Radiation Heat Transfer

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Lecture 5 Radiation exchange between surfaces (View factor, Blackbody radiation exchange)

The View Factor Integral

The view factor F_{ij} is defined as the *fraction of the radiation leaving surface i that is intercepted by surface j*. To develop a general expression for F_{ij}, we consider the arbitrarily oriented surfaces A_i and A_j of Figure 13.1. Elemental areas on each surface, dA_i and dA_j, are connected by a line of length R, which forms the polar angles θ_i and θ_j, respectively, with the surface normals $\mathbf{n}_i$ and $\mathbf{n}_j$. The values of R, θ_i, and θ_j vary with the position of the elemental areas on A_i and A_j.

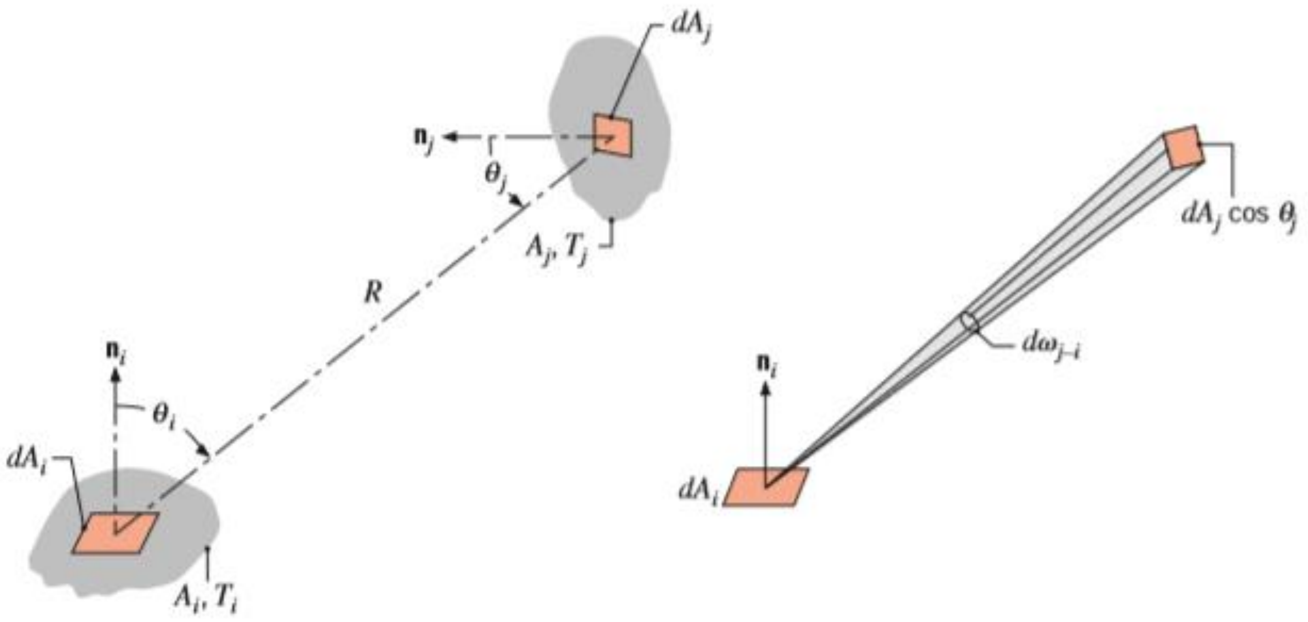


FIGURE 13.1 View factor associated with radiation exchange between elemental surfaces of area dA_i and dA_j.

the rate at which radiation *leaves* dA_i and is *intercepted* by dA_j may be expressed as

$$dq_{i\to j} = I_{e+r,i} \cos\theta_i \, dA_i \, d\omega_{j-i}$$

where $I_{e+r,i}$ is the intensity of radiation leaving surface i by emission and reflection and $d\omega_{j-i}$ is the solid angle subtended by dA_j when viewed from dA_i. With $d\omega_{j-i} = (\cos\theta_j \, dA_j)/R^2$

$$dq_{i\to j} = I_{e+r,i} \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i \, dA_j$$

Assuming that surface i *emits* and *reflects diffusely* we then obtain

$$dq_{i\to j} = J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

The total rate at which radiation leaves surface i and is intercepted by j may then be obtained by integrating over the two surfaces. That is,

$$q_{i\to j} = J_i \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

where it is assumed that the radiosity J_i is uniform over the surface A_i. From the definition of the view factor as the fraction of the radiation that leaves A_i and is intercepted by A_j,

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i}$$

it follows that

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j \qquad \textbf{(1)}$$

Similarly, the view factor F_{ji} is defined as the fraction of the radiation that leaves A_j and is intercepted by A_i. The same development then yields

$$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j \qquad \textbf{(2)}$$

From the last two expressions we can write $A_i F_{ij} = A_j F_{ji}$ **(3)**

This expression, termed the *reciprocity relation*, is useful in determining one view factor from knowledge of the other.

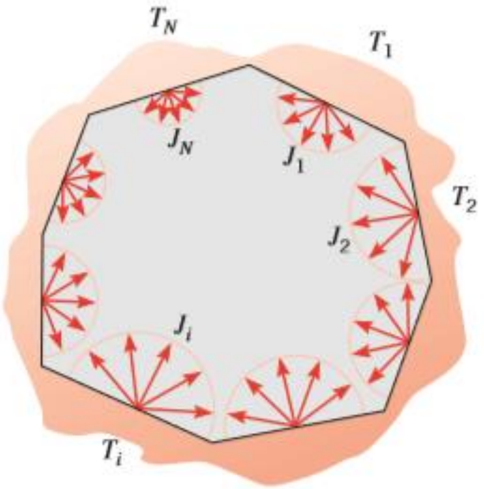


FIGURE 13.2 Radiation exchange in an enclosure.

Another important view factor relation pertains to the surfaces of an *enclosure* (Figure 13.2). From the definition of the view factor, the *summation rule*

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad \textbf{(4)}$$

may be applied to each of the N surfaces in the enclosure. This rule follows from the conservation requirement that all radiation leaving surface i must be intercepted by the enclosure surfaces. The term F_{ii} appearing in this summation represents the fraction of the radiation that leaves surface i and is directly intercepted by i. If the surface is concave, it *sees itself* and F_{ii} is nonzero. However, for a plane or convex surface, $F_{ii} = 0$.

To calculate radiation exchange in an enclosure of N surfaces, a total of N^2 view factors is needed. This requirement becomes evident when the view factors are arranged in the matrix form:

$$\begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & \cdots & F_{2N} \\ \vdots & \vdots & & \vdots \\ F_{N1} & F_{N2} & \cdots & F_{NN} \end{bmatrix}$$

However, all the view factors need not be calculated *directly*. A total of N view factors may be obtained from the N equations associated with application of the summation rule, Equation ~~13~~.4, to each of the surfaces in the enclosure. In addition, $N(N-1)/2$ view factors may be obtained from the $N(N-1)/2$ applications of the reciprocity relation, Equation ~~13~~.3, which are possible for the enclosure. Accordingly, only $[N^2 - N - N(N-1)/2] = N(N-1)/2$ view factors need be determined directly. For example, in a three-surface enclosure this requirement corresponds to only $3(3-1)/2 = 3$ view factors. The remaining six view factors may be obtained by solving the six equations that result from use of Equations ~~13~~.3 and ~~13~~.4.

To illustrate the foregoing procedure, consider a simple, two-surface enclosure involving the spherical surfaces of Figure 13.3. Although the enclosure is characterized by $N^2 = 4$ view factors (F_{11}, F_{12}, F_{21}, F_{22}), only $N(N-1)/2 = 1$ view factor need be determined directly. In this case such a determination may be made by *inspection*. In particular, since all radiation leaving the inner surface must reach the outer surface, it follows that $F_{12} = 1$. The same may not be said of radiation leaving the outer surface, since this surface sees itself. However, from the reciprocity relation, Equation ~~13~~.3, we obtain

$$F_{21} = \left(\frac{A_1}{A_2}\right) F_{12} = \left(\frac{A_1}{A_2}\right)$$

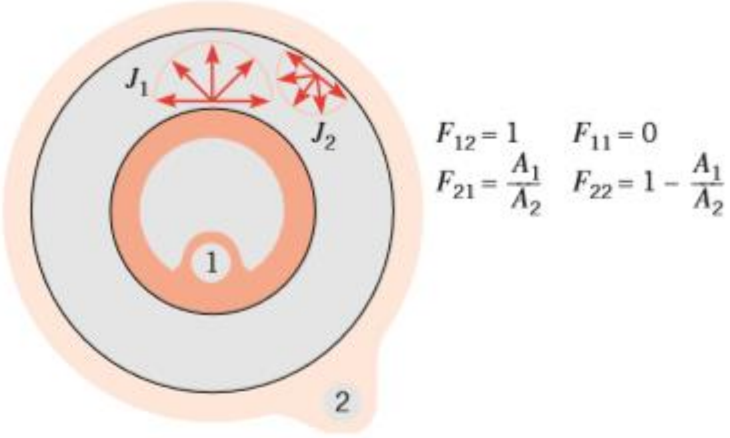


FIGURE 13.3 View factors for the enclosure formed by two spheres.

From the summation rule, we also obtain

$$F_{11} + F_{12} = 1$$

in which case $F_{11} = 0$, and

$$F_{21} + F_{22} = 1$$

in which case

$$F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

EXAMPLE 1

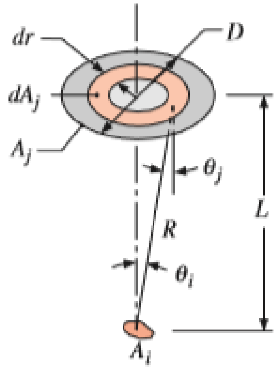
Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i \ll A_j$. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.

SOLUTION

Known: Orientation of small surface relative to large circular disk.

Find: View factor of small surface with respect to disk, F_{ij}.

Schematic:



Assumptions:

1. Diffuse surfaces.
2. $A_i \ll A_j$.
3. Uniform radiosity on surface A_i.

Analysis: The desired view factor may be obtained from Equation 13.1.

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_i \, dA_j$$

Recognizing that θ_i, θ_j, and R are approximately independent of position on A_i, this expression reduces to

$$F_{ij} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_j$$

or, with $\theta_i = \theta_j \equiv \theta$,

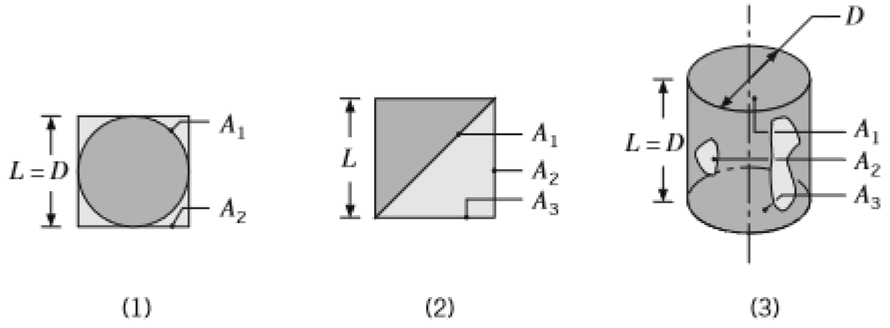
$$F_{ij} = \int_{A_j} \frac{\cos^2\theta}{\pi R^2}\, dA_j$$

With $R^2 = r^2 + L^2$, $\cos\theta = (L/R)$, and $dA_j = 2\pi r\, dr$, it follows that

$$F_{ij} = 2L^2 \int_0^{D/2} \frac{r\, dr}{(r^2 + L^2)^2} = \frac{D^2}{D^2 + 4L^2}$$

Example 2

Determine the view factors F_{12} and F_{21} for the following geometries:



1. Sphere of diameter D inside a cubical box of length $L = D$.
2. One side of a diagonal partition within a long square duct.
3. End and side of a circular tube of equal length and diameter.

Solution

Known: Surface geometries.

Find: View factors.

Assumptions: Diffuse surfaces with uniform radiosities.

Analysis: The desired view factors may be obtained from inspection, the reciprocity rule, the summation rule, and/or use of the charts.

1. Sphere within a cube:

By inspection, $F_{12} = 1$ ◁

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{\pi D^2}{6L^2} \times 1 = \dfrac{\pi}{6}$ ◁

2. Partition within a square duct:

From summation rule, $F_{11} + F_{12} + F_{13} = 1$

where $F_{11} = 0$

By symmetry, $F_{12} = F_{13}$

Hence $F_{12} = 0.50$ ◁

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{\sqrt{2}L}{L} \times 0.5 = 0.71$ ◁

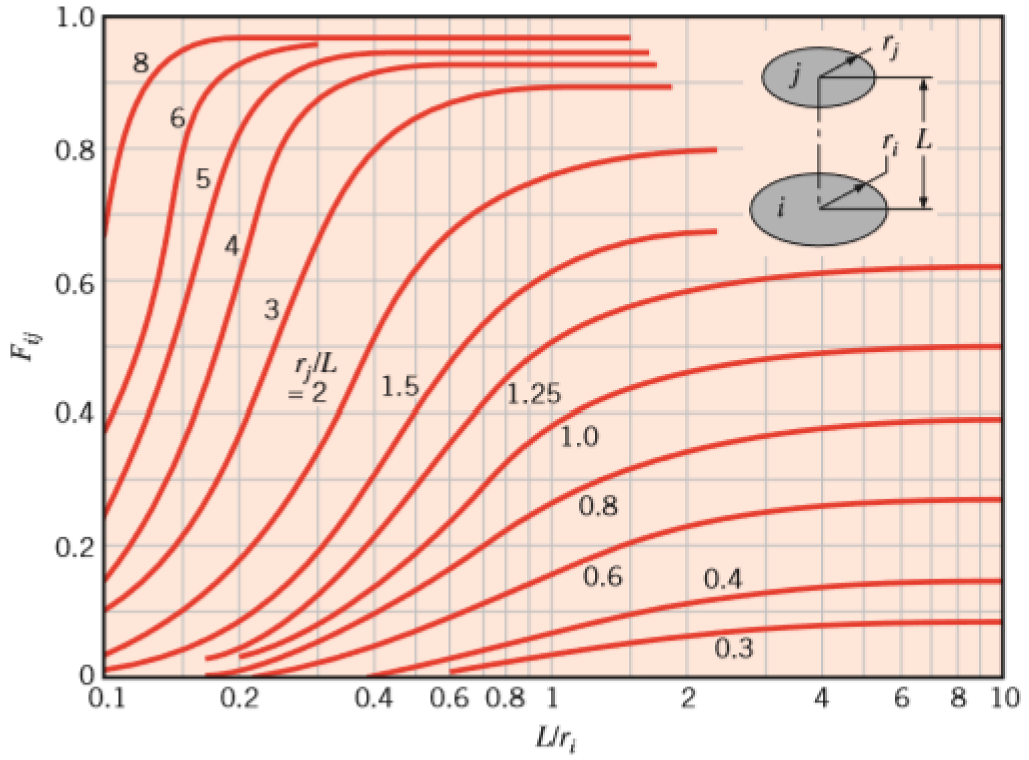
3. Circular tube:

From ~~Table 13.2 or~~ Figure ~~13~~.5, with $(r_3/L) = 0.5$ and $(L/r_1) = 2$, $F_{13} = 0.172$

From summation rule, $F_{11} + F_{12} + F_{13} = 1$

or, with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.828$ ◁

From reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{\pi D^2/4}{\pi D L} \times 0.828 = 0.207$ ◁



View factor for coaxial parallel disks.

Blackbody Radiation Exchange

In general, radiation may leave a surface due to both reflection and emission, and on reaching a second surface, experience reflection as well as absorption. However, matters are simplified for surfaces that may be approximated as blackbodies, since there is no reflection. Hence energy leaves only as a result of emission, and all incident radiation is absorbed.

Consider radiation exchange between two black surfaces of arbitrary shape (Figure 13.8). Defining $q_{i\to j}$ as the rate at which radiation *leaves* surface i and is *intercepted* by surface j, it follows that

$$q_{i\to j} = (A_i J_i) F_{ij}$$

or, since radiosity equals emissive power for a black surface ($J_i = E_{bi}$),

$$q_{i\to j} = A_i F_{ij} E_{bi}$$

Similarly,

$$q_{j\to i} = A_j F_{ji} E_{bj}$$

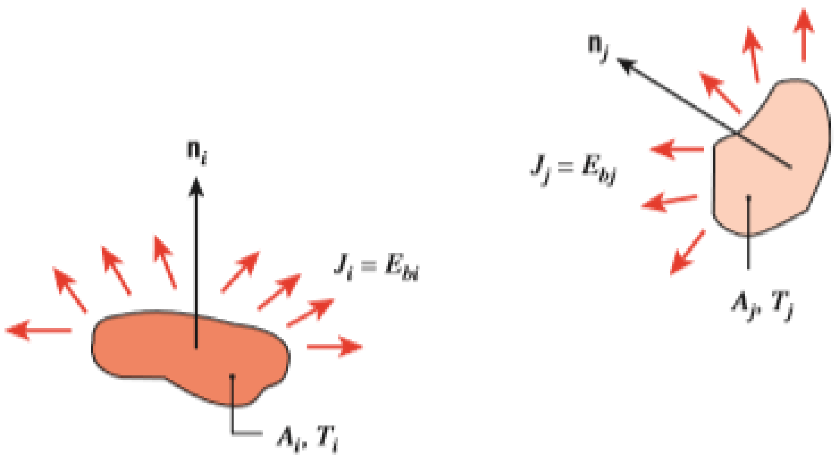


FIGURE 13.8 Radiation transfer between two surfaces that may be approximated as blackbodies.

The *net radiative exchange* between the two surfaces may then be defined as

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

from which it follows that

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

$$q_{ij} = A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

This Equation provides the *net* rate at which radiation *leaves* surface i as a result of its interaction with j, which is equal to the *net* rate at which j *gains* radiation due to its interaction with i.

The foregoing result may also be used to evaluate the net radiation transfer from any surface in an *enclosure* of black surfaces. With N surfaces maintained at different temperatures, the net transfer of radiation from surface i is due to exchange with the remaining surfaces and may be expressed as

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

Assignment

A furnace cavity, which is in the form of a cylinder of 50-mm diameter and 150-mm length, is open at one end to large surroundings that are at 27°C. The bottom of the cavity is heated independently, as are three annular sections that comprise the sides of the cavity. All interior surfaces of the cavity may be approximated as blackbodies and are maintained at 1650°C. What is the required electrical power input to the bottom surface of the cavity? What is the electrical power to the top, middle, and bottom sections of the cavity sides? The backs of the electrically heated surfaces are well insulated.

Thank you