Theory of Machines & Mechanism

MEC 302

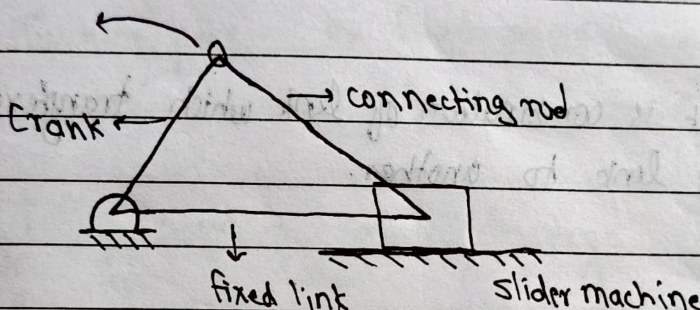
Introduction to Mechanism

Mechanics of Machines

- kinematics of machine (measurements of motion without considering cause)
- kinetics of machine (cause of motion)

In rotation, kinematics : α, θ, s, a, v

kinetics : τ, F



Slider - Crank mechanism

I.C. engine (Internal Combustion)

The 4 links in I.C

Crank - complete rotational motⁿ

Slider - sliding motion

Connecting rod - Combined translational

Fixed link - no motion

Combination of 2 links producing a motion is called kinematic pair.

Combination of kinematic pair is kinematic chain.

Motion transfer:

i. Through link
ii. —"— gear
iii. —"— pulley system
iv. —"— hydrolics

Gear: i. simple gear tail
ii. epicycle —"—
iii. reverted —"—
iv. compound —"—

Mechanism:

It is combination of link which transfers motion from one link to another.

Machine:

It is particular mechanism which transfers motion from one link to another link through useful workdone.

imp All machines are mechanism but not vice versa. Justify

Machines are made of mechanical mechanisms. They don't transfer work but only the motion. But a mechanism may not transfer motion through useful workdone. Hence every time, mechanism may not be a machine

Types of links:

Rigid link

~~primary~~ singular

binary

ternary

qualernary

* Kinematic pair

Two links of a machine when in contact with each other are said to form a pair. A kinematic pair consists of 2 links which have relative motion bet^n them.

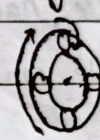
Kinematic pair can be classified as -

1) Type of relative motion
2) 〃 〃 contact
3) — mechanical constraint

1) Kinematic pair according to relative motion

Sliding pair - ex rectangular bar in rectangular hole

Turning pair - ex. Crank shaft turning in a bearing
Cycle wheel revolving over axis

Rolling pair - ex. Ball bearing 

Screw pair - hellical motion ex. screw & jack
rotational + rectilinear

Spherical pair - ex shoulder joint

2) According to type of contact
Surface contact - lower order
point / line contact - higher pair

3) According to type of mechanical constraint,

closed pair ex ball bearing
unclosed pair ex. cam & follower

* Kinematic chains:

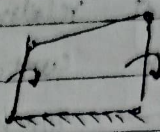
Chain of mechanical links

Fourbar chain

Slider Crank chain

$N = 2P - 4$

$= \frac{2(J+2)}{3}$



N = 4

P = 4

J - 4

N - no. of links

P - No. of pairs

J - No. of joints

* Degree of freedom

A particle can have max 3 d.o.f. in translational motion

A particle can have max 3 dof in rotational motion

A stationary object has 0 d.o.f.

Rectangular bar in rectangular hole has 1 d.o.f

Circular bar in circular hole - 2 dof

Ball & socket joint, pen stand - 3 dof

F - degree of freedom

N - no. of links

P_1 - kinematic pair with 1 d.o.f.

P_2 - " double note

$$F = 3(N-1) - 2P_1 - P_2$$

For fourbar mechanism : $N = 4$ $P_1 = 4$ $P_2 = 0$

$F = 1$

for fivebar mechanism $N = 5$ $P_1 = 5$ $P_2 = 0$

$F = 2$

for slider Crank mechanism $N = 4$ $P_1 = 4$ $P_2 = 0$

$F = 1$

Mechanism

planar mechanism	Spatial mechanism
All points of mechanism move in parallel plane	They don't
ex. slider-crank	ex. robotic manipulator

D.O.F. :

Minimum no of co-ordinates req. to completely specify the relative moment of the links of the mechanism in space

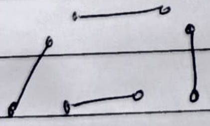
for n bar mechanism, d.o.f. = n-1

order = no. of joints - 1

Kutzbach Mechanism

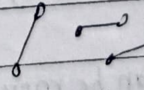
For defining a position & orientation of a 2-D object, 3 independent parameters are req.

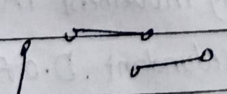
x, y & angle of orientation about Z axis



all 4 are mobile in 2D

dof = 3x4 generalized - 3n

for mechanism, atleast 1 is fixed  = 3(n-1) = dof



3(n-1) - 2 i.e. loss of 2 dof due to 1 joint equivalent

when there are j no. of first order joints,

$$dof = 3(n-1) - 2j$$

second order joint = j_2
third order joint = j_3

j is equivalent first order joint

$$j = j_1 + 2j_2 + 3j_3 + 4j_4$$

Imp

$$dof = 3(n-1) - 2j$$

If dof 0 ⇒ not a mechanism. it is statically indeterminate structure

dof < 0 ⇒ —"—

Grubler's Criterion - dof = 1 ⇒ constrained mechanism

* Equivalent linkages

Higher order pairs are replaced by equivalent lower o mechanisms.

For h no. of higher order pairs, dof is given by,

$$d.o.f. = 3(n-1) - 2j - h$$

If there are F_r no. of redundant D.O.F.,

$$dof = 3(n-1) - 2j - h - F_r$$

If a link can be moved without causing any movement in the mechanism, the link is said to have redundant D.O.F.

D.O.F. of Spatial Mechanism

$$F = 6(n-1) - 5(R+P+H) - 4C - 3S - F_r$$

R = no. of revolute pair
P = no. of prismatic pair
H = no. of screw pair
C = no. of cylindrical pair
S = no. of spherical pair

* Grashof's law

for a planar 4 bar linkage the sum of shortest & longest link lengths can't be greater than the sum of the remaining two link lengths if there is to be continuous relative rotation betn two members.

$$s + l \leq p + q$$

when $s+l < p+q$ ⇒ Grashof linkage Class I kinematic chain
$s+l = p+q$ ⇒ —"— Class III —"—
$s+l > p+q$ ⇒ Non —"— Class II —"—

$s+l > p+q$ i.e. In Class II kinematic chain, only double rocker mechanism is possible

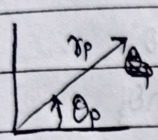
for ~~Grashof's linkage one link~~ continuously has ~~to be in~~ rotation

* Index of merit
Transmission angle: Acute angle betn coupler & follower
When transmission force is 90°, we will get maximum force responsible for torque
Transmission angle value determines the quality of motion transmission

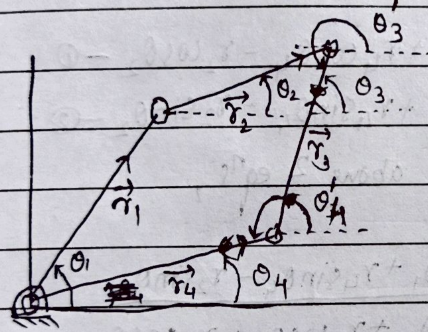
Kinematic Analysis of Planar Mechanisms

Aim:

To establish the relationship betⁿ motⁿ of various components or links or elements of mechanism.



	polar form	Cartesian form
	$r_p \angle \theta_p$	$\hat{i}\, r_p \cos\theta_p + \hat{j}\, r_p \sin\theta_p$
complex no. representⁿ	$r_p e^{i\theta_p}$	$r_p \cos\theta_p + j\, r_p \sin\theta_p$



$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 = 0$$

.... loop closure eqⁿ

$$\vec{r}_1 = r_1 \cos\theta_1\, \hat{i} + r_1 \sin\theta_1\, \hat{j}$$

& so on

$$\vec{r}_2 = r_2 \cos\theta_2\, \hat{i} + r_2 \sin\theta_2\, \hat{j}$$

$$\vec{r}_3 = r_3 \cos\theta_3'\, \hat{i} + r_3 \sin\theta_3'\, \hat{j}$$

$$\vec{r}_4 = r_4 \cos\theta_4'\, \hat{i} + r_4 \sin\theta_4'\, \hat{j}$$

Angles are only to be measured between the +ve x axis & tail of the vector

$$\theta_3' = 180 + \theta_3 \qquad \theta_4' = 180 + \theta_4$$

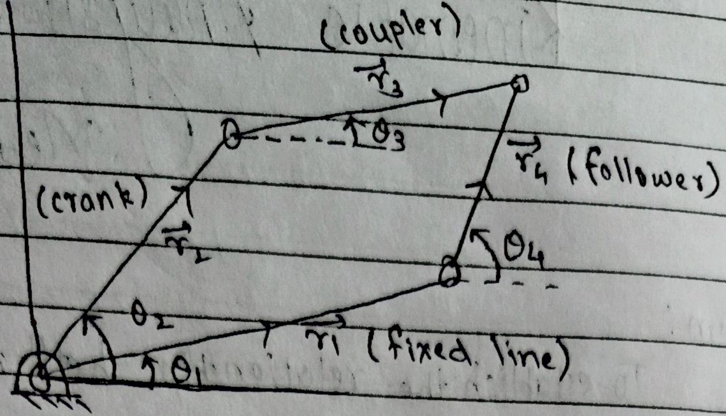
$$\vec{r}_3 = -[r_3 \cos\theta_3 + r_3 \sin\theta_3]$$

$$\vec{r}_4 = -[r_4 \cos\theta_4 + r_4 \sin\theta_4]$$

$$[r_1 \cos\theta_1 + r_2 \cos\theta_2]\hat{i} + [r_1 \sin\theta_1 + r_2 \sin\theta_2]\hat{j} = [r_4 \cos\theta_4 + r_3 \cos\theta_3]\hat{i} + [r_4 \sin\theta_4 + r_4 \sin\theta_3]\hat{j}$$

$$\therefore r_1 \cos\theta_1 + r_2 \cos\theta_2 = r_3 \cos\theta_3 + r_4 \cos\theta_4$$

$$r_1 \sin\theta_1 + r_2 \sin\theta_2 = r_3 \sin\theta_3 + r_4 \sin\theta_4$$



for this standard fig with standard notⁿs,

$$\vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$$

$$r_1\cos\theta_1 + r_4\cos\theta_4 = r_2\cos\theta_2 + r_3\cos\theta_3$$
$$r_1\sin\theta_1 + r_4\sin\theta_4 = r_2\sin\theta_2 + r_3\sin\theta_3$$

squaring & adding ~~above~~ 2 eqⁿs,

$$r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2 \quad -(1)$$
$$r_3\sin\theta_3 = r_1\sin\theta_1 + r_4\sin\theta_4 - r_2\sin\theta_2 \quad -(2)$$

dividing above 2 eqⁿs,

$$\tan\theta_3 = \frac{r_1\sin\theta_1 + r_4\sin\theta_4 - r_2\sin\theta_2}{r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2}$$

squaring & adding (1) & (2),

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 + 2r_1r_4(\cos\theta_1\cos\theta_4 + \sin\theta_1\sin\theta_4)$$
$$- 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) - 2r_4r_2(c\theta_2 c\theta_4 + s\theta_2 s\theta_4)$$

$$\therefore \overbrace{(2r_1r_4\cos\theta_1 - 2r_2r_4\cos\theta_2)}^{A_1}\cos\theta_4 + \overbrace{(2r_1r_4\sin\theta_1 - 2r_2r_4\sin\theta_2)}^{B_1}\sin\theta_4$$
$$+ \underbrace{(r_1^2 + r_2^2 + r_4^2 - r_3^2 - 2r_1r_2\cos(\theta_1 - \theta_2))}_{C_1} = 0$$

$$\therefore A_1\cos\theta_4 + B_1\sin\theta_4 + C_1 = 0$$

$$\therefore A_1\left[\frac{1-\tan^2\theta_4/2}{1+\tan^2\theta_4/2}\right] + B_1\left[\frac{2\tan\theta_4/2}{1+\tan^2\theta_4/2}\right] + C_1$$

$\therefore A_1[1-\tan^2\theta_4/2] + B_1[2\tan\theta_4/2] + C_1[1+\tan^2\theta_4/2] = 0$

$\therefore (C_1 - A_1)\tan^2\theta_4/2 + 2B_1\tan\theta_4/2 + (C_1 + A_1) = 0$

$\therefore A\tan^2\theta_4/2 + B\tan\theta_4/2 + C = 0$

$\therefore \tan\theta_4/2 = \dfrac{-B \pm \sqrt{B^2-4AC}}{2A}$ $\quad \because A = C_1 - A_1,\ B = 2B_1,\ C = C_1 + A_1$

$\therefore \theta_4 = 2\tan^{-1}\left[\dfrac{-B \pm \sqrt{B^2-4AC}}{2A}\right]$

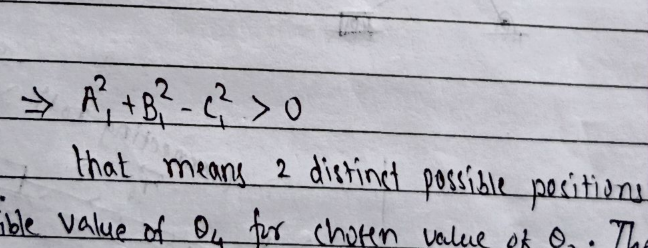
$\ldots \boxed{-\pi \leq \theta_4 \leq \pi}$

$B^2 - 4AC < 0 \Rightarrow 4B_1^2 - 4(C_1^2 - A_1^2) < 0 \Rightarrow A_1^2 + B_1^2 - C_1^2 < 0$

$\Downarrow$

imaginary roots

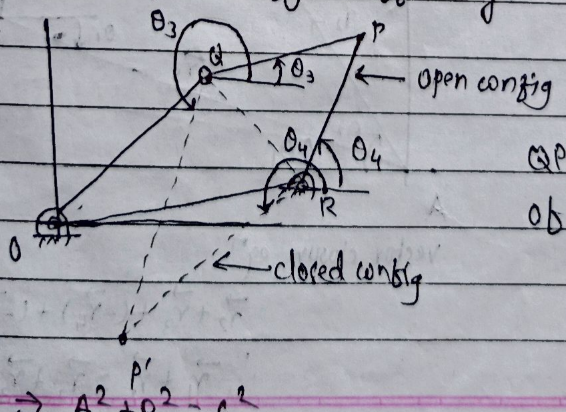
that means mechanism can't be assembled for the specified value of θ_2. The link lengths chosen are not capable of connect^n for the chosen value of θ_2



$B^2 - 4AC > 0 \Rightarrow A_1^2 + B_1^2 - C_1^2 > 0$

$\Downarrow$

distinct real roots that means 2 distinct possible positions are there i.e 2 possible value of θ_4 for chosen value of θ_2. They are called as crossed & open configur^n of linkage.



QP'R is mirror img of QRR wrt line QR

$B^2 - 4AC = 0 \Rightarrow A_1^2 + B_1^2 = C_1^2$

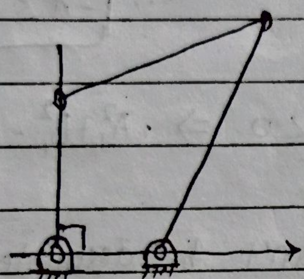
Only unique pos^n is possible for given θ_2

1] A fourbar linkage with fixed link length 1 unit, crank length is 2 unit, couplar length 3.5 unit, follower length is 4 unit & $\theta_1 = 0^\circ$

find follower angle & couplar angle when crank is in the

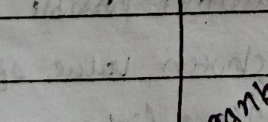
a. position of 90° as well as 180°.

b. If crank angular velocity is 10 rad/sec & angular ~~velocity~~ acceln is 0, find $\dot{\theta}_3$, $\dot{\theta}_4$, $\ddot{\theta}_3$, $\ddot{\theta}_4$ for both position

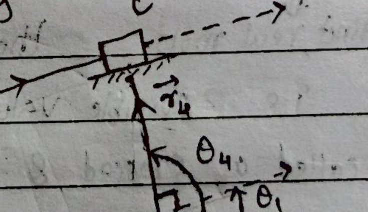


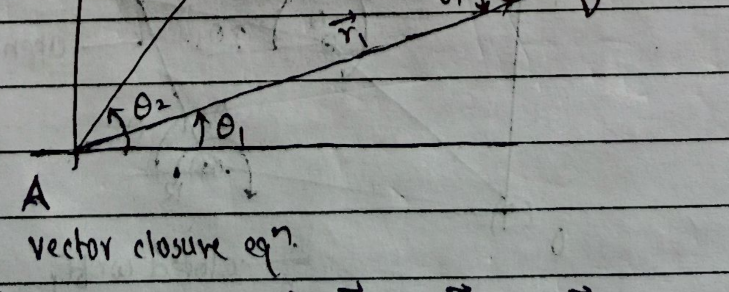
* Slider Crank Mechanism

Centred slider crank



Offset slider crank mech





vector closure eqn.

$$\vec{r}_2 + \vec{r}_3 + (-\vec{r}_4) + (-\vec{r}_1) = 0$$

$$\therefore \vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$$

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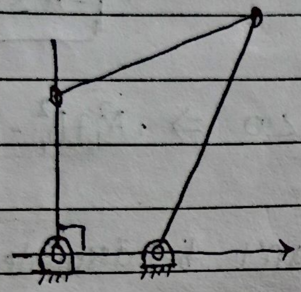
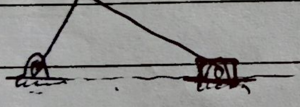
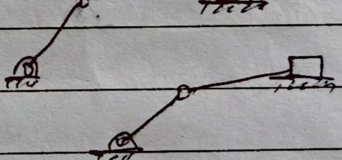


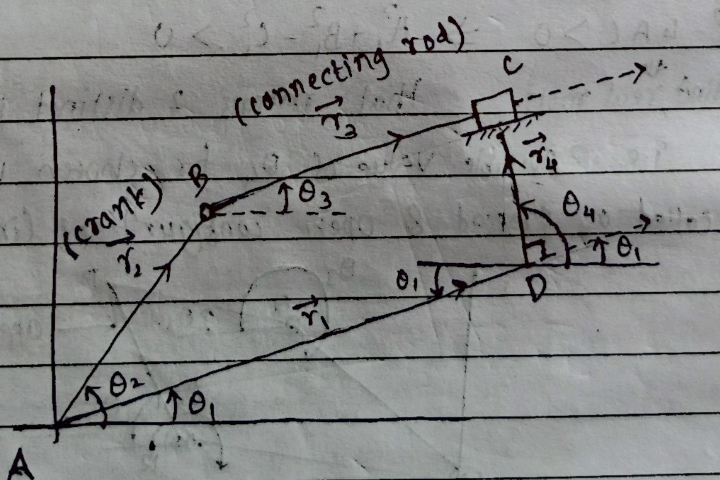
Slider Crank Mechanism

Centred slider crank



Offset slider crank mech





vector closure eqn.

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$$\therefore \vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$$

$[r_2, r_3, r_4, \theta_1, \theta_4$ are constants- $]$ for slider crank

from fig, $\theta_4 = 90° + \theta_1$

θ_1 is constant $\therefore \theta_4$ is constant

r_1 is variable

scalar form of loop closure eq^n:

$$r_2\cos\theta_2 + r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4$$
$$r_2\sin\theta_2 + r_3\sin\theta_3 = r_1\sin\theta_1 + r_4\sin\theta_4$$

1] In slider crank mechanism, $\theta_2 = 45°$, $\dot{\theta}_2 = 10$ rad/sec, $\ddot{\theta}_2 = 0$. The link lengths r_2 & r_3 are 5 unit & 8 unit resp. The line of mot^n of point C is along line AC. Find pos^n, velocity & accel^n of C ~~& the angular velocity & accel^n of link 3.~~

* 4-Bar mechanism ($r_1, r_2, r_3, r_4, \theta_1$ are const)

Differentiating loop closure eq^n,

$$\vec{\dot{r}}_1 + \vec{\dot{r}}_2 + \vec{\dot{r}}_3 + \vec{\dot{r}}_4 = 0$$

$\therefore$ eq^n in scalar form

$$r_2\dot{\theta}_2\sin\theta_2 + r_3\dot{\theta}_3\sin\theta_3 = r_4\dot{\theta}_4\sin\theta_4$$
$$r_2\dot{\theta}_2\cos\theta_2 + r_3\dot{\theta}_3\cos\theta_3 = r_4\dot{\theta}_4\cos\theta_4$$

where $\dot{\theta} = \omega$

$\cancel{V = \vec{r}\omega = \vec{r} \times \dot{\theta}}$

linear $V = \vec{\dot{r}}$

$$\vec{r}_1 = r_1\cos\theta_1\hat{i} + r_1\sin\theta_1\hat{j}$$

ex. $V_4 = \vec{\dot{r}}_4$

$$= -r_4\dot{\theta}_4\sin\theta_4\hat{i} + r_4\dot{\theta}_4\cos\theta_4\hat{j}$$
$$= r_4\dot{\theta}_4(-\sin\theta_4\hat{i} + \cos\theta_4\hat{j})$$

Again differentiating velocity eq^n,

$$\ddot{\vec{r}}_1 + \ddot{\vec{r}}_2 + \ddot{\vec{r}}_3 + \ddot{\vec{r}}_4 = 0$$

$$r_2\ddot{\theta}_2 \sin\theta_2 + r_2\dot{\theta}_2^2 \cos\theta_2 + r_3\ddot{\theta}_3 \sin\theta_3 + r_3\dot{\theta}_3^2 \cos\theta_3 = r_4\ddot{\theta}_4 \sin\theta_4 + r_4\dot{\theta}_4^2 \cos\theta_4$$

$$r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2 + r_3\ddot{\theta}_3 \cos\theta_3 - r_3\dot{\theta}_3^2 \sin\theta_3 = r_4\ddot{\theta}_4 \cos\theta_4 - r_4\dot{\theta}_4^2 \sin\theta_4$$

linear $a = \dot{\vec{v}} = \ddot{\vec{r}}$

ex. $a_2 = \ddot{\vec{r}}_2$

$$= (-r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2)\hat{i} + (r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2)\hat{j}$$

Slider Crank Mechanism ($r_2, r_3, r_4, \theta_1, \theta_4$ const)

differentiating loop closure eq^n,

$$\dot{\vec{r}}_2 + \dot{\vec{r}}_3 = \dot{\vec{r}}_1 + \dot{\vec{r}}_4$$

scalar form:

$$-r_2\dot{\theta}_2 \sin\theta_2 - r_3\dot{\theta}_3 \sin\theta_3 = \dot{r}_1 \cos\theta_1$$

$$-\dot{r}_1 \sin\theta_1 + r_3\dot{\theta}_3 \cos\theta_3 = -r_2\dot{\theta}_2 \cos\theta_2$$

linear velocity of r_2 =

$$v = \dot{r}_2 = r_2\dot{\theta}_2(-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j})$$

differentiating again,

$$\ddot{\vec{r}}_1 + \ddot{\vec{r}}_2 + \ddot{\vec{r}}_3 + \ddot{\vec{r}}_4 = 0$$

scalar form:

$$-r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2 - r_3\ddot{\theta}_3 \sin\theta_3 - r_3\dot{\theta}_3^2 \cos\theta_3 = \ddot{r}_1 \cos\theta_1$$

$$r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2 + r_3\ddot{\theta}_3 \cos\theta_3 - r_3\dot{\theta}_3^2 \sin\theta_3 = \ddot{r}_1 \sin\theta_1$$

linear acceln $a = \vec{\dot{v}} = \vec{\ddot{r}}$

ex. $\vec{\ddot{r}}_2 = (r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2)\hat{i} + (r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2)\hat{j}$

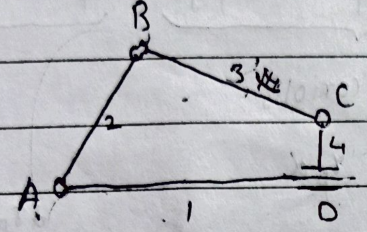
vimp

vector position of slider $= \vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$

velocity vector of slider $= \vec{v}_1 + \vec{v}_4 = \vec{\dot{r}}_1 + \vec{\dot{r}}_4 = \vec{v}_2 + \vec{v}_3 = \vec{\dot{r}}_2 + \vec{\dot{r}}_3$

acceln vector of slider $= \vec{a}_1 + \vec{a}_4 = \vec{\dot{v}}_1 + \vec{\dot{v}}_4 = \vec{\ddot{r}}_1 + \vec{\ddot{r}}_4 = \vec{a}_2 + \vec{a}_3 = \vec{\dot{v}}_2 + \vec{\dot{v}}_3 = \vec{\ddot{r}}_2 + \vec{\ddot{r}}_3$

* Inversion of slider Crank mechanism



1 is fixed - direct acting engine machine / IC engine / Reciprocating engine

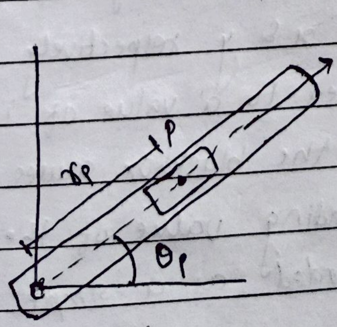
2 is fixed - Slotted lever quick return mechanism

3 is fixed - Oscillating cylinder engine mechanism

4 is fixed - ~~hand~~ hand pump mechanism

* Coriolis acceln

When a sliding joint is present on a rotating link, an additional component of acceln will be present called as Coriolis acceln.



$\vec{r}_p = r_p(\cos\theta_p \hat{i} + \sin\theta_p \hat{j})$

velocity of point P $= \vec{V}_p = \vec{\dot{r}}_p$

r_p & θ_1 will vary with time

$$\therefore V_p = \underbrace{r_p\dot{\theta}_p(-\sin\theta_p \hat{i} + \cos\theta_p \hat{j})}_{\text{rotatory velocity}} + \underbrace{\dot{r}_p(\cos\theta_p \hat{i} + \sin\theta_p \hat{j})}_{\text{translatory velocity } V_{p\,slip}}$$

Imp

Accel^n at point P $= \vec{a}_p = \ddot{\vec{r}}_p = \dot{\vec{V}}_p$

$$\vec{a} = r_p\ddot{\theta}_p(-\sin\theta_p\hat{i} + \cos\theta_p\hat{j}) + r_p\dot{\theta}_p^2(-\cos\theta_p\hat{i} - \sin\theta_p\hat{j})$$
$$+ \dot{r}_p\dot{\theta}_p(-\sin\theta_p\hat{i} + \cos\theta_p\hat{j}) + \dot{r}_p\dot{\theta}_p(-\sin\theta_p\hat{i} + \cos\theta_p\hat{j})$$
$$+ \ddot{r}_p(\cos\theta_p\hat{i} + \sin\theta_p\hat{j})$$

$$\therefore \vec{a} = \overbrace{r_p\ddot{\theta}_p(-\sin\theta_p\hat{i} + \cos\theta_p\hat{j})}^{a_{tangential}} + \overbrace{[-r_p\dot{\theta}_p^2(\cos\theta_p\hat{i} + \sin\theta_p\hat{j})]}^{a_{rotational}}$$
$$+ \underbrace{2\dot{r}_p\dot{\theta}_p(-\sin\theta_p\hat{i} + \cos\theta_p\hat{j})}_{a_{coriolis}} + \underbrace{\ddot{r}_p(\cos\theta_p\hat{i} + \sin\theta_p\hat{j})}_{a_{slip}}$$

$\therefore |a_{tangent}| = r_p\ddot{\theta}_p = r_p\alpha$

$\frac{dr_p}{dt} = V_p$

rotational i.e normal or centripetal $|a_{rotational}| = r_p\dot{\theta}_p^2 = r_p\omega^2$

$\frac{d\theta_p}{dt} = \omega$

$|a_{coriolis}| = 2\dot{r}_p\dot{\theta}_p = 2V\omega$

$\frac{d^2\theta_p}{dt^2} = \alpha$

$|a_{slip}| = \ddot{r}_p = \dot{V}_p = dV_p/dt$

* function generat^n problem

In fun^n gen^n mot^n of input & output links must be corelated. The kinematic synthesis may be designed to corelate input & output such that - as the input moves by 'x' output moves by $y = f(x)$ for the range $x_0 \le x \le x_{n+1}$

In case of rotary input & output the angle of rot^n θ_2 & θ_4 are always in linear rel^n with x & y respectively.

When the input is rotated to a value of independent parameter 'x' the mechanism in the blackbox causes the output link to turn to the corresponding value of the dependent variable $y = f(x)$. This may be regarded as a simple case of a mechanical analog computer.

Dimensional synthesis

Exact synthesis: By exact synthesis, we mean that the generated funⁿ by the physical mechanism fits the desired funⁿ at all points in the interval.

Approximate synthesis: By approximate synthesis, we mean that the generated funⁿ by the physical mechanism fits the desired funⁿ at the finite no of points in the interval.

Accuracy points / Precision points:
The points at which the generated & desired functions agree

Structural error:
Defined as the theoretical difference between the funⁿ generated by the synthesized linkage & the funⁿ originally prescribed
Structural error is inherent in approximate synthesis

Chebyshev's spacing of accuracy points
For 'n' precision positions in the range $x_0 \leq x \leq x_{n+1}$, the Chebyshev spacing is

$$x_j = \frac{x_{n+1}+x_0}{2} - \frac{x_{n+1}-x_0}{2}\cos\left(\frac{(2j-1)\pi}{2n}\right)$$

for $j = 1,2,3,4$

Scale factor for input & output motⁿ
mechanized variables θ_2 & θ_4
functional variables x & y

The orientation of driver link (θ_2) represents the independent variables 'x'. The orientatⁿ of driven link (θ_4) represents the dependent variable 'y'

x & θ_2 and y & θ_4 have linear relation b/w each other let $\theta_2^{(i)}$ be initial value of θ_2 representing x_0

$\theta_4^{(i)}$ be ———— θ_4 " $y_0 = f(x_0)$

The input & output scale factors m_x & m_y are -

$$m_x = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0} = \frac{\theta_2 - \theta_2^{(i)}}{x - x_0}$$

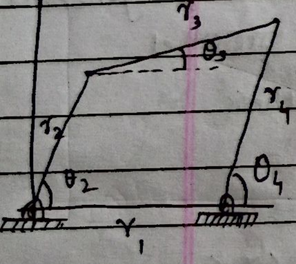
$$m_y = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0} = \frac{\theta_4 - \theta_4^{(i)}}{y - y_0}$$

the linear rel^n's are given by

$$\theta_2 - \theta_2^{(i)} = m_x (x - x_0)$$

$$\& \quad \theta_4 - \theta_4^{(i)} = m_y (y - y_0)$$

* Displ. analysis of 4R linkage
Freudenstein method



$$r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_1 \cos\theta_1 + r_4 \cos\theta_4$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_1 \sin\theta_1 + r_4 \sin\theta_4$$

as $\theta_1 = 0$

$$\therefore \quad r_3 \cos\theta_3 = r_1 + r_4 \cos\theta_4 - r_2 \cos\theta_2$$

$$r_3 \sin\theta_3 = r_4 \sin\theta_4 - r_2 \sin\theta_2$$

squaring & adding,

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 + 2r_1r_4\cos\theta_4 - 2r_1r_2\cos\theta_2 - 2r_2r_4\cos\theta_2\cos\theta_4 - 2r_2r_4\sin\theta_2\sin\theta_4$$

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\cos\theta_2 + 2r_1r_4\cos\theta_4 - 2r_2r_4\cos(\theta_2 - \theta_4)$$

$$\therefore 2r_2r_4\cos(\theta_2 - \theta_4) = 2r_1r_4\cos\theta_4 - 2r_1r_2\cos\theta_2 + r_1^2 + r_2^2 + r_4^2 - r_3^2$$

$$\therefore \cos(\theta_2 - \theta_4) = \frac{r_1}{r_2}\cos\theta_4 - \frac{r_1}{r_4}\cos\theta_2 + \frac{r_1^2 + r_2^2 + r_4^2 - r_3^2}{2r_2r_4}$$

$$\therefore \quad K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)$$

where $K_1 = r_1/r_2 \quad K_2 = r_1/r_4 \quad K_3 = \dfrac{r_1^2 + r_2^2 + r_4^2 - r_2^2}{2r_2r_4}$

1] Determine the lengths of links of a 4 bar linkage to generate $y = \log_{10} x$ in the interval $1 \le x \le 10$. The length of smallest link is 5 cm. Use 3 accuracy points with Chebyshev spacing given $\theta_2^{(i)} = 45^\circ$ $\theta_2^{(f)} = 105^\circ$ $\theta_4^{(i)} = 135^\circ$ $\theta_4^{(f)} = 225^\circ$

$n = 3$ $x_0 = 1$ $x_4 = 10$ $x_{n+1} = x_4 = 10$

$$x_j = \frac{x_{n+1} + x_0}{2} - \frac{x_{n+1} - x_0}{2} \cos\left(\frac{(2j-1)\pi}{2n}\right)$$

This gives $x_1 = 5.5 - 4.5 \cos\pi/6 = 1.6$ $y_1 = \log_{10}(1.6) = 0.204$

$x_2 = 5.5 - 4.5 \cos\pi/2 = 5.5$ $y_2 = \log_{10}(5.5) = 0.741$

$x_3 = 5.5 - 4.5 \cos 5\pi/6 = 9.4$ $y_3 = \log_{10}(9.4) = 0.974$

$x_0 = 1$ $x_4 = 10$ $y_0 = 0$ $y_4 = 1$

$$m_x = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_4 - x_0} = \frac{105-45}{10-1} = 60/9$$

$$m_y = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_4 - y_0} = \frac{225-135}{1-0} = 90$$

$\theta_2 - 45 = \frac{60}{9}(x-1)$ $\therefore \theta_2 = \frac{60}{9}(x-1) + 45$

$\theta_4 - 135 = 90(y-0)$ $\theta_4 = 90y + 135$

Freudenstein eqn

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)$$

before that,

posn	x_j	$\theta_2^{(j)}$	y_j	$\theta_4^{(j)}$
1	1.6	49°	0.204	153.36°
2	5.5	75°	0.741	201.69°
3	9.4	101°	0.974	222.66°

using eqn

$K_1 \cos(153.36^\circ) - K_2 \cos(49^\circ) + K_3 = \cos(49^\circ - 153.36^\circ)$

$K_1 \cos(201.69^\circ) - K_2 \cos(75^\circ) + K_3 = \cos(75^\circ - 201.69^\circ)$

$K_1 \cos(222.66^\circ) - K_2 \cos(101^\circ) + K_3 = \cos(101^\circ - 222.66^\circ)$

three eqns simultaneously give

$K_1 = 2$ $K_2 = -0.7015$ $K_3 = 1.081$

i.e $\frac{r_1}{r_2} = 2$ $\frac{r_1}{r_4} = -0.7015$ $\left|\frac{r_1}{r_4}\right| = 0.7015$

$$K_3 = \frac{r_1^2 + r_2^2 + r_4^2 - r_3^2}{2 r_2 r_4} = \frac{r_1^2 + \left(\frac{r_1}{2}\right)^2 + \left(\frac{r_1}{0.7015}\right)^2 - r_3^2}{2\left(\frac{r_1}{2}\right)\left(\frac{-r_1}{0.7015}\right)} = 1.081$$

$\therefore 1 + \frac{1}{4} + \left(\frac{1}{0.7015}\right)^2 - \left(\frac{r_3}{r_1}\right)^2 = \frac{-1.081}{0.7015}$

$\therefore \frac{r_3}{r_1} = 2.1962$

$\therefore \frac{r_1}{r_3} = 0.462 \qquad \frac{r_1}{r_2} = 2 \qquad \frac{r_1}{r_4} = 0.7015$

$r_3 > r_1 \qquad r_1 > r_2 \qquad r_4 > r_1$

$r_4 > r_1 > r_2 \qquad \therefore r_2 > r_1$

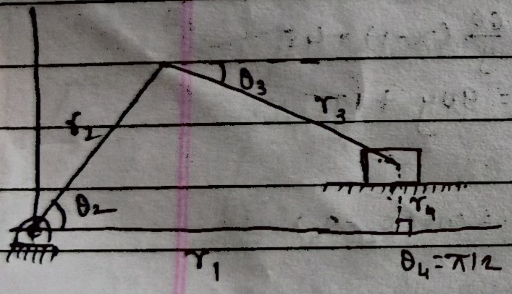
$r_3 > r_1 > r_2 \qquad r_2 > r_3$

$r_2 > r_4$

$\therefore r_2 = 5\,cm \quad \therefore r_1 = 10\,cm, \quad r_4 = 14.2\,cm \quad r_3 = 21.85\,cm$

* Displ. analysis of slider crank mechanism

Freudenstein method



$r_2\cos\theta_2 + r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4$

$r_2\sin\theta_2 + r_3\sin\theta_3 = r_1\sin\theta_1 + r_4\sin\theta_4$

$\theta_1 = 0 \quad \theta_4 = \pi/2$

$\therefore r_3\cos\theta_3 = r_1 - r_2\cos\theta_2$

$r_3\sin\theta_3 = r_4 - r_2\sin\theta_2$

squaring & adding,

$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\cos\theta_2 - 2r_2r_4\sin\theta_2$

i.e. $2r_1r_2\cos\theta_2 + 2r_2r_4\sin\theta_2 - (r_2^2 - r_3^2 + r_4^2) = r_1^2$

$\therefore K_1 s\cos\theta_2 + K_2\sin\theta_2 - K_3 = s^2$

where $K_1 = 2r_2 \qquad K_2 = 2r_2r_4$

$K_3 = r_2^2 - r_3^2 + r_4^2 \qquad s = r_1$ (sliding)

1) Design a slider crank mechanism in which slider displ. is proportional to the square of the crank's angular displacement in the interval $45^\circ \leq \theta_2 \leq 135^\circ$. The initial & final value of slider posn are 10 cm & 3 cm resp. from the frame. Use the 3 point Chebyshev spacing

Slider displacement $= s - s^{(i)}$

Crank angular displacement $= \theta - \theta^{(i)}$

it is given that $s - s^{(i)} \propto |\theta - \theta^{(i)}|^2$

when $s = s^{(f)}$, $\theta = \theta^{(f)}$

$\therefore \quad s^{(f)} - s^{(i)} \propto |\theta^{(f)} - \theta^{(i)}|^2$

$\therefore \quad \cancel{10-3} \; 3 - 10 = c\,|135 - 45|^2$

$$c = \frac{-7}{(90^\circ)^2} = \frac{-7}{(\pi/2)^2} = -2.837$$

$$\therefore \quad S - 10 = -2.837\left(\theta - \frac{\pi}{4}\right)$$

$$\theta_2^j = \frac{\theta_{n+1} + \theta_0}{2} - \frac{\theta_{n+1} - \theta_0}{2} \cos\left(\frac{(2j-1)\pi}{2n}\right)$$

$\theta_2^1 =$ find the values $S_1 =$ find s for

$\theta_2^2 =$ of θ according to $S_2 =$ respective θ

$\theta_2^3 =$ formula in radian $S_3 =$ according to formula

$$K_1 s\cos\theta_2 + K_2 \sin\theta_2 - K_3 = s^2$$

put & get 3 eqns to solve for K_1, K_2 & K_3 & thereby r_1, r_2, r_3

Gears & Gear Mechanisms

Types of Gear:

Spur gear:

Spur gears have straight teeth parallel to the axis of rot^n Mounted on two parallel shafts to transmit the motion.
As teeth are parallel to axis, spur gears are not subjected to axial thrust due to teeth load.

Helical gear:

Teeth are inclined at an angle called helix angle with axis rot^n. Helix angle $\approx 15^\circ$ to 25°
Load applied is gradual resulting in low impact stresses & less noise Hence they can be used in higher velocities than spur gears
However helical gears are subjected to axial thrust which is major drawback of them

Double helical / Herringbone gear

Axial thrust which occurs in case of helical gear is eliminated by the use of double helical gear

Bevel / Straight Bevel gear:

When teeth formed on the truncated cones are straight the gears are called bevel gear. The cross sect^n of the teeth gradually decreases.

Worm & Worm wheel:

Used to transmit rotary motⁿ betⁿ non parallel & non intersecting shafts. There are skew threads on the worm & teeth on the worm wheel

Rack & pinion:

It can convert rotary motⁿ into linear motion & vice versa

1) 2 spur gears have a velocity ratio 4:1 Driven gear has 80 teeth of 8mm module & rotates at 240rpm find no. of teeth, speed of driver & pitch line velocities

$T_2 = 80$ $\qquad \frac{\omega_1}{\omega_2} = \frac{4}{1}$

$N_2 = 240$

$m = 8$ $\qquad \frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{N_1}{240} = \frac{T_2}{T_1} = \frac{d_2}{d_1}$

$\therefore N_1 = 240 \times 4 = 960 \text{ rpm}$

$T_1 = \frac{80}{4} = 20$

$m = \frac{d_1}{T_1} = \frac{d_2}{T_2}$ $\qquad \therefore 8 = \frac{d_1}{20}$ $\qquad d_1 = 160 \text{ mm}$

$\omega_1 = \frac{2\pi N_1}{60}$ $\qquad \omega_2 = \frac{2\pi N_2}{60}$

pitch line velocity $= V_p = \omega_1 r_1 = \frac{\omega_1 d_1}{2}$

$= \frac{\pi N_1 d_1}{60} = \frac{\pi \times 240 \times 4 \times 160}{60}$

$= \cancel{80384 \text{ mm/s}}$ 8042.477 mm/s

2) The no. of teeth of a spur gear is 40 & it rotates at 200rpm. What will be its pitch line if it has module of 2mm

$N_1 = 200$ rpm $\quad T_1 = 40$

$m = 2 = \frac{d_1}{T_1}$

$2 = \frac{d_1}{40} \quad d_1 = 80$

$V_p = \frac{2\pi N_1 d_1}{60 \times 2} = \frac{\pi \times 200 \times 80}{60}$

$= 837.758$ mm/s

✳ Gear terminology

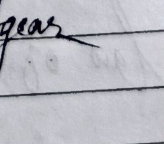
circular pitch (p) $= \frac{\pi d}{T}$ $\quad$ d - diameter of pitch circle, T - no. of teeth

diametrical pitch (P) $= \frac{T}{d}$

module $= m = \frac{d}{T}$ $\quad$ d - diameter in mm of pitch circle

diameter of adendum $= d + 2m$

diameter of dedendum $= d - 2m$



Velocity ratio : angular velocity of driving gear / angular velocity of driven gear

$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$

Gear ratio : Ratio of number of teeth on the gear (T) to the no. of teeth on pinion (t)

$G = T/t$

470.32

Max. velocity ratio

spur gear → 6:1

helical gear → 10:1

worm gear → 100:1

Involute profile path

locus of point on a st. line which rolls without slipping on circumference of circle.

It is the path traced by the end of a taut cord (string) as it is unwound from a stationary cylinder or circle. The string is always tangent to cylinder or circle

The circle on which the st. line rolls or from which the string is unwound is known as base circle

pressure line / line of act^n

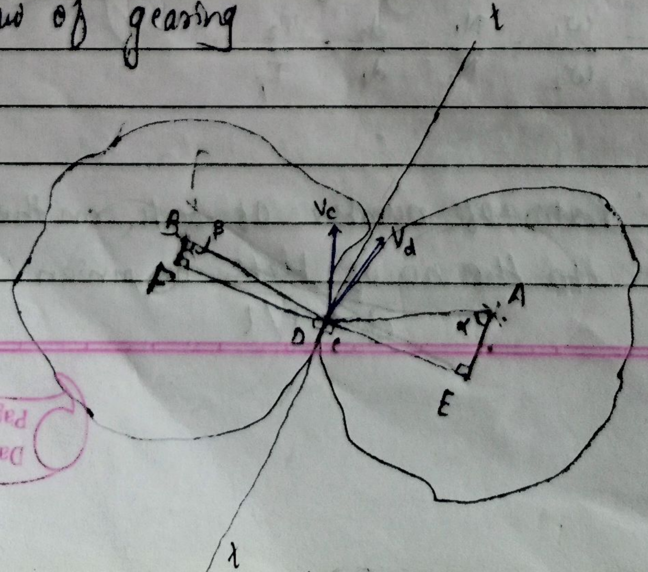
It is the common tangent to the base circles of mating gears.

pressure angle / angle of obliquity

Angle bet^n pressure line & common tangent to the pitch circle is known as pressure angle

$$\text{angle of } act^n = \frac{\text{arc of contact}}{\text{pitch circle radius}}$$

* law of gearing



A & B are centres of gear

t is the common tangent

FD & CE are normals to line t

V_c & V_d are dir of velocity of C & D resp. V_c & V_d dir are ⊥ to AC & BD resp

$\angle V_c D t$ =

- Addendum circle: Circle through top land of the teeth, concentric to pitch circle
- Dedendum circle: Circle through bottom land of the teeth concentric to pitch circle
- Clearance ~~y~~ circle: Circle ~~through~~ touching addendum circle of meshing gear concentric to pitch circle
- Addendum: Tooth height from pitch circle to top of tooth i.e. radial dist betn pitch circle & addendum circle
- Dedendum: Tooth depth from pitch circle to bottom of tooth i.e. radial dist. pitch circle & dedendum circle
- Clearance: The amount by which the dedendum of a gear exceeds the addendum of the mating gear or radial distance b/w addendum & dendendum circle of meshing gears.
- Full depth of teeth: total radial depth of the tooth space
  full depth = addendum + dedendum
- Working depth of teeth: The maximum depth to which a tooth penetrates into the tooth space of the mating gear
  = sum of addendums of two mating gears
- Tooth thickness - Thickness of tooth measured along pitch circle
- Space width - Width of tooth space along the pitch circle
- Backlash - The diff. b/w space width & the tooth thickness along the pitch circle
  = space width - tooth thickness
- Face width: The length or width of the tooth parallel to gear axis
- Epicycloid: Curve traced by a point on the circumference of a circle which rolls outside of pitch circle without slipping
- Hypocycloid: Curve traced by a point on the circumference of a circle which rolls inside of pitch circle without slipping
- Path of contact: locus of point of contact of two mating teeth from the beginning of engagement to the end is known as path of contact or contact length. The pitch point P is always one point on path of contact
- Path of approach: Portn of path of contact from beginning of engagement to the pitch point
- Path of recess: Portn of path of contact from pitch point to the end of engagement
- Arc of contact: Locus of points on the pitch circle from beginning to end of engagement of two mating gears

- Arc of approach: Port^n of arc of contact from beginning of engagement to pitch po
- Arc of recess: Port^n of arc of contact from pitch point to end of engagement
- Angle of act^n: It is an angle turned by gear from the beginning of engagement to the end of engagement of a pair of teeth. i.e. angle turned by arcs of contact of respective gears.

The angle will have different values for driving & driven gears

$$(\delta)\ \text{Angle of act}^n = \frac{\text{Arc of contact}}{\text{Pitch circle radius}}$$

$$\delta = \text{angle of approach } (\alpha) + \text{angle of recess } (\beta)$$

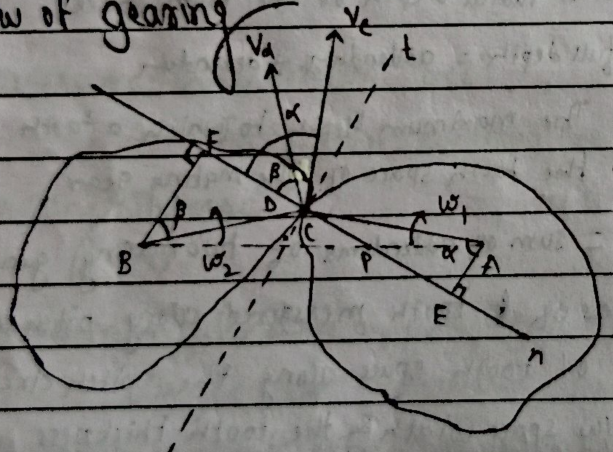
- Contact ratio: It is the ratio of angle of act^n & pitch angle

$$\text{contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}$$

$$\text{also contact ratio} = \frac{\text{arc of contact}}{\text{circular pitch}}$$

as angle of action is the angle subtended by arc of contact & pitch angle is the angle subtended by circular pitch at the centre of pitch circle, contact ratio is the ratio of arc of contact to the circular pitch

* law of gearing



It states "the common normal to the tooth profile at the point of contact should always pass through a fixed point, in order to obtain constant angular velocity ratio b/w two gears

fixed point is P here

V_c is dir. of linear velocity of point C

V_d " " " " D

dir of $V_c \perp AC$ dir of $V_d \perp BD$ line n $\perp$ line t

If the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along common tangent t. The relative motion b/w the surfaces along the common normal n must be 0 to avoid the separ^n or penetrat^n of the 2 teeth into each other

$\omega_1 AC \cdot \frac{AE}{AC} = \omega_2 BD \frac{BF}{BD}$ $\Leftarrow$ $\therefore \boxed{V_c \cos\alpha = V_d \cos\beta}$ $V_c = \omega_1 AC$ $V_d = \omega_2 BD$

$\omega_1 AE = \omega_2 BF$ Velocity of sliding $= V_c \sin\alpha - V_d \sin\beta$

$\frac{BF}{AE} = \frac{FP}{EP}$ .... similar Δs $= \omega_1 AC \cdot \frac{EC}{AC} - \omega_2 BD \cdot \frac{FD}{BD} = \omega_1 EC - \omega_2 FD$

$\therefore \frac{\omega_1}{\omega_2} = \frac{PF}{EP}$ $= \omega_1(EP+PC) - \omega_2(FP-PD)$

$= (\omega_1+\omega_2)PC + \omega_1 EP - \omega_2 PF$

$= (\omega_1+\omega_2)PC$

C & D are same points

PC = PD

* Rel^n in base circle diameter & pitch circle diameter

base circle diameter (d_b) = pitch circle diameter (d) × $\cos\phi$

ϕ is pressure angle

* Path of contact

r - pitch circle radius of pinion

R - pitch circle radius of gear

r_a - addendum circle radius of pinion

R_a - dedendum circle radius of gear

path of approach $= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$ — ①

path of recess $= \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$ — ②

path of contact = ① + ②

path of contact $= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$ — ③

* arc of contact

$$\text{arc of contact} = \frac{\text{path of contact}}{\cos\phi}$$

* Contact ratio : no. of pairs of teeth in contact

$$n = \frac{\text{arc of contact}}{\text{circular pitch}} = \frac{\text{path of contact}}{p\cos\phi}$$

1] Each of 2 gears in a mesh has 48 teeth & a module of 8 mm. The teeth are of 20 degree involute profile. [The arc of contact is 2.25 times circular pitch] Determine the addendum.

contact ratio = 2.25 असा देऊन विचारले असते

$\phi = 20^\circ$ $t = T = 48$ $m = 8\,mm$ $m = \frac{d}{t}$

$r = R = \frac{mT}{2} = 192\,mm$; $R_a = r_a$

arc of contact = 2.25 × circular pitch

$= 2.25 \times \frac{\pi d}{t} = 2.25\pi m = 56.55\,mm$

path of contact = arc of contact × $\cos\phi$ = $56.55\cos 20^\circ$ = 53.14 mm

$53.14 = eq^n$ ③ $\Rightarrow R_a = 202.6\,mm$

$R_a = R$ + addendum ∴ addendum $= R_a - R$

$= 10.6\,mm$

2] Pinion gear having 40 teeth drives a gear having 90 teeth. Profile of gears is Involute with 20° pressure angle, 10mm module & addendum equal to one module. Find path of contact, arc of contact, contact ratio

$\phi = 20°$ $m = 10\,mm$ $t = 40$ $T = 90$

$m = \frac{d}{t}$ $10 = \frac{d}{40}$ $d = 400$ $r = 200\,mm$

$m = \frac{D}{T}$ $10 = \frac{D}{90}$ $D = 900$ $R = 450\,mm$

$r_a = r + \text{addendum} = r + m = 210\,mm$

$R_a = R + \text{addendum} = R + m = 460\,mm$

path of contact $= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$= 52.465\,mm$

arc of contact $= \frac{52.465}{\cos 20°} = 55.832\,mm$

contact ratio $= \frac{\text{arc of contact}}{\text{circular pitch}} = \frac{55.832}{\pi \frac{d}{t}} = 1.778$

vimp Contact ratio 1.778 means 1 pair of teeth is always in contact whereas two pairs of teeth are in contact for 77.8 % of the time

3] The involute spur gears having pressure angle 20° & module are 6mm in mesh. The spur ratio is 3 & the nos of teeth on pinion is 24. Pitch line velocity is 1.5 & addendum = 1 module find path of contact, arc of contact, angle of actn of contact & max velocity of sliding

$t = 24$ Gear ratio $= G = \frac{T}{t} = 3$

$T = 72$ $m = 6 = \frac{d}{t} = \frac{D}{T}$ $d = 144\,mm$

$r = 72\,mm$

$\phi = 20°$ $D = 432\,mm$ $R = 216\,mm$

$r_a = r + m = 78\,mm$

$R_a = R + m = 222\,mm$

Path of contact $= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$= 30.22$

arc of contact $= \frac{30.22}{\cos 20} = 32.16$