Radiation Heat Transfer

Dr. Sayantan Sengupta
Dept. of Mechanical Engineering
NIT Durgapur

Lecture 6 Radiation exchange between opaque, diffuse, gray surfaces in an enclosure

Schematic diagram

In general, radiation may leave an opaque surface due to both reflection and emission, and on reaching a second opaque surface, experience reflection as well as absorption. In an enclosure, such as that of Figure 13.9*a*, radiation may experience multiple reflections off all surfaces, with partial absorption occurring at each.

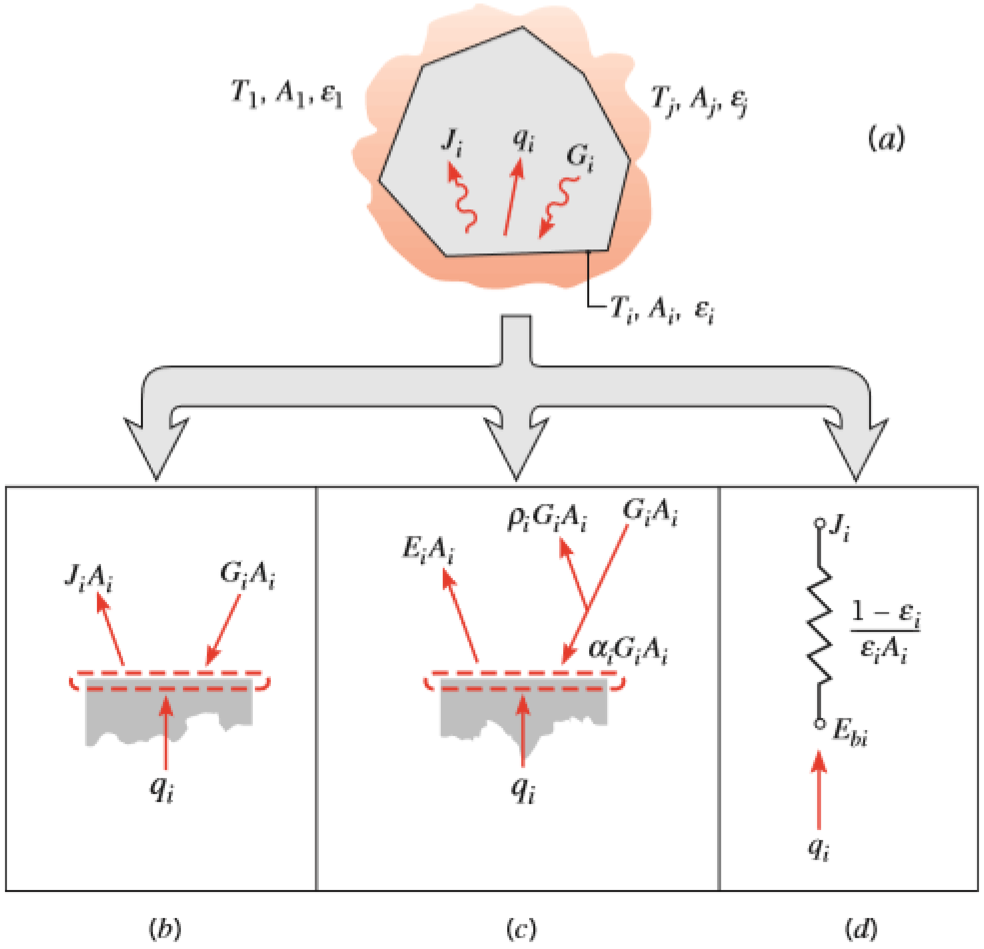


FIGURE 13.9 Radiation exchange in an enclosure of diffuse, gray surfaces with a nonparticipating medium. (*a*) Schematic of the enclosure. (*b*) Radiative balance according to Equation 13.15. (*c*) Radiative balance according to Equation 13.17. (*d*) Resistance representing net radiation transfer from a surface.

Assumptions

Analyzing radiation exchange in an enclosure may be simplified by making certain assumptions. Each surface of the enclosure is assumed to be *isothermal* and to be characterized by a *uniform radiosity* and a *uniform irradiation*. The surfaces are also assumed to be *opaque* ($\tau = 0$) and to have emissivities, absorptivities, and reflectivities that are independent of direction (the surfaces are *diffuse*) *and* independent of wavelength (the surfaces are *gray*). It was shown that under these conditions the emissivity is equal to the absorptivity, $\varepsilon = \alpha$ (a form of Kirchhoff's law). Finally, the medium within the enclosure is taken to be *nonparticipating*. The problem is generally one in which either the temperature T_i or the net radiative heat flux q''_i associated with each of the surfaces is known. The objective is to use this information to determine the unknown radiative heat fluxes and temperatures associated with each of the surfaces.

Net Radiation Exchange at a Surface

The term q_i, which is the *net* rate at which radiation *leaves* surface i, represents the net effect of radiative interactions occurring at the surface (Figure 13.9*b*). It is the rate at which energy would have to be transferred to the surface by other means to maintain it at a constant temperature. It is equal to the difference between the surface radiosity and irradiation and may be expressed as

$$q_i = A_i(J_i - G_i)$$

Using the definition of the radiosity, $J_i \equiv E_i + \rho_i G_i$

the net radiative transfer from the surface may be expressed as $q_i = A_i(E_i - \alpha_i G_i)$ where use has been made of the relationship $\alpha_i = 1 - \rho_i$ for an opaque surface.

Noting that $E_i = \varepsilon_i E_{bi}$ and recognizing that $\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$ for an opaque, diffuse, gray surface, the radiosity may also be expressed as $J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i$

Solving for G_i → $q_i = A_i\left(J_i - \dfrac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right)$ or $q_i = \dfrac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i}$

the net radiative heat transfer is associated with the driving potential $(E_{bi} - J_i)$ and a *surface radiative resistance* of the form $(1 - \varepsilon_i)/\varepsilon_i A_i$.

Reflections

(1) Hence, if the emissive power that the surface would have if it were black exceeds its radiosity, there is net radiation heat transfer from the surface; if the inverse is true, the net transfer is to the surface.

(2) It is sometimes the case that one of the surfaces is very large relative to the other surfaces under consideration. For example, the system might consist of multiple small surfaces in a large room. In this case, the area of the large surface is effectively infinite ($A_i \to \infty$), and we see that its surface radiative resistance, $(1 - \varepsilon_i)/\varepsilon_i A_i$, is effectively zero, just as it would be for a black surface ($\varepsilon_i = 1$). Hence, $J_i = E_{bi}$, and *a surface which is large relative to all other surfaces under consideration can be treated as if it were a blackbody.*

Radiation Exchange Between Surfaces

The irradiation of surface i can be evaluated from the radiosities of all the surfaces in the enclosure. In particular, from the definition of the view factor, it follows that the total rate at which radiation reaches surface i from all surfaces, including i, is

$$A_i G_i = \sum_{j=1}^{N} F_{ji} A_j J_j$$

or from the reciprocity relation,

$$A_i G_i = \sum_{j=1}^{N} A_i F_{ij} J_j$$

Canceling the area A_i

$$q_i = A_i\left(J_i - \sum_{j=1}^{N} F_{ij} J_j\right)$$

or, from the summation rule,

$$q_i = A_i\left(\sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j\right)$$

Hence

$$q_i = \sum_{j=1}^{N} A_i F_{ij}(J_i - J_j) = \sum_{j=1}^{N} q_{ij}$$

this Equation equates the net rate of radiation transfer from surface i, q_i, to the sum of components q_{ij} related to radiative exchange with the other surfaces. Each component may be represented by a *network element* for which $(J_i - J_j)$ is the driving potential and $(A_i F_{ij})^{-1}$ is a *space* or *geometrical resistance* (Figure 13.10).

Radiation Network Approach

$$\frac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$

As shown in Figure 13.10, this expression represents a radiation balance for the radiosity *node* associated with surface i. The rate of radiation transfer (current flow) to i through its surface resistance must equal the net rate of radiation transfer (current flows) from i to all other surfaces through the corresponding geometrical resistances.

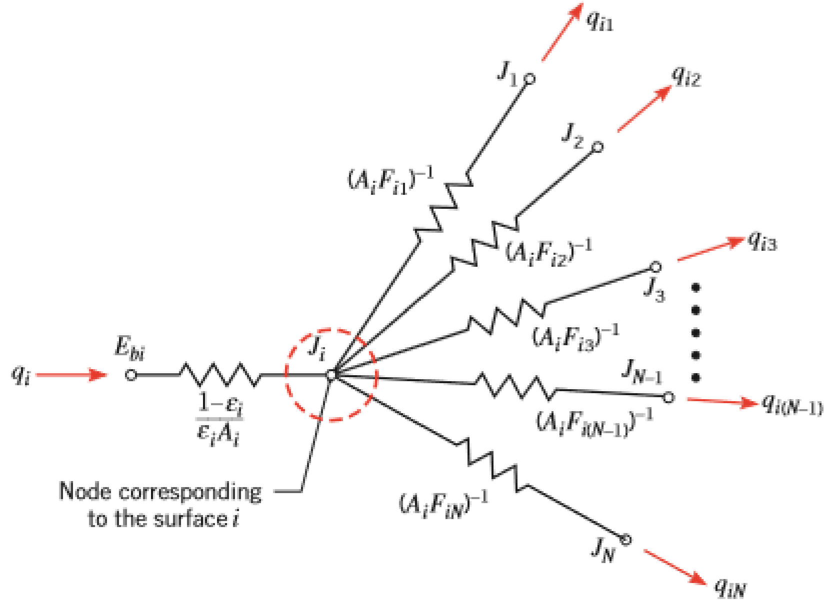


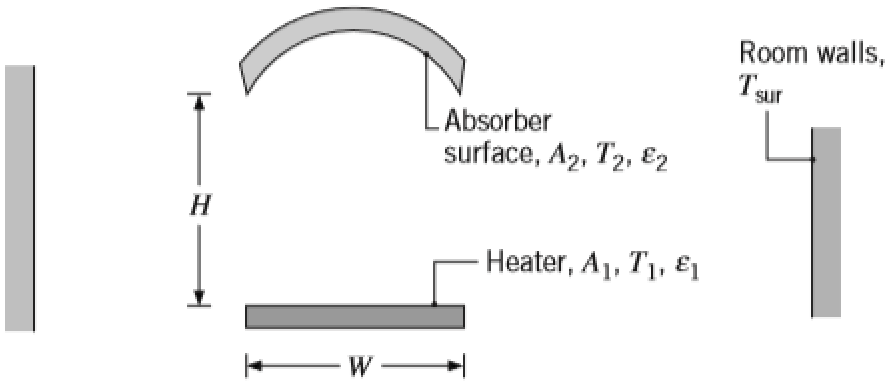
FIGURE 13.10 Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure.

Note that previous Equation is especially useful when the surface temperature T_i (hence E_{bi}) is known. Although this situation is typical, it does not always apply. In particular, situations may arise for which the net radiation transfer rate at the surface q_i, rather than the temperature T_i, is known. In such cases the preferred form of the radiation balance is rearranged as

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$

Assignment 1

In manufacturing, the special coating on a curved solar absorber surface of area $A_2 = 15\ \mathrm{m}^2$ is cured by exposing it to an infrared heater of width $W = 1$ m. The absorber and heater are each of length $L = 10$ m and are separated by a distance of $H = 1$ m. The upper surface of the absorber and the lower surface of the heater are insulated.



The heater is at $T_1 = 1000$ K and has an emissivity of $\varepsilon_1 = 0.9$, while the absorber is at $T_2 = 600$ K and has an emissivity of $\varepsilon_2 = 0.5$. The system is in a large room whose walls are at 300 K. What is the net rate of heat transfer to the absorber surface?

The Two-Surface Enclosure

The simplest example of an enclosure is one involving two surfaces that exchange radiation only with each other. Such a two-surface enclosure is shown schematically in Figure 13.11*a*. Since there are only two surfaces, the net rate of radiation transfer *from* surface 1, q_1, must equal the net rate of radiation transfer *to* surface 2, $-q_2$, and both quantities must equal the net rate at which radiation is exchanged between 1 and 2. Accordingly,

$$q_1 = -q_2 = q_{12}$$

From Figure 13.11*b* we see that the total resistance to radiation exchange between surfaces 1 and 2 is comprised of the two surface resistances and the geometrical resistance. Hence, the net radiation exchange between surfaces may be expressed as

$$q_{12} = q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

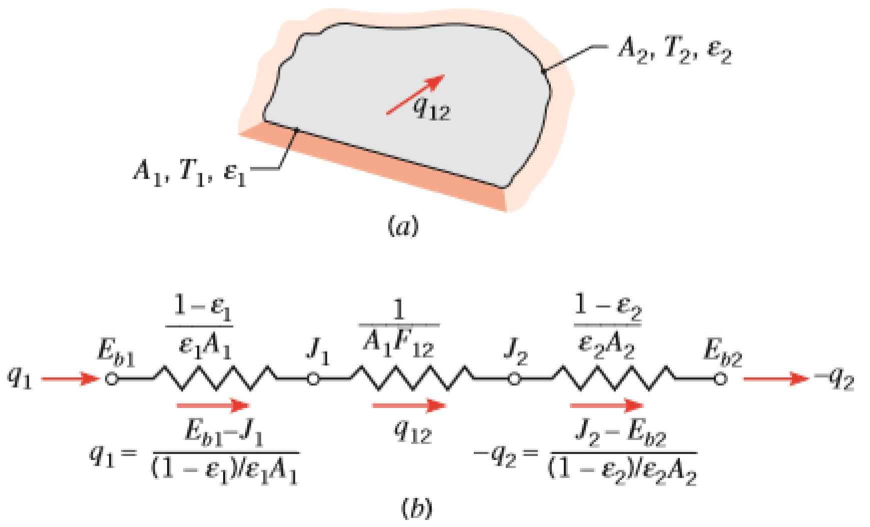
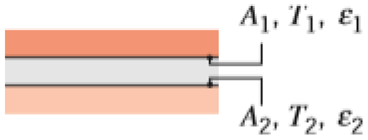


FIGURE 13.11 The two-surface enclosure. (*a*) Schematic. (*b*) Network representation.

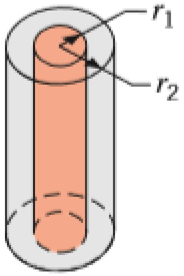
The foregoing result may be used for any two isothermal diffuse, gray surfaces that *form an enclosure* and *are each characterized by uniform radiosity and irradiation.* Important special cases are summarized in Table 13.3.

TABLE 13.3 Special Diffuse, Gray, Two-Surface Enclosures

Large (Infinite) Parallel Planes

	$A_1 = A_2 = A$ $F_{12} = 1$	$q_{12} = \dfrac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$

Long (Infinite) Concentric Cylinders

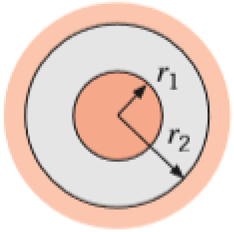


$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

Concentric Spheres

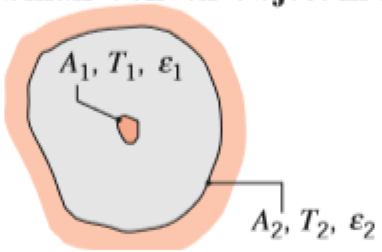


$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

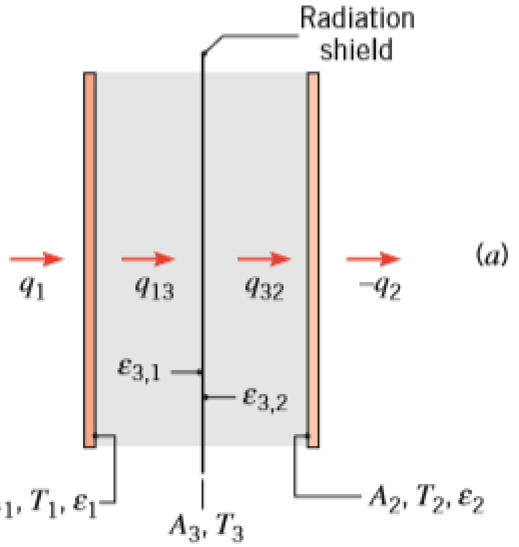
Radiation Shields

Radiation shields constructed from low emissivity (high reflectivity) materials can be used to reduce the net radiation transfer between two surfaces. Consider placing a radiation shield, surface 3, between the two large, parallel planes of Figure 13.12*a*.

with the radiation shield, additional resistances are present, as shown in Figure 13.12*b*, and the heat transfer rate is reduced. Note that the emissivity associated with one side of the shield ($\varepsilon_{3,1}$) may differ from that associated with the opposite side ($\varepsilon_{3,2}$) and the radiosities will always differ. Summing the resistances and recognizing that $F_{13} = F_{32} = 1$, it follows that

$$q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} + \dfrac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}}$$

Note that the resistances associated with the radiation shield become very large when the emissivities $\varepsilon_{3,1}$ and $\varepsilon_{3,2}$ are very small.



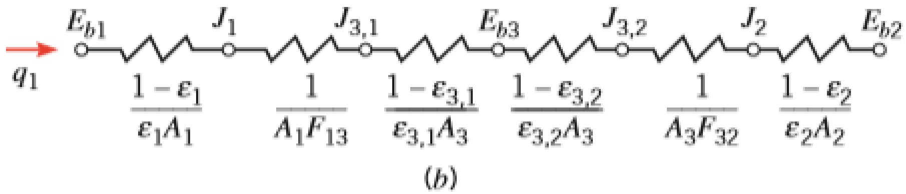


FIGURE 13.12 Radiation exchange between large parallel planes with a radiation shield. (*a*) Schematic. (*b*) Network representation.

The foregoing procedure may readily be extended to problems involving multiple radiation shields. In the special case for which all the emissivities are equal, it may be shown that, with N shields,

$$(q_{12})_N = \frac{1}{N+1}(q_{12})_0$$

where $(q_{12})_0$ is the radiation transfer rate with no shields ($N = 0$).

The Reradiating Surface

The assumption of a *reradiating surface* is common to many industrial applications. This idealized surface is characterized by *zero* net radiation transfer ($q_i = 0$). It is closely approached by real surfaces that are well insulated on one side *and* for which convection effects may be neglected on the opposite (radiating) side. With $q_i = 0$, it follows from

$$q_i = A_i(J_i - G_i) \quad (1) \qquad \text{and} \qquad q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} \quad (2)$$

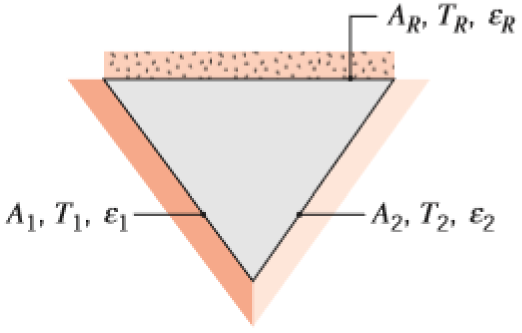
that $G_i = J_i = E_{bi}$. Hence, if the radiosity of a reradiating surface is known, its temperature is readily determined. In an enclosure, the equilibrium temperature of a reradiating surface is determined by its interaction with the other surfaces, and it is *independent of the emissivity of the reradiating surface.*

A three-surface enclosure, for which the third surface, surface R, is reradiating, is shown in Figure 13.13*a*, and the corresponding network is shown in Figure 13.13*b*. Surface R is presumed to be well insulated, and convection effects are assumed to be negligible. Hence, with $q_R = 0$, the net radiation *transfer* from surface 1 must equal the net radiation transfer to surface 2. The network is a simple series–parallel arrangement, and from its analysis it is readily shown that

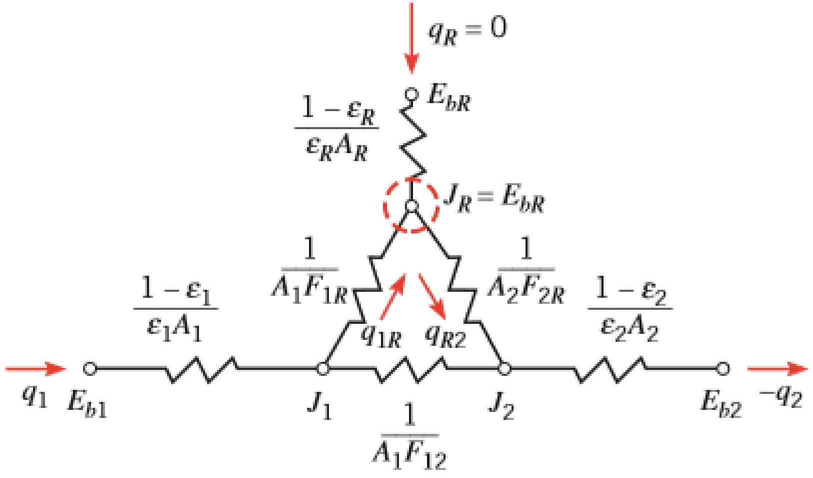
$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \quad (3)$$

Knowing $q_1 = -q_2$, Equation (2) may be applied to surfaces 1 and 2 to determine their radiosities J_1 and J_2. Knowing J_1, J_2, and the geometrical resistances, the radiosity of the reradiating surface J_R may be determined from the radiation balance

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} - \frac{J_R - J_2}{(1/A_2F_{2R})} = 0 \quad (4)$$



(*a*)



(*b*)

FIGURE 13.13 A three-surface enclosure with one surface reradiating. (*a*) Schematic. (*b*) Network representation.

The temperature of the reradiating surface may then be determined from the requirement that $\sigma T_R^4 = J_R$.

Assignment 2

A paint baking oven consists of a long, triangular duct in which a heated surface is maintained at 1200 K and another surface is insulated. Painted panels, which are maintained at 500 K, occupy the third surface. The triangle is of width $W = 1$ m on a side, and the heated and insulated surfaces have an emissivity of 0.8. The emissivity of the panels is 0.4. During steady-state operation, at what rate must energy be supplied to the heated side per unit length of the duct to maintain its temperature at 1200 K? What is the temperature of the insulated surface?

Thank you