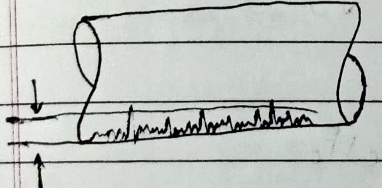
Dimensional Analysis

Reynold no. (Re) = $\frac{\rho u D}{\mu}$

u is average ~~density~~ velocity
μ is ~~velocity~~ viscosity

Re < 2000 ⇒ flow is Laminar.
2000 < Re < 4000 ⇒ transition state
Re > 4000 ⇒ Turbulant flow

Geometric similary
Kinematic similarity
Dynamic similarity
Model & prototype concept
Buckingham Pi theorem
Numerical problems



ρ, μ, D, P, T, K are factors deciding flow of fluid in a bounded pipe

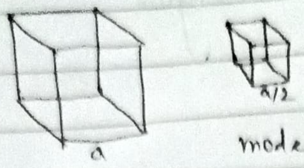
K or ε
absolute roughness

$\frac{K}{D}$ or $\frac{\varepsilon}{D}$ is called relative roughness

* Physically similar systems
Geometrically Similar systems - Geometrically same (shapewise same)
Kinetically similar - Similar motion
Dynamically similar - Similar force

Fluid mechanics is based on space & time
When it is free from time its called steady flow
———— " ———— space " ———— uniform flow

Geometrically similar



a   a/2

model (m)

prototype (p)

Scale factor $= \frac{L_m}{L_p} = \frac{a/2}{a} = \frac{1}{2}$

Scale factor $= \frac{u_m t_m}{u_p t_p} = \frac{u_m}{u_p} \times \frac{t_m}{t_p}$

$=$ Velocity ratio $\times$ time ratio

$\therefore$ velocity ratio $= \frac{\text{scale factor}}{\text{time ratio}}$

* forces on which the flow of liquid depends:
  - Pressure force (F_p)
  - Viscous force (F_v)
  - Gravitational force (F_g)
  - Surface Tension force (F_c) (capillary force)
  - Elastic force (F_e)

$F_{resultant} = F_r = F_p + F_v + F_g + F_c + F_e$

According to Newton's law of motion, inertia force is in opp. dir. but same in magnitude as that of F_r

$\therefore F_r = -F_i$

$\therefore F_p + F_v + F_g + F_c + F_e + F_i = 0$

Mach no. $= M_a = \frac{V}{C}$

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A fluid element of length l, velocity u, density ρ & time of applied force t,

Thus $F_p = \Delta p l^2$

$F_v = \mu \cdot \frac{u}{l} \cdot l^2 = \mu u l$

$F_i = \rho l^3 \times \frac{u}{t} = \rho l^3 \times \frac{u}{l/u} = \rho l^2 u^2$

$F_g = \rho l^3 g \qquad F_c = \sigma l \qquad F_e = E l^2$

① Dynamic Similarity with viscous force only

$$\frac{\text{Inertia force}}{\text{viscous force}} = \frac{\rho l^2 u^2}{\mu \cdot u l} = \frac{\rho u l}{\mu}$$

→ This is called Reynold's no. (Re)

used to determine laminar / turbulent flow

② Dynamic similarity with gravity force only

$$\frac{\text{Inertial force}}{\text{Gravity force}} = \frac{\rho l^2 u^2}{\rho l^3 g} = \frac{u^2}{g l}$$

→ (Froude no.)^2

$$F_r = \frac{u}{\sqrt{g l}}$$

$F_r = 1$ Critical flow

$F_r < 1$ Subcritical flow

$F_r > 1$ Supercritical flow

③ Dynamic Similarity with pressure force only

$$\frac{\text{Inertia force}}{\text{pressure force}} = \frac{\rho l^2 u^2}{\Delta p l^2} = \frac{\rho u^2}{\Delta p} = \frac{1}{E_u}$$

→ i.e Euler's no.

$$\text{Euler's no.} = E_u = \frac{\Delta p}{\rho u^2}$$

③ Dynamic similarity with Elastic force

$$\frac{|\text{Inertia force}|}{|\text{Elastic force}|} = \frac{\rho l^2 u^2}{E l^2} = \frac{\rho u^2}{c^2 \rho} = \frac{u^2}{c^2} = (\text{Mach no.})^2$$

$$Ma = u/c \qquad \sqrt{E/\rho} = c$$

④ Dynamic similarity with surface tension force

$$\frac{|\text{Inertia force}|}{|\text{Surface tension force}|} = \frac{\rho l^2 u^2}{\sigma l} = \frac{\rho l u^2}{\sigma}$$

→ (Weber no.)^2

$$W_e = u\sqrt{\frac{\rho l}{\sigma}}$$

* Application of dynamic similarity

Buckingham Pi theorem :

If there are 'n' no. of variables whether dependent or independent & if there are 'm' no. of fundamental dimensions then n no. of variables can be arranged in (n-m) no. of dimensionless variables & these dimensionless variables are called as π terms.

$$f(x_1, x_2, x_3, x_4) \Rightarrow F(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

$$\pi_1 = x_2^{a_1}\, x_3^{b_1}\, x_4^{c_1}\, x_1$$

$$\pi_2 = x_2^{a_2}\, x_3^{b_2}\, x_4^{c_2}\, x_5$$

$$\pi_3 = x_2^{a_3}\, x_3^{b_3}\, x_4^{c_3}\, x_6$$

$$\pi_4 = x_2^{a_4}\, x_3^{b_4}\, x_4^{c_4}\, x_7$$

$\vdots$

$$\pi_{n-m} = x_2^{a_{n-m}}\, x_3^{b_{n-m}}\, x_4^{c_{n-m}}\, x_n$$

x_2, x_3, x_4 are called repeating variables.

Each π term will contain (m+1) variables i.e. if there are 3 fundamental dimensions, each π term contains (3+1) i.e. 4 no. of variables.

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Selecting repeating variable:

- Dependent variable shouldn't be considered as repeating variables.
- Select 1st one from the geometric property i.e. diameter, length, height
- Select 2nd one from the kinematic property i.e. velocity, acceln, etc.
- Select 3rd one from the fluid property like density, viscosity

1) Pressure diff ΔP in a pipe of diameter d & length l, due to turbulent flow depends on velocity V, viscosity μ, density ρ & absolute roughness k. Using Buckingham Pi theorem, obtain an eqn for ΔP.

$\Delta P = f(d, l, V, \mu, \rho, k)$

i.e. $F(\Delta P, d, l, V, \mu, \rho, k) = 0$

Thus no. of variables $= 7$

No. of fundamental dimensions $= 3$

$\therefore$ No of π terms $= 7 - 3 = 4$

variable	dimension
ΔP	$ML^{-1}T^{-2}$
d	L
l	L
V	LT^{-1}
μ	$ML^{-1}T^{-1}$
ρ	ML^{-3}
k	L

considering d, V, ρ as repeating variables,

now, $\pi_1 = (L)^{a_1}(LT^{-1})^{b_1}(ML^{-3})^{c_1} ML^{-1}T^{-2}$

π_1 is dimensionless

$\therefore a_1 + b_1 - 3c_1 - 1 = 0 \quad c_1 + 1 = 0 \quad -b_1 - 2 = 0 \Rightarrow a_1 = 0, b_1 = -2, c_1 = -1$

$\therefore \pi_1 = D^0 V^{-2} \rho^{-1} \Delta P \quad \therefore \pi_1 = \frac{\Delta P}{\rho V^2}$

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$\pi_2 = d^{a_2} V^{b_2} \rho^{c_2} l$ $\qquad \pi_2 = M^0 L^0 T^0$

putting dimensions & equating powers,

$\pi_2 = d^{-1} V^0 \rho^0 L \qquad \therefore \pi_2 = \frac{l}{d}$

$\pi_3 = d^{a_3} V^{b_3} \rho^{c_3} \mu$

$\therefore \pi_3 = \frac{\mu}{d \rho V}$

$\pi_4 = \frac{k}{d}$

$\because F(\pi_1, \pi_2, \pi_3, \pi_4) = 0$

thus $F\left(\frac{\Delta P}{\rho V^2}, \frac{l}{d}, \frac{\mu}{d\rho V}, \frac{k}{d}\right)$

i.e. $\frac{\Delta P}{\rho V^2} = f\left(\frac{l}{d}, \frac{\mu}{d\rho V}, \frac{k}{d}\right)$

$\therefore \Delta P = \rho V^2 f\left(\frac{l}{d}, \frac{\mu}{d\rho V}, \frac{k}{d}\right)$

$= \rho V^2 f\left(\frac{l}{d}, \frac{1}{Re}, \frac{k}{d}\right)$

7) Derive an expression to present the thrust developed by an act of propellor system. The thrust P depends on ω, dynamic viscosity μ, density ρ, ~~elasticity of fluid medium denoted by~~ speed of sound C, diameter D, speed of advance V

$P = f(\omega, V, D, \mu, \rho, C)$

No. of variables $= 7$

No. of π terms $= 7 - 3 = 4$

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variable	dimension
thrust P	MLT^{-2}
ω	T^{-1}
V	LT^{-1}
D	L
μ	$ML^{-1}T^{-1}$
ρ	ML^{-3}
C	ML^{-1}

let D, V & ρ be independent variables

Thus, $\pi_1 = L^a [LT^{-1}]^b [ML^{-3}]^c \, T^{-1}$ .... ω

$\therefore a + b - 3c = 0$

$c = 0$

$-b - 1 = 0 \quad b = -1 \quad a = +1$

$\therefore \pi_1 = D^{+1} V^{-1} \rho^0 \omega = \frac{D\omega}{V}$

$\pi_2 = L^a [LT^{-1}]^b [ML^{-3}]^c [ML^{-1}T^{-1}]$ .... μ

$a + b - 3c = 1$

$c + 1 = 0 \quad c = -1$

$-b - 1 = 0 \quad b = -1 \quad a = -1$

$\therefore \pi_2 = D^{-1} V^{-1} \rho^{-1} \mu = \frac{\mu}{V\rho D}$

$\pi_3 = L^a [LT^{-1}]^b [ML^{-3}]^c [LT^{-1}]$ C

$\therefore a + b - 3c + 1 = 0$

$c = 0$

$-b - 1 = 0 \quad b = -1 \quad a = 0$

$\pi_3 = D^0 V^{-1} \rho^0 C = \frac{C}{V}$

$\pi_4 = L^a [LT^{-1}]^b [ML^{-3}]^c [M^1 L^1 T^{-2}]$ .... P

$a + b - 3c + 1 = 0$

$c + 1 = 0 \quad c = -1$

$-b - 2 = 0 \quad b = -2 \quad a = -2$

$$\pi_4 = D^{-2} V^{-2} \rho^{-1} P = \frac{P}{D^2 V^2 \rho}$$

thus, $f\left(\frac{D\omega}{V}, \frac{\mu}{V\rho D}, \frac{C}{V}, \frac{P}{D^2 V^2 \rho}\right) = 0$

$$\frac{P}{D^2 V^2 \rho} = \phi\left(\frac{D\omega}{V}, \frac{\mu}{\rho V D}, \frac{C}{V}\right)$$

$$\therefore P = D^2 V^2 \rho \, \phi\left(\frac{D\omega}{V}, \frac{1}{R_e}, \frac{1}{M_a}\right)$$

A 5 metre ship model was tested in water with density 1000 kg/m³. The measurement show resistance of 60N when model moved with 2.5 m/s. Determine velocity of 80 metre prototype & force req to drive the prototype at this speed through sea water with density 1025 kg/m3

model

$L_m = 5\,m$

$\rho_m = 1000\,kg/m^3$

$F_m = 60N$

$V_m = 2.5\,m/s$

prototype

$L_p = 80\,m$

$\rho_p = 1025\,kg/m^3$

$F = ?$

$V_p = ?$

$\because$ Problem is related to open channel flow, Froude no. is considered

$$(F_r)_m = (F_r)_p$$

$$\therefore \frac{V_p}{\sqrt{gL_p}} = \frac{V_m}{\sqrt{gL_m}} \quad \therefore \frac{V_p}{\sqrt{80}} = \frac{2.5}{\sqrt{5}}$$

$$V_p = 10\,m/s$$

$$\text{Force} = ma = \rho L^3 \times \frac{V}{t} = \rho L^3 \times \frac{V}{V/L}$$

$$= \rho L^3 \times \frac{V}{L/V} = \rho L^2 V^2$$

$$\therefore \frac{F_p}{F_m} = \frac{\rho_p V_p^2 L_p^2}{\rho_m V_m^2 L_m^2} \Rightarrow F_p = 60 \times \frac{1025 \times 10^2 \times 80^2}{1000 \times 2.5^2 \times 5^2}$$

$$=$$

Compressible fluid - Mach no.
Capillary tube - Weber no.
Open channel flow - Froude no.

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4) A wind tunnel is used to test 5:1 scale model of a car. Velocity of prototype is 60km/hr & for dynamic similar conditions, model drag is ~~250~~ 240N. If air is used with model & prototype, determine drag & power req. for prototype

$$\text{scale factor} = \frac{L_m}{L_p} = 5$$

$V_p = 60$ $\quad$ $V_m = ?$

$F_p = ?$ $\quad$ $F_m = 240$

$Power_p = ?$

~~using~~ Equaling Froude no.,

$$(F_r)_m = (F_r)_p$$

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \Rightarrow V_m^2 = 5 \times 60^2$$

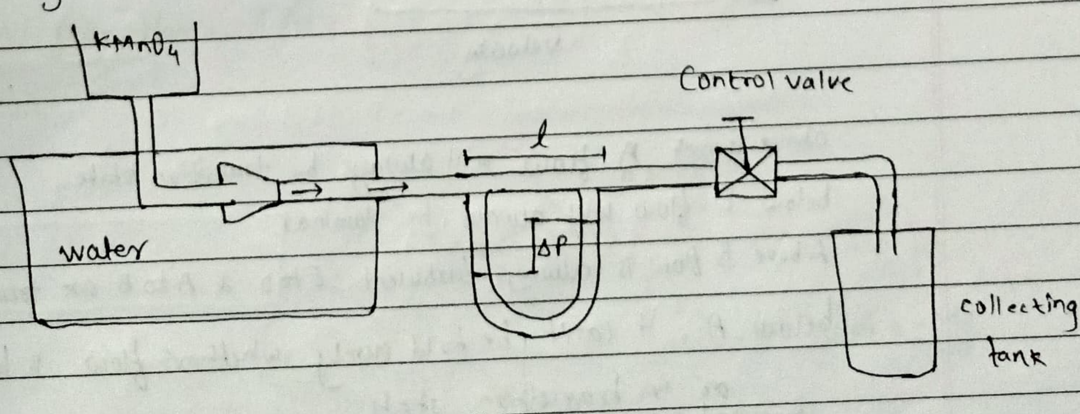
$$V_m = 60\sqrt{5}$$

$$\frac{F_p}{F_m} = \frac{\rho_p V_p^2 L_p^2}{\rho_m V_m^2 L_m^2} = \frac{1}{5 \times 25}$$

$$F_p = \frac{240}{75} = 32\text{ N}$$

$$Power_p = F_p U_p = 32 \times 60 = 1920\text{ W}$$

* Viscous fluid flow
Reynold's experiment (in 1839 by Oswald Reynold)

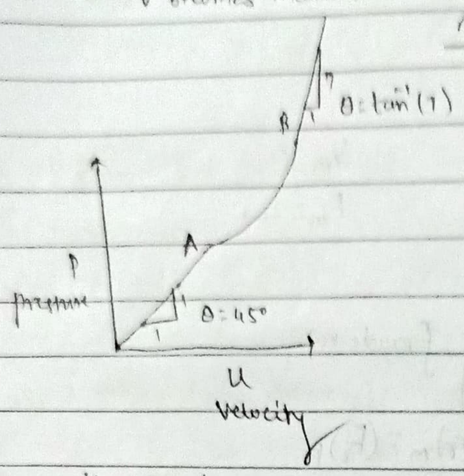


when value is open,

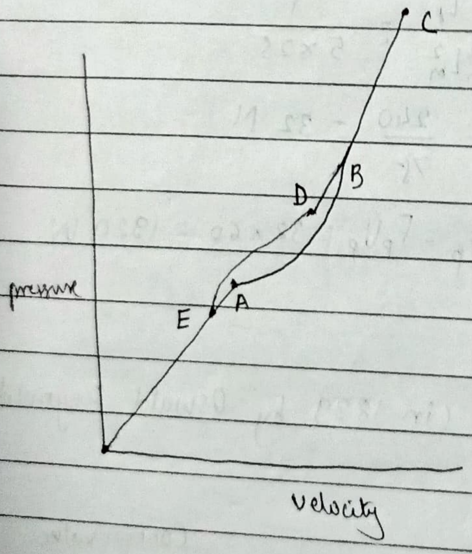
initially velocity of fluid v is very low i.e. laminar

afterwards, the flow increases & thereby V increases

V becomes medium — Transition



if we decrease velocity from certain value, its graph would be somewhat different.



above point A flow will always be transition state

below E flow will always be laminar

Above B flow is always turbulent. E to D & A to B are transition states

below A, it can't be told surely whether flow is laminar or in transition state

ocity at E corresponds to lower critical Re no.

ocity at A corresponds to higher critical Re no.

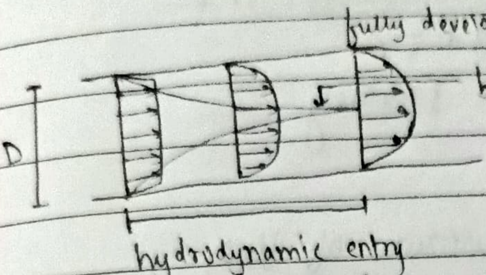
Velocity profile, shear stress profile, thermal boundary profile remains constant for a flow beyond entry length. The corresponding flow is called as fully developed flow

Imp

Re < 2000 ⇒ laminar flow

Re > 4000 ⇒ turbulent flow

2000 < Re < 4000 ⇒ transition state



$L_h = 0.05\, R_e D$

D is diameter of pipe

initially there is high hindrance to the flow because of acting shear force. Boundary layer there, has very less thickness. The — gradually goes on decreasing & boundary level thickness widens & flow gets fully developed over entry length L_h

* Frictional loss in laminar flow
(Hagen Poiseulli's eqn)

Assumptn

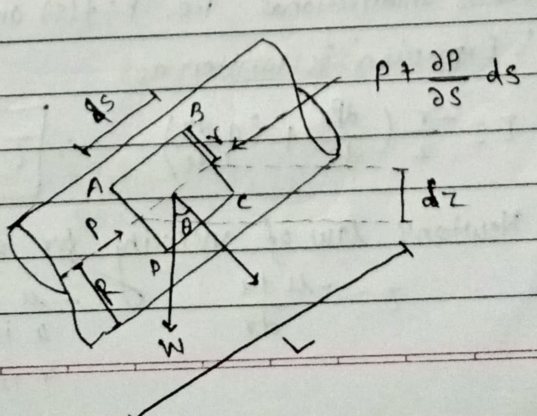
i. flow is steady, uniform & axisymmetric

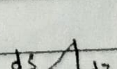
ii. One directional flow

iii. Laminar flow

iv. Fluid is real & incompressible

v. fully developed flow





$\sin\theta = \frac{dz}{ds}$

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forces acting on fluid element :

pressure force in upstream side = $P\pi r^2$

pressure force in downstream side = $(P + \frac{\partial P}{\partial s} ds)\pi r^2$

shear stress on element = $2\pi\, ds\, \tau r$

Body force of fluid element = $\rho \pi r^2 ds\, g$

Resolving the forces in direction of flow,

this is $\sin\theta$

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial s} ds\right)\pi r^2 - 2\pi r \delta s\, \tau - \rho \pi r^2 ds\, g \cdot \frac{\delta z}{\delta s}$$

According to Newton law's of motion, the resultant force must be equal to the product of mass & acceln. Since the flow is steady & uniform, thus acceln is 0 ∵ both local & convective terms are zero.

Imp

∴ The sum of 4 terms written above is 0.

$$\therefore -\frac{\partial P}{\partial s} ds\, \pi r^2 - 2\pi r \delta s\, \tau - \rho g \pi r^2 ds \frac{\delta z}{\delta s} = 0$$

$$\therefore \tau = \frac{-r}{2}\left(\frac{\partial P}{\partial s} + \rho g \frac{\delta z}{\delta s}\right)$$

at limiting conn i.e. when $\delta s \to 0$,

$$\tau = \frac{-r}{2}\left(\frac{\partial P}{\partial s} + \rho g \frac{\partial z}{\partial s}\right)$$

τ_{max} is at $r = R$

$$\boxed{\tau_{max} = \frac{-R}{2} \frac{d(P + \rho g z)}{ds}}$$

∵ flow is unidirectional i.e. $P = f(s)$ only

∴ Expression is written as

$$\tau = \frac{-r}{2}\left(\frac{dP}{ds} + \rho g \frac{dz}{ds}\right) \quad \therefore \boxed{\tau = \frac{-r}{2} \frac{d}{ds}(P + \rho g z)}$$

from Newton's law of viscosity for axisymmetric flow,

$$\tau = \frac{-\mu\, du}{dr}$$

... μ is viscosity
u is velocity
r is radius

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$$\therefore \frac{r}{2}\frac{d}{ds}(P+\rho g z) = \frac{\mu\, du}{dr}$$

$$\therefore du = \frac{1}{2\mu}\frac{d}{ds}(P+\rho g z)\, r\,dr$$

$$u = \frac{r^2}{4\mu}\frac{d}{ds}(P+\rho g z)+A$$ --- On integrating

when $r=R$, $u=0$,

$$\therefore A = \frac{-R^2}{4\mu}\frac{d}{ds}(P+\rho g z).$$

Governing eq^n is given as

$$\boxed{u = -\frac{R^2-r^2}{4\mu}\frac{d}{ds}(P+\rho g z)}$$

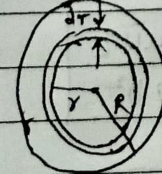
for finding u_{max}, $\frac{du}{dr}=0$

$$\therefore \frac{r}{2\mu}\frac{d}{ds}(P+\rho g z) = 0 \Rightarrow r=0$$

$$\boxed{u_{max} = \frac{-R^2}{4\mu}\frac{d}{ds}(P+\rho g z)}$$

$$\frac{u}{u_{max}} = 1-(r/R)^2 \qquad \boxed{u = u_{max}\left(1-\frac{r^2}{R^2}\right)}$$

lets consider an annular ring of thickness dr at a radius r.
The vol. flow rate is given by $dQ = 2\pi r dr . u$

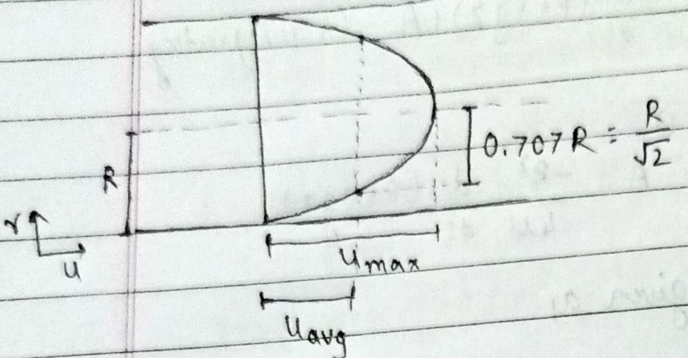


$$\therefore Q = \int_0^R 2\pi r u\,dr = \frac{-2\pi}{4\mu}\frac{d}{ds}(P+\rho g z)\int_0^R (R^2-r^2)\, r\,dr$$

imp

$$\boxed{Q = \frac{-\pi R^4}{8\mu}\frac{d}{ds}(P+\rho g z)}$$

$$\bar{u} = u_{avg} = \frac{Q}{\pi R^2} \qquad \boxed{\begin{aligned}\bar{u} &= u_{avg} = \frac{-R^2}{8\mu}\frac{d}{ds}(P+\rho g z) \\ \bar{u} &= u_{avg} = \frac{1}{2} u_{max}\end{aligned}}$$



$$\therefore d(P+\rho g z) = \frac{-8\mu \, ds}{R^2}$$

Integrating

$$\therefore P_2 - P_1 + \rho g (Z_2 - Z_1) = \frac{-8\mu\bar{u}(S_2 - S_1)}{R^2}$$

$$\therefore \boxed{\frac{P_1}{\rho g} + Z_1 = \frac{P_2}{\rho g} + Z_2 + \frac{8\mu\bar{u}L}{\rho g R^2}} \qquad \ldots S_2 - S_1 = L$$

Again Bernaulli's eq^n for real fluid is,

$$\frac{P_1}{\rho g} + Z_1 = \frac{P_2}{\rho g} + Z_2 + \frac{8\mu\bar{u}L}{\rho g R^2} + h_f$$

h_f is loss of head due to frict^n in laminar flow

Imp

comparing above 2 eq^{n}s,

$$\boxed{h = \frac{8\mu\bar{u}L}{\rho g R^2} = \frac{32\mu\bar{u}L}{\rho g D^2}}$$

pressure gradient is given by $\boxed{\frac{dP}{ds} = \frac{\Delta P}{L} = \frac{8\mu\bar{u}}{R^2}}$

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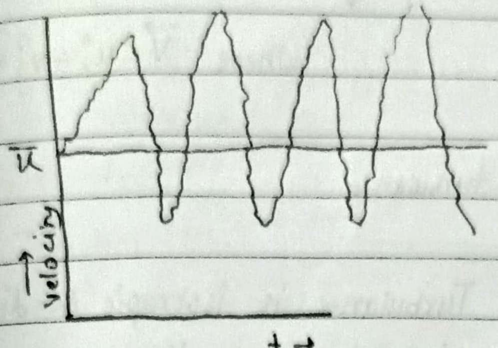
* Turbulant flow



u - instantaneous velocity
$\bar{u}$ - time average mean
u' - fluctuating velocity

$u = \bar{u} + u'$

similarly $v = \bar{v} + v'$ } other components of velocity
$w = \bar{w} + w'$
$p = \bar{p} + p'$ (pressure)

Turbulant flow is characterized by irregular & random motion of fluid particles. In turbulant flow, both pressure & velocity fluctuate wrt time

Time average mean velocity

$$\bar{u} = \frac{1}{t}\int_0^t u\,dt \qquad \bar{w} = \frac{1}{t}\int_0^t w\,dt$$

$$\bar{v} = \frac{1}{t}\int_0^t v\,dt$$

Root mean square of the fluctuating component is

$$\bar{u}' = \left[\frac{1}{t}\int_0^t u'^2\,dt\right]^{1/2} \qquad \bar{w}' = \left[\frac{1}{t}\int_0^t w'^2\,dt\right]^{1/2}$$

$$\bar{v}' = \left[\frac{1}{t}\int_0^t v'^2\,dt\right]^{1/2}$$

Magnitude of fluctuating component in all directions -

$$\sqrt{\frac{1}{3}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)}$$

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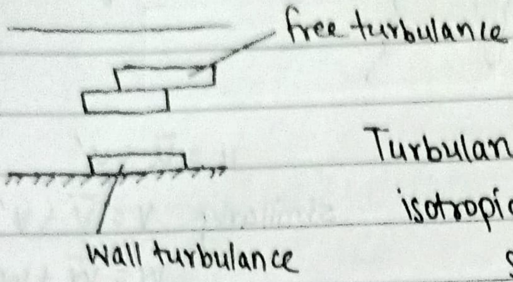
Turbulant intensity $= \dfrac{\sqrt{\frac{1}{3}(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})}}{\overline{V}}$

where $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$



Turbulance is isotropic & diffusive

isotropic means - its properties remain same throughout i.e. if it is turbulant, it won't change nature unless velocity is reduced

diffusive: due to transverse velocity component, fluid layer is diffused in with other fluid layers

Reynold's theory of turbulance, -

$$\tau = \rho u'v'$$

Boussinesq's theory of turbulance, -

$$\tau = \eta \frac{d\bar{u}}{dy} + \mu \frac{du}{dy}$$

η - eddy viscosity

turbulant part, laminar part

Prandtl mixing length theory -

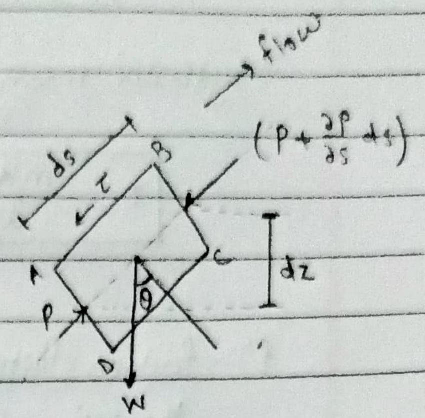
$$\tau = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$$

, l - mixing length

* Head loss due to turbulent flow (Darcy-Weisbach eq^n)

Assumpt^n

1) Steady, uniform & anisymmetric
2) One dimensional flow
3) fully developed flow
4) Turbulent flow
5) Incompressible & real fluid



area = dA

wetted perimeter = p

resolving forces in direct^n of flow,

$$p.dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \tau p ds - W \sin\theta = \text{mass} \times \text{accel}^n$$

however accel^n = 0

∴ RHS = 0

$$\therefore -\frac{\partial p}{\partial s} ds\, dA - \tau p ds - W \sin\theta = 0$$

$$W = mg = \rho . ds . dA . g$$

$$\sin\theta = dz / ds$$

$$\therefore -\frac{\partial p}{\partial s} ds\, dA - \tau p ds - \rho g\, ds\, dA \frac{dz}{ds} = 0$$

$$\therefore -\frac{\partial p}{\partial s} dA - \tau p - \rho g\, dA \frac{dz}{ds} = 0$$

$$\therefore \tau p = -\frac{\partial p}{\partial s} dA - \rho g\, dA \frac{dz}{ds}$$

∵ Pressure is fun^n of s only

$$\therefore \tau p = -\frac{dp}{ds} dA - \rho g\, dA \frac{dz}{ds}$$

$$\therefore \frac{\tau p}{dA} = \frac{-d}{ds}(p + \rho g z)$$

$$\therefore \tau = \frac{dA}{P}\frac{d}{ds}(P+\rho g z)$$

hydraulic mean depth $= m = \frac{dA}{P} = \frac{\text{area}}{\text{wetted perimeter}}$

$$\boxed{\tau = \frac{-m\,d}{ds}(P+\rho g z)}$$

f' - friction factor or coefficient of frict^n which is given as the ratio of wall shear force & dynamic pressure based on average velocity

$$\therefore f' = \frac{\tau}{\frac{1}{2}\rho\bar{u}^2}$$ $\bar{u}$ - average velocity

$$\therefore \boxed{\tau = \frac{1}{2}\rho\bar{u}^2 f'}$$

Equating the 2 eq^{n}s,

$$-m\frac{d}{ds}(P+\rho g z) = \frac{1}{2}\rho\bar{u}^2 f'$$

$$\therefore d(P+\rho g z) = -\frac{\rho\bar{u}^2 f'}{2m}\,ds$$

Integrating,

$$P_2 - P_1 + \rho g(z_2 - z_1) = -\frac{\rho\bar{u}^2 f'}{2m}(s_2 - s_1)$$

$$\therefore \frac{P_1}{\rho g} + z_1 = \frac{P_2}{\rho g} + z_2 + \frac{\rho\bar{u}^2 f' L}{2m\rho g}$$

comparing with Bernaulli's eq^n of real fluid,

$$\boxed{h_f = \frac{\bar{u}^2 f' L}{2mg}}$$ m is hydraulic mean depth

h_f - head loss due to frict^n in turbulent flow & it is valid for any cross sect^n of pipe

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★ for circular pipe of diameter D,

$$m = \frac{\text{area}}{\text{wetted peri}} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

$$\therefore h_f = \frac{4f'\bar{u}^2 L}{2Dg}$$

★ for square pipe of side a,

$$m = \frac{a^2}{4a} = \frac{a}{4}$$

$$h_f = \frac{4f'\bar{u}^2 L}{2ag}$$

i.e. for circular pipe,

$$h_f = \frac{4f'\bar{u}^2 L}{2gD} = \frac{f\bar{u}^2 L}{2gD} \quad \text{where } f = 4f'$$

f' - fanning's frictn factor
f - Darcy's frictn factor

$$\boxed{h_f = \frac{f\bar{u}^2 L}{2gD}}$$

Imp

$h_{f\,laminar} = \frac{32\mu\bar{u}L}{\rho g D^2}$	$\frac{\Delta P}{l} \propto \bar{u} \Rightarrow$ laminar flow
$h_{f\,turbulent} = \frac{f\bar{u}^2 L}{2gD}$	$\frac{\Delta P}{l} \propto \bar{u}^2 \Rightarrow$ turbulent flow

frictn factor (f) $= f\left(Re, \frac{k}{D}\right)$ $\quad \frac{k}{D}$ is relative roughness

fluid flowing through 2 pipes of same diameter
one is laminar & other is turbulent.
Velocity & frictn factor is also same

then, $h_{f\,laminar} = h_{f\,turbulent}$ $\quad \therefore \frac{32\mu\bar{u}L}{\rho g D^2} = \frac{f\bar{u}^2 L}{2gD}$

vimp

$$\boxed{f = \frac{64\mu}{\rho\bar{u}D} = \frac{64}{Re}}$$

i.e. frictn factor doesn't depend on any other factor

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* Friction factor dependency on absolute roughness

we know, $\frac{\Delta P}{\rho V^2} = f\left(\frac{l}{d}, R_e, \frac{k}{D}\right)$

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = \frac{l}{D}\phi\left(R_e, \frac{k}{D}\right) \quad \text{--- (1)}$$

Now, Bernoulli's eqn for real fluid for uniform cross section & horizontal pipe

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \qquad \therefore h_f = \frac{P_2 - P_1}{\rho g} = \frac{\Delta P}{\rho g}$$

$$\therefore \frac{\Delta P}{\rho} = h_f g \quad \text{--- (2)}$$

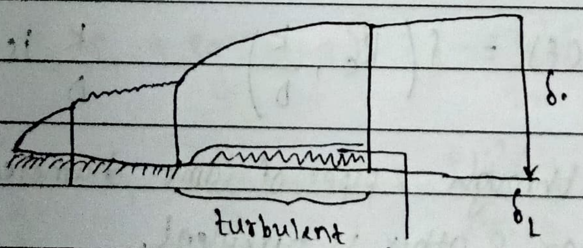
from (1) & (2),

$$\frac{2 h_f g D}{V^2 l} = \phi\left(R_e, \frac{k}{D}\right)$$

$V = \bar{u}$ (average velocity)

we know, $f = \frac{2 h_f g D}{\bar{u}^2 l}$

$$\therefore f = \phi\left(R_e, \frac{k}{D}\right)$$ here, h_f is $h_{f\,turbulent}$



(acts as good lubricant)

for smooth commercial pipe ($\delta_L > k$)

$$\frac{1}{\sqrt{f}} = 2 \log R_e \sqrt{f} - 0.8$$

For rough pipe ($\delta_L < 0.3k$)

$$\frac{1}{\sqrt{f}} = 2\log\frac{D}{k} + 1.14$$

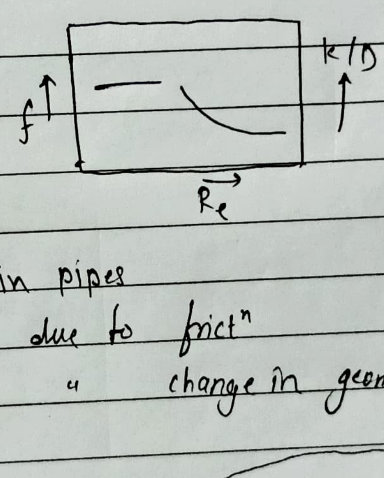
for intermediate stage ($\delta_L > 0.3k$)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right)$$

One of the popular eq^n is Jain & Swami eq^n

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{k}{D} + \frac{21.25}{R_e^{0.9}}\right)$$

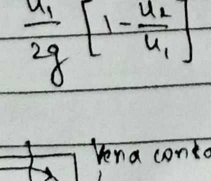
using Moody's diamter

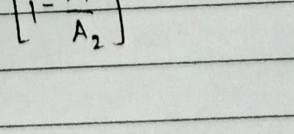


losses due to flow in pipes

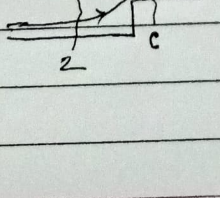
major loss : mainly due to frict^n

minor loss - " " change in geometry, sudden expansion

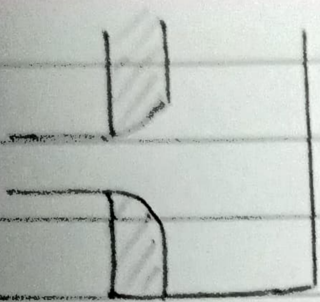




$$h_f = \frac{u_1^2}{2g}\left[1 - \frac{u_2}{u_1}\right]^2 = \frac{u_1^2}{2g}\left[1 - \frac{A_1}{A_2}\right]^2$$



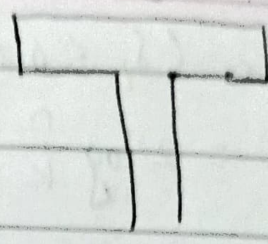
$$h_e = \frac{u_2^2}{2g}\left[\frac{u_c}{u_2} - 1\right]^2$$



Entry loss :

$k \to 0.5 - 0.8$

Tee



$k = 1.8$

Elbow

$k = 0.9$

Globe value $\to$ 10

Gate value $\to$ 0.2

Foot value $\to$ 1.5