# National Institute of Technology Durgapur

Mathematics-I

## MAC01

Full Marks: 60 Time: 2 Hr (15 minutes extra for uploading)

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**Important Instructions**

- Answer all questions
- Symbols have their usual meanings
- Write your name, roll number, and paper code on the top of the first page of your answer script
- Make a single pdf file of the entire answer script. Give the file name as 'ROLL-NUMBER.pdf' and upload.

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1. Verify Green's theorem in the plane for $\int_C (2xy - x^2)\,dx + (x + y^2)\,dy$, where $C$ is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ having anti-clockwise sense of description. 6

2. Evaluate $\iiint \frac{dx\,dy\,dz}{x^2+y^2+(z-2)^2}$ over the spherical region $x^2 + y^2 + z^2 \leq 1$. 6

3. Let $S$ be a closed surface enclosing a volume $V$ and $\mathbf{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$, where $a$, $b$ and $c$ are constants. Derive a relation between $\iint_S (\mathbf{A} \cdot \mathbf{n})dS$ and the volume $V$. 6

4. Examine the convergence of the following series: 3 + 3

   (a)
   $$\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}.$$

   (b)
   $$\sum_{n=3}^{\infty} \frac{\sqrt{2n^2 - 5n + 1}}{4n^3 - 7n^2 + 2}.$$

5. Suppose $f(x) = (1 - x)^{\frac{5}{2}}$ and it has a representation of the form:
   $$f(h) = f(0) + h.f'(0) + \frac{h^2}{2} f''(\theta h), \quad 0 < \theta < 1.$$
   If $h = 1$, then find the value of $\theta$. 6

6. Check the continuity of the function

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$$

at $(0,0)$. Also check the existence of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(0,0)$. 6

7. Find a point in the plane $x + 2y + 3z = 13$ nearest to the point $(1,1,1)$ using the method of Lagrange's multiplier. 6

8. (a) Evaluate 3

$$\int_{-a}^{a} \frac{dx}{\sqrt{a^2 - x^2}},$$

where $a$ is a real constant.

(b) Evaluate 3

$$\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^7\theta d\theta.$$

9. Evaluate

$$\iint_R \cos(x + 2y)\, dxdy$$

where $R = [0,\pi] \times [0,\frac{\pi}{2}]$. 6

10. Find the volume of the solid generated by revolving the curve $xy^2 = 4(2-x)$ about the $y$-axis. 6

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