
Mathematics - II

April 30, 2022

1 PROBABILITY

1.1 PROBLEM SET

1. A parking lot has 10 parking spaces arranged in a row. There are 7 cars parked. Assume that each car owner has picked at a random a parking place among the spaces available. Determine the probability that the three empty places are adjacent to each other.
2. The probability that a visit to a particular car dealer results in neither buying a second-hand car nor a Japanese car is 55%. Of those coming to the dealer, 25% buy a second-hand car and 30% buy a Japanese car. What is the probability that a visit leads to buying a second-hand Japanese car?
3. In a high school class, 35% of the students take Spanish as a foreign language, 15% take French as a foreign language, and 40% take at least one of these languages. What is the probability that a randomly chosen student takes French given that the student takes Spanish?
4. A professor gives only two types of exams, “easy” and “hard.” You will get a hard exam with probability 0.80. The probability that the first question on the exam will be marked as difficult is 0.90 if the exam is hard and is 0.15 otherwise. What is the probability that the first question on your exam is marked as difficult. What is the probability that your exam is hard given that the first question on the exam is marked as difficult?
5. A family is chosen at random from all three-child families. What is the probability that the chosen family has one boy and two girls if the family has a boy among the three children? Use Bayes’ rule in odds form to answer this question.
6. In 10 independent throws of a defective die, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. Find

the probability that an even number will not appear at all in 10 independent throws of the die.

7. The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates, at least 4 passed the examination?
8. In a shooting competition, the probability of a man hitting a target is $\frac{1}{5}$. If he fires 5 times, what is the probability of hitting the target at least twice?
9. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease what is the probability that (a) at least 10 survive? (b) from 3 to 8 survive (c) exactly 5 survive.
10. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms? (Given that area to the left of $z = 1.5$ is 0.9332).
11. In an industrial process, the diameter of a ball bearing is specified as 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process, the diameter of a ball bearing has a normal distribution with mean is equal to 3.0 and standard deviation is equal to 0.005. On the average, what % of manufactured ball bearings will be scrapped? Given that area to the right of $z = 2.0$ is 0.0228.
12. Given a normal distribution with mean $\mu = 40$, and s.d. $\sigma = 6$. Find the value of x that has 45% of the area to the left.
13. If you use pure guessing on a 10-question true/false exam, what is the probability that you get them all right?
Ans: $\frac{1}{2^{10}} = \frac{1}{1024}$.
14. If you use pure guessing on a 10-question true/false exam, what is the probability that you get at least 7 out of 10?
Ans: $\frac{11}{64}$.
15. There is a box containing 30 bulbs of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs chosen are defective?
Ans: $\frac{2}{87}$.
16. A box of 30 diodes is known to contain five defective ones. If two diodes are selected at random without replacement, what is the probability that at least one of these diodes is defective?
Ans: 0.31.
17. A random variable x has the following probability function:

x:	-2	-1	0	1	2	3
p(x):	0.1	k	0.2	2k	0.3	k

Find the value of k and calculate the mean and variance.

18. A die is tossed thrice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes.
19. Find the standard deviation for the following discrete distribution:

x:	8	12	16	20	24
p(x):	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

20. The diameter x of an electric cable is assumed to be a continuous variate with possible probability density function $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$. Verify whether f is a probability density function. Also find the mean and variance.
21. Two cards are drawn successively with replacement from a well-shuffled pack of 52

playing cards. Find the probability distribution of the number of aces.

22. Find the probability distribution of the number of green balls drawn when three balls are drawn one by one without replacement from a bag containing three greens and five white balls.
23. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease? Ans: $\frac{53}{3125}$
24. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that (i) exactly two will strike the target and (ii) at least two will strike the target. Ans: 0.24576, 0.34478
25. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that
 - a) exactly two pens will be defective
 - b) at least two pens will be defective
 - c) none will be defective.
26. The scores in a competitive examination is normally distributed with mean 400 and standard deviation 80. Out of 10,000 candidates appeared in the examination, it is desired to pass 350 candidates. What should be the lowest score permitted for passing the examination?
27. Find the value of z in each of the cases
 - a) Area between 0 and z is 0.3770 Ans: $z = \pm 1.16$
 - b) Area to the left of z is 0.8621. Ans: $z = 1.09$
28. For a normally distributed variate X with mean 1 and standard deviation 3, find out the probability that
 - a) $3.43 \leq x \leq 6.19$ Ans: 0.1672
 - b) $-1.43 \leq x \leq 6.19$. Ans: 0.7492

Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored more than 60 marks? Ans: 50%
29. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find
 - a) how many students score between 12 and 15? Ans: 444
 - b) how many score above 18? Ans: 55
 - c) how many score below 8? Ans: 8
 - d) how many score 16? Ans: 116
30. A manufacturer of envelopes knows that the weight of the envelopes is normally distributed with mean 1.9 gm and variance 0.01 gm. Find how many envelopes weighing (i) 2 gm or more, (ii) 2.1 gm or more, can be expected in a given packet of 1000 envelopes. Ans: 159, 23
31. The life of army shoes is 'normally' distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? Ans: 4886
32. The mean height of 500 students is 151 cm and the standard deviation is 15 cm. As-

suming that the heights are normally distributed, find the number of students whose heights lie between 120 and 155 cm. Ans: 294

33. In an examination taken by 500 candidates the average and standard deviation of marks obtained (normally distributed) are 40% and 10%. Find approximately
 - a) How many will pass if 50 is fixed as a minimum? Ans: 79
 - b) What should be minimum score if 350 candidates are to be declared as pass? Ans: 35%
 - c) How many candidates have scored marks above 60%? Ans: 11
34. The marks obtained by the number of students for a certain subject are assumed to be approximately distributed with mean value 65 and with a standard deviation of 5. If three students are taken at random from this set of students, what is the probability that exactly two of them will have marks over 70? Ans: 0.06357
35. If you use pure guessing on a 10-question true/false exam, what is the probability that you get at least 7 out of 10?
36. There is a box containing 30 bulbs of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs chosen are defective?
37. An urn I contains 3 white and 4 red balls and an urn II contains 5 white and 6 red balls. One ball is drawn at random from one of the urns and is found to be white. Find the probability that it was drawn from urn I .
38. Three urns contains 6 red, 4 black; 4 red, 6 black; 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn.
39. In a bolt factory, machines A , B and C manufacture respectively 25%, 35% and 40% of the total. If their output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ?
40. A box of 30 diodes is known to contain five defective ones. If two diodes are selected at random without replacement, what is the probability that at least one of these diodes is defective?
41. Toss a coin for 12 times. What is the probability of getting exactly 7 heads?