# MEC403 HEAT & MASS TRANSFER

## Topics :

1). Relⁿ b/w thermo. & heat tr.

2). Various Conservation eqⁿ.

3). Newton's law of Cooling.

4). Basic Convective heat tr. Configuration.

5). Concept of Boundary layer.

6). Dimensional analysis & corelation in Convection heat tr.

7). Visualisation of Convection.

8). Entropy generation minimization (EGM) as a general heat tr. obj.

9). Analysis of heat exchangers.

10). Term Projects.

11). Assignments

## Convection :

Latin terms → { Convecto - are ; Convecto - vehere } ⇒ to bring together, to carry into one place.

Vector Calculations : Divergence, Gradient, analytical, scaling analysis. (MATLAB) mass gain by vel $\Rightarrow$ $\boxed{m = \dfrac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$

rest mass

moving mass

$$\Rightarrow \quad m = m_0\left[1-\left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} = m_0\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2+\frac{1}{2}\left(\frac{1}{2}+1\right)\cancel{\left(\frac{v}{c}\right)^4}+\cdots\right] \quad \approx 0$$

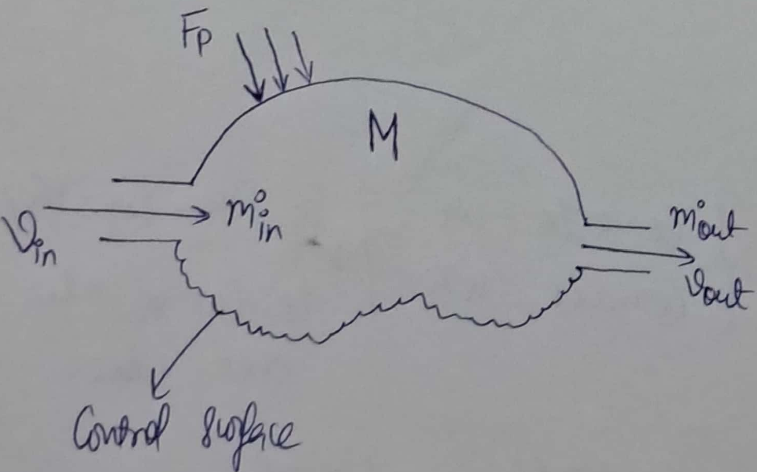
$$\Rightarrow \quad (m-m_0)c^2 = \frac{1}{2}mv^2 \quad \Rightarrow \quad \Delta mc^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad F \propto \frac{d}{dt}(mv)$$

$$\Rightarrow \quad F = K\frac{d}{dt}(mv) = Km\frac{dv}{dt} = Kma$$

## Momentum Theorem :

Picture-book representation $\rightarrow$

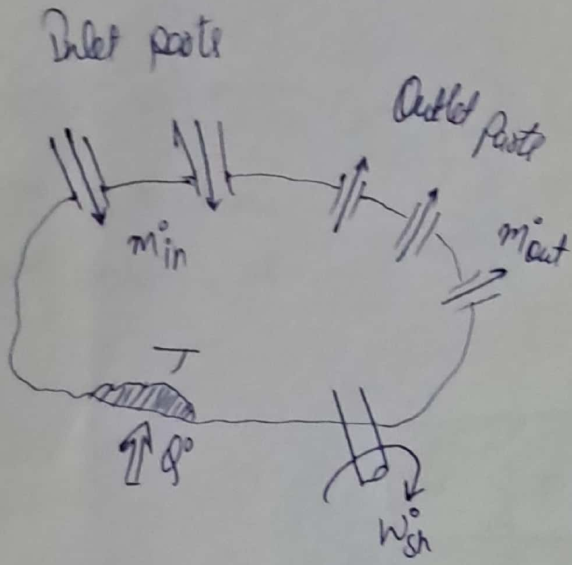



$\Rightarrow$ Transport Eqn :-

$$\frac{\partial}{\partial t}(MV_x)_{cv} = \sum F_x + \sum_{in}(\dot{m}V_x) - \sum_{out}(\dot{m}V_x)$$

$\left(\frac{mv}{m} \rightarrow \text{specific momentum}\right)$

# First law of Thermodynamics & Second law for Open & Control Surface:




$\Rightarrow$ Entropy transfer rate $= \frac{\dot{Q}}{T}$

Momentum Conservation $\rightarrow$

$$\frac{\partial (MV_x)_{cv}}{\partial t} = \sum_{in} (\dot{m}v_x) - \sum_{out} (\dot{m}v_x) + \sum F_x$$

$\rightarrow$ Mass Conservation $\Rightarrow$ $\frac{\partial}{\partial t}(M) = \sum_{in} \dot{m} - \sum_{out} \dot{m}$

$\rightarrow$ $\frac{\partial E}{\partial t} = \sum_{in} (\dot{m}e) - \sum_{out} (\dot{m}e) + \dot{Q} - \dot{W}_{sh}$

$\rightarrow$ $e = h + \frac{1}{2}V^2 + gz$

$\rightarrow$ Stagnation Enthalpy $= h + \frac{1}{2}V^2$

$\rightarrow$ Methalpy $= h + \frac{1}{2}V^2 + gz$

$\rightarrow$ $Q =$ Quantity of heat

$\rightarrow$ $T =$ Quality of heat

$\Rightarrow$ $S = \frac{Q}{T}$ , $W \xrightarrow{100\%} Q$

$Q \xrightarrow{<100\%} W$

$\rightarrow \frac{\partial E}{\partial t} = \sum_{in} \dot{m} e - \sum_{out} \dot{m} e + \dot{Q} - \dot{W}_{sh}$

$\rightarrow \frac{\partial S}{\partial t} \geq \sum_{in} \dot{m} s - \sum_{out} \dot{m} s + \frac{\dot{Q}}{T} - 0 \longrightarrow$ (as 100% conversion of energy no residual of energy)

$>$ Irreversible

$=$ Reversible

$\rightarrow \boxed{\dot{S}_{gen} = \frac{\partial S}{\partial T} - \frac{\dot{Q}}{T} - \sum_{in} \dot{m} s + \sum_{out} \dot{m} s}$

Entropy generation

Degree of Irreversibility

$\rightarrow$ $m =$ Const.

$E_2 - E_1 = \int_1^2 \delta Q - \int_1^2 \delta W$

$\rightarrow S_2 - S_1 \geq \int_1^2 \frac{\partial Q}{\partial t}$

$\rightarrow S_{gen.} = (S_2 - S_1) - \int_1^2 \frac{\delta Q}{\delta t} \geq 0$

<u>Consequence of Entropy</u> <u>(The pr. of fact)</u> :-

$\rightarrow S = \frac{Q}{T}$

$Q \rightarrow$ Physical quantity

$S \rightarrow$ Non-phy. Entity

1) Flow

2). On the Verge of flow

→ $Q \xrightarrow{< 100\%} W$ → It cannot be converted 100% to W

∴ It is called "degree of Imperfection".

→ $S_{gen}$ = Degree of Irreversibility

1). Psychological issue

2). Philosophical Issue

3). Engineering issue

→ Mental principle ⇌ physical pr.

→ Consciousness → Thought → Language → Reality

→ Thought Exp. → To allow the flow in thought plane by consciousness plane.

→ Thermodynamics ~ Heat transfer

→ Laws of Thermodynamics :

1. Zeroth law
2. First law
3. Second law
4. Third law
5. Constructal law (Adrian Bejan, 1996)
6. Law of Motive force

$\rightarrow$ $Q \xrightarrow{<100\%} W \rightarrow$ high grade energy

$\rightarrow$ $Q_{\text{upgradation}} + Q_{\text{degradation}} = Q_{\text{Total}}$

$$\Rightarrow \quad W + T\left(\frac{Q}{T}\right) = Q$$

$$\Rightarrow \quad W + TS = Q$$

## An estimate of Lost Available Work (LAW):

(The Gouy-Stodola th.)

(Shear work)

$$\rightarrow \frac{\partial E}{\partial t} = \sum_{in} \dot{m}\left(h + \frac{1}{2}v^2 + gz\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}v^2 + gz\right) + \dot{Q} - \dot{W}_{sh} \quad \text{----(1)}$$

$$\rightarrow \frac{\partial S}{\partial t} \geq \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \frac{\dot{Q}}{T_0} \quad \text{----(2)}$$

$$\rightarrow \dot{S}_{gen} = \frac{\partial S}{\partial t} - \frac{\dot{Q}}{T_0} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s \quad \text{----(3)}$$

eliminate $\dot{Q}$ b/w (1) & (2),

$$\Rightarrow (1) - (2) \times T_0$$

$$\Rightarrow \dot{W}_{sh} \leq \sum_{in} \dot{m}\left(h + \frac{1}{2}v^2 + gz - T_0 s\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}v^2 + gz - T_0 s\right) - \frac{\partial}{\partial t}(E - T_0 S) \quad \text{----(4)}$$

$$\dot{W}_{sh_{max}} = \sum_{in} \dot{m}\left(h+\frac{1}{2}V^2+gz-T_0 s\right) - \sum_{out} \dot{m}\left(h+\frac{1}{2}V^2+gz-T_0 s\right) - \frac{\partial}{\partial t}(E-T_0 S) \quad \text{---(5)}$$

It is max. for Reversible pr.

$$\rightarrow \dot{W}_{lost} = \underset{\uparrow (5)}{\dot{W}_{sh_{max}}} - \underset{\uparrow (1)}{\dot{W}_{sh}} \quad \text{----(6)}$$

$$\rightarrow \dot{W}_{lost} = T_0 \underbrace{\left(\frac{\partial S}{\partial t} - \frac{\dot{Q}}{T_0} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s\right)}_{\uparrow (3)} \quad \text{---(7)}$$

$$\rightarrow \boxed{\dot{W}_{lost} = T_0 \dot{S}_{gen}} \quad \text{----(8)}$$

$$\rightarrow \boxed{\dot{W}_{lost} \propto \dot{S}_{gen}} \quad \text{---(9)}$$

(As $T_0 \rightarrow$ Environmental temp. & it is const.)

(Gouy - stodola )

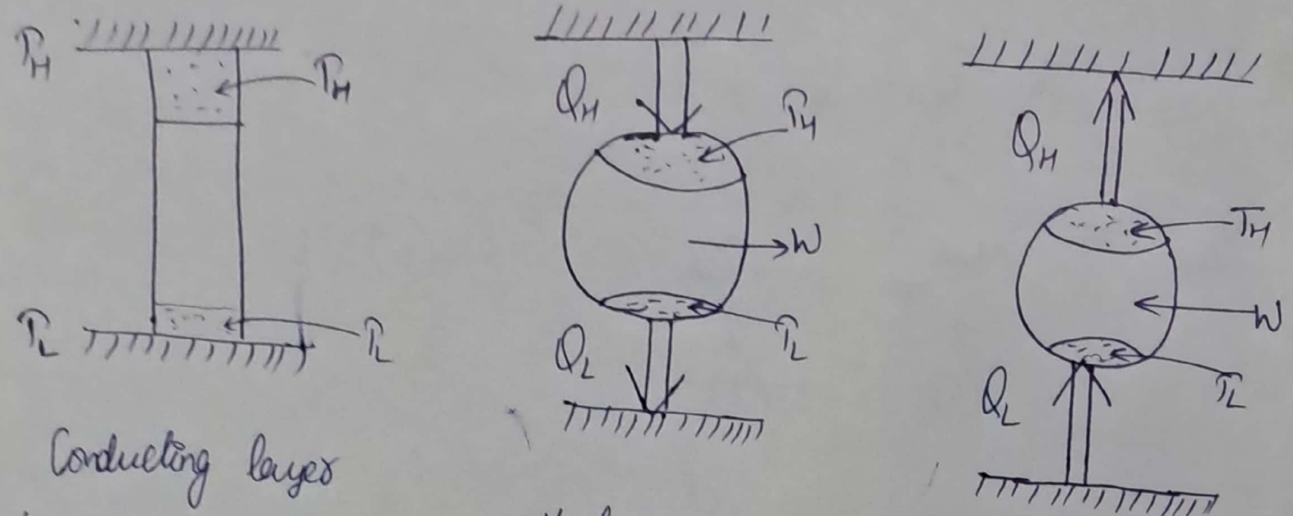
$\rightarrow$ In Convection $\Rightarrow$ $W_{sh} \downarrow$ (the input work should be min. & takes max. amount of heat).

Entropy Generation Minimization : (EGM)

Finite time Thermodynamics (FTT):

EGM $\equiv$ FTT

On the qualitative similarity of heat tr. objectives




Conducting layer
$(T_H > T_L)$

Heat Engine

Refrigerator / Heat Pump

Dissipative structure (above three)

World of words:

Entropy ⇒ Ancient greek : (I) Turning Inward
(II)

→ Inward Manifestation
→ Outward Manifestation

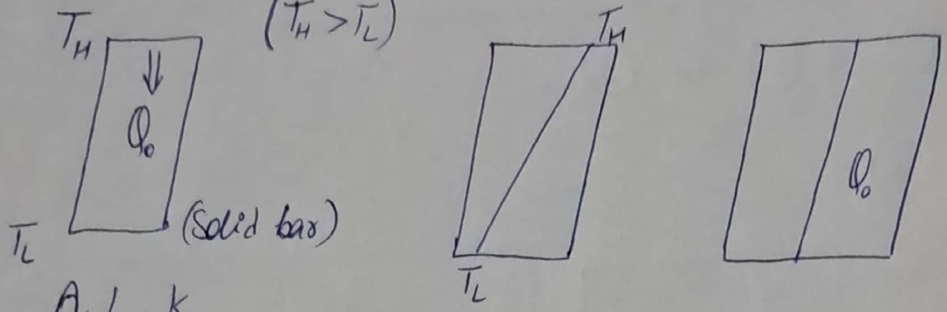
⇒ Modern Greek : (I) Turn (II) Not transformation
(II) change

⇒ Heat flow ⟶ Fluid flow
flow ⟶ field

heat transmission $\longrightarrow$ information transmission

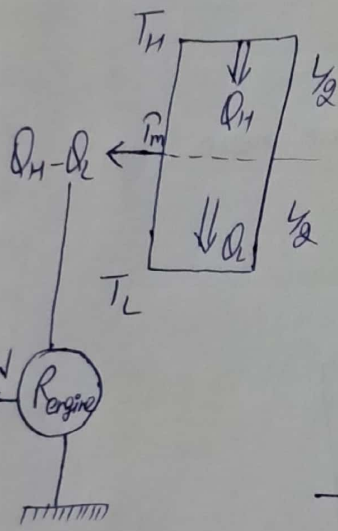
$\rightarrow$ Entropy generation minimization in a general heat tr.

Objectives :




A, L, k

Physical solⁿ $\rightarrow$




$\Rightarrow Q_H - Q_L = W$

$\Rightarrow Q_L \rightarrow Q_H \implies W \rightarrow 0$

By fourier law of cond.,

$$\rightarrow Q_0 = \frac{KA}{L}(T_H - T_L) \quad \text{----} (1)$$

$$\rightarrow S_{gen.,0} = Q_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad \text{----} (2) \quad \left\{ S_{gen.} = S_{out} - S_{in} \right\}$$

$$\rightarrow S_{gen.} = \frac{Q_L}{T_L} + \frac{Q_H - Q_L}{T_m} - \frac{Q_H}{T_H}$$

$$= Q_L \left( \frac{1}{T_L} - \frac{1}{T_m} \right) + Q_H \left( \frac{1}{T_m} - \frac{1}{T_H} \right) \quad \text{-----} (3)$$

$$\rightarrow \quad Q_H = \frac{KA}{(L/2)}(T_H - T_m) \quad \text{----} \; (4)$$

$$\rightarrow \quad Q_L = \frac{KA}{(L/2)}(T_m - T_L) \quad \text{----} \; (5)$$

By putting (4) & (5) in (3),

$$\Rightarrow \quad S_{gen} = S_{gen}(T_m)$$

$$\Rightarrow \quad \text{for Eqm} \Rightarrow \boxed{\frac{dS_{gen}}{dT_m} = 0}$$

$\Rightarrow$

~~Sgen.~~ After doing diff.,

$$\boxed{T_{m,opt} = \sqrt{T_H T_L}} \quad \text{---} \; (6)$$

($T_{m,opt} \rightarrow$ optimum mean temp.)

put (6) in (3),

$$\Rightarrow \quad \boxed{S_{gen,min.} = 4\left[\sqrt[4]{\frac{T_H}{T_L}} - \sqrt[4]{\frac{T_L}{T_H}}\right]^2 \frac{KA}{L}} \quad \text{----} \; (7)$$

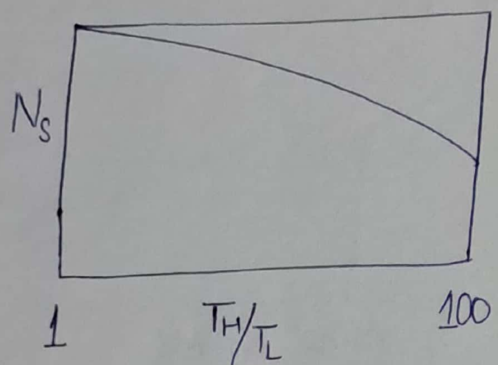
(7) ÷ (2)

$$\Rightarrow \quad \boxed{N_s = \frac{S_{gen,min.}}{S_{gen,0}} < 1} \quad \text{----} \; (8)$$

Entropy gen. no.

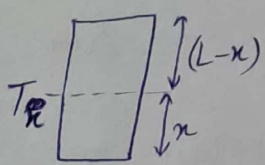
$$\left( N_s = \frac{\text{EG in engineered system}}{\text{EG in pre-engineered system}} \right)$$

$$\Rightarrow N_s = \frac{4\sqrt{\frac{T_H}{T_L}}}{\left(\sqrt{\frac{T_H}{T_L}}+1\right)^2} < 1 \quad \text{----} \quad (9)$$




1) Draw this graph in MATLAB

2) Repeat prob. 1 taking,




$S_{gen.} = S_{gen.}(T_x, x)$

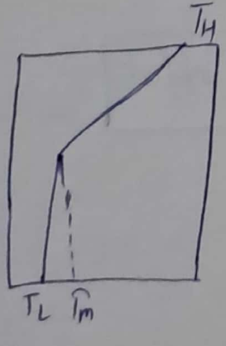
first derive & have to diff. 2 times in this.

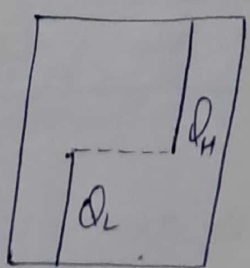
$$\Rightarrow \frac{\partial S_{gen}}{\partial T_x} = 0 \Rightarrow \text{Non-Linear Algebric eq}^n.$$

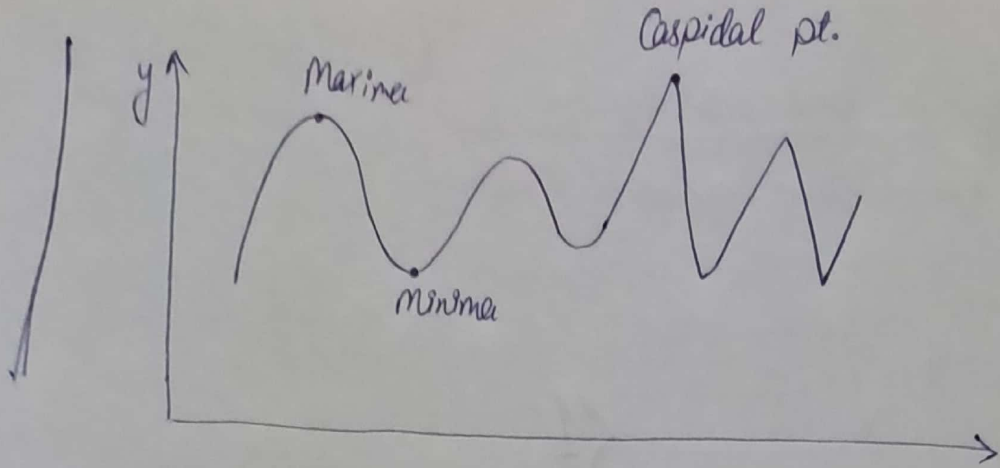
$$\Rightarrow \frac{\partial S_{gen.}}{\partial x} = 0 \Rightarrow \text{" \quad " \quad " \quad "}.$$

Simultaneous sol$^n$. (write a code)

(using Newton raphson method)






for extremum pt. $\Rightarrow \frac{dy}{dx} = 0$

for Caspidal pt. $\Rightarrow \frac{dy}{dx} \to \infty$

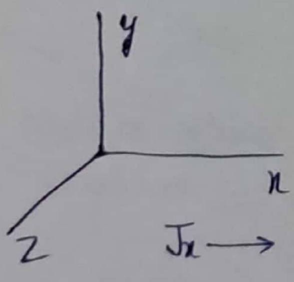
## Againt Method :

$\Rightarrow$ Flow of heat $\longrightarrow$ flow of AI , Mind & M/c

Ludwig Wittgentein $\Rightarrow$ Philosophy of language
Philosophy of mind

flux

$\rightarrow \quad J = \hat{i} J_x + \hat{j} J_y + \hat{k} J_z$




'Orthogonal'

$J_x \Rightarrow$ In flux

Efflux (Out)

$\boxed{\text{Net Efflux} = \text{Efflux} - \text{Influx}}$

## Taylor's Series :

$\rightarrow$ $f(x+h) = f(x) + hf'(x) + \frac{h^2}{\lfloor 2} f''(x) + \dots\dots$

Putting $x = J_x$ & $h = dx$

$\rightarrow$ $J_{x+dx} = J_x + \frac{\partial J_x}{\partial x} dx$

$\Rightarrow$ Net Efflux per unit Vol. = (Efflux − Influx) / Vol.

Total efflux per unit Vol. $= \left[ \left\{ \left(J_x + \frac{\partial J_x}{\partial x} dx\right) dydz + \left(J_y + \frac{\partial J_y}{\partial y} dy\right) dxdz + \left(J_z + \frac{\partial J_z}{\partial z} dz\right) dxdy \right\} - \left\{ (J_x dydz) + (J_y dxdz) + (J_z dxdy) \right\} \right] / dxdydz$

$$\boxed{\text{Net efflux per unit vol.} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}}$$

$\rightarrow$ Net efflux per unit vol. $= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i} J_x + \hat{j} J_y + \hat{k} J_z\right)$

$= \nabla \cdot J = \text{div. } J$

$\Rightarrow$ $\text{grad } \phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$

$q = -kA \frac{dT}{dx}$ or $q = -kA \text{ grad } T$

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## The Generalized Diff. eq$^n$ for a Conservation principle :

Change = Transfer ------ (1)

Change > Transfer ---- (2)

Change $\geq$ Transfer ---- (3)

Change = Transfer + $\epsilon$ --- (4)

$\epsilon \geq 0$

from (1) & (2)

from (2) & (3)

$\Rightarrow$ $S = R$ ------- (1)

same no. of st.

rate

One to One Correspondence,

$D \geq C$

$D = C$ ------ (2)

$R + C = D + S$

rate

Convection

Diffusion

source

$\Rightarrow$ Rate + faster motion (Convection) = slower motion (diffusion) + source

→ $\phi \Rightarrow$ Intensive

$\rho \Rightarrow$ local density , $\rho\phi \rightarrow$ Total $\phi$ for CV → Extensive pr.

$\Rightarrow R = \frac{\partial}{\partial t}(\rho\phi)$

$\Rightarrow C = div(\rho\phi \vec{u})$ → velocity of fluid

(div. → Net efflux per unit vol.)

$\Rightarrow D = div(\Gamma_\phi \, grad\, \phi)$ $(\Gamma_\phi = \text{diff. Coeff.})$

$\Rightarrow S = S_\phi$

→ $R + C = D + S$

→ $$\boxed{\frac{\partial}{\partial t}(\rho\phi) + div(\rho\phi u) = div(\Gamma_\phi \, grad\, \phi) + S_\phi}$$

Ex-1 : Conservation of a chemical species :

Mass fraction, $m_L = \dfrac{M_L}{M_a + M_b + M_c}$

Step 1 → $\phi = m_L$

$J_L = -\Gamma_L \, grad\, m_L$ , $J_L \propto grad\, m_L$

$= |\Gamma_L| \, |grad\, m_L|$

"Fick's law of Diffusion"

→ $S_\phi = R_L$

$\rightarrow$ $A + B = C + D$ , $S_\phi = R_L = 0$

$A + B = C + D + E$ , $S_\phi = R_L = 1$

$A + B = C$ , $S_\phi = R_L = -1$

$$\Rightarrow \frac{\partial}{\partial t}(\rho m_L) + \text{div}(\rho m_L v) = \text{div}(\Gamma_L \text{ grad } m_L) + R_L$$

No. of new space,
Created $= 1$
destroyed $= -1$
neither Created nor destroyed $= 0$

Ex-2: Conservation of Mass:

$\rightarrow$ $m_L = 1$ , $R_L = 0$

$$\Rightarrow \frac{\partial}{\partial t}(\rho) + \text{div}(\rho v) = 0$$

Difficult Conversation :-

$\rightarrow$ Flow of heat $\longrightarrow$ Flow of pent-up emotion

$$R = S \quad \text{---(1)}$$
$$C = D \quad \text{---(2)}$$
$$R + C = S + D \quad \text{---(3)}$$

$\rightarrow$ $$\boxed{\frac{\partial}{\partial t}(\rho\phi) + \text{div}(\rho\phi v) = \text{div}(\Gamma_\phi \text{ grad } \phi) + S_\phi}$$ "Master eq$^n$"

→ Y, N, M → logical eqn

→ Schematic Dif. eqn

→ Procedure :

1) Meaning $\phi$.

2) Law of diffusion

3) Nature of source term

Ex-3 Conservation of momentum :- (Navier-Stoke

→ R + C = D + S

diffusion coff.

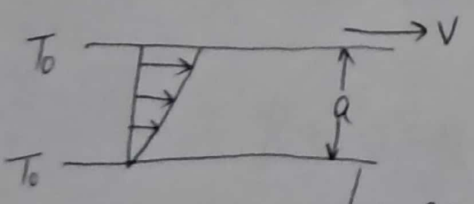
$$\frac{\partial}{\partial t}(\rho\phi) + div(\rho\phi u) = div(\Gamma_\phi \, grad\,\phi) + S_\phi$$

$\Rightarrow$ $\frac{momentum}{mass} = \frac{mv}{m} = v$ → specific momentum

Step-I : $\phi = \frac{mv}{m} = u$

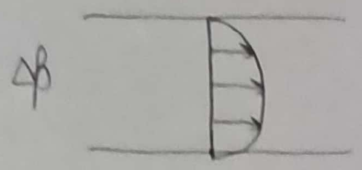
Step II : Law of momentum diffusion:

$J_u = -\mu \, grad\, u = \mu \,|grad\, u|$ "Newton's law of Velocity"




↳ 2D, stat., $\infty$

→ Couette flow / Shear driven flow



→ Haigen – Poiseuille

Body force

surface force

$$\Rightarrow \text{III}: \boxed{S_u = -\frac{\partial p}{\partial x} + B_x + V_x}$$

$$\frac{\partial}{\partial t}(\rho u) + \operatorname{div}(\rho u u) = \operatorname{div}(\mu \operatorname{grad} u) - \frac{\partial p}{\partial x} + B_x + V_x$$

**Ex – 4 :** Conservation of Energy :

$$R + C = D + S$$

$$\frac{\partial}{\partial t}(\rho\phi) + \operatorname{div}(\rho\phi u) = \operatorname{div}(\Gamma_\phi \operatorname{grad}\phi) + S_\phi$$

**Step I :** $\cancel{\phi = h + px}$ $\phi = h = u + pV$

**Step II :** Enthalpy diffusion

$$J_x = -K \operatorname{grad} T$$

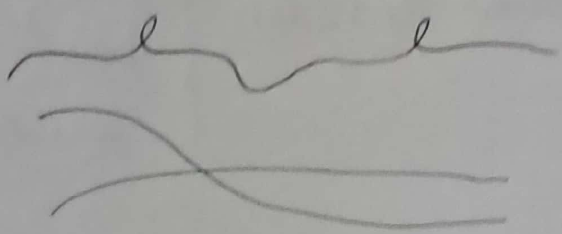
Fourier law of Conduction

**Step – III :** $S_\phi \equiv S_h$

$$\Rightarrow \frac{\partial}{\partial t}(\rho h) + \operatorname{div}\underbrace{(\rho h u)}_{0\ \text{(If convection not happen)}} = \operatorname{div}(K \operatorname{grad} T) + S_h$$

# Ex-5 : Turbulance

Leonardo da Vinci



Fluid Filament

→ Conservation of $\overset{\text{turbulent}}{\wedge}$ KE :

I : $\phi = k \equiv$ specific turbulent KE $= \dfrac{\frac{1}{2}mv^2}{m} = v^2/2$

⇒ $\boxed{\text{Growth} + \text{Decay} = \text{Const.}}$

(II) :

$$J_k = -\Gamma_k \text{ grad } k$$

III : $\boxed{S_k = G - \rho\epsilon}$

→ kinematic rate of dissipation of turbulent KE

$\rho\epsilon$ → Dynamic " " "
" " "

rate of gen. of tur. KE

$(S_k > 0)$

Buckling

$$\Rightarrow \frac{\partial}{\partial t}(\rho\phi) + \text{div}(\rho\phi u) = \text{div}(\Gamma_\phi \text{ grad}\phi) + S_\phi$$

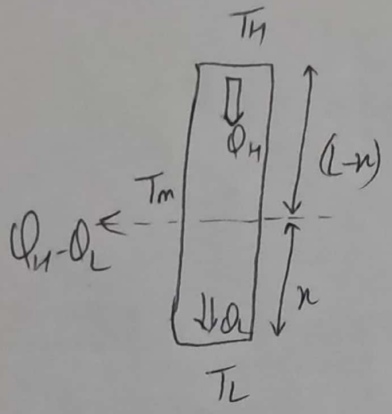
$$\Rightarrow \frac{\partial}{\partial t}(\rho k) + \text{div.}(\rho k u) = \text{div.}(\Gamma_k \text{ grad } k) + G - \rho\epsilon$$

(k-ε model of Turbulence)

(I): Interpretation of order related observation:

(II): Unified description

} natural philosophy




→ $T_H, T_L, x, L$

→ DOF ⟹ Reduction of DOF

⟹ Non-dimensional parameter,

$$\tilde{T}_H = \frac{T_H}{T_L}, \quad \tilde{T}_m = \frac{T_m}{T_L}, \quad \xi = \frac{x}{L}$$

$$\frac{\partial}{\partial x}(S_{gen.}) = 0, \quad \frac{\partial}{\partial T_m}(S_{gen.}) = 0$$

$$\Rightarrow T_H = \tilde{T}_H T_L, \quad x = \xi L$$

$$\Rightarrow \frac{\partial}{\partial \tilde{T}_m}(S_{gen.}) = 0, \quad \frac{\partial}{\partial \xi}(S_{gen.}) = 0$$

(assump. $\tilde{T}_H >> \tilde{T}_L$)

<u>Sadhana</u>

→ Take home exam

⟹ 1) Term project

CA **

J: The second law analysis in fundamental Convective heat tr. problm.

Graph → Matlab.

2) Assignment

J: A challange to Physicist.

1-page synopsis, 1942, 1946 Bridgman

Lack of wishful thinking

⇒ Flow of Heat → Flow of Life

1 Rabindranath Tagore, Sadhana, Macmillian, New York, 1913

Basic Convective Configuration:

(I) External Convection

(II) Internal Convection → occur over hollow surfaces or → through pipe.

(III) Mixed Convection

(III) Entry region Convection

→

(I) Forced Convection

(II) Free (Natural) Convection

(III) Mixed Convection

(I) Laminar Convection

(II) Turbulent Convection

(III) Convection in two phase

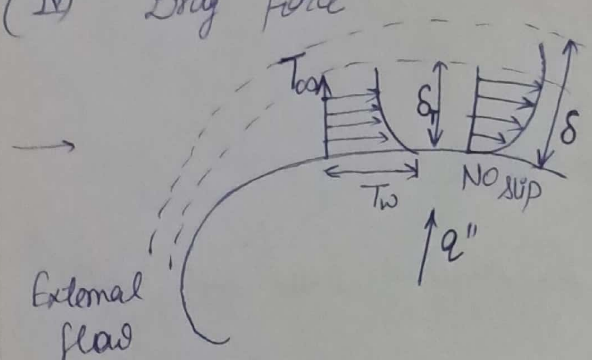
<u>Objectives</u> :

(I) Flow field

(II) Temperature field

(III) Convective heat tr. Coeff.

(IV) Drag force




↳ means solid object

(Tangential vel. distribution)

$T_w$ = Temp of solid wall
or
wall Temp.

$T_\infty \rightarrow$ Temp. of fluid that is pulling the obj.

→ <u>Free stream</u> ⇒ fluid is far away from solid obj.

→ $T_w > T_\infty$  Assymphotic

Boundary layer thick. > Thermal boundary layer ($\delta_T$)

⇒ Hydrodynamic boundary layer thick.

⇒ External flow,

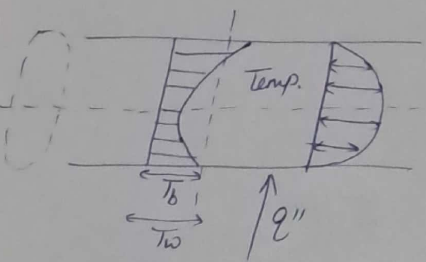
$$q'' = h\,(T_w - T_\infty) \quad \text{---- (1)}$$

[Newtons law of cooling]

($h$ = convective heat tr. coeff.)

## Internal flow

$T_b \rightarrow$ Bulk temp.
Cross sec. based avg. temp.




Longitudinal vel. distr.

$$q'' = h\,(T_w - T_b) \quad \text{---} (2)$$

H.T.
1. $T_b$ for pipe flow
2. Vel. distr. in parabolic - proof

J.P.H

$$\Rightarrow \quad q'' = -K \left(\frac{\partial T}{\partial y}\right)_{y=0^+} \quad \text{--} (3) \quad K \rightarrow \text{th. Cond. of fluid}$$

$\Rightarrow$ Eliminate $q''$ b/w (1) & (3)

$$\Rightarrow \quad h = -\frac{K}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial y}\right)_{y=0^+} \quad \text{---} (4) \quad (\text{Ext.})$$

Eliminate $q''$ b/w (2) & (3)

$$h = \frac{-K}{(T_w - T_b)} \left(\frac{\partial T}{\partial y}\right)_{y=0^+} \quad \text{----} (5) \quad (\text{Int.})$$

$$\rightarrow \quad \frac{dT}{dt} = b\,(T - T_\infty) \quad \text{----} (6)$$

$\left(b = \frac{h}{C}\right)$ $\rightarrow$ sp. heat

## Arbitrary of a thought :

$\Rightarrow q \propto \Delta T$

$\Rightarrow q = h\Delta T \Rightarrow h = \frac{q}{\Delta T}$

| | | |
|---|---|---|
| $\Rightarrow q = \left(\frac{q}{\Delta T}\right)\Delta T$ | $y = mx$ | $y = f(x)$ |
| $\Rightarrow q = f(\Delta T)$ | $m = y/x$ | Vertical Thinking |
| $\Rightarrow \Delta T = f(q)$ | $y = (y/x)x$ | |

## Correlation in Convective heat tr. :

$\Rightarrow h = \frac{hL}{k} \rightarrow$ Non-dim. no.

### (a) Forced Convection :

$h = f(L, u, \rho, \mu, k, C_p)$ , M, L, T, $\theta$ (T → time, $\theta$ → temp.)

$$h = A L^a u^b \rho^c \mu^d k^e C_p^f \quad \text{----- (1)}$$

$$MT^{-3}\theta^{-1} = AL^a(LT^{-1})^b(ML^{-3})^c(ML^{-1}T^{-1})^d(MLT^{-3}\theta^{-1})^e(L^2T^{-2}\theta^{-1})^f \quad \text{--- (2)}$$

$\Rightarrow$ L: $a + b - 3c - d + e + 2f = 0$ ----- (3)

$\Rightarrow$ T: $-b - d - 3e - 2f = -3$ ----- (4)

$\Rightarrow$ M: $c + d + e = 1$ --- (5)

$\theta: \quad -e-f=-1 \quad \text{------} (6)$

<u>Solⁿ</u>: (3), (4), (5), (6)

$$a = c-1, \; b = c, \; d = -c+f, \; e = 1-f \quad \text{----} (7)$$

Put (7) in (1)

$$\Rightarrow h = A L^{c-1} u^{c} \rho^{c} \mu^{-c+f} k^{1-f} C_p^{f}$$

$$\Rightarrow \frac{hL}{k} = A\left(\frac{\rho L u}{\mu}\right)^{c}\left(\frac{\mu C_p}{k}\right)^{f}$$

$$\Rightarrow \underset{Nu}{\frac{hL}{k}} = A\underset{Re}{\left(\frac{\rho L u}{\mu}\right)^{c}}\underset{Pr}{\left(\frac{\mu C_p}{k}\right)^{f}} \quad \text{------} (8)$$

$$\Rightarrow \boxed{Nu = A\, Re^{c}\, Pr^{f}}$$

Nu = Nusselt no.
Re = Reynold no.
Pr = Prandtl no.

(b) <u>Natural Convection</u>:

→ Natural Conv. is driven by Buoyancy.

$\Rightarrow PV = RT \Rightarrow P = \rho RT \Rightarrow \rho = \rho(T, P) \equiv$ 2-parameter model

$\Rightarrow \rho_\infty \to$ reference density, $T_\infty \to$ reference temp., $P_0 \to$ ref. pres.

$\Rightarrow \rho_\infty = \rho(T_\infty, P_0)$ ——— (1)

$\Rightarrow \rho \approx \rho_\infty + \left(\frac{\partial \rho}{\partial T}\right)_P (T - T_\infty) + \cancel{\left(\frac{\partial \rho}{\partial P}\right)_T}^{0} (P - P_0) + \cdots$ --- (2)

**Coff. of Vol. th. exp. :**

$\beta = \frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial T}\right)_P$  $\vartheta$ = specific vol.

$\Rightarrow$ rate of change of vol. per unit vol. per unit change in temp. at const. press.

$\Rightarrow \vartheta = \frac{1}{\rho}$ , $d\vartheta = \frac{-1}{\rho^2} d\rho$

$\Rightarrow \beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$ ----- (4)

$\Rightarrow$ From (2), $\rho \approx \rho_\infty + \left(\frac{\partial \rho}{\partial T}\right)_P (T - T_\infty)$

$\Rightarrow \rho \approx \rho_\infty - \rho_\infty \beta (T - T_\infty) + \cdots$ (3)

$\Rightarrow \rho = \rho_\infty [1 - \beta(T - T_\infty) + \cdots]$

$\Rightarrow 1 >> \beta(T - T_\infty)$ [Oberbeck - Boussinesq approximation]

$$\Rightarrow (\rho-\rho_\infty)g = \rho_\infty g\beta\Delta T \quad \text{-----} (5)$$
$$\sim g\beta\Delta T$$

$$\Rightarrow h = h(L, k, C_p, \rho, \mu, g\beta\Delta T) \quad \text{---} (6)$$

(u will not come as 'u' will not depend

$$\Rightarrow \underbrace{\frac{hL}{k}}_{Nu} = A\Big(\underbrace{\frac{g\beta\Delta T L^3\rho^2}{\mu^2}}_{\text{Grashoff's no. }(Gr)}\Big)^a\Big(\underbrace{\frac{\mu C_p}{k}}_{Pr}\Big)^b \quad \text{---} (7)$$

$$\Rightarrow \boxed{Nu = A\,Gr^a\,Pr^b} \quad \text{---} (8)$$

(unknown = A, a, b)

(for large no. of experiment ⇒ a = b)

A Poetic Rejoice :

$$\rightarrow R + C = D + S$$

$$\frac{\partial}{\partial t}(\rho\phi) + div(\rho\phi u) = div(\Gamma_\phi\, grad\phi) + S_\phi$$

Steady one-dimensional convection and diffusion problem :

$$\Rightarrow \cancel{R}^{\,0} + C = D + \cancel{S}^{\,0}$$

$$\Rightarrow \cancel{\frac{\partial}{\partial t}(\rho\phi)}^{\,0} + div(\rho\phi u) = div(\Gamma_\phi\, grad\,\phi) + \cancel{S_\phi}^{\,0}$$

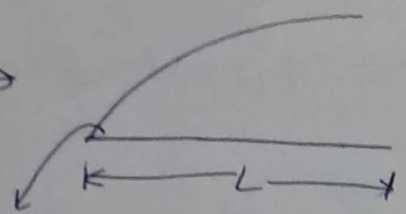
$\Longrightarrow \quad \frac{d}{dx}(\rho\phi u) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)$ ----- (1) [Second order ODE]

→ CE (Continuity eqn) : $\frac{d}{dx}(\rho u) = 0$

$\therefore \quad \rho u = \text{Const.}$ ---- (2)

$\Gamma = \text{Const.}$

$$\Longrightarrow \boxed{\frac{d\phi}{dx} = \left(\frac{\Gamma}{\rho u}\right)\frac{d^2\phi}{dx^2}}$$

→ 

(Flat plate) $\quad 0 \le x \le L$

B.C. ⇒

At $x = 0$, $\phi = \phi_0$ ---- (3)

At $x = L$, $\phi = \phi_L$ ---- (4)

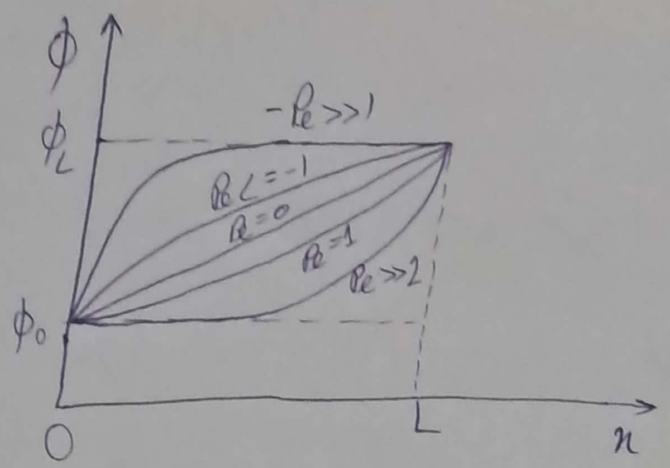
$$\Longrightarrow \quad \frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp\left(P_e \frac{x}{L}\right) - 1}{\exp(P_e) - 1} \quad \text{---} (5)$$

$$P_e = \text{Peclet No.} = \frac{\rho u L}{\Gamma} \quad \text{---} (6)$$

H.T 1 : Analytical soln

H.T.2 : Numerical soln

H.T3 : Plot (MATLAB)




If $Pe \downarrow \Rightarrow$ either $u\downarrow$ or $\Gamma\uparrow$ "Diffusion".

If $Pe \uparrow \Rightarrow$ Either $u\uparrow$ or $\Gamma\downarrow$ "Convection"

$\Rightarrow$ If $Pe = -ve \Rightarrow$ Means it is a back flow.

$\rightarrow$ Apples $\bar{c}$ Oranges

$\Rightarrow$ Heat lines for visualising Convection.

$\Rightarrow$ $R + C = D + S$ [div. $\rightarrow$ Net efflux per unit vol.]

$\Rightarrow$ $\frac{\partial}{\partial t}(\rho\phi) + div(\rho\phi u) = div(\Gamma_\phi\, grad\,\phi) + S_\phi$

$\Rightarrow$ CE: $\frac{\partial}{\partial t}(\rho\phi) + div.(\rho\phi u) = 0$

$\Rightarrow$ Incompressible : $\rho = Const.$

$\Rightarrow$ steady : $\frac{\partial}{\partial t}() = 0$

$div.(\rho u) = 0$ or $div(u) = 0$

$\Rightarrow$ CE in 2-D : $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$ ---- (1)

(for incomp., steady)

$\Rightarrow$ $\psi(x,y) \rightarrow$ stream $f^n$

$$\Rightarrow \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad \text{----} (2)$$

$\Rightarrow$ Putting (2) in (1).

$$\Rightarrow \quad \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

[$\psi$ is cont. in domain of $x, y$]

→ <u>Energy</u> <u>eq$^n$</u> : (EE)

$$\xcancel{\frac{\partial}{\partial t}(\rho h)}^{\,0} + \text{div.}(\rho h v) = \text{div}(k \text{ grad } T) + \cancel{S_h}^{\,0}$$

(steady)

$\Rightarrow$ $h = C_p T$ [Considering ideal fluid] , $h \sim T$

$$\Rightarrow \underline{\underline{2\text{-}D}}: \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad \text{----} (3)$$

$\alpha \rightarrow$ thermal diffusivity , $\alpha = \dfrac{k}{\rho C_p}$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(\rho C_p V_x T - k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho C_p V_y T - k \frac{\partial T}{\partial y}\right) = 0 \quad \text{..} (4)$$

$$\left[ \frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \right]$$

$\Rightarrow T = (T - T_{ref.}) = $ Const. $=$ Lowest temp.

$\Rightarrow$ Stream $[\psi(x,y)]$ = Heat $f^n$ – $[H(x,y)]$

$$\Rightarrow \frac{\partial H}{\partial y} = \rho C_p u (T - T_{ref.}) - k\frac{\partial T}{\partial x} \quad \text{--- (5)}$$

$$\Rightarrow -\frac{\partial H}{\partial x} = \rho C_p v (T - T_{ref.}) - k\frac{\partial T}{\partial y} \quad \text{--- (6)}$$

$\rightarrow$ (5) & (6) identically specified $eq^n$ (4),

$$\frac{\partial^2 H}{\partial x \partial y} - \frac{\partial^2 H}{\partial y \partial x} = 0$$

[Both are equal when H is cont in domain of x,y]

[H- Lines or Heat lines]

$\Rightarrow$ For No convection (neither force & natural) $\Rightarrow$ $v = 0$ (vel.)

$$\Rightarrow \frac{\partial}{\partial x}\left(\rho C_p \overset{0}{\cancel{u}} T - k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho C_p \overset{0}{\cancel{v}} T - k\frac{\partial T}{\partial y}\right)$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \Rightarrow \boxed{\nabla^2 T = 0} \quad \text{Laplace } eq^n$$

$\Rightarrow$ Const. temp. Line $\Rightarrow$ Family of Isotherms

$\Rightarrow$ Convective heat tr. process is depicted by Heat Lines

$\Rightarrow$ $\boxed{\nabla^2 T = 0} \longrightarrow$ Laplace $eq^n$

$\Rightarrow$ $\boxed{\nabla^2 T = -M} \longrightarrow$ Poisson's $eq^n$

→ Isy = Const temp. line

→ It describe heat is flowing from high temp. to low temp.