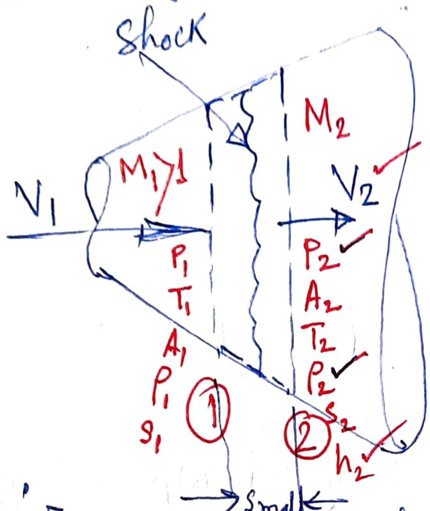
Standing Normal Shock :-

Normal shock: Normal shock is an abrupt discontinuity in 1-d Supersonic ($M > 1$) flow ~~and~~ where the wave front, created by compression process, is perpendicular to the direction of flow.

It is characterised by abrupt changes in flow properties such as pressure, temperature, density, velocity, entropy etc. across the 'shock wave'



Assumptions :-

- steady 1-d flow
- Adiabatic across the shock ; - $\delta q = 0$; $ds_e = 0$
- No shaft work , $\delta w_s = 0$
- constant area , $A_1 = A_2 = A$
- No potential $dz = 0$
- No wall shear

Working equation :-

Applying conservation equation on cv shown :-

① Continuity : $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\boxed{\rho_1 V_1 = \rho_2 V_2} \quad (\because A_1 = A_2) \text{ ——— } ①$$

② Energy : $h_{t1} + \cancel{q}^{0} = h_{t2} + \cancel{w_s}^{0}$

$$\therefore h_{t1} = h_{t2}$$

$$\text{or} \quad h_1 + \frac{V_1^2}{2} + \cancel{gz_1}^{0} = h_2 + \frac{V_2^2}{2} + \cancel{gz_2}^{0}$$

$$\therefore \boxed{h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}} \text{ - - - } ②$$

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Momentum :- $\sum F_x = \dot{m}\left(V_{out\,x} - V_{in\,x}\right)$.

$$\sum F_x = P_1A_1 - P_2A_2 + \cancel{\frac{P_1+P_2}{2}\left(A_2 - A_1\right)} = \left(P_1 - P_2\right)A$$

$$\therefore \left(P_1 - P_2\right)A = \rho A V\left(V_2 - V_1\right)$$

$$P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2$$

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2} \quad \text{--- (3)}$$

Therefore, equ^n (1), (2) & (3) are the three ~~governing~~ equations governing the standing Normal shock.

Generally, P_2, ρ_2, h_2 & V_2 are the four unknowns

No. of equations $= 3$

Another equ^n, equat^n of state $\Rightarrow$ $P = \rho R T$ is used.

Working Equations :-

1. Continuity $\Rightarrow \rho_1 V_1 = \rho_2 V_2 \Rightarrow \boxed{\dfrac{P_1 M_1}{P_2 M_2} = \sqrt{\dfrac{T_1}{T_2}}}$

2. Energy equ^n $\Rightarrow h_{t1} = h_{t2} \Rightarrow T_{t1} = T_{t2}$

From $\Rightarrow T_t = T\left(1 + \frac{\gamma-1}{2}M^2\right)$

we have $\boxed{\dfrac{T_1}{T_2} = \dfrac{1 + \frac{\gamma-1}{2}M_2^2}{1 + \frac{\gamma-1}{2}M_1^2}}$

3. Momentum

$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \Rightarrow P_1 + \left(\dfrac{P_1}{RT_1}\right)\left(M_1^2 \gamma R T_1\right) = P_2 + \left(\dfrac{P_2}{RT_2}\right)\cdot\left(M_2^2 \gamma R T_2\right)$ ρ_2 V_2^2

$$\therefore \boxed{P_1\left(1 + \gamma M_1^2\right) = P_2\left(1 + \gamma M_2^2\right)}.$$

putting values of P_1/P_2 & $\sqrt{\frac{T_1}{T_2}}$ in continuity equ^n

we get, $\left(\dfrac{1+\gamma M_2^2}{1+\gamma M_1^2}\right)\dfrac{M_1}{M_2} = \left(\dfrac{1 + ((\gamma-1)/2)M_2^2}{1 + ((\gamma-1)/2)M_1^2}\right)^{1/2}$

$M_2 = f(M_1, \gamma)$, for $M_1 = M_2$ solution is trivial.

$AM_2^4 + BM_2^2 + C = 0 \Rightarrow$

$$\left(\frac{1+\gamma M_2^2}{1+\gamma M_1^2}\right)^2 \frac{M_1^2}{M_2^2} = \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

$$\text{or, } \left(1+2\gamma M_2^2+\gamma^2 M_2^4\right) M_1^2 \left(1+\frac{\gamma-1}{2}M_1^2\right) = \left(1+2\gamma M_1^2+\gamma^2 M_1^4\right) M_2^2 \left(1+\frac{\gamma-1}{2}M_2^2\right)$$

$$\text{or, } \left(M_1^2 + \cancel{2\gamma M_1^2 M_2^2} + \gamma^2 M_2^4 M_1^2\right) + \frac{\gamma-1}{2}M_1^4 + \frac{2\gamma(\gamma-1)}{2}M_1^4 M_2^2 + \cancel{\frac{\gamma^2(\gamma-1)}{2}M_2^4 M_1^4}$$
$$- M_2^2 - \cancel{2\gamma M_1^2 M_2^2} - \gamma^2 M_1^4 M_2^2 - \frac{\gamma-1}{2}M_2^4 - \frac{\gamma-1}{2}\cdot 2\gamma M_1^2 M_2^4 - \cancel{\gamma^2\frac{(\gamma-1)}{2}M_1^4 M_2^4} = 0$$

$$\left(M_1^2 - M_2^2\right) + \gamma^2 M_1^2 M_2^2\left(M_2^2 - M_1^2\right) + \frac{\gamma-1}{2}\left(M_1^2 - M_2^2\right)\left(M_1^2 + M_2^2\right) + \gamma(\gamma-1)M_1^2 M_2^2\left(M_1^2 - M_2^2\right) = 0$$

$$\left(M_2^2 - M_1^2\right)\left(-1 + \gamma^2 M_1^2 M_2^2 - \frac{\gamma-1}{2}\left(M_1^2+M_2^2\right) - \gamma(\gamma-1)M_1^2 M_2^2\right) =$$

$$\therefore M_2^2 = M_1^2 \Rightarrow M_2 = M_1$$

$$\text{or, } -\frac{\gamma-1}{2}\left(M_1^2+M_2^2\right) + \gamma^2 M_1^2 M_2^2 - \gamma(\gamma-1)M_1^2 M_2^2 = 1$$

$$\text{or, } -\frac{\gamma-1}{2}M_1^2 - \frac{\gamma-1}{2}M_2^2 + M_1^2 M_2^2\left(\cancel{\gamma^2} - \cancel{\gamma^2} + \gamma\right) = 1$$

$$\text{or, } -\frac{\gamma-1}{2}M_1^2 - \frac{\gamma-1}{2}M_2^2 + \gamma M_1^2 M_2^2 = 1$$

$$\text{or, } -\frac{\gamma-1}{2}M_2^2 + \gamma M_1^2 M_2^2 = 1 + \frac{\gamma-1}{2}M_1^2$$

$$\text{or, } M_2^2\left(-\frac{\gamma-1}{2} + \gamma M_1^2\right) = 1 + \frac{\gamma-1}{2}M_1^2$$

$$\therefore M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{-\frac{\gamma-1}{2} + \gamma M_1^2}$$

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{-\frac{\gamma-1}{2}+\gamma M_1^2} =$$

$$= \frac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}$$

$$\therefore M_2^2 = \frac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1} = 1-\frac{\gamma+1}{2}\cdot\frac{\overset{>0 \text{ for } M_1>1}{M_1^2-1}}{M_1^2-1+\frac{\gamma+1}{2\gamma}}$$

$\therefore$ M_2 always less unity for $M_1>1$.

$$\frac{s_2-s_1}{R} = \frac{1}{\gamma-1}\ln\left(1+\frac{2\gamma}{\gamma+1}\overset{>0}{(M_1^2-1)}\right)+\frac{\gamma}{\gamma-1}\ln\left(1\overset{+?}{-}\frac{2}{\gamma+1}\overset{>0}{\left(1-\frac{1}{M_1^2}\right)}\right)$$

$s_2>s_1$ for $M_1>1$ & $s_2<s_1$ for $M_1<1$.

or, $\underline{s_2-s_1} = R\ln\frac{P_{t1}}{P_{t2}}$

stronger the shock $\left(\frac{P_2-P_1}{P_1}\right)$ or more initial Mach No. the greater will be loss of stagnation pressure.

SNS-3

Solving we ~~get~~ get :-

$M_2 = M_1$ & $\boxed{M_2 = \left(\frac{M_1^2 + 2/(\gamma-1)}{[2\gamma/(\gamma-1)]M_1^2 - 1}\right)^{1/2}} \Rightarrow M_2 = f(M_1, \gamma)$

Implies no shock

For shock.

Putting M_2 in terms of M_1 we can have all other quantities:-

$$\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} = \frac{1+\gamma M_1^2}{1+\gamma \frac{M_1^2 + \frac{2}{\gamma-1}}{2\gamma(\gamma-1)M_1^2 - 1}} = \frac{(1+\gamma M_1^2)\left(\frac{2\gamma}{\gamma-1}M_1^2 - 1\right)}{\frac{2\gamma}{\gamma-1}M_1^2 - 1 + \gamma M_1^2 + \frac{2\gamma}{\gamma-1}} \checkmark$$

$$= \boxed{\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}}.$$

$$\frac{(\ )(\ )}{\frac{2\gamma+\gamma^2-\gamma}{\gamma-1}M_1^2 + \frac{2\gamma-\gamma+1}{\gamma-1}} = \frac{(\ )(\ )}{\frac{\gamma(\gamma+1)}{\gamma-1}M_1^2 + \frac{\gamma+1}{\gamma-1}} = \frac{(\ )(\ )}{\gamma M_1^2 + \frac{\gamma+1}{\gamma-1}}$$

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

$$\therefore \boxed{\frac{T_2}{T_1} = \frac{2(\gamma-1)}{(\gamma+1)^2 M_1^2}\left\{1+\left[(\gamma-1)/2\right]M_1^2\right\}\left\{\left[\frac{2\gamma}{\gamma-1}\right]M_1^2 - 1\right\}}.$$

$\frac{\rho_2}{\rho_1}$ = combining equ^n of $\frac{P_2}{P_1}$ & $\frac{T_2}{T_1}$ we get equ^n of $\frac{\rho_2}{\rho_1}$

$$P = \rho R T \Rightarrow \frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1}$$

$$\therefore \boxed{\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}}$$

$$\frac{T_{t2}}{T_{t1}} = 1 \quad (\text{adiabatic steady}).$$

$$\frac{P_{t2}}{P_{t1}} = \underbrace{\left(\frac{P_{t2}}{P_2}\right)}_{\text{isentropic}} \cdot \underbrace{\left(\frac{P_1}{P_{t1}}\right)}_{\text{isentropic}} \times \underbrace{\left(\frac{P_2}{P_1}\right)}_{\text{Normal shock}} = \left(\frac{P_{t2}}{P_2}\right) \times \left(\frac{P_1}{P_{t1}}\right) \times \frac{P_2}{P_1}$$

$$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1}\left(\frac{1+\left(\frac{\gamma-1}{2}\right)M_2^2}{1+\left((\gamma-1)/2\right)M_1^2}\right)^{\gamma/\gamma-1} =$$

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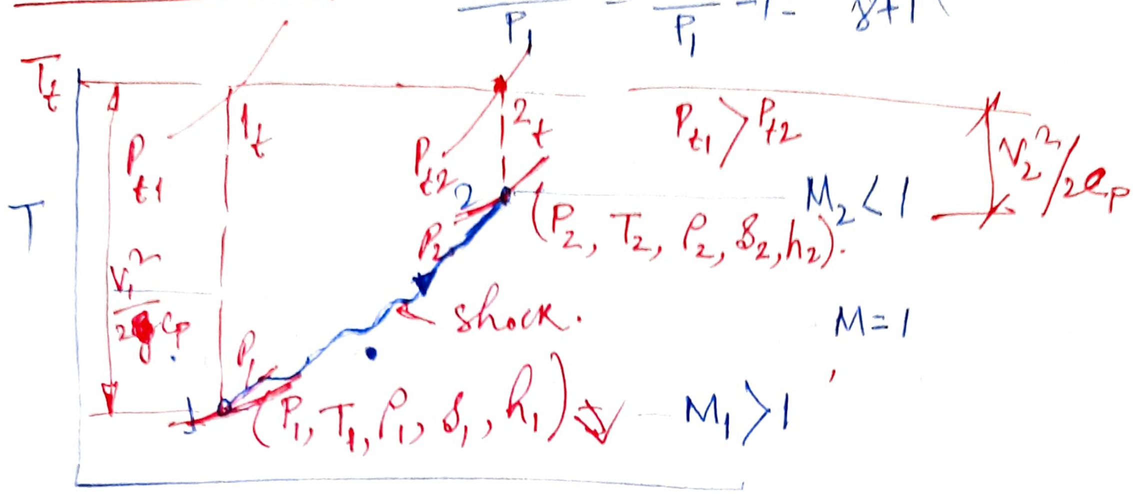
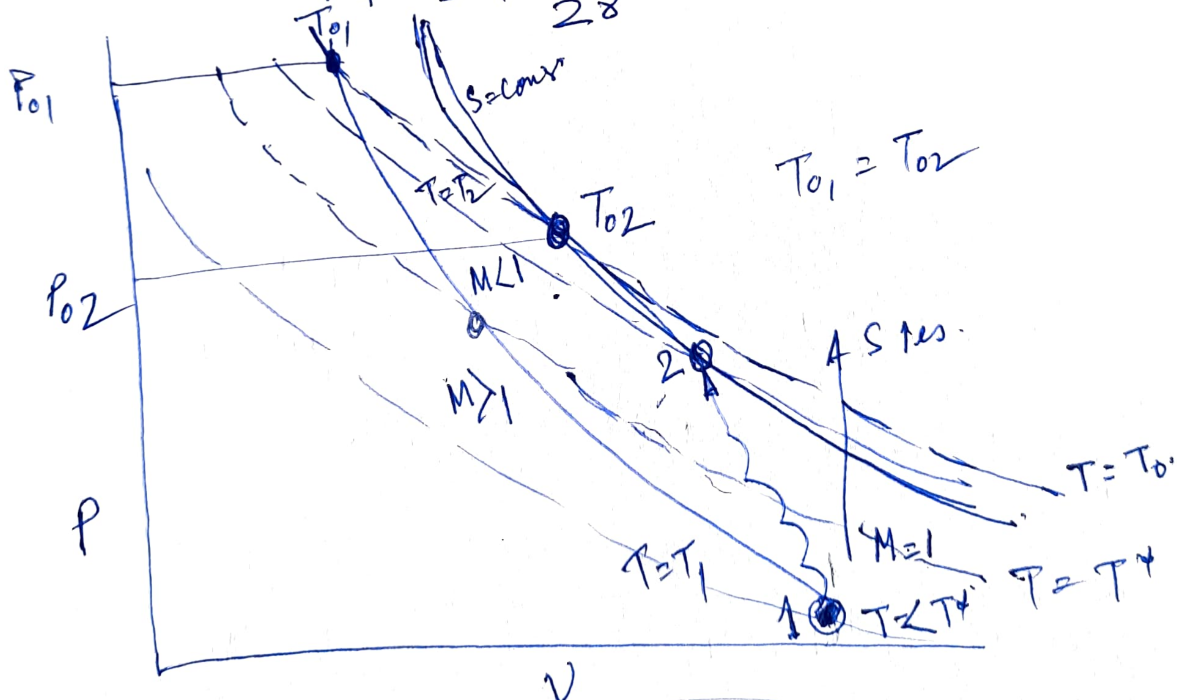
$$\frac{P_{t2}}{P_{t1}} = \left(\frac{[(\gamma+1)/2]M_1^2}{1+[(\gamma-1)/2]M_1^2}\right)^{\gamma/\gamma-1}\left[\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right]^{1/(\gamma-1)} = f(M_1,\gamma)$$

Shock strength :- $\frac{P_2-P_1}{P_1} = \frac{P_2}{P_1} - 1 = \frac{2\gamma}{\gamma+1}(M_1^2-1)$.



T-s diagram for normal shock.

$$M_2^2 = 1 - \frac{\gamma+1}{2\gamma}\cdot\frac{M_1^2-1}{M_1^2-1+\frac{\gamma+1}{2\gamma}}.$$



M_1,	M_2,	$\frac{T_2}{T_1}$,	$\frac{P_2}{P_1}$,	$\frac{\rho_2}{\rho_1}$,	$\frac{P_{t2}}{P_{t1}}$,	$\frac{P_{t2}}{P_1}$