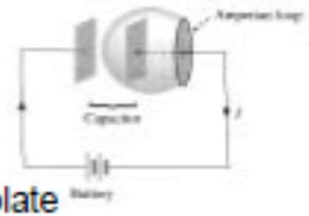
How?. Inside the capacitor there is a changing E



A changing electric field induces a magnetic field

The Displacement Current

How the displacement current resolves the paradox of the charging capacitor.



The electric field between the two capacitor plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{Q}{A}$$

Q ← the charge on the plate

A ← the area of the plate

$$\varepsilon_0 \frac{\partial E}{\partial t} = \frac{1}{A}\frac{\partial Q}{\partial t} = \frac{I}{A} = J$$

$$\mathbf{J}_{tot} = \mathbf{J} + \mathbf{J}_d \begin{cases} \mathbf{J}=\mathbf{J},\ \mathbf{J}_d=0 \text{ at the flat surface} \\ \mathbf{J}=0,\ \mathbf{J}_d=\mathbf{J} \text{ at the balloon-shaped surface} \end{cases}$$

Maxwell's Equations

Maxwell's equations in the traditional way.

$$\begin{cases} \nabla\cdot\mathbf{E} = \dfrac{1}{\varepsilon_0}\rho & \text{(Gauss's law)} \\ \nabla\cdot\mathbf{B} = 0 & \text{(no name)} \\ \nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t} & \text{(Faraday's law)} \\ \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\dfrac{\partial\mathbf{E}}{\partial t} & \text{(Ampere's law with Maxwell's correction)} \end{cases}$$

Lorentz force law $\quad \mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

Continuity equation $\quad \nabla\cdot\mathbf{J} = -\dfrac{\partial\rho}{\partial t}$

Electromagnetic Waves in Vacuum
The Wave Equation for **E** and **B**

In regions of space where there is no charge or current, Maxwell's equations read

$$\text{(i)}\ \nabla\cdot\mathbf{E}=0 \qquad \text{(iii)}\ \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\text{(ii)}\ \nabla\cdot\mathbf{B}=0 \qquad \text{(iV)}\ \nabla\times\mathbf{B}=\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\times(\nabla\times\mathbf{E})=-\frac{\partial(\nabla\times\mathbf{B})}{\partial t} \quad\Rightarrow\quad \nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=-\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$\nabla\times(\nabla\times\mathbf{B})=\mu_0\varepsilon_0\frac{\partial(\nabla\times\mathbf{E})}{\partial t} \quad\Rightarrow\quad \nabla(\nabla\cdot\mathbf{B})-\nabla^2\mathbf{B}=-\mu_0\varepsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$\nabla^2 f=\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \quad\Leftarrow\quad \text{since}\ \begin{cases}\nabla\cdot\mathbf{E}=0\\ \nabla\cdot\mathbf{B}=0\end{cases} \quad\Rightarrow\quad \begin{cases}\nabla^2\mathbf{E}=\mu_0\varepsilon_0\dfrac{\partial^2\mathbf{E}}{\partial t^2}\\[2ex] \nabla^2\mathbf{B}=\mu_0\varepsilon_0\dfrac{\partial^2\mathbf{B}}{\partial t^2}\end{cases}$$

Maxwell's equations imply that empty space supports the propagation of electromagnetic waves, traveling at a speed

$$v=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=3\times10^8\ \text{m/s} \longleftarrow \text{the speed of light}$$

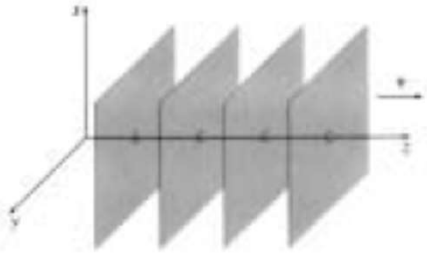
$$\nabla^2 E_x-\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}=0$$

Monochromatic Plane Waves

Consider a monochromatic wave of frequency ω and the wave is traveling in the z direction and has no x or y dependence, called plane waves.
Plane waves: the fields are uniform over every plane perpendicular to the direction of propagation.

Are these waves common? Yes, very common.

$$\begin{cases} \tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)} \\ \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)} \end{cases}$$ where $\tilde{\mathbf{E}}_0$ and $\tilde{\mathbf{B}}_0$ are the complex amplitudes.



Transverse Electromagnetic Waves

Q: What is the relation between $\mathbf{E}$ and $\mathbf{B}$?

$$\nabla\cdot\mathbf{E}=0 \qquad \frac{\partial E_z}{\partial z} = (\tilde{E}_0)_z ike^{i(kz-\omega t)} = 0 \qquad \Rightarrow (\tilde{E}_0)_z = 0$$

$$\nabla\cdot\mathbf{B}=0 \qquad \frac{\partial B_z}{\partial z} = (\tilde{B}_0)_z ike^{i(kz-\omega t)} = 0 \qquad \Rightarrow (\tilde{B}_0)_z = 0$$

That is, electromagnetic waves are transverse: the electric and magnetic fields are perpendicular to the direction of propagation. Moreover, Faraday's law $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$

$$\hat{\mathbf{x}}: \quad \frac{\partial(E_0)_z}{\partial y} - \frac{\partial(E_0)_y}{\partial z} = -\frac{\partial(B_0)_x}{\partial t} \quad \Rightarrow \quad k(\tilde{E}_0)_y = -\omega(\tilde{B}_0)_x$$

$$\hat{\mathbf{y}}: \quad \frac{\partial(E_0)_x}{\partial z} - \frac{\partial(E_0)_z}{\partial x} = -\frac{\partial(B_0)_y}{\partial t} \quad \Rightarrow \quad k(\tilde{E}_0)_x = \omega(\tilde{B}_0)_y$$

$$\hat{\mathbf{z}}: \quad \frac{\partial(E_0)_y}{\partial x} - \frac{\partial(E_0)_x}{\partial y} = -\frac{\partial(B_0)_z}{\partial t} \quad \Rightarrow \quad 0 = 0$$

Transverse Electromagnetic Waves (II)

Ampere's law with Maxwell's correction: $\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$

$$\hat{\mathbf{x}}: \quad \frac{\partial (B_0)_z}{\partial y} - \frac{\partial (B_0)_y}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial (E_0)_x}{\partial t} \quad \Rightarrow \quad k(\tilde{B}_0)_y = \mu_0 \varepsilon_0 \omega (\tilde{E}_0)_x$$

$$\hat{\mathbf{y}}: \quad \frac{\partial (B_0)_x}{\partial z} - \frac{\partial (B_0)_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial (E_0)_y}{\partial t} \quad \Rightarrow \quad k(\tilde{B}_0)_x = -\mu_0 \varepsilon_0 \omega (\tilde{E}_0)_y$$

$$\hat{\mathbf{z}}: \quad \frac{\partial (B_0)_y}{\partial x} - \frac{\partial (B_0)_x}{\partial y} = \mu_0 \varepsilon_0 \frac{\partial (E_0)_z}{\partial t} \quad \Rightarrow \quad 0 = 0$$

In free space, the speed of light is $c = \dfrac{\omega}{k} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

More compactly, $\tilde{\mathbf{B}}_0 = \dfrac{k}{\omega}(\hat{\mathbf{z}} \times \tilde{\mathbf{E}}_0) = \dfrac{1}{c}(\hat{\mathbf{z}} \times \tilde{\mathbf{E}}_0) \Rightarrow \mathbf{E} \perp \mathbf{B}$

amplitude relation: $B_0 = \dfrac{1}{c} E_0$

The Index of Refraction

Electromagnetic waves propagate through a linear homogeneous medium at a speed

$$\begin{cases} \nabla^2 \mathbf{E} = \mu\varepsilon \dfrac{\partial^2 \mathbf{E}}{\partial t^2} \\ \nabla^2 \mathbf{B} = \mu\varepsilon \dfrac{\partial^2 \mathbf{B}}{\partial t^2} \end{cases} \Rightarrow \nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n}$$

$$n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$$

The index of refraction of the material

For most material, μ is very close to μ_0, so $n \cong \sqrt{\dfrac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$

Since ε_r is almost always greater than 1, light travels more slowly through matter.

Production of Electromagnetic Waves

Since a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field, once sinusoidal fields are created they can propagate on their own.

These propagating fields are called electromagnetic waves.

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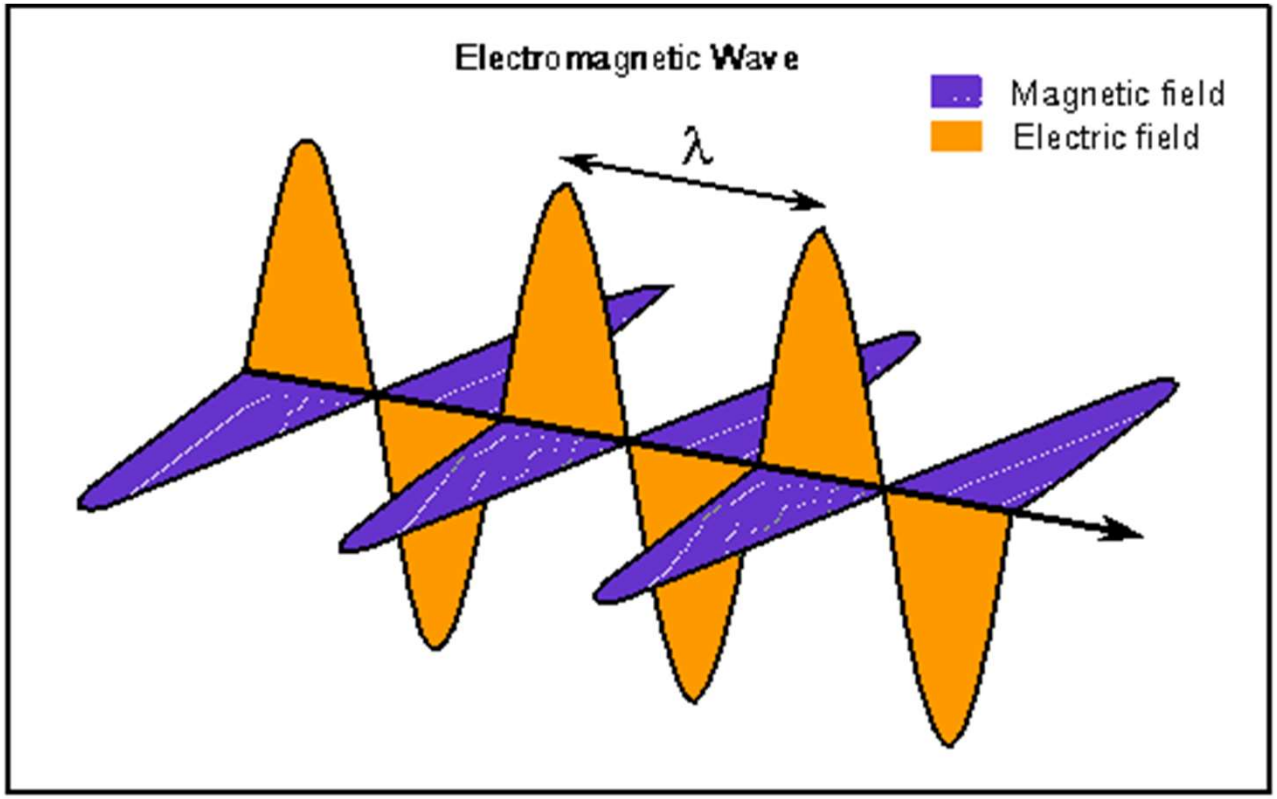
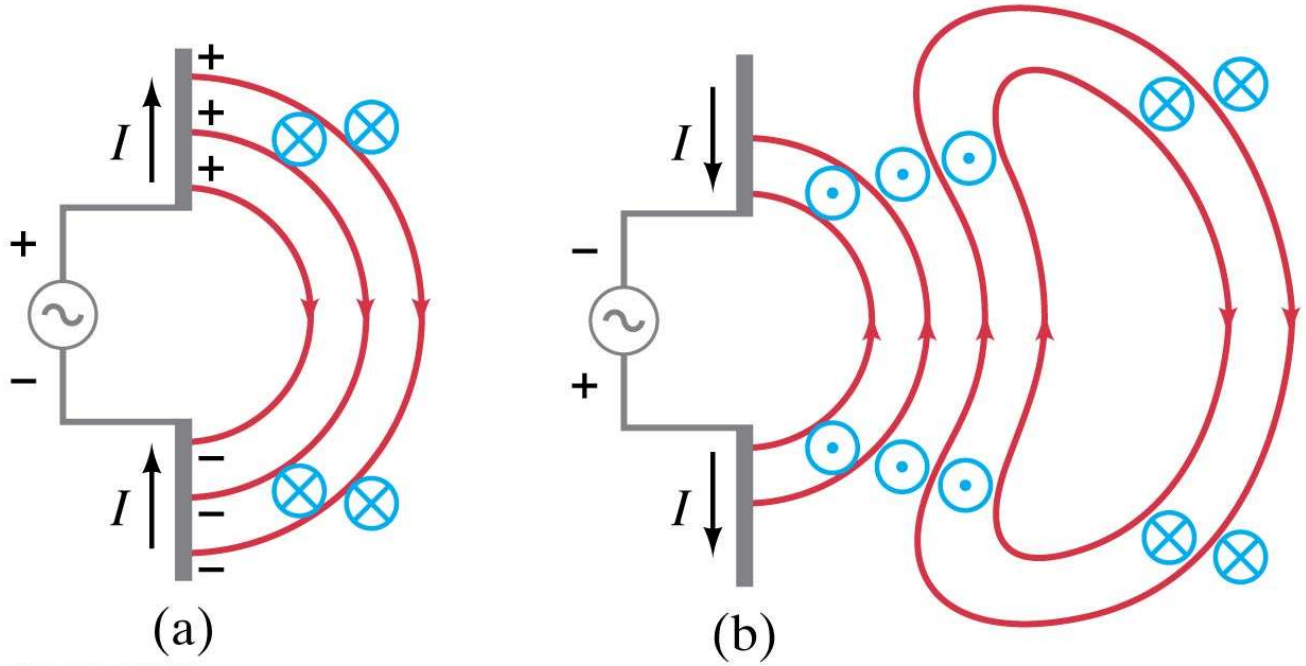


Production of Electromagnetic Waves

Oscillating charges will produce electromagnetic waves:



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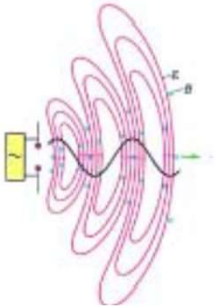
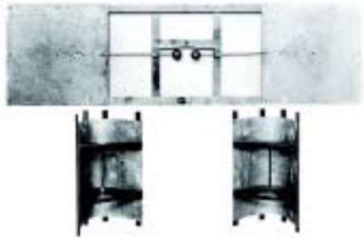
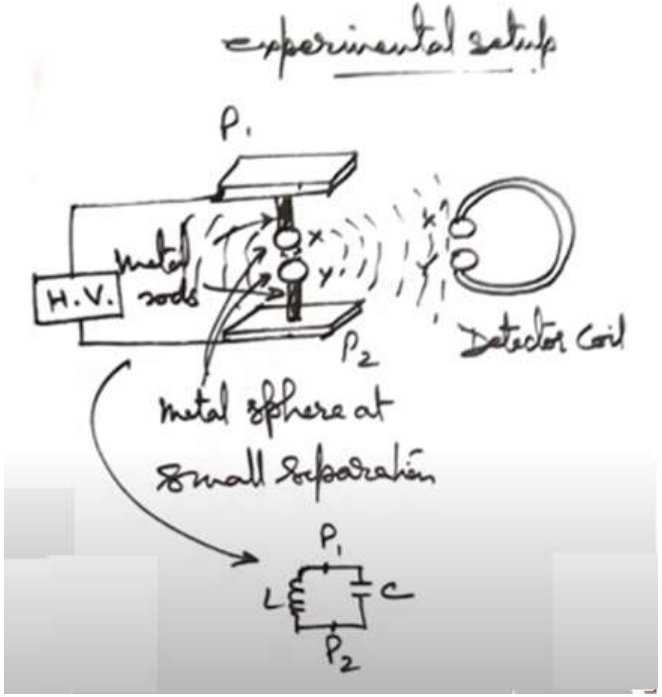
Hertz's Experiment

When Maxwell's work was published in 1867 it did not receive immediate acceptance. It is Hertz who conclusively demonstrated the existence of electromagnetic wave.







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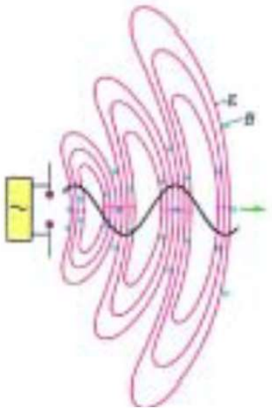
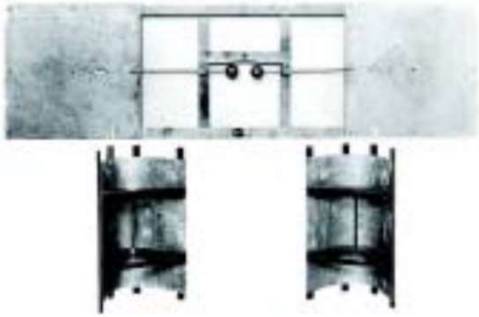
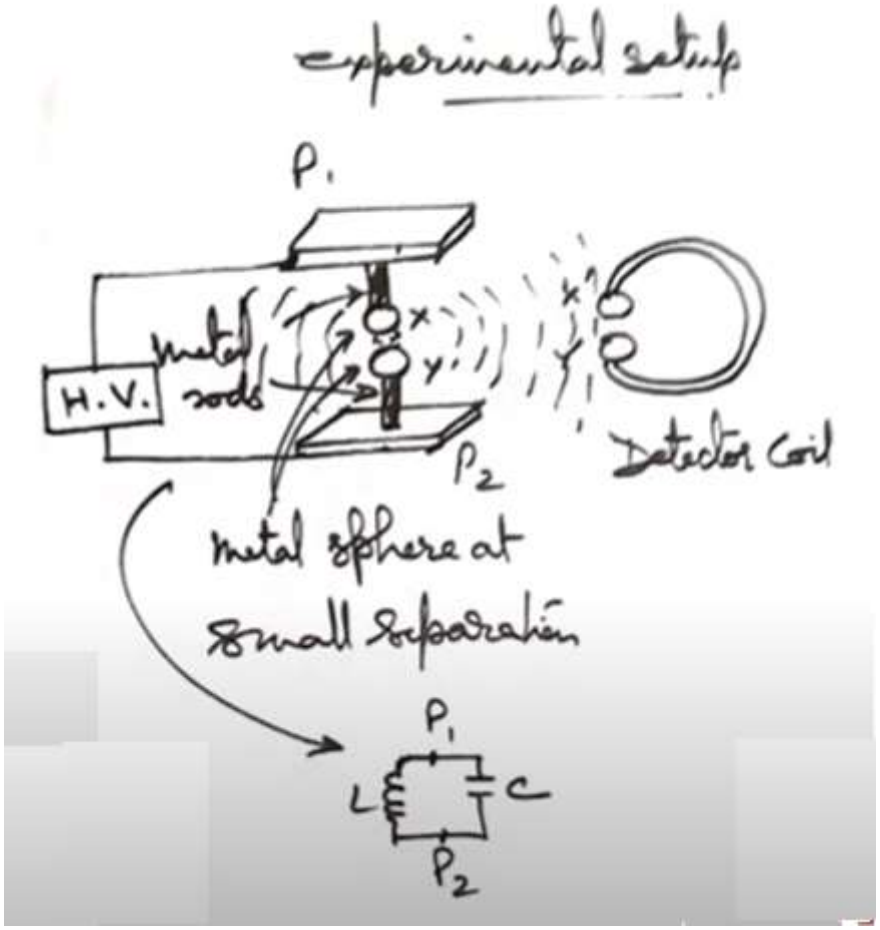
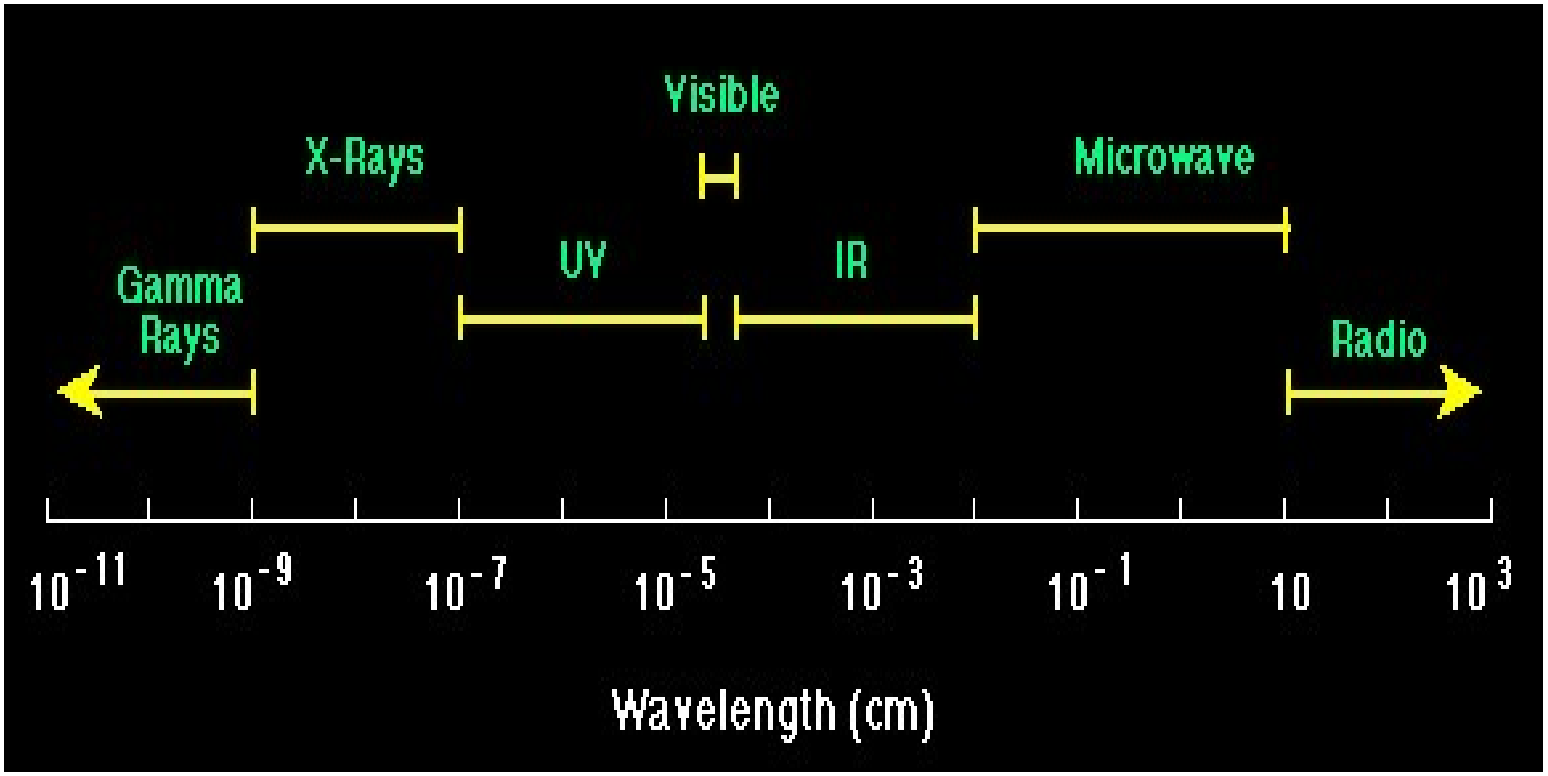
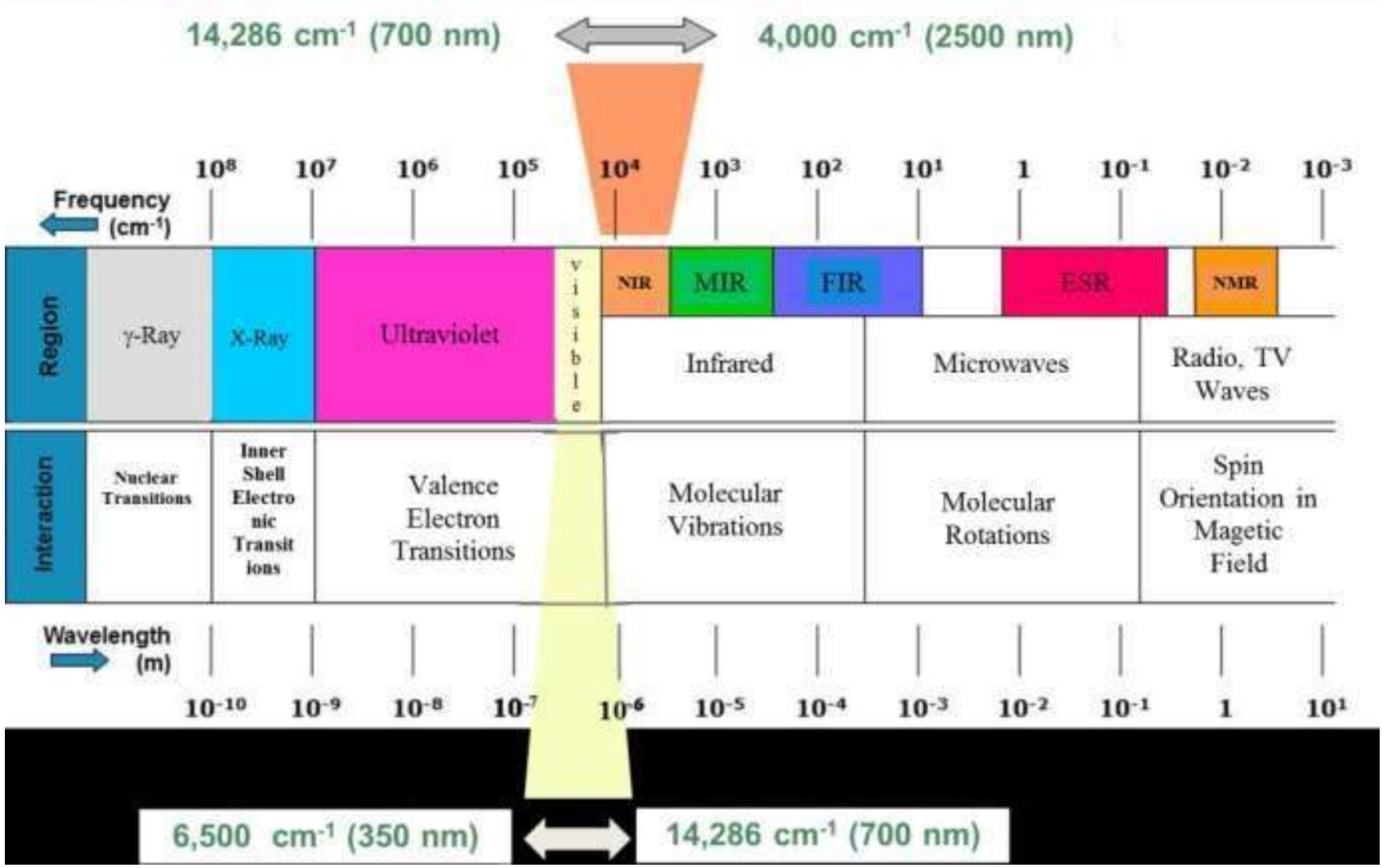







Visible
X-Rays
Microwave
UV
IR
Gamma Rays
Radio
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
10
10^{3}
Wavelength (cm)


14,286 cm-1 (700 nm)
4,000 cm-1 (2500 nm)
Frequency (cm-1)
10⁸ 10⁷ 10⁶ 10⁵ 10⁴ 10³ 10² 10¹ 1 10-1 10-2 10-3
Region
γ-Ray
X-Ray
Ultraviolet
visible
NIR
MIR
FIR
ESR
NMR
Infrared
Microwaves
Radio, TV Waves
Interaction
Nuclear Transitions
Inner Shell Electronic Transitions
Valence Electron Transitions
Molecular Vibrations
Molecular Rotations
Spin Orientation in Magetic Field
Wavelength (m)
10-10 10-9 10-8 10-7 10-6 10-5 10-4 10-3 10-2 10-1 1 10¹
6,500 cm-1 (350 nm)
14,286 cm-1 (700 nm)