Simple Harmonic motion

- $F = -kx$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \ldots (2) \quad \text{(equation of motion in SHM)}$$

(1) Trigonometric Solution

$$x_1 = a\sin\omega t \quad \ldots (3)$$

$$x_2 = b\cos\omega t \quad \ldots (4)$$

$$x = a\sin\omega t + b\cos\omega t \quad \ldots (5)$$

(2) Exponential

$$x_1(t) = c_1 e^{i\omega t} \quad \ldots (6)$$

$$x_2(t) = c_2 e^{-i\omega t} \quad \ldots (7)$$

$$x(t) = x_1 + x_2 = c_1 e^{i\omega t} + c_2 e^{i\omega t} \quad \ldots (8)$$

Linear Superposition ~~Position~~ Principle

The resultant of simultaneous is simply the aljebric sum of individual displacement.

x_1 $\quad$ x_2

$$x = x_1 + x_2$$

- way of finding SHM

→ Analitical

→ Graphical

→ Method using complex quantities

$$\frac{d^2x}{dt^2} = -\omega^2 x + \alpha x^2 + \beta x^2 \quad \ldots (1)$$

non linear diff equation

x_1, x_2

$A = B$

$-A = -B$

$$\frac{d^2x_1}{dt^2} = -\omega^2 x_1 + \alpha x_1^2 + \beta x_1^3 \quad \ldots \text{(2)}$$

$$\frac{d^2x_2}{dt^2} = -\omega^2 x_2 + \alpha x_2^2 + \beta x_2^3 \quad \ldots \text{(3)}$$

$$x = x_1 + x_2$$

$$\frac{d^2(x_1+x_2)}{dt^2} = -\omega^2(x_1+x_2) + \alpha(x_1+x_2)^2 + \beta(x_1+x_2)^3 + \ldots \quad \text{(4)}$$

$$\text{(2)} + \text{(3)} \Rightarrow \frac{d^2x}{dt^2} + \frac{d^2x_2}{dt^2} = -\omega^2 x_1 - \omega^2 x_2 + \alpha x_1^2 + \alpha x_2^2 + \beta x_1^3 + \beta x_2^3 + \ldots$$

$$= -\omega^2 x_1 - \omega^2 x_2 + \alpha(x_1^2 + x_2^2) + \beta(x_1^3 + x_2^3) + \ldots \quad \text{(5)}$$

$$4 = 5 \quad \checkmark \begin{cases} \dfrac{d^2(x_1+x_2)}{dt^2} = \dfrac{d^2x_1}{dt^2} + \dfrac{d^2x_2}{dt^2} \quad \ldots \text{(6)} \\ -\omega^2(x_1+x_2) = -\omega^2 x_1 - \omega^2 x_2 \quad \ldots \text{(7)} \end{cases}$$

True always

$$\times \begin{cases} \alpha(x_1+x_2)^2 = \alpha(x_1^2+x_2^2) \quad \ldots \text{(8)} \\ \beta(x_1+x_2)^2 = \beta(x_1^2+x_2^2) \quad \ldots \text{(9)} \end{cases}$$

not true always,

only when

$\alpha = \beta = 0$