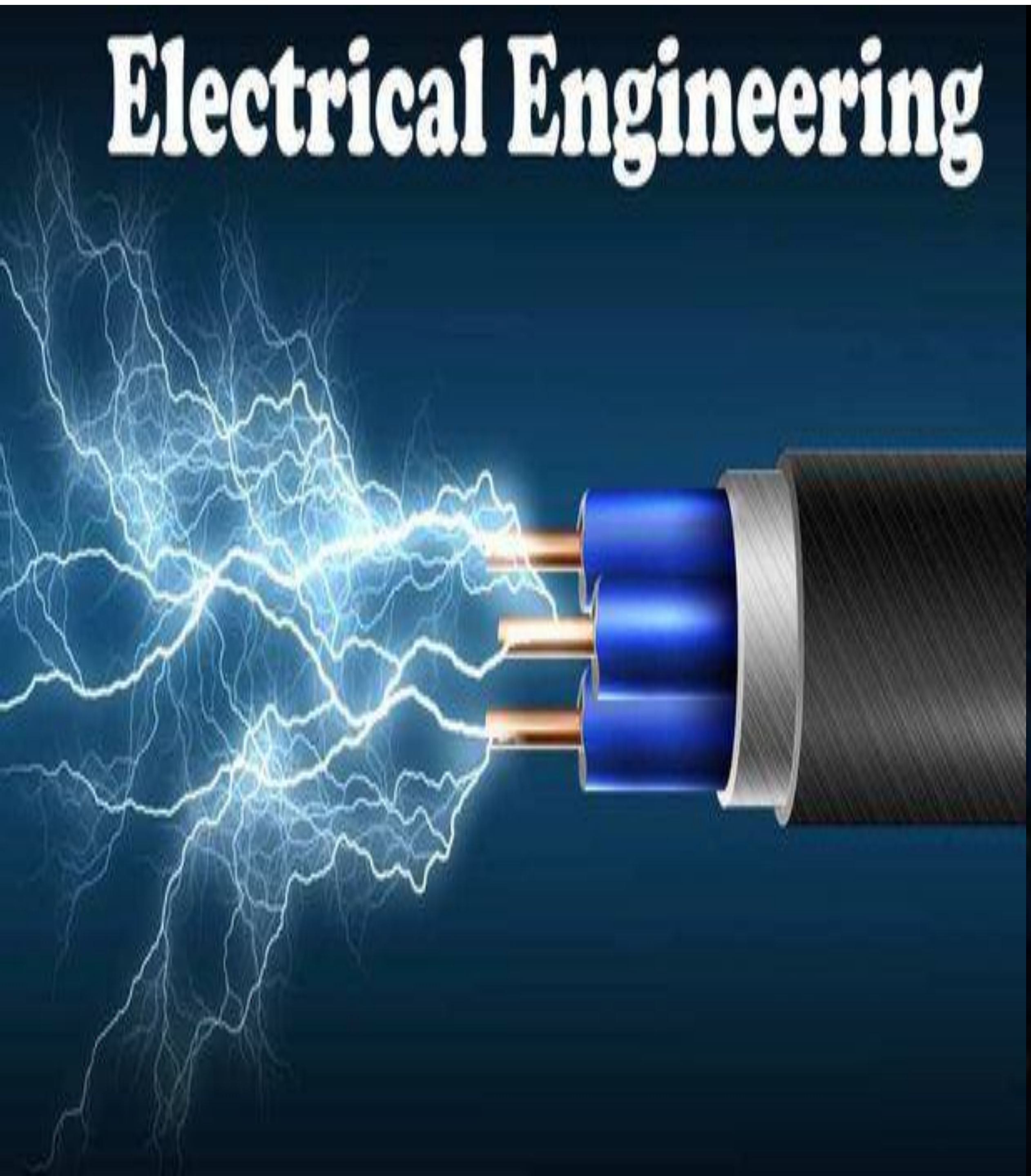
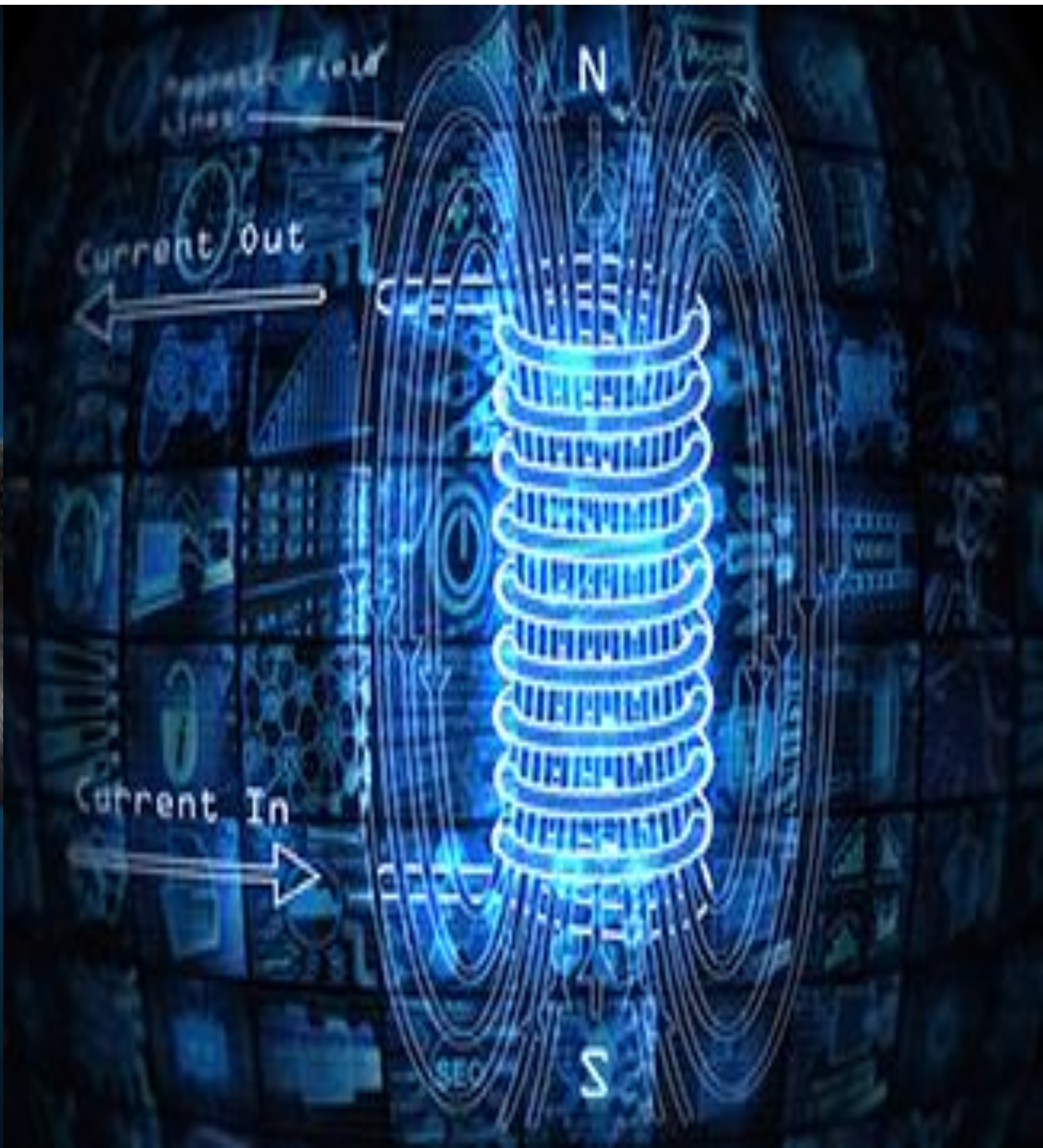

Electrical Engineering

N
Current Out
Current In
S

LECTURE 9

Electromagnetic Induction

- It is known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. It can be said that when electrons are in motion, they produce a magnetic field.

- It is also known that when a magnetic field embracing a conductor moves relative to the conductor, it produces a flow of electrons in the conductor. The phenomenon whereby an e.m.f. and hence current is induced in any conductor which is cut across or is cut by a magnetic flux is known as **electromagnetic induction**.

Production of Induced E.M.F. and Current

- ✓ An insulated coil whose terminals are connected to a sensitive galvanometer *G* as shown in **Fig. 3.18** .

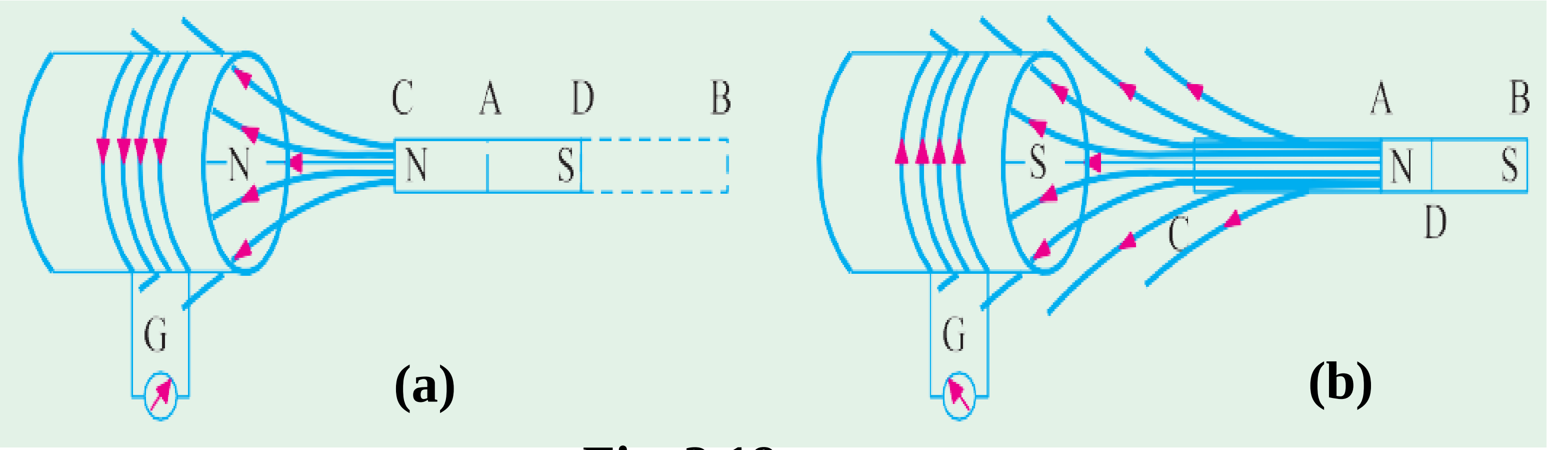


Fig. 3.18

Electromagnetic Induction

➢ Production of Induced E.M.F. and Current

- ✓ The coil is placed close to a stationary bar magnet initially at position *AB* *as* shown dotted in **Fig. 3.18(a)**.
- ✓ Some flux from the *N*-pole of the magnet is linked with or threads through the coil but there is no deflection in the galvanometer.
- ✓ Now, the magnet is suddenly brought closer to the coil in position *CD* as shown in **Fig. 3.18(a)**. Then, it is found that there is a jerk or a sudden but a momentary deflection in the galvanometer.
- ✓ The deflection lasts so long as the magnet is in motion relative to the coil, not otherwise.
- ✓ The deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*.
- ✓ It should be noted that due to the approach of the magnet, flux linked with the coil is increased.
- ✓ Next, the magnet is suddenly withdrawn away from the coil as shown in **Fig. 3.18(b)**.
- ✓ It is found that again there is a momentary deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary.
- ✓ It is important to note that this deflection is in a direction opposite to that of **Fig. 3.18(a)**. Obviously flux linkage with the coil is decreased due to the withdrawal of the magnet.

Electromagnetic Induction

- **Production of Induced E.M.F. and Current**

  - ✓ The deflection of the galvanometer indicates the production of e.m.f. in the coil.
  - ✓ The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of flux linking with the coil.
  - ✓ This e.m.f. exists so long as the change in flux exists. Even strong stationary flux will never induce any e.m.f. in a stationary conductor.
  - ✓ The same result can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.
  - ✓ The direction of the electromagnetically-induced e.m.f. is shown in **Fig. 3.18(a)** and **Fig. 3.18(b)** respectively.

Electromagnetic Induction

➢ Production of Induced E.M.F. and Current

- ✓ The production of the electromagnetically-induced e.m.f. is also illustrated by considering a conductor *AB* lying within a magnetic field and connected to a galvanometer as shown in **Fig. 3.19**.
- ✓ It is found that whenever the conductor is moved up or down, *a* momentary deflection is produced in the galvanometer.
- ✓ It means that some transient e.m.f. is induced in *AB.*
- ✓ The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) depends on the quickness of the movement of AB.

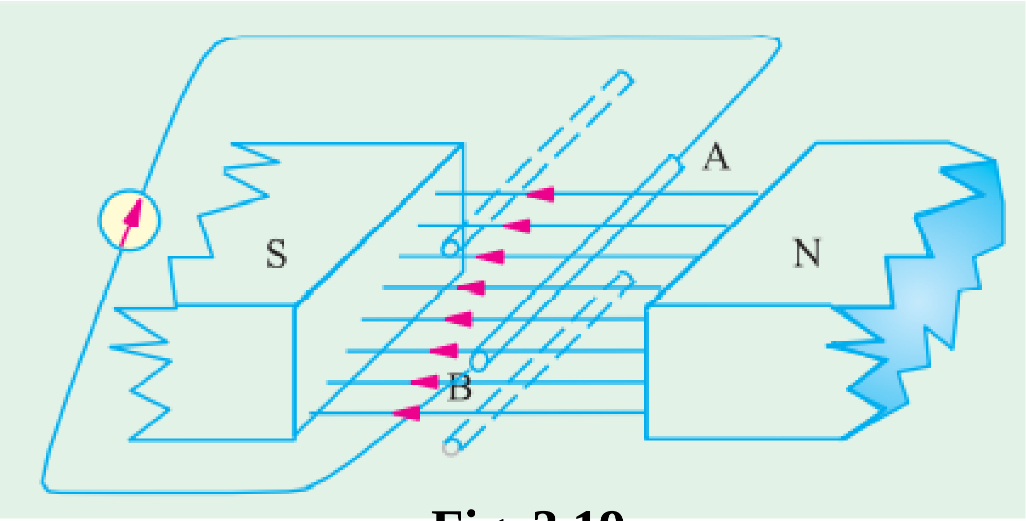


Fig. 3.19

- ✓ It is seen from the experiment that whenever a conductor cuts or *shears* the magnetic flux, an e.m.f. is always induced in it.
- ✓ It is also found that if the conductor is moved parallel to the direction of the flux so that it does not cut it, then no e.m.f. is induced in it.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - Faraday summarized the phenomenon of Electromagnetic Induction into two laws, known as Faraday's Laws of Electromagnetic Induction.

    First Law: Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

    or Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

    Second Law: The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

  - **Explanation.**

    - Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds.

      Initial flux linkages = $N\Phi_1$ and Final flux linkages = $N\Phi_2$

      $\therefore$ induced *e.m.f.* $e = \dfrac{N\Phi_2 - N\Phi_1}{t}$ Wb/s or volt or, $e = N\dfrac{\Phi_2 - \Phi_1}{t}$ volt

      The expression in its differential form $e = \dfrac{d}{dt}(N\,\Phi) = N\dfrac{d\Phi}{dt}$ volt

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Explanation.**
    - The induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it. So, a minus sign is given to the right-hand side expression

$$e = -N\frac{d\emptyset}{dt} \text{ volt}$$

  - **Induced e.m.f.**
    - Induced e.m.f. is classified as two types

1. dynamically induced e.m.f.
2. statically induced e.m.f.

Dynamically Induced E.M.F.: The field is stationary and conductors cut across it (as in d.c. generators).

Statically Induced E.M.F.: The conductors or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - **Explanation.**

    - The induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it. So, a minus sign is given to the right-hand side expression

$$e = -N\frac{d\emptyset}{dt} \text{ volt}$$

  - **Direction of induced e.m.f. and currents**

    - The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or Flat-hand rule or Lenz's Law.

1. Fleming's rule is used where induced e.m.f. is due to flux-cutting (*i.e.*, dynamically induced e.m.f.)
2. Lenz's law is used where induced e.m.f. is due to change of flux-linkages (*i.e.*, statically induced e.m.f.).

Electromagnetic Induction

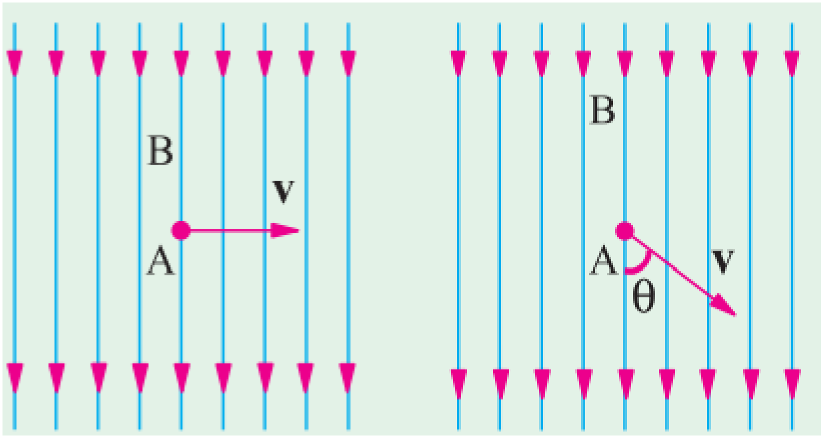
- **Faraday's Laws of Electromagnetic Induction**
  - **Dynamically induced e.m.f.**
    - In **Fig. 3.20,** a conductor *A* is shown in cross-section, lying within a uniform magnetic field of flux density *B* Wb/m^2.
    - The arrow attached to *A* shows its direction of motion.
    - The conductor *A* cuts the flux at right angles as shown in **Fig. 3.20 (a)** .

    Suppose '*l*' is its length lying within the field and let it move a distance *dx* in time *dt*.

    Then area swept by it is = *ldx*.

    Hence, flux cut = *l.dx* × *B* webers.

    Change in flux = *Bldx* weber; Time taken = *dt* second



(a) (b)

Fig. 3.20

$\therefore$ Rate of change of flux linkages $= \dfrac{B\ l\ dx}{dt} = Bl\dfrac{dx}{dt} = Blv$ volt, where, $\dfrac{dx}{dt}$ = velocity

According to Faraday's Laws, e.m.f = Rate of change of flux linkages

This e.m.f. is known as Dynamically Induced e.m.f.

Electromagnetic Induction

- **Faraday’s Laws of Electromagnetic Induction**
  - ✓ **Dynamically induced e.m.f.**
    - If the conductor A moves at an angle θ with the direction of flux as shown in **Fig. 3.20 (b),** then the induced e.m.f. is

$$e = Blv \sin\theta \quad \text{volts}$$

$$= lv \times B \quad (i.e.\ \text{as cross product vector } v \text{ and } B\ )$$

    - The direction of the induced e.m.f. is given by Fleming’s Right-hand rule or Flat-hand rule.
    - Generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

❖ **Fleming’s Right-hand Rule**

The right hand is held with the thumb, index finger and middle finger mutually perpendicular to each other (at right angles), as shown in the **Fig. 3.21.** The thumb is pointed in the direction of the motion of the conductor relative to the magnetic field, the index finger is pointed in the direction of the magnetic field (north to south), then the middle finger represents the direction of the induced or generated current within the conductor

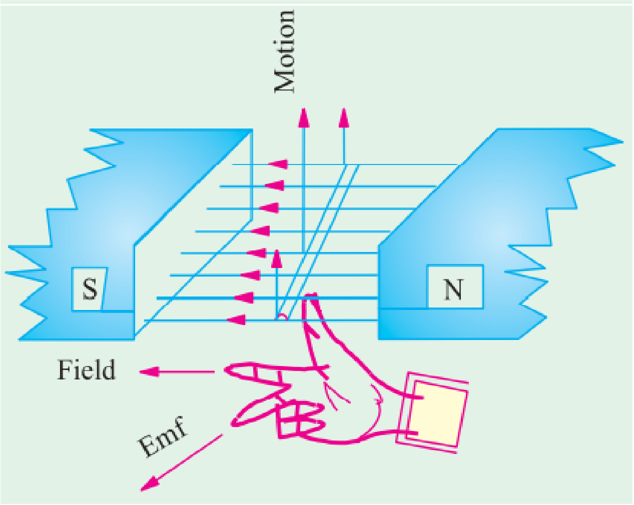


Fig. 3.21

Electromagnetic Induction

➢ **Faraday's Laws of Electromagnetic Induction**

✓ **Dynamically induced e.m.f.**

❖ **Right Flat-hand rule**

In **Fig. 3.22,** the front side of the right hand is held perpendicular to the incident flux with the thumb pointing in the direction of the motion of the conductor. The direction of the fingers give the direction of the induced e.m.f. and current.

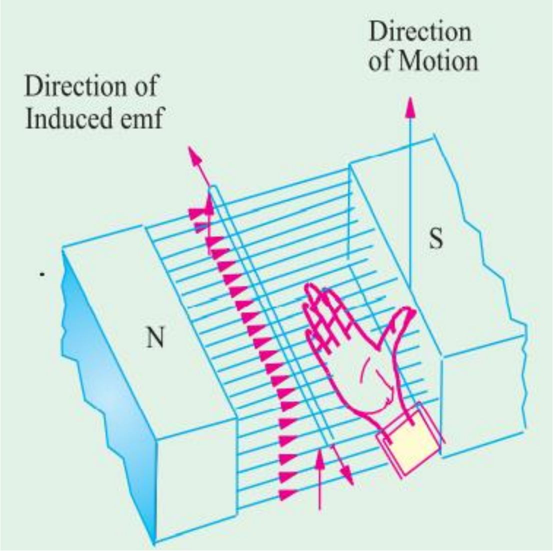


Fig. 3.22

✓ **Statically Induced E.M.F.**

There are two types of statically induced e.m.f. which are following:

(a) mutually induced e.m.f. and

(b) self-induced e.m.f.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Statically Induced E.M.F.**
    - **Mutually-induced e.m.f.**
      - Consider two coils *A* and *B* lying close to each other as shown in **Fig.3.23**.
      - Coil *A* is joined to a battery, a switch and a variable resistance *R* whereas coil *B* is connected to a sensitive voltmeter *V*.

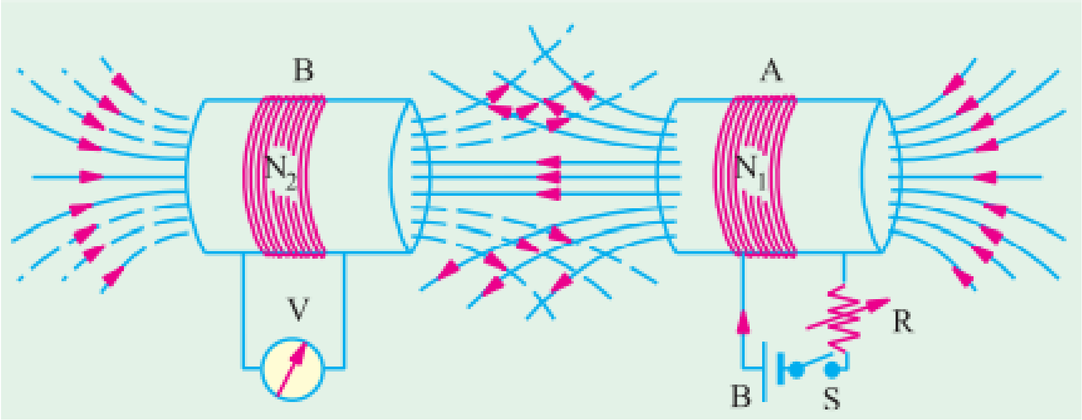


Fig. 3.23

- When current through *A* is established by closing the switch, its magnetic field is set up which partly links with or threads through the coil *B*.
- As current through *A* is changed, the flux linked with *B* is also changed.
- Hence, mutually induced e.m.f. is produced in *B* whose magnitude is given by Faraday's Laws and direction by Lenz's Law .

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Statically Induced E.M.F.**
    - **Mutually-induced e.m.f.**
      - Now, battery is connected to *B* and the voltmeter across *A* as shown in **Fig. 3.24.**
      - Now a change of current in *B* will produce mutually-induced e.m.f. in *A*.
      - In the examples, there is no movement of any conductor.
      - The flux variations being brought about by variations in current strength only.
      - An e.m.f. is induced in one coil by the influence of the other coil is called mutually induced e.m.f.

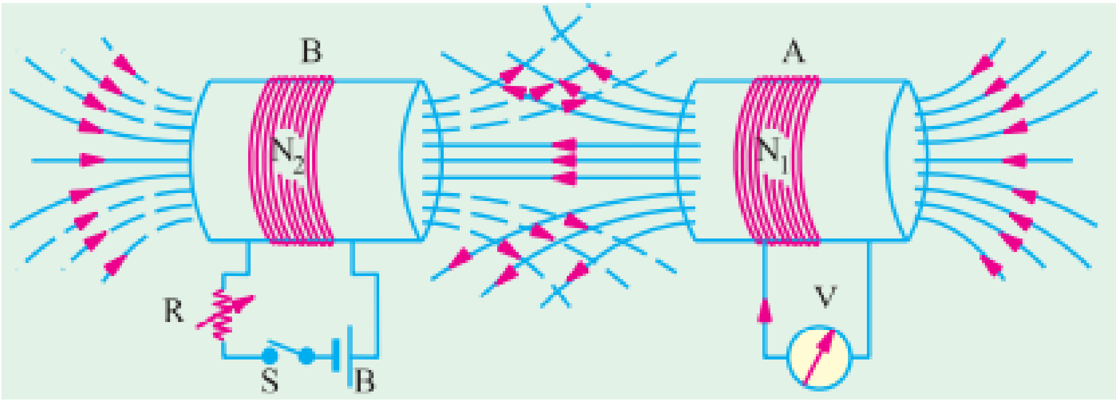


Fig. 3.24

Electromagnetic Induction

- **Lenz's Law**
  - ✓ The Lenz's law was formulated by Lenz in 1835.
  - ✓ This law states that electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

Explanation

- It is found that when N-pole of the bar magnet approaches the coil as shown in **Fig. 3.18(a),** the induced current set up by induced e.m.f. flows in the anti-clockwise direction in the coil as seen from the magnet side. The result is that face of the coil becomes a N-pole and so tends to oppose the onward approach of the *N*-Pole of the magnet. The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.
- When the magnet is withdrawn as shown in in **Fig. 3.18(b)**, the induced current flows in the clockwise direction, thus making the face of the coil a *S*-pole. Therefore, the *N*-pole of the magnet has to withdrawn against this attractive force of the *S*-pole of coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

Electromagnetic Induction

- **Lenz's Law**

  - **Explanation**

    - It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy.
    - Imagine a moment, when *N*-pole of the magnet as shown in **Fig. 3.18(a)** approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then, due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy.
    - It means that electric energy is created out of nothing which is denied by the inviolable Law of Conservation of Energy.
    - The relative motion of the magnet with respect to the coil is the cause of the production of the induced current.
    - Hence, the induced current always flows in such a direction that it oppose this relative motion *i.e.*, the approach or withdrawal of the magnet.

Electromagnetic Induction

➢ **Faraday's Laws of Electromagnetic Induction**

✓ **Statically Induced E.M.F.**

▪ **Self-induced e.m.f.**

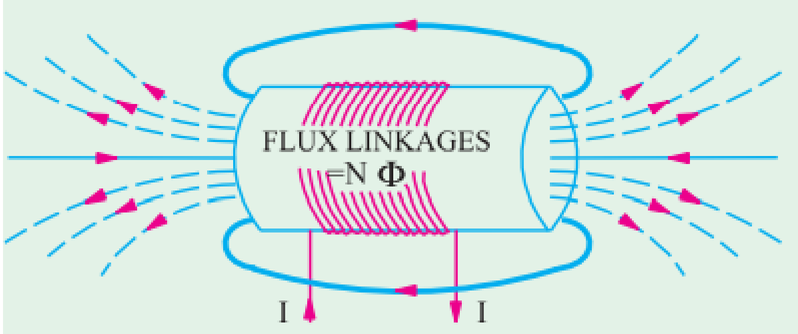


Fig. 3.25

- The e.m.f. induced in a coil due to the change of its own flux linked with it.
- If current through the coil as shown in **Fig. 3.25** is changed, then the flux linked with its own turns will also change, which will produce e.m.f. in it. This e.m.f. is called self-induced e.m.f.
- The direction of this induced e.m.f. as given by Lenz's law would be such as to oppose any change of flux which is, in fact, the very cause of its production.
- Hence, it is also known as the opposing or counter e.m.f. of self-induction.

Electromagnetic Induction

➢ Faraday's Laws of Electromagnetic Induction

✓ Self-inductance

- Imagine, the coil as shown in **Fig. 3.25** is connected to a battery through a rheostat.
- It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction.
- Energy required to overcome this opposition is supplied by the battery.
- If an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction.
- This property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as **self-inductance**. It is quantitatively measured in terms of coefficient of self induction L.
- This property is analogous to inertia in a material body.

It is difficult to set a heavy body into motion, but once it is in motion, it is equally difficult to stop it. Similarly, for a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it.

Hence, self-induction is sometimes analogously called electrical inertia or electromagnetic inertia.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - ✓ **Coefficient of Self-induction (L)**

It may be defined in any one of the three ways given below:

- **First Method for L**

  - The coefficient of self-induction of a coil is defined as the weber-turns per ampere in the coil.

    The 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked.

    Consider a solenoid having N turns and carrying a current of I amperes. If the flux produced is Φ webers, the weber-turns are $N\Phi$. Hence, weber-turns per ampere are $N\Phi/I$.

By definition, $$L = \frac{N\Phi}{I}$$ The unit of self-induction is henry.

If $N\Phi = 1$ Wb-turn, $I = 1$ ampere, then $L = 1$ henry (H)

Hence a coil is said to have a self-inductance of one henry if a current of 1 ampere when flowing through it produced flux-linkages of 1 Wb-turn in it.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Coefficient of Self-induction (L)**
    - **Second Method for L**

We know $\Phi = \dfrac{NI}{l/\mu_0\mu_r A}$ Now $L = N\dfrac{\Phi}{I} = N \cdot \dfrac{N}{l/\mu_0\mu_r A} H = \dfrac{N^2}{l/\mu_0\mu_r A}$

$\therefore \dfrac{\Phi}{I} = \dfrac{N}{l/\mu_0\mu_r A}$ $\therefore \quad L = \dfrac{\mu_0\mu_r A N^2}{l}$ H

- **Third Method for L**

We know $L = N\dfrac{\Phi}{I}$ $\therefore N\,\Phi = L\,I$ or, $-N\,\Phi = -L\,I$

Differentiating both sides, we get $-\dfrac{d}{dt}(N\Phi) = -L \cdot \dfrac{dI}{dt}$ (assuming L to be constant)

$$\text{or,} \quad -N \cdot \frac{d\Phi}{dt} = -L \cdot \frac{dI}{dt}$$

$$\therefore \; e_L = -L\frac{di}{dt} \qquad \because \; -N \cdot \frac{d\Phi}{dt} = e_L = \text{self-induced e.m.f.}$$

If $\dfrac{dI}{dt} = 1$ ampere/second and e = 1 volt, then L = 1 H

Electromagnetic Induction

➢ **Faraday's Laws of Electromagnetic Induction**

✓ **Mutual Inductance**

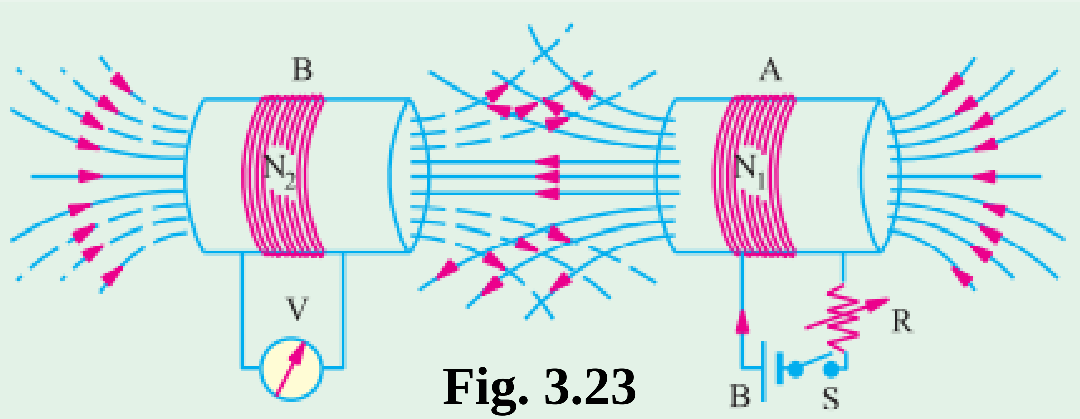


Fig. 3.23

- It is found that in the **Fig. 3.23,** any change of current in coil *A* is accompanied by the production of mutually-induced e.m.f. in coil *B*.
- Similarly, in the **Fig. 3.23,** the change of current in coil *B* induce an e.m.f. in coil *A*.
- Mutual inductance is defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes.
- The mutual inductance is measured in terms of the coefficient of mutual induction *M*.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - **Coefficient of Mutual Inductance (M)**

    It is also defined in any one of the three ways given below:

    - **First Method for M**

      - The coefficient of mutual-inductance between the two coils is defined as the weber-turns in one coil due to one ampere current in the other.
        - Two coils A and B having N_1 and N_2 turns are magnetically-coupled as shown in **Fig. 3.23.**
        - Let a current I_1 ampere when flowing in the coil A produce a flux Φ_1 webers in it. It is supposed that whole of this flux links with the turns of the coil B. Then, flux-linkages i.e., webers-turns in the coil B for unit current in the coil A is $N_2 \Phi_1/I_1$.
        - Hence, by definition

$$M = \frac{N_2 \emptyset_1}{I_1} \qquad \text{If} \quad N_2\,\Phi_1/I_1 = 1 \qquad \text{then,} \quad M = 1\text{H.}$$

        - Hence, two coils are said to have a mutual inductance of 1 henry when one ampere current flowing in one coil produces flux-linkages of one Wb-turn in the other.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Coefficient of Mutual Inductance (M)**
    - **Second Method for M**

The coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil $= \emptyset_1 = \dfrac{N_1 I_1}{l/\mu_0 \mu_r A}$ Wb

Flux/ampere $= \dfrac{\emptyset_1}{I_1} = \dfrac{N_1}{l/\mu_0 \mu_r A}$

Assume that whole of the flux produced by coil A is linked with coil B having N_2 turns.

So, the weber-turns in coil B due to the flux/ampere in coil A is

$$M = \frac{N_2 \emptyset_1}{I_1} = \frac{N_2 N_1}{l/\mu_0 \mu_r A} \qquad \therefore M = \frac{\mu_0 \mu_r A N_2 N_1}{l} \text{ H}$$

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Coefficient of Mutual Inductance (M)**
    - **Third Method for M**

We know $M = \dfrac{N_2\varnothing_1}{I_1}$ $\quad \therefore \; MI_1 = N_2\varnothing_1$

or, $-MI_1 = -N_2\varnothing_1$

Differentiating both sides, we get $-\dfrac{d}{dt}(N_2\varnothing_1) = -M\dfrac{dI}{dt}$ (assuming M to be constant)

Now, $-\frac{d}{dt}(N_2\varnothing_1)$ = mutually induced e.m.f. in coil $B = e_M$ $\quad \therefore e_M = -M\dfrac{dI_1}{dt}$

If $\dfrac{dI_1}{dt} = 1$ A/s $\quad e_M = 1$ volt, then $M = 1$ H

Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other

LECTURE 10

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - **Coefficient of Coupling**

Consider two magnetically-coupled coils *A* and *B* having N_1 and N_2 turns respectively. Their individual coefficients of self-induction are,

$$L_1 = \frac{N_1^2}{l/\mu_0\mu_r A} \qquad \text{and} \qquad L_2 = \frac{N_2^2}{l/\mu_0\mu_r A}$$

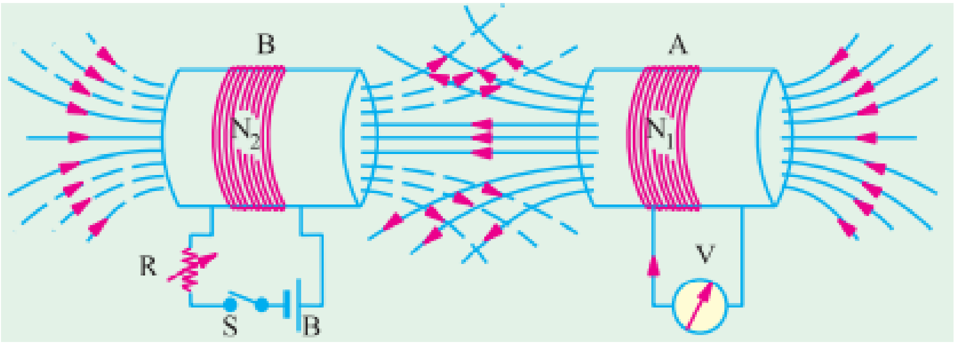


Fig. 3.23

The flux Φ_1 produced in *A* due to a current I_1 ampere is

$$\emptyset_1 = \frac{N_1 I_1}{l/\mu_0\mu_r A}$$

Suppose a fraction k_1 of this flux *i.e.* $k_1\Phi_1$ is linked with coil *B*.

Then $$M = \frac{k_1\emptyset_1 \times N_2}{I_1} \quad \text{where } k_1 \ll 1$$

Substituting the value of Φ_1, we get, $$\therefore M = k_1 \times \frac{N_1 N_2}{l/\mu_0\mu_r A}$$

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**

  - ✓ **Coefficient of Coupling**

Similarly, the flux Φ_2 produced in *B* due to I_2 ampere in it is $\emptyset_2 = \dfrac{N_2 I_2}{l/\mu_0\mu_r A}$

Suppose a fraction k_2 of this flux *i.e.* $k_2\Phi_2$ is linked with *A*.

Then $M = \dfrac{k_2\emptyset_2 \times N_1}{I_2}$ where $k_2 \ll 1$ $\therefore M = k_2 \times \dfrac{N_1 N_2}{l/\mu_0\mu_r A}$

$$M^2 = k_1 k_2 \frac{N_1^2}{l/\mu_0\mu_r A} \times \frac{N_2^2}{l/\mu_0\mu_r A} = k_1 k_2 L_1 L_2$$

Putting $\sqrt{k_1 k_2} = k$ $\therefore M = k\sqrt{L_2 L_2}$ or, $k = \dfrac{M}{\sqrt{L_2 L_2}}$

The constant k is called the coefficient of coupling. It is defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value.

If the flux due to one coil completely links with the other, then value of k is unity. If the flux of one coil does not link with the other, then k = 0. In the first case, when k = 1, coils are said to be tightly coupled and when k = 0, the coils are magnetically isolated from each other.

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Inductances in Series**

I. Let the two coils be joined in series as shown in **Fig. 3.26.** The fluxes produced by two coils are additive *i.e.*, in the same direction.

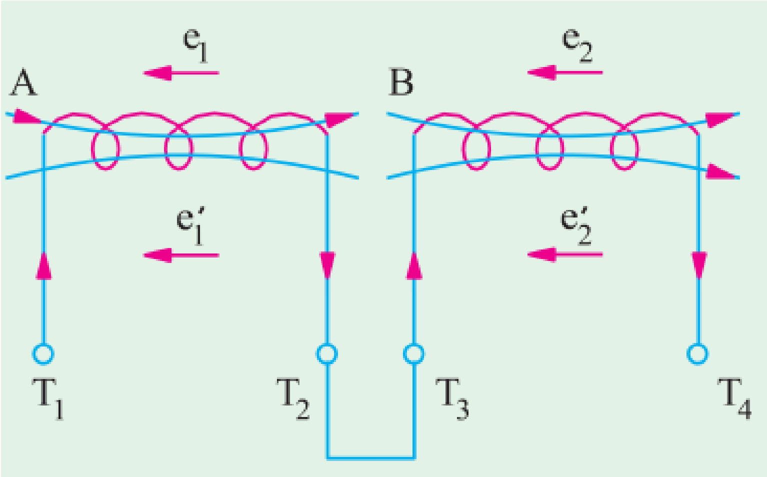


Fig. 3.26

Let M = coefficient of mutual inductance

L_1 = coefficient of self-inductance of 1st coil

L_2 = coefficient of self-inductance of 2nd coil

Then, self-induced e.m.f. in *A* is $e_1 = -L_1 \dfrac{di}{dt}$

Mutually-induced e.m.f. in *A* due to change of current in *B* is $e'_1 = -M\dfrac{di}{dt}$

Self-induced e.m.f. in *B* is $e_2 = -L_2 \dfrac{di}{dt}$

Mutually-induced e.m.f. in *B* due to change of current in *A* is $e'_2 = -M\dfrac{di}{dt}$

(All have –ve sign, because both self and mutally induced e.m.fs. are in opposition to the applied e.m.f.)

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Inductances in Series**

Total induced e.m.f. in the combination is $-(L_1 + L_2 + 2M)\dfrac{di}{dt}$

If L is the equivalent inductance,

Then total induced e.m.f. in that single coil is $-L\dfrac{di}{dt}$

$$\therefore L = (L_1 + L_2 + 2M)$$

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - **Inductances in Series**

II. Let the two coils be joined in series as shown in **Fig. 3.27.** The fluxes produced by two coils are in the opposite direction.

Self-induced e.m.f. in A is $e_1 = -L_1 \frac{di}{dt}$

Mutually-induced e.m.f. in A due to change of current in B is

$$e_1' = +M \frac{di}{dt}$$

Self-induced e.m.f. in B is $e_2 = -L_2 \frac{di}{dt}$

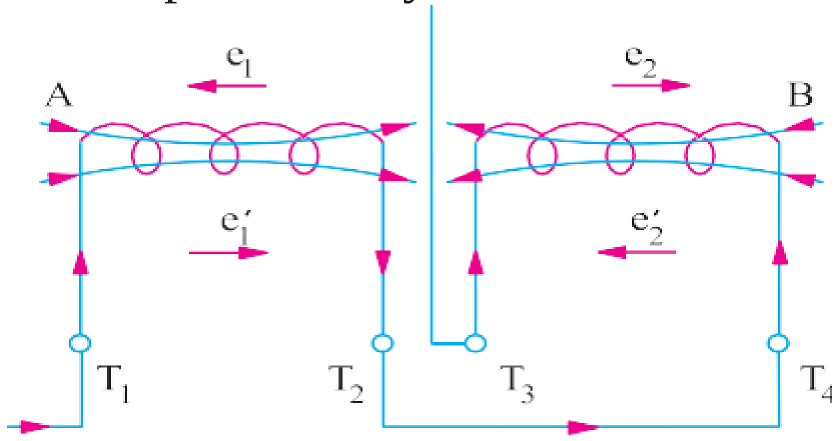


Fig. 3.27

Mutually-induced e.m.f. in B due to change of current in coil A is $e_2' = +M \frac{di}{dt}$

Total induced e.m.f. in the combination is $-(L_1 + L_2 - 2M)\frac{di}{dt}$

Then total induced e.m.f. in that single coil is $-L\frac{di}{dt}$

$$\therefore L = (L_1 + L_2 - 2M)$$

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Inductance in Parallel**

Two inductances of values L_1 and L_2 henry are connected in parallel as shown in **Fig. 3.28**. The coefficient of mutual inductance between the two be M.

Let i be the main supply current and i_1 and i_2 be the branch currents

$$i = i_1 + i_2$$

$$\therefore \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

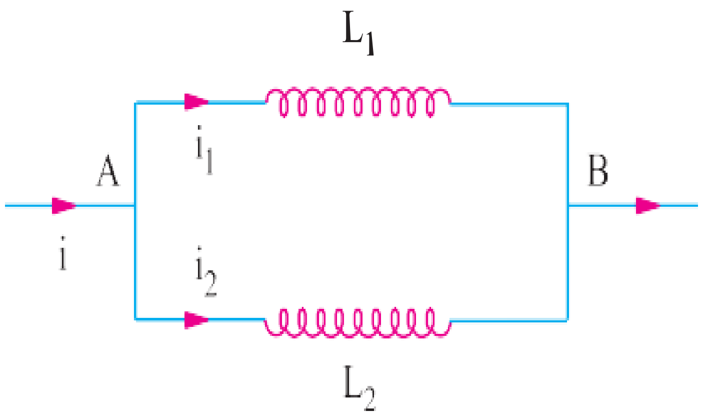


Fig. 3.28

In each coil, both self and mutually induced e.m.fs. are produced. Since the coils are in parallel, the net e.m.f. produced across each coin is equal.

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Inductance in Parallel**

$$\therefore \quad L_1\frac{di_1}{dt}+M\frac{di_2}{dt}=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$

or $$\frac{di_1}{dt}(L_1-M)=\frac{di_2}{dt}(L_2-M)$$

$$\therefore \quad \frac{di_1}{dt}=\left(\frac{L_2-M}{L_1-M}\right)\frac{di_2}{dt}$$

$$\because \quad \frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$$

$$\frac{di}{dt}=\left[\left(\frac{L_2-M}{L_1-M}\right)+1\right]\frac{di_2}{dt}$$

If L is the equivalent inductance, then

$e=L\dfrac{di}{dt}$ = induced e.m.f. in the parallel combination

= induced e.m.f. in any one coil

$$=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$\therefore \quad \frac{di}{dt}=\frac{1}{L}\left(L_1\frac{di_1}{dt}+M\frac{di_2}{dt}\right)=\frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]\frac{di_2}{dt}$$

Electromagnetic Induction

- **Faraday's Laws of Electromagnetic Induction**
  - ✓ **Inductance in Parallel**

$$\therefore \left(\frac{L_2 - M}{L_1 - M}\right) + 1 = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$$

or, $$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L}\left[\frac{L_1 L_2 - M^2}{L_1 - M}\right]$$

$$\therefore \quad L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$ when mutual field assists the separate fields.

Similarly, $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$ when the two fields oppose each other

Electromagnetic Induction

➢ Faraday's Laws of Electromagnetic Induction

- ✓ The energy supplied to exciting coils of an electromagnet is spent in two ways

  (i) part of it goes to meet I^2R loss and is lost

  (ii) part of it goes to create flux and is stored in the magnetic field as potential energy.

- ✓ When current through an inductive coil is gradually changed from zero to maximum value I, then every change of it is opposed by the self-induced e.m.f.

- ✓ So, energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil.

  Let, i = instantaneous value of current ;

  e = induced e.m.f. at that instant = $L\dfrac{di}{dt}$

- ✓ Then, work done in time dt to overcome this opposition is $dW = e\;i\;dt = Li\;di$

- ✓ Total work done for changing the current from zero to $I = \int_0^W dW = \int_0^I Li\;di$ $\quad\therefore\; W = \frac{1}{2}LI^2$

- ✓ This work is stored as the energy of the magnetic field $\quad\therefore\; E = \frac{1}{2}LI^2$

Electromagnetic Induction

Example – P3.5

A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate (i) the inductance of the coil (ii) the e.m.f. induced in the coil when a current of 5 A is switched off, assuming the current to fall to zero in 1 milli second and (iii) the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.

Electromagnetic Induction

Solution of Example – P3.5

Inductance of the first coil = $\frac{N\emptyset}{I} = \frac{900 \times 0.5 \times 10^{-3}}{3} = 0.5$ H

e.m.f. induced $e_1 = L\frac{di}{dt} = 0.15 \times \frac{(5-0)}{1 \times 10^{-3}} = 750$ V

$$M\frac{N_2\emptyset_1}{I_1} = \frac{600 \times 0.5 \times 10^{-3}}{3} = 0.1 \text{ H}$$

Example – P3.6

Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

Electromagnetic Induction

Solution of Example – P3.6

Now, $$e_M = M\frac{dI_1}{dt}$$

$$\therefore M = \frac{e_M}{dI_1/dt} = \frac{11.25}{1500} = 7.5\times10^{-3}\ \text{H} = 7.5\ \text{mH}$$

Now, $$L_1 = \frac{N_1\phi_1}{I_1} \qquad \therefore \frac{\phi_1}{I_1} = \frac{L_1}{I_1} = \frac{15\times10^{-3}}{750} = 2\times10^{-5}\ \text{Wb/A}$$

Now, $$k = \frac{M}{\sqrt{L_1L_2}} = \frac{7.5\times10^{-3}}{\sqrt{15\times10^{-3}\times15\times10^{-3}}} \quad [\because L_1 = L_2]$$

$$= 0.5$$

So, the percentage of the flux which links the turns of B $= 50\%$

Example – P3.7

Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.

LECTURE 10

Capacitor

- ✓ A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called dielectric. The conducting surfaces may be in the form of either circular (or rectangular) plates or be of spherical or cylindrical shape.
- ✓ The purpose of a capacitor is to store electrical energy by electrostatic stress in the dielectric.
- ✓ A parallel-plate capacitor is shown in **Fig. 4.1**. One plate is joined to the positive end of the supply and the other to the negative end or is earthed.
- ✓ There is a momentary flow of electrons from *A* to *B*. As negatively-charged electrons are withdrawn from *A*, it becomes positive and as these electrons collect on *B*, it becomes negative.
- ✓ Hence, a p.d. is established between plates *A* and *B*.
- ✓ The transient flow of electrons gives rise to charging current.
- ✓ The strength of the charging current is maximum when the two plates are uncharged.
- ✓ It then decreases and finally ceases when p.d. across the plates becomes slowly and slowly equal and opposite to the battery e.m.f.

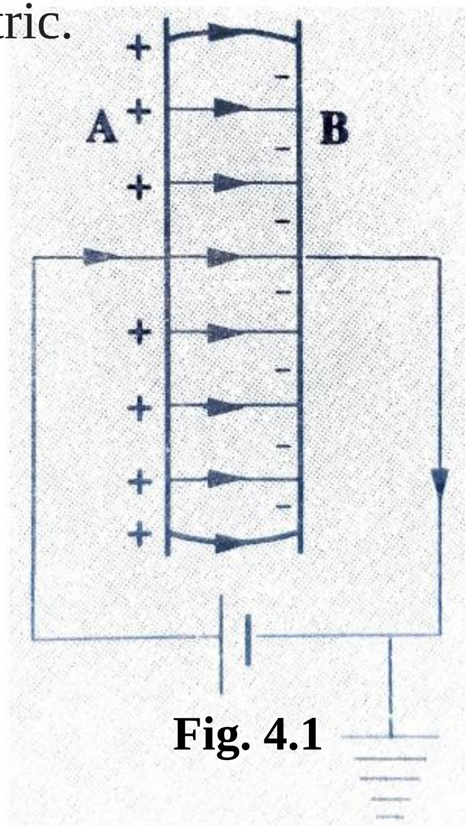


Fig. 4.1

Capacitor

➢ Capacitance

- ✓ The property of a capacitor to 'store electricity' is called its capacitance. Capacitance is the property of a capacitor which delays the change of voltage across it.
- ✓ The capacitance of a capacitor is defined as "the amount of charge required to create a unit p.d. between its plates."

Suppose, Q coulomb of charge is put on one of the two plates of a parallel plate capacitor. So, a p.d. of V volt is established between the two. Then its capacitance C is

$$C = \frac{Q}{V} = \frac{\text{charge}}{\text{potential difference}}$$

Fig. 4.2.

- ✓ The unit of capacitance is coulomb/volt which is also called farad

1 farad = 1 coulomb/volt

- ✓ smaller units are microfarad (μF), nanofarad (nF) and micro-microfarad ($\mu\mu$F) or picofarad (pF)

$$1\ \mu\text{F} = 10^{-6}\ \text{F};\ 1\ \text{nF} = 10^{-9}\ \text{F};\ 1\ \text{pF} = 10^{-12}\ \text{F};$$

Capacitor

➢ Capacitance of an Isolated Sphere

- ✓ Consider a charged sphere of radius r metres having a charge of Q coulomb placed in a medium of relative permittivity ϵ_r as shown in **Fig. 4.3**.

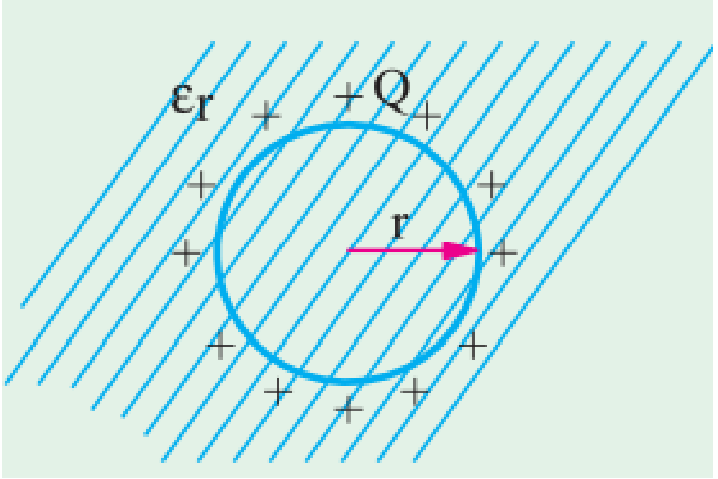


Fig. 4.3

- ✓ The free surface potential V of such a sphere with respect to infinity (in practice, earth) is given by

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r} \qquad \therefore \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$$

By definition, Q/V = capacitance C

$\therefore$ $= 4\pi\varepsilon_0\varepsilon_r r$ F – in a medium

$= 4\pi\varepsilon_0 r$ F – in air

- ✓ An isolated sphere act as a capacitor. The surface potential V of one plate is measured with reference to infinity (actually earth). So, the capacitance $4\pi\varepsilon_0 r$ exists between the surface of the sphere and earth.

Capacitor

➢ Capacitance of Spherical Capacitor

(a) When outer sphere is earthed

- ✓ Consider a spherical capacitor consisting of two concentric spheres of radii '*a*' and '*b*' metres as shown in **Fig.4.4 (a)**.

- ✓ Suppose, the inner sphere is given a charge of + Q coulombs.

- ✓ It will induce a charge of – Q coulombs on the inner surfaces and a charge of + Q coulombs on the outer surfaces which will go to earth.

- ✓ If the dielectric medium between the two spheres has a relative permittivity of ε_r, then the free surface potential of the inner sphere due to its own charge $Q/4\pi\varepsilon_0\varepsilon_r a$ volts.

- ✓ The potential of the inner sphere due to $-Q$ charge on the outer sphere is $-Q/4\pi\varepsilon_0\varepsilon_r b$ (remembering that potential anywhere inside a sphere is the same as that its surface).

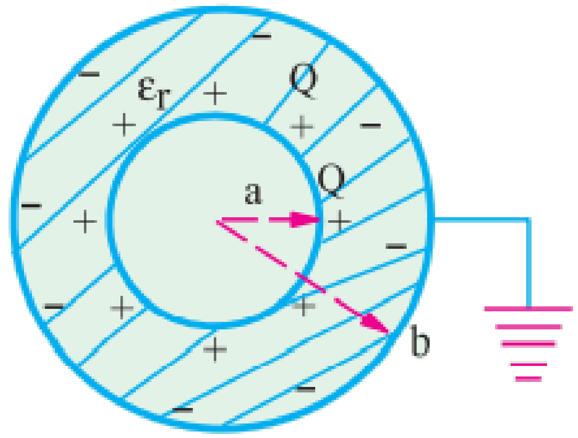


Fig. 4.4 (a)

Capacitor

➢ Capacitance of Spherical Capacitor

✓ Total potential difference between two surfaces is

$$V = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r a} - \frac{Q}{4\pi \varepsilon_0 \varepsilon_r b}$$

$$= \frac{Q}{4\pi \varepsilon_0 \varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V} = \frac{4\pi \varepsilon_0 \varepsilon_r ab}{b-a} \qquad \therefore\ C = 4\pi \varepsilon_0 \varepsilon_r \frac{ab}{b-a}\ \text{F}$$

Capacitor

➢ Capacitance of Spherical Capacitor

(b) When inner sphere is earthed

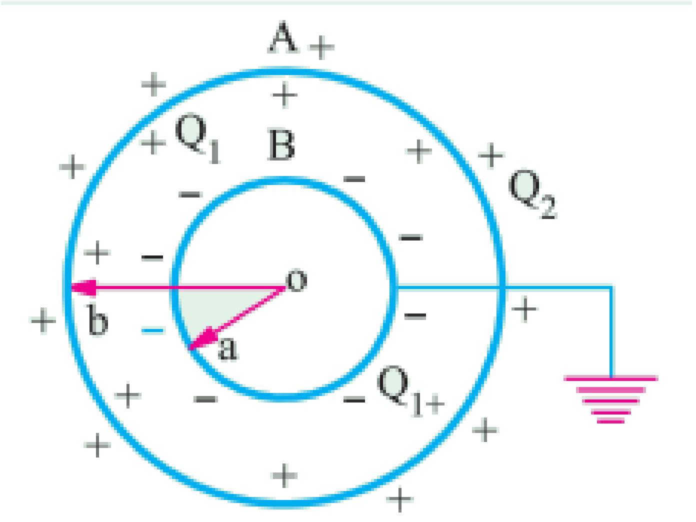


Fig. 4.4 (b)

- ✓ In **Fig. 4.4 (b)** a charge of + Q coulombs is given to the outer sphere *A*, it will distribute itself over both its inner and outer surfaces.
- ✓ Some charge Q_2 coulomb will remain on the outer surface of *A* because it is surrounded by earth all around.
- ✓ Also, some charge + Q_1 coulombs will shift to its inner side because there is an earthed sphere *B* inside *A*.

Obviously, $$Q = Q_1 + Q_2$$

- ✓ The inner charge + Q_1 coulomb on *A* induces – Q_1 coulomb on *B* but the other induced charge of + Q_1 coulomb goes to earth.
- ✓ There are two capacitors connected in parallel :
  - (i) One capacitor consists of the inner surface of A and the outer surface of B.
  - (ii) The second capacitor consists of outer surfaces of A and earth. Its capacitance is $C_2 = 4\pi\epsilon_0 b$

Total capacitance $C = C_1 + C_2$

Capacitor

✓ Capacitance of Parallel-plate Capacitor

(i) Uniform Dielectric-Medium

✓ A parallel-plate capacitor consisting of two plates M and N, each of area A m^2 separated by d metres in a medium having relative permittivity of ε_r is shown in **Fig. 4.5**.

✓ If a charge of + Q coulomb is given to plate M, then flux passing through the medium is $\psi = Q$ coulomb.

✓ Flux density in the medium is $D = \frac{\psi}{A} = \frac{Q}{A}$

✓ Electric intensity $E = V/d$ and electric flux density $D = \varepsilon\, E$

or, $\frac{Q}{A} = \varepsilon \frac{V}{d}$ $\therefore$ $\frac{Q}{V} = \frac{\varepsilon A}{d}$

$\therefore$ $C = \frac{\varepsilon_0\, \varepsilon_r\, A}{d}$ farad – in a medium

$= \frac{\varepsilon_0\, A}{d}$ farad – with air as medium

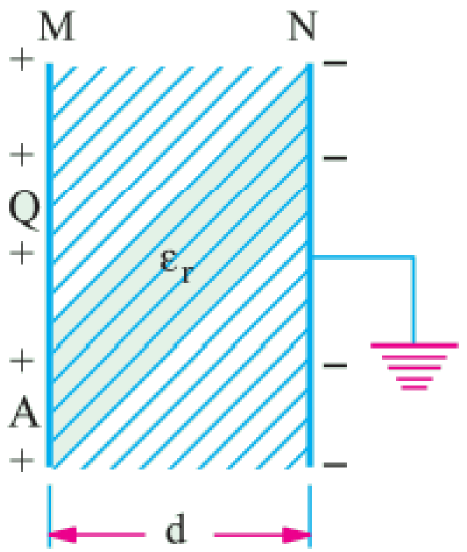


Fig. 4.5

✓ Capacitance of Parallel-plate Capacitor

(ii) Medium Partly Air

- ✓ The medium consists partly of air and partly of parallel-sided dielectric slab having thickness t and relative permittivity ε_r as shown in **Fig. 4.6.**
- ✓ The electric flux density $D = Q/A$ is the same in both medium but electric intensities are different.

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_r} \quad \text{... in the medium}$$

$$E_2 = \frac{D}{\varepsilon_0} \quad \text{... in air}$$

p.d. between plates,

$$V = E_1 \cdot t + E_2 (d - t)$$

$$= \frac{D}{\varepsilon_0 \varepsilon_r} t + \frac{D}{\varepsilon_0}(d - t) = \frac{D}{\varepsilon_0}\left(\frac{t}{\varepsilon_r} + d - t\right)$$

$$= \frac{Q}{\varepsilon_0 A}[d - (t - t/\varepsilon_r)]$$

or

$$\frac{Q}{V} = \frac{\varepsilon_0 A}{[d - (t - t/\varepsilon_r)]} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{[d - (t - t/\varepsilon_r)]}$$

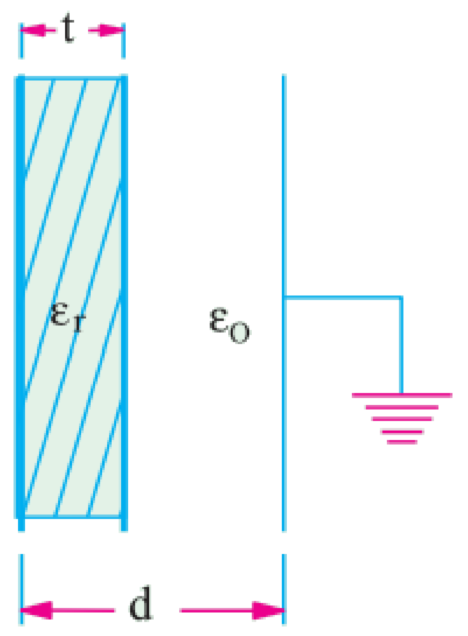


Fig. 4.6

Capacitor

✓ Capacitance of Parallel-plate Capacitor

(ii) Medium Partly Air

- ✓ If the medium were totally air, then capacitance would have been $C = \varepsilon_0 A/d$
- ✓ It is obvious that when a dielectric slab of thickness t and relative permittivity ε_r is introduced between the plates of an air capacitor, then its capacitance increases.
- ✓ The distance between the plates is effectively reduces by $(t - t/\varepsilon_r)$.
- ✓ To bring the capacitance back to its original value, the capacitor plates will have to be further separated so that the new separation between the two plates would be = $[d + (t - t / \varepsilon_r)]$

Capacitor

✓ Capacitance of Parallel-plate Capacitor

(iii) Composite Medium

If V is the total potential difference across the capacitor plates and V_1, V_2, V_3, the potential differences across the three dielectric slabs, then

$$V = V_1 + V_2 + V_3 = E_1 t_1 + E_2 t_2 + E_3 t_3$$

$$= \frac{D}{\varepsilon_0\,\varepsilon_{r1}} . t_1 + \frac{D}{\varepsilon_0\,\varepsilon_{r2}} . t_2 + \frac{D}{\varepsilon_0\,\varepsilon_{r3}} . t_3$$

$$= \frac{D}{\varepsilon_0}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right) = \frac{Q}{\varepsilon_0 A}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right)$$

$$\therefore \boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right)}}$$

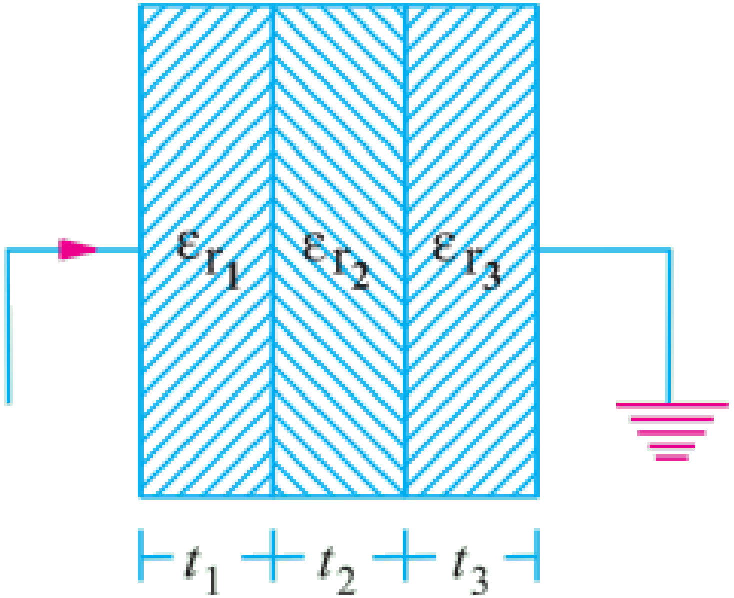


Fig. 4.7

LECTURE 11

Capacitor

➢ Capacitance of Cylindrical Capacitor

- ✓ A single-core cable or cylindrical capacitor consisting of two co-axial cylinders of radii *a* and *b* metres, is shown in **Fig. 4.8**.
- ✓ Let the charge per metre length of the cable on the outer surface of the inner cylinder be + Q coulomb and on the inner surface of the outer cylinder be – Q coulomb.
- ✓ The charge + Q coulomb/metre on the surface of the inner cylinder can be supposed to be located along its axis.
- ✓ Let ε_r be the relative permittivity of the medium between the two cylinders.
- ✓ The outer cylinder is earthed.
- ✓ Consider an imaginary co-axial cylinder of radius x metres and length one metre between the two given cylinders as shown in **Fig. 4.8**.
- ✓ Total flux coming out radially from the curved surface of this imaginary cylinder is Q coulomb.
- ✓ Area of the curved surface = $2\pi\, x \times 1 = 2\,\pi\, x$ m^2.

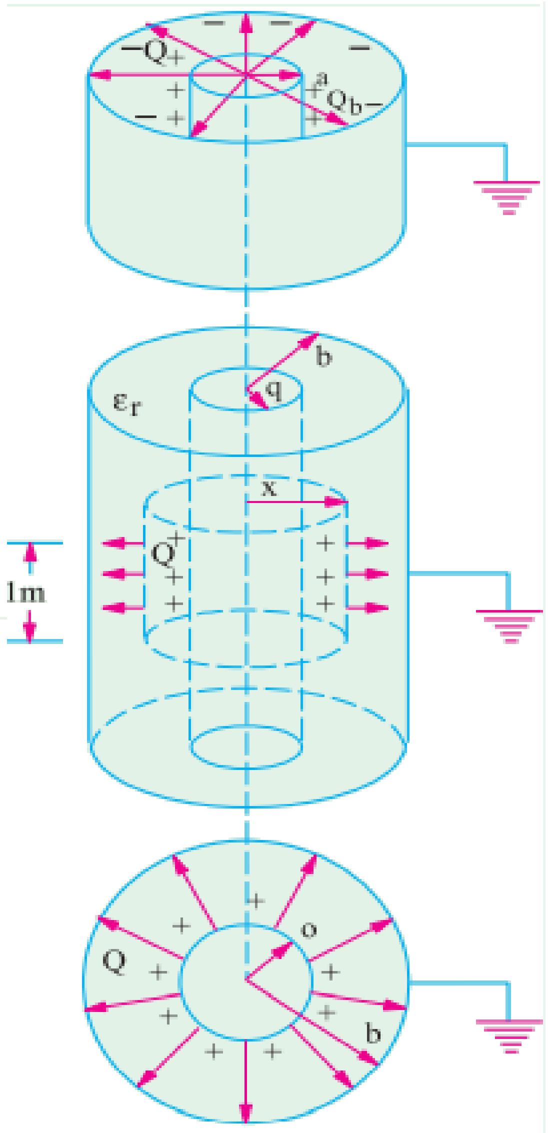


Fig. 4.8.

Capacitor

➢ Capacitance of Cylindrical Capacitor

✓ Hence, the value of electric flux density on the surface of the imaginary cylinder is

$$D = \frac{\text{flux in coulomb}}{\text{area in metre}^2} = \frac{\Psi}{A} = \frac{Q}{A}\ \text{C/m}^2 \quad \therefore D = \frac{Q}{2\pi x}\ \text{C/m}^2$$

The value of electric intensity is

$$E = \frac{D}{\varepsilon_0\,\varepsilon_r} \quad \text{or} \quad E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x}\ \text{V/m}$$

Now, $dV = -E\,dx$

or
$$V = \int_b^a -E\,.dx = \int_b^a -\frac{Q\,dx}{2\pi\varepsilon_0\,\varepsilon_r\,x}$$

$$= \frac{-Q}{2\pi\varepsilon_0\,\varepsilon_r}\int_b^a \frac{dx}{x} = \frac{-Q}{2\pi\varepsilon_0\,\varepsilon_r}\left|\log x\right|_b^a$$

$$= \frac{-Q}{2\pi\varepsilon_0\,\varepsilon_r}(\log_e a - \log_e b) = \frac{-Q}{2\pi\varepsilon_0\,\varepsilon_r}\log_e\left(\frac{a}{b}\right) = \frac{Q}{2\pi\varepsilon_0\,\varepsilon_r}\log_e\left(\frac{a}{b}\right)$$

$$\frac{Q}{V} = \frac{2\pi\varepsilon_0\,\varepsilon_r}{\log_e\left(\frac{b}{a}\right)} \quad \therefore C = \frac{2\pi\varepsilon_0\,\varepsilon_r}{2.3\log_{10}\left(\frac{b}{a}\right)}\ \text{F/m}\left(\log_e\left(\frac{b}{a}\right) = 2.3\log_{10}\left(\frac{b}{a}\right)\right)$$

The capacitance of l metre length of this cable is $C = \dfrac{2\pi\varepsilon_0\,\varepsilon_r\,l}{2.3\log_{10}\left(\frac{b}{a}\right)}$ F

Capacitor

- **Capacitance of Cylindrical Capacitor**

  - ✓ The capacitance of *l* metre length of this cable is

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F}$$

  - ✓ If the capacitor has compound dielectric, the capacitance is

$$C = \frac{2\pi\varepsilon_0 l}{\Sigma \log_e\left(\frac{b}{a}\right)/\varepsilon_r} \text{ F}$$

  - ✓ The capacitance of 1 km length of the cable in μ F can be found by putting $l = 1$ km in the above expression.

$$C = \frac{2\pi\times 8.854\times 10^{-12}\times\varepsilon_r\times 1000}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F/km} = \frac{0.024\,\varepsilon_r}{\log_{10}\left(\frac{b}{a}\right)}\ \mu\text{ F/km}$$

Capacitor

- **Potential Gradient in a Cylindrical Capacitor**

  - ✓ We know, in a cable capacitor $E = \dfrac{Q}{2\pi\varepsilon_0\varepsilon_r\, x}$ V/m

    where x is the distance from cylinder axis to the point under consideration.

Now $E = g \quad \therefore\ g = \dfrac{Q}{2\pi\varepsilon_0\varepsilon_r\, x}$ V/m

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\frac{b}{a}\right) \qquad \text{or} \qquad Q = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\frac{b}{a}\right)}$$

Substituting this value of Q, we get

$$g = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\frac{b}{a}\right)\times 2\pi\varepsilon_0\varepsilon_r\, x}\ \text{V/m} \quad \text{or} \quad g = \frac{V}{x\log_e\left(\frac{b}{a}\right)}\ \text{V/m} \quad \text{or} \quad g = \frac{V}{2.3\, x\log_{10}\left(\frac{b}{a}\right)}\ \text{volt/metre}$$

Capacitor

- **Potential Gradient in a Cylindrical Capacitor**

  - ✓ Obviously, potential gradient varies inversely as x.
  - ✓ Hence maximum value of potential gradient at minimum value of $x = a$, $g_{max} = \dfrac{V}{2.3a\log_{10}\left(\frac{b}{a}\right)}$ V/m

Similarly, $g_{min} = \dfrac{V}{2.3b\log_{10}\left(\frac{b}{a}\right)}$ V/m

- ✓ The relation is used to obtain most economical dimension of a cable. Greater the value of permissible maximum stress E_{max}, smaller the cable for given value of V. E_{max} is dependent on the dielectric strength of the insulating material. If V and E_{max} are fixed, then

$$E_{max} = \frac{V}{a\,\mathrm{logh}_e\left(\frac{b}{a}\right)} \text{ or } a\,\mathrm{logh}\left(\frac{b}{a}\right) = \frac{V}{E_{max}} \therefore \frac{b}{a} = e^{k/a} \text{ or } b = a\,e^{k/a}$$

- ✓ For most economical cable $db/da = 0$

$$\therefore \quad \frac{db}{da} = 0 = e^{k/a} + a\,(-k/a^2)e^{k/a} \quad \text{or} \quad a = k = V/E_{max} \text{ and } b = ae = 2.718\,a$$

Capacitor

- **Capacitance Between Two Parallel Wires**
  - ✓ The capacitance between two parallel wires of overhead transmission lines is of practical importance of cylindrical capacitor.
  - ✓ The simplest 2-wire system is either d.c. or a.c.
  - ✓ In the case of a.c. system, if the transmission line is long and voltage is high, the charging current drawn by the line due to the capacitance between conductors is appreciable and affects its performance considerably.
  - ✓ Consider a 2-wire transmission line as shown in **Fig. 4.9**

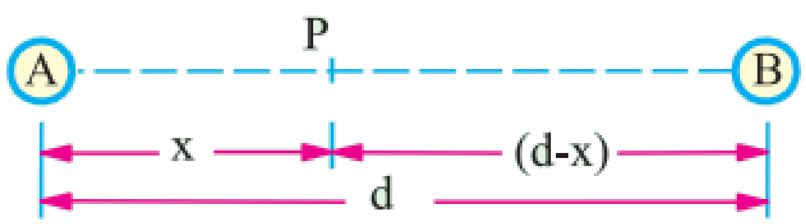


Fig. 4.9

where, d = distance between centres of the wires A and B

r = radius of each wire $(\leq d)$

Q = charge in coulomb/metre of each wire

Capacitor

- **Capacitance Between Two Parallel Wires**

  - ✓ Now, let us consider electric intensity at any point *P* between conductors *A* and *B*.
  - ✓ Electric intensity at *P* due to charge + *Q* coulomb/metre on *A* is

$$E_A = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x}\,\text{V/m}$$

  - ✓ Electric intensity at *P* due to charge – *Q* coulomb/metre on *B* is

$$E_B = \frac{Q}{2\pi\varepsilon_0\varepsilon_r (d-x)}\,\text{V/m}$$

  - ✓ Total electric intensity at *P*,

$$E = E_A + E_B = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{x}+\frac{1}{d-x}\right)$$

Capacitor

- **Capacitance Between Two Parallel Wires**

  - ✓ Hence, potential difference between the two wires is

$$V = \int_r^{d-r} E.dx = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\int_r^{d-r}\left(\frac{1}{x}+\frac{1}{d-x}\right)dx$$

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left|\log_e x - \log_e(d-x)\right|_r^{d-r} = \frac{Q}{\pi\varepsilon_0\varepsilon_r}\log_e\frac{d-r}{r}$$

$$\text{Now } C = Q/V \quad \therefore C = \frac{\pi\varepsilon_0\varepsilon_r}{\log_e\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)} \text{ F/m (approx.)}$$

  - ✓ The capacitance for a length of *l* metres, $C = \frac{\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{d}{r}\right)}$ F

  - ✓ The capacitance per kilometer, $C = \frac{\pi\times 8.854\times 10^{-12}\times 100\times 10^{6}}{2.3\log_{10}\left(\frac{d}{r}\right)} = \frac{0.012\varepsilon_r}{\log_{10}\left(\frac{d}{r}\right)}$ μ F/km

Capacitor

- **Capacitors in Series**

  ✓ The capacitors C_1, C_2 and C_3 are connected is series as shown in **Fig. 4.10**

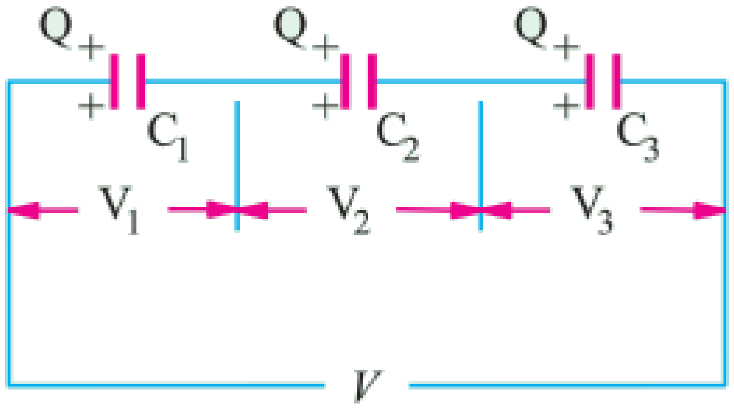


Fig. 4.10

C_1, C_2, C_3 = Capacitances of three capacitors
V_1, V_2, V_3 = p.ds. across three capacitors.
V = applied voltage across combination
C = combined or equivalent or joining capacitance.

  ✓ In series combination, charge on all capacitors is the same but p.d. across each is different

$$\therefore \quad V = V_1 + V_2 + V_3$$

$$\text{or} \quad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitor

- **Capacitors in Series**

  - ✓ For a changing applied voltage,

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{dV_3}{dt}$$

  - ✓ We can also find values of V_1, V_2 and V_3 in terms of V.
  - ✓ Now, $Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = CV$

where $$C = \frac{C_1 C_2 C_3}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{C_1C_2C_3}{\Sigma C_1C_2}$$

$\therefore$ $$C_1V_1 = C V \text{ or } V_1 = V \frac{C}{C_1} = V \cdot \frac{C_2 C_3}{\Sigma C_1C_2}$$

Similarly, $$\boxed{V_2 = V \cdot \frac{C_1 C_3}{\Sigma C_1 C_2}} \text{ and } \boxed{V_3 = V \cdot \frac{C_1 C_2}{\Sigma C_1 C_2}}$$

Capacitor

- **Capacitors in Parallel**

  - ✓ The capacitors C_1, C_2 and C_3 are connected is parallel as shown in **Fig. 4.11**

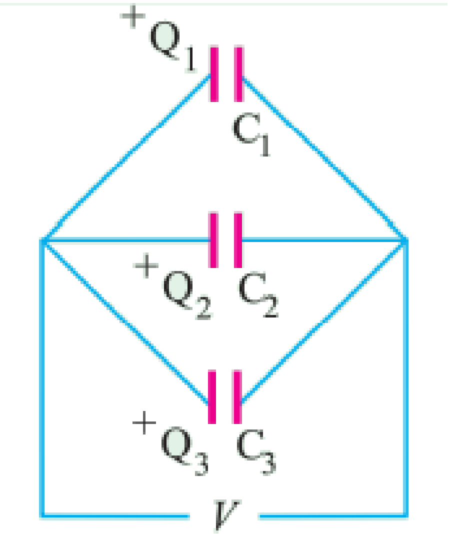


Fig. 4.11

C_1, C_2, C_3 = Capacitances of three capacitors

Q_1, Q_2, Q_3 = Charges of three capacitors.

V = applied voltage across combination

C = combined or equivalent or joining capacitance.

  - ✓ In this case, p.d. across each is the same but charge on each is different as shown in Fig. **4.11.**

$$\therefore Q = Q_1 + Q_2 + Q_3$$

$$\text{or } CV = C_1V + C_2V + C_3V$$

$$\text{or } C = C_1 + C_2 + C_3$$

Capacitor

- **Insulation Resistance of a Cable Capacitor**
  - ✓ A cable capacitor in which the current flows along the axis of the core as shown in **Fig. 4.12**.
  - ✓ There is always present some leakage of current. This leakage is radial *i.e.* at right angles to the flow of useful current.
  - ✓ The resistance offered to this radial leakage of current is called insulation resistance of the cable.
  - ✓ If cable length is greater, then leakage is also greater.
  - ✓ It means that more current will leak. In other words, insulation resistance is decreased.
  - ✓ Hence, we find that insulation resistance is inversely proportional to the cable length.

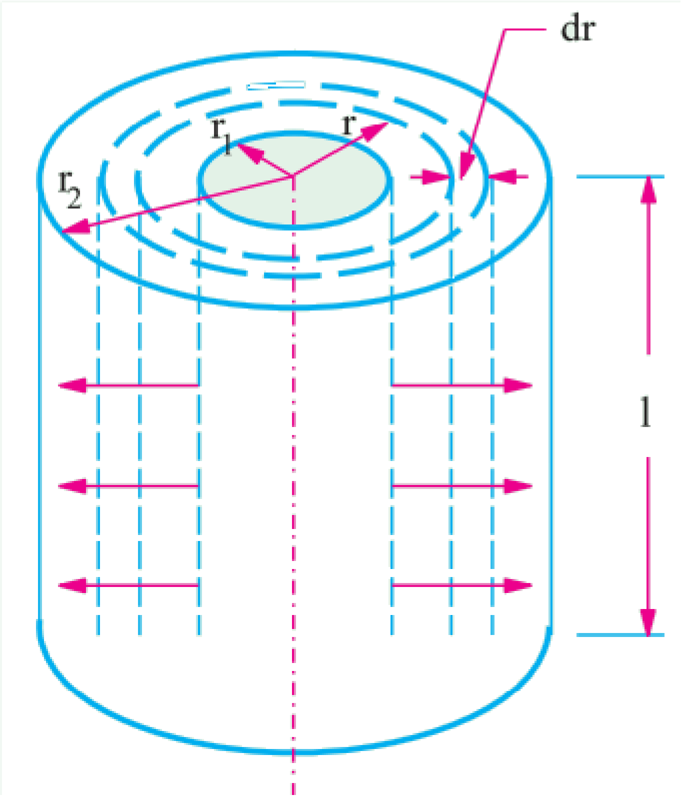


Fig. 4.12

Capacitor

- **Insulation Resistance of a Cable Capacitor**

  - ✓ Consider a single-core cable having length of l metre, inner and outer radius of r_1 and r_2 respectively as shown in **Fig. 4.12**.

  - ✓ Imagine an annular ring of radius 'r' and radial thickness 'dr'.

  - ✓ If resistivity of insulating material is ρ, then resistance of resistance of the this narrow ring is

$$dR = \frac{\rho\, dr}{2\pi r \times l} = \frac{\rho dr}{2\pi r l}$$

∴ Insulation resistance of l metre length of cable is

$$\int dR = \int_{r1}^{r2} \frac{\rho dr}{2\pi\, r l} \quad or \quad R = \frac{\rho}{2\pi l} \left| \log_e (r) \right|_{r1}^{r2}$$

$$R = \frac{\rho}{2\,\pi l} \log_e (r_2/r_1) = \frac{2.3\,\rho}{2\,\pi\, l} \log_{10} (r_2/r_1)\ \Omega$$

  - ✓ It should be noted

    i. Resistance of cable is inversely proportional to the cable length

    ii. Resistance of cable depends upon the ratio r_2/r_1 , not on the thickness of insulator itself.

Capacitor

- **Energy Stored in a Capacitor**
  - ✓ Charging of a capacitor always involves some expenditure of energy by the charging agency.
  - ✓ This energy is stored up in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.
  - ✓ Little work is done in transferring charge from one plate to another when the capacitor is uncharged.
  - ✓ Further instalments of charge is to be carried against the repulsive force due to the charge already collected on the capacitor plates.
  - ✓ Let the energy E spent in charging a capacitor of capacitance C to a voltage V.
  - ✓ Suppose at any stage of charging, the p.d. across the plates is v.
  - ✓ It is equal to the work done in shifting one coulomb from one plate to another.
  - ✓ If 'dq' charge is transferred next, then the work done $dW = v.dq$

Now $\quad q = Cv \qquad \therefore\ dq = C.dv \qquad \therefore\ dW = Cv.dv$

Capacitor

- **Energy Stored in a Capacitor**
  - Total work done in giving *V* units of potential is

$$W = \int_0^v Cv.dv = C\left|\frac{v^2}{2}\right|_0^v \quad \therefore\; W = \frac{1}{2}CV^2$$

  - If C is in farads and V is in volts, then

$$W = \frac{1}{2}CV^2 \text{ joules} = \frac{1}{2}QV \text{ joules} = \frac{Q^2}{2C} \text{ joules}$$

  - If *Q* is in coulombs and *C* is in farads, the energy stored is given in joule
  - Energy stored in a capacitor is $E = \dfrac{1}{2}CV^2$ joules
  - The energy per unit volume of dielectric medium for a capacitor having plate area of *A* m^2 and dielectric thickness of *d* metre,

$$\frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\frac{\epsilon A}{d}.\frac{V^2}{Ad} = \frac{1}{2}\varepsilon\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}DE = \frac{D^2}{2\varepsilon} \text{ joules/m}^3$$

  - The formula $\frac{1}{2}DE$ is similar to the expression $\frac{1}{2}$ ×stress × strain which is used for calculating the mechanical energy stored per unit volume of a body subjected to elastic stress.

Capacitor

- **Force of Attraction Between Oppositely-Charged Plates**
  - ✓ Two parallel conducting plates *A* and *B* carrying constant charges of + *Q* and – *Q* coulombs respectively as shown in **Fig. 4.13**.
  - ✓ Let the force of attraction between the two be *F* newtons.
  - ✓ If one of the plates is pulled apart by distance *dx*, then work done

$$= F \times dx \text{ joules}$$

  - ✓ Since the plate charges remain constant, no electrical energy comes into the arrangement during the movement *dx*.

$$\therefore \quad \text{Work done} = \text{change in stored energy}$$

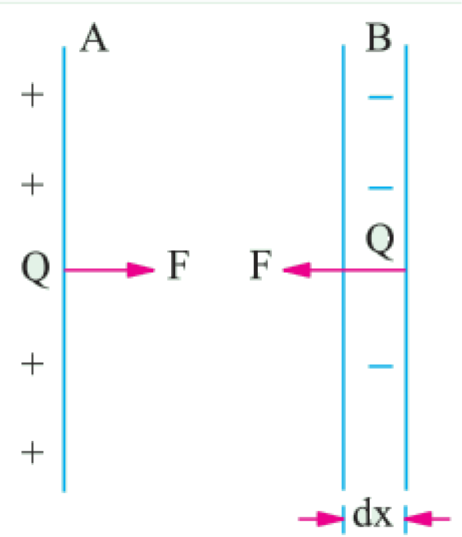


Fig. 4.13

  - ✓ Initial stored energy = $\frac{1}{2}\frac{Q^2}{C}$ joules
  - ✓ Final stored energy = $\frac{1}{2}\frac{Q^2}{(C-dC)} = \frac{1}{2}\frac{Q^2}{C}\frac{1}{\left(1-\frac{dC}{C}\right)} = \frac{1}{2}\frac{Q^2}{C}\left(1+\frac{dC}{C}\right)$ joules if $dC \ll C$

Capacitor

- **Force of Attraction Between Oppositely-Charged Plates**

$\therefore$ Change in stored energy $= \frac{1}{2}\frac{Q^2}{C}\left(1+\frac{dC}{C}\right) - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{C^2}\,.\,dC$ joules

$$\therefore \quad F.dx = \frac{1}{2}\frac{Q^2}{C^2}\,.\,dC$$

$$F = \frac{1}{2}\frac{Q^2}{C^2}\,.\,\frac{dC}{dx} = \frac{1}{2}V^2\,.\,\frac{dC}{dx} \qquad (\because V = Q/C)$$

$$C = \frac{\varepsilon A}{x} \quad \therefore \quad \frac{dC}{dx} = -\frac{\varepsilon A}{x^2}$$

$$\therefore \quad F = -\frac{1}{2}V^2.\frac{\varepsilon A}{x^2} = -\frac{1}{2}\varepsilon A\left(\frac{V}{x}\right)^2 \text{ newtons} = -\frac{1}{2}\varepsilon A E^2 \text{ newtons}$$

- ✓ This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of V volts. The negative sign shows that it is a force of attraction.

Capacitor

Example – P4.1

A parallel-plate air capacitor is charged to 100 V. Its plate separation is 2 mm and the area of each of its plate is 120 cm^2. Calculate and account for the increase or decrease of stored energy when plate separation is reduced to 1 mm (a) at constant voltage (b) at constant charge.

Capacitor

Solution of Example – P4.1

Capacitance is the first case

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{8.854\times10^{-12}\times120\times10^{-4}}{2\times10^{-3}} = 53.1\times10^{-12}\ \text{F}$$

Capacitance in the second case *i.e.* with reduced spacing

$$C_2 = \frac{8.854\times10^{-12}\times120\times10^{-4}}{1\times10^{-3}} = 106.2\times10^{-12}\ \text{F}$$

(a) When Voltage is Constant

Change in stored energy $dE = \frac{1}{2}C_2V^2 \quad \frac{1}{2}C_1V^2$

$$= \frac{1}{2}\times100^2\times(106.2-53.1)\times10^{-12} = 26.55\times10^{-8}\ J$$

This represents an increase in the energy of the capacitor. This extra work has been done by the external supply source because charge has to be given to the capacitor when its capacitance increases, voltage remaining constant

Capacitor

Solution of Example – P4.1

(b) When Charge Remains Constant

Energy in the first case $E_1 = \frac{1}{2}\frac{Q^2}{C_1}$ Energy in the second case $E_2 = \frac{1}{2}\frac{Q^2}{C_2}$

Change in energy is
$$dE = \frac{1}{2}Q^2\left(\frac{1}{53.1} - \frac{1}{106.2}\right)\times 10^{12}\ \text{J}$$
$$= \frac{1}{2}(C_1V_1)^2\left(\frac{1}{53.1} - \frac{1}{106.2}\right)\times 10^{12}\ \text{J}$$
$$= \frac{1}{2}(53.1\times 10^{-12})^2\times 10^4\times 0.0094\times 10^{12}$$
$$= 13.3\times 10^{-8}\ \text{J}$$

Hence, there is a decrease in the stored energy. The reason is that charge remaining constant, when the capacitance is increased, then voltage must fall with a consequent decrease in stored energy

$$(E = \frac{1}{2}QV)$$


Thank you