CHAPTER 12

Q. 12.3

Fundamentals of Heat and Mass Transfer [EXP-7090]
(https://holooly.com/sources/fundamentals-of-heat-and-mass-transfer-exp-7090/)

Determine an expression for the net radiative heat flux at the surface of the small solid object of Figure 12.1 in terms of the surface and surroundings temperatures and the Stefan Boltzmann constant. The small object is a blackbody.

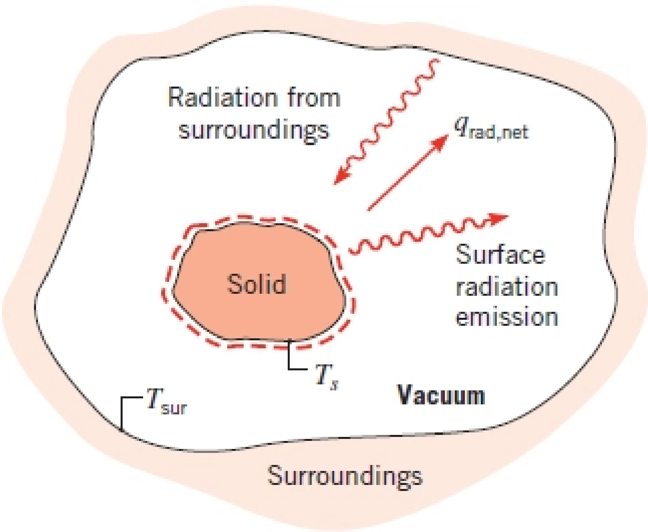

FIGURE 12.1 Radiation cooling of a hot solid.

Step-by-Step

Verified Answer

Known: Surface temperature of a small blackbody, T_s and the surroundings temperature, T_{sur} .

Find: Expression for the net radiative flux at the surface of the small object, q''_{rad}

Assumptions: Small object experiences blackbody irradiation.

Analysis: Since none of the irradiation is reflected from the small object, Equation 12.28 may be written as

$$q''_{rad} = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi) \cos\theta \sin\theta d\theta d\phi d\lambda \quad (12.28)$$

$$- \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) \cos\theta \sin\theta d\theta d\phi d\lambda \quad (1)$$

The intensity emitted by the small object corresponds to that of a blackbody. Hence

$$I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,b}(\lambda,T_s) \quad (2)$$

The intensity corresponding to the irradiation is also black. Therefore

$$I_{\lambda,i}(\lambda,\theta,\phi) = I_{\lambda,b}(\lambda,T_{\text{sur}}) \quad (3)$$

Since the blackbody intensity is diffuse, it is independent of angles θ and φ. Therefore, substituting Equations 2 and 3 into Equation 1 yields

$$q''_{rad} = \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi \times \int_0^{\infty} I_{\lambda,b}(\lambda,T_{sur}) d\lambda$$

$$- \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi \times \int_0^{\infty} I_{\lambda,b}(\lambda,T_s) d\lambda$$

$$= \pi \left[\int_0^{\infty} I_{\lambda,b}(\lambda,T_{\text{sur}}) d\lambda - \int_0^{\infty} I_{\lambda,b}(\lambda,T_s) d\lambda\right]$$

Substituting from Equations 12.32 and 12.33 yields

$E_b = \sigma T^4$ (12.32)

$I_b = \frac{E_b}{\pi}$ (12.33)

$q''_{rad} = \sigma \left(T_s^4 - T_{sur}^4\right)$

which is identical to Equation 1.7 with $\varepsilon = 1$.

$q''_{rad} = \frac{q}{A} = \varepsilon E_b \left(T_s\right) - \alpha G = \varepsilon \sigma \left(T_s^4 - T_{sur}^4\right)$ (1.7)