Undergraduate Graduate Course

on

Theory of Mechanisms & Machines

(Mechanisms Synthesis & Analysis)

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&

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Books

1. Ghosh, Mallik, "*Theory of Mechanisms and Machines* ", EWP.
2. Uicker, Pennock, Shigley, "*Theory of Machines and Mechanisms*", Oxford University Press.
3. Erdman & Sandor, "Mechanism Design: Analysis & Synthesis" 'Vol. 1', PHI.

Prerequisite Subject Knowledge

- ✓ Engineering Mechanics (*Rigid body/ Multi-body mechanics*)
- ✓ Theory of Mechanisms & Machines
- ✓ Basics of Machine Drawing
- ✓ Engineering Mathematics

Topics to be Covered

Chapter	Topics	Books
Introduction to Mechanisms	Linkages, Mechanisms and machines; Kinematic pair, element, chains and inversions; degrees of freedom, mobility and Gruebler's criterion; four bar mechanisms and slider crank mechanisms Special Mechanisms - Indicator Diagram Mechanisms, Steering Mechanism, Hookes Joint	
Kinematics of Rigid Bodies	Frame of reference in general motion, General plane motion, absolute and relative velocity in plane motion, Instantaneous center of rotation in plane motion	
Kinetics of Rigid Bodies	Plane motion of rigid bodies: Force and accelerations methods, Energy and momentum methods	
Kinematic Analysis of Planar Linkages	Position & displacement analysis, Velocity analysis, Acceleration analysis	
Gears & Gear trains	Fundamental law of gearing, gear tooth terminology, gear type, contact ratio & Kinematics analysis, Kinematic analysis of Gear trains: Velocity ratio and sense of rotation; simple, compound and epicyclic gear trains	
Cam Mechanisms	Cam terminology, displacement diagram, graphical layout of cam profile.	
Computer Aided Mechanism Analysis	Displacement, Velocity & Acceleration analysis of planar linkages using vector loop closure method	
Kinematic Synthesis of Planar Linkages	Type, number and dimensional synthesis, Body guidance, path and function generation, Analytical linkage synthesis	

Topics to be Covered

Chapter	Topics	Books
Dynamic Force Analysis of Machines	Dynamic force analysis for slider crank mechanism; inertia forces in reciprocating parts; primary and secondary inertia forces; simple engine mechanism – gas force, piston effort, gudgeon pin load, crank effort or turning moment; single and double acting engine; inertia force analysis considering mass of the connecting rod; force analysis for a four bar mechanism	
Flywheels:	Turning moment diagram, indicator diagrams – mean effective pressures for suction, compression, expansion and exhaust strokes; overall mean effective pressure for the cycle; mean resisting torque; fluctuation of energy and speed; flywheel	
Governor Mechanisms:	Types, characteristics of centrifugal governors; conical pendulum type governors – Watt, Porter, and Proell; Spring loaded type of governors – Hartnell; controlling force, effort, power, sensitiveness, isochronism, stability and hunting of governors	

Analysis & Synthesis of Mechanisms

Keywords:

- ANALYSIS → Mechanics
- SYNTHESIS → Design
- MECHANISM

Introduction

Rigid body Mechanics → The branch of scientific analysis that deals with geometry, motion, time & forces.

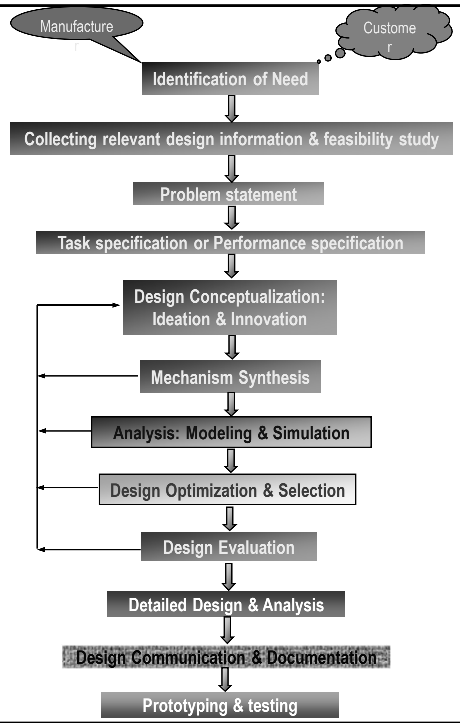
Mechanics
- Statics
- Dynamics
  - Kinematics
  - Kinetics

Assumption of Rigidity

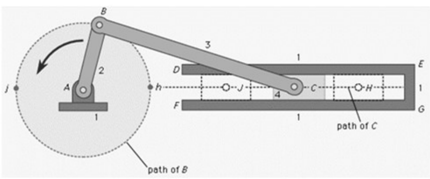
- The assumption of rigidity is that there is no relative motion (or change in distance) between two arbitrary chosen points on the same link.

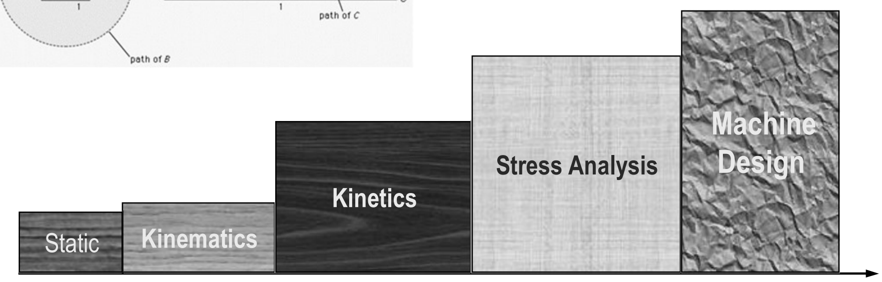
- It is very important assumption that allows the separation of dynamics into KINEMATICS and KINETICS.

Stages of Machine Design



Steps in Machine Design or Robot Design





Mechanisms: An Introduction

Theory of Machine is an Applied science which is used

- to understand the relationships between the geometry & motions of the parts of the mechanisms/machines.

- To estimate the forces required to produce these motions

The study of relative motion between the various parts of a machine & the study of the forces that act on those parts

Learning Objectives:

Are to provide the engineers the necessary tools to systematically synthesize a system which means scientifically arriving at the best possible configurations & kinematic dimensions of the bodies constituting the system

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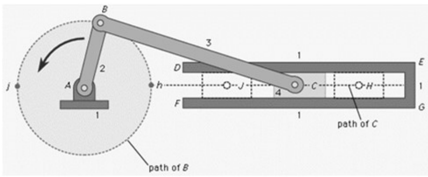
Basics of Mechanism

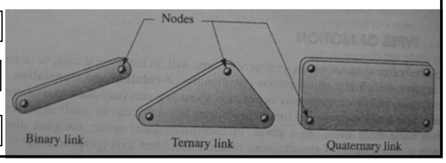
Kinematic Link → Kinematic Pair → Kinematic Chain → (One link fixed) → Mechanism → Machine



Kinematic Link - **Each part of a mechanism which has motion relative to some other part**

Types of Rigid Links

Singular link	connected to only one other link
Binary link	connected to two other links
Ternary link	connected to three other links
Quaternary link	connected to four other links



Kinematics Links

Rigid Link — It does not undergo any significant deformation while transmitting motion.

Flexible Link — It is one which is partly deformed in a manner not to affect the transmission of motion .

Fluid Link — It is deformed by having fluid in a closed vessel & the motion is transmitted through the fluid by pressure.

Types of Rigid Links

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- **Binary link** — connected to two other links
- **Ternary link** — connected to three other links
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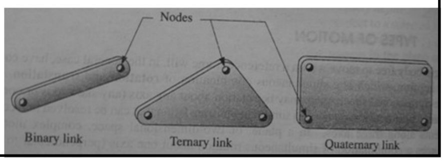


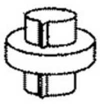
Basics of Mechanism (contd...)

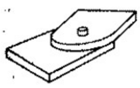
Kinematic Pair - is a joint of two or more links having relative motion between them

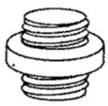
Classification of Joints or Kinematic pairs

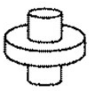
Types of Kinematic Pairs

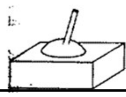
- **According to Nature of Contact**
  - Lower Pair — Surface or area contact between links
  - Higher Pair — Line or point contact between links
- **According to Nature of Relative Motion**
  - Sliding/ Prismatic Pair
  - Turning/ Revolute Pair
  - Screw Pair
  - Cylindrical Pair
  - Spherical Pair

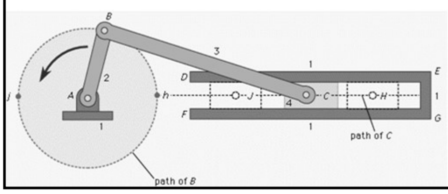












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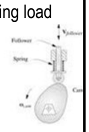
Basics of Mechanism (contd...)

Classification of Joints or Kinematic pairs

Types of Joints/ Kinematic Pairs

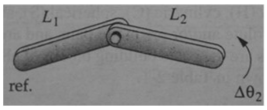
- **According to type of physical closure of the joint**
  - Form closed — The contact is ensured by the geometry or form of the links
  - Forced closed — The contact is ensured by an external load, such as spring load



- **According to number of links joined (order of joint)**
  - First order joint
  - Second order joint
  - ⋮
  - Higher order joint

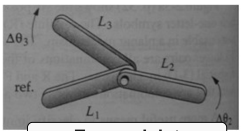
- Joint order is defined as the number of links joined minus one.

First order joint



Binary joint

Second order joint



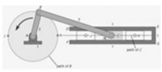
Ternary joint

Basics of Mechanism (contd...)

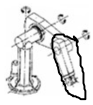
Kinematic Link → Kinematic Pair → Kinematic Chain → (One link fixed) → Mechanism → Machine

Kinematic Chain - A combination of kinematic pairs joined in such a way the relative motion of any point on a link w.r.t the any other point on other link follows a definite law.

- *Closed Chain*



- *Open Chain*



Mechanism - A kinematic chain in which one link fixed & used for transmitting & transforming motion.

Mechanism Vs. Machine

Mechanism → transfers / transforms → Motion

Primary Objectives

Machine → transfers / transforms → Energy

All Machines are Mechanism but all Mechanisms are not Machine

All Machine Tools are Machine but all Machines are not Machine Tool

A mechanism is a combination of rigid or restraining bodies so shaped and connected that they move upon each other with a definite relative motion.

A machine is a mechanism or a collection of mechanisms which transmits force from the source of power to the resistance to be overcome,and thus perform a mechanical work.

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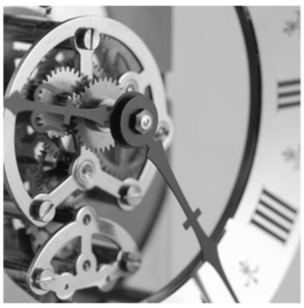
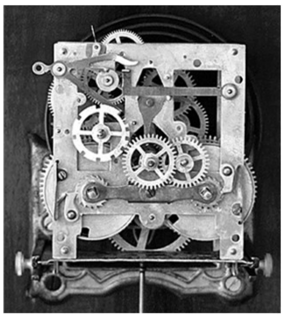
Basics of Mechanism (contd…)

Mechanism

- **assemblage of resistant bodies, connected by joints, to form a kinematic chain with one link fixed & having the purpose of transmitting & transforming motion.**
- **provides the definite motion of the parts of a machine.**





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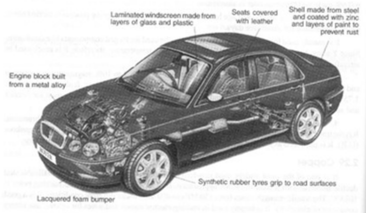
Basics of Mechanism (contd...)

Mechanism
- assemblage of resistant bodies, connected by joints, to form a kinematic chain with one link fixed & having the purpose of transmitting & transforming motion.
- provides the definite motion of the parts of a machine.

Machine
- Combination of resisting bodies having definite motions & capable of performing useful work.
- Combination of resistant bodies with successfully constrained relative motions which is used to transform, transmit & modify available energy to do some useful work



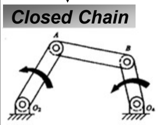


Basics of Mechanism (contd...)

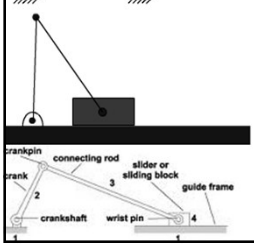
Classification of Mechanism

- **Planar Mechanism** — All the points of a mechanism move in parallel planes
  - *Closed Chain*
  - *Open Chain*
- **Spatial Mechanism** — All the points of a mechanism do not move in parallel planes
  - *Closed Chain*
  - *Open Chain*

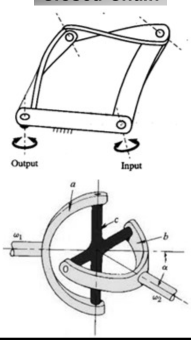
Closed Chain



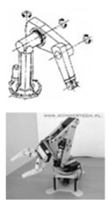
Open Chain

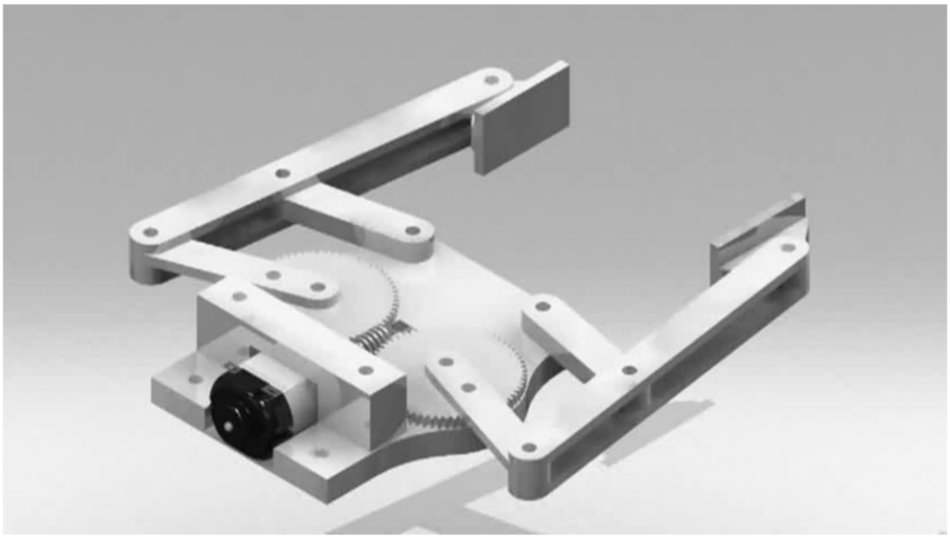
Closed Chain



Open Chain




Planar Mechanism



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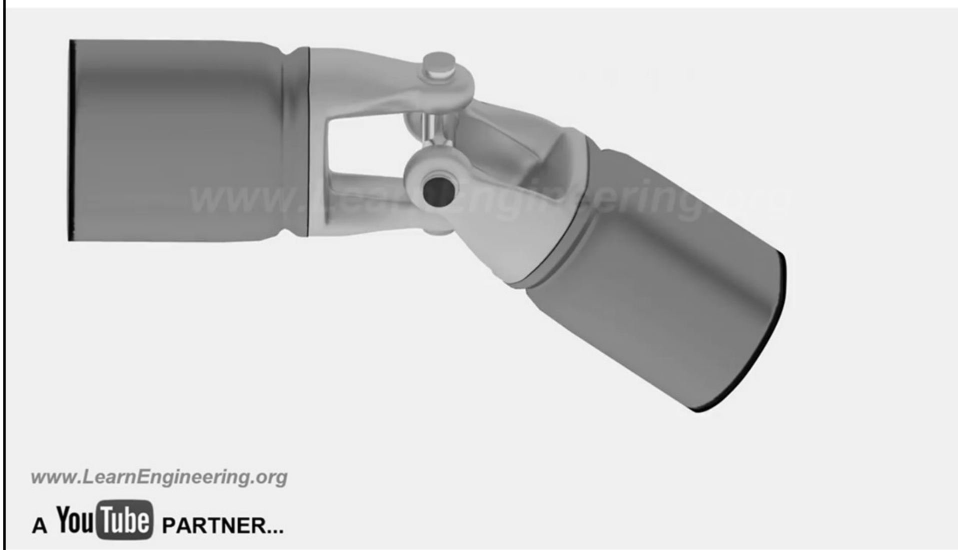
Spatial Mechanism



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Basics of Mechanism (contd...)

Degrees of Freedom (DOF)

- ***Number*** of ***Independent co-ordinates/variables*** **required to completely specify the configuration of the mechanism in motion in space**
- ***Minimum number*** of ***co-ordinates/variables*** **required to describe a system completely**
- ***Minimum number*** of ***co-ordinates/variables*** **required to completely specify the relative movement of the links of the mechanism in space**

Lower Pairs

Kinematic Pair	*Symbol*	*Pair variable*	*DOF*
Revolute	R	θ	1
Prismatic	P	d	1
Screw or Helical	H	θ or d	1
Cylindrical	C	θ & d	2
Spherical	S	θ, φ, ψ	3

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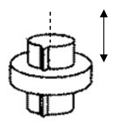
Basics of Mechanism (contd...)

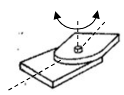
Kinematic Pairs in Planar Kinematic Chain

- Prismatic Pair (P)



1 DOF

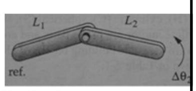
- Revolute Pair (R)

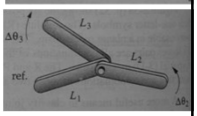


1 DOF

Order of joint or joint order

- First order joint — 1 DOF
- Second order joint — 2 DOF
- $(L-1)^{th}$ order joint — (L-1) DOF





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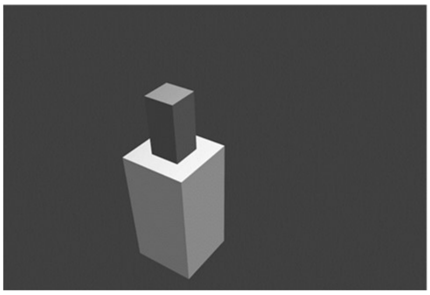
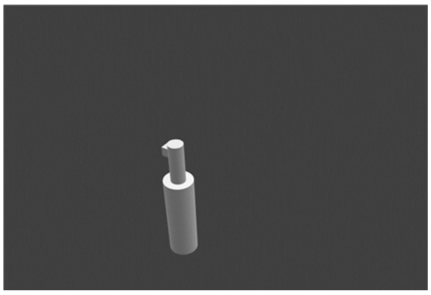
Kinematic Pairs in Planar Kinematic Chain





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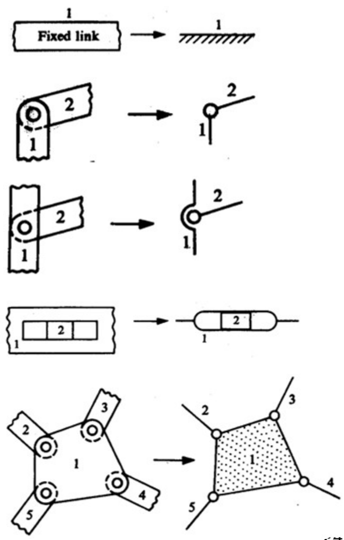
Basics of Mechanism (contd...)

Kinematic Diagram

- As a result of the assumption of rigidity, many of the intricate details of the actual part shapes are irrelevant when studying the kinematics of a mechanism.
- For this reason it is common practice to draw highly simplified schematic diagrams, which contain important features of the shape of each link, such as the relative locations of pair elements, but which completely subdue the real geometry of the manufactured parts. This simplified schematic diagram is known as **KINEMATIC DIAGRAM**. Such a diagram depicts the essential kinematic features of the mechanism.



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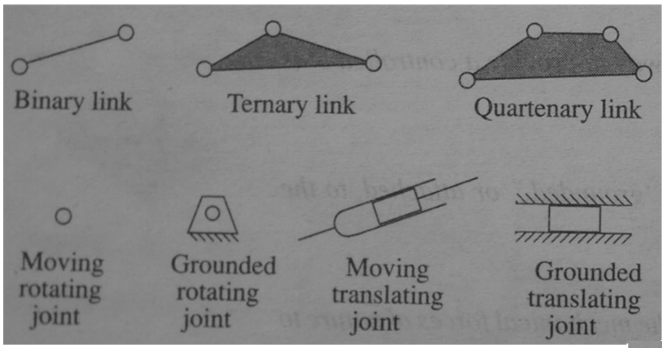
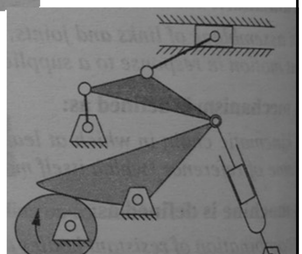
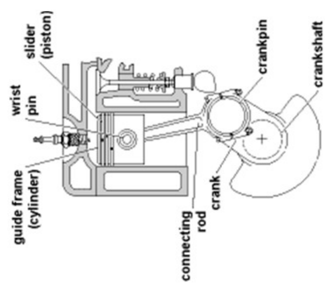
Kinematic Diagram



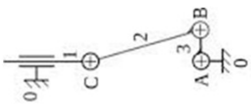


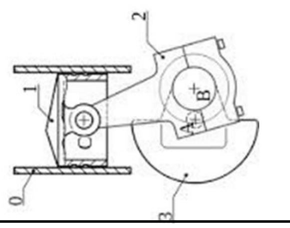
Example

Basics of Mechanism (contd…)



Kinematic Diagram





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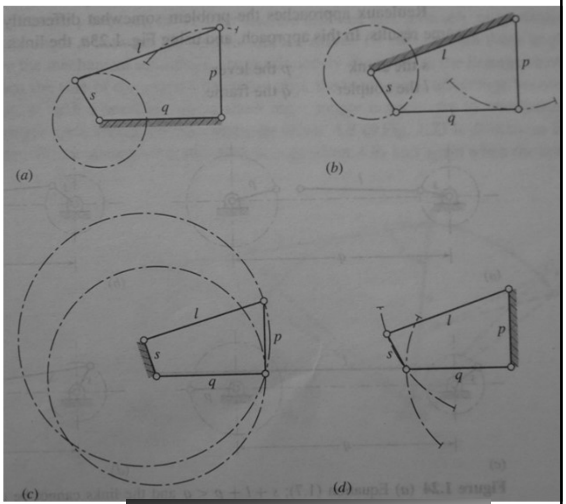
Basics of Mechanism (contd...)

Kinematic Inversion

- Every mechanism has a fixed link called the frame.
- When different links are chosen as the frame for a given kinematic chain, the relative motions between the various links are not altered, but their absolute motions (those measured w.r.t the frame) may be changed drastically.

The process of fixing different links of the same closed kinematic chain to produce different mechanisms is called Kinematic Inversion.



Four Inversion of 4 bar linkage

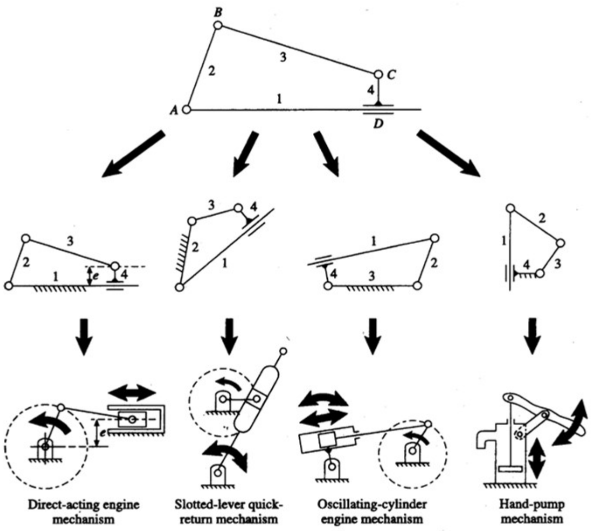
Basics of Mechanism (contd...)

Kinematic Inversion

It is the process of fixing different links of the same closed kinematic chain to produce different mechanisms.

3R1P Mechanism

RRRP Mechanism



Basics of Mechanism (contd...)

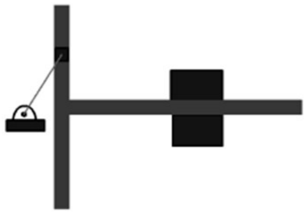
Kinematic Inversion

It is the process of fixing different links of the same closed kinematic chain to produce different mechanisms.

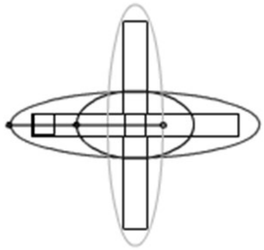
2R2P Mechanism

RRPP Mechanism

Scotch Yoke



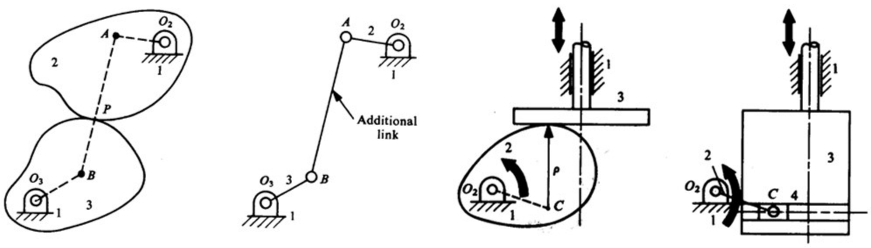
Elliptic Trammel



Basics of Mechanism (contd...)

Equivalent Linkages

- A mechanism with higher pairs can be replaced by an Equivalent Mechanism with lower pairs.
- This equivalence is valid for studying only the instantaneous characteristics.



Basics of Mechanism (contd…)

Degrees of Freedom (DOF)

- ***Number*** **of** ***Independent co-ordinates/variables*** **required to completely specify the configuration of the mechanism in motion in space**
- ***Minimum number*** **of** ***co-ordinates/variables*** **required to describe a system completely**
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Lower Pairs

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Cylindrical	C	θ & d	2
Spherical	S	θ, φ, ψ	3

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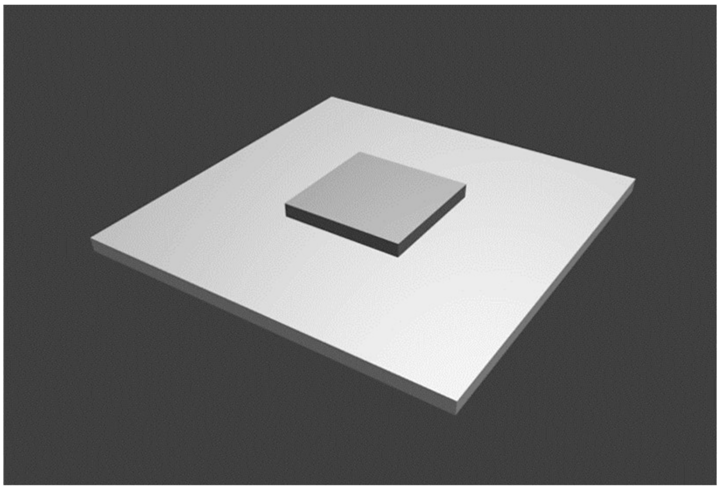

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Basics of Mechanism (contd…)

Mobility & Degrees of Freedom

- An arbitrary no. of links connected by a no. of kinematic pairs do not result in a mechanism. Some conditions must be satisfied for a system of interconnected links to serve as a useful mechanism.
- The foremost thing which has to be investigated is the mobility of a mechanism in terms of the number of DOF.

***Number* of *Independent co-ordinates/variables* required to completely specify the configuration of the mechanism in motion in space**

Kutzbach Equation — **For Planar Mechanisms**

DOF $F=3(n-1) - 2j$

n=number of links

j=number of simple hinges or first order joints or equivalent first order joints=($j_1+2j_2+3j_3+\ldots$..)

j_i is the nos. of i^{th} order joint.

i^{th} order joint is equivalent to 'i' nos. of first order joint or simple hinge

Grubler's Criteria — **For Planar Mechanisms**

If F=1, the mechanism is said to be Constrained.

$$2j - 3n + 4=0$$

DOF of a mechanism having higher pairs

$F=3(n-1) - 2j - h$ h=number of higher pairs

Basics of Mechanism (contd…)

Equivalent Linkages

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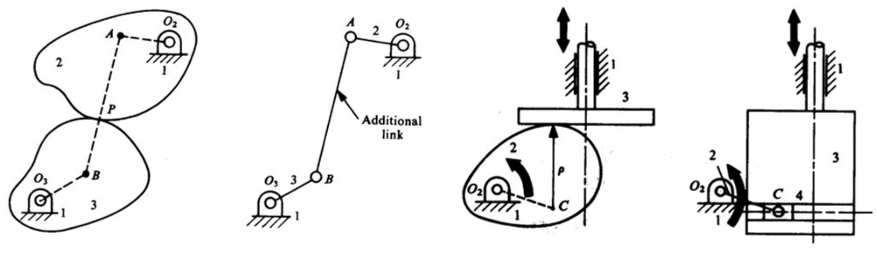
- This equivalence is valid for studying only the instantaneous characteristics.



Basics of Mechanism (contd...)

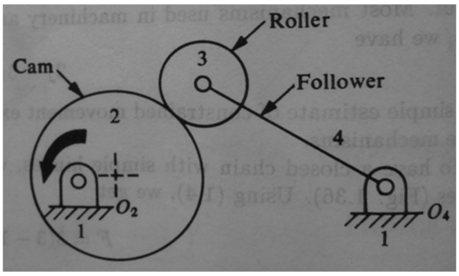
Degrees of Freedom

Effective DOF of a mechanism having higher pairs & redundant DOF

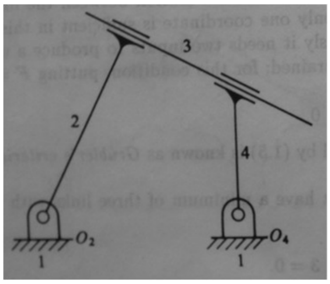
$$F_e = 3(n-1) - 2j - h - F_r$$

Redundant DOF

- If a link can be moved without causing any movement in the rest of the mechanism, then the link is said to be have redundant DOF.



- Roller 3 can rotate without causing any movement in the rest of the mechanism.



- Rod 3 can slide without causing any movement in the rest of the mechanism.

Basics of Mechanism (contd...)

Degrees of Freedom of Spatial Mechanism

$$F = 6(n-1) - 5(R+P+H) - 4C - 3S - F_r$$

R=number of Revolute pair
P=number of Prismatic pair
H=number of Screw pair
C=number of Cylindrical pair
S=number of Spherical pair

Kinematic Pair in Space	*Symbol*	*No. of Constraints*	*DOF*
Revolute	**R**	**5**	**1**
Prismatic	**P**	**5**	**1**
Screw or Helical	**H**	**5**	**1**
Cylindrical	**C**	**4**	**2**
Spherical	**S**	**3**	**3**

Basics of Mechanism (contd...)

Degrees of Freedom of Planar Mechanism

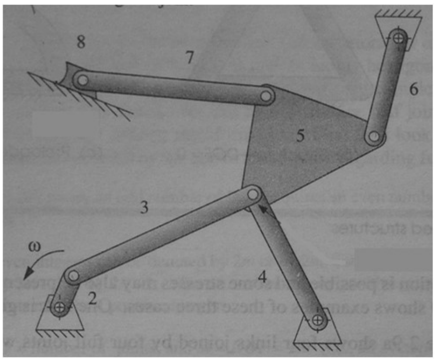
Kinematic Pair in Plane	*Symbol*	*No. of Constraints*	*DOF*
Revolute	R	2	1
Prismatic	P	2	1
Screw or Helical	H	2	1
Cylindrical	C	1	2
Spherical	S	0	3

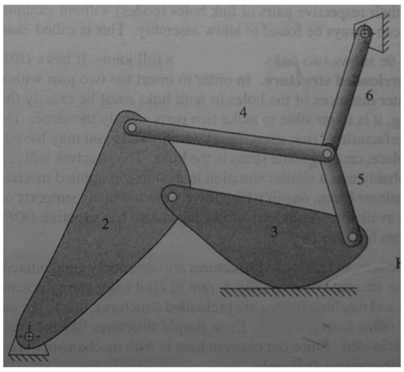
Degrees of Freedom of Spatial Mechanism

Kinematic Pair in Space	*Symbol*	*No. of Constraints*	*DOF*
Revolute	R	5	1
Prismatic	P	5	1
Screw or Helical	H	5	1
Cylindrical	C	4	2
Spherical	S	3	3

Basics of Mechanism (contd...)

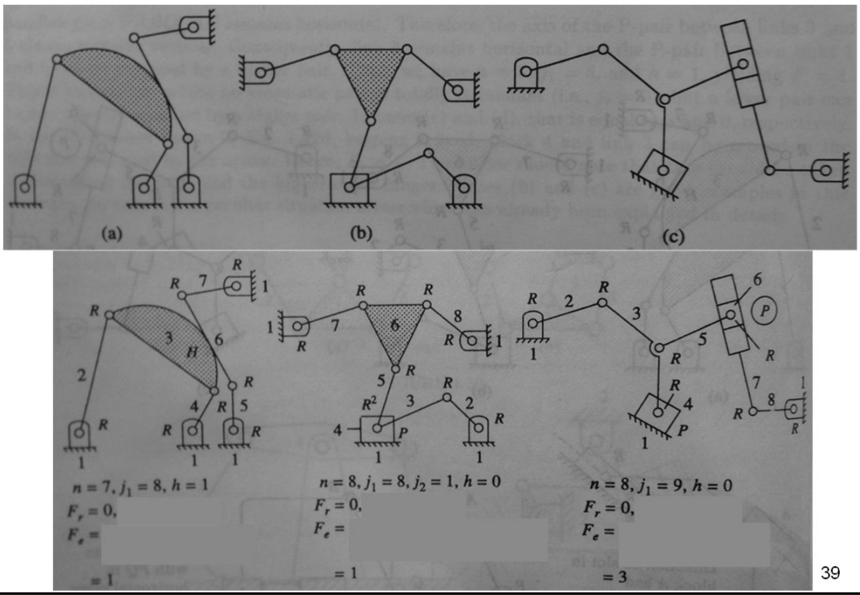
Degrees of Freedom





Basics of Mechanism (contd...)

Degrees of Freedom



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Basics of Mechanism (contd...)

Grashof's Law

- For a planar four-bar linkage, the sum of the shortest and longest link lengths can not be greater than the sum of the remaining two link lengths if there is to be continuous relative rotation between two members.

$$s + l \leq p + q$$

s=*length* of the shortest link
l=length of the longest link
p, q= length of the other two links

Grashof Linkage Class-I Kinematic Chain $s + l < p + q$

The linkage which satisfies inequality given in Equation (1.7) is Grashof linkage. In this at least one link will be capable of making a full revolution with respect to the ground plane. This is called Class-I kinematic chain.

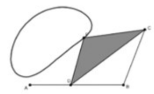
Note that above statements apply regardless of the order of assembly of the links. That is, the determination of the Grashof condition can be made on a set of unassembled links. Assembling of the link into a kinematic chain in *S, L, P, Q* or *S, P, L, Q* or any other order, will not change the Grashof condition.

Non-Grashof Linkage Class-II Kinematic Chain $s + l > p + q$

If the inequality (1.7) is not true, then the linkage is non-Grashof and no link will be capable of a complete revolution relative to any other link. This is called Class-II kinematic chain.

Grashof Linkage Class-III Kinematic Chain $s + l = p + q$

Basics of Mechanism (contd…)



Grashof's Law

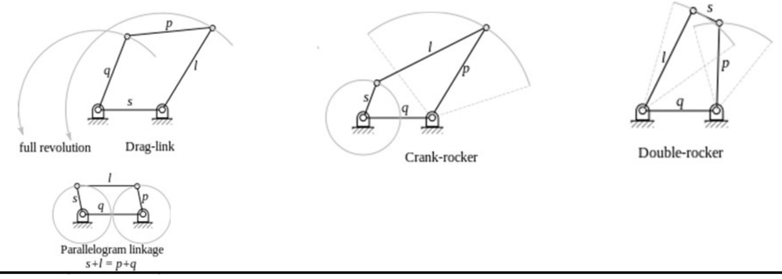
- For a planar four-bar linkage, the sum of the shortest and longest link lengths can not be greater than the sum of the remaining two link lengths if there is to be continuous relative rotation between two members.

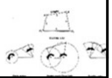
$$s + l \leq p + q$$

s=*length* of the shortest link
l=length of the longest link
p, q= length of the other two links

Inversion of Grashof's linkage

- **Double-crank or drag-link mechanism**
- **Crank-rocker mechanism**
- **Double-rocker mechanism**



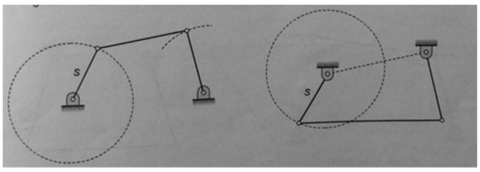


For the Class-I case : $S + L < P + Q$

(i) Ground either link adjacent to the shortest to get a crank-rocker mechanism shortest link will fully rotate and the other link pivoted to ground will oscillate.

(ii) Ground the shortest link to get a double-crank. Both links pivoted to ground make complete revolutions as does the coupler.

(iii) Ground the link opposite the shortest to get a Grashof double-rocker. Both links pivoted to ground will oscillate and only the coupler makes a full revolution.



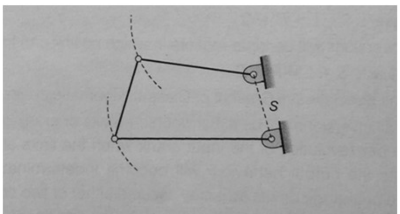


Fig. 1.10 Double-crank inversion (coupler rotates)

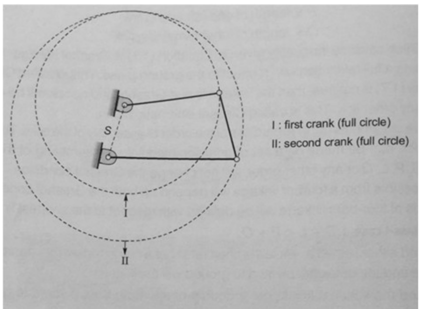
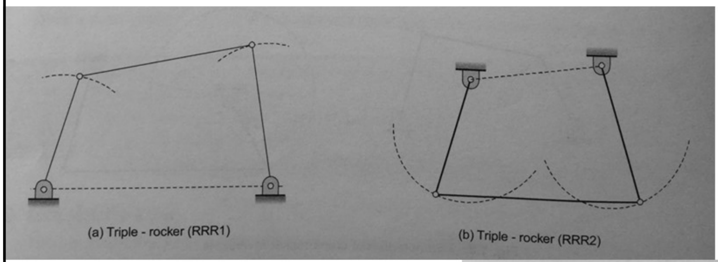


Fig. 1.9 Double-crank inversion (drag link mechanisms)

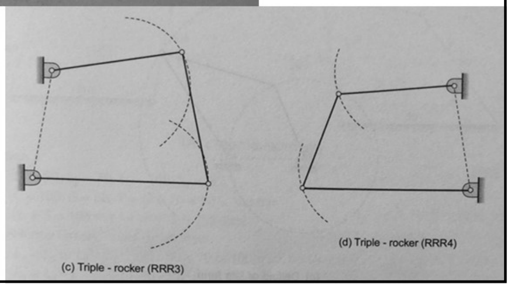
For the Class-II case : $S + L > P + Q$

In this class, all inversions will be triple-rockers in which no link can fully rotate.



(a) Triple - rocker (RRR1)

(b) Triple - rocker (RRR2)



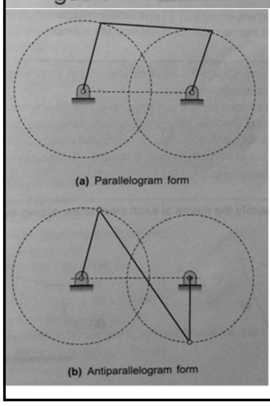
(c) Triple - rocker (RRR3)

(d) Triple - rocker (RRR4)

For the Class-III case : $S + L = P + Q$

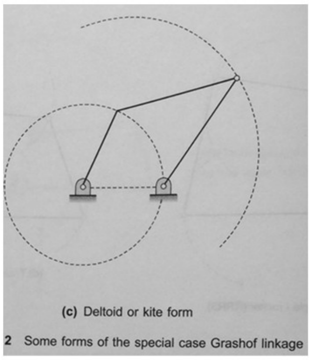
(i) It is referred as special-case Grashof or Class-III kinematic chain.

(ii) In this class, all inversions will be either double-cranks or crank-rockers but will have "change points" twice per revolution of the input crank when the links all become collinear. At these change points the output behaviour will become indeterminate. The linkage behaviour is unpredictable at change points as it may assume either of two configurations. Its motion must be limited to avoid reaching the change points or an additional, out of phase link provided to guarantee a carry through of the change points.



(a) Parallelogram form

(b) Antiparallelogram form



(c) Deltoid or kite form

2 Some forms of the special case Grashof linkage

(i) The fourbar linkage is the simplest possible pin-jointed mechanism for single degree of freedom controlled motion.

(ii) The Grashof condition is a very simple relationship which predict the rotation behaviour or rotatability of a fourbar linkage's inversions based only on the link lengths.

(iii) The parallelogram linkage is quite useful as it exactly duplicates the rotary motion of the driver crank at the driven crank. **Example :** Windshield mechanism on an automobile, drafting machine etc.

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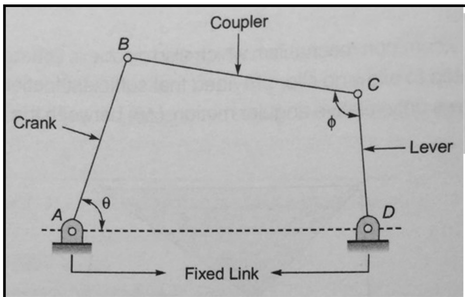

Link *AD* is fixed : Frame

Link *AB* is input : Crank or driver link

Link *BC* is coupler : Connecting rod

Link *CD* is lever : Rocker or follower link

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Basics of Mechanism (contd…)

Index of Merit

- There are some parameters of the mechanisms, proposed by various researchers, that tell us whether a mechanism is good one or poor one.
- Such indices of merit are Mechanical Advantage, Transmission angle etc.

Mechanical Advantage

- It is the ratio of the output torque exerted by the driven link (follower) to the necessary input torque required at the driver (crank).

Mechanical Advantage= $\frac{T_4}{T_2} = \frac{R_{CD}\sin Y}{T_{BA}\sin \beta}$

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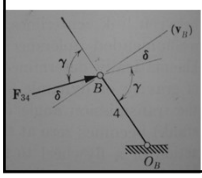
Basics of Mechanism (contd…)

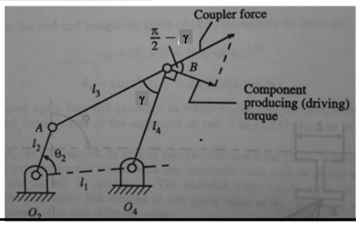
Transmission Angle

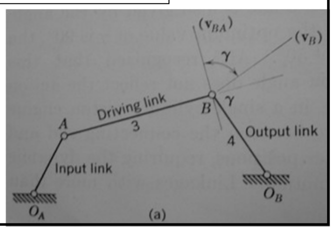
- In practice, besides satisfying kinematic requirements, a mechanism should also move freely. A complete dynamic force analysis is necessary to check this free-running quality.
- However, even at the stage of kinematic design, we should ensure that the output member receives a large component of the force or torque from the driving member along its direction of movement.
- we can express the free-running quality of simple mechanisms through an index known as the Transmission Angle.
- Its optimum value is 90 deg.

For a 4R linkage, the Transmission Angle is defined as the acute angle between the coupler and the follower.

If $\angle ABO_B$ is acute, then $\gamma=\angle ABO_B$	If $\angle ABO_B$ is obtuse, then $\gamma=\pi-\angle ABO_B$



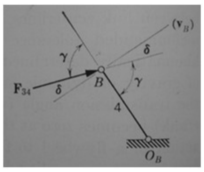




Basics of Mechanism (contd…)

Deviation Angle

- Another approach was taken by A. Bock, who suggested working with the directions of the static force and absolute velocity at the point of connection, terming the angle between the directions the deviation angle.
- Its optimum value is 0 deg.



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Analysis vs. Synthesis

Analysis

Input Motions ⇨ ***ANALYSIS*** ⇨ *Output Motions*

Given Mechanism & its Configuration, dimensions

In Kinematic Analysis one is given a mechanism & the task is to determine the various relative motion that can take place in that mechanism.

Synthesis

- **decision –making process**
- **Innovative or creative process**
- **process of creating new mechanism**
- **Selecting optimum/best configuration from no. of existing mechanism**
- **Determination of optimum dimensions of the elements of the mechanism on the basis of analysis**

In Kinematic Synthesis one has to be come up with a design of mechanism to generate prescribed motion characteristic.

Analysis vs. Synthesis

Analysis

- These are the technique that allow the designer to critically examine an already existing design in order to judge its suitability for the task.
- Analysis is simply a scientific tool

Real goal is SYNTHESIS i.e., Design.

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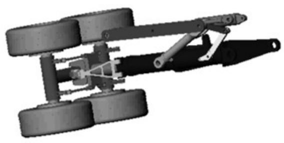
Computational Tools

CAD/CAE software	*Purpose*
ADAMS	*Multi-body dynamic analysis & simulation*
SimDESIGNER Motion with CATIA	*Multi-body dynamic analysis & simulation*
Working Model	*Multi-body dynamic analysis & simulation*



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