GIVEN DATA

$\varepsilon_x = -400 \times 10^{-6}$

$\varepsilon_y = +200 \times 10^{-6}$

$\gamma_{xy} = +800 \times 10^{-6}$

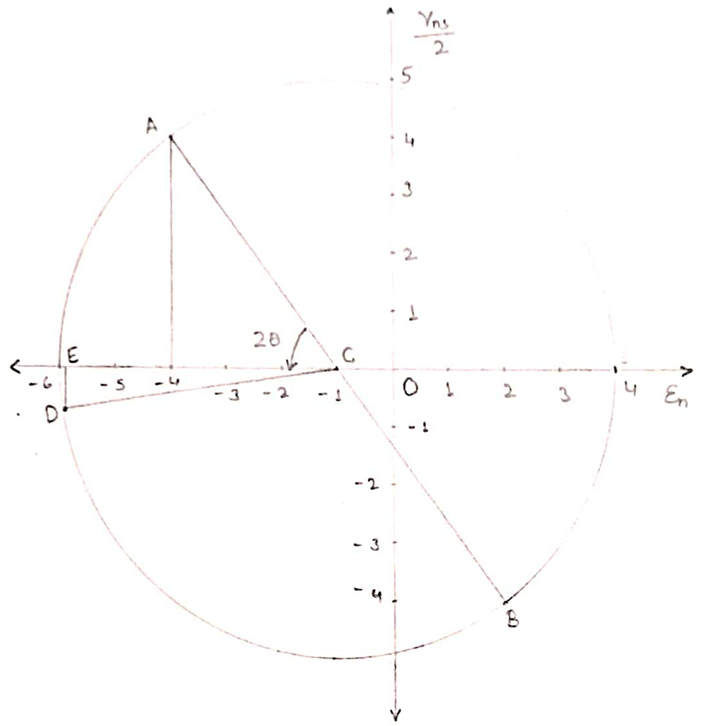
$E = 200$ GPa

$\mu = 0.30$

PROBLEM-1

Mohr's Circle for Strain

Scale : $1\,cm = 100 \times 10^{-6}$



	Magnitude	Direction with x-axis (anti clockwise)
OC_ε	100×10^{-6}	–
R_ε	500×10^{-6}	–
ε_{max}	600×10^{-6}	116.5°
ε_{min}	-400×10^{-6}	26.5°
$\frac{\gamma_{max}}{2}$	500×10^{-6}	161.5°
$\frac{\gamma_{min}}{2}$	-500×10^{-6}	71.5°

State of strain of x-plane is denoted by a point $A\left(\varepsilon_x, \frac{\gamma_{xy}}{2}\right) \Rightarrow (-4,4)$

State of strain of x-plane is denoted by point B $\left(\varepsilon_y, -\frac{\gamma_{xy}}{2}\right) \Rightarrow (2,-4)$

Measure of OC = 1 cm

Measure of R (radius) = 5 cm

Maximum Principle Strain :

$\varepsilon_{max} = OC + R$

$= 6$ cm

Minimum Principle Strain :

$\varepsilon_{min} = OC - R$

$= -4$ cm

Maximum shearing strain:

R = 5 cm

Minimum shearing strain :

−R = −5 cm

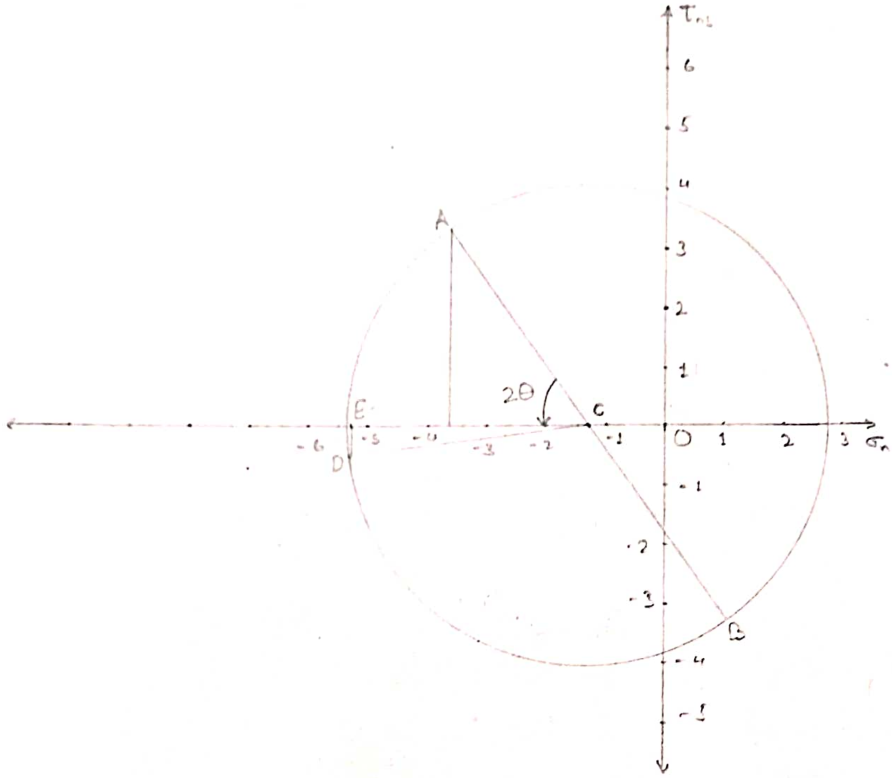
Measure angle of 2θ :

$2\theta = 53°$

$\theta = 26.5$

Mohr's Circle for Stress

Scale : 1 cm = 20 MPa



	Magnitude	Direction with x-axis (anti-clockwise)
OC_σ	28.57 MPa	–
R_σ	76.9 MPa	–
σ_{max}	105.47 MPa	116.5°
σ_{min}	-48.33 MPa	26.5°
τ_{max}	76.9 MPa	161.5°
τ_{min}	-76.9 MPa	71.5°

Calculations :

$$R_\sigma = R_\epsilon \times \frac{E}{1+\mu}$$

$$= 500 \times 10^{-6} \times \frac{200 \times 10^3}{1+0.3}$$

$R_\sigma = 76.9$ MPa

$$OC_\sigma = OC_\epsilon \frac{E}{1-0.3}$$

$$= 100 \times 10^{-6} \times \frac{200 \times 10^3}{1-0.3}$$

$OC_\sigma = 28.57$ MPa

Measure of OC = 1.4 cm

Measure of R = 3.8 cm

Maximum Principle Stress

$\sigma_{max} = OC + R$
$= 5.2$ cm

Minimum Principle Stress

$\sigma_{min} = OC - R$
$= 2.4$ cm

Maximum Shearing Stress

$\tau_{max} = 3.8$ cm

Minimum Shearing Stress

$\tau_{min} = -3.8$ cm

PROBLEM-2(a)

GIVEN DATA

$\varepsilon_a = 400 \times 10^{-6}$

$\varepsilon_b = -200 \times 10^{-6}$

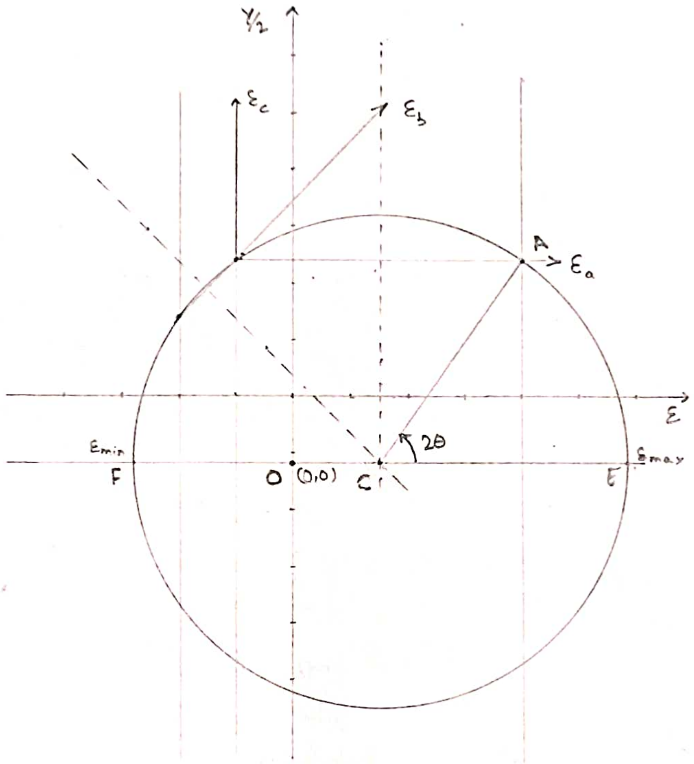
$\varepsilon_c = -100 \times 10^{-6}$

$E = 200$ GPa

$\mu = 0.30$

Mohr's Circle for Strain

Scale :- 1 cm = 100×10^{-6}



Measure of OC = 1.5 cm

Measure of R_ε = 4.4 cm

Measure of ε_{max} = 5.9 cm

Measure of ε_{min} = 2.8 cm

Measure of $\frac{\gamma_{max}}{2}$ = 4.4 cm

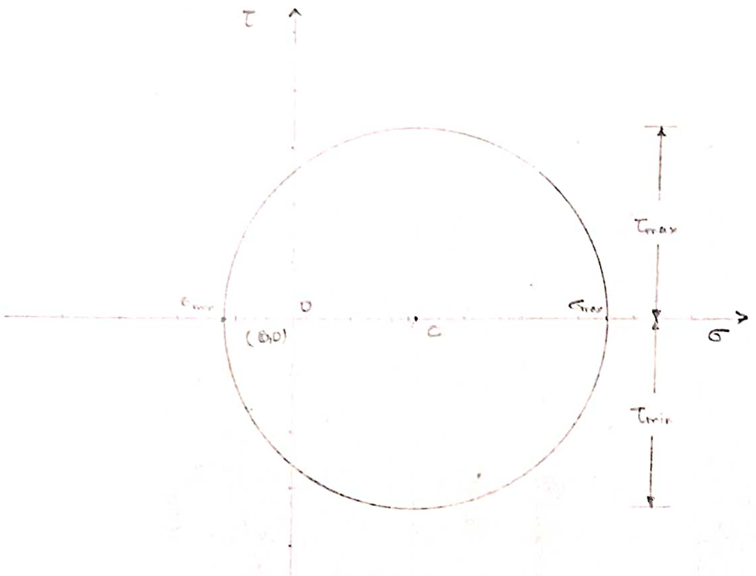
$2\theta = 55^\circ$

$\theta = 27.5$

	Magnitude	Direction with x-axis (anticlockwise)
OC_ε	150×10^{-6}	–
R_ε	440×10^{-6}	–
ε_{max}	590×10^{-6}	152.5°
ε_{min}	-280×10^{-6}	62.5°
$\frac{\gamma_{max}}{2}$	$+440 \times 10^{-6}$	17.5°
$\frac{\gamma_{min}}{2}$	-440×10^{-6}	107.5°

Mohr's Circle for Stress

Scale :- 1 cm = 20 MPa



	Magnitude	Direction with x-axis (anticlockwise)
OC_σ	42.85 MPa	-
R_σ	67.69 MPa	-
σ_{max}	110 MPa	152.5°
σ_{min}	24 MPa	62.5°
τ_{max}	67.69 MPa	17.5°
τ_{min}	- 67.69 MPa	107.5°

Calculations :-

$$R_\sigma = R_\varepsilon \times \frac{E}{1+\mu}$$

$$= 440\times10^{-6} \times \frac{200\times10^{9}}{1+0.3}$$

$$= 67.69 \text{ MPa}$$

$$OC_\sigma = OC_\varepsilon \times \frac{E}{1-0.3}$$

$$= 150 \times10^{-6} \times \frac{200\times10^{9}}{1-0.3}$$

$$= 42.85 \text{ MPa}$$

Measure of OC = 2.14 cm

Measure of R = 3.38 cm

Measure of σ_{max} = 5.5 cm

Measure of σ_{min} = 1.2 cm

Measure of τ_{max} = 3.38 cm

GIVEN DATA

$\varepsilon_a = 300 \times 10^{-6}$

$\varepsilon_b = 600 \times 10^{-6}$

$\varepsilon_c = 100 \times 10^{-6}$

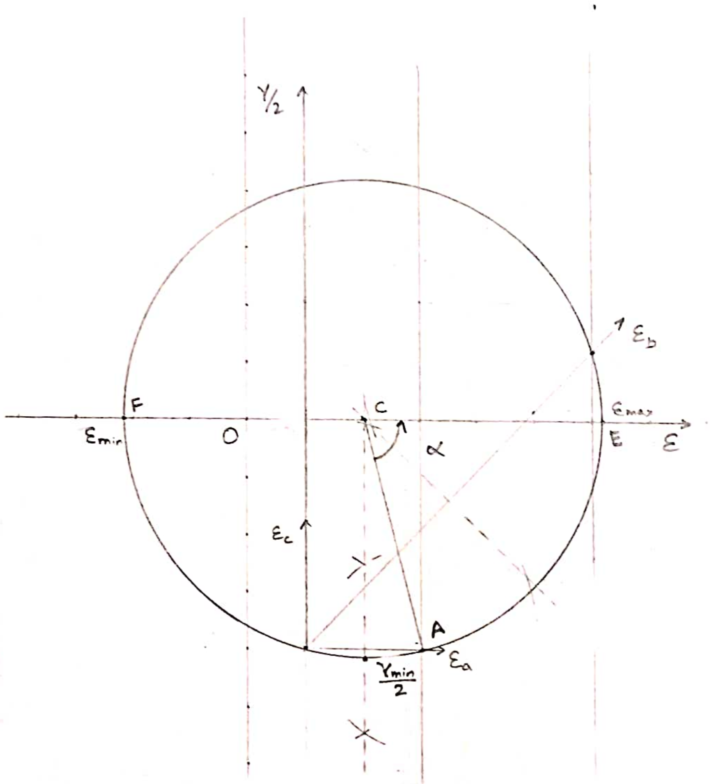
$E = 200$ GPa

$\mu = 0.30$

PROBLEM-2(b)

Mohr's Circle for Strain

Scale :- 1cm = 100×10^{-6}



Measure of OC = 1 cm

Measure of R_ε = 4.2 cm

Measure of ε_{max} = 6.2 cm

Measure of ε_{min} = 2.1 cm

Measure of $\frac{\gamma_{max}}{2}$ = 4.2cm

$\alpha = 76°$

$2\theta = 360 - \alpha$

$\therefore \theta = 142°$

	Magnitude	Direction with x-axis (anticlockwise)
OC_ε	100×10^{-6}	-
R_ε	420×10^{-6}	-
ε_{max}	620×10^{-6}	38°
ε_{min}	-210×10^{-6}	-52°
$\frac{\gamma_{max}}{2}$	420×10^{-6}	-97°
$\frac{\gamma_{min}}{2}$	-420×10^{-6}	-7°

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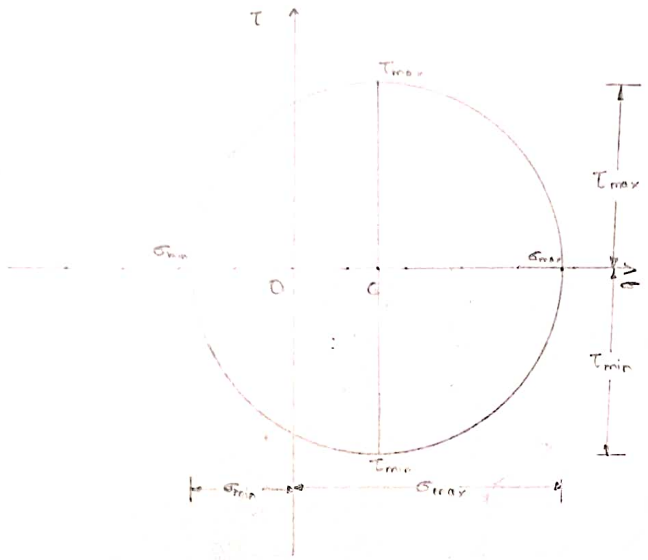
Mohr's Circle for Stress

Scale :- 1 cm = 20 MPa



	Magnitude	Direction with x-axis (anticlockwise)
OC_σ	28.57 MPa	-
R_σ	64.61 MPa	-
σ_{max}	94 MPa	38°
σ_{min}	- 36 MPa	- 52°
τ_{max}	64.61 MPa	-97°
τ_{min}	-64.61 MPa	-7°

Calculations

$$R_\sigma = R_\varepsilon \times \frac{E}{1+\mu}$$

$$= 420 \times 10^{-6} \times \frac{200 \times 10^9}{1+0.3}$$

$$= 64.61 \text{ MPa}$$

$$OC_\sigma = OC_\varepsilon \times \frac{E}{1-\mu}$$

$$= 100 \times 10^{-6} \times \frac{200 \times 10^9}{1-0.3}$$

$$= 28.57 \text{ MPa}$$

Measure of R = 3.23 cm

Measure of OC = 1.42 cm

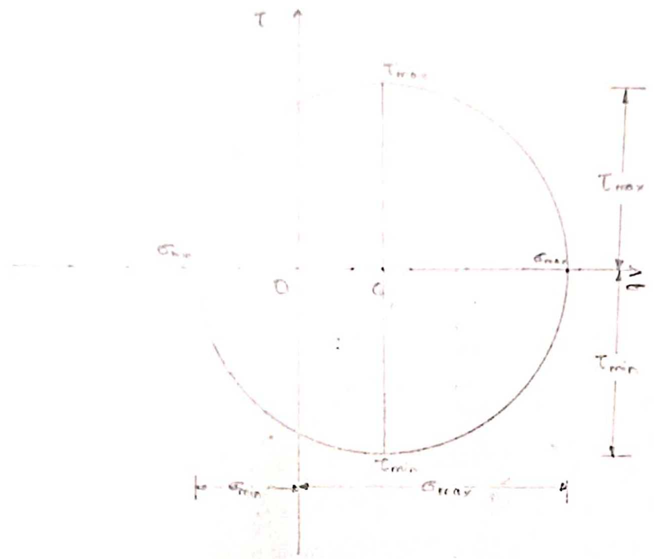
Measure of σ_{max} = 4.7 cm

Measure of σ_{min} = 1.8 cm

Measure of τ_{max} = 3.23 cm

Mohr's Circle for Stress

Scale :- 1 cm = 20 MPa



	Magnitude	Direction with x-axis (anticlockwise)
OC_σ	28.57 MPa	-
R_σ	64.61 MPa	-
σ_{max}	94 MPa	38°
σ_{min}	- 36 MPa	- 52°
τ_{max}	64.61 MPa	-97°
τ_{min}	-64.61 MPa	-7°

Calculations:

$$R_\sigma = R_\varepsilon \times \frac{E}{1+\mu}$$

$$= 420\times10^{-6} \times \frac{200\times10^9}{1+0.3}$$

$$= 64.61 \text{ MPa}$$

$$OC_\sigma = OC_\varepsilon \times \frac{E}{1-\mu}$$

$$= 100\times10^{-6} \times \frac{200\times10^9}{1-0.3}$$

$$= 28.57 \text{ MPa}$$

Measure of R = 3.23 cm

Measure of OC = 1.42 cm

Measure of σ_{max} = 4.7 cm

Measure of σ_{min} = 1.8 cm

Measure of τ_{max} = 3.23 cm

PROBLEM-2 (c)

MOHR'S CIRCLE FOR STRAIN

SCALE :- 1cm = 100×10^{-6}

GIVEN DATA

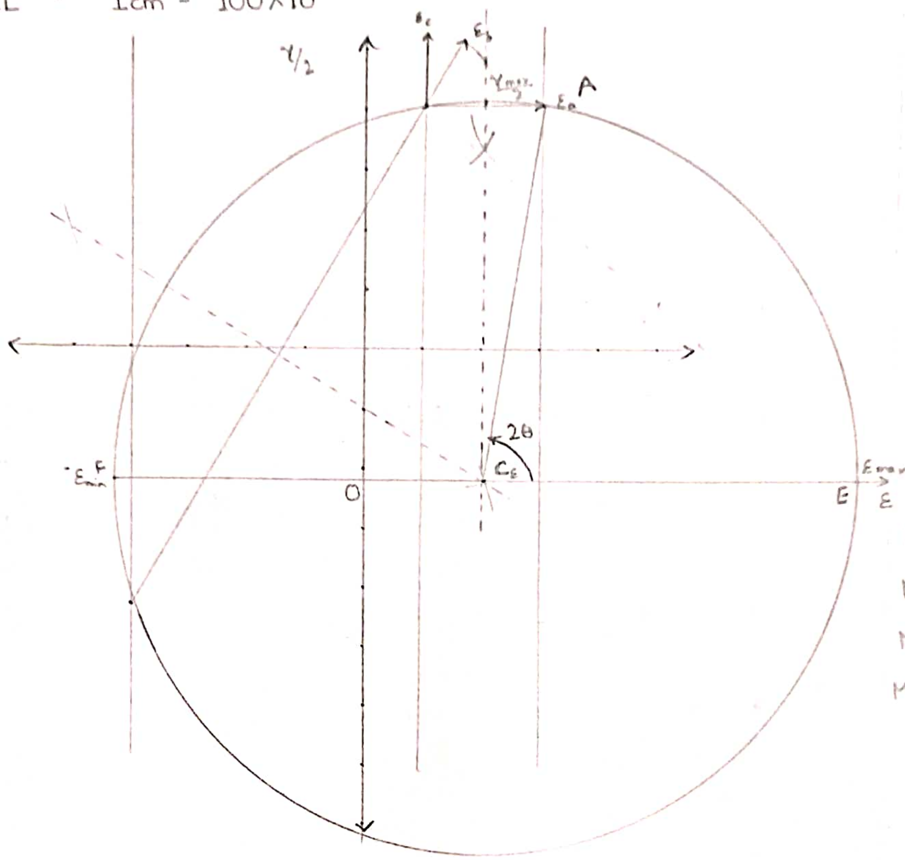
$\varepsilon_a = 300 \times 10^{-6}$

$\varepsilon_b = -400 \times 10^{-6}$

$\varepsilon_c = 100 \times 10^{-6}$

$E = 200$ GPa

$\mu = 0.30$



Measure of $OC_\varepsilon = 2$cm

Measure of $R = 6.4$cm

Measure of $\varepsilon_{max} = 8.4$cm

Measure of $\varepsilon_{min} = 4.3$cm

Measure of $\frac{\gamma_{max}}{2} = 6.4$cm

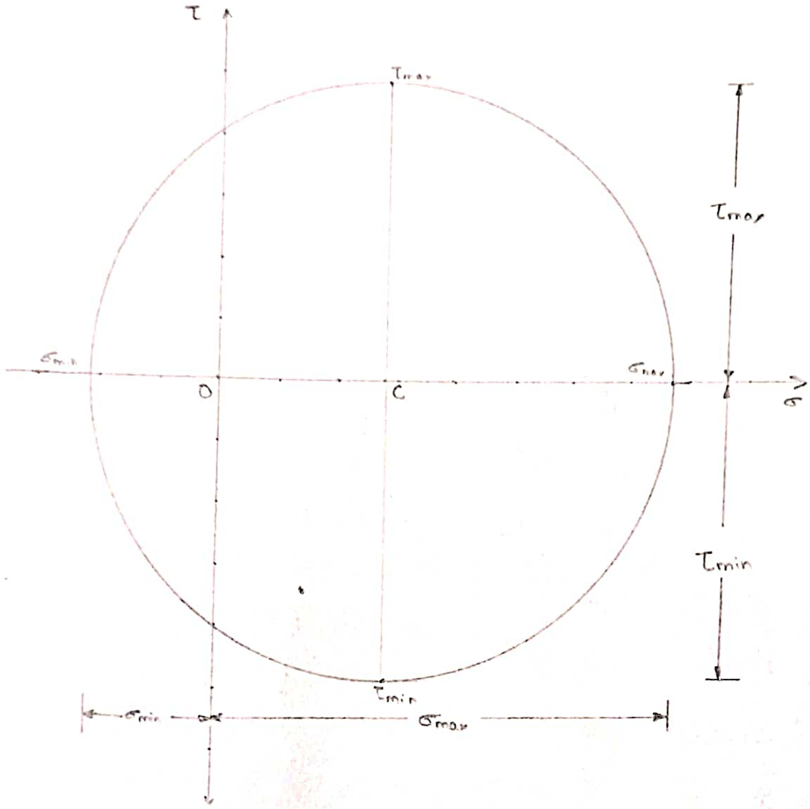
$2\theta = 80°$

$\theta = 40°$

	Magnitude	Direction with x-axis (anticlockwise)
OC_ε	200×10^{-6}	–
R_ε	640×10^{-6}	–
ε_{max}	840×10^{-6}	140°
ε_{min}	430×10^{-6}	50°
$\frac{\gamma_{max}}{2}$	640×10^{-6}	5°
$\frac{\gamma_{min}}{2}$	-640×10^{-6}	95°

MOHR'S CIRCLE FOR STRESS

SCALE :- 1 cm = 20 MPa



	Magnitude	Direction with x-axis
OC_σ	57.142 MPa	–
R_σ	98.461 MPa	–
σ_{max}	154 MPa	140°
σ_{min}	– 42 MPa	50°
τ_{max}	98.461 MPa	5°
τ_{min}	-98.461 MPa	95°

CALCULATIONS

$$R_\sigma = R_\epsilon \times \frac{E}{1+\mu}$$

$$= 640 \times 10^{-6} \times \frac{200 \times 10^9}{1.3}$$

$$= 98.461 \text{ MPa}$$

$$OC_\sigma = OC_\epsilon \times \frac{E}{1-\mu}$$

$$= 200 \times 10^{-6} \times \frac{200 \times 10^9}{0.7}$$

$$= 57.142 \text{ MPa}$$

Measure of R = 4.9 cm

Measure of OC = 2.8 cm

Measure of σ_{max} = 7.7 cm

Measure of σ_{min} = 2.1 cm

Measure of τ_{max} = 4.9 cm

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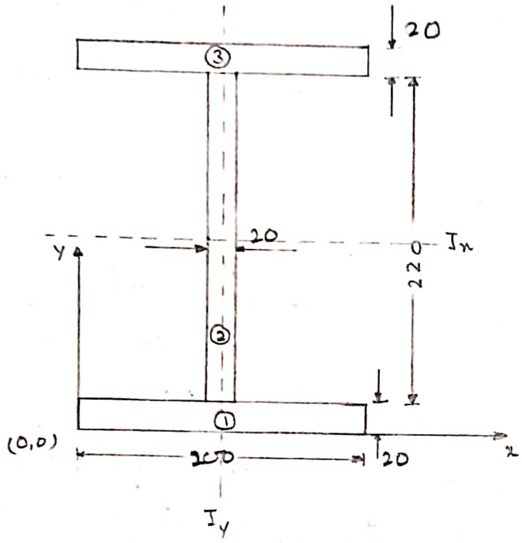
PROBLEM - 3(a)

SPACE DIAGRAM

SCALE :- 10 mm = 40 mm

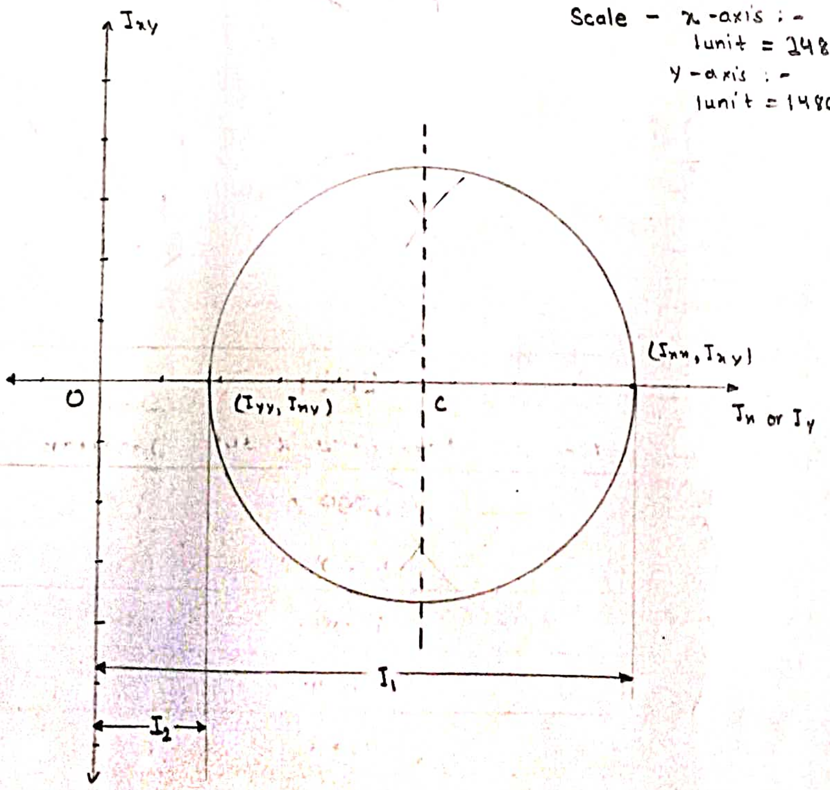


MOHR'S CIRCLE

A (I_{xx}, I_{xy}) , B (I_{yy}, $-I_{xy}$)

A (29293.33, 0), B (2681.32 , 0) ,

Scale - x-axis :-
1 unit = 1480.14 cm^4
y-axis :-
1 unit = 1480.14 cm^4



CALCULATIONS

$x_1 = 100 mm$ $y_1 = 10 mm$

$x_2 = 100 mm$ $y_2 = 130 mm$

$x_3 = 100 mm$ $y_3 = 250 mm$

$A_1 = 200 \times 20 = 4000 mm^2$

$A_2 = 220 \times 20 = 4400 mm^2$

$A_3 = 200 \times 20 = 4000 mm^2$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{4000 + 100 + 4400 \times 100 + 4000 \times 100}{4000 + 4400 + 4000}$$

$$\bar{x} = 100 mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{4000 \times 10 + 4400 \times 130 + 4000 \times 250}{4000 + 4400 + 4000}$$

$$\bar{y} = 130 mm$$

∴ centroid (100, 130)

$$I_{xx} = \frac{1}{12} 200 \times 260^3 - 2 \times \frac{1}{12} \times 90 \times 220^3$$

$$= 29293.33 - 15972.0$$

$$I_{xx} = 13321.33 cm^4$$

$$I_{yy} = 2 \times \frac{1}{12} \times 20 \times 200^3 + \frac{1}{12} \times 220 \times 20^3$$

$$= 2666.66 + 14.66$$

$$= 2681.32 cm^4$$

$I_{xy} = 0$ (since symmetry)

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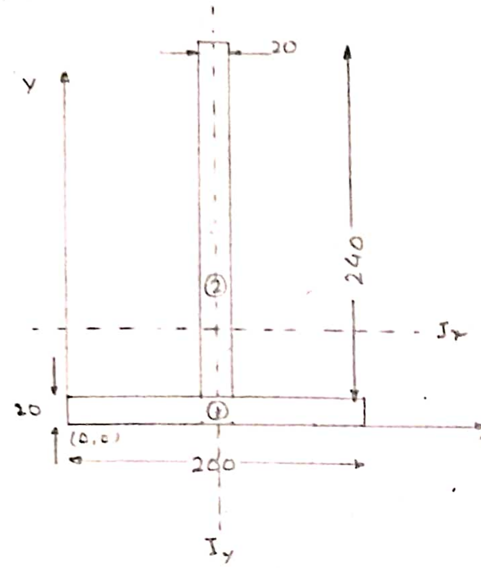
Result

Component	Value $\times 1480.14 cm^4$	Direction
OC	8140.77	-
I_1	13321.26	0°
I_2	2679.05	90°
P_1	5328.5	45°
P_2	-5328.5	135°
R	5328.5	-

PROBLEM-3(b)

SPACE DIAGRAM

SCALE :- 10 mm = 40mm



MOHR'S CIRCLE

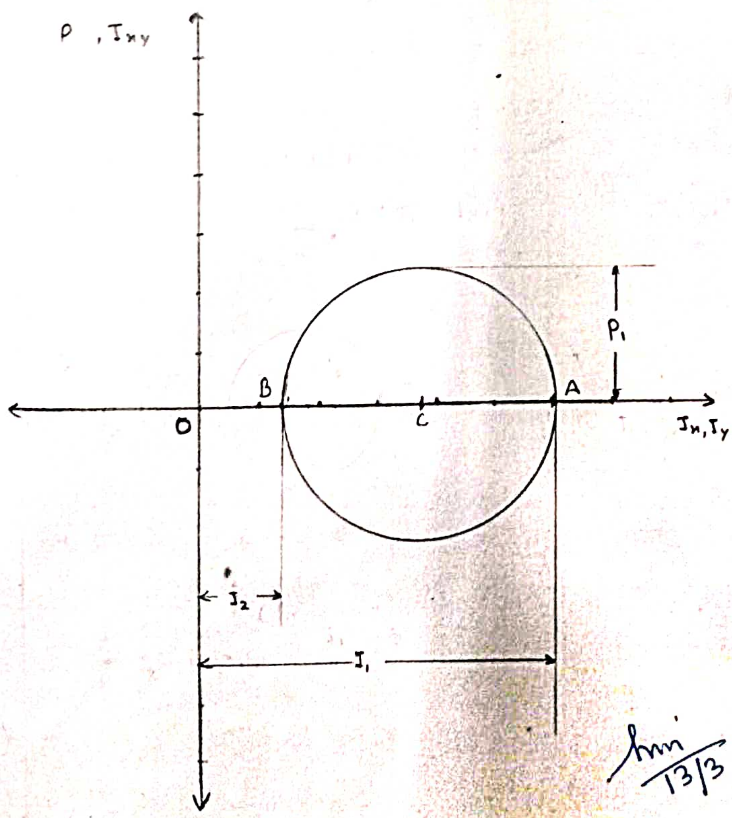
A (I_{xx}, I_{xy}) B (I_{yy}, $-I_{xy}$)

A (6, 0) , B (1.34, 0)

Scale :-

x-axis: 1unit = 10^3 cm^4

y-axis = 1unit = 10^3 cm^4



CALCULATIONS

x_1 = 100 mm y_1 = 10 mm

x_2 = 100 mm y_2 = 130 mm

A_1 = 20×200 = 4000 mm^2

A_2 = 240×20 = 4800 mm^2

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{4000\times100 + 4800\times100}{4000 + 4800}$$

$$\bar{x} = 100 \text{ mm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{4000\times10 + 4800\times140}{4000+4800}$$

$$= 80.9 \text{ mm}$$

$$I_{yy} = \frac{1}{12}\times 20\times200^3 + \frac{1}{12}\times20^3\times240$$

$$= 1333.33 + 16$$

$$I_{yy} = 1349.33 \text{ cm}^4$$

$$I_{xx} = \frac{1}{12}\times20^3\times200 + 20\times200\times(80.9-10)^2 + \frac{1}{12}\times240^3\times20 + 20\times240\times(140-80.9)^2$$

$$= 6004.7 \text{ cm}^4$$

I_{xy} = 0 (as centroid of both rectangle lies on centroidal axis)

RESULT

Component	Value × 10^3 cm^4	Direction
OC	3800 cm^4	–
I_1	6004.7 cm^4	0°
I_2	1349.33 cm^4	90°
P_1	2300 cm^4	45°
P_2	-2300 cm^4	135°
R	2300 cm^4	–

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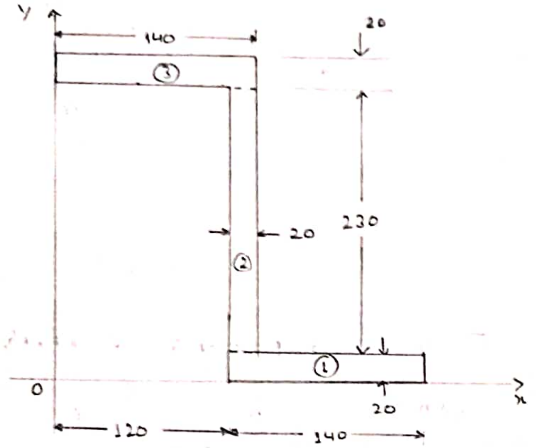
PROBLEM - 3 (c)

SPACE DIAGRAM

SCALE :- 10 mm = 40 mm



MOHR'S CIRCLE

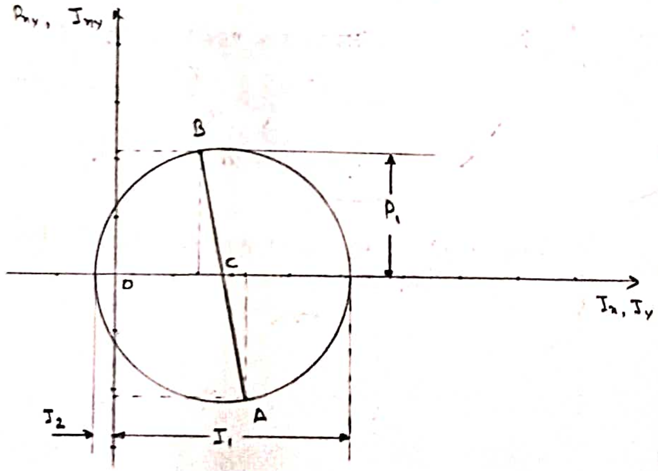
A (I_{xx}, I_{xy}), B $(I_{yy}, -I_{xy})$

A (2.192, − 2.1), B (1.4725, 2.1)

Scale :

I-axis : 1cm = 2×10^3 cm^4

I_{xy} axis : 1cm = 2×10^3 cm^4



RESULT

Component	value x 2×10^3 cm^4	Direction
OC	3800 cm^4	—
I_1	8200 cm^4	23.5°
I_2	800 cm^4	113.5°
P_1	4400 cm^4	68.5°
P_2	− 4400 cm^4	158.5
R	4400 cm^4	—

CALCULATIONS :-

x_1 = 190 mm $\quad$ x_3 = 70 mm

x_2 = 130 mm

y_1 = 10 mm $\quad$ y_3 = 260 mm

y_2 = 135 mm

A_1 = 2800 mm^2

A_2 = 4600 mm^2

A_3 = 2800 mm^2

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{2800 \times 190 + 4600 \times 130 + 2800 \times 70}{2800 + 4600 + 2800}$$

$\bar{x}$ = 130 mm = 13 cm

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{2800 \times 10 + 4600 \times 135 + 2800 \times 260}{2800 + 4600 + 2800}$$

$\bar{y}$ = 135 mm = 13.5 cm

∴ centroid (13, 13.5)

$$I_{xx} = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= \frac{140 \times 20^3}{12} + 2800(10-135)^2 + \frac{20 \times 230^3}{12} + 4600(135-135)^2 + \frac{140 \times 20^3}{12} + 2800(260-135)^2$$

I_{xx} = 4.384×10^3 cm^4

$$I_{yy} = I_{y_1} + I_{y_2} + I_{y_3}$$

$$= \frac{20 \times 140^3}{12} + 2800(190-130)^2 + \frac{230 \times 20^3}{12} + \frac{20 \times 140^3}{12} + 2800(70-130)^2$$

I_{yy} = 2.945×10^3 cm^4

$$I_{xy} = I_{xy_1} + I_{xy_2} + I_{xy_3}$$

$$= 2800(190-130)(10-135) + 4600(130-130)(135-135) + 2800(70-130)(260-135)$$

I_{xy} = -4.2×10^3 cm^4

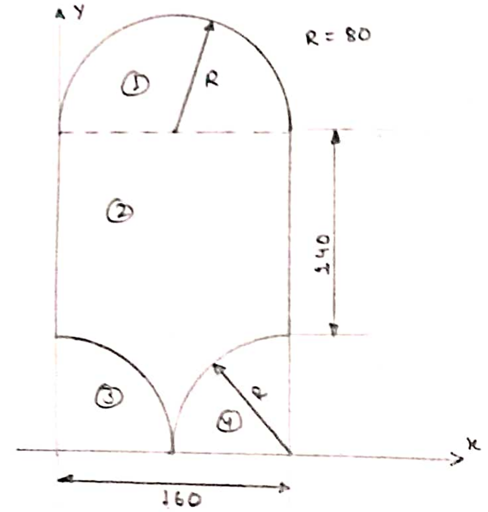
$$\tan 2\theta_p = \frac{2 I_{xy}}{I_x - I_y} = \frac{-2 \times 4.2 \times 10^3}{4.384 \times 10^3 - 2.945 \times 10^3} = \cdot$$

θ_p = 23.5°

PROBLEM - 3 (d)

SPACE DIAGRAM

SCALE :- 10 mm = 40 mm

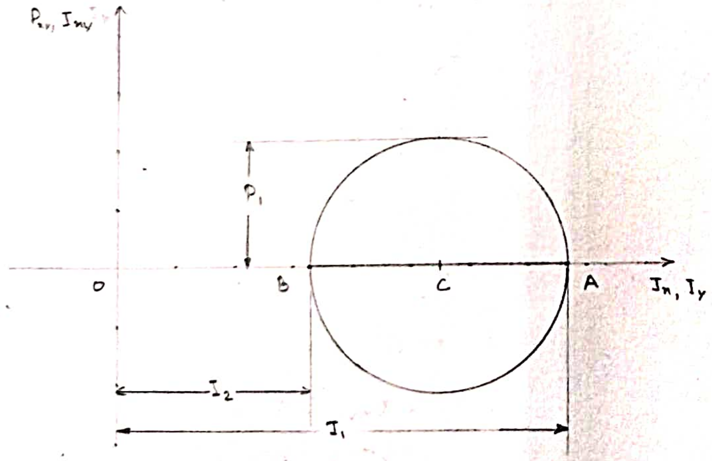


MOHR'S CIRCLE

A (I_{xx} , I_{xy}) , B (I_{yy} , $-I_{xy}$)

A (7.655 , 0) , B (3.27 , 0)

Scale :
I-axis : 1cm = 2×10^3 cm^4
I_{xy} axis : 1cm = 2×10^3 cm^4



RESULT

Component	value $\times 2 \times 10^3$ cm^4	Direction
OC	11000 cm^4	—
I_1	15310 cm^4	0°
I_2	6540 cm^4	90°
P_1	4400 cm^4	45°
P_2	− 4400 cm^4	135°
R	4400 cm^4	—

CALCULATIONS :-

$x_1 = 80$ mm $\qquad x_3 = 33.95$ mm

$x_2 = 80$ mm $\qquad x_4 = 126.05$ mm

$y_1 = 253.95$ mm $\qquad y_2 = 110$ mm

$y_3 = 33.95$ mm $\qquad y_4 = 33.95$ mm

$A_1 = 10053.1$ mm^2 $\qquad A_2 = 35200$ mm^2

$A_4 = A_3 = \frac{\pi}{4} \times 80^2 = 5026.55$ mm^2

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{10053.1 \times 80 + 35200 \times 80 + 5026.55 \times (33.95 + 126.05)}{10053.1 + 35200 + 5026.55 + 5026.55}$$

$\bar{x} = 80$ mm $= 8$ cm

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{10053.1 \times 253.95 + 35200 \times 110 + 5026.55 (33.95 + 33.95)}{10053.1 + 35200 + 2 \times 5026.55}$$

$\bar{y} = 172.88$ mm $= 17.288$ cm

$I_{xx} = I_{x_1} + I_{x_2} + I_{x_3} + I_{x_4}$

$= 0.11 \times 80^4 + 10053.1 \times (253.95 - 172.8)^2 + \frac{160 \times 220^3}{12} + 35200 \, (110 - 172.8)^2 - 0.055 \times 80^4 - 5026.55 \, (33.95 - 172.8)^2 - 0.055 \times 80^4 - 5026.55 \, (33.95 - 172.8)^2$

$I_{xx} = 15.31 \times 10^3$ cm^4

$I_{yy} = I_{y_1} + I_{y_2} + I_{y_3} + I_{y_4}$

$= 0.393 \times 80^4 + \frac{220 \times 160^3}{12} - 0.055 \times 80^4 \times 2 - 5026.55 \, (33.95 - 80)^2 - 5026.55 \, (126.05 - 80)^2$

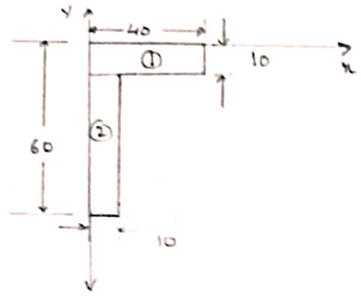
$I_{yy} = 6.54 \times 10^3$ cm^4

$I_{xy} = 0$ (as centroid of all shapes lies on centroidal axis)

PROBLEM - 3 (e)

SPACE DIAGRAM

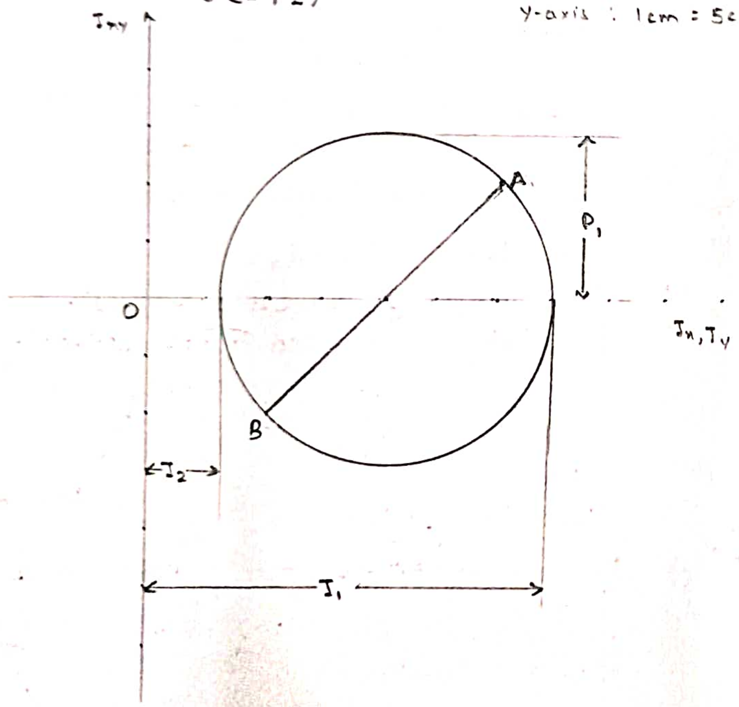
SCALE : 10 mm = 20 mm



MOHR'S CIRCLE

A (6, 2) B (2, 2)

Scale : x-axis : 1cm = 5 cm⁴
y-axis : 1cm = 5 cm⁴



RESULT

Component	Value x 5	Direction (Anti clockwise)
OC	20.5 cm⁴	–
I_1	35 cm⁴	157.5°
I_2	6.25 cm⁴	67.5°
P_1	14.5 cm⁴	22.5°
P_2	– 14.5 cm⁴	112.5°
R	14.5 cm⁴	–

CALCULATIONS :-

$x_1 = 20mm$ $x_2 = 5\ mm$

$y_1 = 5\ mm$ $y_2 = 35\ mm$

$A_1 = 400\ mm^2$ $A_2 = 500\ mm^2$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$\bar{x} = \frac{400 \times 20 + 500 \times 5}{400 + 500}$$

$$\bar{x} = 11.67\ mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{400 \times 5 + 500 \times 35}{400 + 500}$$

$$\bar{y} = 21.67\ mm$$

$$I_{xx} = I_{x_1} + I_{x_2}$$

$$= \frac{40 \times 10^3}{12} + 400(5 - 21.67)^2 + \frac{10 \times 50^3}{12} + 500(35 - 21.67)^2$$

$$I_{xx} = 31\ cm^4$$

$$I_{yy} = I_{y_1} + I_{y_2}$$

$$= \frac{10 \times 40^3}{12} + 400(20 - 11.67)^2 + \frac{10^3 \times 50}{12} + 500(5 - 11.67)^2$$

$$I_{yy} = 11\ cm^4$$

$$I_{xy} = I_{xy_1} + I_{xy_2}$$

$$= 400(8.3)(16.67) + 500(-6.67)(-13.33)$$

$$I_{xy} = 10\ cm^4$$

$$\tan 2\theta_p = \frac{2 I_{xy}}{I_x - I_y} = 45°$$

$$\theta_p = 22.5°$$

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