sequence :- A sequence is a function from $N \longrightarrow R$

$1, 2, 3 \cdots\cdots$

$\left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots\cdots \right\}$

$\{x_n\}_n \qquad n \in N$

A sequence is convergent if there exists a finite real number l as its limit.

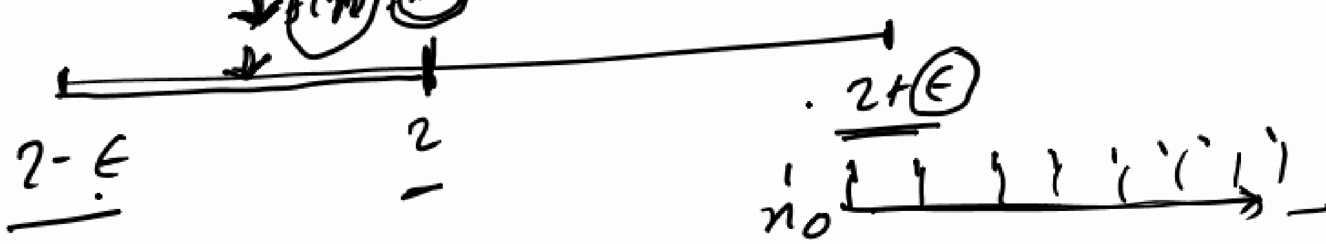
limit

:

$|x_n - l| < \epsilon \qquad \forall n \geq n_0$

$n \geq n_0 \qquad n \geq n_0 \qquad x_n \rightarrow$

$f(n)\ x_n$



n_0

if $\lim_{n \to \infty} \frac{2n+1}{n+1} = 2$ convergent

$x_n = f(n)$ 2

$$\left| \frac{2n+1}{n+1} - 2 \right| < \epsilon$$

$$\left| \frac{2n+1-2n-2}{n+1} \right| \text{ or } \frac{1}{n+1} < \epsilon$$

$$n+1 > \frac{1}{\epsilon}$$

or $n > \frac{1}{\epsilon} - 1$ $\quad n_0 = \left(\frac{1}{\epsilon} - 1\right) + 1$

Let $\{a_n\}$ be a sequence of real numbers the symbolic expression

$a_1 + a_2 \ldots + a_n$

$$\sum_{n=1}^{\infty} a_n \qquad \sum a_n$$

that is called infinite series.

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$$\sum_{r=1}^{n} a_r$$ = partial Sum.

Series

- Convergent (tends to a finite limit)
- Divergent ( )
- oscillating
  - ( )

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$$

$$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots$$

$$= 1 - \frac{1}{n+1} \qquad n \to \infty$$

$$= 1$$

$$\sum n = 1 + 2 + 3 \cdots = \frac{n(n+1)}{2} \qquad n \to \infty$$

$$\sum_{r=1}^{\infty} (-1)^{r+1} = 1 - 1 + 1 - 1 + 1 - 1 \cdots$$

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$$= 1$$

$$\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p}$$

$p > 1$

$p < 1$

$$= 1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}$$

$$1 + \frac{1}{2^{p-1}} + \frac{1}{2^{2(p-1)}} = 1 + \frac{2}{2^p} + \frac{4}{4^p}$$

$p > 1$

$$\frac{1 - r^n}{1 - r}$$

$$\frac{1}{1 - \frac{1}{2^{(p-1)}}}$$

$p > 1$ convergen

$$\lim_{n\to\infty} \sqrt[n]{a_n} = l \qquad l < 1 \text{ converges}$$

$$l > 1 \text{ diverges}$$

$$l = \lim_{n\to\infty} a_n^{\frac{1}{n}} < 1 \qquad l - \epsilon \longleftarrow l \longrightarrow l + \epsilon$$

$$(a_n)^{\frac{1}{n}}$$

$$l - \epsilon < a_n^{\frac{1}{n}} < \underbrace{l + \epsilon}_{r}$$

$$a_n < r^n \qquad a_n < r^n$$

$$\sum_{n=1}^{\infty} r^n \qquad r < 1 \qquad \underbrace{1 + r^1 + r^2 + r^3}_{r < 1}$$