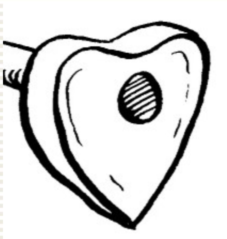
Cam Mechanism



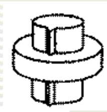
Dr. S. S. Roy
Professor
Department of Mechanical Engineering

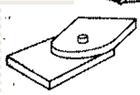
Basics of Mechanism

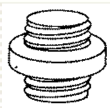
Kinematic Pair - is a joint of two or more links having relative motion between them

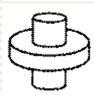
Classification of Joints or Kinematic pairs

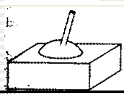
Types of Kinematic Pairs
- According to Nature of Contact
  - Lower Pair — Surface or area contact between links
  - Higher Pair — Line or point contact between links
- According to Nature of Relative Motion
  - Sliding/ Prismatic Pair
  - Turning/ Revolute Pair
  - Screw Pair
  - Cylindrical Pair
  - Spherical Pair

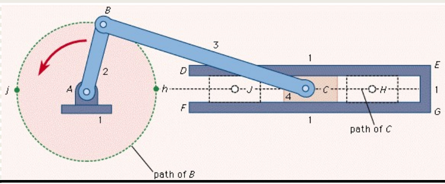




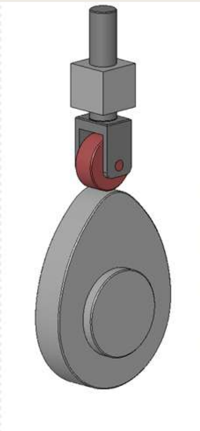


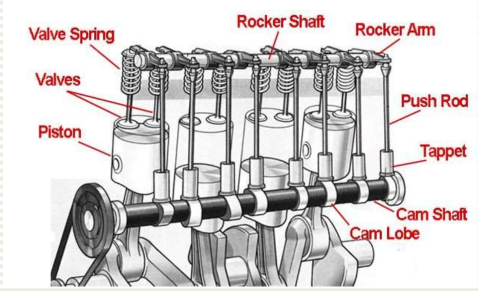






Higher Pair





Cam & Follower : A higher pair

A cam is a machine element used to impart desired/specified motion to another element , called follower, by direct contact.

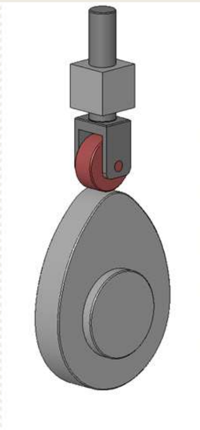
- ✓ The driving member is called Cam
- ✓ The driven member is called Follower

A ***cam*** may be defined as a machine element having a curved outline or a curved groove, which, by its rotation or reciprocating motion, gives a predetermined specified motion to another element called the ***follower***

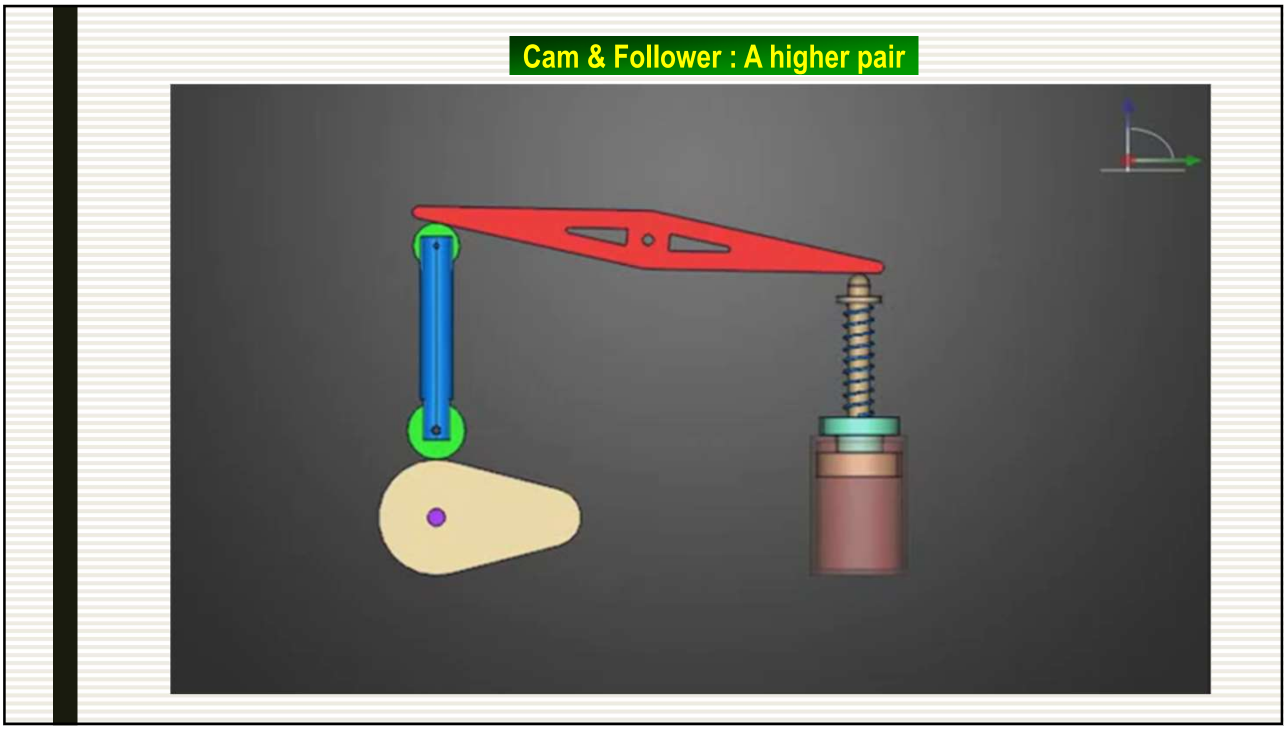
The frame which supports the cam & guides the follower

Motion of cam: either rotating or reciprocating

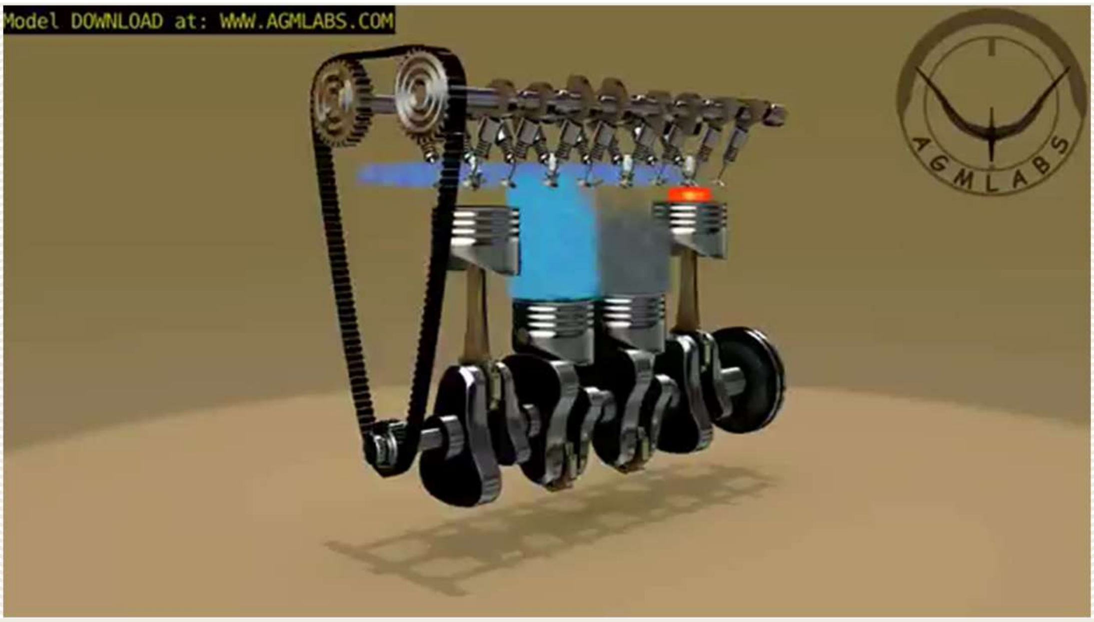
Motion of follower: Reciprocating
Oscillating
Complex coordinated movement




SINGER


Cam & Follower : A higher pair

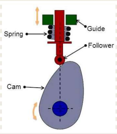
Cam & Follower : A higher pair

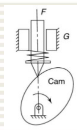


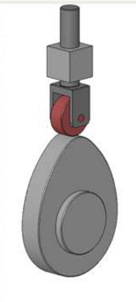
Type of Cams

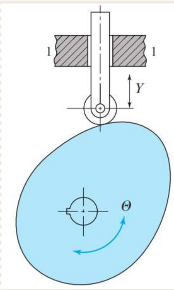
According to shape

- Plate cam/Radial cam/ Disc cam

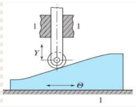


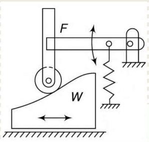




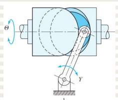


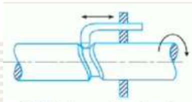
- Wedge cam/ Flat cam

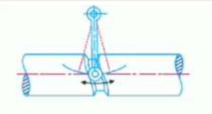




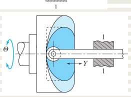
- Cylindrical cam/ Barrel cam



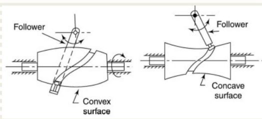

Cylindrical cam with reciprocating follower.


Cylindrical cam with oscillating follower.

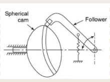
- Face cam/ End cam



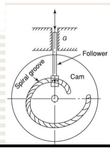
- Globoidal cam



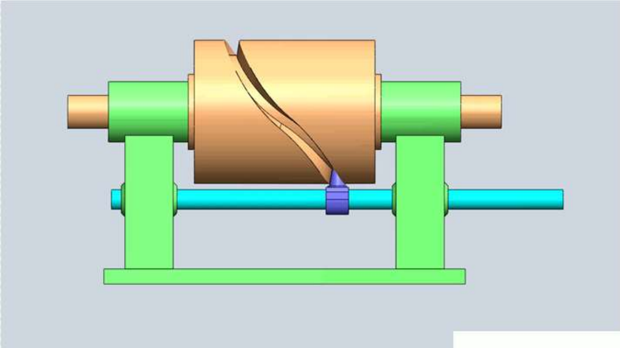
- Spherical cam

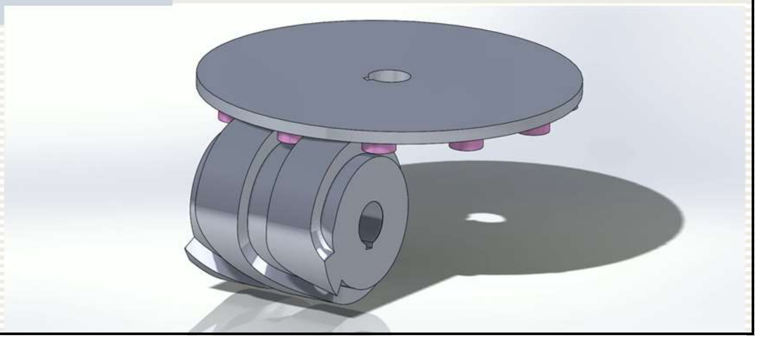


- Spiral cam

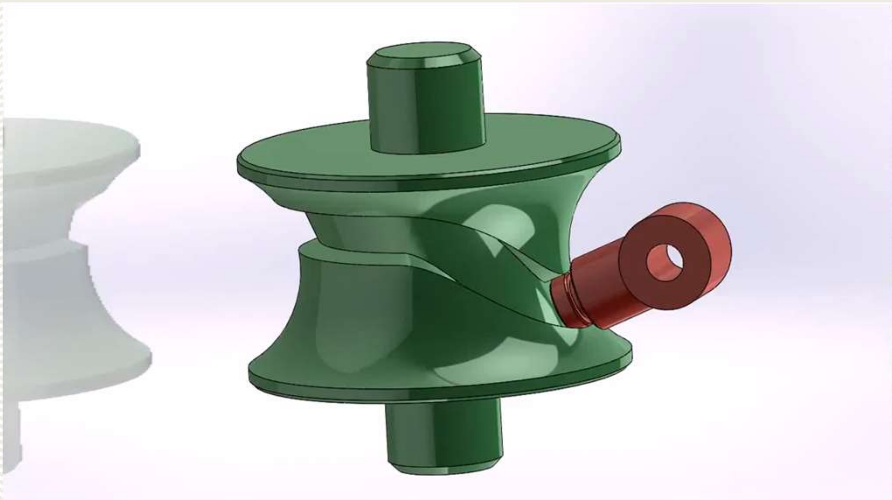


Cylindrical cam/ Barrel cam





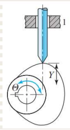
Globoidal cam

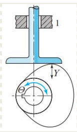


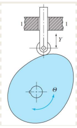
Classification of Followers

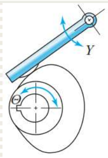
Types of Followers

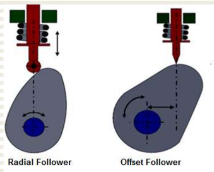
- **According to shape or the nature of surface in contact**
  - Knife-edge follower
  - Flat-face follower
  - Roller follower
  - Spherical-face /curved-shoe follower
- **According to the characteristic of output motion**
  - Reciprocating or translating followers
    - Radial follower
    - Offset follower
  - Oscillating or rotating followers

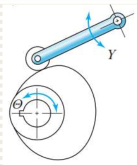










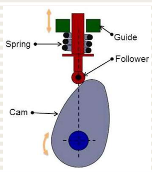


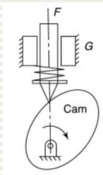
According to manner of constraint of the follower

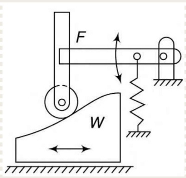
To transmit desired motion from the cam to the follower, it is necessary that the two remain in touch at all speeds & at all times. The cams can be classified according to the manner in which this is achieved.

Spring pre-loaded Cam

A pre-loaded compression spring is used for the purpose of keeping the contact between the cam & the follower.

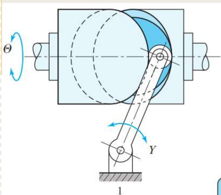






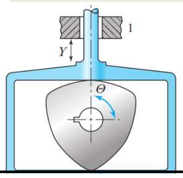
Constraint or Positive-drive Cam

In this type, constant touch between the cam & the follower is maintained by a roller follower operating in the groove of a cam .



Gravity constrained cam

The weight of the follower keeps them in contact. This is called a gravity constraint cam



Application of Cam & Follower

- ✓ IC engine valve opening / closing mechanism
- ✓ Sewing machine
- ✓ printing control mechanisms
- ✓ Machine tools
- ✓ Automatic machines

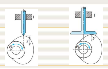
Advantages of Cam & Follower

- ✓ simple & inexpensive
- ✓ have few moving parts
- ✓ occupy very little space
- ✓ follower motions having almost any desired characteristics are not difficult to design
- ✓ Automatic machines

Radial Cam Nomenclature

Trace point

- is a theoretical point on the follower & its motion describing the movement of the follower.
- For a knife-edge follower, the trace point is at the knife-edge.
- For a roller follower, the trace point is at the roller centre
- For flat-face follower, it is at the point of contact between the follower & the cam surface

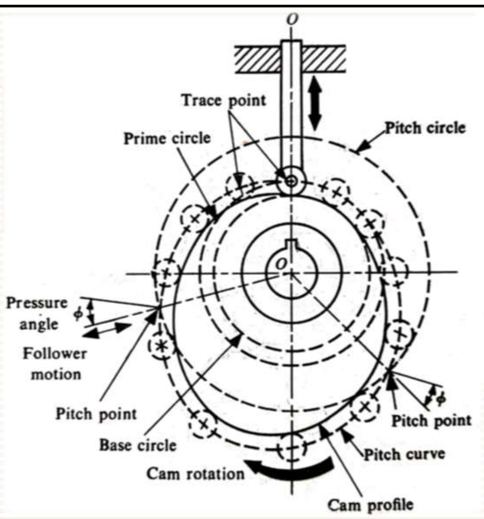


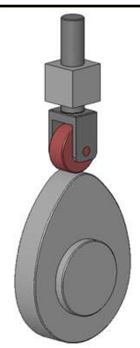
Base circle

- is the smallest circle that can be drawn with center at the cam rotation axis and tangent to the cam profile. The base circle decides the overall size of a cam

Pitch curve

- Is the locus generated by the trace point as the follower moves relative to the cam (after holding the cam fixed).
- For a knife-edge follower, the pitch curve & cam profile are identical.
- For roller follower, they are separated by the radius of the roller.





Pressure Angle

- The angle between the direction of the follower movement & the normal to the pitch curve at any point is referred to as the pressure angle.
- During a complete rotation, the pressure angle varies from its maximum to its minimum value.
- The greater the pressure angle, the higher will be the side thrust & consequently the chances of the translating follower jamming in its guide will increase.
- In case of low-speed cam mechanisms with translating followers, the highest permissible value of the pressure angle is 30°
- For a given motion requirement, the pressure angle can be reduced by increasing the cam size. However, a bigger cam requires more space & is more prone to unbalance at high speeds

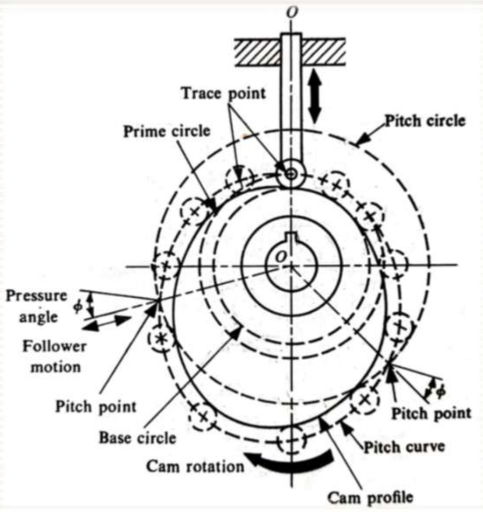
Radial Cam Nomenclature

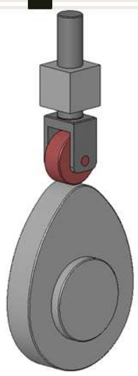
Pitch point & Pitch circle

A pitch point corresponds to the point of maximum pressure angle and a circle drawn with its centre at the cam centre, to pass through the pitch point, is known as the pitch circle.

Prime circle

- is the smallest circle that can be drawn with center at the cam rotation axis and tangent to the pitch curve.





Stroke or Throw

- The greatest distance or angle through which the follower moves or rotates. .
- It is also called as Lift of the Cam

Cam Profile

- The surface in contact with the follower .

Angle of ascent

- Is the angle of rotation of the cam during which the follower rises up.

Angle of Dwell

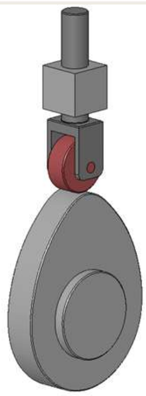
- Is the angle through which the cam turns while the follower remains stationary at the highest or lowest position.

Angle of descent

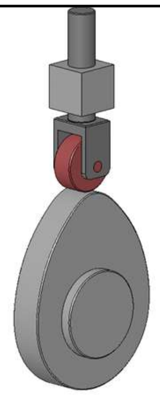
-angle of rotation of cam during which the follower returns to its initial position

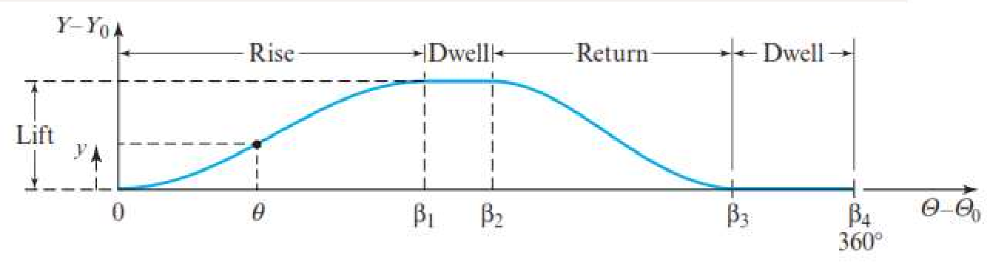
Angle of Action

- is the total angle moved by the cam during the time, between the beginning of rise & the end of the return of the follower

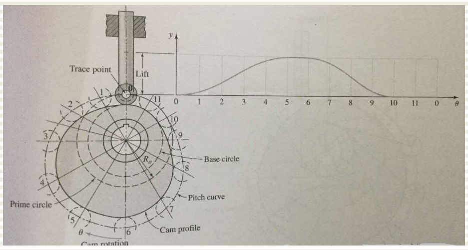


Description of the Follower Movement

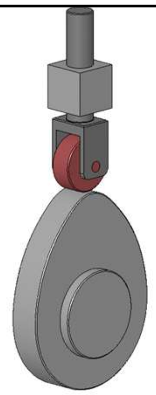


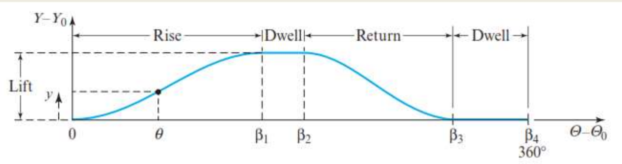


Displacement Diagram of Radial Cam



- Usually, cam system is a single DOF device.
- It is driven by a known input motion, usually a shaft that rotates at constant speed & it is intended to produce a certain desired output motion for the follower.





DISPLACEMENT DIAGRAMS

During the rotation of the cam through one cycle of input motion, the follower executes a series of events as demonstrated in graphical form in the Displacement Diagram

Rise: is the portion of the displacement diagram in which the motion of the follower is away from the cam centre. **The maximum rise is called Lift.**

Return: is the portion of the displacement diagram in which the motion of the follower is toward the cam centre

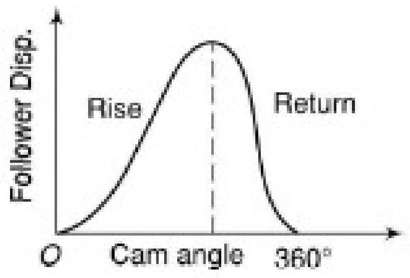
Dwell: is the portion of the cycle during which the follower is at rest.

According to Follower Movement

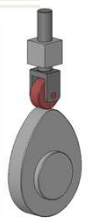
The motions of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.

Cams are classified according to the motions of the followers in the following ways:

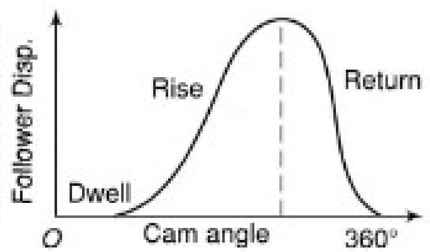
1. ***Rise-Return-Rise*** **(R-R-R)** In this, there is alternate rise and return of the follower with no periods of dwells Its use is very limited in the industry. The follower has a linear or an angular displacement.



(a)

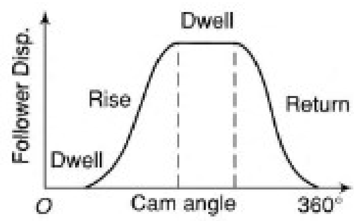


2. ***Dwell-Rise-Return-Dwell (D-R-R-D)*** In such a type of cam, there is rise and return of the follower after a dwell This type is used more frequently than the R-R-R type of cam.



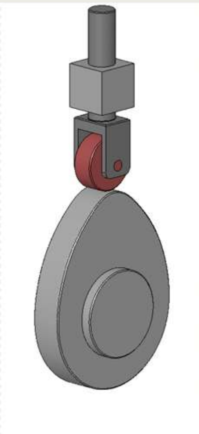
(b)

3. ***Dwell-Rise-Dwell-Return-Dwell*** **(D-R-D-R-D)** It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell



(c)

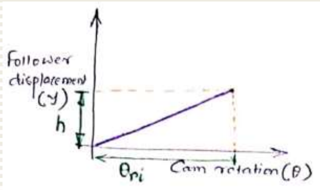
Kinematic Analysis of the Follower Motion



Motion of the Follower

- Uniform motion or Constant velocity & its modification
- Uniform Acceleration & deceleration motion /Parabolic motion
- Simple Harmonic Motion
- Cycloidal motion

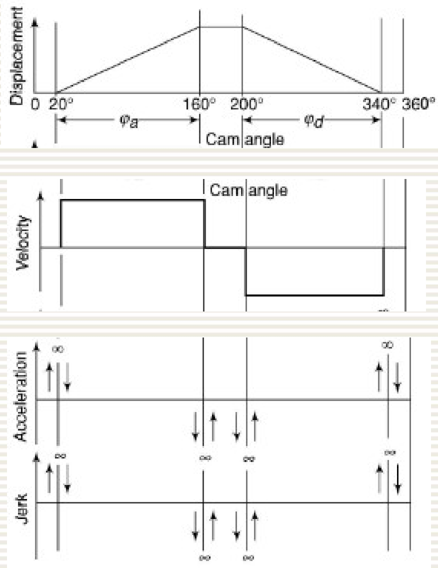
Uniform motion or Motion with Constant velocity



By uniform motion,
We mean that the Velocity of the follower is Constant.

$$\frac{dy}{dt} = \text{Constant} = C_1$$

(Slope of the displacement Curve is Constant)



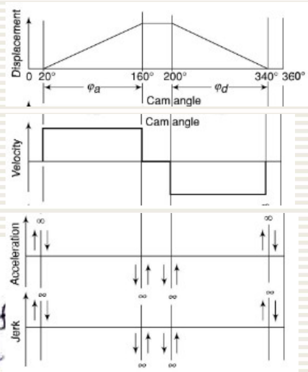
the displacement of the follower is proportional to the angle of cam rotation (Linear relationship)

$$y \propto \theta$$

$$y = c\theta \quad \text{where } c = \text{Constant of proportionality}$$

$$\theta = \text{Angle of cam rotation} = \omega t$$

$$y = c\theta$$



Since, the follower displacement is from $y=0$ to $y=h$ when the cam rotates from $\theta = 0$ to $\theta = \theta_{ri}$

$$\therefore \quad h = c.\theta_{ri}$$

where h is called 'follower rise or Lift'

$$\therefore \quad c = \frac{h}{\theta_{ri}}$$

θ_{ri} is the angle through which the Cam is to rotate to lift the follower by 'h'

Angle of ascent

= Cam rotation angle for the maximum follower displacement

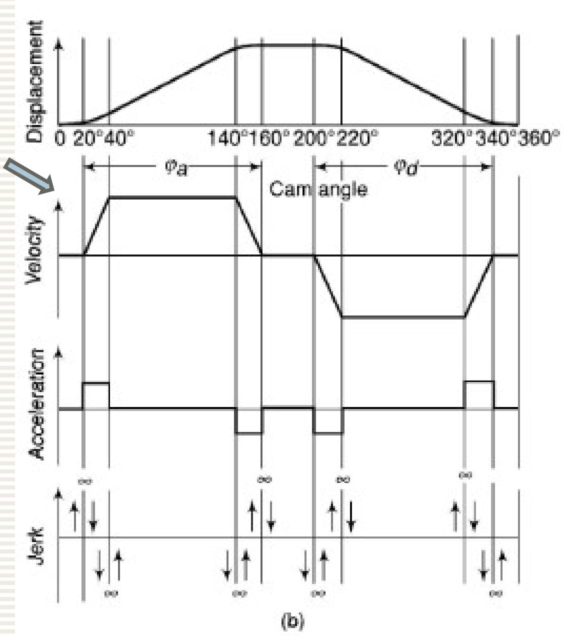
$$\therefore \quad y = \left(\frac{h}{\theta_{ri}}\right).\theta \qquad \text{Where } 0 \leq \theta \leq \theta_{ri}$$

$$y = \left(\frac{h}{\theta_{ri}}\right).\omega t$$

Velocity $v = \frac{dy}{dt} = \left(\frac{h}{\theta_{ri}}\right)\omega$ $\quad$ ω = Angular Velocity of Cam

Acceleration $a = \frac{dv}{dt} = 0$

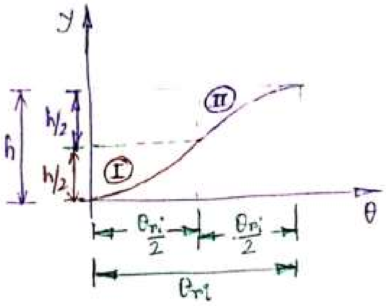
A modified programme for the follower motion can be evolved in which the accelerations are reduced to finite values. This can be done by rounding the sharp corners of the displacement curve so that the velocity changes are gradual at the beginning and end of the follower motion. During these periods, the acceleration may be assumed to be constant and of finite values. A modified constant velocity programme is shown in

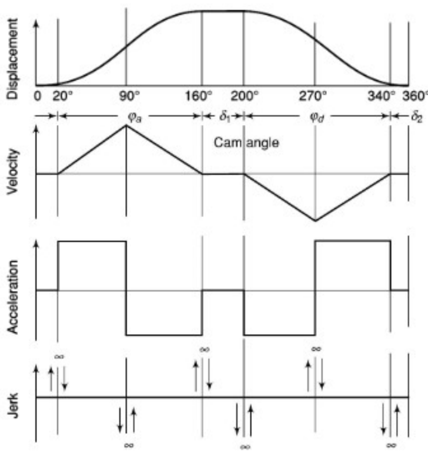


(b)

Uniform Acceleration Motion or Parabolic Motion

In the parabolic motion, the displacement of the follower is proportional to the square of the angle of cam rotation.





Uniform Acceleration & deceleration motion /Parabolic motion

In parabolic motion, there is two period during rise

(I) Acceleration period of rise

(II) Retardation period of rise

(I) Acceleration period of rise

$$y \propto \theta^2$$

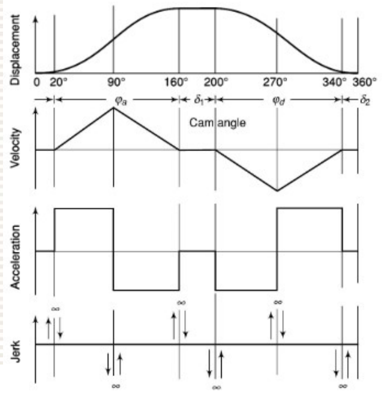
$$y = c\theta^2 \quad \text{where } 0 \le \theta \le \frac{\theta_{ri}}{2}$$

Since $y = h/2$ when $\theta = \frac{\theta_{ri}}{2}$, we get

$$\frac{h}{2} = c\left(\frac{\theta_{ri}}{2}\right)^2$$

$$\therefore c = \frac{2h}{\theta_{ri}^2}$$

$$\therefore \quad y = 2h\left(\frac{\theta}{\theta_{ri}}\right)^2 \quad \text{where } \theta = \omega.t \; ; \; \omega = \text{Angular Velocity of Cam}$$



Velocity of the follower

$$v = \frac{dy}{dt} = \frac{2h}{\theta_{ri}^2} \cdot \frac{d}{dt}(\theta^2) = \frac{2h \cdot 2\theta}{\theta_{ri}^2} \cdot \frac{d\theta}{dt} = \frac{4h\theta.\omega}{\theta_{ri}^2}$$

$$\dot{y} = v = \left(\frac{4h\omega}{\theta_{ri}^2}\right)\theta \longleftarrow \text{Linear equation}$$

The max^m velocity of the follower will be at $\theta = \frac{\theta_{ri}}{2}$

$$v_{max} = \frac{2h\omega}{\theta_{ri}}$$

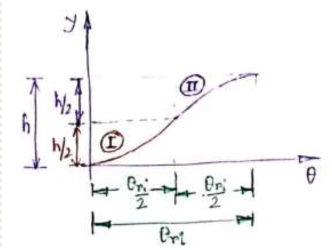
$$v = \left(\frac{4h\omega}{\theta_{ri}^2}\right)\theta$$

Acceleration of the follower

$$a = \frac{dv}{dt} = \frac{4h\omega}{\theta_{ri}^2} \cdot \frac{d\theta}{dt}$$

$$\ddot{y} = a = \frac{4h\omega^2}{\theta_{ri}^2} = \text{Constant}$$

(II) Retardation period of rise



$$y = c_1 + c_2\theta + c_3\theta^2 \quad \text{where } \frac{\theta_{ri}}{2} \le \theta \le \theta_{ri}$$

Apply boundary condition to obtain three constants c_1, c_2 & c_3

When $\theta = \theta_{ri}$; $y = h$

$\theta = \theta_{ri}$; $\dot{y} = 0$ (assuming a dwell at the end of the rise)

$\theta = \frac{\theta_{ri}}{2}$; $\dot{y} = v = \frac{2h\omega}{\theta_{ri}}$ (maintaining the velocity continuity at $\theta = \frac{\theta_{ri}}{2}$)

$$\dot{y} = c_2\frac{d\theta}{dt} + 2c_3\theta\frac{d\theta}{dt}$$

$$\dot{y} = c_2\omega + 2c_3\omega\theta$$

$y = c_1 + c_2\theta + c_3\theta_{,}^{2}$ where $\frac{\theta_{ri}}{2} \le \theta \le \theta_{ri}$

$\dot{y} = c_2\frac{d\theta}{dt} + 2c_3\theta\frac{d\theta}{dt}$

$\dot{y} = c_2\omega + 2c_3\omega\theta$

when $\theta = \theta_{ri}$, $\dot{y} = 0$

$\therefore \quad 0 = c_2\omega + 2c_3\omega.\theta_{ri}$ ------ (A)

when $\theta = \frac{\theta_{ri}}{2}$, $\dot{y} = \frac{2h\omega}{\theta_{ri}}$

$\frac{2h\omega}{\theta_{ri}} = c_2\omega + 2c_3\omega.\frac{\theta_{ri}}{2}$ ------ (B)

(B) − (A)

$c_3 = -\frac{2h}{\theta_{ri}^2}$ | from (A) $c_2 = \frac{4h}{\theta_{ri}}$

When $\theta = \theta_{ri}$; $y = h$

$h = c_1 + \frac{4h}{\theta_{ri}}.\theta_{ri} - \frac{2h}{\theta_{ri}^2}.\theta_{ri}^2$

$c_1 = -h$

$\therefore \quad y = -h + \frac{4h}{\theta_{ri}}.\theta - \frac{2h}{\theta_{ri}^2}.\theta^2$

$y = c_1 + c_2\theta + c_3\theta_{,}^{2}$ where $\frac{\theta_{ri}}{2} \le \theta \le \theta_{ri}$

$$y = h\left[-1 + \frac{4\theta}{\theta_{ri}} - \frac{2\theta^2}{\theta_{ri}^2}\right]$$

$$y = h\left[1 - 2\left\{1 - \frac{2\theta}{\theta_{ri}} + \left(\frac{\theta}{\theta_{ri}}\right)^2\right\}\right]$$

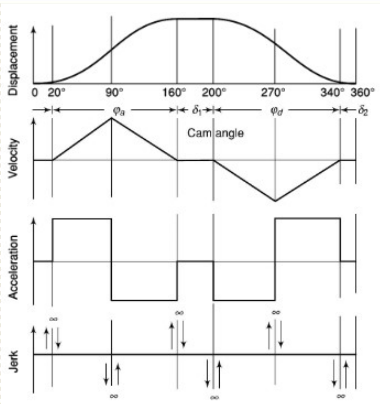
$$y = h\left[1 - 2\left(1 - \frac{\theta}{\theta_{ri}}\right)^2\right]$$

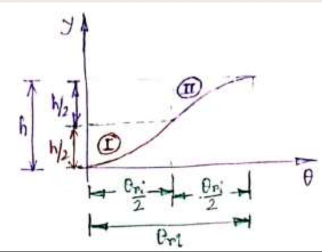
Velocity of the follower

$$\dot{y} = v = \frac{4h\omega}{\theta_{ri}}\left(1 - \frac{\theta}{\theta_{ri}}\right)$$

Acceleration of the follower

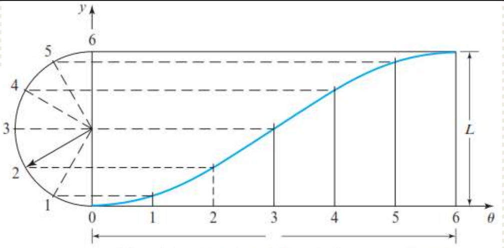
$$\ddot{y} = a = -\frac{4h\omega^2}{\theta_{ri}^2}$$



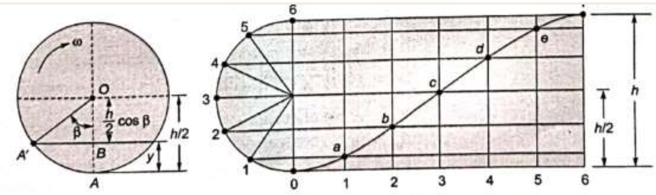


Simple Harmonic Motion (SHM)

y = displacement of the follower (Instantaneous)
h = maximum displacement of the follower = Lift or rise
β = angle on the harmonic Circle
θ = Cam rotation angle (instantaneous)
θ_{ri} = Cam rotation angle for the maximum follower displacement



Simple harmonic motion displacement diagram:



(b) Displacement curve

At any instant, displacement of the follower is given by

$$y = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$y = \frac{h}{2}(1 - \cos\beta)$$

At any instant, displacement of the follower is given by

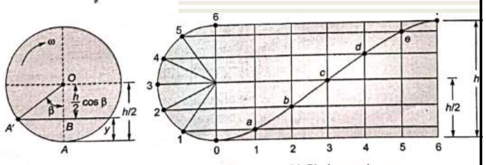
$$y = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$y = \frac{h}{2}(1 - \cos\beta)$$

For the rise (ascent) or fall (descent) [h] of the follower displacement, the Cam is rotated through an angle θ_{ri}, whereas a point on the harmonic semicircle traverses an angle π.
The Cam rotation is proportional to the angle turned by the point on the harmonic semicircle

$$\beta = \frac{\pi}{\theta_{ri}} \cdot \theta$$

$$\therefore \quad y = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\theta_{ri}}\right) \qquad 0 \le \theta \le \theta_{ri}$$



(a) (b) Displacement curve

$y = \frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\theta_{ri}}\right)$ where ω = Angular velocity of the cam

Velocity of the follower

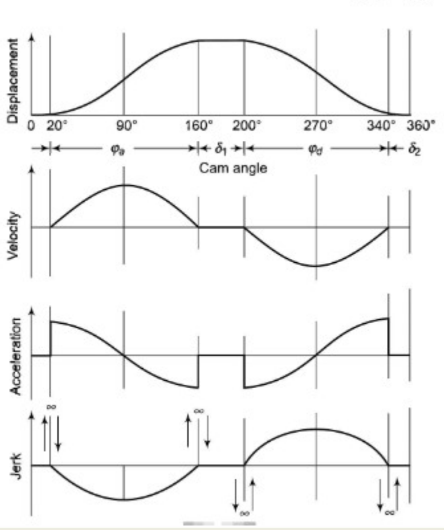
$$\dot{y} = \frac{dy}{dt} = \frac{d}{dt}\left[\frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\theta_{ri}}\right)\right]$$

$$v = \frac{h}{2} \cdot \frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\omega t}{\theta_{ri}}\right) = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\theta}{\theta_{ri}}\right)$$

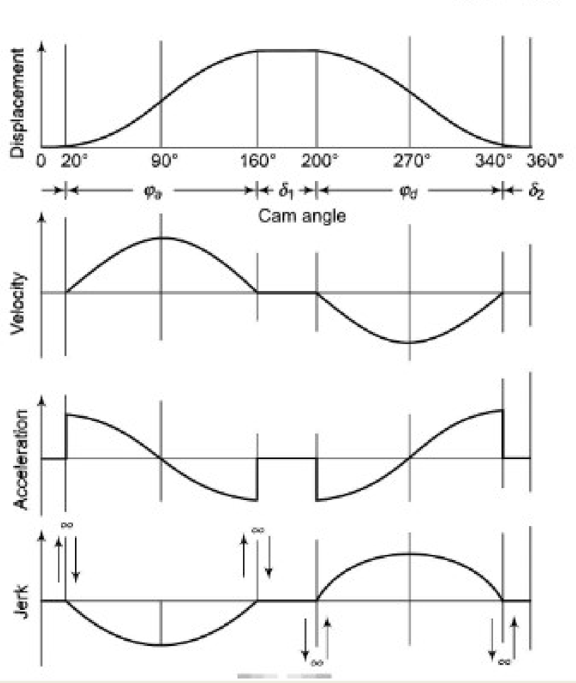
Acceleration of the follower

$$\ddot{y} = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2 \cos\left(\frac{\pi\omega t}{\theta_{ri}}\right)$$

$$a = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2 \cos\left(\frac{\pi\theta}{\theta_{ri}}\right)$$



- It can be observed that there is an abrupt change of acceleration from zero to maximum at the beginning of the follower motion & also from maximum (negative) to zero at the end of the follower motion.
- The same pattern is repeated during descent.
- This lead to jerk, vibration & noise etc.
- SHM should be adopted only for low to moderate speeds



Velocity of the follower

$$\dot{y} = \frac{dy}{dt} = \frac{d}{dt}\left[\frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\theta_{ri}}\right)\right]$$

$$v = \frac{h}{2}\cdot\frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\omega t}{\theta_{ri}}\right) = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\theta}{\theta_{ri}}\right)$$

find out v_{max}

At v is max^m $\quad \frac{dv}{d\theta} = 0$

$$\frac{d}{d\theta}\left[\frac{h}{2}\frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\theta}{\theta_{ri}}\right)\right] = 0$$

$$\frac{h}{2}\frac{\pi\omega}{\theta_{ri}}\frac{\pi}{\theta_{ri}} \cos\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0$$

$$\cos\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0 = \cos\frac{\pi}{2} \text{ as } \frac{h}{2}\frac{\pi\omega}{\theta_{ri}^2}\cdot\pi \neq 0$$

$$\therefore \frac{\pi\theta}{\theta_{ri}} = \frac{\pi}{2} \quad \therefore \theta = \frac{\theta_{ri}}{2}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}} \sin\left(\frac{\pi\theta_{ri}}{2\theta_{ri}}\right)$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}}$$

find out a_{max}

At 'a' is max^m $\quad \frac{da}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2 \cos\left(\frac{\pi\theta}{\theta_{ri}}\right)\right] = 0$

$$\therefore \sin\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0 = \sin 0^\circ$$

$$\therefore \theta = 0^\circ$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2$$

Problem # 1

A disc cam is to give SHM to a knife edge follower during out-stroke of 50 mm. the angle of ascent is 120 degree, dwell 60 degree & angle of descent 90 degree. Calculate the maximum velocity & acceleration during ascent when the cam shaft revolves at 200 rpm.

Given Data:

Stroke or lift (h) = 50 mm

Angle of ascent (θ_{ri}) = 120 degree $=120\pi/180$ rad

RPM of cam (N) = 200 rpm

Angular velocity of cam (ω) = $2\pi N/60$ rad/s

$= 2\pi\times200/60$ rad/s

$=20.93$ rad/s

At any instant, displacement of the follower is given by

$$y = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$\beta = \frac{\pi}{\theta_{ri}}.\theta$$

$$\therefore \quad y = \frac{h}{2}\left(1-\cos\frac{\pi\theta}{\theta_{ri}}\right) \qquad 0 \le \theta \le \theta_{ri}$$

$$y = \frac{h}{2}\left(1-\cos\frac{\pi\omega t}{\theta_{ri}}\right)$$

Velocity of the follower

$$\dot{y} = \frac{dy}{dt} = \frac{d}{dt}\left[\frac{h}{2}\left(1-\cos\frac{\pi\omega t}{\theta_{ri}}\right)\right]$$

$$v = \frac{h}{2}.\frac{\pi\omega}{\theta_{ri}}\sin\left(\frac{\pi\omega t}{\theta_{ri}}\right) = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}}\sin\left(\frac{\pi\theta}{\theta_{ri}}\right)$$

Acceleration of the follower

$$\ddot{y} = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2\cos\left(\frac{\pi\omega t}{\theta_{ri}}\right)$$

$$a = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2\cos\left(\frac{\pi\theta}{\theta_{ri}}\right)$$

Find out v_{max}

At v is max^m $\quad \frac{dv}{d\theta} = 0$

$$\frac{d}{d\theta}\left[\frac{h}{2}\frac{\pi\omega}{\theta_{ri}}\sin\left(\frac{\pi\theta}{\theta_{ri}}\right)\right] = 0$$

$$\frac{h}{2}\frac{\pi\omega}{\theta_{ri}}\frac{\pi}{\theta_{ri}}\cos\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0$$

$$\cos\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0 = \cos\frac{\pi}{2} \text{ as } \frac{h}{2}\frac{\pi\omega}{\theta_{ri}^2}.\pi \neq 0$$

$$\therefore \frac{\pi\theta}{\theta_{ri}} = \frac{\pi}{2} \qquad \therefore \theta = \frac{\theta_{ri}}{2}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}}\sin\left(\frac{\pi\theta_{ri}}{2\theta_{ri}}\right)$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\theta_{ri}}$$

Find out a_{max}

At 'a' is max^m $\quad \frac{da}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2\cos\left(\frac{\pi\theta}{\theta_{ri}}\right)\right] = 0$

$$\therefore \sin\left(\frac{\pi\theta}{\theta_{ri}}\right) = 0 = \sin 0^\circ$$

$$\therefore \theta = 0^\circ$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_{ri}}\right)^2$$

Max. velocity during ascent (v_{max}) = $h\pi\omega/(2\theta_{ri})$ mm/s

$= 50\pi 20.93/(2\times120\pi/180)$ mm/s

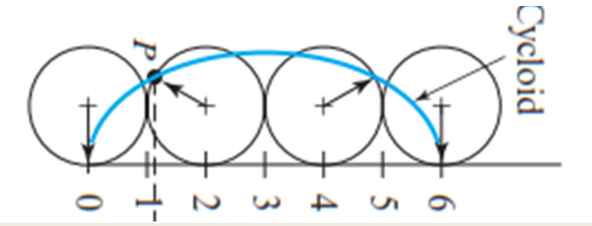
$=785$ mm/s

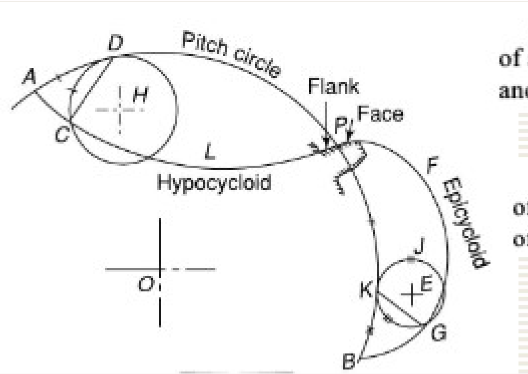
Max. Accel. during ascent (a_{max}) $=(h/2)\times(\pi\omega/\theta_{ri})^2$ mm/s^2

$= (50/2)\times(\pi 20.93\times180/120\pi)^2$ mm/s^2

$=24640$ mm/s^2

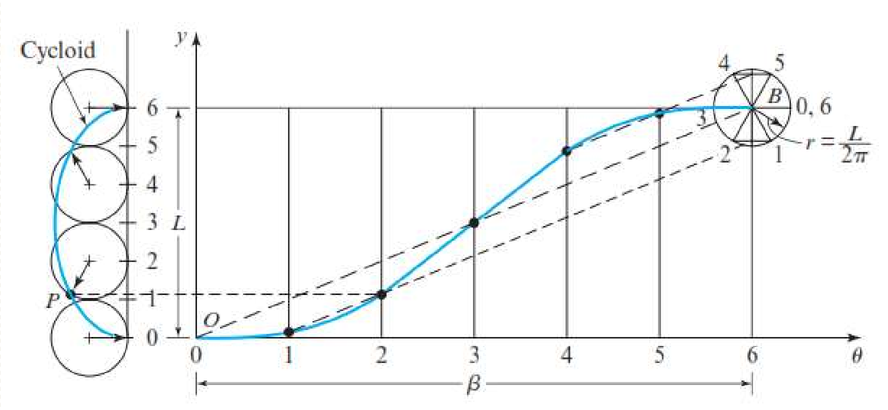
A *cycloid* is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.





An *epicycloid* is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.

A *hypocycloid* is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.

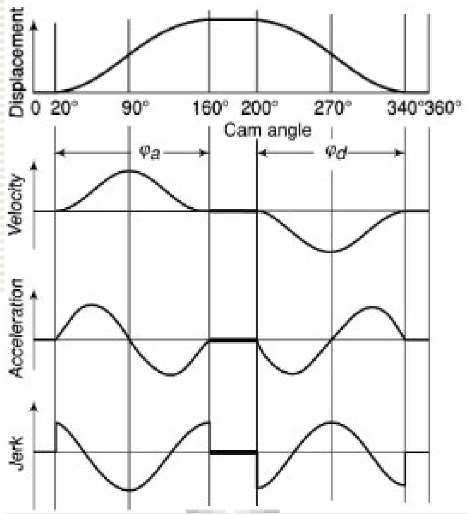


Cycloidal motion displacement diagram

Cycloidal Motion

Mathematically, a cycloid is expressed by

$$y = \frac{h}{\pi}\left[\frac{\pi\theta}{\theta_{ri}} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$



Velocity of the follower

$$\dot{y} = v = \frac{dy}{dt} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt}$$

$$v = \omega\left[\frac{h}{\theta_{ri}} - \frac{h}{\pi}\cdot\frac{1}{2}\,\frac{2\pi}{\theta_{ri}}\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

$$v = \frac{h.\omega}{\theta_{ri}}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

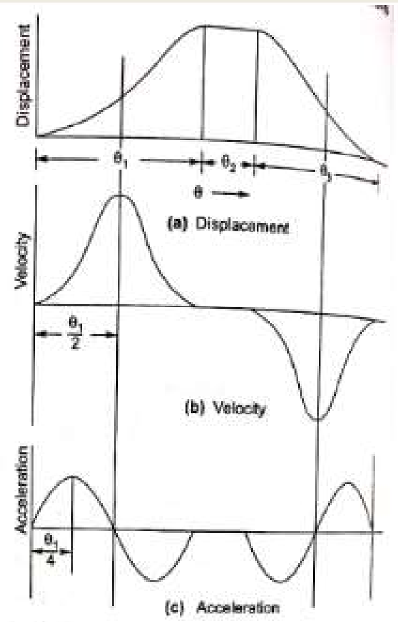
$$\boxed{v = \frac{h.\omega}{\theta_{ri}}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]}$$

Acceleration of the follower

$$\ddot{y} = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$$

$$a = \omega\left[\frac{h\omega}{\theta_{ri}}\cdot\frac{2\pi}{\theta_{ri}}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

$$a = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)$$



$$\boxed{v = \frac{h.\omega}{\theta_{ri}}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]}$$

For v_{max}:

$$\frac{dv}{d\theta} = 0 \Rightarrow \frac{h\omega}{\theta_{ri}} \cdot \frac{2\pi}{\theta_{ri}} \sin\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0$$

$$\sin\frac{2\pi\theta}{\theta_{ri}} = \sin\pi$$

$$\therefore \frac{2\pi\theta}{\theta_{ri}} = \pi \Rightarrow \theta = \theta_{ri}/2$$

$$\therefore v_{max} = \frac{h.\omega}{\theta_{ri}}\left[1 - \cos\left(\frac{2\pi}{\theta_{ri}} \cdot \frac{\theta_{ri}}{2}\right)\right]$$

$$v_{max} = \frac{2h.\omega}{\theta_{ri}}$$

For a_{max}:

$$\boxed{a = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2 \sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)}$$

$$\frac{da}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2 \sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right] = 0$$

$$\left[2h\pi \cdot \left(\frac{\omega}{\theta_{ri}}\right)^2 \frac{2\pi}{\theta_{ri}}\right] \cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0$$

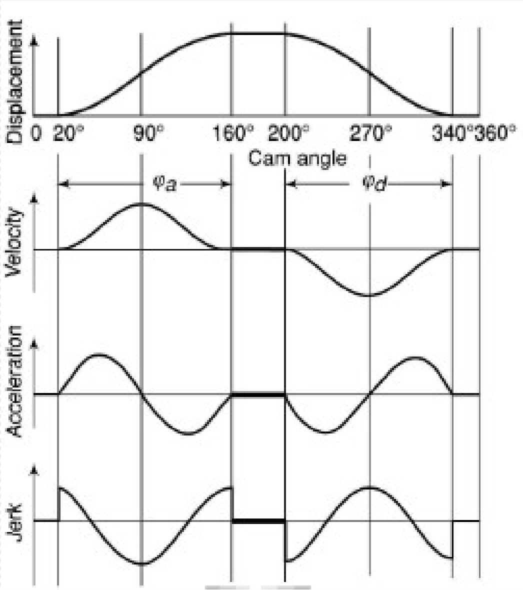
$$\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0 \quad \text{as} \left[2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2 \frac{2\pi}{\theta_{ri}}\right] \neq 0$$

$$\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = \cos\pi/2$$

$$\therefore \theta = \frac{\theta_{ri}}{4}$$

$$\therefore a_{max} = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2 \sin\left(\frac{2\pi\theta_{ri}}{\theta_{ri} \cdot 4}\right)$$

$$a_{max} = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2.$$



- It can be observed that there are no abrupt changes in the velocity & the acceleration at any stage of the motion.
- It is most suitable for high speed follower motion

Problem # 2

It is required to set out the profile of a cam to give motion to the follower in such a way that it rises through 31.4 mm during 180 degree of cam rotation with cycloidal motion & returns with cycloidal motion during 180 degree of cam rotation. Determine the maximum velocity & acceleration of the follower during the outstroke when the cam rotates at 1800 rpm clockwise.

Given Data:

Lift (h) = 31.4 mm

Angle of ascent (θ_{ri}) = 180 degree =180π/180 rad=π rad

RPM of cam (N) = 1800 rpm

Angular velocity of cam (ω) = $2\pi N/60$ rad/s

= $2\pi \times 1800/60$ rad/s

=188.5 rad/s

Mathematically, a cycloid is expressed by

$$y = \frac{h}{\pi}\left[\frac{\pi\theta}{\theta_{ri}} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

Velocity of the follower

$$\dot{y} = v = \frac{dy}{dt} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt}$$

$$v = \omega\left[\frac{h}{\theta_{ri}} - \frac{h}{\pi}\cdot\frac{1}{2}\,\frac{2\pi}{\theta_{ri}}\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

$$v = \frac{h.\omega}{\theta_{ri}}\left[1-\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

Acceleration of the follower

$$\ddot{y} = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$$

$$a = \omega\left[\frac{h\omega}{\theta_{ri}}\cdot\frac{2\pi}{\theta_{ri}}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

$$a = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)$$

For v_{max}:

$$\frac{dv}{d\theta} = 0 \Rightarrow \frac{h\omega}{\theta_{ri}}\cdot\frac{2\pi}{\theta_{ri}}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0$$

$$\sin\frac{2\pi\theta}{\theta_{ri}} = \sin\pi$$

$$\therefore \frac{2\pi\theta}{\theta_{ri}} = \pi \Rightarrow \theta = \theta_{ri}/2$$

$$\therefore v_{max} = \frac{h.\omega}{\theta_{ri}}\left[1-\cos\left(\frac{2\pi}{\theta_{ri}}\cdot\frac{\theta_{ri}}{2}\right)\right]$$

$$v_{max} = \frac{2h\omega}{\theta_{ri}}$$

Max. velocity during outstroke (v_{max}) = $2h\omega/\theta_{ri}$ mm/s
= $(2\times31.4\times188.5)/\pi$ mm/s
=3770 mm/s

Acceleration of the follower

$$\ddot{y} = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$$

$$a = \omega\left[\frac{h\omega}{\theta_{ri}}\cdot\frac{2\pi}{\theta_{ri}}\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right]$$

$$a = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)$$

For a_{max}:

$$\frac{da}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\sin\left(\frac{2\pi\theta}{\theta_{ri}}\right)\right] = 0$$

$$\left[2h\pi\cdot\left(\frac{\omega}{\theta_{ri}}\right)^2\frac{2\pi}{\theta_{ri}}\right]\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0$$

$$\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = 0 \quad \text{as} \left[2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\frac{2\pi}{\theta_{ri}}\right] \neq 0$$

$$\cos\left(\frac{2\pi\theta}{\theta_{ri}}\right) = \cos\pi/2$$

$$\therefore \theta = \frac{\theta_{ri}}{4}$$

$$\therefore a_{max} = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2\sin\left(\frac{2\pi\,\theta_{ri}}{\theta_{ri}\cdot 4}\right)$$

$$a_{max} = 2h\pi\left(\frac{\omega}{\theta_{ri}}\right)^2.$$

Max. Accel. during ascent (a_{max}) =$(2h\pi)\times(\omega/\theta_{ri})^2$ mm/s^2
= $(2\times31.4\times\pi)\times(188.5/\pi)^2$ mm/s^2
=710645 mm/s^2

Problem # 3

A cam with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below:

To move outwards through 40 mm during 100° rotation of the cam,
To dwell for next 80°,
To return to its starting position during next 100°,
To dwell for the rest period of a revolution i.e. 80°,

The displacement of the follower is to take place with uniform acceleration & uniform retardation. Determine the maximum velocity and acceleration of the follower during ascent when the cam shaft rotates at 850 RPM.

Problem # 4

A cam is to be designed for a knife edge follower with the following data. Cam lift is 40 mm during 90° of cam rotation with Simple harmonic Motion (SHM), dwell for the next 50°, during the next 60° of cam rotation, the follower return to its original position with SHM, next dwell during the remaining 160°. The radius of the base circle of the cam is 40 mm. The cam rotates at 250 rpm. Determine the maximum velocity and acceleration of the follower during ascent and descent.

1. The pitch point on a cam is
(a) any point on the cam profile
(b) any point on base circle
(c) the point on pitch curve with maximum pressure angle
(d) a point at a distance equal to pitch circle radius from the centre

2. The jerk in cam motion is
(a) rate of change of displacement
(b) rate of change of velocity
(c) rate of change of acceleration
(d) rate of change of pressure angle

3. Follower motion best suitable for high speed application
(a) SHM (b) cycloidal
(c) parabolic (d) depends on the data

4. Which of the following is constant acceleration cam?
(a) polynomial (b) circular arc
(c) cycloidal (d) parabolic

6. The locus of the trace point if the follower is moved around the cam is known as
(a) prime circle (b) pitch curve
(c) base circle (d) cam circle

7. The size of the cam does not depend on
(a) pitch circle
(b) base circle
(c) pressure angle
(d) radius of curvature of cam profile

8. In its simplest or equivalent form, a cam mechanism consists of following number of links
(a) 4 (b) 3
(c) 2 (d) 1