Kinematic Synthesis of Planar Mechanisms

(Mechanisms Synthesis)

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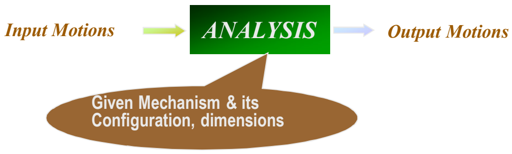
Analysis vs. Synthesis

Analysis



In Kinematic Analysis one is given a mechanism & the task is to determine the various relative motion that can take place in that mechanism.

Synthesis

- decision –making process
- Innovative or creative process
- process of creating new mechanism
- Selecting optimum/best configuration from no. of existing mechanism
- Determination of optimum dimensions of the elements of the mechanism on the basis of analysis

In Kinematic Synthesis one has to be come up with a design of mechanism to generate prescribed motion characteristic.

Kinematics Synthesis of Plane Mechanisms or Linkages

Aim:

Design or creation of a mechanism to obtain a desired set of motion characteristics.

Objective

- design of mechanisms to satisfy certain kinematic specification.

- In other words, motion characteristics are given & the mechanism is to be found

Kinematic Synthesis Problems

- Type Synthesis
- Number Synthesis
- Dimensional Synthesis

Kinematic Synthesis of Mechanisms

Synthesis Problems

Kinematic Synthesis

- **Type Synthesis** → Refers to the kind of mechanism selected
- **Number Synthesis** → Deals with the number of links & nos. of joints that are required to obtain a certain mobility
- **Dimensional Synthesis**
  - Motion generation /Body Guidance
  - Path generation
  - Function generation

By Dimensional Synthesis, we mean the determination of kinematic dimensions of the individual links of a mechanism to satisfy specified motion characteristics or specified tasks.

Clasiification of Dimensional Synthesis Problems

Depending on the required kinematic characteristics to be satisfied by the designed mechanism or linkage, dimensional synthesis problems can be broadly classified as given below:

Motion generation /Body Guidance

In this general class of synthesis problem, the linkage has to be so designed that a rigid body (i.e., one link of the mechanism, for example the coupler of a 4R linkage) can be guided in a prescribed manner.

The guidance may or may not be coordinated with the input motion

Path generation

If a point on the floating link (i.e. link not connected to the frame , like coupler) of a mechanism has to be guided along a prescribed path, then such a problem is classified as a path- generation problem .

This refers to a problem in which a coupler point is to generate a path having a prescribed shape

The generation of a prescribed path may or may not be coordinated with the input motion

Function generation

In this class of problem, the motion parameters (displacement, velocity, acceleration etc.) of the output & input links are to be coordinated so as to satisfy a prescribed functional relationship.

The output & input motion characteristics have to maintain a specified functional relationship

Steps in Kinematics Synthesis of Plane Mechanisms

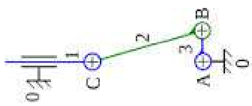
Type Synthesis

↓

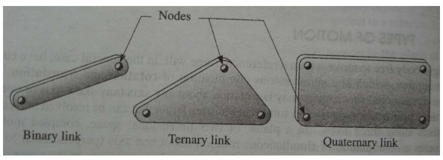
Number Synthesis

↓

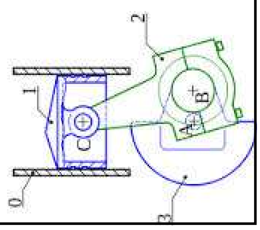
Dimensional Synthesis

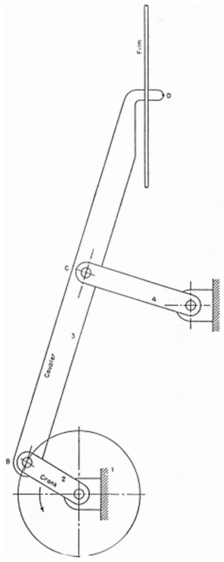


What is Kinematic dimensions?



Node-node distance or joint centre to centre distance etc.

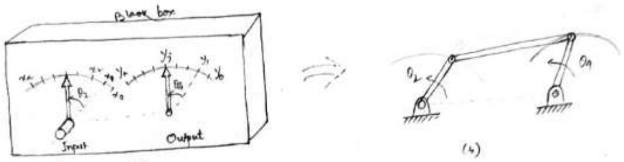




Path generation

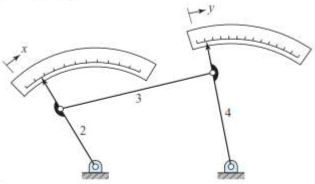
Function generation problem

In function generation, rotation (or translation) motion of input and output links must be correlated. The kinematic synthesis task may be to design a linkage to correlate input and output such that as the input moves by 'x', the output moves by $y = f(x)$ for the range $x_0 \leq x \leq x_{n+1}$



(a) (b)

Fig. 1 Function-generator mechanism (a) exterior view, (b) Schematic of the mechanism inside.
(i.e four-bar linkage function generator)



In the case of rotary input and output, the angles of rotation θ_2 & θ_4 are the linear analogs of x and y respectively.

When the input is rotated to a value of the independent parameter 'x', the mechanism in the 'black box' causes the output link to turn to the corresponding value of the dependent variable $y = f(x)$. This may be regarded as a simple case of a mechanical analog computer.

Function generation problem

Fig. 2 Shows a Six-link function generator mechanism in which two four-link mechanisms are joined in a series. The objective in this linkage is to provide a measure of flow discharge rate (i.e y) through the weir where the input is the vertical translation 'x' of the water level.

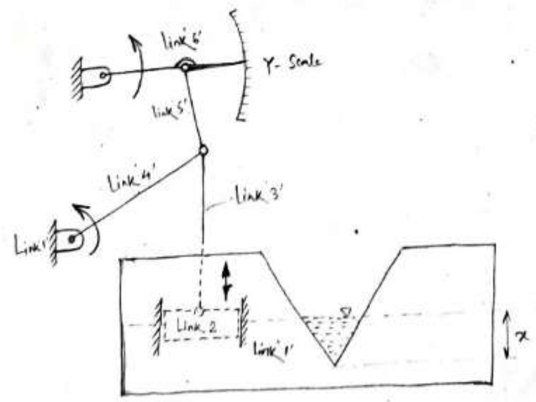


Fig. 2: Flow rate indicator mechanism, $y = f(x)$

Dimensional Synthesis Problems

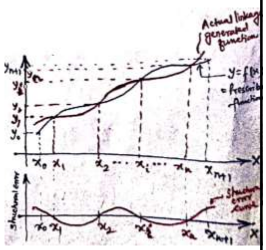
Function generation problem

Synthesis

- **Exact Synthesis** → By exact synthesis, we mean that the generated function by the physical mechanism fits the desired function at all points in the interval
- **Approximate Synthesis** → By approximate synthesis, we mean that the generated function by the physical mechanism fits the desired function at a finite number of points in the interval

Accuracy points / Precision points

The points at which the generated and desired functions agree



Structural Error

It is defined as the theoretical difference between the function generated by the synthesized linkage & the function originally prescribed

Structural error is inherent in approximate synthesis

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Chebyshev's Spacing of Accuracy Points

Let $y=f(x)$ be the function desired to be generated in an interval $x_0 \le x \le x_{n+1}$:

Let the mechanism generated function be $F(x, R_1, R_2, \ldots, R_k)$ where $R_1, R_2, \ldots, R_k$ are design parameters

Structural Error

$$E(x)=f(x)-F(x, R_1, R_2, \ldots, R_k)$$

The best choice for the spacing of accuracy points will be that which gives the min. value of E(x) between any two adjacent points:

However, Chebyshev's spacing of accuracy points can always be taken as a first approximation

A very good trial for the spacing of these precision positions is called Chebyshev Spacing

Chebyshev's Spacing of Accuracy Points

For 'n' precision positions in the range $x_0 \le x \le x_{n+1}$, the Chebyshev's spacing is

$$x_j = \left(\frac{x_{n+1}+x_0}{2}\right) - \frac{(x_{n+1}-x_0)}{2}\cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } j=1,2,\ldots n.$$

Determine the three accuracy points with Chebyshev's spacing for a 4 bar linkage to generate the function $y=x^{0.8}$ in the interval $1 \le x \le 3$,

Fig. 1 Function-generator mechanism (a) exterior view, (b) Schematic of the mechanism inside.
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Determine the three accuracy points with Chebyshev's spacing for a 4 bar linkage to generate the function $y=x^{0.8}$ in the interval $1 \le x \le 3$,

Here $n=3$; $x_0 = 1$; $x_{3+1} = x_4 = 3$

$$x_j = \left(\frac{x_{n+1}+x_0}{2}\right) - \left(\frac{x_{n+1}-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{2n}\right\} \quad \text{where } j=1,2,3$$

$$x_j = \left(\frac{x_4+x_0}{2}\right) - \left(\frac{x_4-x_0}{2}\right)\cos\left\{\frac{(2j-1)\pi}{6}\right\}$$

$$x_1 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(2-1)\pi}{2\times 3}\right\} = 2 - \cos \pi/6 = 1.134$$

$$x_2 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(4-1)\pi}{2\times 3}\right\} = 2 - \cos \pi/2 = 2$$

$$x_3 = \left(\frac{3+1}{2}\right) - \left(\frac{3-1}{2}\right)\cos\left\{\frac{(6-1)\pi}{6}\right\} = 2 - \cos 5\pi/6 = 2.866$$

Accuracy pts.

The corresponding values of 'y' to be

$y_1 = x^{0.8} = (1.134)^{0.8} = 1.106$

$y_2 = (2)^{0.8} = 1.741$

$y_3 = (2.866)^{0.8} = 2.322$.

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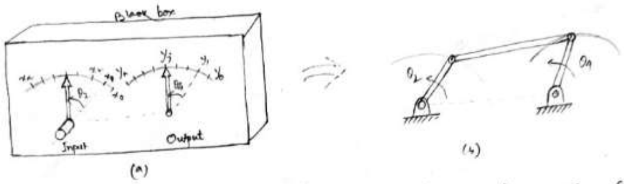

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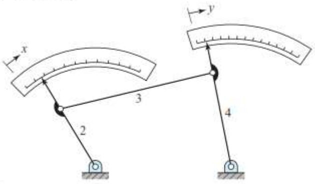

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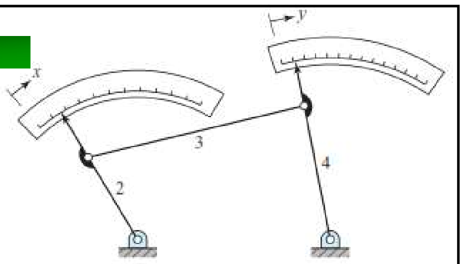
Scale Factor for Input & Output motion

Mechanized variables: θ_2 & θ_4

Functional variables: 'x' & 'y'



The orientation of the driver link (θ_2) represents the independent variable 'x'.

The orientation of the driven link (θ_4) represents the dependent variable 'y'.

The mechanized variables θ_2 & θ_4 are proportional to the functional variables 'x' & 'y'.

The relation betⁿ Δx and $\Delta\theta_2$ & that betⁿ Δy and $\Delta\theta_4$ is usually assumed to be linear.

With the mappings betⁿ function variable space (x, y) and mechanism joint space (θ_2, θ_4) known, we can map the three function precision points to corresponding precision joint angles.

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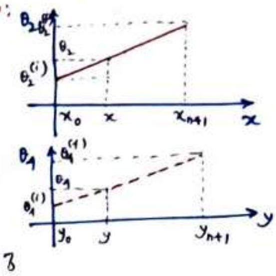
Let $\theta_2^{(i)}$ be the initial value of θ_2 representing x_0

$\theta_4^{(i)}$ be " " " of θ_4 " $y_0 = f(x_0)$

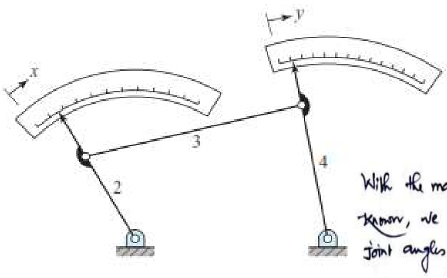
The input & output scale factors m_x & m_y resp. are defined as:

$$\therefore \text{Scale factor } m_x = \frac{\Delta\theta_2}{\Delta x} = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0} = \frac{\theta_2 - \theta_2^{(i)}}{x - x_0}$$

$$m_y = \frac{\Delta\theta_4}{\Delta y} = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0} = \frac{\theta_4 - \theta_4^{(i)}}{y - y_0}$$



The superscripts 'i' & 'f' denote the initial & final values of θ_2 & θ_4.



$$\theta_2 - \theta_2^{(i)} = m_x(x - x_0) \Rightarrow \theta_2 = \theta_2^{(i)} + m_x(x - x_0) \text{ where } m_x = \frac{\theta_2^{(f)} - \theta_2^{(i)}}{x_{n+1} - x_0}$$

$$\theta_4 - \theta_4^{(i)} = m_y(y - y_0) \Rightarrow \theta_4 = \theta_4^{(i)} + m_y(y - y_0) \text{ where } m_y = \frac{\theta_4^{(f)} - \theta_4^{(i)}}{y_{n+1} - y_0}$$

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