

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{s}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

* Drift velocity :

Average velocity acquired by an electron of a metal in a particular dir. on application of an electric field

* Relaxation time:

Time taken by free electrons to reach equilibrium from disturbed one under in presence of electric field.

* Mean free path: Average dist. travelled by an electron betw 2 successive collisions.

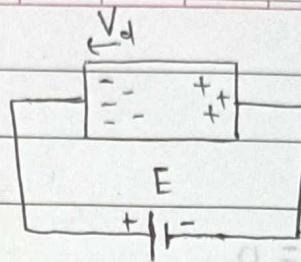
* Assumptions for free electron model

- i. A metal contains large no. of free electrons which are free to move like the gas molecules in a container.
- ii. The free electrons move in random direction & collide with the charges or among themselves.
- iii. Velocity & energy distribution of free electron obey the classical Boltzmann-Maxwell distribution
- iv. Electrons move randomly in absence of electric field. Whereas electrons get drifted in a particular directn, opp to electric field dir. when applied.
- v. The free electrons obey classical mechanics law

force acting on free electron = eE

by Newton's second law, $F = m_e a$

$$\therefore eE = m_e a$$



$$\therefore a = \frac{V_d}{\tau} = \frac{eE}{m_e}$$

$$\therefore V_d = \frac{eE\tau}{m_e}$$

$$V_d = \frac{n e E A}{m_e c}$$

$$I = nAV_d e$$

n - electrons per unit volume

A - area

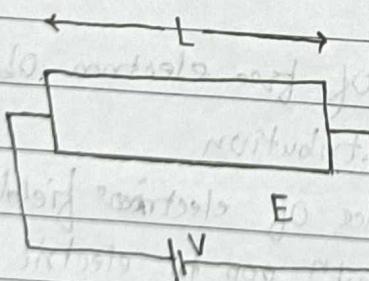
$$I = \frac{de}{dt} = nAV_d e$$

$$de = nAV_d e dt$$

$$J = \frac{I}{A} = \frac{nV_d e}{A}$$

$$J = \frac{nE\lambda e^2}{C m_e}$$

$$J = \frac{nEe^2\tau}{m_e}$$



$$V = EL$$

$$V = IR \quad J = \frac{I}{A} = \frac{n^2 E \tau}{m_e}$$

$$EL = JAR$$

$$\therefore R = \frac{E}{J} \times \frac{L}{A}$$

$$J = E \times \frac{L}{A} \times \frac{1}{R}$$

$$R = \rho \frac{L}{A} \quad \therefore \frac{L}{A} \frac{1}{R} = \frac{1}{\rho}$$

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$V_{avg} = \sqrt{\frac{8kT}{\pi m}}$$

$$V_{mp} = \sqrt{\frac{2kT}{m}}$$

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$$\therefore J = E \frac{1}{\rho} = E\sigma$$

: (σ) ρ - resistivity

σ - Conductivity

We know, $J = \frac{n e^2 \tau E}{m_e} = \sigma E$

$$\therefore \text{conductivity} = \sigma = \frac{n e^2 \tau}{m_e}$$

Order of conductivity of a conductor is around 10^7
insulator is around 10^{-1}

- 1) find drift velocity of electrons in a metal wire of radius 10^{-4} m carrying current of 2 A. find root mean square velocity or thermal velocity at room temp.

We know, $I = n A V_d e$

$$V_d = \frac{2}{10^{28} \times \pi \times 10^{-4} \times 10^{-4} \times 1.6 \times 10^{-19}} = 0.398 \times 10^{-11} \text{ m/s}$$

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 298}{9.11 \times 10^{-31}}} = 1.16 \times 10^5 \text{ m/s}$$

2) $V_d = 1.16 \times 10^5 \text{ m/s}$

* Mobility (μ):

Defined as drift velocity per unit electric field

$$\mu = \frac{V_d}{E}$$

$$\mu = \frac{eET}{Em_e} = \frac{eT}{m_e}$$

* Relaxation time τ :

It is the time that is required for the drift velocity to lower down by the factor of e

$$V_d = V_0 e^{-t/\tau}$$

at $t = \tau$, $V_d = V_0/e$

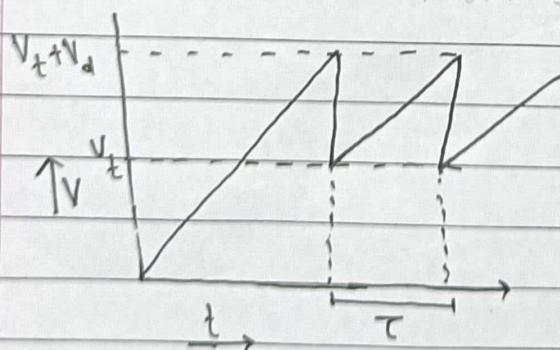
V is drift velocity, τ is relaxation time

All electrons move with RMS velocity according to Maxwell-Boltzmann distribution i.e. with thermal velocity

1) find relaxation time for electron-ion collision. Take $n = 10^{28} \text{ m}^{-3}$

$$\sigma = 10^7 \text{ cm}^2 \text{ m}^{-1} \cdot 0.1 \times 8 \pi \epsilon_0 =$$

$$\sigma = \frac{n e^2 \tau}{m_e} = \frac{\sigma m_e}{n e^2} = \frac{10^7 \times 9.11 \times 10^{-31}}{10^{28} \times 1.6 \times 1.6 \times 10^{-38}} = 3.55 \times 10^{-14} \text{ sec}$$



$$\text{average kinetic energy} = \frac{1}{2} m \langle v \rangle^2 = \frac{3}{2} k_B T$$

$$\langle v \rangle = \sqrt{\frac{3 k_B T}{M}}$$

$$\tau = \frac{\lambda}{\langle v \rangle} = \frac{\lambda}{\sqrt{\frac{3 k_B T}{M}}}$$

$$\therefore \tau = \sqrt{\frac{M}{3k_B T}} \lambda$$

$$\sigma = \frac{n e^2 \tau}{m_e} = \frac{n e^2 \lambda}{m_e} \sqrt{\frac{M_e}{3k_B T}}$$

M is mass of
e- i.e. m_e

$$= \frac{n e^2 \lambda}{\sqrt{3k_B T m_e}}$$

* Merits of free electron model

Explains Ohm's law

Explains thermal electrical conductivity

* Debye Temperature (T_D)

$$T_D = \frac{h\nu_0}{k_B}$$

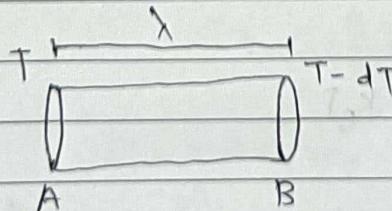
ν_0 - Maximum frequency of the vibration of a solid atom

k_B - Boltzmann const.

$$C_V = C_{V \text{ lattice}} + C_{V \text{ electronic}}$$

$$C_{V \text{ lattice}} = \begin{cases} 3R & \dots T > T_D \\ BT^3 & \dots T < T_D \end{cases}$$

B is some const.



$$\text{Average KE at } A = \frac{3}{2} k_B T$$

$$\text{Average KE at } B = \frac{3}{2} k_B (T - dT)$$

Equipartition of energy : Equal division of energy of a system in thermal equilibrium bet' different degrees of freedom

$$\text{Equipartition of energy} = \frac{1}{2} k_B T$$

$$n_e = \frac{\text{no. of electrons}}{\text{Volume}}$$

now, degrees of freedom = 6 (3 translational & 3 rotational)

let v be the velocity of e^-

$$\therefore \frac{1}{6} n_e v = \text{no. of } e^- \text{ per unit area per unit time for each d.o.f}$$

for 1 electron

$$\text{Excess KE} = \frac{3}{2} K_B T$$

\therefore Excess of energy carried from A to B per unit area in unit

$$\text{time is given by} = \frac{3}{2} K_B T \cdot \frac{1}{6} n v$$

$$Q = \frac{1}{4} K_B n v dT \quad (\text{i.e. no. of } e^- \times \text{excess energy for } 1 e^-)$$

K is thermal conductivity
unit - W/mK

$$\therefore K = \frac{1}{4} n v k_B \lambda \quad \lambda \text{ is } \propto \text{ here}$$

$$= \frac{1}{4} n v k_B \lambda$$

$$= \frac{1}{4} n v^2 k_B T$$

$$\text{average energy per molecule} = \langle E \rangle = \frac{3}{2} K_B T$$

$$\text{avg. energy per mole} = \langle E \rangle = \frac{3}{2} K_B N_A T = \frac{3}{2} R T$$

$$K_B = R / N_A$$

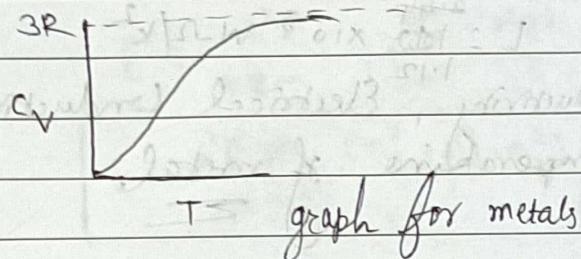
$$\therefore C_V^{\text{electronic}} = \frac{\langle E \rangle}{T} = \frac{3}{2} R$$

$$C_V^{\text{Fattice}} = \begin{cases} 3R & \dots T > T_D \\ BT^3 & \dots T < T_D \end{cases}$$

$$\therefore C_V = C_V \text{ lattice} + C_V \text{ el.}$$

$$= 3R + \frac{3}{2} R$$

$$= 4.5R$$



$$K = \frac{1}{4} n v^2 k_B T$$

~~$\propto \frac{n e^2 z}{m_e}$~~

~~Thermal conductivity~~ ~~electric conductivity~~

$$\frac{K_e}{K} = \frac{1}{4} \frac{v^2 k_B m_e}{e^2}$$

Thermal conductivity of electron = $K_{el} = \frac{1}{3} C_V \langle v \rangle \lambda \frac{n}{N_A}$

$$= \frac{1}{3} \frac{3}{2} R \langle v \rangle \langle v \rangle T \frac{n}{N_A}$$

$$= \frac{1}{2} \frac{R \langle v \rangle^2 n m_e z}{N_A m_e}$$

$$= \frac{1}{2} m_e v^2 \cdot \frac{n R z}{N_A m_e}$$

now $\frac{1}{2} m_e v^2 = \frac{3}{2} k_B T$

$$k_{el} = \frac{3}{2} k_B T \cdot \frac{n \cdot k_B N_A T}{N_A m_e} = \frac{3}{2} \frac{k_B^2 T n z}{m_e}$$

$$\sigma = n e^2 z / m_e$$

$$\frac{k_{el}}{\sigma} = \frac{3}{2} \frac{k_B^2}{e^2} T = L T$$

L is Lorentz no.
T is temp

$$k_B = 1.38 \times 10^{-23}$$

$$\boxed{L = \frac{3}{2} \frac{k_B^2}{e^2}}$$

$$L = 1.12 \times 10^{-8} \text{ W} \cdot \text{m}^2 \text{ K}^{-2}$$

$$P = i \hbar \frac{\partial \Psi}{\partial n}$$

$$T + V = E$$

$$\frac{P^2}{2m} + V = E$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi = E\Psi \quad - \text{Schrodinger's eqn}$$

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* Wiedemann-Franz law

$$K \propto T \quad \sigma \propto \frac{1}{T}$$

$$\therefore \frac{K}{\sigma} \propto T \quad \frac{K}{\sigma} = LT$$

$$L = \frac{2.44}{1.12} \times 10^{-8} \text{ W} \cdot \text{K}^2 / \text{A}^2$$

Thermal conductivity, Electrical conductivity,
absolute temperature of metal,
constant of proportionality

* Schrodinger Eqn

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi = E\Psi$$

$$V = 0$$

$$\therefore -\frac{\hbar^2}{2m} \nabla^2 \Psi = E\Psi \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\therefore -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = E\Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2mE}{\hbar^2} \Psi = 0$$

$$\Psi = A e^{ikx}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi$$

$$\left(-k^2 + \frac{2mE}{\hbar^2} \right) \Psi = 0$$

$$\therefore E = \frac{k^2 \hbar^2}{2m}$$

$$\therefore k = \pm \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Psi = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$= A_1 e^{i\sqrt{2mE/\hbar^2} x} + A_2 e^{-i\sqrt{2mE/\hbar^2} x}$$

$$\text{at } x=0 \quad \Psi = A \cos(kx) + i \sin(kx)$$

$$\Psi(0) = 0 \quad \therefore A = 0$$

$$\Psi(L) = 0 \quad \therefore B \sin(kL) = 0$$

$$B \neq 0 \quad \therefore \sin(kL) = 0$$

$$\therefore kL = n\pi$$

$$\therefore [k = n\pi/L]$$

$$\Psi(x, y, z) = B \sin k_x L \sin k_y L \sin k_z L$$

$$\sin k_x = \frac{n_x \pi}{L} \quad k_y = \frac{n_y \pi}{L} \quad k_z = \frac{n_z \pi}{L}$$

$$E = \frac{\hbar^2 R^2}{2m} = \frac{\hbar^2}{2m} \cdot \frac{n^2 \pi^2}{L^2} = \frac{\hbar^2}{8\pi^2 m} \cdot \frac{n^2 \pi^2}{L^2}$$

$$E = \frac{\hbar^2 (n_x^2 + n_y^2 + n_z^2)}{8\pi^2 m L^2}$$

$$\text{Maxwell-Boltzmann energy distribution: } f(E) = \frac{1}{e^{(E_i - E_f)/k_B T}}$$

$$(E_i R^2 / \pi^2) \frac{1}{8} = \text{number of states}$$

$$\text{Fermi-Dirac: } f(E) = \frac{1}{e^{(E_i - E_f)/k_B T} + 1}$$

$$\text{Fermi Energy Level: } [E_{\text{F}}(k_B T) \pi^2 / 3]^{1/3} = E_F$$

The energy of the state of electron which accommodates the 'N' total electrons.

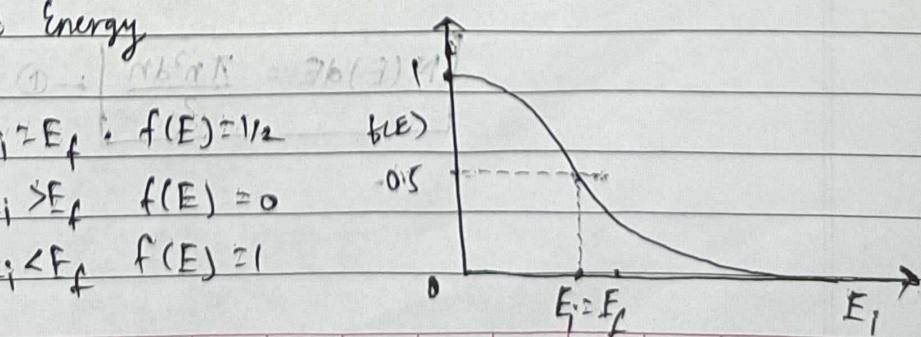
for N total electrons, one energy level accommodates 2 electrons the total no. of thus participating energy states will be $N/2$. The energy of $(N/2)^{\text{th}}$ level at 0°K is called Fermi Energy level & the corresponding level is called Fermi Energy level.

In other words, energy of lowest occupied molecular orbitals is called Fermi Energy.

$$T > 0\text{K} \quad E_i > E_F \quad f(E) = 1/2$$

$$T = 0\text{K} \quad E_i < E_F \quad f(E) = 0$$

$$T = 0\text{K} \quad E_i < E_F \quad f(E) = 1$$



* Fermi Surface: It is the surface in reciprocal space which separates occupied from unoccupied e^- states at zero temperature. Energy of e^- on a Fermi surface is same all over the surface.

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* Density of States: No. of available e^- states / volume

in energy interval E & $E+dE$

It is denoted by $N(E)dE$

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$$\text{Volume of sphere} = \frac{4}{3}\pi n^3$$

n is n_x, n_y, n_z all of which are the integers

we're to consider one octant will have $(\frac{1}{8}, \frac{1}{8}, \frac{1}{8})$

$$\text{Vol of that octant} = \frac{1}{8} \left(\frac{4}{3}\pi n^3 \right)$$

similarly no. of available energy states within one octant of the sphere of radius ' $n+dn$ ' corresponding to energy $E+dE$ is given by

$$E+dE = \frac{1}{8} \left[\frac{4}{3}\pi (n+dn)^3 \right]$$

No of available energy states within one octant betw the shells of radius n & $n+dn$ i.e. betw the energy values E & $E+dE$

$$N(E)dE = \frac{1}{8} \left(\frac{4\pi}{3} \right) (n+dn)^3 - \frac{1}{8} \left(\frac{4\pi}{3} \right) n^3 = \frac{4\pi}{24} [(n+dn)^3 - n^3]$$

neglecting dn^3 & $d(n^3)$, we get

$$N(E)dE \approx \frac{\pi}{6} 3n^2 dn = \frac{\pi n^2 dn}{2}$$

$$N(E)dE = \frac{\pi n^2 dn}{2} \quad \text{--- (1)}$$

$$N(E) dE = Z(E) f(E) dE$$

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$$E = \frac{n^2 h^2}{8mL^2} \quad \therefore n = \left(\frac{8mL^2 E}{h^2} \right)^{1/2}$$

differentiating above eq 7,

$$ndn = \frac{8}{2} \cdot \frac{mL^2 dE}{h^2}$$

putting these values in eq 7,

$$N(E) dE = \frac{\pi}{2} \left(\frac{8mL^2 E}{h^2} \right)^{1/2} \times \left(\frac{8mL^2 dE}{2h^2} \right)$$

$$\boxed{N(E) dE = \frac{\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} E^{1/2} dE} \quad \dots \text{for } E_i < E_f, f(E) = 1$$

density of states is given by no. of energy states per volume

$$Z(E) dE = \frac{N(E) dE}{V} = \frac{N(E) dE}{\frac{4}{3} \pi R^3 L^3}$$

$$\therefore \boxed{Z(E) dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE}$$

According to Pauli's exclusion principle two electrons of opp-spins can occupy each state & hence no. of energy states available for electron occupancy is given by

$$N(E) dE = \frac{2\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} E^{1/2} \frac{dE}{e^{E_i - E_f + 1}}$$

N = total no. of electrons

$$N = \int_0^{E_F} N(E) dE = \int_0^{E_F} \frac{2\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} E^{1/2} dE \quad \dots E_i < E_F, T=0$$

$$= \frac{2\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} \frac{2}{3} [E^{3/2}]_0^{E_F}$$

$$n = \frac{N}{V} = \frac{2\pi}{6} \left(\frac{8m}{h^2} \right)^{3/2} E_F^{3/2}$$

$$E_F^{3/2} = \left(\frac{h^2}{8m} \right)^{3/2} \frac{6n}{2\pi}$$

$$\boxed{E_F = \frac{h^2}{2m} \left(\frac{3n}{8\pi} \right)^{2/3}}$$

Q] A gold crystal has FCC lattice with 4 effective atomic no.

Lattice constant $\approx 4.08 \text{ \AA}$. Every gold atom has 1 valence e^- . find e^- fermi energy, fermi velocity & e^- wavelength

$$n = N/V \quad N = 4 \times 1 = 4 \text{ atoms}$$

$$E_F = \frac{\hbar^2}{2m_e} \left(\frac{3\pi}{8\pi} \right)^{2/3} n^{2/3}$$

$$n = \text{no. of } e^- / \text{vol.}$$

$$n = \frac{4}{(4.08)^3 \times 10^{-30}} = 0.0589 \times 10^{30}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_F = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} \left(\frac{3 \times 0.0589 \times 10^{30}}{\sqrt{8\pi}} \right)^{2/3}$$

$$= 2.405 \times 10^{-17} (0.00703)^{2/3}$$

$$= 2.405 \times 10^{-17} \times 1.6476 \times 10^{-5} \quad \begin{matrix} 0.03669 \\ 0.08826 \end{matrix}$$

$$V_F = 3.962 \times 10^{-22} \quad 0.08826 \times 10^{-17} \quad 0.08826 \times 1.6 \times 10^{-19} \text{ ev}$$

$$V_F = 1.39 \times 10^6 \text{ m/s} \quad V_F = \frac{h}{m_e E_F} \quad E_F = \frac{1}{2} m V_F^2$$

$$\lambda = \frac{h}{V_F} = \frac{h}{m_e V_F}$$

* Thermal conductivity arises due to collisions of charges in metal

$$\chi = \frac{1}{NS}$$

N = no. of ions per vol.

S = Electrons or energy scattering cross section

y is amplitude of vibration of lattice atom

$B = By^2$ B is constant of proportionality

$$\lambda = \frac{B_1}{N y^2}$$

$$B_1 = \frac{1}{B}$$

$$V_F \tau_F = \frac{B_1 M K_B T^2}{N \hbar^2 T}$$

$$y^2 \perp \frac{\hbar^2 T}{M K_B T_D^2}$$

$$\tau_F = \frac{B_1}{N V_F} \times \frac{M K_B T_D^2}{T \hbar^2}$$

T_D - debye temp

T - temp.

M - mass of lattice

$$\text{we know, } \sigma = \frac{n e^2 \tau_F}{m_e}$$

$$= B_1 \left(\frac{e^2}{m_e V_F} \right) \left(\frac{M K_B T_D^2}{\hbar^2 T} \right)$$

$$\sigma = \frac{1}{S}$$

resistivity = conductivity

$$\therefore S(T) = B \left(\frac{m_e V_F}{e^2} \right) \left(\frac{\hbar^2 T}{M K_B T_D^2} \right)$$

This is Sommerfeld model

But actually it is found to be

$$\sigma_i = \sigma_0 + \sigma(T)$$

\uparrow \uparrow Sommerfeld result
Constant

$$\lambda = \frac{v}{N y^2}$$

$$B_1 = \frac{1}{B}$$

$$v_F^2 F = \frac{B_1 M K_B T_D^2}{N \hbar^2 T}$$

$$\tau_F = \frac{B_1}{N v_F} \times \frac{M K_B T_D^2}{T \hbar^2}$$

$$y^2 = \frac{\hbar^2 T}{M K_B T_D^2}$$

T_D - Debye temp
 T - temp.

M - mass of lattice

we know, $\sigma = \frac{n e^2 \tau_F}{m_e}$

$$= B_1 \left(\frac{e^2}{m_e v_F} \right) \left(\frac{M K_B T_D^2}{\hbar^2 T} \right)$$

$$\sigma = \frac{1}{S}$$

$$\text{resistivity} = \frac{1}{\text{conductivity}}$$

$$\therefore S(T) = B_1 \left(\frac{m_e v_F}{e^2} \right) \left(\frac{\hbar^2 T}{M K_B T_D^2} \right)$$

This is Sommerfeld model

But actually it is found to be

$$\sigma_i = \sigma_0 + \sigma(T)$$

↑ ↑ Sommerfeld result
Constant

* Demerits of Drude model

- i. It failed to explain Crompton effect, photoelectric effect blackbody radiation
- ii. failed to explain electrical conductivity of semiconductors & insulators
- iii. Did not explain specific heat capacity of metals
- iv. Ferromagnetism was not explained by this theory

* Kamerlingh Onnes superconductivity theory

Mercury below 4.2 K acts like a superconductor

Superconductors have resistance close to 0.

Most of the inner transition elements ⁽²⁰⁾ are found to act as superconductors at lower temp

The temp. below which an element acts as a conductor
is called critical temp.

Niobium 9.2 K ← highest critical temp

Mercury 4.2 K

Iridium 0.14 K ← lowest critical temp.

Besides some compounds are also found to exhibit the same nature

ex. $HgBa_2Ca_2Cu_3O_8 \Rightarrow 125K$

Generally it is found experimentally that superconductors are poor electricity conductors at room temp

And good conductors of electricity cannot show superconducting behaviour.

Adding to that, magnetism & superconductivity are found to be mutually exclusive, i.e. entity showing one of the characters doesn't exhibit other.

Superconductor acts as a perfect demagnet.

Electric field is 0

* BCS theory

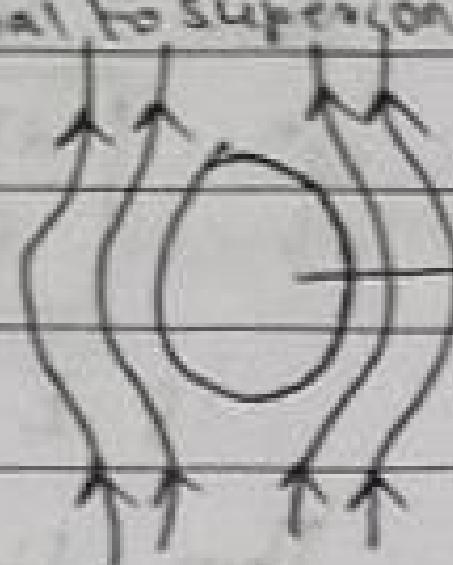
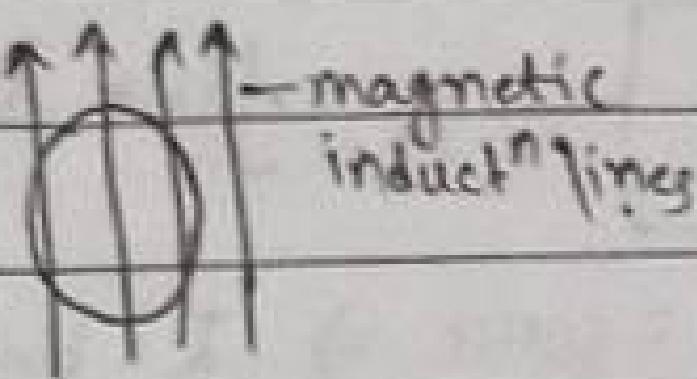
Bardeen - Cooper - Schrieffer theory

* Meissner effect : Below critical temp. magnetic inductⁿ lines are 0 for

inside

a semiconductor material
when a material makes the transition from normal to superconducting state, it actively

excludes the magnetic field lines from its interior
The expulsion of magnetic field lines from the interior of a material in the process of being superconductor that is losing its resistance to the flow of electric current when cooled below certain temp called critical temp T - Temp T_c : critical temp usually close to absolute zero



$T < T_c$

Intensity of magnetization / Magnetising field (H)

$$H = \frac{B_o}{\mu_0}$$

μ_0 - permeability of free space

Magnetic susceptibility χ

$$\chi = \frac{I}{H} \quad I - \text{intensity of magnetization}$$

$$H = \frac{B}{\mu_0} - M$$

$$\mu_0 = 4\pi \times 10^{-7}$$

μ_0 - absolute permeability of vacuum

μ - permeability of a medium

μ_r - relative permeability

$$\mu_r = \frac{\mu}{\mu_0}$$

* Two fluid model : The model proposed that e- within a superconductor appear as 2 different types normal & superconducting. i.e. some of the e- behave normally & follow Ohm's law while others are responsible for superconducting nature of material.

n_n - electron density of normal e-

n_s - .. . " super e- $J = J_n + J_s$

v_n - velocity of normal e-

$$= n_n v_n e + n_s v_s e$$

v_s - .. . super e-

$$m \frac{dV_s}{dt} = eE$$

$$\frac{dJ_s}{dt} = e n_s \frac{dV_s}{dt} = e n_s \frac{eF}{m} = \frac{e^2 E n_s}{m}$$

for semiconductor, below critical temp, at 0 resistance,

finite current exists in absence of electric field

in eqn $\boxed{\frac{dJ_s}{dt} = \frac{e^2 E n_s}{m}}$ ④ first London eqn

$$\text{if } E = 0 \Rightarrow J_s = \text{const} \Rightarrow I = \text{finite}$$

Thus finite current exists for absence of E

Thus it can be verified

from Maxwell's eqn, $\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times \vec{E} = ②$

$$\vec{\nabla} \times \vec{H} = J_s + \frac{\partial \vec{B}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \left(J_s + \frac{\partial \vec{B}}{\partial t} \right)$$

$\frac{\partial \vec{B}}{\partial t}$ is negligible almost

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 J_s - ③$$

taking del operator on both sides of eqn ①

$$\vec{\nabla} \frac{d\vec{J}_s}{dt} = \frac{e^2 n_s}{m} \vec{\nabla} \times \vec{E}$$

$$= - \frac{e^2 n_s}{m} \frac{\partial \vec{B}}{\partial t}$$

$$\frac{d}{dt} \left[\vec{\nabla} \times \vec{J}_s + \frac{n_s e^2 \vec{B}}{m} \right] = 0$$

$\therefore \boxed{\vec{\nabla} \times \vec{J}_s + \frac{n_s e^2 \vec{B}}{m} = \text{constant}} = 0$

second London eqn

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$$

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Taking del operator on both sides of eqⁿ ③

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla} \times \vec{J}_s$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2 \vec{B} = \mu_0 \vec{\nabla} \times \vec{J}_s$$

$$-\vec{\nabla}^2 \vec{B} = -\frac{\mu_0 n_s e^2}{m} \vec{B}$$

$$\vec{\nabla}^2 \vec{B} = \frac{\mu_0 n_s e^2}{m} \vec{B}$$

trial solⁿ of this partial differential eqⁿ is

$$B = B_0 e^{-\frac{r}{\lambda}}$$

on solving gives

$$\text{coherence length} = \text{London penetrat length} = \lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

* Cooper pair:

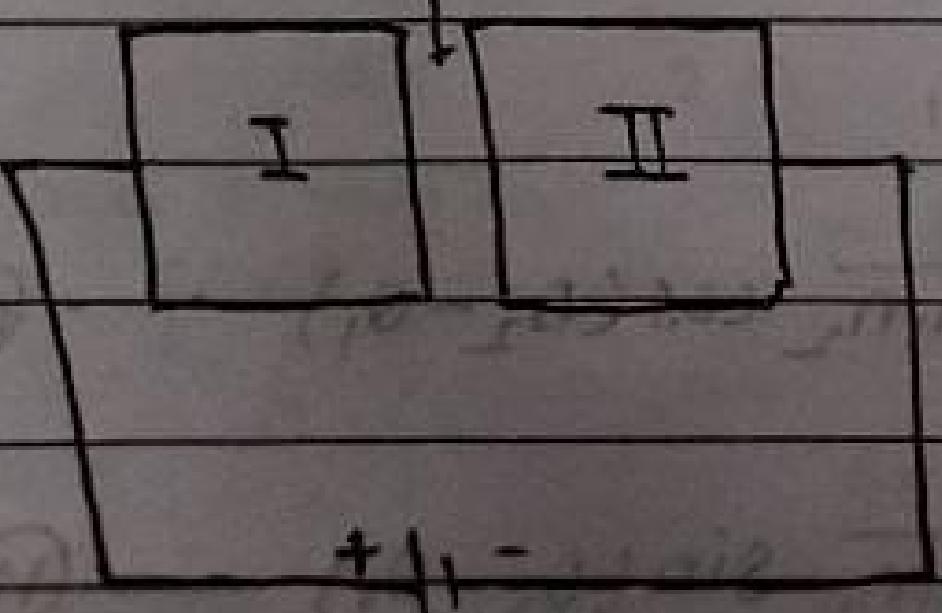
In condensed matter physics, cooper pair or BCS pair is a pair of electrons bound together at low temp. They are the pair of e⁻ in superconductor that are attractively bound & have equal & opposite momentum & spin.

* BCS theory (Bardeen - Cooper - Schrieffer)

The attractive electron-electron interaction mediated by the phonons give rise to cooper pairs i.e. bound states formed by two e⁻ of opposite spins & momenta.

Electrons can combine into pairs (Cooper pairs) at low temp in order to travel through superconductors without resistance. However the e⁻ normally repel each other, & some intermediary is req to induce them to pair.

Very thin insulating layer ($\approx 1 \text{ nm}$)



* Josephson effect : Two semi superconductors are placed & a very thin ($1\text{nm}-5\text{nm}$) wide insulator is introduced in between. This is known as Josephson junction. The effect causes a supercurrent to flow even without applying voltage

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FOR DC FLOW

Ψ_1 - prob of finding e^- in I

Ψ_2 - " " II

Schrodinger eqn for two state quantum system,

$$i\hbar \frac{\partial \Psi}{\partial t} = \hbar T \Psi_2 \quad (1) \quad T - \text{measure of leakage}$$

$$i\hbar \frac{\partial \Psi_2}{\partial t} = \hbar T \Psi_1 \quad (2)$$

let $\Psi_1 = \sqrt{n_1} e^{i\theta_1}$ - (3) where n is cooper pair density

$\Psi_2 = \sqrt{n_2} e^{i\theta_2}$ - (4) & θ is phase angle of cooper pair

be the trial sol's

putting values in original eqn's,

$$i\hbar \cdot \frac{1}{2\sqrt{n_1}} \frac{dn_1}{dt} \cdot e^{i\theta_1} = \hbar \sqrt{n_1} e^{i\theta_1} \frac{d\theta_1}{dt} = \hbar T \sqrt{n_1} e^{i\theta_2} \quad (5)$$

$$i\hbar \cdot \frac{1}{2\sqrt{n_2}} \frac{dn_2}{dt} \cdot e^{i\theta_2} = \hbar \sqrt{n_2} e^{i\theta_2} \frac{d\theta_2}{dt} = \hbar T \sqrt{n_1} e^{i\theta_1} \quad (6)$$

(5) $\times \sqrt{n_1} e^{-i\theta_1}$ gives

$$\frac{i\hbar}{2} \frac{dn_1}{dt} - \hbar n_1 \frac{d\theta_1}{dt} = \hbar T \sqrt{n_1 n_2} e^{i(\theta_2 - \theta_1)} \quad (7)$$

(6) $\times \sqrt{n_2} e^{-i\theta_2}$ gives

$$\frac{i\hbar}{2} \frac{dn_2}{dt} - \hbar n_2 \frac{d\theta_2}{dt} = \hbar T \sqrt{n_1 n_2} e^{i(\theta_1 - \theta_2)} \quad (8)$$

comparing real & imaginary part in eqn (7),

$$\frac{\hbar}{2} \frac{dn_1}{dt} = \hbar T \sqrt{n_1 n_2} \sin(\theta_2 - \theta_1) \quad (9)$$

$$-\hbar n_1 \frac{d\theta_1}{dt} = \hbar T \sqrt{n_1 n_2} \cos(\theta_2 - \theta_1) \quad (10)$$

similarly, for eqn (8),

$$n_2 \frac{d\theta_2}{dt} = -T \sqrt{n_1 n_2} \cos(\theta_2 - \theta_1) \quad (11)$$

$$\frac{1}{2} \frac{dn_2}{dt} = -T \sqrt{n_1 n_2} \sin(\theta_2 - \theta_1) \quad (12)$$

- applications of Joseph effect:
- i. Superconducting Quantum Interference Device (SQUID) is used to measure very weak magnetic fields even upto $10^{-18} T$ (magnetometre)
 - ii. Accurate reproducible conversion rate in relⁿ to freq & voltage is possible
 - iii. Superconducting single electron based transistors have an application of this effect
 - iv. Superconducting tunnel junction detectors (STT) have dependency on Josephson phenomenon
 - v. It defines a voltage standard metrology
 - vi. AC Josephson is used to measure very low temp based on variation of freq on emitted radiation with temperature

we can verify, for identical superconductor, $n_1 = n_2$

$$\therefore \frac{d\theta_1}{dt} = \frac{d\theta_2}{dt} \quad \text{i.e. } \frac{d(\theta_1 - \theta_2)}{dt} = 0$$

$$\& \frac{dn_1}{dt} = -\frac{dn_2}{dt} \quad \text{i.e. } \frac{d(n_1 + n_2)}{dt} = 0$$

FOR AC FLOW

Ψ_1 - prob of finding e^- in I

Ψ_2 - prob of finding e^- in II

Schrodinger eqn for two state quantum system,

$$i\hbar \frac{\partial \Psi_1}{\partial t} = \hbar T \Psi_2 - eV \Psi_1 \quad \text{--- (1)}$$

$$i\hbar \frac{\partial \Psi_2}{\partial t} = \hbar T \Psi_1 + eV \Psi_2 \quad \text{--- (2)}$$

let $\Psi_1 = \sqrt{n_1} e^{i\theta_1}$ --- (3) where n is cooper pair density

$\Psi_2 = \sqrt{n_2} e^{i\theta_2}$ --- (4) θ is phase angle of cooper pair

be the trial sol's

putting values in original eqn's,

$$i\sqrt{n_1} e^{i\theta_1} \frac{d\theta_1}{dt} + \frac{1}{2\sqrt{n_1}} e^{i\theta_1} \frac{dn_1}{dt} = -\sqrt{n_2} e^{i\theta_2} iT + \frac{eV}{\hbar} \sqrt{n_1} e^{i\theta_1} \quad \text{--- (5)}$$

$$i\sqrt{n_2} e^{i\theta_2} \frac{d\theta_2}{dt} + \frac{1}{2\sqrt{n_2}} e^{i\theta_2} \frac{dn_2}{dt} = -iT\sqrt{n_1} e^{i\theta_1} - \frac{ieV}{\hbar} \sqrt{n_2} e^{i\theta_2} \quad \text{--- (6)}$$

eqn (5) $\times \sqrt{n_1} e^{-i\theta_1}$ gives

$$n_1 \frac{d\theta_1}{dt} - \frac{i}{2} \frac{dn_1}{dt} = -T\sqrt{n_1 n_2} e^{i(\theta_2 - \theta_1)} + \frac{eV n_1}{\hbar} \quad \text{--- (7)}$$

comparing real & imaginary parts,

$$n_1 \frac{d\theta_1}{dt} = -T\sqrt{n_1 n_2} \cos(\theta_2 - \theta_1) + \frac{eV n_1}{\hbar} \quad \text{--- (8)}$$

$$\frac{dn_1}{dt} = T\sqrt{n_1 n_2} \sin(\theta_2 - \theta_1) \quad \text{--- (9)}$$

eqn ⑥ $\times \sqrt{n_2} e^{-i\theta_2}$ gives

$$n_2 \frac{dn_2}{dt} - i \frac{dn_2}{2dt} = -T \sqrt{n_1 n_2} e^{i(\theta_1 - \theta_2)} - \frac{eV n_2}{\hbar} \quad (1)$$

Comparing real & imaginary part,

$$\frac{n_2 d\theta_2}{dt} = -T \sqrt{n_1 n_2} \cos(\theta_1 - \theta_2) - \frac{eV n_2}{\hbar} \quad (11)$$

$$-\frac{dn_2}{2dt} = -T \sqrt{n_1 n_2} \sin(\theta_2 - \theta_1) \quad (12)$$

for ideal superconductors $n_1 = n_2$

Comparing eqn ⑧, ⑨, ⑪ & ⑫,

$$\frac{dn_1}{dt} = -\frac{dn_2}{dt} \quad d(\theta_2 - \theta_1) = -\frac{2eVdt}{\hbar} \quad : \quad \int d(\theta_2 - \theta_1) = \int -\frac{2eVdt}{\hbar}$$

δ is phase difference

$$\therefore \delta(t) - \delta(0) = -\frac{2eVt}{\hbar}; \quad \boxed{\delta(t) = \delta(0) - \frac{2eVt}{\hbar}} \quad \text{current} = \boxed{I = I_0 \sin(\delta(t))}$$

current oscillates with freq

This is Josephson's effect

$$\omega = \frac{2eV}{\hbar}$$

* Dielectric

At high E , e^- is shifted then small dipole is produced

$$\vec{P} = q \vec{d}$$

electric field at any random point is ..

$$\vec{E} = \frac{3(\vec{P} \cdot \vec{r}) \vec{r} - \gamma^2 \vec{P}}{r^5}$$

$$\vec{D} = \epsilon_r \vec{E}$$

D - displacement current

$$\epsilon = \bullet \frac{1}{4\pi \times 9 \times 10^9}$$

$$\vec{D} = \epsilon_r \vec{E} = \vec{E} \epsilon_r + \vec{P}$$

$$\begin{aligned} \vec{P} &= \epsilon_0 (\epsilon_r - 1) \vec{E} \\ &= \epsilon_0 \chi_e \vec{E} \end{aligned}$$

$$\text{Capacitance in vacuum } C_{\text{vac}} = \epsilon_0 \frac{A}{d}$$

A - area of capacitor plate

$$\text{Capacitance in dielectric } C_{\text{di}} = \epsilon_r \epsilon_0 (A/d) \quad d - \text{distance in parallel plates}$$

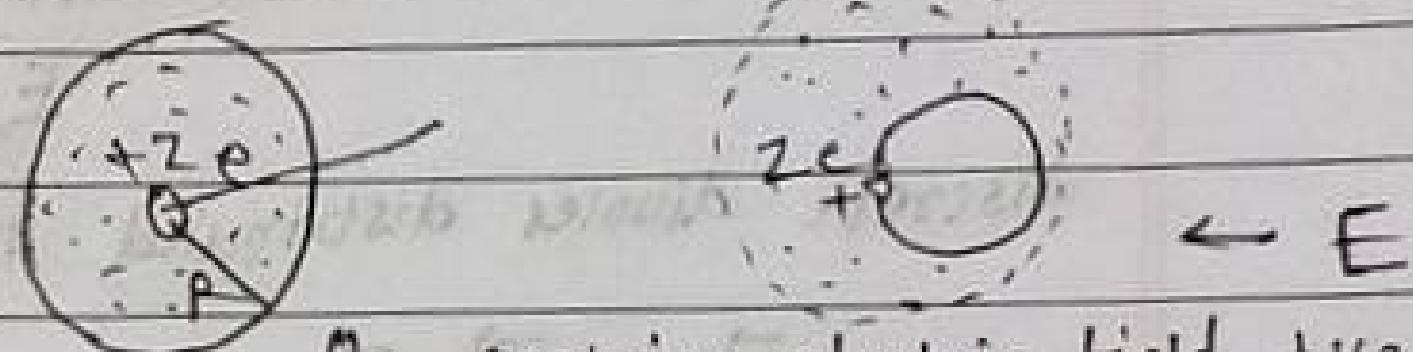
$$\text{dielectric constant } (\epsilon_r) = C_{\text{di}} / C_{\text{vac}}$$

Types of polarization

- 1) deformational / distortional
- 2) orientation / dipolar

deformational polarizability :

electronic polarization



ionic polarization (α_i)

$$\text{charge density } (\rho) = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

On applying electric field, +ve charge is going in opp. direction of field & -ve charge is in dir. of field

$$M_e = \alpha_e E \quad M_e - \text{electronic dipole moment}$$

$$P_e = N M_e \quad \kappa_e - \text{electronic polarizability}$$

$$= N \alpha_e E \quad N - \text{no. of dipoles per unit vol.}$$

$$= E \epsilon_0 (\epsilon_r - 1) \cdot P_e \quad P_e - \text{electric polarization}$$

$$= E \epsilon_0 \chi$$

$$\epsilon_r - 1 = \chi = \frac{\alpha_e N}{\epsilon_0}$$

ionic polarizability α_i

$$M_i = \alpha_i E$$

Total dipole moment (P_t) = $M_i + M_{\text{induced}}$

$$P = N \alpha_e E$$

$$D = \epsilon_r E = E \epsilon_0 + P = E \epsilon_0 + N \alpha_e E$$

$$D = \epsilon_0 E \left(1 + \frac{N \alpha_e}{\epsilon_0} \right) = \epsilon_0 E \epsilon_r$$

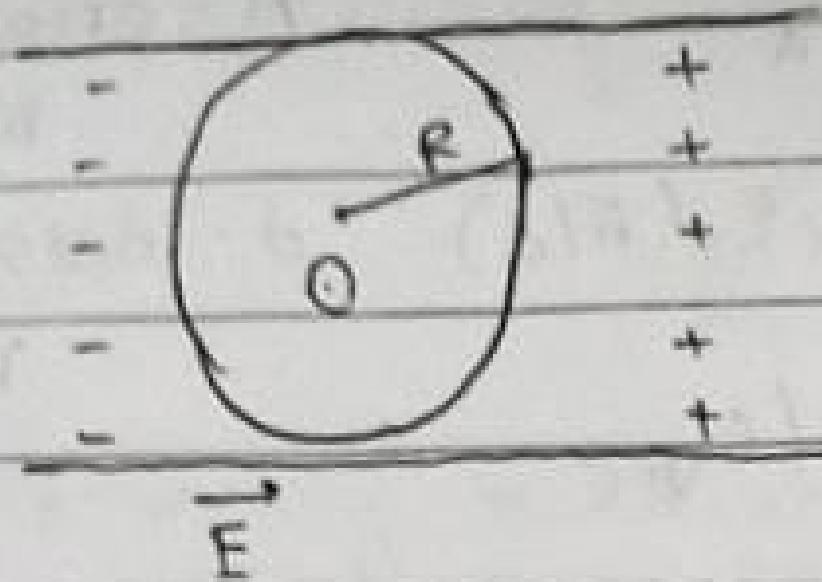
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$$D = \epsilon_0 E (1 + \chi)$$

ϵ_r - macroscopic

χ - microscopic

R Lorentz



$$\vec{E}_{\text{Local}}(\vec{r}) = \vec{E}_0(\vec{r}) + \vec{E}_{\text{dep}}(\vec{r}) + \vec{E}_{\text{surf}}(\vec{r}) + \vec{E}_{\text{dip}}(\vec{r})$$

E_0 - field intensity due to charge on the plates of the capacitor

E_{dep} - depolarization field is the field at the atom due to the polarized charge (induced charge) on the plane surfaces of the dielectric

E_{surf} - field due to polarized charges on the surface of the spherical cavity

E_{dip} - field at the centre of cavity due to discrete dipoles distributed inside spherical cavity (Lorentz cavity)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{E}_0 = \frac{\vec{D}_0}{\epsilon_0} = \vec{E} + \frac{\vec{P}}{\epsilon_0}$$

$$\vec{E}_{\text{dep}} = -\frac{\vec{P}}{\epsilon_0} \quad (\text{Infinite slab})$$

$$\vec{E}_{\text{surf}} = \int_0^R \frac{-P \cos \theta}{4\pi \epsilon_0 R^2} \cos \theta (2\pi R \cdot R \sin \theta d\theta) \\ = \frac{\vec{P}}{3\epsilon_0}$$

$$\vec{E}_{\text{dip}} = 0$$

adding all $\vec{E}_{\text{Local}} = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$

E_{Local} - microscopic
 E_{ext} - macroscopic

Lorentz field

We know, $\vec{P} = \alpha E_L N$

$$E_{\text{local}} = E_{\text{ext}} + \frac{\vec{P}}{3\epsilon_0} = E_{\text{ext}} + \frac{N \alpha E_{\text{local}}}{3\epsilon_0}$$

$$\therefore E_{\text{ext}} = E_{\text{local}} \left(1 - \frac{N \alpha}{3\epsilon_0} \right)$$

$$\frac{E_{\text{local}}}{E_{\text{ext}}} = 1 - \frac{N\alpha}{3\epsilon_0}$$

$$\chi_c = \frac{\vec{P}}{\epsilon_0 E_{\text{ext}}} = \frac{N\alpha}{\epsilon_0 \left(1 - \frac{N\alpha}{3\epsilon_0}\right)}$$

$$\epsilon_r = 1 + \chi_c$$

$$= 1 + \frac{N\alpha}{\epsilon_0 \left(1 - \frac{N\alpha}{3\epsilon_0}\right)} = 1 + \frac{3N\alpha}{3\epsilon_0 - N\alpha}$$

$$= \frac{3\epsilon_0 - N\alpha + 3N\alpha}{3\epsilon_0 - N\alpha}$$

$$\epsilon_r = \frac{3\epsilon_0 + 2N\alpha}{3\epsilon_0 - N\alpha}$$

$$\text{now, } \epsilon_{r-1} = \frac{N\alpha}{3\epsilon_0 - N\alpha}$$

$$\epsilon_{r+2} = 3 + \frac{3N\alpha}{3\epsilon_0 - N\alpha} = \frac{9\epsilon_0}{3\epsilon_0 - N\alpha}$$

$$\therefore \boxed{\begin{aligned} \epsilon_{r-1} &= N\alpha \\ \epsilon_{r+2} &= 3\epsilon_0 \end{aligned}}$$

Clausius Mosotti rel.

$$\frac{\epsilon_{r-1} - M}{\epsilon_{r+2} - S} = \frac{N\alpha - M}{3\epsilon_0 - S}$$

$$\frac{NM}{S} = N_A \quad N - \text{molecules per vol.}$$

M - molecular wt

S - charge density

N_A - avogadro no.

molar polarizn

$$\frac{\epsilon_{r-1} - M}{\epsilon_{r+2} - S} = \frac{N\alpha}{3\epsilon_0} = P_m$$

$$P_m = \frac{N\alpha}{3\epsilon_0} (\alpha_e + \alpha_i + \alpha_o)$$

e - electronic

i - ionic

o - orientational

D - deformational

$$\boxed{P_m = \frac{N}{3\epsilon_0} (\alpha_D + \alpha_O)}$$

Debye eq^n

* Lorentz formula

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

μ_0 - magnetic permeability
 ϵ_0 - absolute permittivity

$$v_w = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\frac{1}{n} = \frac{\text{Velocity of wave in medium}}{\text{Velocity of wave in vacuum}} = \frac{\frac{1}{\sqrt{\mu \epsilon}}}{\frac{1}{\sqrt{\mu_0 \epsilon_0}}} = \frac{\sqrt{\epsilon_r}}{\sqrt{\mu_0 \epsilon_0}}$$

$$\therefore n = \frac{\text{velocity in vacuum}}{\text{velocity in medium}} = \left(\frac{\mu \epsilon}{\mu_0 \epsilon_0} \right)^{1/2} = \sqrt{\epsilon_r}$$

for non magnetic medium, $\mu = \mu_0$

$$\therefore n^2 = \epsilon_r$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} \cdot \frac{M}{g} = \frac{N \alpha}{3 \epsilon_0}$$

$$\therefore \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N \alpha}{3 \epsilon_0} \times \frac{g}{M}$$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N \alpha g}{3 \epsilon_0 M}$$

- Q) polarizability of Neon is 0.35×10^{-42} Farad/m² if gas contains 2.7×10^{25} atm/m³ at 0°C at 1 atm, find its relative dielectric const.

$$\epsilon_r = \frac{3 \epsilon_0 + 2 N \alpha}{3 \epsilon_0 - N \alpha} \quad N = 2.7 \times 10^{25} \text{ atm/m}^3$$

$$\alpha = 35 \times 10^{-42} \text{ Farad/m}^2$$

$$\epsilon_0 = 8.84 \times 10^{-12} \text{ Farad/m}$$

$$= 26.52 \times 10^{12} - 189 \times 10^{17}$$

$$26.52 \times 10^{12} - 94.5 \times 10^{17}$$

$$= \frac{26.25 \times 10^5 - 189}{26.52 \times 10^5 - 94.5} = \frac{2.625 \times 10^4}{2.652 \times 10^6}$$

$$\approx 0.9897$$