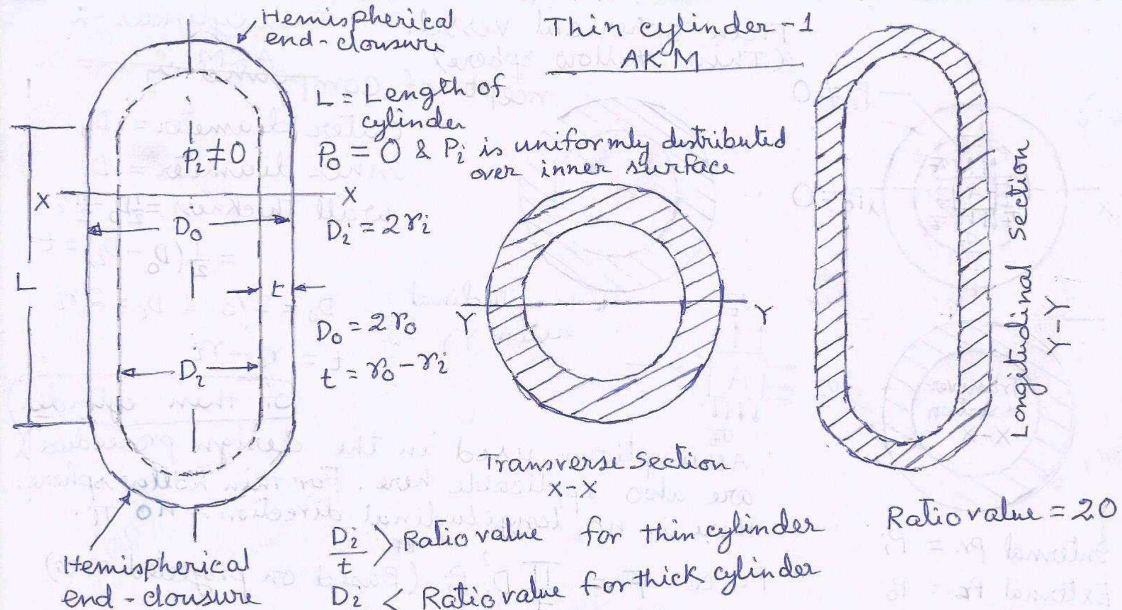
Pressure Vessels – Thin cylinder and Thick Cylinder



Ratio value = 20

$\frac{D_i}{t}$ > Ratio value for thin cylinder

$\frac{D_i}{t} \leq$ Ratio value for thick cylinder

For the design of thin cylinder ~~pressure~~ subjected to internal pressure only, the design procedure has been simplified through the use of following two assumptions.

i) Effect radial stress in thin cylinder is negligible.

ii) Tangential stress σ_t and longitudinal stress σ_l are uniformly distributed over wall thickness of cylinder.

Analysis of longitudinal stress: σ_l

Based on Transverse section and forces in longitudinal direction. stresses are assumed to ~~unifo~~ be uniformly distributed.

Force $F = \frac{\pi}{4} D_i^2 P_i$ (Based on projected area)

crossectional Area $A = \frac{\pi}{4} D_o^2 - \frac{\pi}{4} D_i^2 \approx \pi D_i t$

Longitudinal stress $= \frac{F}{A} = \sigma_l = \frac{P_i D_i}{4t}$

Analysis of tangential stress: σ_t

Based on longitudinal section passing through axis of symmetry.

Force $F = D_i P_i$ and Area $= 2t$ (For unit length)

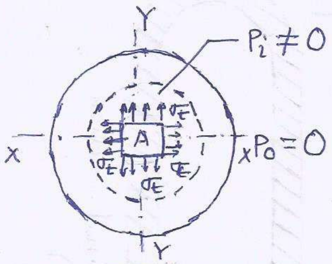
Trangential stress $\sigma_t = \frac{\text{Force}}{\text{Area}} = \frac{D_i P_i}{2t}$

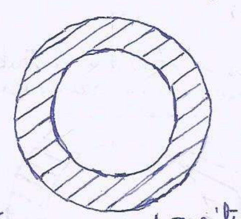
Design will be based on trangential stress σ_t, t can be calculated using allowable tensile stress $[\sigma_t]$. $[\sigma_t] \geq \sigma_t$; $\therefore t \geq \frac{P_i D_i}{2[\sigma_t]}$

Brittle: $\sigma_{ut}/F.S. = [\sigma_t]$; Ductile: $[\sigma_t] = \sigma_{yield}/F.S.$

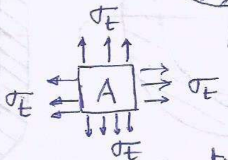
Wall thickness of cylinder (t): calculated value of t + corrosion allowance (Rounded)

Thin spherical vessel.
(Thin Hollow sphere)





Longitudinal section YY



Outer diameter $= D_o$
Inner diameter $= D_i$
wall Thickness $= \frac{1}{2}D_o - \frac{1}{2}D_i$
$= \frac{1}{2}(D_o - D_i) = t$

$D_o = 2r_o$ & $D_i = 2r_i$
$t = r_o - r_i$

Internal Pr. $= P_i$
External Pr. $= P_o$
Two Principal stress : σ_r and σ_t
σ_r is not considered.

Assumptions used in the design procedure (of thin cylinder) are also applicable here. For thin hollow sphere, there is no longitudinal direction - no σ_l.

Force $F = \frac{\pi}{4} D_i^2 P_i$ (Based on projected area)

Cross sectional Area $A = \frac{\pi}{4}D_o^2 - \frac{\pi}{4}D_i^2 \cong \pi D_i t$

Tangential stress $\sigma_t = \frac{F}{A} = \frac{P_i D_i}{4t}$

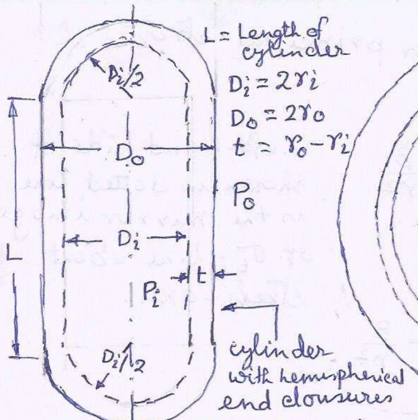
This concept is applicable for design of hemispherical end closures. Students are advised to study the concepts of semi-ellipsoidal enclosure and Torispherical end closure from the chapter "END CLOSURES" of prescribed text book. Students are also advised to study "GASKETS" and "GASKETED JOINT" from ~~to~~ prescribed text book.

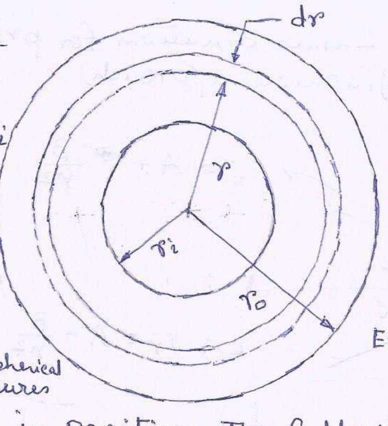
Discussion on thick-walled hollow sphere is not included in syllabus.

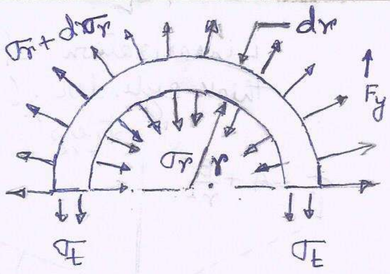
Practise :-
1) worked-out examples
2) Numerical exercise Problems.
3) Theoretical Problems.

Corrosion Allowance :- Every pressure vessel has a life (in years) specified by its designer. It is expected that the Pressure vessel will function throughout the span of life without onset of any failure. It is observed that the inner surface and outer surface of pressure vessel are corroded nonuniformly by the corrosive action of the fluid stored inside the vessel and by the corrosive action of the atmospheric fluid ~~surround~~ surrounding the vessel respectively. The corrosion experts ⊛

⊛ can estimate the probable maximum reduction of wall ~~thickness~~ thickness of the vessel. To avoid any failure, extra thickness is added to the calculated value of wall thickness.







Enlarged transverse section

The tensile stress is positive. The following derivation is based on positive tensile stress concept.

$$F_y = -2\sigma_t dr - 2r\sigma_r + 2(r+dr)(\sigma_r + d\sigma_r) = 0$$

$$\Rightarrow -\sigma_t dr - r\sigma_r + (r+dr)(\sigma_r + d\sigma_r) = 0$$

$$\Rightarrow -\sigma_t dr - r\sigma_r + r\sigma_r + r d\sigma_r + \sigma_r dr + dr d\sigma_r = 0$$

$$\Rightarrow -\sigma_t dr + \sigma_r dr + r d\sigma_r = 0 \quad [\text{Putting } dr d\sigma_r \cong 0]$$

$$\Rightarrow \sigma_r - \sigma_t + r\frac{d\sigma_r}{dr} = 0 \quad \ldots\ldots (a)$$

Applying generalized Hooks in ~~log~~ longitudinal direction,

$$\varepsilon_l = \frac{\sigma_l}{E} - \frac{\mu}{E}(\sigma_t + \sigma_r); \Rightarrow \sigma_r + \sigma_t = \frac{E}{\mu}\left(\frac{\sigma_l}{E} - \varepsilon_l\right) = \cancel{2A} \quad (b)$$

$\sigma_r + \sigma_t = 2A$ .... (c) for in text book, $A = C_1$.

$$\sigma_r - \sigma_t + r\frac{d\sigma_r}{dr} = 0$$

$$\sigma_r + \sigma_t = 2A$$

$$2\sigma_r + r\frac{d\sigma_r}{dr} = 2A$$

$$\sigma_r + \sigma_t = 2A; \Rightarrow \sigma_t = 2A - \sigma_r$$

$$\sigma_t = 2A - A + \frac{B}{r^2} = A + \frac{B}{r^2}$$

$$2\sigma_r + r\frac{d\sigma_r}{dr} = 2A$$

$$2r\sigma_r + r^2\frac{d\sigma_r}{dr} = 2Ar$$

$$\frac{d}{dr}(r^2\sigma_r) = 2Ar$$

Integrating w.r.to r,

$$r^2\sigma_r = \frac{2Ar^2}{2} - B \quad \left(\text{In text book } B = C_2\right)$$

$$\sigma_r = A - \frac{B}{r^2}.$$

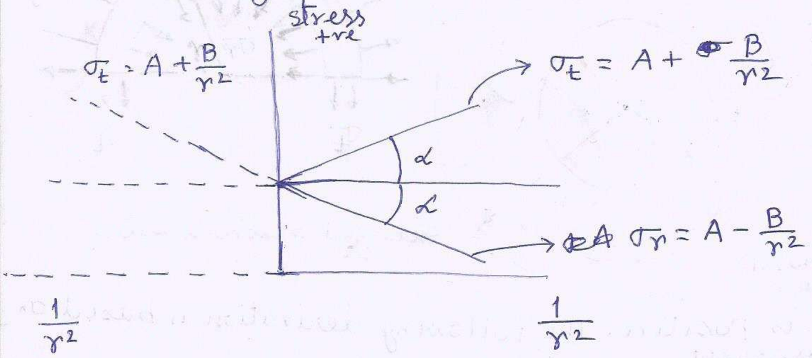
$$\left.\begin{aligned}\sigma_r &= A - \frac{B}{r^2}\\ \sigma_t &= A + \frac{B}{r^2}\end{aligned}\right\}$$ Lame's equations of ~~for~~ principal stresses ~~of~~ for thick cylinder

These two equations can be linearized in following form.

$\sigma_r = A - BX$ & $\sigma_t = A + BX$

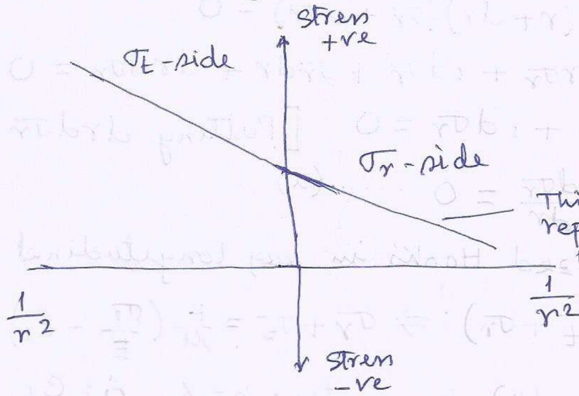
By Putting $\frac{1}{r^2} = X$

Linearization of Lame's equations for principal stresses of thick cylinder. (Graphical approach)



Left-hand side of shown in dotted line is the mirror image of σ_t - line about stress - axis.

Taking mirror image line for σ_t and neglecting orginal σ_t line



This is a single-line representation for Lames equations of Thick cylinder. This line is called Lame's Line.

Thick cylinder - Page 2
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Mathematical approach:

At $r = r_i$, $\sigma_r = -P_i$ and $A - \frac{B}{r_i^2}$

$$A - \frac{B}{r_i^2} = -P_i \quad \cdots (i)$$

At $r = r_o$, $\sigma_r = -P_o$

$$A - \frac{B}{r_o^2} = -P_o \quad \cdots (ii)$$

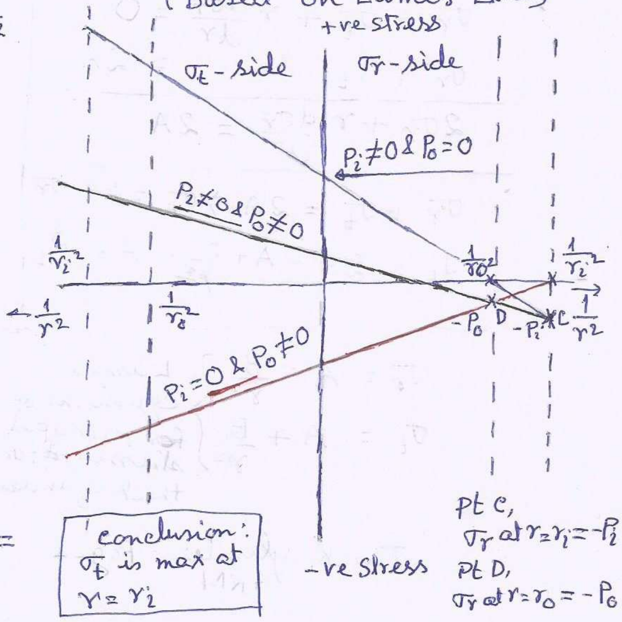
Solving eqn (i) and eqn (ii)

$$A = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2}$$

$$B = \frac{(P_i - P_o) r_o^2 r_i^2}{r_o^2 - r_i^2}$$

$\sigma_t = A + \frac{B}{r^2}$; $\sigma_r = A - \frac{B}{r^2}$; $\sigma_l = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}$

Graphical Approach (Based on Lame's Line)



$\boxed{\text{conclusion: } \sigma_t \text{ is max at } r = r_i}$

Pt C, σ_r at $r = r_i = -P_i$
Pt D, σ_r at $r = r_o = -P_o$

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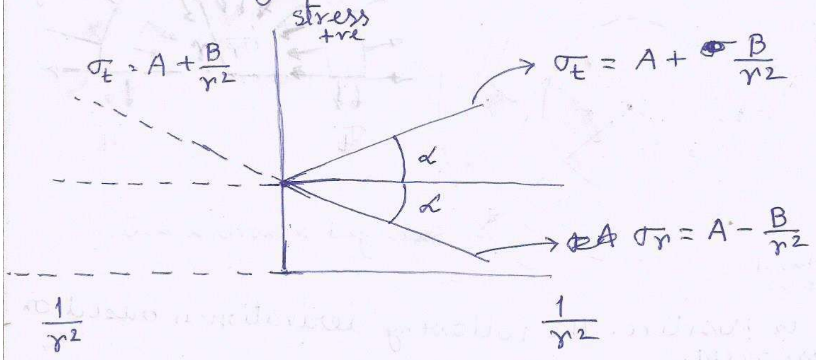
Linearization of Lame's equations for principal stresses of thick cylinder. (Graphical approach)



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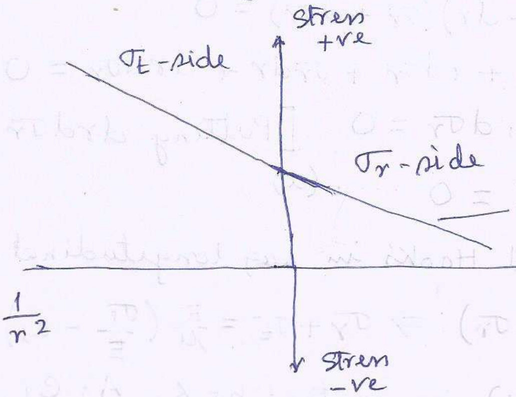

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$$A - \frac{B}{r_i^2} = -P_i \quad \cdots (i)$$

At $r = r_o$, $\sigma_r = -P_o$

$$A - \frac{B}{r_o^2} = -P_o \quad \cdots (ii)$$

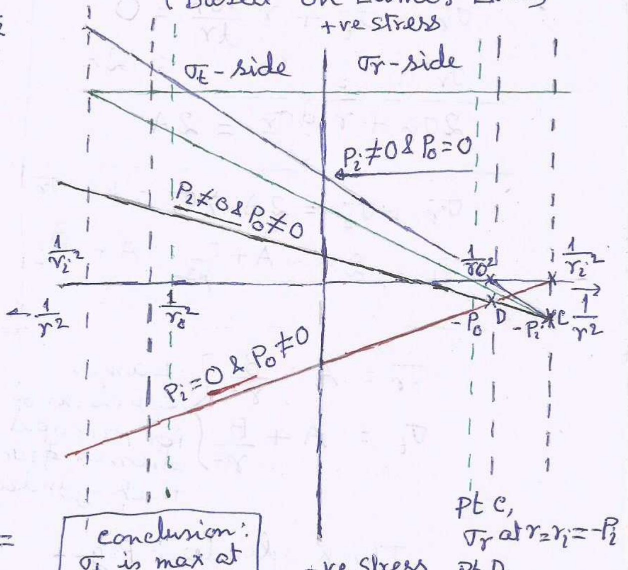
Solving equ (i) and eqn (ii)

$$A = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2}$$

$$B = \frac{(P_i - P_o) r_o^2 r_i^2}{r_o^2 - r_i^2}$$

$\sigma_t = A + \frac{B}{r^2}$; $\sigma_r = A - \frac{B}{r^2}$; $\sigma_l = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}$

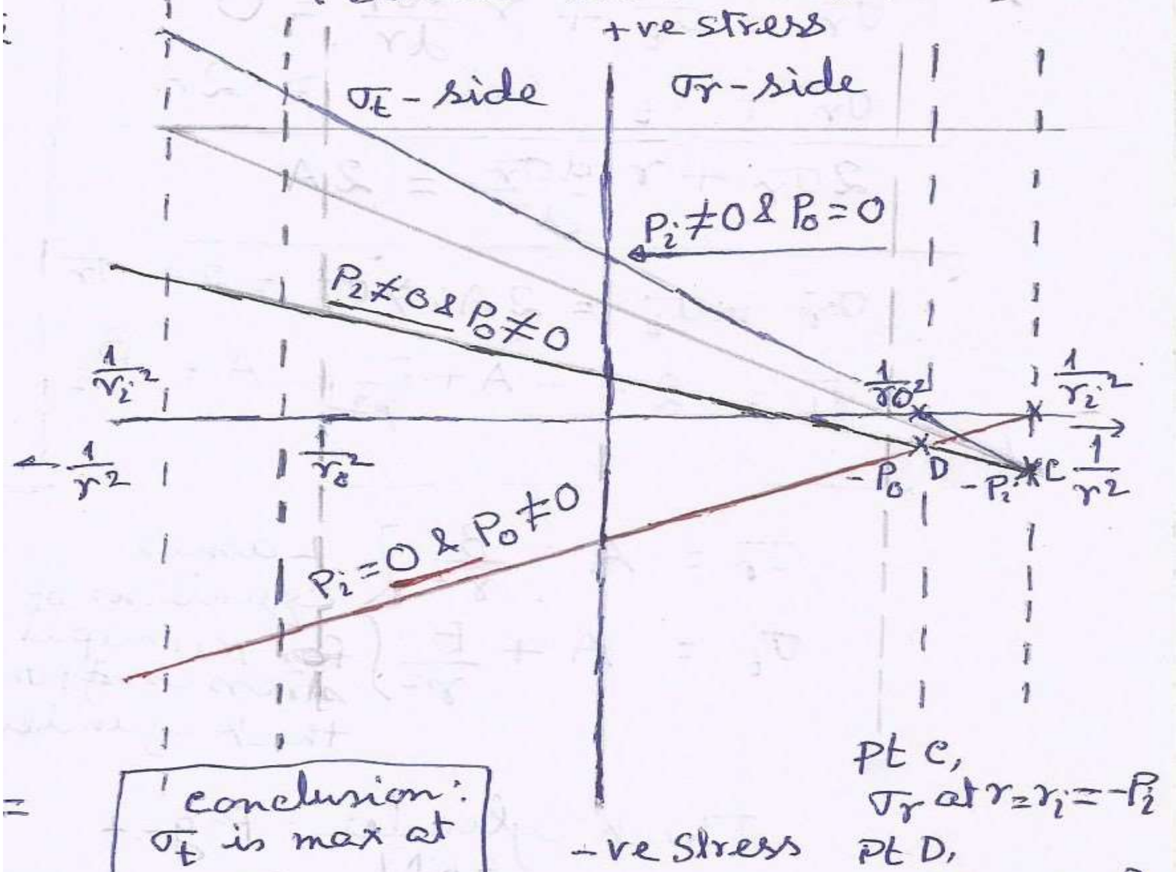
Graphical Approach
(Based on Lame's Line)



$\boxed{\text{Conclusion: } \sigma_t \text{ is max at } r = r_i}$

Pt C,
σ_r at $r = r_i = -P_i$
Pt D,
σ_r at $r = r_o = -P_o$

Graphical Approach
(Based on Lamé's Line)



conclusion:
σ_t is max at $r = r_i$

Pt C,
σ_r at $r = r_i = -P_i$

Pt D,
σ_r at $r = r_o = -P_o$

For straight cylinder with hemispherical end-closures at both ends,

$$\sigma_r = A - \frac{B}{r^2} \qquad A = \frac{D_i^2 P_i - D_o^2 P_o}{D_o^2 - D_i^2} = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2}$$

$$\sigma_t = A + \frac{B}{r^2} \qquad B = \frac{(P_i - P_o) D_o^2 D_i^2}{4(D_o^2 - D_i^2)} = \frac{(P_i - P_o) r_o^2 r_i^2}{r_o^2 - r_i^2}$$

$$\sigma_l = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2} = \frac{D_i^2 P_i - D_o^2 P_o}{D_o^2 - D_i^2}$$

Most critically stressed location is inner surface for which $r = r_i$. This is shown through graphical approach based on Lame's line.

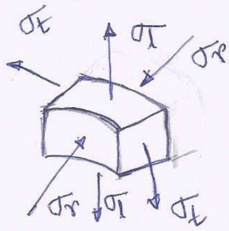
If $P_i \neq 0$ and $P_o = 0$, i.e. outside atmospheric pressure is negligible, stresses at $r = r_i$.

$$\sigma_r = -P_i$$

$$\sigma_t = \frac{P_i (r_o^2 + r_i^2)}{r_o^2 - r_i^2} = \frac{P_i (D_o^2 + D_i^2)}{D_o^2 - D_i^2}$$

$$\sigma_l = \frac{P_i r_i^2}{r_o^2 - r_i^2} = \frac{P_i D_i^2}{D_o^2 - D_i^2}.$$

at $r = r_i$



Infinestesimal free body. curved surface will be Plane.

$$\sigma_t > \sigma_l > \sigma_r \quad \text{and} \quad |\sigma_t| > |\sigma_l| > |\sigma_r|$$

σ_t, σ_l and σ_r are principal stresses.

Base on the above stresses, Lame's equation for the wall thick ~~ns~~ of thick cylinder can be derived for brittle materials.

$$t \geq \frac{D_i}{2}\left[\sqrt{\frac{[\sigma_t] + P_i}{[\sigma_t] - P_i}} - 1\right] \text{ or } r_i\left[\sqrt{\frac{[\sigma_t] + P_i}{[\sigma_t] - P_i}} - 1\right]$$

where $[\sigma_t] = \frac{\sigma_{ut}}{F.S.}$

Maximum Principal stress theory of failure.

Based on above stresses, clavarino's equation and Birnie's equation for the wall thickness of thick cylinder can be derived for ductile material.

$$t \geq \frac{D_i}{2}\left[\sqrt{\frac{[\sigma_t] + (1-2\mu)P_i}{[\sigma_t] - (1+\mu)P_i}} - 1\right] \text{ or}$$

$$t \geq \frac{D_i}{2}\left[\sqrt{\frac{[\sigma_t] + (1-\mu)P_i}{[\sigma_t] - (1+\mu)P_i}} - 1\right]$$

Maximum strain theory of failure. $[\sigma_t] = \frac{\sigma_{yield}}{F.S.}$; $[\tau] = \frac{\tau_{yield}}{F.S.}$

Application of Max. shear stress theory

$$\tau_{max} = \frac{1}{2}\left|\frac{P_i (r_o^2 + r_i^2)}{r_o^2 - r_i^2} + P_i\right| = \frac{r_o^2 P_i}{r_o^2 - r_i^2}$$

$$[\tau] \geq \tau_{max} \; ; \Rightarrow [\tau] \geq \frac{r_o^2 P_i}{r_o^2 - r_i^2}$$

$$\frac{r_o^2}{r_i^2} \geq \frac{[\tau]}{[\tau] - P_i} \; ; \Rightarrow \frac{r_o}{r_i} \geq \sqrt{\frac{[\tau]}{[\tau] - P_i}}$$

$$\frac{r_i + t}{r_i} \geq \sqrt{\frac{[\tau]}{[\tau] - P_i}} \; ; \; t \geq r_i\left[\sqrt{\frac{[\tau]}{[\tau] - P_i}} - 1\right]$$

For brittle material, design is done on the basis of maximum principal stress theory of failure.

For safe design, $[\sigma_t] \geqslant \sigma_t$; $[\sigma_t] \geqslant \dfrac{P_i(r_o^2+r_i^2)}{r_o^2-r_i^2}$ where $[\sigma_t] = \dfrac{\sigma_{ut}}{F.S.}$

$$[\sigma_t](r_o^2-r_i^2) \geqslant P_i(r_o^2+r_i^2)\ ; \quad [\sigma_t]r_o^2-[\sigma_t]r_i^2 \geqslant P_i r_o^2 + P_i r_i^2\ ;$$

$$[\sigma_t]r_o^2 - P_i r_o^2 \geqslant [\sigma_t]r_i^2 + P_i r_i^2\ ; \quad ([\sigma_t]-P_i)r_o^2 \geqslant ([\sigma_t]+P_i)r_i^2\ ;$$

$$\frac{r_o^2}{r_i^2} \geqslant \frac{[\sigma_t]+P_i}{[\sigma_t]-P_i}\ ; \quad \frac{r_o}{r_i} \geqslant \sqrt{\frac{[\sigma_t]+P_i}{[\sigma_t]-P_i}}\ ; \quad \frac{t+r_i}{r_i} \geqslant \sqrt{\frac{[\sigma_t]+P_i}{[\sigma_t]-P_i}}$$

$$\frac{t}{r_i}+1 \geqslant \sqrt{\frac{[\sigma_t]+P_i}{[\sigma_t]-P_i}}\ ; \quad t \geqslant r_i\left[\sqrt{\frac{[\sigma_t]+P_i}{[\sigma_t]-P_i}}-1\right] \quad \text{where } r_i = \frac{D_i}{2}$$

For ductile material, design is done on the basis of maximum strain theory of failure. Allowable failure strain $[\epsilon_t] = \epsilon_{yield}/FS = \sigma_{yield}/(E \times FS)$

For safe design, $[\epsilon_t] \geqslant \epsilon_{max}$; $[\epsilon_t] \geqslant \epsilon_t$ where $\epsilon_t = \dfrac{\sigma_t}{E} - \dfrac{\mu}{E}(\sigma_r+\sigma_l)$

$$\epsilon_t \leqslant [\epsilon_t]\ ; \quad \frac{\sigma_t}{E} - \frac{\mu}{E}(\sigma_r+\sigma_l) \leqslant \frac{[\sigma_t]}{E}\ ; \quad \sigma_t - \mu\sigma_r - \mu\sigma_l \leqslant [\sigma_t]$$

$$\frac{P_i(r_o^2+r_i^2)}{r_o^2-r_i^2} + \mu P_i - \frac{\mu P_i r_i^2}{r_o^2-r_i^2} \leqslant [\sigma_t]\ ; \quad \frac{(r_o^2+r_i^2)+\mu(r_o^2-r_i^2)-\mu r_i^2}{r_o^2-r_i^2} \leqslant \frac{[\sigma_t]}{P_i}$$

$$\frac{r_o^2(1+\mu)+r_i^2(1-2\mu)}{r_o^2-r_i^2} \leqslant \frac{[\sigma_t]}{P_i}\ ; \quad \frac{[\sigma_t]}{P_i} \geqslant \frac{r_o^2(1+\mu)+r_i^2(1-2\mu)}{r_o^2-r_i^2}$$

$$[\sigma_t](r_o^2-r_i^2) \geqslant r_o^2(1+\mu)P_i + r_i^2(1-2\mu)P_i\ ;$$

$$[\sigma_t]r_o^2 - [\sigma_t]r_i^2 \geqslant r_o^2(1+\mu)P_i + r_i^2(1-2\mu)P_i\ ;$$

$$[\sigma_t]r_o^2 - r_o^2(1+\mu)P_i \geqslant [\sigma_t]r_i^2 + r_i^2(1-2\mu)P_i$$

$$\left[[\sigma_t]-(1+\mu)P_i\right]r_o^2 \geqslant \left[[\sigma_t]+(1-2\mu)P_i\right]r_i^2$$

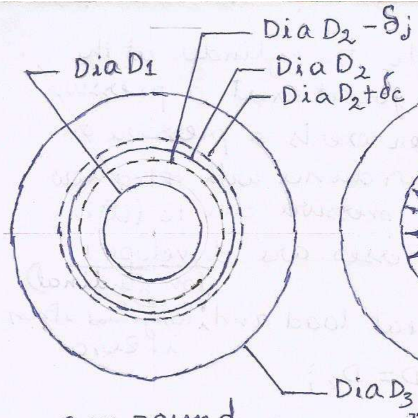
$$\frac{r_o^2}{r_i^2} \geqslant \frac{[\sigma_t]+(1-2\mu)P_i}{[\sigma_t]-(1+\mu)P_i}\ ; \quad \frac{r_o}{r_i} \geqslant \sqrt{\frac{[\sigma_t]+(1+2\mu)P_i}{[\sigma_t]-(1+\mu)P_i}}$$

$$t \geqslant r_i\left[\sqrt{\frac{[\sigma_t]+(1-2\mu)P_i}{[\sigma_t]-(1+\mu)P_i}}-1\right] \quad \text{where } r = \frac{D_i}{2} \text{ and } [\sigma_t] = \frac{\sigma_{yield}}{F.S.}$$

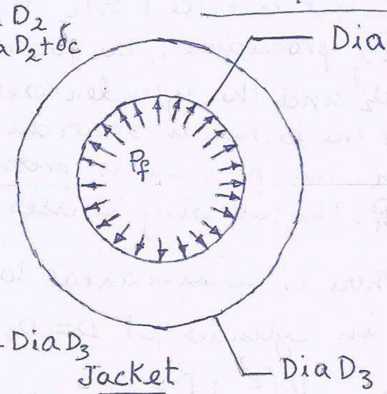
This formula is applicable where both the ends of the cylinder are closed. When the ends of cylinder are open, $\sigma_l = 0$. Then $\sigma_t - \mu\sigma_r \leqslant [\sigma_t]$. From the equation, following formula can be derived.

$$t \geqslant r_i\left[\sqrt{\frac{[\sigma_t]+(1-\mu)P_i}{[\sigma_t]-(1+\mu)P_i}}-1\right]$$

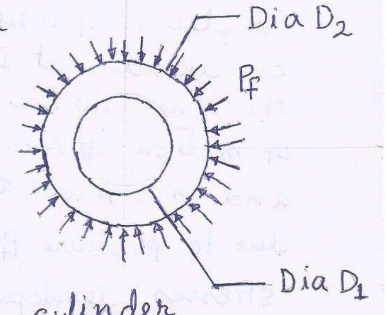
Concept of compound cylinder



compound cylinder



Jacket



cylinder

Principle of autofrettage:

Before fitting of jacket onto the cylinder, the inner diameter of Jacket is less than the outer diameter of cylinder. with the application of any process of autofrettage, the assembly is done. During the process of ~~an~~ assembly, the outer surface of cylinder and inner surface of jacket become identical. In this process, outer diameter of cylinder decreases to D_2 inner diameter of the jacket increases to D_2. Let us consider that the outer diameter of cylinder decreases by amount δ_c to D_2 and inner diameter of jacket increases by ~~an amount~~ amount δ_j to D_2. So, before assembly, the inner diameter of Jacket was $D_2 - \delta_j$ and before assembly, the outer dimeter of cylinder was $D_2 + \delta_c$. See Graphical approach (Based on Lamé's line)

So, clearance amount $= (D_2 - \delta_j) - (D_2 + \delta_c) = -(\delta_j + \delta_c) = -\delta$

with the negative value of clearance, natural insertion is not possible. It is called interference problem and $\delta = (\delta_j + \delta_c)$ is called diametral interference.

As a result of assembly, tangential strain ϵ_t occurs in the ~~jacket~~ jacket as well as in the cylinder.

ϵ_t = (Final Perimeter − initial perimeter)/ initial Perimeter

$$\epsilon_t^c \text{ at } D = D_2 \text{ for cylinder} = \frac{\pi D_2 - \pi(D_2 + \delta_c)}{\pi(D_2 + \delta_c)} = -\frac{\delta_c}{D_2 + \delta_c} \approx -\frac{\delta_c}{D_2} \text{ for } \delta_c \lll D_2$$

$$\delta_c = -D_2 \epsilon_t^c$$

$$\epsilon_t^j \text{ at } D = D_2 \text{ for jacket} = \frac{\pi D_2 - \pi(D_2 - \delta_j)}{\pi(D_2 - \delta_j)} = \frac{\delta_j}{D_2 - \delta_j} \approx \frac{\delta_j}{D_2} \text{ for } \delta_j \lll D_2$$

$$\delta_j = D_2 \epsilon_t^j \qquad \delta = \delta_j + \delta_c = D_2(\epsilon_t^j - \epsilon_t^c)$$

compound cylinder - 1

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When the jacket is fitted onto the cylinder at the completion of assembly procedure, the jacket exert a pressure on cylinder at $D = D_2$ and the cylinder exerts a pressure on the inner surface of the jacket in accordance with third law of motion of Newton. This pressure is pressure due to fitting and is referred as P_f. The following stresses are developed due to pressure P_f. There is no ~~axi~~ axial load and longitudinal stress is zero.

Stresses developed in cylinder at $D = D_2$,

$$\sigma_r^c = -P_f \qquad \sigma_t^c = -\frac{P_f(D_2^2 + D_1^2)}{D_2^2 - D_1^2} \quad \text{and} \quad \sigma_l^c = 0$$

Stresses developed in jacket at $D = D_2$,

$$\sigma_r^j = -P_f \qquad \sigma_t^j = \frac{P_f(D_3^2 + D_2^2)}{D_3^2 - D_2^2} \quad \text{and} \quad \sigma_l^j = 0.$$

So, at $D = D_2$,

$$\epsilon_t^c = \frac{\sigma_t^c}{E} - \frac{\mu}{E}(\sigma_r^c + \sigma_l^c) = -\frac{P_f}{E}\left(\frac{D_2^2 + D_1^2}{D_2^2 - D_1^2}\right) + \frac{\mu}{E}P_f$$

$$\epsilon_t^j = \frac{\sigma_t^j}{E} - \frac{\mu}{E}(\sigma_r^j + \sigma_l^j) = \frac{P_f}{E}\left(\frac{D_3^2 + D_2^2}{D_3^2 - D_2^2}\right) + \frac{\mu}{E}P_f$$

So,

$$\delta = D_2(\epsilon_t^j - \epsilon_t^c) = \frac{D_2 P_f}{E}\left[\frac{D_3^2 + D_2^2}{D_3^2 - D_2^2} + \mu + \frac{D_2^2 + D_1^2}{D_2^2 - D_1^2} - \mu\right]$$

$$= \frac{D_2 P_f}{E}\left(\frac{D_3^2 + D_2^2}{D_3^2 - D_2^2} + \frac{D_2^2 + D_1^2}{D_2^2 - D_1^2}\right) = \frac{2 P_f D_2^3}{E}\left[\frac{D_3^2 - D_1^2}{(D_3^2 - D_2^2)(D_2 - D_1^2)}\right]$$

$$\delta = \frac{2 P_f D_2^3}{E}\left[\frac{D_3^2 - D_1^2}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)}\right] = \frac{4 P_f r_2^3}{E}\left[\frac{r_3^2 - r_1^2}{(r_3^2 - r_2^2)(r_2^2 - r_1^2)}\right]$$

The above relation corelates diametral interference δ and pressure P_f due to fitting. Graphical approach based on Lame's line may be used as a useful tool for the analysis and design of compound cylinder.

Stress calculation method for compound cylinder:

Step 1: Calculate the distribution of stresses along radius for a single cylinder subjected to internal pressure P_i only. The wall thickness and inner dimeter of the cylinder are equal to those of compound cylinder — no jacket and no pressure P_f for fitting. Outside atmospheric pressure is negligible.

Step 2: Calculate the distribution of stresses along radius in compound cylinder subjected to pressure P_f due to fitting — no internal pressure and no external pressure.

Step 3: Superimpose the two stress distributions calculated in step 1 and step 2.

Design guidelines for compound cylinder: (1) cylinder and all jackets are safe from failure. (2) Maximum tangential stress σ_t in cylinder and jackets are same.

Compound cylinder - 2

AKM