Question:

Engineering Heat & Mass Transfer [EXP-717] (https://holooly.com/sources/engineering-heat-mass-transfer-exp-717/)

A surface emits as a blackbody at 1500 K. Calculate the rate of emission per unit area, if radiation corresponds to 0° ≤ θ ≤ 60° and wavelength interval 2 μm ≤ λ ≤ 4 μm.

Step-by-Step

Verified Answer

Given : Black body emission

$T_s = 1500K$

$\lambda_1 = 2\mu m, \lambda_2 = 4\mu m$

$\varphi_1 = 0^\circ$ and$\varphi_2 = 60^\circ = \frac{\pi}{3}$

To find : Rate of emission per unit area.

Analysis : The emission from the black surface may be obtained by using eqn. (12.50)

within limits

$\lambda_1 = 2\mu m \ \ to \ \ \lambda_2 = 4\mu m, \theta = 0^\circ \ \ to \ \ 2\pi, and\varphi = 0 \ \ to \ \ \pi/3.$

$\Delta E_b = \int_{2\mu m}^{4\mu m} \int_0^{2\pi} \int_0^{\pi/3} I_{b\lambda}$cos θ sin θ dθ dφ dλ

For diffuse black body

$\Delta E_b = \int_{2\mu m}^{4\mu m} I_{b\lambda} d\lambda \int_0^{2\pi} \int_0^{\pi/3}$cos θ sin θ dθ dφ

$= \int_{2\mu m}^{4\mu m} I_{b\lambda} d\lambda \left[2\pi \frac{sin^2\theta}{2}\right]_0^{\pi/3}$

$$= 0.75\int_{2\mu m}^{4\mu m}\pi I_{b\lambda}d\lambda$$

We know $E_{b\lambda} = \pi I_{b\lambda}$. Thus

$$\Delta E_b = 0.75\int_{2\mu m}^{4\mu m} E_{b\lambda}d\lambda$$

$$= 0.75E_b\int_{2\mu m}^{4\mu m}\frac{E_{b\lambda}d\lambda}{E_b}$$

$$= 0.75E_b(f_{0\text{–}4}\text{–}f_{0\text{–}2})$$

From Table 12.2.

$$\lambda_1 T = 2\mu m \times 1500K$$

$$= 3000\mu mK \rightarrow f_{0\text{–}2} = 0.273$$

$$\lambda_2 T = 4\mu m \times 1500K$$

$$= 6000\mu mK \rightarrow f_{0\text{–}4} = 0.738$$

$$\Delta E_b = 0.75 \times 5.67 \times 10^{-8} \times (1500)^4 \times (0.738\text{–}0.273)$$

$$= 100.16 \times 10^3 W/m^2.$$

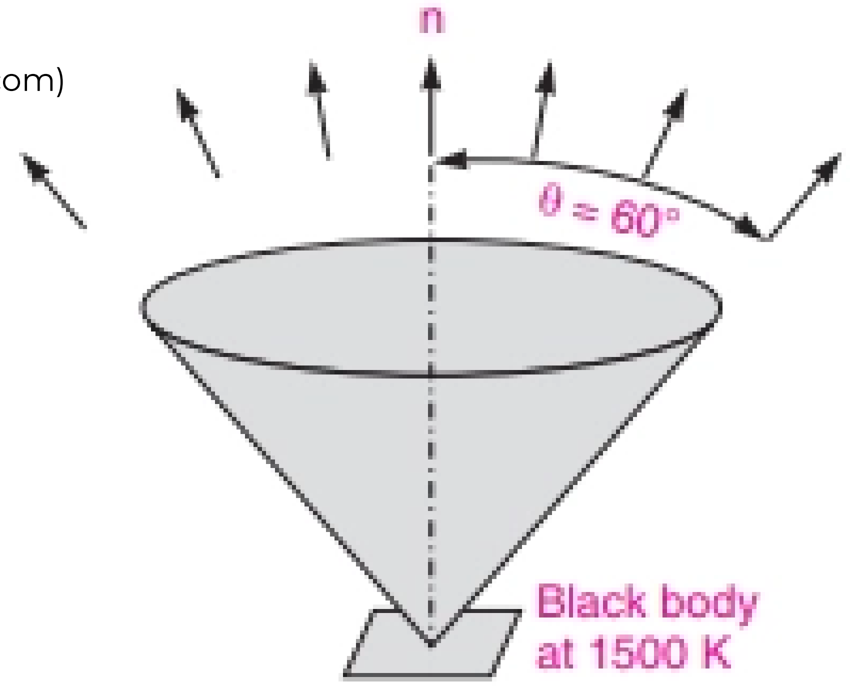

TABLE 12.2. Black body radiation functions

λT (μm K)	$f_{0-\lambda}$	λT (μm K)	$f_{0-\lambda}$	λT (μm K)	$f_{0-\lambda}$
100	0.000000	4800	0.607589	9500	0.903124
200	0.000000	4900	0.620937	9600	0.905490
300	0.000000	5000	0.633777	9700	0.907782
400	0.000000	5100	0.646127	9800	0.910002
500	0.000000	5200	0.658001	9900	0.912153
600	0.000000	5300	0.669417	10000	0.914238
700	0.000002	5400	0.680392	11000	0.931929
800	0.000016	5500	0.690940	12000	0.945138
900	0.000087	5600	0.701079	13000	0.955179
1000	0.000321	5700	0.710824	14000	0.962938
1100	0.000911	5800	0.720192	15000	0.969021
1200	0.002134	5900	0.729196	16000	0.973855
1300	0.004317	6000	0.737852	17000	0.977741
1400	0.007791	6100	0.746173	18000	0.980901
1500	0.012850	6200	[illegible]	19000	0.983494
1600	0.019720	6300	0.761869	20000	0.985643
1700	0.028535	6400	0.769268	21000	0.987437
1800	0.039344	6500	0.776386	22000	0.988947
1900	0.053111	6600	[illegible]	23000	[illegible]

1900	0.052111	6600	0.783234	23000	0.990227
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2100	0.083058	6800	0.796164	25000	0.992256
2200	0.100895	6900	0.802268	26000	0.993064
2300	0.120037	7000	0.808144	27000	0.993765
2400	0.140266	7100	0.813803	28000	0.994376
2500	0.161366	7200	0.819253	29000	0.994911
2600	0.183132	7300	0.824504	30000	0.995381
2700	0.205370	7400	0.829563	35000	0.997044
2800	0.227904	7500	0.834439	40000	0.998008
2900	0.250577	7600	0.839139	45000	0.998605