National Institute of Technology Durgapur

Department of Mathematics

End-term Examination

Subject: Mathematics-III

Code: MAC331

Full Marks: 60 Time: 2 hours

Symbols have their usual meanings. Brief and to the point answers are preferred. Upload the answer script with filename as **ROLLNUMBER.pdf**. In the front page of the same you must write your name & roll number.

Answer all the questions.

1. The general solution of $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$ is given by

   (a) $F(x^2 + y^2 + z^2, y^2 - 2yz - z^2) = 0$

   (b) $F(x^2 - y^2 + z^2, y^2 + 2yz - z^2) = 0$

   (c) $F(x^2 - y^2 - z^2, y^2 + 2yz + z^2) = 0$

   (d) $F(x^2 - y^2 + 2xy, y^2 - z^2) = 0$ [**2**]

2. Find the value of
$$\int_0^{\pi/2} \sqrt{\sin x}\, dx$$
taking $n = 6$ correct up to four significant figures using trapezoidal rule. [**2**]

3. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{z^{n^2}}{2^n}$. [**2**]

4. The value of the integral $\int_C \frac{e^z}{(z-1)(z+3)^2}\, dz$, where $C : |z| = \frac{3}{2}$ and the integral is taken in the positive sense is

   (a) 0

   (b) $\frac{\pi e i}{8}$

   (c) -$\frac{\pi e i}{8}$

   (d) $\frac{\pi e(e-5e^{-3})}{8}$ [**2**]

5. The PDE $(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$ can be classified as

(a) Elliptic

(b) Parabolic

(c) Hyperbolic

(d) Elliptic if $x < 0, y < 0$, hyperbolic if $x > 0, y > 0$, parabolic if $x = 0 = y$. [**2**]

6. The characteristics of the PDE

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \cos^2 x \frac{\partial^2 z}{\partial x^2} + 2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 0,$$

when it is of hyperbolic type are

(a) $y = x + \cos x + c_1, y = -x + \cos x + c_2$

(b) $y - x + \cos x = c_1, y - x - \cos x = c_2$

(c) $y + x - \cos x = c_1, y + x - \sin x = c_2$

(d) None of these. [**2**]

7. Let $f : \mathbb{R} \to \mathbb{R}$ be given and it has a real root. Can we apply Bisection method to find that real root? Justify. [**2**]

8. What is the appropriate iterative scheme to find the m-th root of the real number R by Newton-Raphson method?

(a) $x_{n+1} = \frac{(m-1)x_n^m + R}{mx_n^{m-1}}$

(b) $x_{n+1} = \frac{mx_n^m + R}{(m+1)x_n^{m-1}}$

(c) $x_{n+1} = \frac{(m-1)x_n^{m-1} + R}{mx_n^m}$

(d) None of these [**2**]

9. A basic feasible solution of the system of equations

$$x + 2z = 1, y + z = 4, x \geq 0, y \geq 0, z \geq 0$$

is

(a) $(1, 4, 0)$

(b) $(0, 4, 1)$

(c) $(4, 1, 1)$

(d) $(0, 0, 4)$ **[2]**

10. The sets X and Y are defined as follows:

$$(i) \quad X = \{(x, y) \in \mathbb{R}^2 : 4x^2 + 9y^2 \geq 36\} \quad (ii) \quad Y = \{(x, y) \in \mathbb{R}^2 : x + 2y = 5\}.$$

Choose the correct alternative.

(a) (i) and (ii) are both convex

(b) both (i) and (ii) are not convex

(c) (i) is non-convex and (ii) is convex

(d) (i) is convex and (ii) is non-convex **[2]**

11. Consider the initial value problem $y' = \frac{x-y}{2}$, $y(0) = 1$. Find $y(0.2)$ with step length $h = 0.1$ using modified Euler method, correct upto 4-decimal places. **[5]**

12. Solve the PDE $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$. **[5]**

13. Solve the initial and boundary value problem:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial t^2}, \quad 0 \leq x \leq 1$$

satisfying (i) $T \to 0$ as $t \to \infty$; (ii) $T(0, t) = 0 = T(1, t)$ for all t; (iii) $T(x, 0) = 2x$ when $0 \leq x \leq 1/2$ and $T(x, 0) = 2(1 - x)$ when $1/2 \leq x \leq 1$. **[5]**

14. Find the nonnegative integer values of n for which the real-valued function $u(x, y) = x^n - y^n$ is harmonic and obtain the corresponding harmonic conjugate functions in each case. **[5]**

15. If $f(z)$ is analytic at ∞ and has a zero of order 2 then find $\text{Res}\,[f(z); \infty]$. **[5]**

16. Evaluate $\displaystyle\int_{|z|=3} \frac{e^{1/(z-1)}}{z - 2} dz$. **[5]**

17. A company owns mines; mine A produces 1 ton of high grade ore, 3 tons of medium grade ore and 5 tons of low grade ore each day; and mine B produces 3 tons of each of the three grades of ore each day. The company needs 80 tones of high grade ore, 160 tones of medium grade ore and 200 tons of low grade ore. If it costs Rs. 200 per day to work each mine, find the number of days each mine has to be operated for producing the required output with minimum total cost. [**5**]

18. Find the real root of $2x - \log_{10} x = 7$ near 3.8 correct up to three decimal places. [**5**]