Solid Mechanics (ME 301)

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Books:

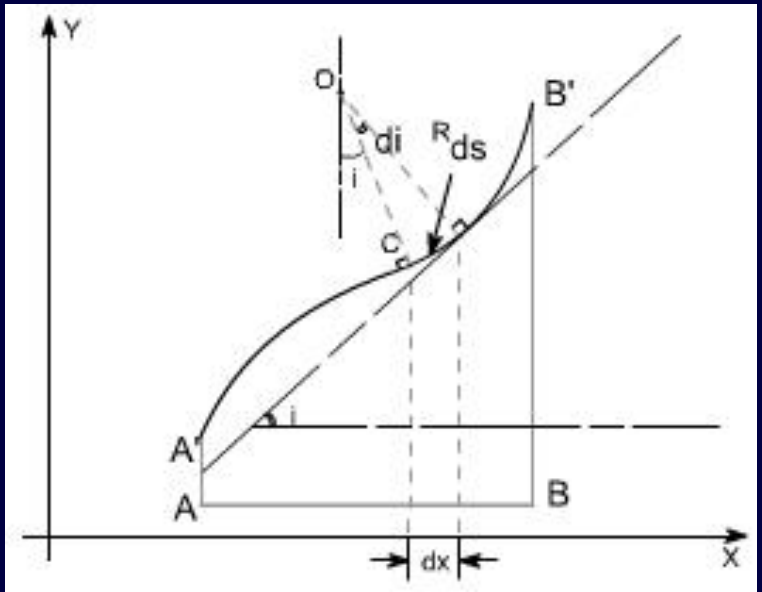
1. Strength of Materials: Part I, II, S. Timoshenko, CBS Publishers, 1985.
2. Engineering Mechanics of Solids, E. P. Popov, PHI, 1993.
3. Introduction to Solid Mechanics, I. H. Shames and J. M. Pittariesi, PHI, 2003.
4. Strength of Materials, F. L. Singer and A. Pytel, HarperCollins Publishers, 1991

Deflection of Beams

Assumptions:

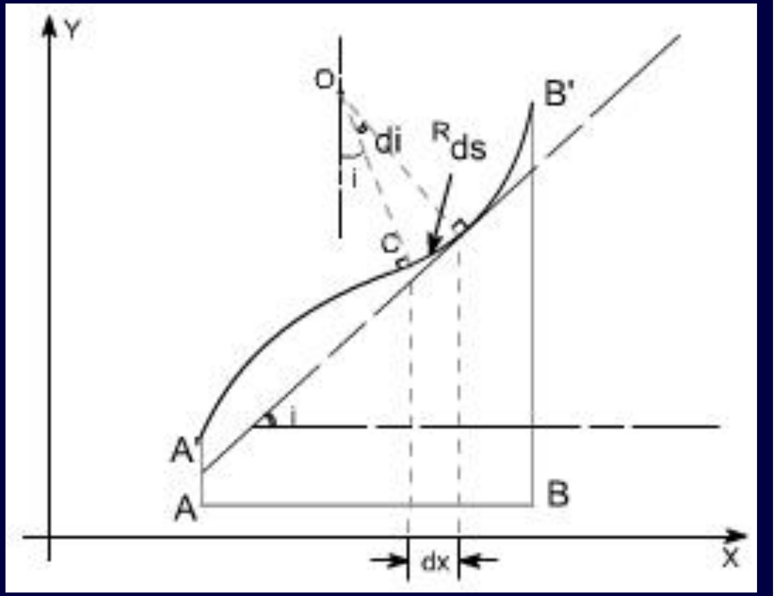
- Stress is proportional to strain i.e. hooks law applies. Thus, the equation is valid only for beams that are not stressed beyond the elastic limit.
- The curvature is always small.
- Any deflection resulting from the shear deformation of the material or shear stresses is neglected.

Consider a beam AB which is initially straight and horizontal when unloaded. If under the action of loads the beam deflect to a position A'B' under load or in-fact we say that the axis of the beam bends to a shape A'B'. It is customary to call A'B' the curved axis of the beam as the elastic line or deflection curve.



To express the deflected shape of the beam in rectangular co-ordinates let us take two axes x and y, x-axis coincide with the original straight axis of the beam and the y axis shows the deflection.

Futher, let us consider an element ds of the deflected beam. At the ends of this element let us construct the normal which intersect at point O denoting the angle between these two normal be di



$\tan i = \dfrac{dy}{dx}$ ......(1) or $i = \dfrac{dy}{dx}$ Assuming $\tan i = i$

Futher

$ds = R\,di$

however,

$ds = dx$ [usually for small curvature]

Hence

$ds = dx = R\,di$

or $\boxed{\dfrac{di}{dx} = \dfrac{1}{R}}$

substituting the value of i, one get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{R} \text{ or } \frac{d^2y}{dx^2} = \frac{1}{R}$$

From the simple bending theory

$$\frac{M}{I} = \frac{E}{R} \text{ or } M = \frac{EI}{R}$$

so the basic differential equation governing the deflection of beams is

$$\boxed{M = EI\frac{d^2y}{dx^2}}$$

Relationship between shear force, bending moment and deflection

$$\frac{dM}{dx} = EI\frac{d^3y}{dx^3} \quad \text{Recalling } \frac{dM}{dx} = F$$

Thus,

$$F = EI\frac{d^3y}{dx^3}$$

$$\text{i.e } w = -\frac{dF}{dx}$$

$$w = -EI\frac{d^4y}{dx^4}$$

Therefore if 'y' is the deflection of the loaded beam, then the following important relations can be arrived at

$$\boxed{\text{slope} = \frac{dy}{dx}}$$

$$\boxed{B.M = EI\frac{d^2y}{dx^2}}$$

$$\boxed{\text{Shear force} = EI\frac{d^3y}{dx^3}}$$

$$\boxed{\text{load distribution} = EI\frac{d^4y}{dx^4}}$$

Methods for finding the deflection

Direct integration method: The governing differential equation is defined as

$$M = EI\frac{d^2y}{dx^2} \quad \text{or} \quad \frac{M}{EI} = \frac{d^2y}{dx^2}$$

on integrating one get,

$$\frac{dy}{dx} = \int \frac{M}{EI}\,dx + A \text{ - - - - this equation gives the slope}$$

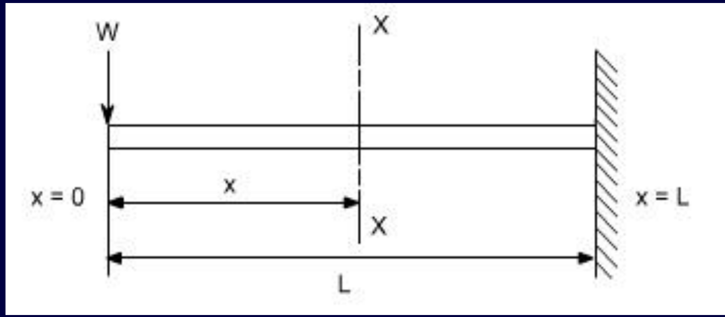
of the loaded beam.

Integrate once again to get the deflection.

$$y = \int\int \frac{M}{EI}\,dx + Ax + B$$

Where A and B are constants of integration to be evaluated from the known conditions of slope and deflections for the particular value of x.

Case 1: Cantilever Beam with Concentrated Load at the end



End Criteria / Boundary Conditions:

i.e at x= L ; y= 0 ------------------ (1)

at x = L ; dy/dx = 0 ------------------ (2)

$S.F|_{x-x} = -W$

$B.M|_{x-x} = -W.x$

Therefore $M|_{x-x} = -W.x$

the governing equation $\frac{M}{EI} = \frac{d^2y}{dx^2}$

substituting the value of M interms of x then integrating the equation one get

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = -\frac{Wx}{EI}$$

$$\int \frac{d^2y}{dx^2} = \int -\frac{Wx}{EI}dx$$

$$\frac{dy}{dx} = -\frac{Wx^2}{2EI} + A$$

Integrating once more,

$$\int \frac{dy}{dx} = \int -\frac{Wx^2}{2EI}dx + \int A\,dx$$

$$y = -\frac{Wx^3}{6EI} + Ax + B$$

$$A = \frac{Wl^2}{2EI}$$

While employing the first condition yields

$$y = -\frac{WL^3}{6EI} + AL + B$$

$$B = \frac{WL^3}{6EI} - AL$$

$$= \frac{WL^3}{6EI} - \frac{WL^3}{2EI}$$

$$= \frac{WL^3 - 3WL^3}{6EI} = -\frac{2WL^3}{6EI}$$

$$B = -\frac{WL^3}{3EI}$$

Substituting the values of A and B we get

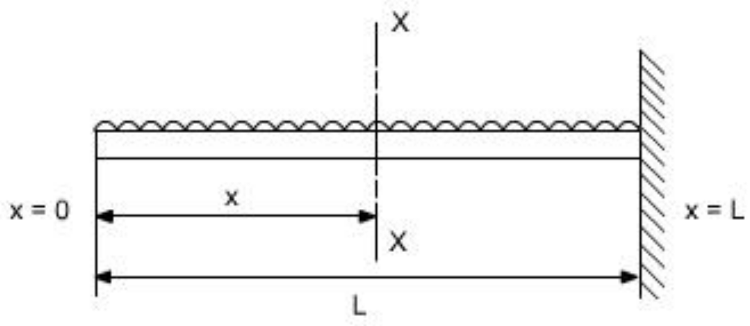
$$y = \frac{1}{EI}\left[-\frac{Wx^3}{6EI} + \frac{WL^2x}{2EI} - \frac{WL^3}{3EI}\right]$$

The slope as well as the deflection would be maximum at the free end hence putting x=0 we get,

$$\boxed{y_{max} = -\frac{WL^3}{3EI}}$$

$$\boxed{(Slope)_{maxm} = +\frac{WL^2}{2EI}}$$

Case 2: A Cantilever with Uniformly distributed Loads



End Criteria / Boundary Conditions:

i.e at x= L ; y= 0 ------------------ (1)

at x = L ; dy/dx = 0 ------------------ (2)

$$S.F|_{x-x} = -w$$

$$B.M|_{x-x} = -w.x.\frac{x}{2} = w\left(\frac{x^2}{2}\right)$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = -\frac{wx^2}{2EI}$$

$$\int\frac{d^2y}{dx^2} = \int -\frac{wx^2}{2EI}dx$$

$$\frac{dy}{dx} = -\frac{wx^3}{6EI} + A$$

$$\int\frac{dy}{dx} = \int -\frac{wx^3}{6EI}dx + \int A\,dx$$

$$y = -\frac{wx^4}{24EI} + Ax + B$$

$$A = +\frac{wx^3}{6EI}$$

whereas the first boundary conditions yields

$$B = \frac{wL^4}{24EI} - \frac{wL^4}{6EI}$$

$$B = -\frac{wL^4}{8EI}$$

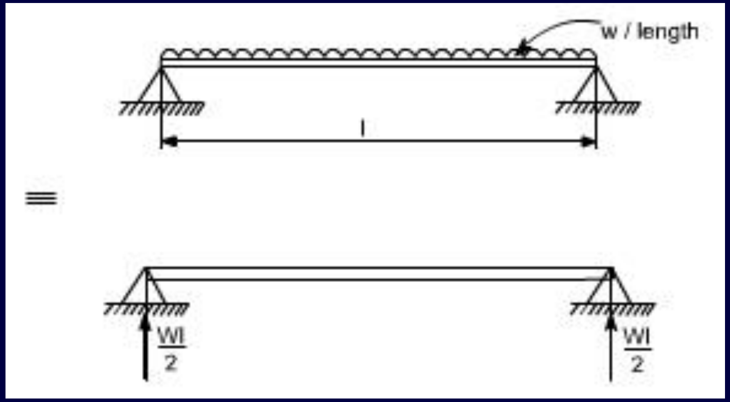
$$\text{Thus, } y = \frac{1}{EI}\left[-\frac{wx^4}{24} + \frac{wL^3x}{6} - \frac{wL^4}{8}\right]$$

So y_{max^m} will be at $x = 0$

$$\boxed{y_{max^m} = -\frac{wL^4}{8EI}}$$

$$\boxed{\left(\frac{dy}{dx}\right)_{max^m} = \frac{wL^3}{6EI}}$$

Case 3: Simply Supported beam with uniformly distributed Loads



End Criteria / Boundary Conditions:

i.e at x = 0 ; y= 0 ------------------ (1)

at x = L ; y= 0 ------------------ (2)

$$S.F|_{x-x} = w\left(\frac{l}{2}\right) - w.x$$

$$B.M|_{x-x} = w.\left(\frac{l}{2}\right).x - w.x.\left(\frac{x}{2}\right)$$

$$= \frac{wl.x}{2} - \frac{wx^2}{2}$$

The differential equation which gives the elastic curve for the deflected beam is

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{EI}\left[\frac{wl.x}{2} - \frac{wx^2}{2}\right]$$

$$\frac{dy}{dx} = \int \frac{wlx}{2EI}dx - \int \frac{wx^2}{2EI}dx + A$$

$$= \frac{wlx^2}{4EI} - \frac{wx^3}{6EI} + A$$

Integrating, once more one gets

$$y = \frac{wlx^3}{12EI} - \frac{wx^4}{24EI} + A.x + B \qquad \text{-----(1)}$$

$$0 = \frac{wl^4}{12EI} - \frac{wl^4}{24EI} + A.l$$

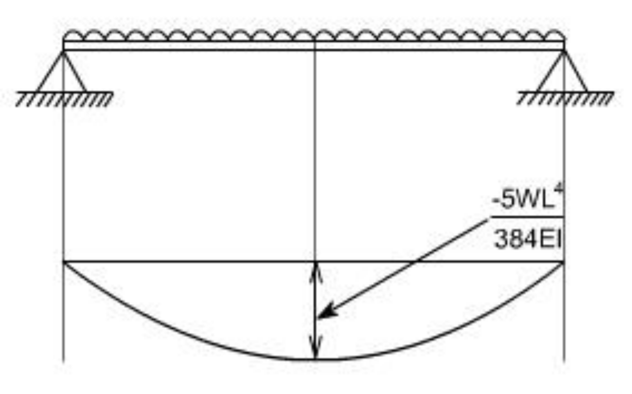
$$A = -\frac{wl^3}{24EI}$$

So the equation which gives the deflection curve is

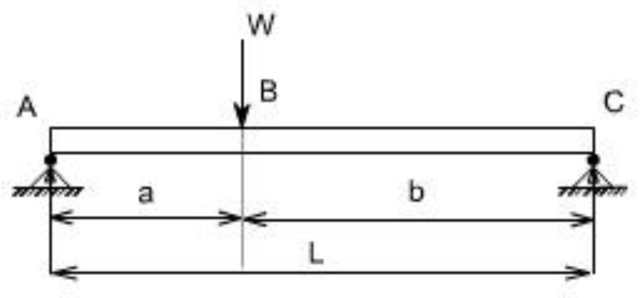
$$y = \frac{1}{EI}\left[\frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24}\right]$$

Then $$y_{max^m} = \frac{1}{EI}\left[\frac{wL}{12}\left(\frac{L^3}{8}\right) - \frac{w}{24}\left(\frac{L^4}{16}\right) - \frac{wL^3}{24}\left(\frac{L}{2}\right)\right]$$

$$\boxed{y_{max^m} = -\frac{5wL^4}{384EI}}$$



Case 4: Simply Supported beam with Concentrated Load



$$R_1 = \frac{Wb}{a+b}$$

Hence,

$$EI\frac{d^2y}{dx^2} = \frac{Wb}{(a+b)}x \qquad 0 \le x \le a \text{ --------(1)}$$

$$EI\frac{d^2y}{dx^2} = \frac{Wb}{(a+b)}x - W(x-a) \qquad a \le x \le l \text{ ---------(2)}$$

integrating (1) and (2) we get,

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 + k_1 \qquad 0 \le x \le a \text{ --------(3)}$$

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 - \frac{W(x-a)^2}{2} + k_2 \qquad a \le x \le l \text{ --------(4)}$$

B.M for the portion AB

$M|_{AB} = R_1.x \quad 0 \le x \le a$

B.M for the portion BC

$M|_{BC} = R_1.x - W(x-a) \quad a \le x \le l$

so the differential equation for the two cases would be,

$$EI\frac{d^2y}{dx^2} = R_1 x$$

$$EI\frac{d^2y}{dx^2} = R_1 x - W(x-a)$$

Boundary Conditions:

at x = 0; y = 0 in the portion AB i.e. $0 \le x \le a$

at x = l; y = 0 in the portion BC i.e. $a \le x \le l$

at x = a; dy/dx, the slope is same for both portion

at x = a; y, the deflection is same for both portion

Using Condition (c)

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 + k \qquad 0 \le x \le a \text{------}(3)$$

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 - \frac{W(x-a)^2}{2} + k \qquad a \le x \le l \text{-------}(4)$$

Integrating agian equation (3) and (4) we get

$$EIy = \frac{Wb}{6(a+b)}x^3 + kx + k_3 \qquad 0 \le x \le a \text{------}(5)$$

$$EIy = \frac{Wb}{6(a+b)}x^3 - \frac{W(x-a)^3}{6} + kx + k_4 \qquad a \le x \le l \text{------}(6)$$

Utilizing condition (a) in equation (5) yields

$$k_3 = 0$$

Utilizing condition (b) in equation (6) yields

$$0 = \frac{Wb}{6(a+b)}l^3 - \frac{W(l-a)^3}{6} + kl + k_4$$

$$k_4 = -\frac{Wb}{6(a+b)}l^3 + \frac{W(l-a)^3}{6} - kl$$

But $a + b = l$,

Thus,

$$k_4 = -\frac{Wb(a+b)^2}{6} + \frac{Wb^3}{6} - k(a+b)$$

Using Condition (d)

Therefore $y|_{\text{from equation 5}} = y|_{\text{from equation 6}}$

or

$$\frac{Wb}{6(a+b)}x^3 + kx + k_3 = \frac{Wb}{6(a+b)}x^3 - \frac{W(x-a)^3}{6} + kx + k_4$$

$$\frac{Wb}{6(a+b)}a^3 + ka + k_3 = \frac{Wb}{6(a+b)}a^3 - \frac{W(a-a)^3}{6} + ka + k_4$$

Thus, $k_4 = 0$;

OR

$$k_4 = -\frac{Wb(a+b)^2}{6} + \frac{Wb^3}{6} - k(a+b) = o$$

$$k(a+b) = -\frac{Wb(a+b)^2}{6} + \frac{Wb^3}{6}$$

$$k = -\frac{Wb(a+b)}{6} + \frac{Wb^3}{6(a+b)}$$

so the deflection equations for each portion of the beam are

$$EIy = \frac{Wb}{6(a+b)}x^3 + kx + k_3$$

$$EIy = \frac{Wbx^3}{6(a+b)} - \frac{Wb(a+b)x}{6} + \frac{Wb^3x}{6(a+b)} \qquad ----\textbf{for } 0 \le x \le a -----(7)$$

and for other portion

$$EIy = \frac{Wb}{6(a+b)}x^3 - \frac{W(x-a)^3}{6} + kx + k_4$$

Substituting the value of 'k' in the above equation

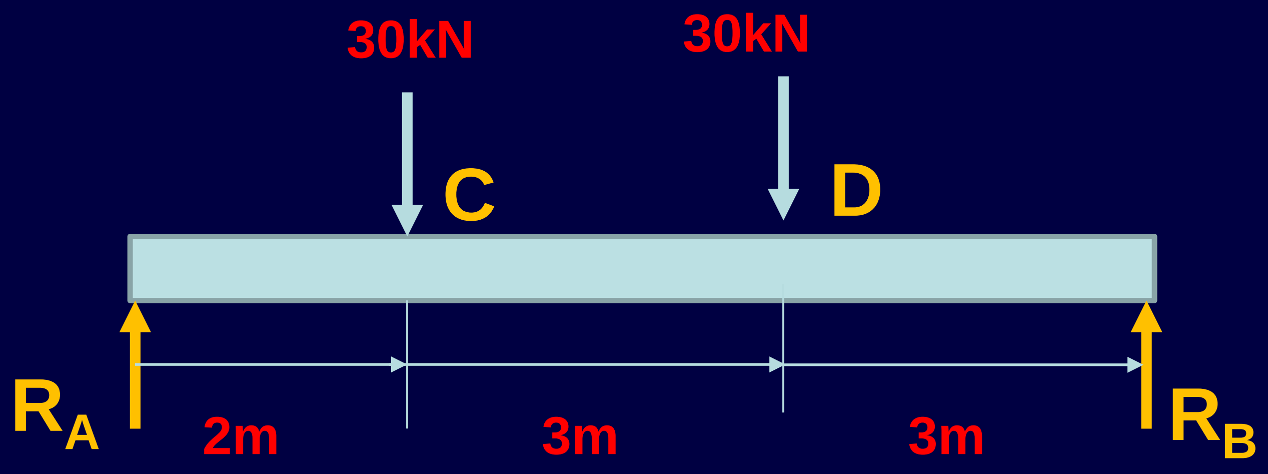
$$EIy = \frac{Wbx^3}{6(a+b)} - \frac{W(x-a)^3}{6} - \frac{Wb(a+b)x}{6} + \frac{Wb^3x}{6(a+b)} \qquad \text{For } \textbf{for } a \le x \le l ----- (8)$$

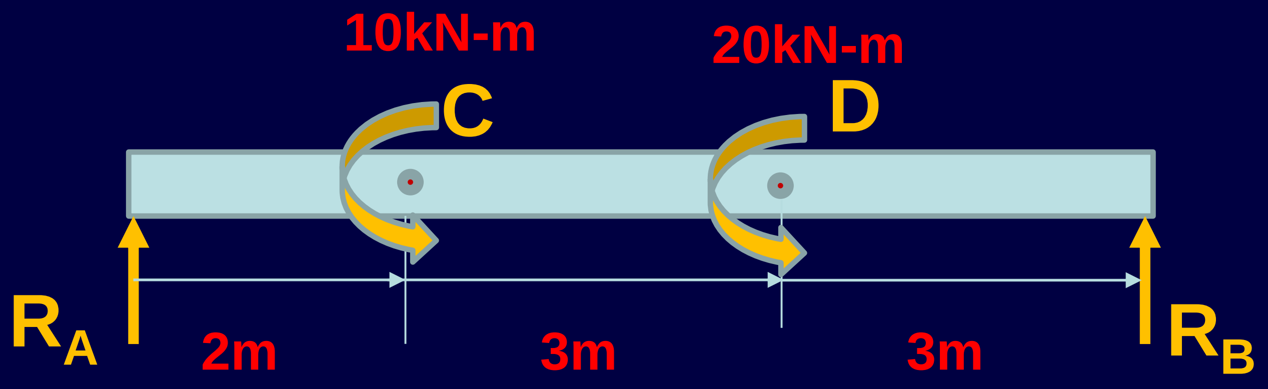
so either of the equation (7) or (8) may be used to find the deflection at $x = a$
hence substituting $x = a$ in either of the equation we get

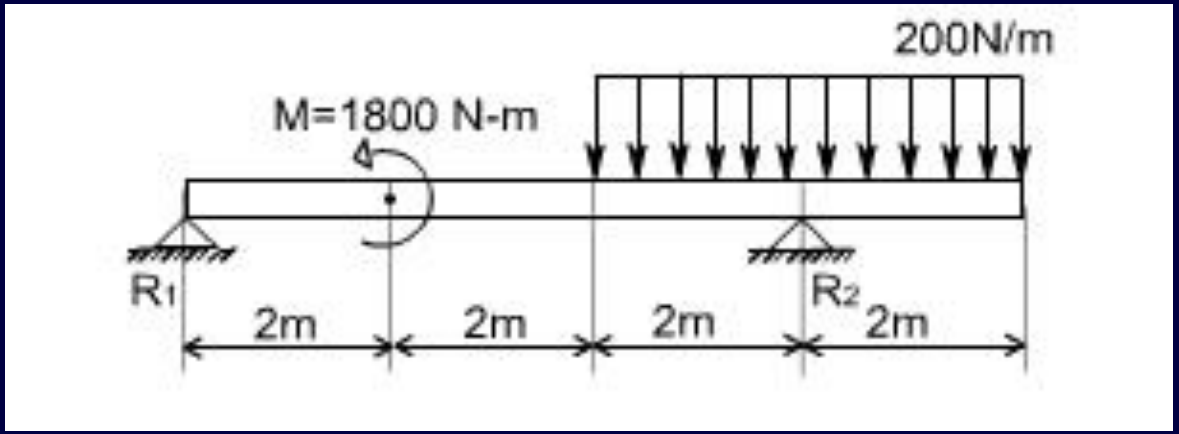
$$Y|_{x=a} = -\frac{Wa^2b^2}{3EI(a+b)}$$

OR if $a = b = l/2$

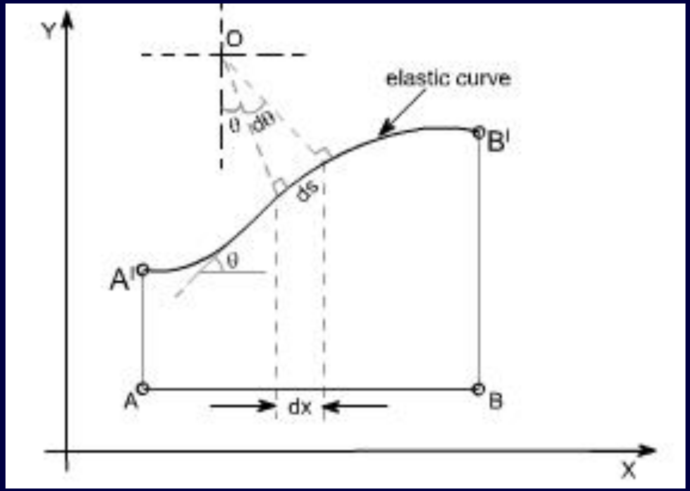
$$\boxed{Y_{maxm} = -\frac{WL^3}{48EI}}$$


30kN
30kN
C
D
RA
2m
3m
3m
RB


10kN-m
20kN-m
C
D
RA
2m
3m
3m
RB


200N/m
M=1800 N-m
R1
R2
2m
2m
2m
2m

THE AREA-MOMENT / MOMENT-AREA METHODS



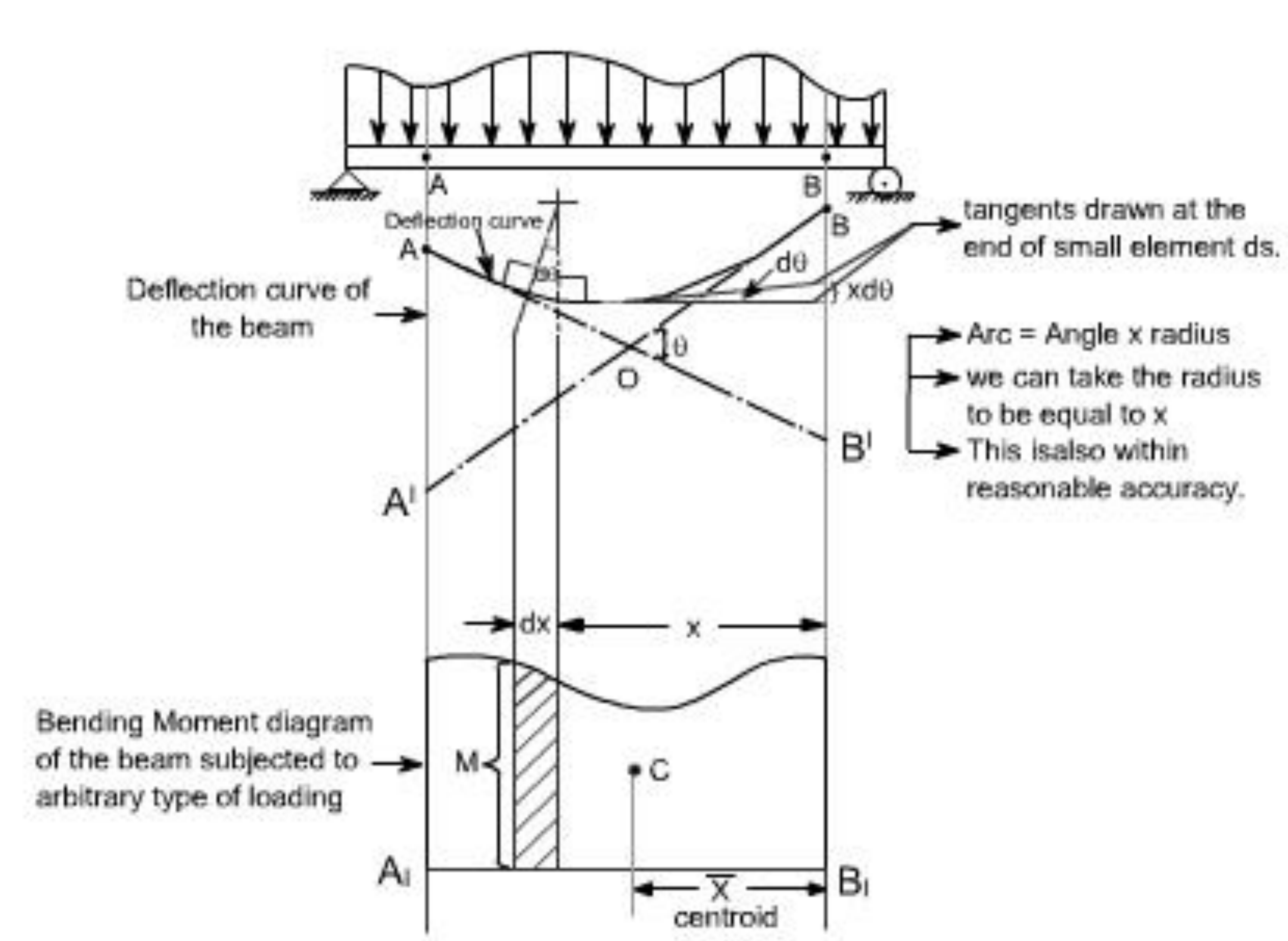
$$\frac{d\theta}{ds} = \frac{1}{R} = \frac{M}{EI}$$

$$\frac{d\theta}{ds} = \frac{M}{EI}$$

But for small curvature[but θ is the angle ,slope is $\tan\theta = \frac{dy}{dx}$ for small angles $\tan\theta \approx \theta$,hence $\theta \cong \frac{dy}{dx}$ so we get $\frac{d^2y}{dx^2} = \frac{M}{EI}$ by putting $ds \approx dx$]

Hence,

$$\frac{d\theta}{dx} = \frac{M}{EI} \text{ or } \boxed{d\theta = \frac{M.dx}{EI}} \text{ - - - - - (1)}$$

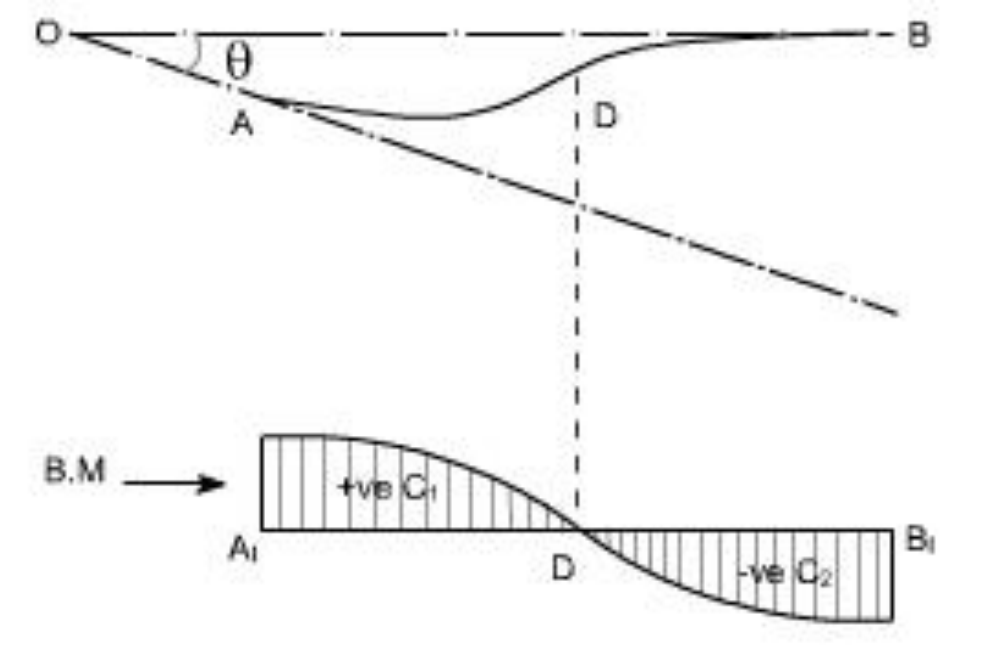


$$\boxed{\theta = \int_A^B \frac{Mdx}{EI} = \frac{1}{EI}\int_A^B Mdx}$$

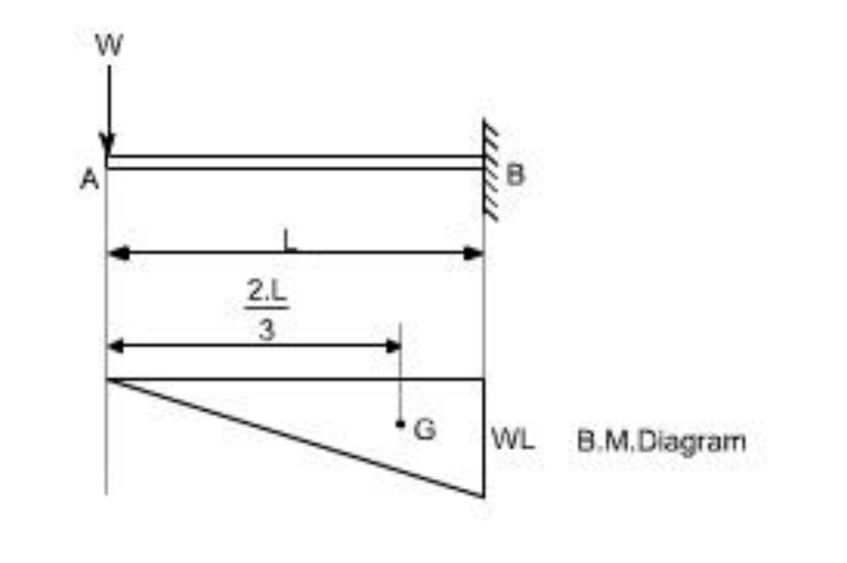
$$\left\{\begin{array}{c}\text{slope or } \theta \\ \text{between any two points}\end{array}\right\} = \left\{\begin{array}{l}\frac{1}{EI} \times \text{area of B.M diagram between} \\ \text{corresponding portion of B.M diagram}\end{array}\right\}$$

Theorem I: The angle θ between tangents at any two points A and B on the elastic line is equal to the total area of corresponding portion of the bending moment diagram, divided by EI.

$$\delta = \int_A^B x\,d\theta = \frac{1}{EI} \times \left\{ \begin{array}{l} \text{first moment of area with respect} \\ \text{to point B, of the total B.M diagram} \end{array} \right\}$$



Theorem II: The deflection of B away from the tangent at A is equal to the statical moment, with respect to B, of the bending moment area between A and B, divided by EI.



$$\text{slope at } A = \frac{1}{EI}\left[\text{Area of B.M diagram between the points A and B}\right]$$

$$= \frac{1}{EI}\left[\frac{1}{2}L.WL\right]$$

$$= \frac{WL^2}{2EI}$$

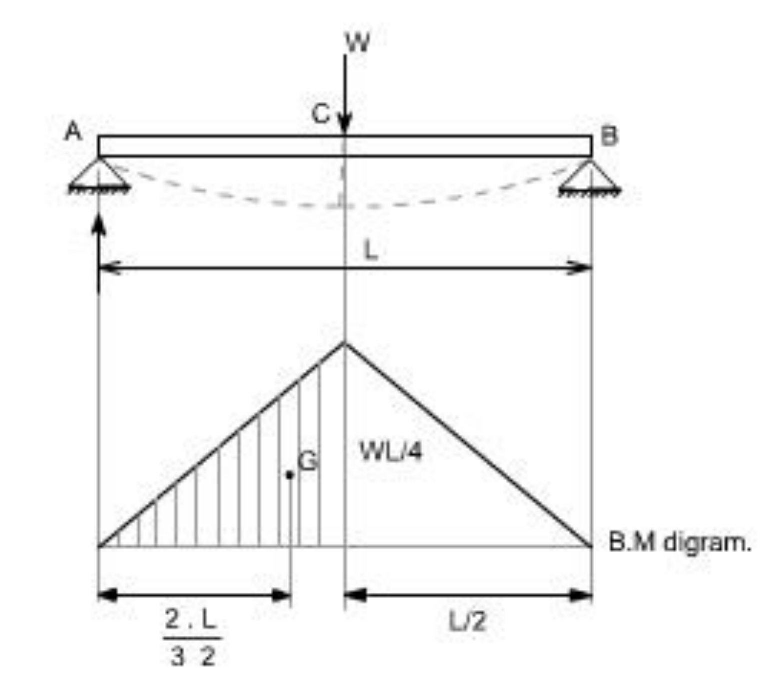
Thus,

$$\delta = \frac{1}{EI}\left[\text{first moment of area of B.M diagram between A and B about A}\right]$$

$$= \frac{1}{EI}\left[A\bar{y}\right]$$

$$= \frac{1}{EI}\left[\left(\frac{1}{2}L.WL\right)\frac{2}{3}L\right]$$

$$= \frac{WL^3}{3EI}$$



slope at A $= \frac{1}{EI}$[Area of B.M diagram between A and C]

$$= \frac{1}{EI}\left[\left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\left(\frac{WL}{4}\right)\right]$$

we are taking half area of the B.M because we have to work out this relative to a zero slope

$$= \frac{WL^2}{16EI}$$

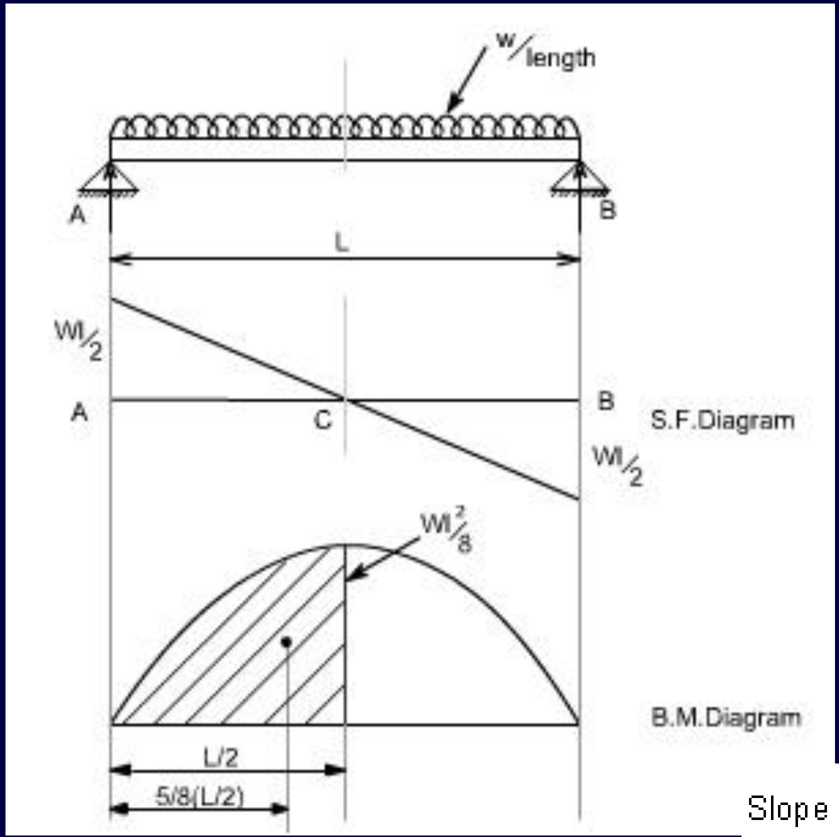
Deflection of A relative to C = central deflection of C

or

$\delta_C = \frac{1}{EI}$[Moment of B.M diagram between points A and C about A]

$$= \frac{1}{EI}\left[\left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\left(\frac{WL}{4}\right)\frac{2}{3}L\right]$$

$$= \frac{WL^3}{48EI}$$



$$\text{Slope at point C w.r.t point A} = \frac{1}{EI}\left[\text{Area of B.M diagram between point A and C}\right]$$

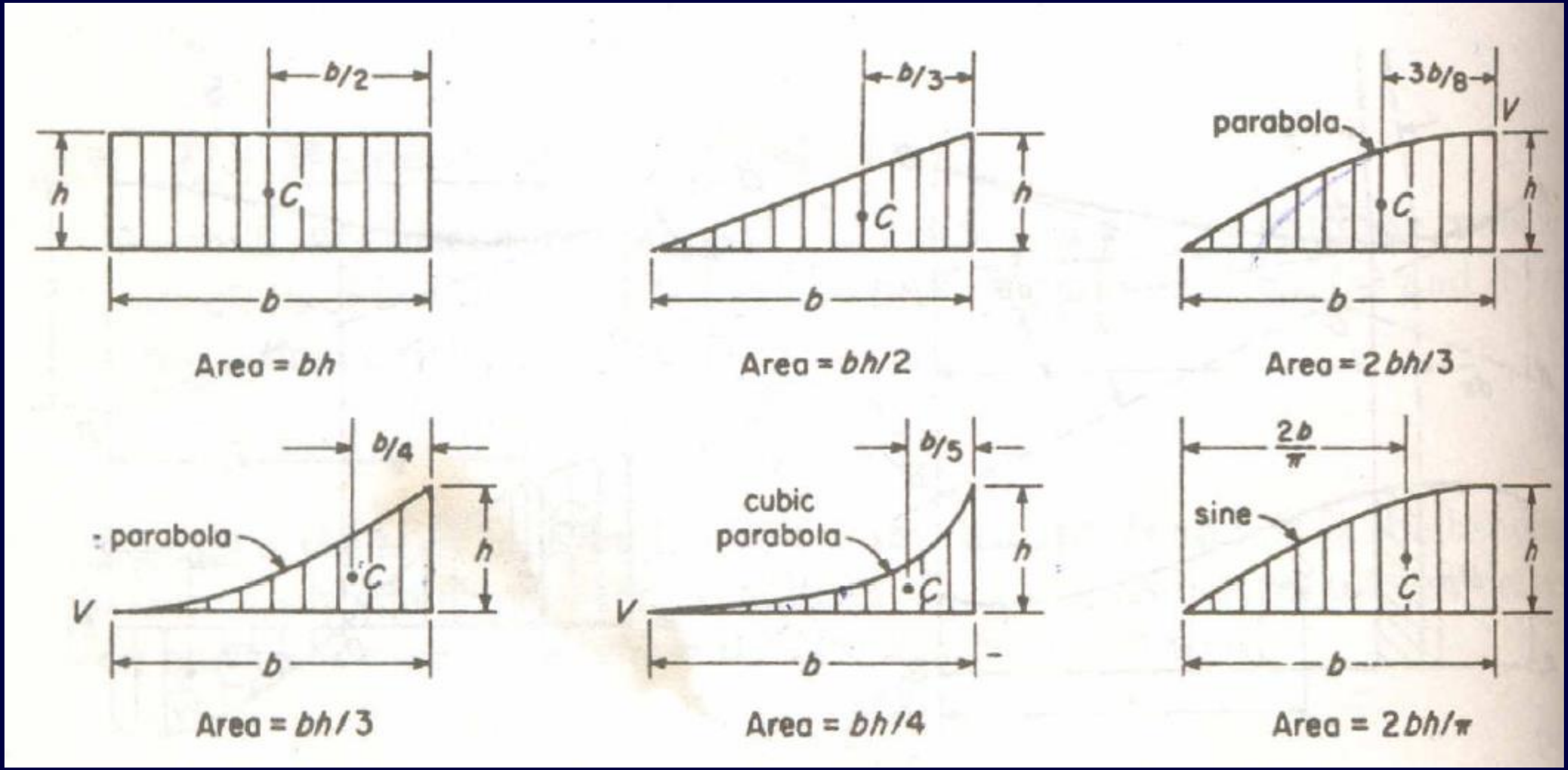
$$= \frac{1}{EI}\left[\left(\frac{2}{3}\right)\left(\frac{WL^2}{8}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{WL^3}{24EI}$$

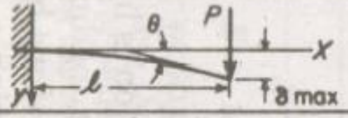
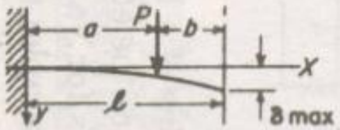
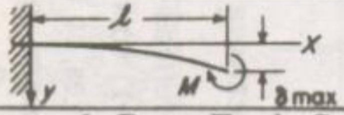
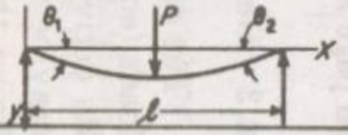
$$\text{Deflection at point C relative to A} = \frac{1}{EI}\left[A\,\bar{y}\right]$$

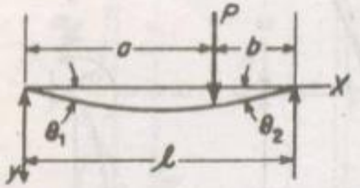
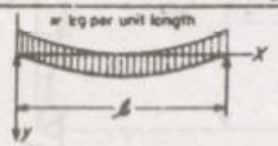
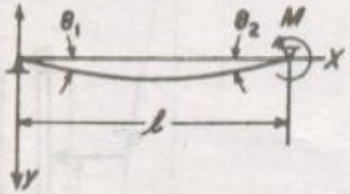
$$= \frac{1}{EI}\left[\left(\frac{WL^3}{24}\right)\left(\frac{5}{8}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{5}{384EI}.WL^4$$

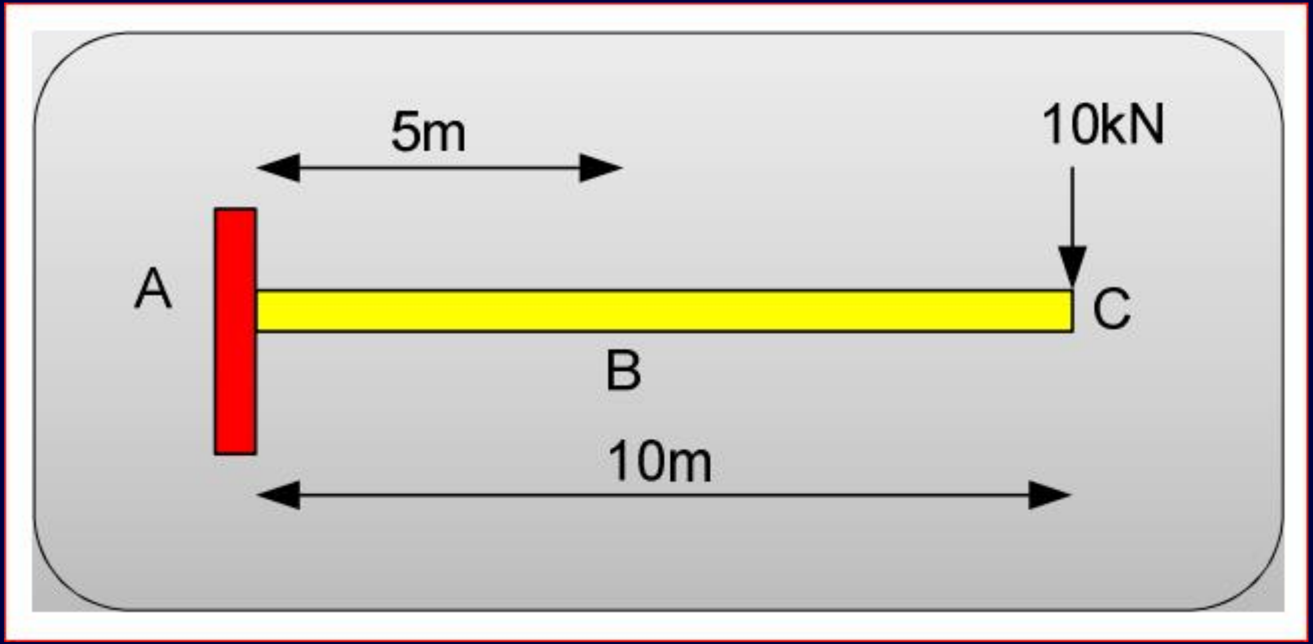


b/2
h
C
b
Area = bh
b/3
h
C
b
Area = bh/2
3b/8
V
parabola
h
C
b
Area = 2bh/3
b/4
parabola
V
h
C
b
Area = bh/3
b/5
cubic
parabola
V
h
C
b
Area = bh/4
2b
π
sine
h
C
b
Area = 2bh/π

Beam Type	Slope at Free End	Deflection at any section in terms of x: y is positive downward	Maximum deflection
1. Cantilever Beam — Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam — Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam — Uniformly distributed load of w kg per unit length			
	$\theta = \frac{wl^3}{6EI}$	$y = \frac{wx^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{wl^4}{8EI}$
4. Cantilever Beam — Uniformly varying load; maximum intensity w kg per unit length			
	$\theta = \frac{wl^3}{24EI}$	$y = \frac{wx^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{wl^4}{30EI}$
5. Cantilever Beam — Couple M applied at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$
6. Beam Freely Supported at Ends — Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$

Beam Type	Slope at Ends	Deflection at any sectoin in terms of x: y is positive downward	Maximum and center deflection
7. Beam Freely Supported at Ends — Concentrated load at any point			
	Left End. $\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ Right End. $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}(l^2 - x^2 - b^2)[0 < x < a]$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x - a)^3 + (l^2 - b^2)x - x^3\right][a < x < l]$	$\delta_{max} = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,l\,EI}$ at $x = \sqrt{\frac{l^2 - b^2}{3}}$ At center, if $a > b$ $\delta = \frac{Pb}{48EI}(3l^2 - 4b^2)$
8. Beam Freely Supported at Ends — Uniformly distributed load of w kg per unit length			
	$\theta_1 = \theta_2 = \frac{wl^3}{24EI}$	$y = \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \frac{5wl^4}{384EI}$
9. Beam Freely Supported at Ends — Couple M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = l/\sqrt{3}$ At center $\delta = \frac{Ml^2}{16EI}$
10. Beam Freely Supported at Ends — Uniformly varying load: max. intensity w			
	$\theta_1 = \frac{7wl^3}{360EI}$ $\theta_2 = \frac{wl^3}{45EI}$	$y = \frac{wx}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = .00652\frac{wl^4}{EI}$ at $x = 0.519l$ At center $\delta = .00651\frac{wl^4}{EI}$

DETERMINE THE SLOPE AT POINTS B AND C OF THE BEAM SHOWN BELOW. TAKE E = 200 GPa AND I = 360 x 10^6 mm^4



$$\theta_B = \theta_{B/A}; \quad \theta_C = \theta_{C/A}$$

- Applying Theorem 1, is equal to the area under the M/EI diagram between points A & B

$$\theta_B = \theta_{B/A} = -\left(\frac{50kNm}{EI}\right)(5m) - \frac{1}{2}\left(\frac{100kNm}{EI} - \frac{50kNm}{EI}\right)(5m)$$

$$= -\frac{375kNm^2}{EI}$$

- Substituting numerical data for E & I

$$-\frac{375kNm^2}{[200(10^6)kN/m^2][360(10^6)(10^{-12})m^4]} = -0.00521rad$$

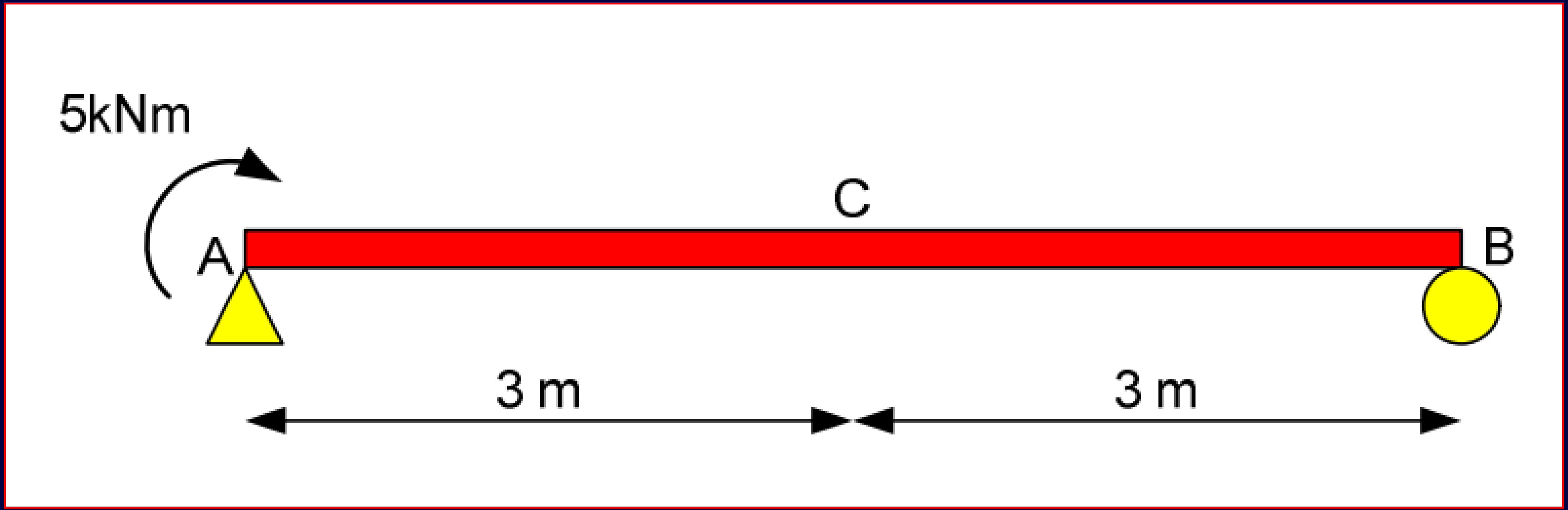
The –ve sign indicates that the angle is measured clockwise from A

$$\theta_C = \theta_{C/A} = \frac{1}{2}\left(-\frac{100\,kNm}{EI}\right)(10m) = -\frac{500\,kNm^2}{EI}$$

Substituti ng numerical values of EI, we have :

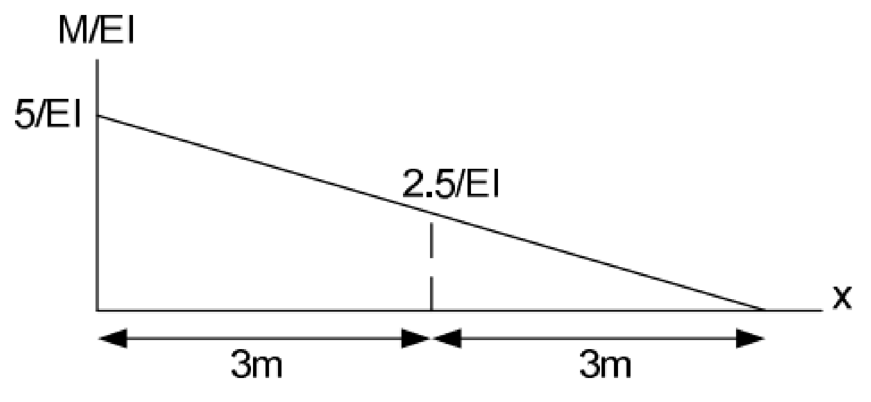
$$\frac{-500\,kNm^2}{[200(10^6)kN/m^2][360(10^6)(10^{-12})m^4]} = -0.00694\,rad$$

Determine the deflection at C of the beam shown as below. Take E = 200GPa and I=360E6 mm4.

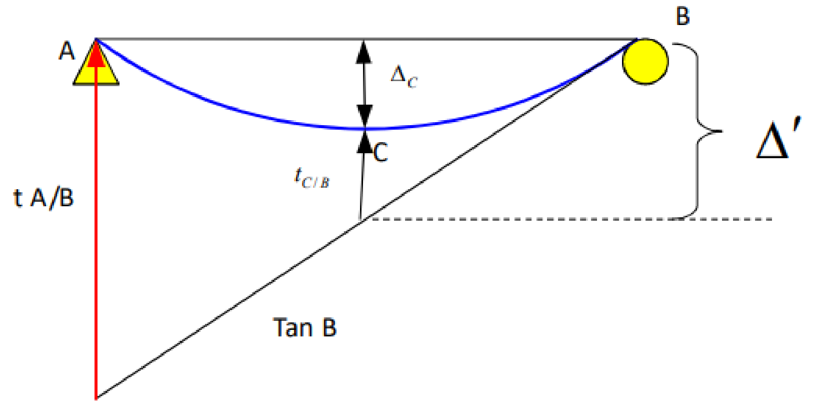


Solution

M/EI Diagram



ELASTIC CURVE



$$\frac{\Delta'}{3} = \frac{t_{A/B}}{6}$$

OR

$$\Delta' = \frac{t_{A/B}}{2}$$

$$\Delta_C = \frac{t_{A/B}}{2} - t_{C/B} \qquad (1)$$

$t_{A/B}$ **is the moment of the M/EI diagram between A and B about point A**

$$t_{A/B} = \left[\frac{1}{3}(6m)\right]\left[\frac{1}{2}(6m)\left(\frac{5kN.m}{EI}\right)\right]$$

$$t_{A/B} = \frac{30kN.m^3}{EI}$$

$t_{C/D}$ **is the moment of the M/EI diagram between C and B about the point C**

$$t_{C/B} = \left[\frac{1}{3}(3m)\right]\left[\frac{1}{2}(3m)\left(\frac{2.5kN.m}{EI}\right)\right]$$

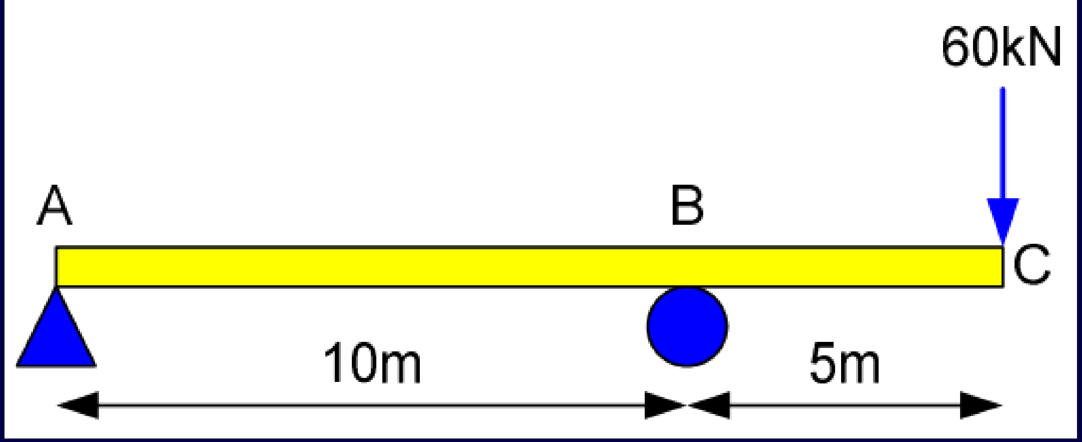
$$t_{C/B} = \frac{3.75kN.m^3}{EI}$$

$$\Delta_C = \frac{1}{2}\left(\frac{30kN.m^3}{EI}\right) - \frac{3.75kN.m^3}{EI}$$

$$\Delta_C = \frac{11.25kN.m^3}{EI}$$

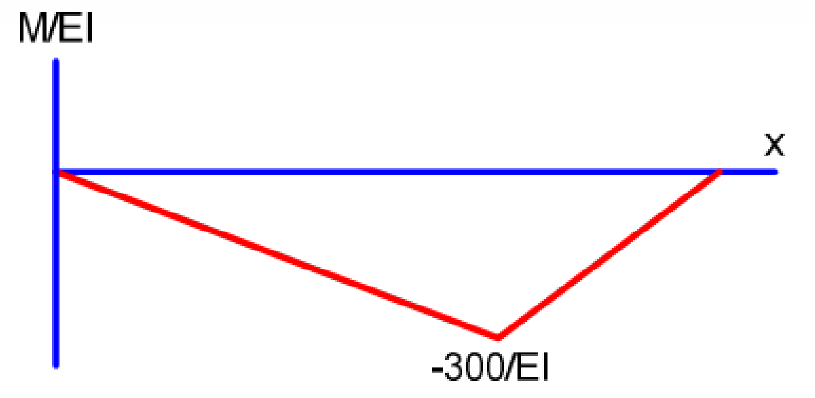
$$\Delta_C = 0.0141m = 14.1mm$$

USE THE MOMENT AREA THEOREM TO DETERMINE THE DEFLECTION AT C. EI IS CONSTANT.

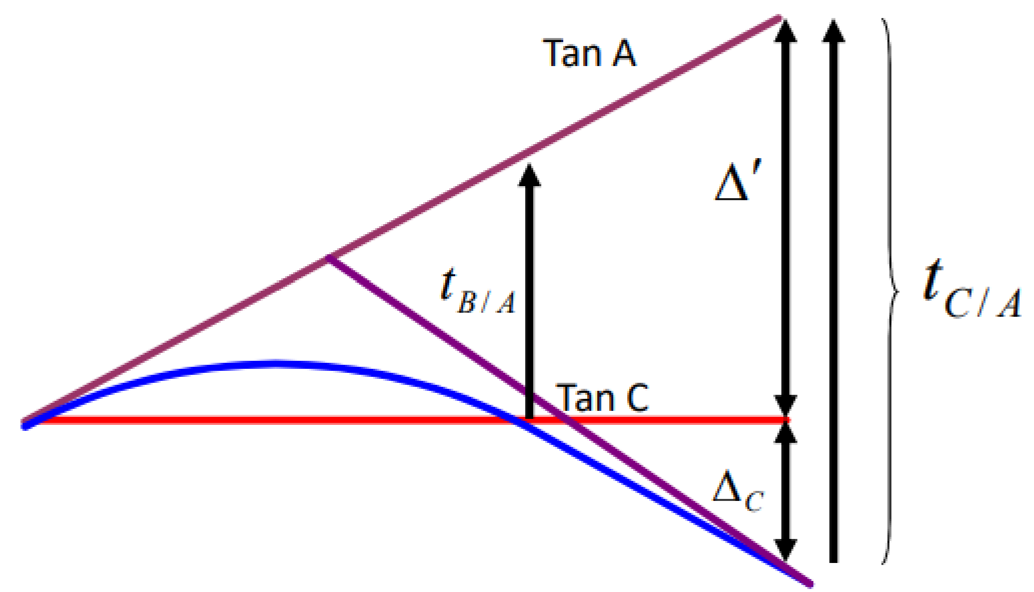


solution

M/EI Diagram



Elastic Curve



From elastic curve,

$$\frac{\Delta'}{15} = \frac{t_{B/A}}{10}$$

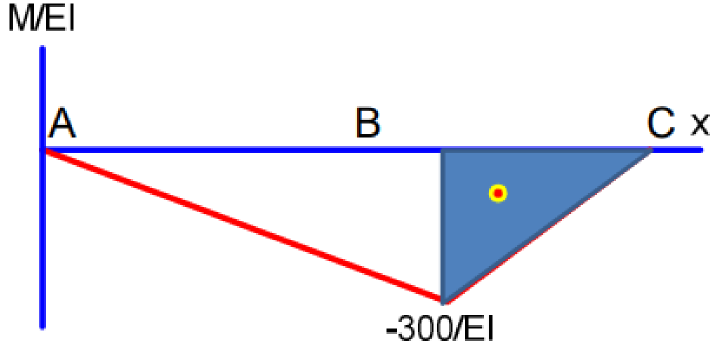
$$\Delta' = \frac{15}{10} t_{B/A}$$

$$\Delta_C = t_{C/A} - \Delta'$$

$$\Delta_C = t_{C/A} - \frac{15}{10} t_{B/A}$$

$$t_{C/A} = \left[\frac{1}{2}\left(\frac{-300}{EI}\right)(5)(3.33)\right] + \left[\frac{1}{2}\left(\frac{-300}{EI}\right)(10)(8.33)\right]$$

$$t_{C/A} = \frac{-15000kN.m^3}{EI}$$



$$t_{B/A} = \left[\frac{1}{2}\left(\frac{-300}{EI}\right)(10)(3.33)\right]$$

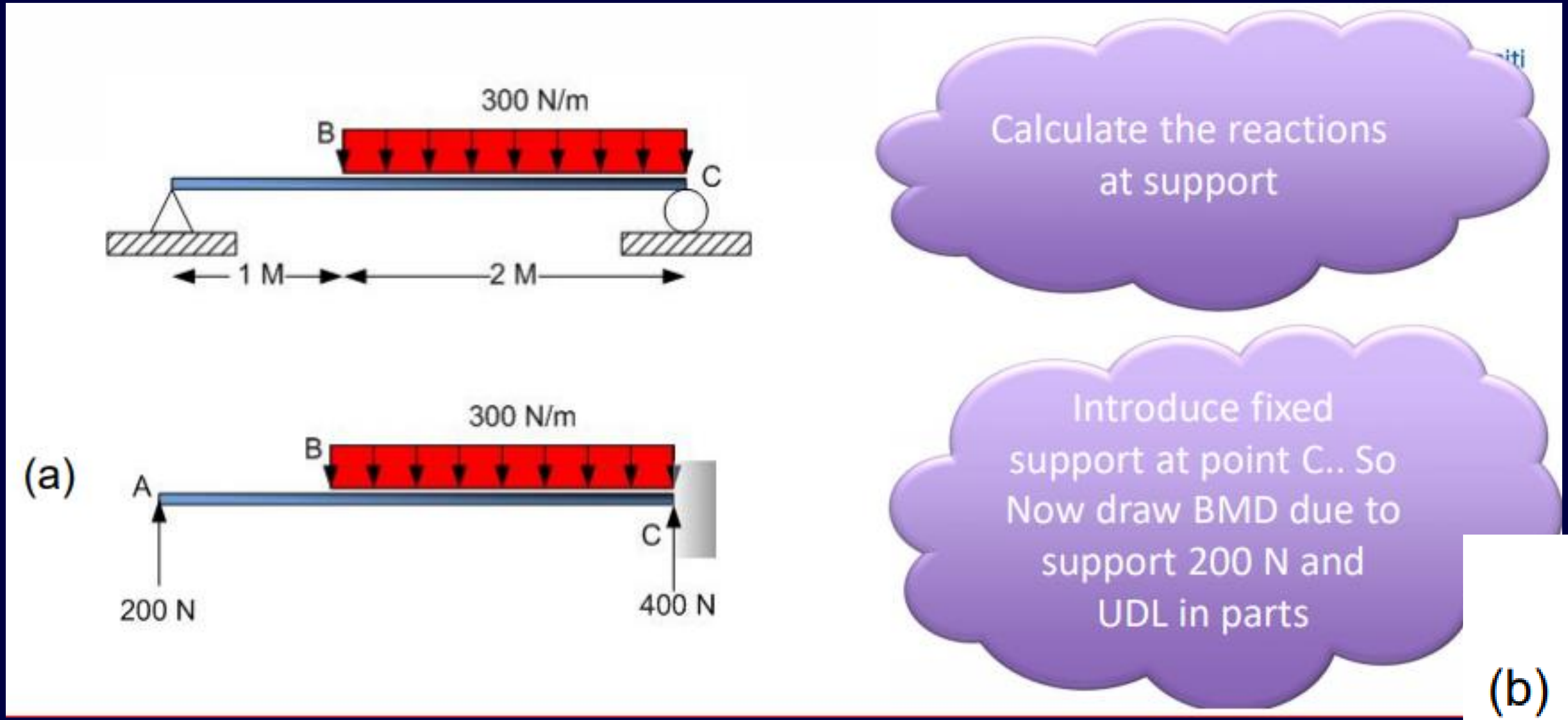
$$t_{C/A} = \frac{-5000kN.m^3}{EI}$$

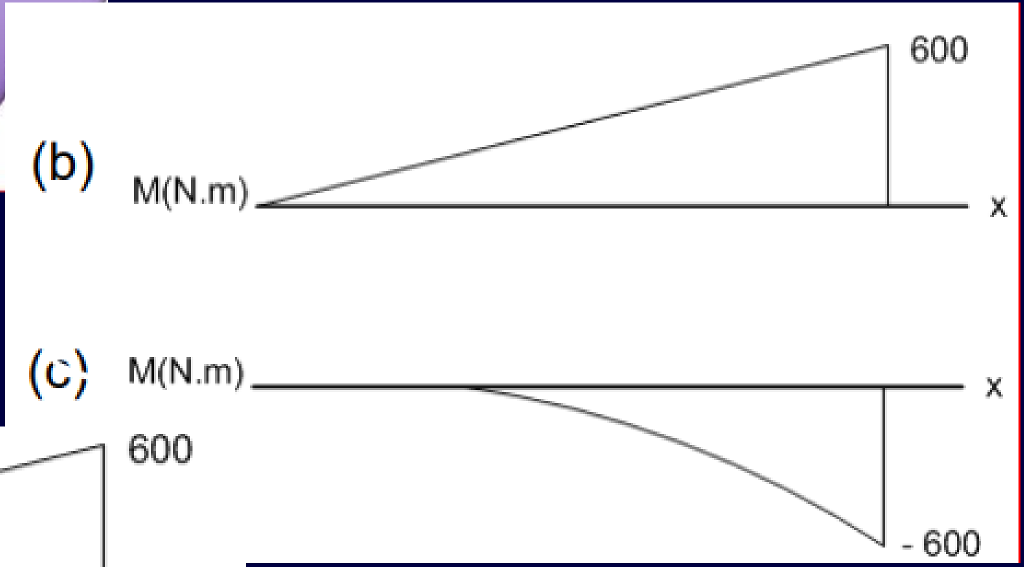
$$\Delta_C = \frac{-15000}{EI} - \frac{15}{10}\left(\frac{-5000}{EI}\right)$$

$$\Delta_C = \frac{7500kN.m^3}{EI}$$

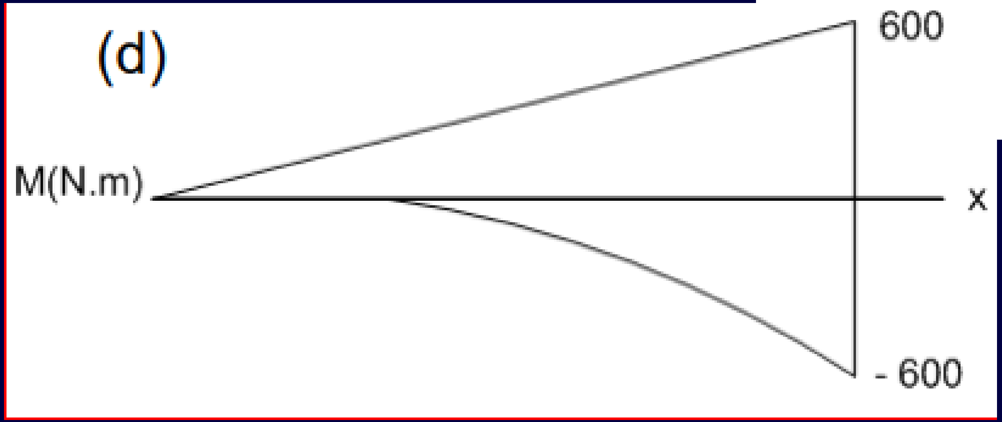
Bending Moment Diagrams by Parts

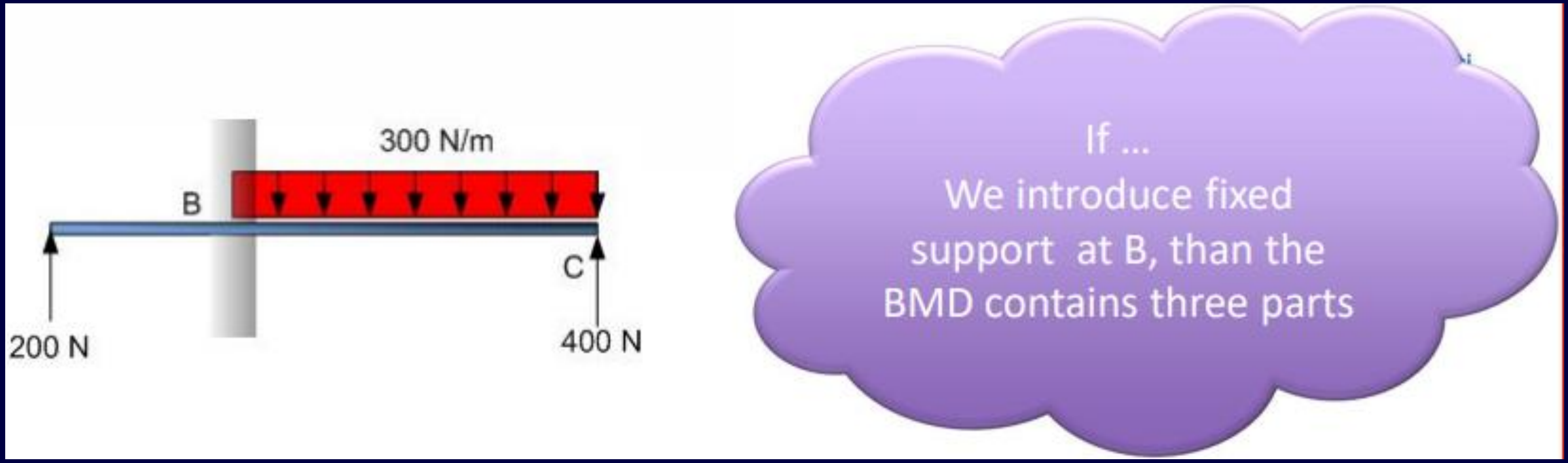
- Application of the moment-area theorems is practically only if the area under the bending moment diagrams and its first moment can be calculated without difficulty.
- The key to simplifying the computation is to divide the BMD into simple geometric shape (rectangles, triangles and parabolas) that have known areas and centroidal coordinates.
- Sometimes the conventional BMD lends itself to such division, but often it is preferable to draw the BMD by parts, with each part of the diagrams representing the effect of one load.
- Calculate the support reactions
- introduce a FIX SUPPORT as a convenient location. A simply support by the original beam is usually a good choice, but sometimes another point is more convenient. The beam is now cantilevered from this support.
- Draw a BMD for each loading (including the support reactions of the original beam. If all the diagrams can be fitted on a single plot, do so. Draw the positive moment above the x-axis and negative moment below the x-axis

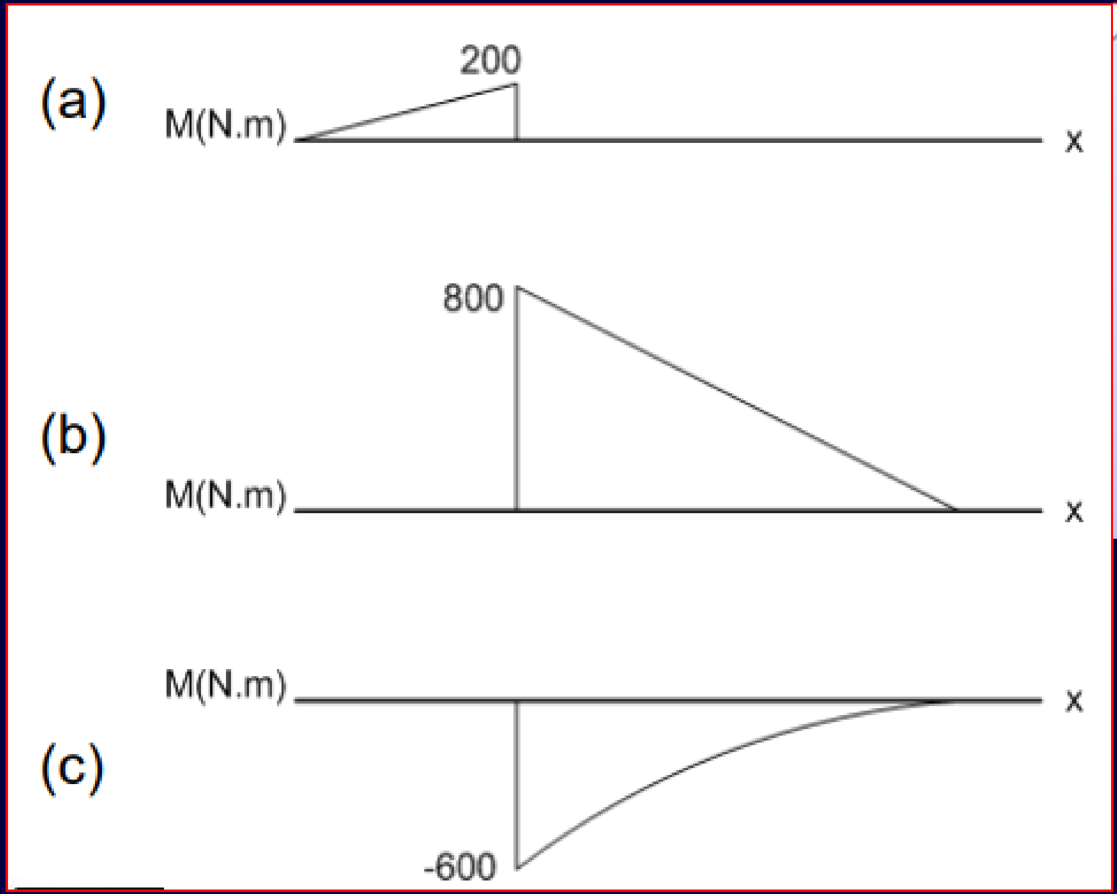


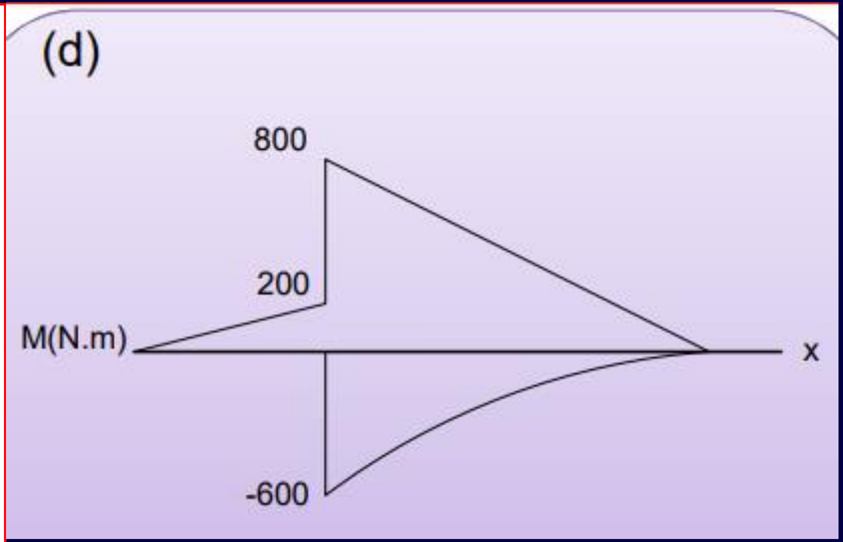


(a) Equivalent beam with fixed support at C,
(b) BMD due to support reaction at A,
(c) BMD due to UDL,
(d) combination BMD by parts



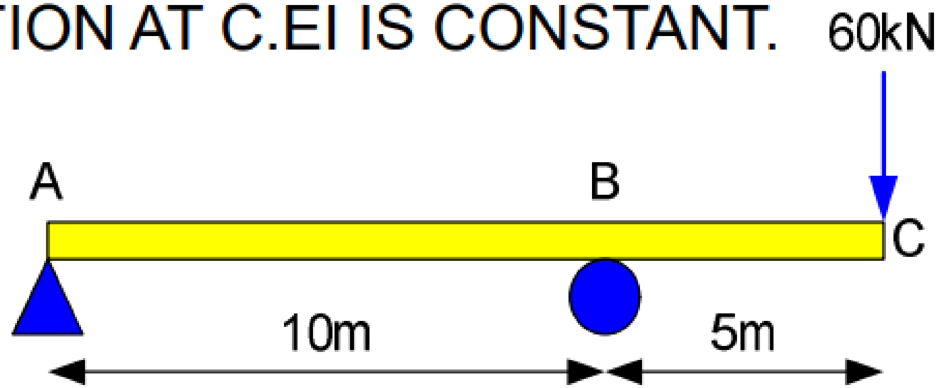


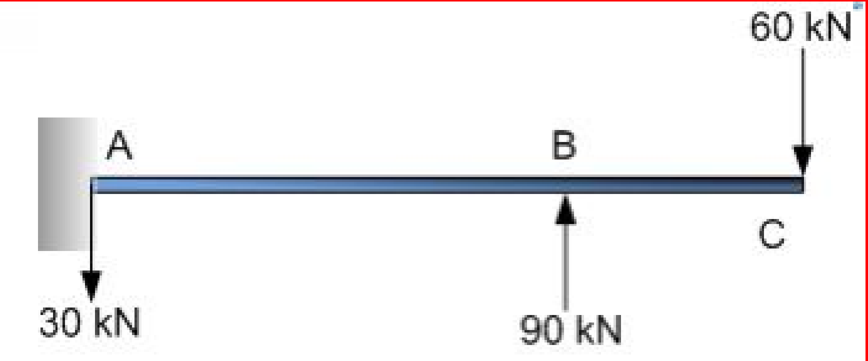


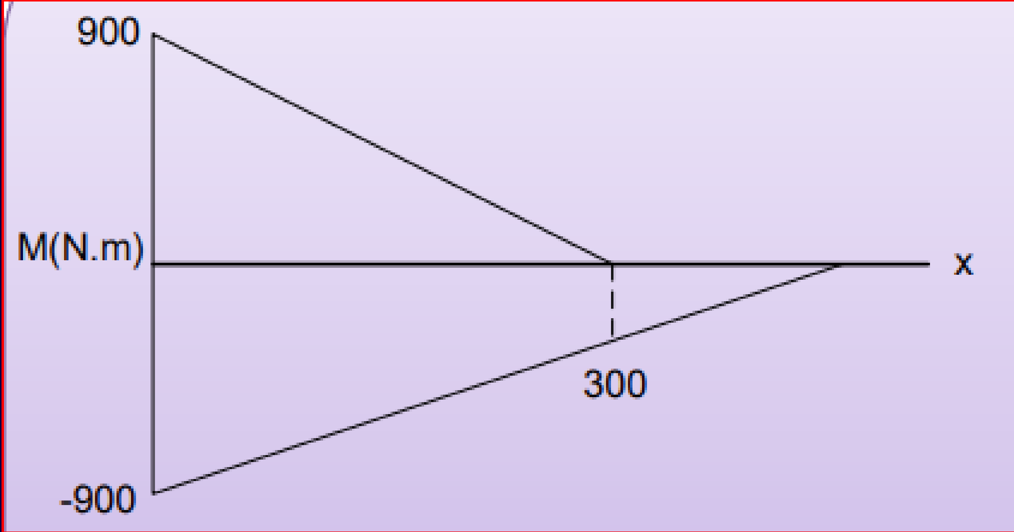


(a) BMD due to reaction at A,
(b) BMD due to support reaction at C,
(c) BMD due to UDL
(d) combination BMD by parts

USE THE MOMENT AREA THEOREM TO DETERMINE THE DEFLECTION AT C.EI IS CONSTANT.

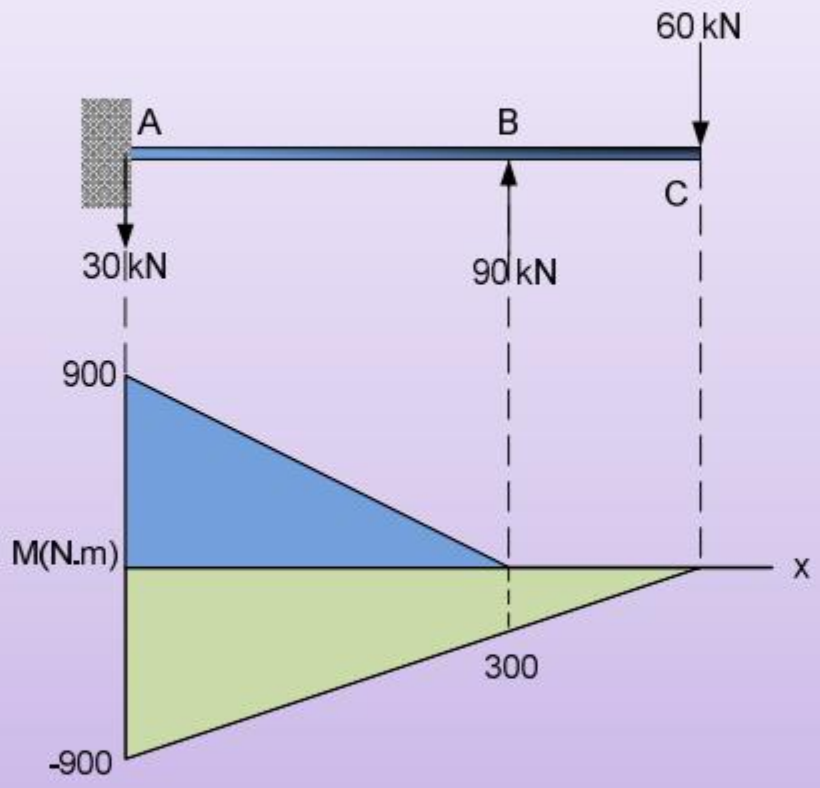






From the elastic curve… refer to the previous example

$$\Delta_C = t_{C/A} - \frac{15}{10} t_{B/A}$$

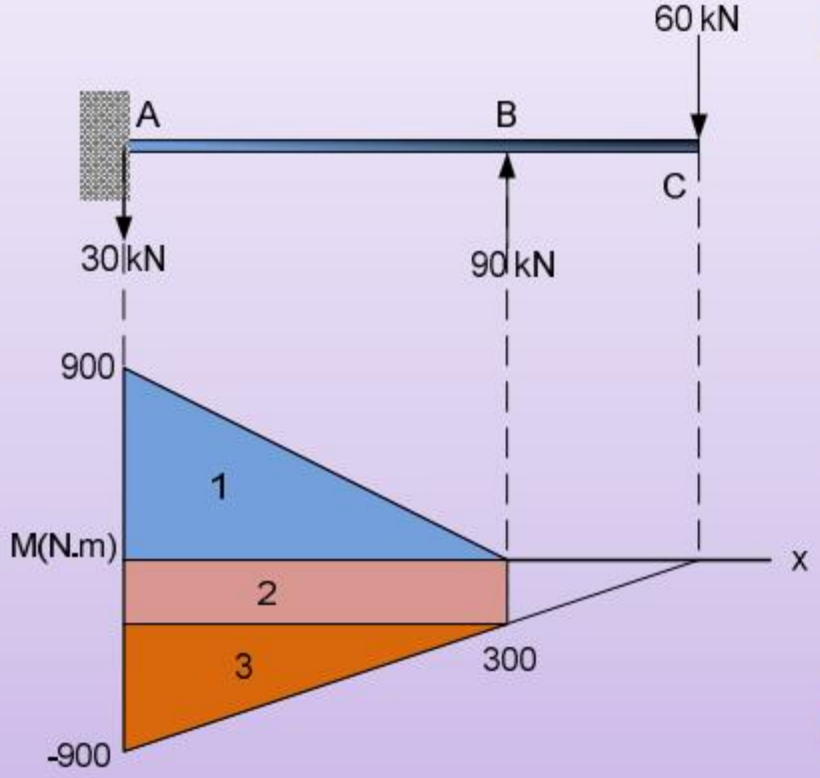


$$t_{C/A} = \frac{1}{2}(900)(10)(11.67)$$

$$+\frac{1}{2}(-900)(15)(\frac{2}{3})(15)$$

$$t_{C/A} = 52500 - 67500$$

$$= -15000$$



$$t_{B/A} = \frac{1}{2}(900)(10)(\frac{2}{3})(10)$$

$$+ (-300)(10)(\frac{1}{2})(10)$$

$$+ \frac{1}{2}(-600)(10)(\frac{2}{3})(10)$$

$$t_{B/A} = 30000 - 15000 - 20000$$

$$= -5000$$

$$\Delta_C = \frac{-15000}{EI} - \frac{15}{10}\left(\frac{-5000}{EI}\right)$$

$$\Delta_C = \frac{7500kN.m^3}{EI}$$