Integral Calculus

Wednesday, December 22, 2021 8:52 AM

$$\int_a^b f(x)\,dx$$

$f(x) \to$ integrand

(i) $f(x)$ is continuous ✓

(ii) $f(x) \geq 0 \quad \forall x \in [a, b]$ ✓

$a, b \in \mathbb{R}$

$a < b < \infty$

Proper integral

Area ?

$A = ?$

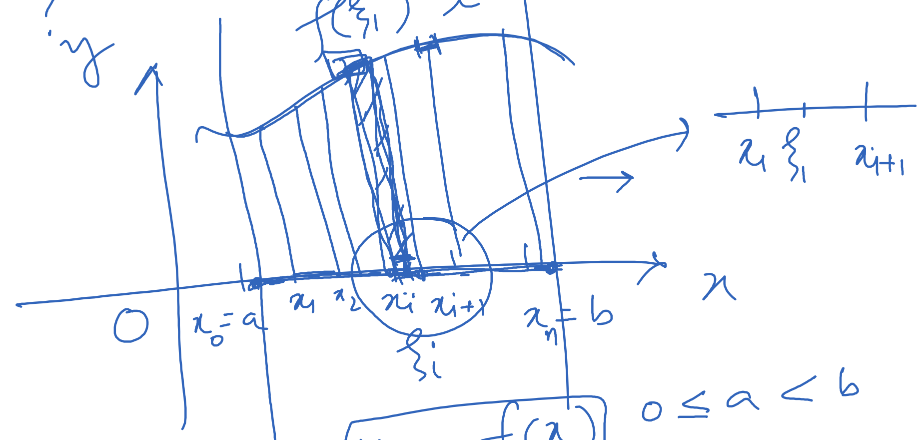
Area

w

$A = lw$

l

$$\int_{x=a}^{b} f(x)\,dx$$



$$\sum_{i=0}^{n-1} \left| (x_{i+1} - x_i) f(\xi_1) \right|$$

$\boxed{y = f(x)}$

$0 \leq a < b$

$b - a$

$f(\xi_i)$

$f(x) \geq 0$

$x \in [a, b]$

$$\lim_{n \to \infty} \sum_{i=0}^{n-1} (x_{i+1} - x_i) f(\xi_i) = \int_a^b f(x)\,dx$$

Improper Integral

on limit

continuity

Depend on limit $(a, \& \ b)$ Type I

$b = \infty$ $a = -\infty$ $a = -\infty$, $b = \infty$

continuity

$f(x)$ Type II

$\underline{a}$ $\underline{b}$ $\underline{a \ \& \ b}$

$$\int_0^1 \frac{dx}{1-x} \qquad f(x) = \frac{1}{1-x} \qquad [0, 1]$$

$= \infty$ $\underline{\text{at } x = 1}$ is not continuous $\underline{\text{at } x = 1}$

$$\int_0^\infty \underline{x\,dx} = ?$$

<u>Type I</u>

$$\int_a^\infty f(x)\,dx = \lim_{B \to \infty} \int_a^B f(x)\,dx$$

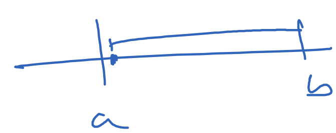
= If limit exists and finite

$$\int_{-\infty}^b f(x)\,dx = \lim_{A \to -\infty} \int_A^b f(x)\,dx$$

$$\underline{\int_{-\infty}^{\infty} f(x)\,dx} = \underline{\int_{-\infty}^{0} f(x)\,dx} + \underline{\int_{0}^{\infty} f(x)\,dx}$$

① $\int_a^b f(x)\,dx$ $f(x)$ is not continuous at $x = b$

$$\lim_{\epsilon \to 0} \int^{b-\epsilon} f(x)\,dx$$



$\lim_{\epsilon \to 0} \int_a f(x)\,dx$ $\quad a \quad b$

(2) $\int_a^b f(x)\,dx \qquad \lim_{\delta \to 0} \int_{a+\delta}^b f(x)\,dx$

(3) $\int_a^b f(x)\,dx$ $\quad$ $f(x)$ is not continuous at $c \in (a,b)$

$\boxed{\infty \;\; \alpha \;\; \infty \;\; \alpha} = \boxed{\int_a^c f(x)\,dx} + \boxed{\int_c^b f(x)\,dx}$

$c_1 < c_2 < c_3 < c_4 < c_5 \in (a,b)$

$$\int_0^\infty \frac{dx}{1+x^2} = \lim_{B \to \infty} \int_0^B \frac{dx}{1+x^2}$$

$$= \lim_{B \to \infty} \left[\tan^{-1} x\right]_0^B$$

$$= \lim_{B \to \infty} \left[\tan^{-1} B - \underbrace{\tan^{-1} 0}_{=0}\right]$$

$$= \lim_{B \to \infty} \tan^{-1} B$$

$$= \pi/2$$

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} f \qquad \underline{\int_{-\infty}^0 \frac{dx}{1+x^2}} + \underline{\int_0^\infty \frac{dx}{1+x^2}}$$

$$= \pi/2 + \pi/2 = \pi \quad \underline{\text{Ans}}.$$

$$\int_0^1 \frac{dx}{1-x} = \lim_{\epsilon \to 0} \int_0^{1-\epsilon} \frac{dx}{1-x}$$

$]^{1-\epsilon}$

$$\int_0 \frac{}{1-x} = \lim_{\epsilon \to 0} \int_0 \frac{}{1-x}$$

divergence

$$= \lim_{\epsilon \to 0} \left[-\log(1-x) \right]_0^{1-\epsilon}$$

$$= \lim_{\epsilon \to 0} \left[-\log \epsilon + 0 \right]$$

$$= \lim_{\epsilon \to 0} \log \frac{1}{\epsilon} \longrightarrow \infty$$