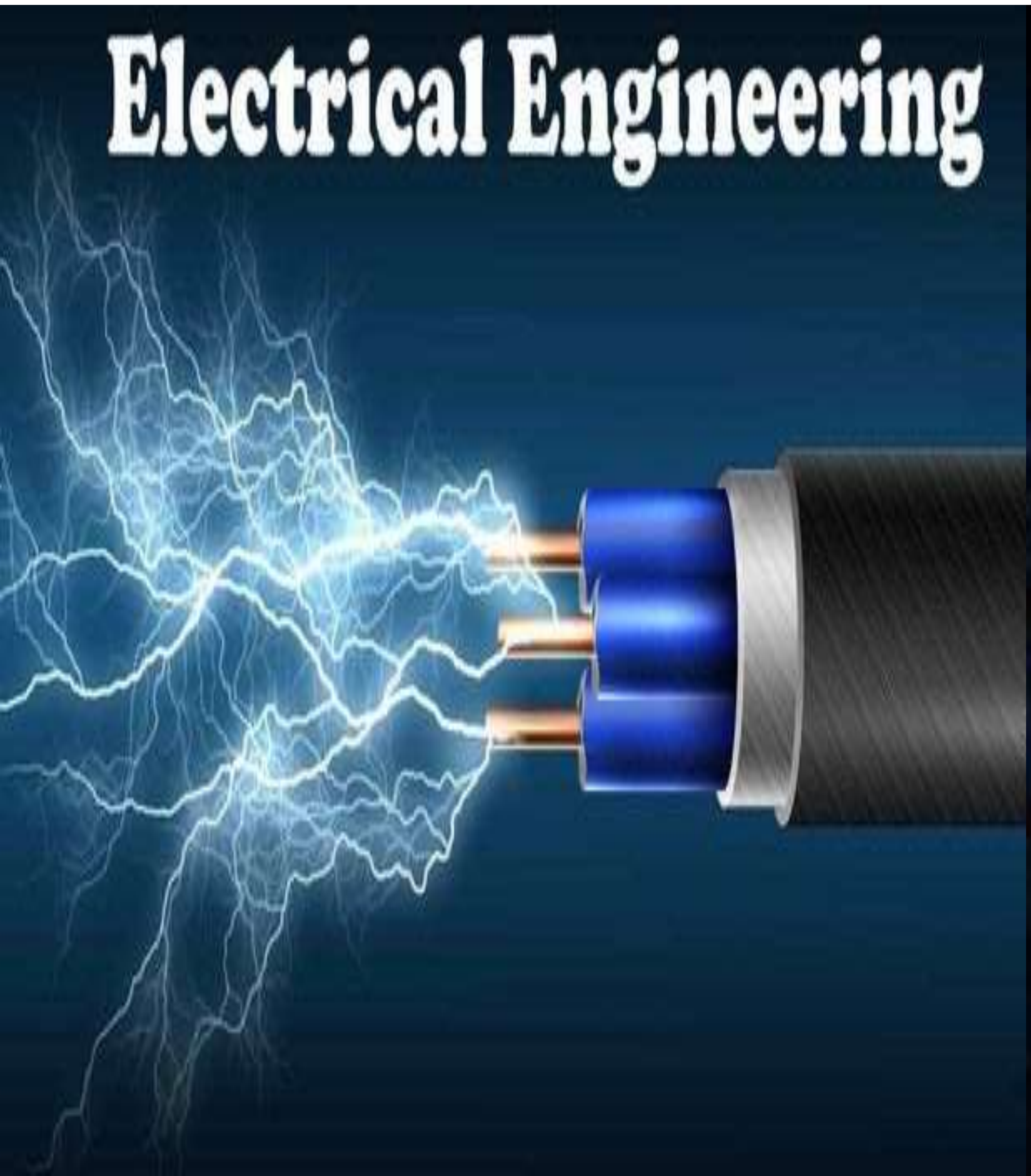
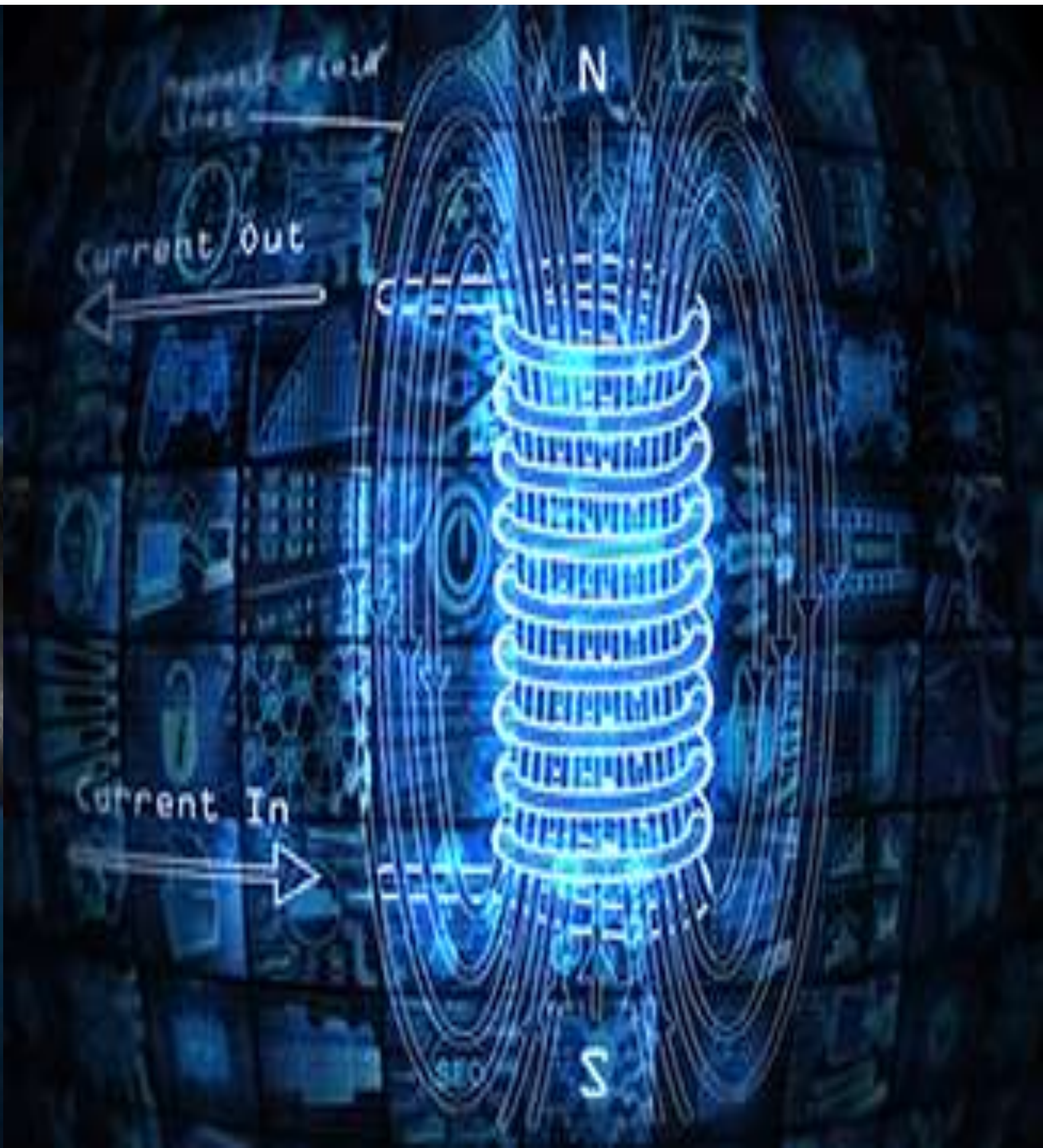

Electrical Engineering

N
Current Out
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S

LECTURE 17

Polyphase Circuits

➢ Introduction

- ✓ It is possible to have any number of sources in a polyphase system and to connect the polyphase source to more number of individual circuits.
- ✓ The increase in the available power is not significant beyond the three-phase system. The power generated by the same machine increases 41.4 percent from single phase to two-phase, and the increase in the power is 50 percent from single phase to three-phase.
- ✓ The maximum possible increase is only seven percent beyond three-phase but the complications are many. So, the increase beyond three-phase does not justify the extra complications.
- ✓ In view of this, more than three phases are used in exceptional cases. Circuits supplied by six, twelve and more phases are used in high power radio transmitter stations. Two-phase systems are used to supply two-phase servo motors in feedback control systems.
- ✓ In general, a three-phase system of voltages (currents) is merely a combination of three single phase systems of voltages (currents) of which the three voltages (currents) differ in phase by 120 electrical degrees from each other in a particular sequence.

Polyphase Circuits

➢ Advantages of Three-phase System

✓ The polyphase, especially three-phase, system has many advantages over the single phase system, both from the utility point of view as well as from the consumer point of view.

Some of the advantages are as under.

1. The power in a single phase circuit is pulsating. The power becomes zero 100 times in a second in a 50 Hz supply when the power factor of the circuit is unity. Therefore, single phase motors have a pulsating torque. Although the power supplied by each phase is pulsating, the total three-phase power supplied to a balanced three-phase circuit is constant at every instant of time. So, three-phase motors have an absolutely uniform torque.
2. A three-phase transmission circuit requires less conductor material than a single-phase circuit to transmit a given amount of power over a given length.
3. In a given frame size, a three-phase motor or a three-phase generator produces more output than its single phase counterpart.
4. Three-phase motors are more easily started than single phase motors. Single phase motors are not self-starting, whereas three-phase motors are.

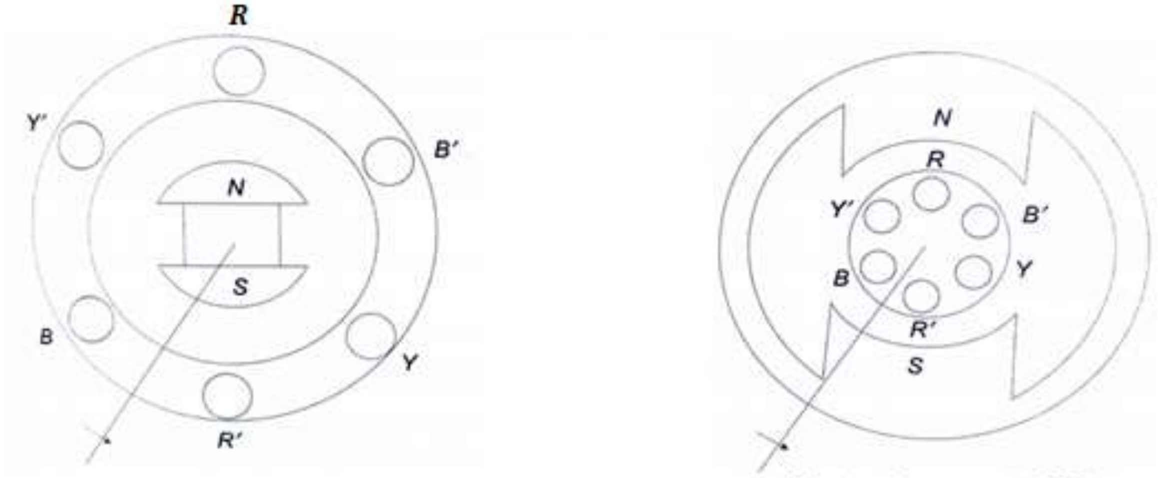
Polyphase Circuits

➢ Advantages of Three-phase System

- ✓ The operating characteristics of a three-phase apparatus are superior to those of a similar single phase apparatus.
- ✓ All three-phase machines are superior in performance.
- ✓ Their control equipments are smaller, cheaper, lighter in weight and more efficient.

➢ Generation of Three-Phase Voltages

- ✓ Three-phase voltages can be generated in a stationary armature with a rotating field structure, or in a rotating armature with a stationary field as shown in **Fig. 7.1 (a)** and **(b)**.



(a) Stationary Armature (b) Stationary Field

Fig. 7.1

Polyphase Circuits

➢ Generation of Three-Phase Voltages

- ✓ Single phase voltages and currents are generated by single phase generators as shown in **Fig. 7.2(a)**.

  The stationary armature of such a generator has only one winding, or one set of coils.

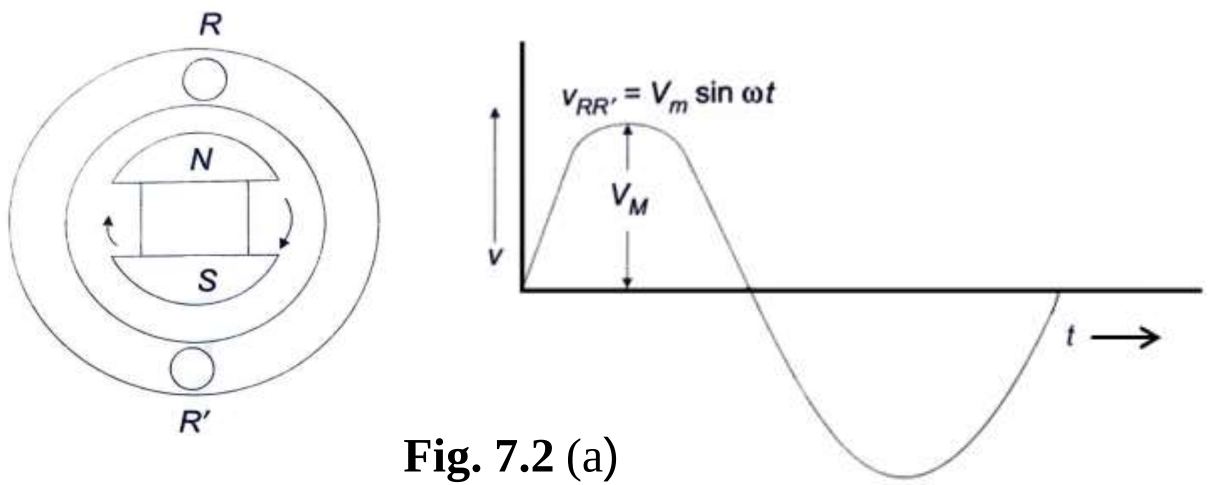


Fig. 7.2 (a)

- ✓ The generated voltages in the two phases have 90 degrees phase displacement as shown in **Fig. 7.2(b).**

  In a two-phase generator the armature has two distinct windings, or two sets of coils that are displaced by 90^0(electrical degrees) apart,

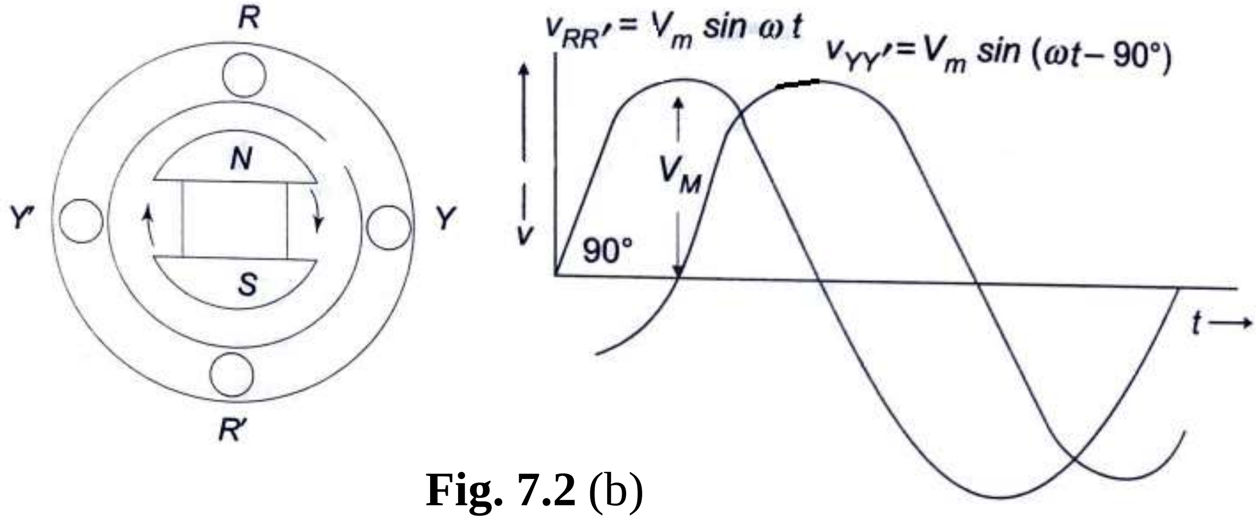


Fig. 7.2 (b)

Generation of Three-Phase Voltages

- Three-phase voltages are generated in three separate but identical sets of windings or coils that are displaced by 120 electrical degrees in the armature, so that the voltages generated in them are 120^0 apart in time phase. This arrangement is shown in **Fig. 7.2(c).**
- Here RR' constitutes one coil (R-phase); YY' another coil (Y phase), and BB' constitutes the third phase (B-phase). The field magnets are assumed in clockwise rotation.

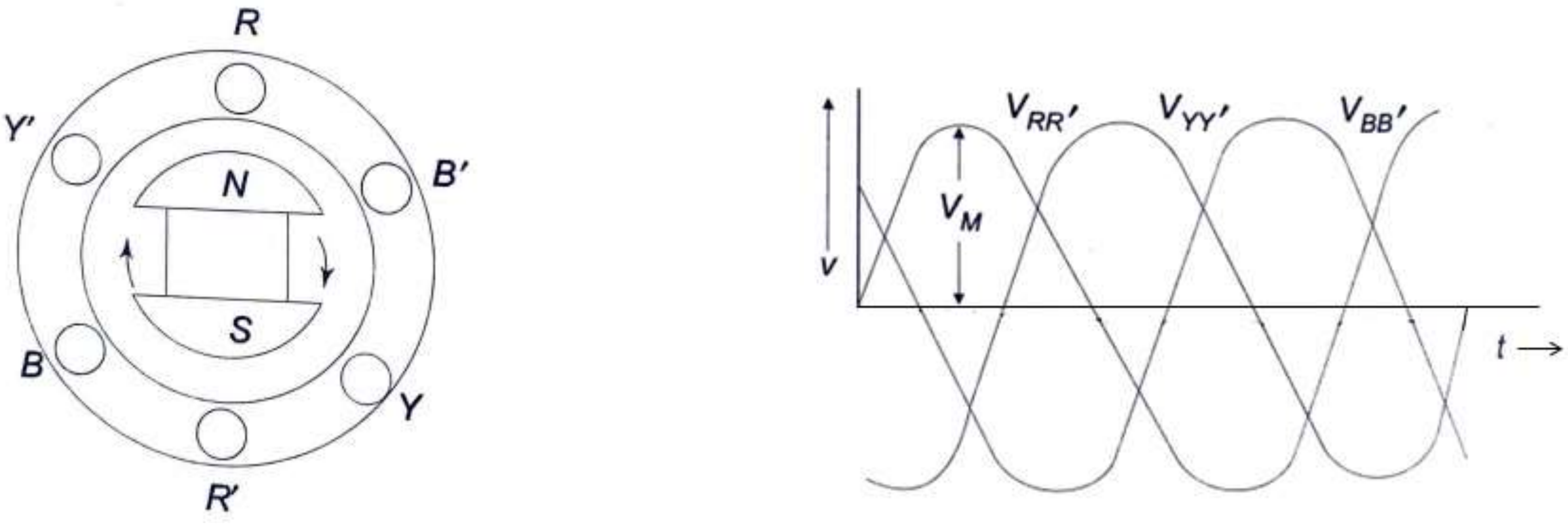


Fig. 7.2 (c)

Polyphase Circuits

➢ Generation of Three-Phase Voltages

- ✓ The voltages generated by a three-phase alternator is shown in **Fig. 7.3**.
- ✓ The three voltages are of the same magnitude and frequency, but are displaced from one another by 120^0.
- ✓ The voltages are assumed to be sinusoidal

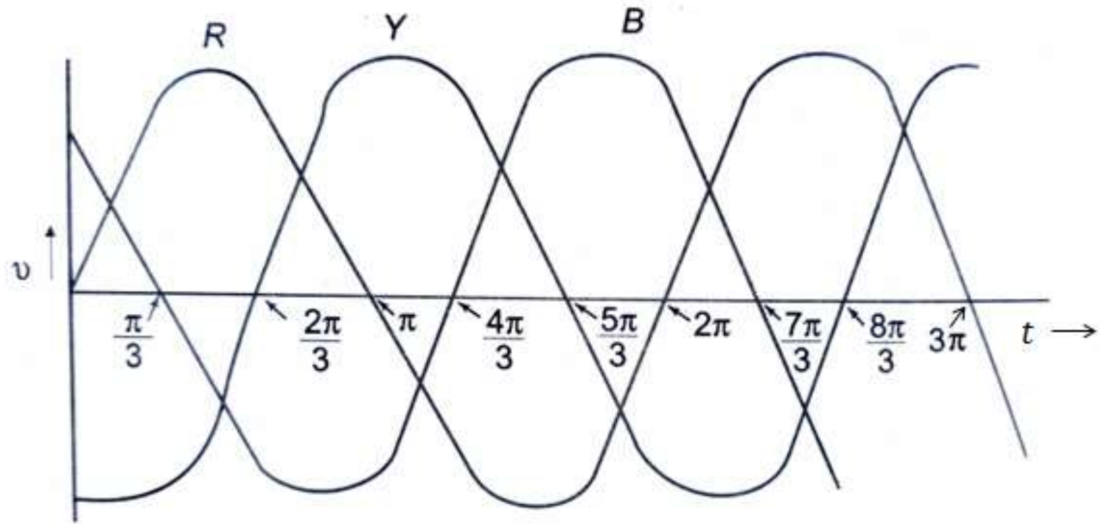


Fig. 7.3

- ✓ Counting the time from the instant when the voltage in phase R is zero. So, the equations for the instantaneous values of the voltages of the three phases are

$$V_{RR'} = V_m \sin \omega t$$
$$V_{YY'} = V_m \sin(\omega t - 120^0)$$
$$V_{BB'} = V_m \sin(\omega t - 240^0)$$

- ✓ At any given instant, the algebraic sum of the three voltages must be zero.

Polyphase Circuits

➢ Phase Sequence

- ✓ The sequence of voltages in the three phases are in the order of $V_{RR'} - V_{YY'} - V_{BB'}$ and they undergo changes one after the other in the above order. This is called the phase sequence.
- ✓ It is observed that this sequence depends on the rotation of the field. If the field system is rotated in the anticlockwise direction, then the sequence of the voltages in the three-phases are in the order

$V_{RR'}\ -V_{YY'} - V_{BB'}$; This sequence is called RBY.

- ✓ The equations can be written as

$$V_{RR'} = V_m \sin \omega t$$
$$V_{YY'} = V_m \sin(\omega t - 120^0)$$
$$V_{BB'} = V_m \sin(\omega t - 240^0)$$

Polyphase Circuits

➢ Inter Connection of Three-phase Sources and Loads

✓ Inter Connection of Three-phase Sources

- In a three-phase alternator, there are three independent phase windings or coils. Each phase or coil has two terminals, viz. start and finish.
- The end connections of the three sets of the coils may be brought out of the machine, to form three separate single phase sources to feed three individual circuits as shown in **Fig. 7.4** (a) and (b).
- The coils are inter-connected to form a wye (Y) or delta (Δ) connected three-phase system to achieve economy and to reduce the number of conductors, and thereby, the complexity in the circuit.
- The three-phase sources so obtained serve all the functions of the three separate single phase sources.

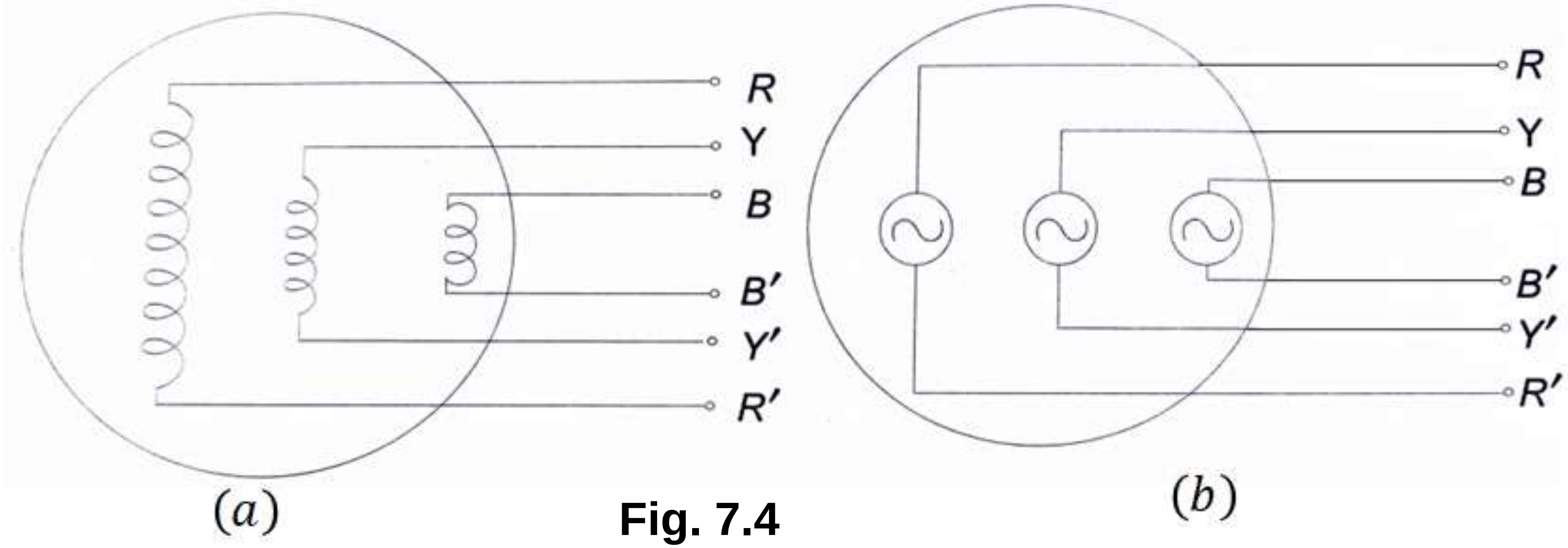


Fig. 7.4

Polyphase Circuits

❖ Wye or Star-Connection

- A three-phase, four-wire, star-connected system is shown **in Fig. 7.5**. In this figure, similar ends (start or finish) of the three phases are joined together within the alternator.
- The common terminal so formed is referred to as the neutral point (N), or neutral terminal.
- Three lines are run from the other free ends (R, Y, B) to feed power to the three-phase load.
- The terminals R, Y, B are called the line terminals of the source.
- The voltage between any line and the neutral point is called the phase voltage (i.e. V_{RN}, V_{YN} and V_{BN}) and the voltages displaced by 120^0 from one another.
- The voltage between any two lines is called the line voltage ($\text{i.e.} V_{RY}, V_{YB}$ or V_{BR}) and the voltages displaced by 120^0 from one another..
- The currents flowing through the phases are called the phase currents, while those flowing in the lines are called the line currents.
- If the neutral wire is not available for external connection, the system is called a three-phase, three-wire, star-connected system.

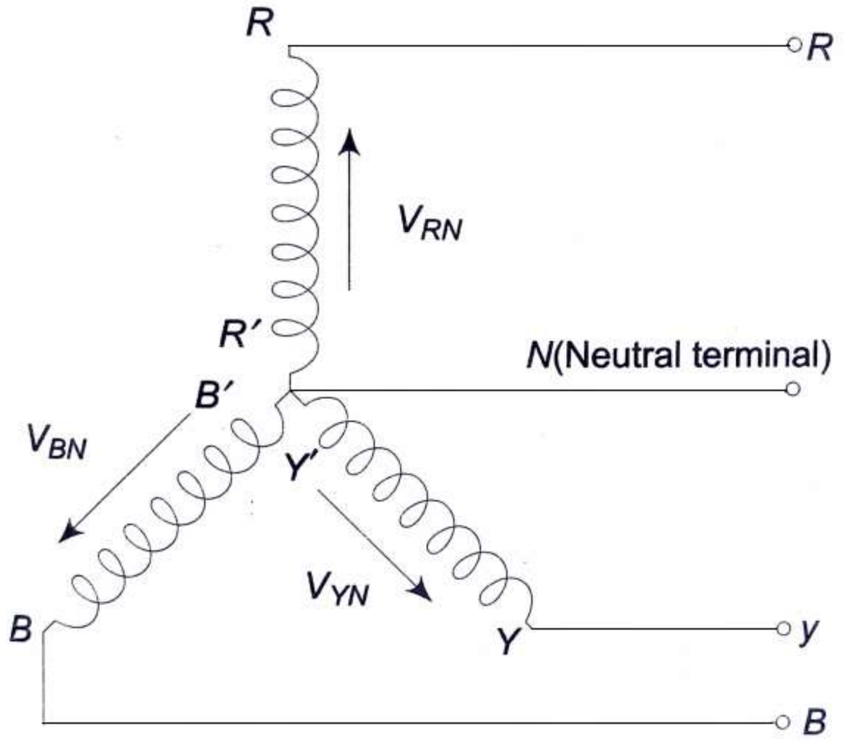


Fig. 7.5

Polyphase Circuits

❖ Delta or Mesh-Connection

- The dissimilar ends of the windings are joined together, i.e. R' is connected to Y, Y' to B and B' to R as shown in **Fig. 7.6.**
- The three conductors are taken from the three junctions of the mesh or delta connection to feed the three-phase load.
- This constitutes a three-phase, three-wire, delta-connected system.
- The line voltages V_{RY}, V_{YB} and V_{BR} are referred to as phase voltages in the delta-connected system.

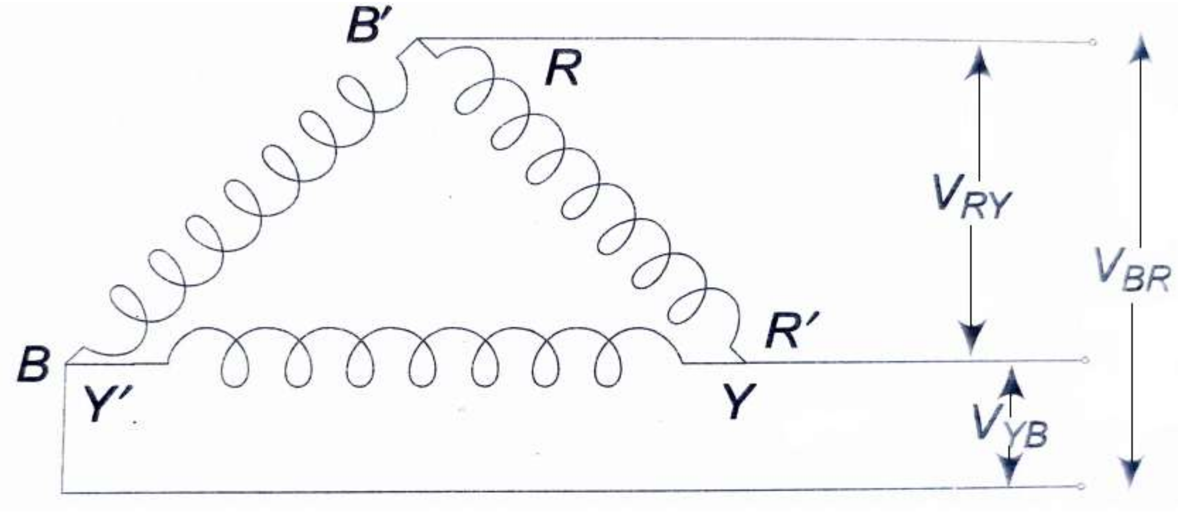


Fig. 7.6

- A three-phase source, star or delta, can be either balanced or unbalanced.
- A balanced three-phase source is one in which the three individual sources have equal magnitude, with 120^0 phase difference as shown in **Fig. 7.6**.

Polyphase Circuits

✓ **Interconnection of Loads**

- A load connected across any two terminals of an active network (source) will draw power from the source.
- The three-phase loads are connected in either star or delta formation and then connected to the three-phase source as shown in **Fig. 7.7**.
- A three-phase star connected load is connected to a three-phase star-connected source, terminal to terminal, and both the terminals are joined with a fourth wire as shown in **Fig. 7.7**(a) .

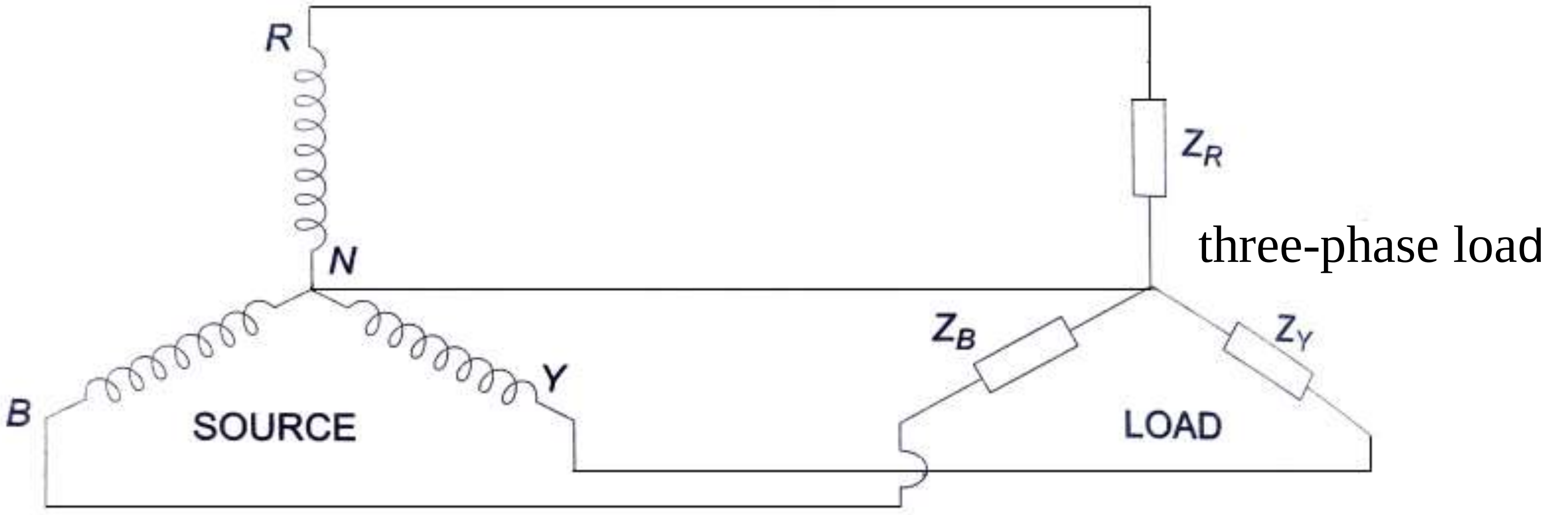


Fig. 7.7 (a)

Polyphase Circuits

✓ Interconnection of Loads

A three-phase delta-connected load is connected to a three-phase star-connected source, terminal to terminal, as shown in **Fig. 7.7** (b).

When either source or load, or both are connected in delta, only three wires will suffice to connect the load to source. So, this is a three-phase, three-wire system.

A balanced three-phase load is one in which all the branches have identical impedances, i.e. each impedance has the same magnitude and phase angle. The resistive and reactive components of each phase are equal.

The load which does not satisfy the requirements of identical impedances is said to be an unbalanced load.

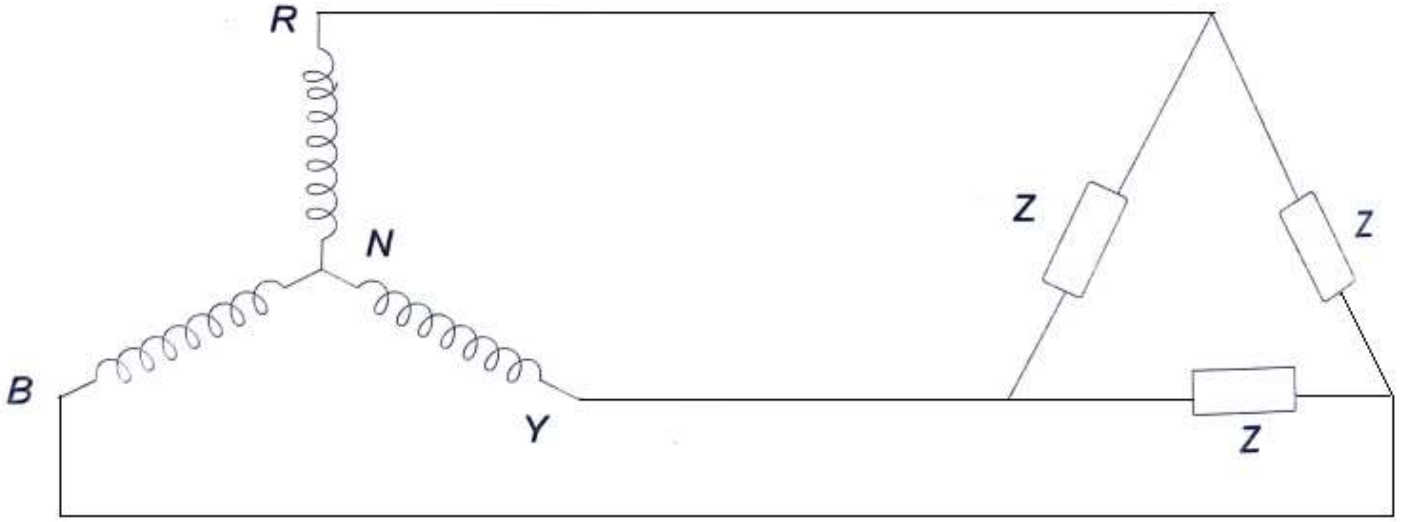


Fig. 7.7 (b).

Polyphase Circuits

- **Voltage, Current and Power in a Star Connected System**

  - **Star-Connected System**

    - A balanced three-phase, Y-connected system is shown in **Fig. 7.8**.
    - The voltage induced in each winding is called the phase voltage (V_{ph}). V_{RN}, V_{YN} and V_{BN} represent the rms values of the induced voltages in each phase.
    - The voltage available between any pair of terminals is called the line voltage (V_L). V_{RY}, V_{YB} and V_{BR} are known as line voltages.
    - V_{RY} indicates a voltage V between points R and Y, with R being positive with respect to point Y during its positive half cycle.

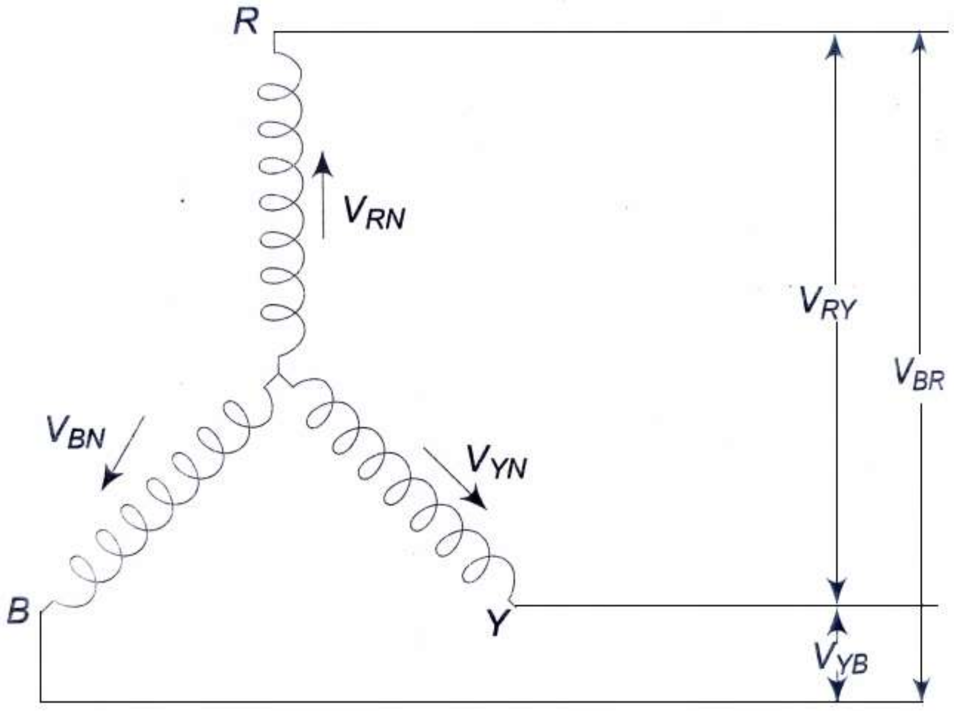


Fig. 7.8

    - Similarly, V_{YB} means that Y is positive with respect to point B during its positive half cycle.
    - The relation of voltage is $V_{RY} = -V_{YR}$.

Polyphase Circuits

- **Star-Connected System**
  - **Voltage Relation**
    - The phasors corresponding to the phase voltages constituting a three-phase system can be represented by a phasor diagram as shown in **Fig. 7.9**.
    - The line voltage V_{RY} is equal to the phasor sum of V_{RN} and V_{NY} which is also equal to the phasor difference of V_{RN} and V_{YN} ($V_{NY} = -V_{YN}$).
    - The two phasors, V_{RN} and $-V_{YN}$, are equal in length and are 60^0 apart.

$$|V_{RN}| = -|V_{YN}| = V_{ph}$$

$$\therefore V_{RY} = 2V_{ph} \cos 60/2 = \sqrt{3}\, V_{ph}$$

    - Similarly, the line voltage V_{YB} is equal to the phasor difference of V_{YN} and V_{BN}, and is equal to $\sqrt{3}V_{ph}$.
    - The line voltage V_{BR} is equal to the phasor difference of V_{BN} and V_{RN}, and is equal to $\sqrt{3}V_{ph}$.

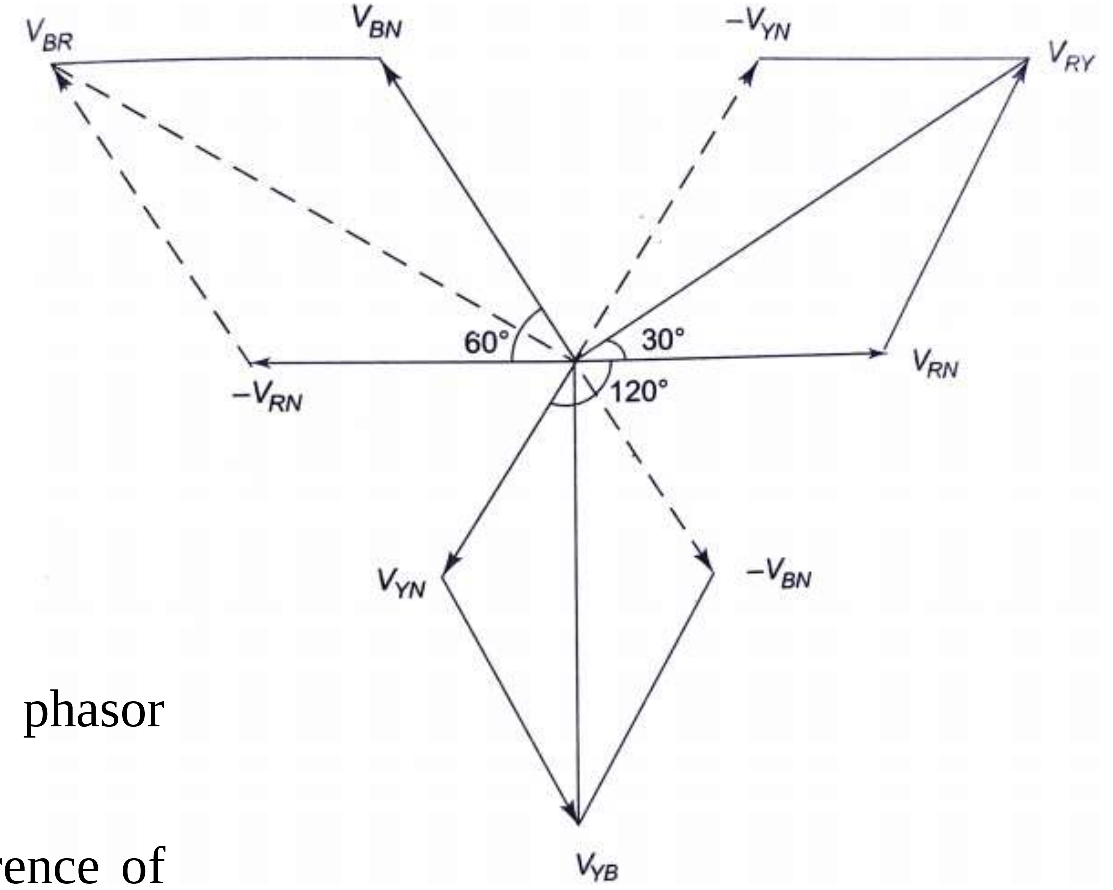


Fig. 7.9

Polyphase Circuits

- **Star-Connected System**
  - **Voltage Relation**
    - Hence, in a balanced star-connected system

1. Line voltage $= \sqrt{3}\, V_{ph}$
2. All line voltages are equal in magnitude and are displaced by 120^0, and
3. All line voltages are 30^0 ahead of their respective phase voltages (from **Fig. 7.9**).

Polyphase Circuits

➢ Star-Connected System

✓ Current Relations

- A balanced three-phase, wye-connected system indicating phase currents and line currents are shown in **Fig. 7.10** (a).
- The arrows placed alongside the currents $I_{R,}$ I_Y and I_B flowing in the three phases indicate the directions of currents when they are assumed to be positive and not the directions at that particular instant.
- It can be observed from **Fig. 7.10** (a) that each line conductor is connected in series its individual phase winding.
- Therefore, the current in a line conductor is the same as that in the phase to which the line conductor is connected.

$$\therefore I_L = I_{ph} = I_R = I_Y = I_B$$

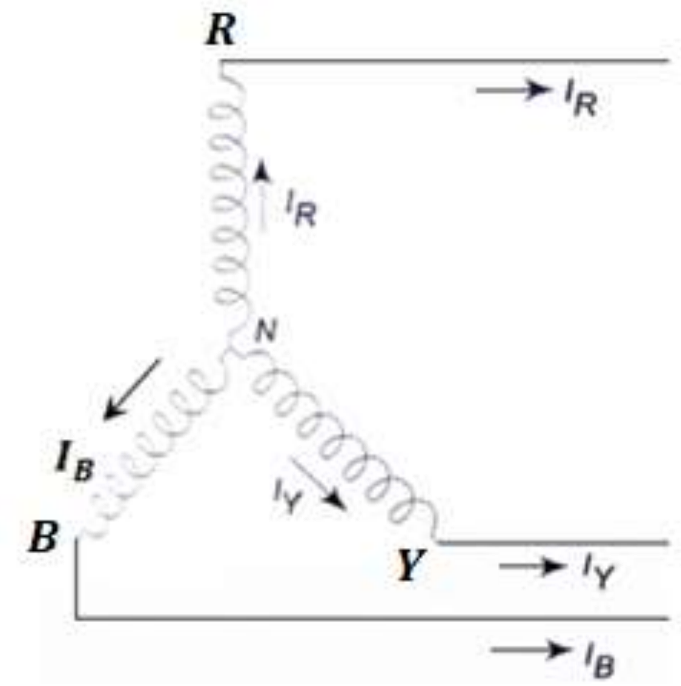


Fig. 7.10 (a)

Polyphase Circuits

- **Star-Connected System**

  ✓ **Current Relations**

  - The phasor diagram for phase currents with respect to their phase voltages is shown in **Fig. 7.10** (b).
  - All the phase currents are displaced by 120^0 with respect to each other.
  - $\emptyset$ is the phase angle between phase voltage and phase current (lagging load is assumed).
  - For a balanced load, all the phase currents are equal in magnitude.

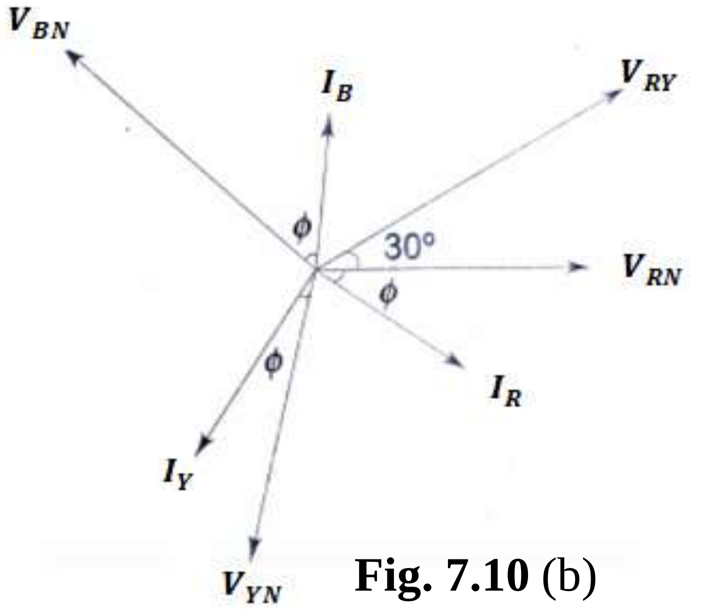


Fig. 7.10 (b)

  - It can be observed from **Fig. 7.10** (b) that the angle between the line (phase) current and the corresponding line voltage is $(30^0 + \emptyset)^0$ for a lagging load.
  - Consequently, if the load is leading, then the angle between the line (phase) current and corresponding line voltage will be $(30^0 - \emptyset)^0$.

Polyphase Circuits

- **Star-Connected System**
  - **Power in the Star-Connected Network**
    - The total active power or true power in the three-phase load is the sum of the powers in the three phases.
    - For a balanced load, the power in each load is the same.
    - Hence total power = 3× power in each phase

$$\text{or } P = 3 \times V_{ph} \times I_{ph} \cos\emptyset$$

    - It is the usual practice to express the three-phase power in terms of line quantities as follows.

$$V_L = \sqrt{3}V_{ph}, \qquad I_L = I_{ph}$$

$$\therefore \quad P = \sqrt{3}V_L I_L \cos\emptyset \quad W$$

So, $\sqrt{3}V_L I_L \cos\emptyset$ is the active power in the circuit.

    - Total reactive power is given by $Q = \sqrt{3} \times V_{ph} \times I_{ph} \sin\emptyset \quad VAR$
    - Total apparent power or volt-amperes = $\sqrt{3}V_L I_L \quad VA$

Polyphase Circuits

- **Star-Connected System**
  - **N-Phase Star System**
    - The star and mesh are general terms applicable to any number of phases but wye and delta are special cases of star and mesh when the system is a three-phase system.
    - Consider an n-phase balanced star system with two adjacent phases as shown in **Fig. 7.11** (a). Its vector diagram is shown in **Fig. 7.11** (b).
    - The angle of phase difference between adjacent phase voltages is $360^0/n$.
    - Let E_{ph} be the voltage of each phase.
    - The line voltage, i.e. the voltage between A and B is equal to $E_{AB} = E_L = E_{AO} + E_{OB}$.

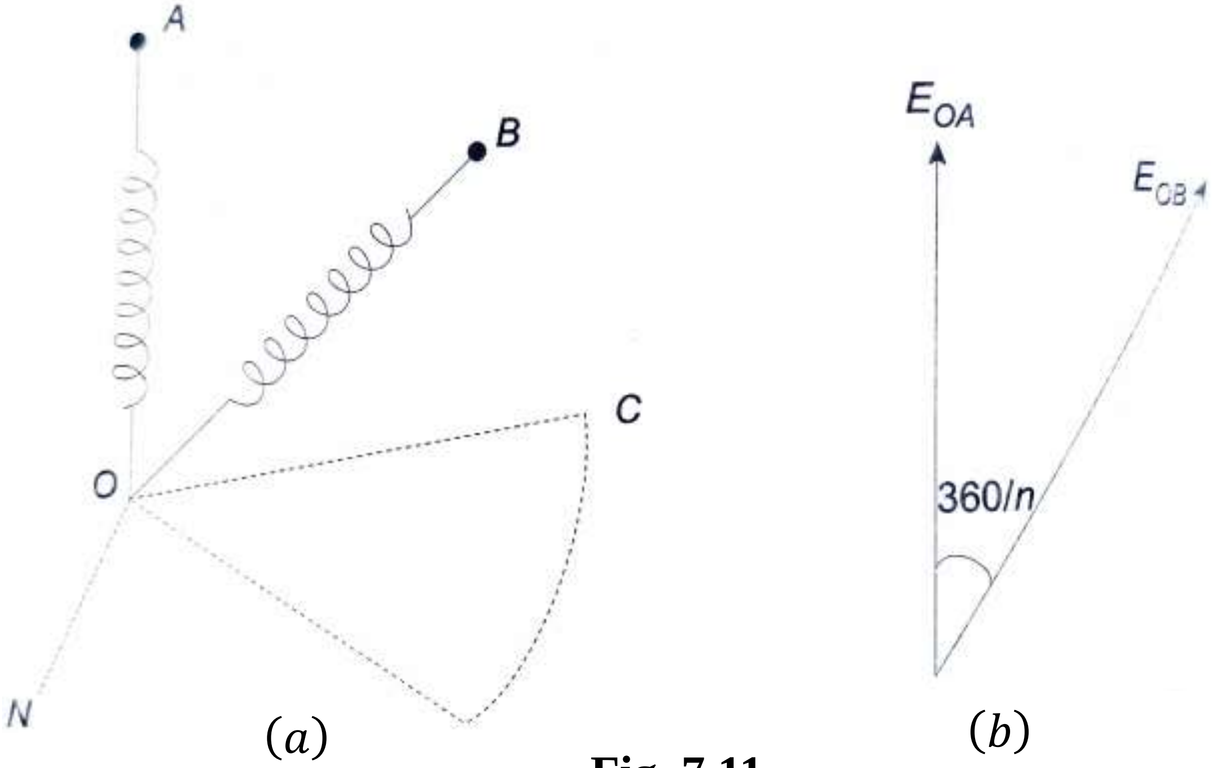


Fig. 7.11

Polyphase Circuits

➢ **Star-Connected System**

- The vector addition is shown in **Fig. 7.11** (c). It is seen that the line current and phase current are same.
- Consider the parallelogram $OABC$

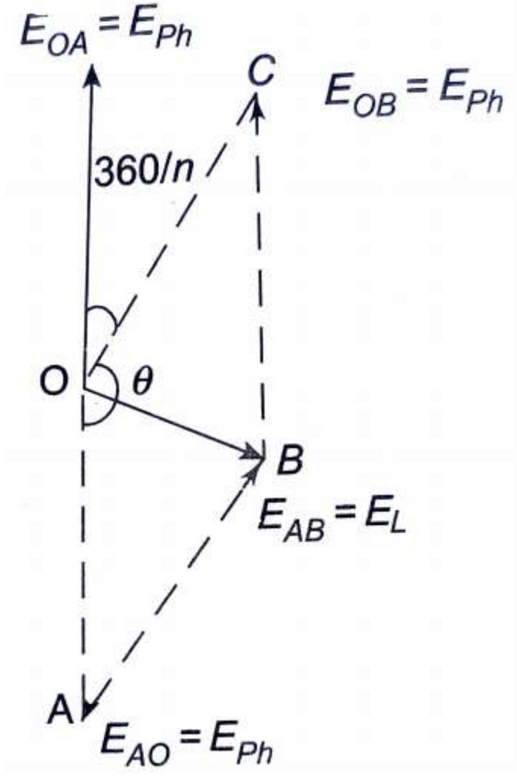
$$OB = \sqrt{OC^2 + OA^2 + 2 \times OA \times OC \times \cos\theta}$$

$$= \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph}^2 \cos\left(180^0 - \frac{360^0}{n}\right)}$$

$$= \sqrt{2E_{ph}^2 - 2E_{ph}^2 \cos\frac{360^0}{n}}$$

$$= \sqrt{2}E_{ph}\sqrt{\left[1 - \cos 2\left(\frac{180^0}{n}\right)\right]}$$

$$= \sqrt{2}E_{ph}\sqrt{2\sin^2\left(\frac{180^0}{n}\right)} = 2E_{ph}\sin\frac{180^0}{n}$$



(c)

Fig. 7.11

- The equation is a general equation for line voltage.
- For example, for a three-phase system, $n = 3$; $E_L = 2\,E_{ph}\sin 60^0 = \sqrt{3}E_{ph}$

Polyphase Circuits

➢ Delta-Connected System

- A balanced three-phase, three-wire, delta-connected system is shown in **Fig. 7.12.**
- This arrangement is referred to as mesh connection because it forms a closed circuit.
- It is also known as delta connection because the three branches in the circuit can also be arranged in the shape of delta (Δ).
- The sum of the three voltages round the closed mesh is zero since the system is balanced.
- No current can flow around the mesh when the terminals are open.

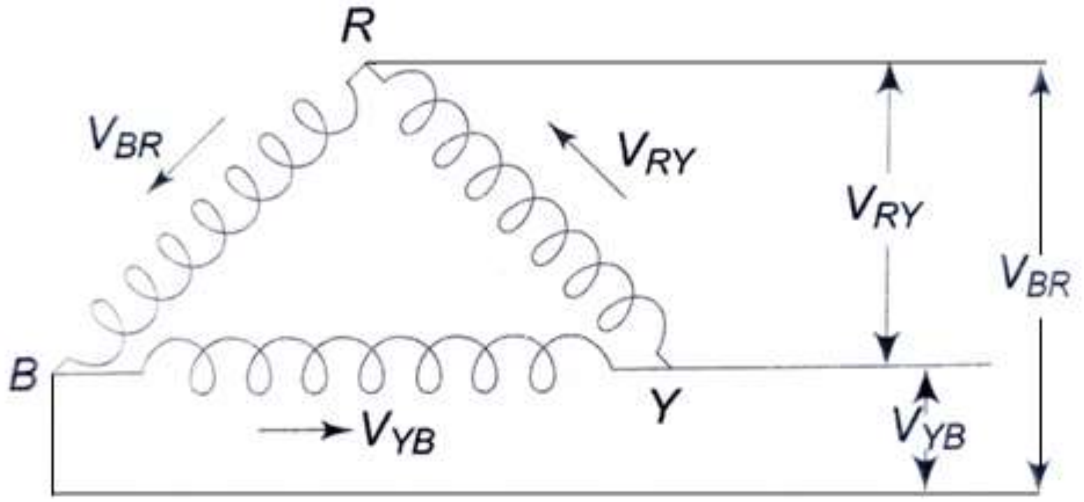


Fig. 7.12

- The arrows placed alongside the voltages, V_{RY}, V_{YB} and V_{BR}, of the three phases indicate that the terminals R, Y and B are positive with respect to Y, B and R, respectively, during their respective positive half cycles.

Delta-Connected System

Voltage Relation

- It is observed from **Fig. 7.12** that only one phase is connected between any two lines. Hence, the voltage between any two lines (V_L) is equal to the phase voltage (V_{ph}).

$$\therefore V_{RY} = V_L = V_{ph}$$

- Since the system is balanced, all the phase voltages are equal, but displaced by 120^0 from one another as shown in the phasor diagram in **Fig. 7.13**.

- The phase sequence RYB is assumed.

$$\therefore |V_{RY}| = |V_{YB}| = |V_{BR}| = V_L = V_{ph}$$

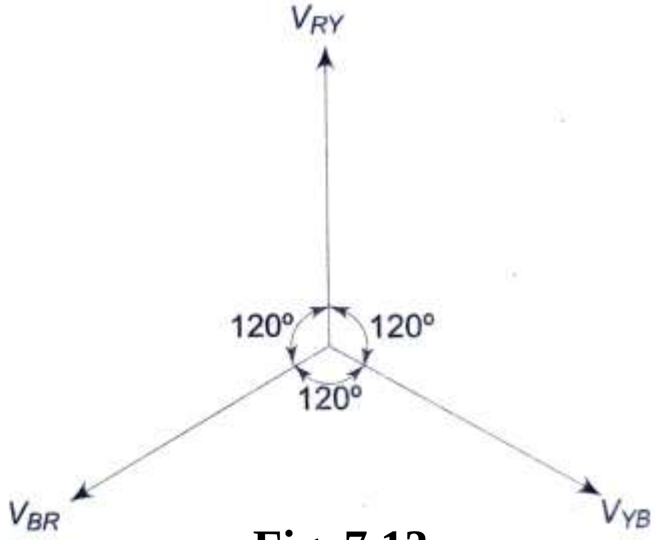


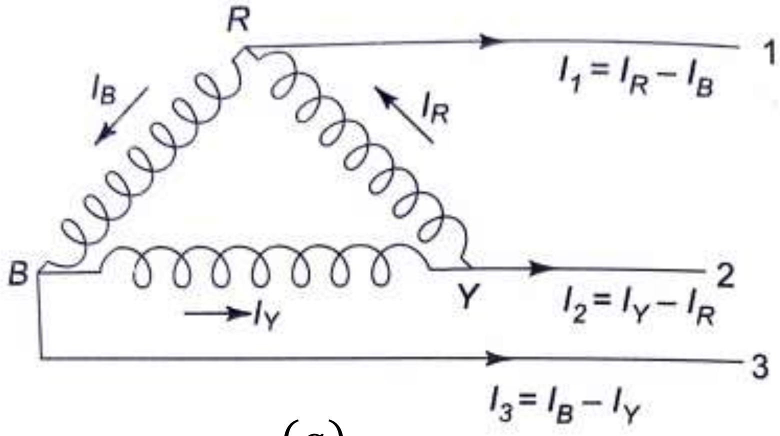
Fig. 7.13

Polyphase Circuits

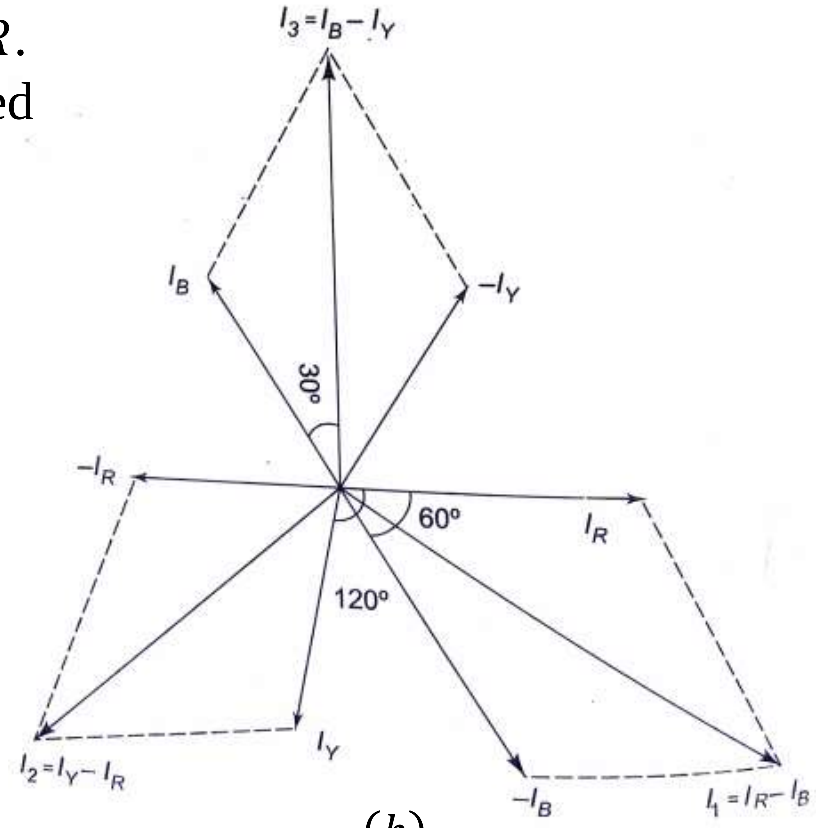
- **Delta-Connected System**

  - **Current Relation**

    - The system as shown in **Fig. 7.14** (a) is balanced. So, the three phase currents (I_{ph}), i.e. I_R, I_Y, I_B are equal in magnitude but displaced by 120^0 from one another as shown in **Fig. 7.14** (b).
    - I_1 is the line current in line 1 connected to the common point of R. Similarly, I_2 and I_3 are the line currents in lines 2 and 3, connected to common points Y and B, respectively.
    - The current in line 1, $I_1 = I_R - I_B$; i.e. the current in any line is equal to the phasor difference of the currents in the two phases attached to that line.
    - Similarly, the current in line 2,
      $$I_2 = I_Y - I_R,$$
    - The current in line 3,
      $$I_3 = I_B - I_Y$$



(a)



(b)

Fig. 7.14

Polyphase Circuits

- **Delta-Connected System**
  - **Current Relation**
    - The phasor addition of these currents is shown in **Fig. 7.14** (b).
    - From the figure,

$$I_1 = I_R - I_B$$

$$I_1 = \sqrt{I_R^2 + I_B^2 + 2I_R I_B \cos 60^0}$$

$$I_1 = \sqrt{3}\, I_{ph} \qquad \text{since } I_R = I_B = I_{ph}$$

- Similarly, the remaining two line currents, I_2 and I_3, are also equal to $\sqrt{3}$ times the phase currents,

i.e. $I_L = \sqrt{3}\, I_{ph}$.

- It is seen from **Fig. 7.14** (b) that

I. all the line currents are equal in magnitude but displaced by 120^0 from one another.
II. the line currents are 30^0 behind the respective phase currents.

Polyphase Circuits

- **Delta-Connected System**
  - **Power in the Delta-Connected System**
    - The power consumed in each phase is same since the load is balanced.
    - Power per phase $= V_{ph}I_{ph}\cos\emptyset$

      Where $\emptyset$ is the phase angle between phase voltage and phase current.
    - Total power in the delta circuit is the sum of the powers in the three phases.

$$\text{Total power } P = 3 \times V_{ph}I_{ph}\cos\emptyset$$

$$P = \sqrt{3} \times V_L I_L \cos\emptyset \;\; W$$

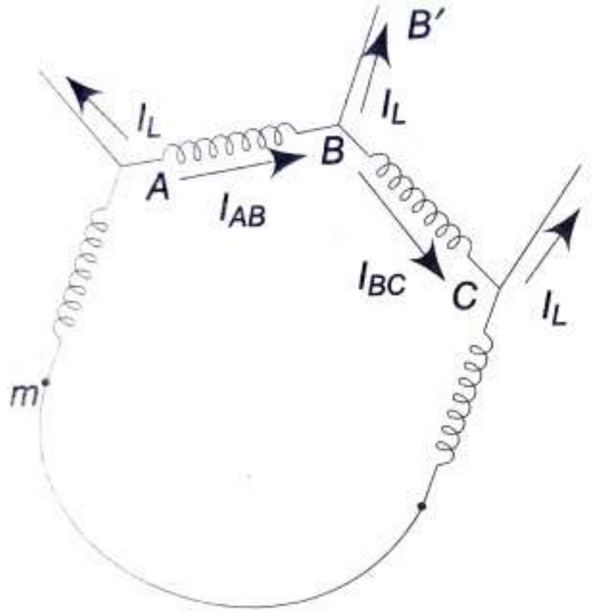
Since $V_{ph} = V_L$ and

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

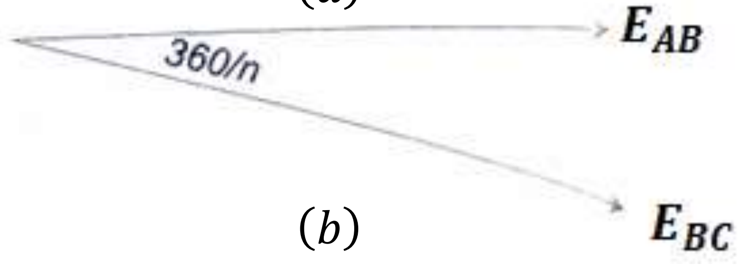
- For a balanced system, whether star or delta, the expression for the total power is the same.

➢ Delta-Connected System

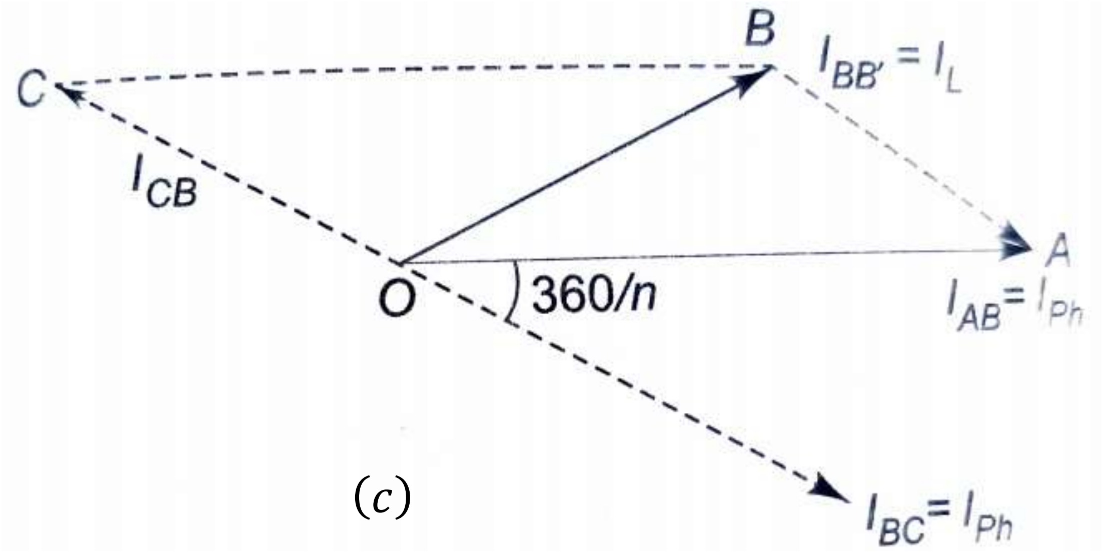
- A part of an n-phase balanced mesh system is shown in **Fig. 7.15** (a) and its vector diagram is shown in **Fig. 7.15** (b).
- Let the current in line BB' be I_L. This is same in all the remaining lines of the n-phase system.
- I_{AB}, I_{BC} are the phase currents in AB and BC phases respectively.
- The vector addition of the line current is shown in **Fig. 7.1**5 (c).



(a)



(b)



(c)

Fig. 7.15

Polyphase Circuits

- **Delta-Connected System**

  - It is seen from the **Fig. 7.15** (b) that the line and phase voltages are equal.

$$I_{BB} = I_L = I_{AB} + I_{CB}$$
$$= I_{AB} - I_{BC}$$

  - Consider the parallelogram $OABC$

$$OB = \sqrt{OA^2 + OC^2 + 2 \times OA \times OC \times \cos\left(180 - \frac{360}{n}\right)}$$
$$= \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \cos\frac{360}{n}}$$
$$= \sqrt{2}I_{ph}\sqrt{1 - \cos 2\left(\frac{180}{n}\right)}$$
$$= \sqrt{2}I_{ph}\sqrt{2\sin^2\frac{180}{n}}\, I_L = 2I_{ph}\sin\frac{180}{n}$$

Example – P 7.1

A symmetrical three-phase, three-wire 400 V, supply is connected to a delta-connected load as shown in **Fig. P 7.1**. Impedances in each branch are $Z_{RY} = 10\angle 30^0\,\Omega$; $Z_{YB} = 10\angle -45^0\ \Omega$ and $Z_{BR} = 2.5\angle 60^0\,\Omega$. Find its equivalent star-connected load; the phase sequence is RYB.

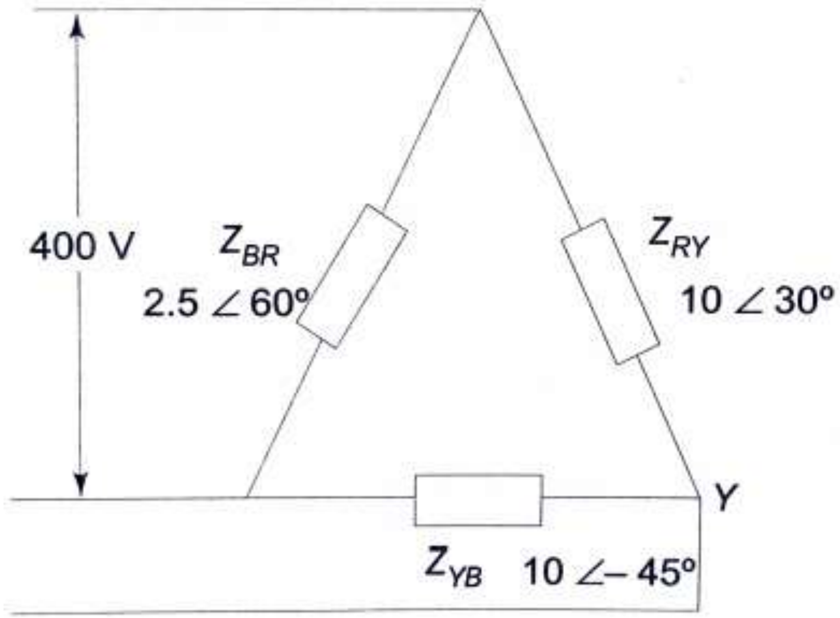


Fig. P 7.1

Solution of Example – P7.1

The $Z_{R,}$ Z_Y and Z_B are as follows:

$$Z_R = \frac{Z_{RY}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$Z_{RY}+Z_{YB}+Z_{BR} = 10\,\angle 30^0\ + 10\,\angle -45^0\ +2.5\,\angle 60^0$$

$$= (8.66 + j5) + (7.07 - j7.07) + (1.25 + j2.17)$$

$$= 16.98 + j0.1 = 16.98\angle 0.33^0\ \Omega$$

$$Z_R = \frac{10\,\angle 30^0\ \times\ 2.5\,\angle 60^0}{16.98\angle 0.33^0} = 1.47\angle 89.67^0 = (0.008 + j1.47)\ \Omega$$

$$Z_Y = \frac{Z_{RY}Z_{YB}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$= \frac{10\,\angle 30^0\ \times\ 10\,\angle -45^0}{16.98\angle 0.33^0} = 5.89\angle -15.33^0\ \ \Omega$$

Solution of Example – P7.1

$$Z_B = \frac{Z_{BR}Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{2.5\,\angle 60^0 \times 10\,\angle -45^0}{16.98\angle 0.33^0} = 1.47\angle 14.67^0\ \Omega$$

The equivalent star impedances are

$Z_R = 1.47\angle 89.67^0\ \Omega$, $Z_Y = 5.89\angle - 15.33^0\ \Omega$ and $Z_B = 1.47\angle 14.67^0\ \Omega$

Polyphase Circuits

Example – P 7.2

A balanced star-connected load of (4 +j3) Ω per phase is connected to a balanced 3-phase 400 V supply. The phase current is 12 A. Find (i) the total active power (ii) reactive power and (iii) total apparent power.

Polyphase Circuits

Solution of Example – P7.2

The voltage given in the data is rms value of the line voltage.

$$\therefore \quad Z_{ph} = \sqrt{4^2 + 3^2} = 5\ \Omega$$

$$PF = \cos\emptyset = \frac{R_{ph}}{Z_{ph}} = \frac{4}{5} = 0.8$$

$$\sin\emptyset = 0.6$$

(i) Active power $= \sqrt{3} V_L I_L \cos\emptyset$ W

$$= \sqrt{3} \times 400 \times 12 \times 0.8 = 6651\ \text{W}$$

(ii) Reactive power $= \sqrt{3} V_L I_L \sin\emptyset$ VAr

$$= \sqrt{3} \times 400 \times 12 \times 0.6 = 4988.36\ \text{VAr}$$

(iii) Apparent power $= \sqrt{3} V_L I_L$

$$= \sqrt{3} \times 400 \times 12 = 8313.84\ \text{VA}$$

Polyphase Circuits

Example – P 7.3

A three-phase, balanced delta-connected load of $(4 + j8)\ \Omega$ is connected across a 400 V, $3 - \emptyset$ balanced supply. Determine the phase currents and line currents. Assume the phase sequence to be RYB. Also calculate the power drawn by the load.

Polyphase Circuits

Solution of Example – P7.3

Taking the line voltage $V_{RY} = V\angle 0^0$ as reference $V_{RY} = 400\angle 0^0$ V; $V_{YB} = 400\angle -120^0$ V, $V_{BR} = 400\angle -240^0$ V

Impedance per phase $= (4 + j8)\ \Omega = 8.94\angle 63.4^0\ \Omega$

Phase currents are:

$$I_R = \frac{400\angle 0^0}{8.94\angle 63.4^0} = 44.74\angle -63.4^0\ \text{A}$$

$$I_Y = \frac{400\angle -120^0}{8.94\angle 63.4^0} = 44.74\angle -183.4^0\ \text{A}$$

$$I_B = \frac{400\angle -240^0}{8.94\angle 63.4^0} = 44.74\angle -303.4^0\ \text{A}$$

Solution of Example – P7.3

The three line currents are

$$I_1 = I_R - I_B = (44.74\angle -63.4^0 - 44.74\angle -303.4^0)$$

$$= (20.03 - j40) - (24.62 + j37.35)$$

$$= (-4.59 - j77.35)\ \text{A}$$

$$= 77.49\ \angle 266.6^0\ \text{A}$$

The line current I_1 is equal to the $\sqrt{3}$ times the phase current and 30^0 behind its respective phase current.

$$I_1 = \sqrt{3} \times 44.74\ \angle(-63.4^0 - 30^0) = 77.49\angle -93.4^0 = 77.49\angle 266.6^0\ \text{A}$$

Solution of Example – P7.3

Similarly,

$$I_2 = I_Y - I_R = \sqrt{3} \times 44.74\ \angle(-183.4^0 - 30^0) = 77.49\angle -213.4^0 = 77.49\angle 146.6^0 \text{ A}$$

$$I_3 = I_B - I_Y = \sqrt{3} \times 44.74\ \angle(-303.4^0 - 30^0) = 77.49\angle -333.4^0 = 77.49\angle 26.6^0 \text{ A}$$

Power drawn by the load is

$$P = 3V_{ph} I_{ph} \cos \emptyset$$

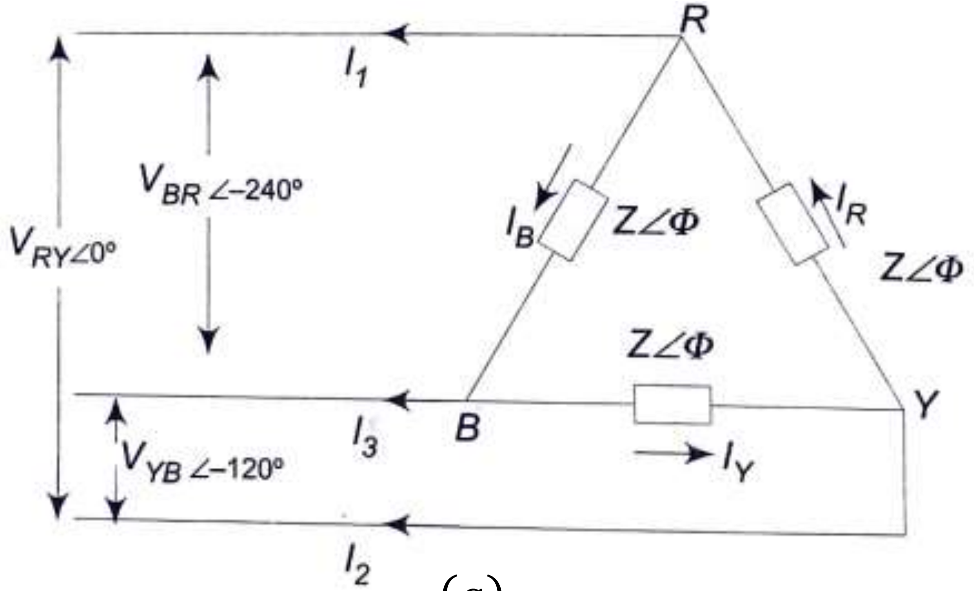
$$= \sqrt{3} \times V_L \times I_L \cos 63.4^0$$

$$= 24.039 \text{ kW}$$

Polyphase Circuits

- **Three-phase Balanced Circuits**
  - ✓ **Balanced Three-Phase System — Delta Load**
    - A three-phase, three-wire, balanced system supply power to a balanced three-phase delta load as shown in **Fig. 7.16** (a). The phase sequence is considered as RYB.
    - Let us assume the line voltage $V_{RY} = V\angle 0^0$ as the reference phasor.
    - Then the three source voltages are given by

$$V_{RY} = V\angle 0^0 \text{ V}$$
$$V_{YB} = V\angle -120^0 \text{ V}$$
$$V_{BR} = V\angle - 240^0 \text{ V}$$



(a)

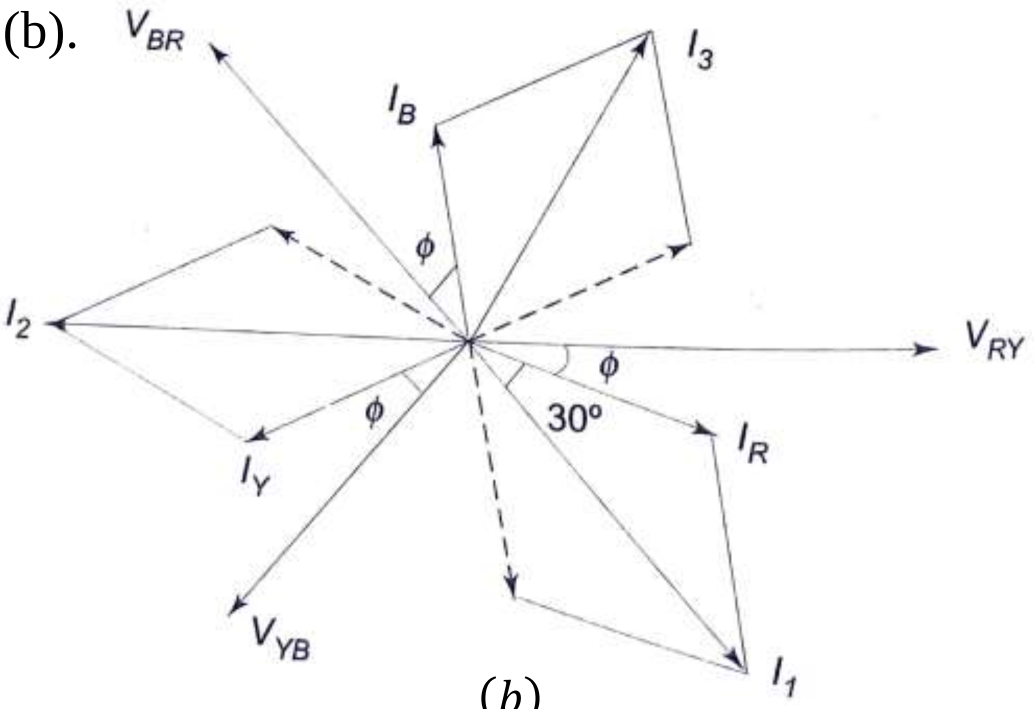
Fig. 7.16

Polyphase Circuits

- **Three-phase Balanced Circuits**

  - **Balanced Three-Phase System — Delta Load**

    - These voltages are represented by phasors in **Fig. 7.16** (b).
    - Since the load is delta-connected, the line voltage of the source is equal to the phase voltage of the load.
    - The current in phase RY, I_R will lag (lead) behind (ahead of) the phase voltage V_{RY} by an angle $\emptyset$ based on the nature of the load impedance.
    - The angle of lag of I_Y with respect to V_{YB}, as well as the angle of lag of I_B with respect to V_{BR} will be $\emptyset$ as the load is balanced.
    - The current flowing in the three load impedances are



(b)

Fig. 7.16

$$I_R = \frac{V_{RY}\angle 0}{Z\angle\emptyset} = \frac{V}{Z}\angle-\emptyset \qquad I_Y = \frac{V_{YB}\angle-120}{Z\angle\emptyset} = \frac{V}{Z}\angle-120-\emptyset \qquad I_B = \frac{V_{BR}\angle-240}{Z\angle\emptyset} = \frac{V}{Z}\angle-240-\emptyset$$

Polyphase Circuits

- **Three-phase Balanced Circuits**
  - **Balanced Three-Phase System — Delta Load**

The line currents are $\sqrt{3}$ times the phase currents, and are 30^0 behind their respective phase currents.

$\therefore$ Current in line 1 is given by

$$I_1 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-\emptyset - 30^0), \qquad \text{or} \qquad I_R - I_B \text{ (phase difference)}$$

Similarly, the current in line 2

$$I_2 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-120 - \emptyset - 30^0), \qquad \text{or} \qquad I_Y - I_R \text{ (phase difference)}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle(-\emptyset - 150^0)$$

The current in line 3

$$I_3 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-240 - \emptyset - 30^0), \qquad \text{or} \qquad I_B - I_Y \text{ (phase difference)}$$

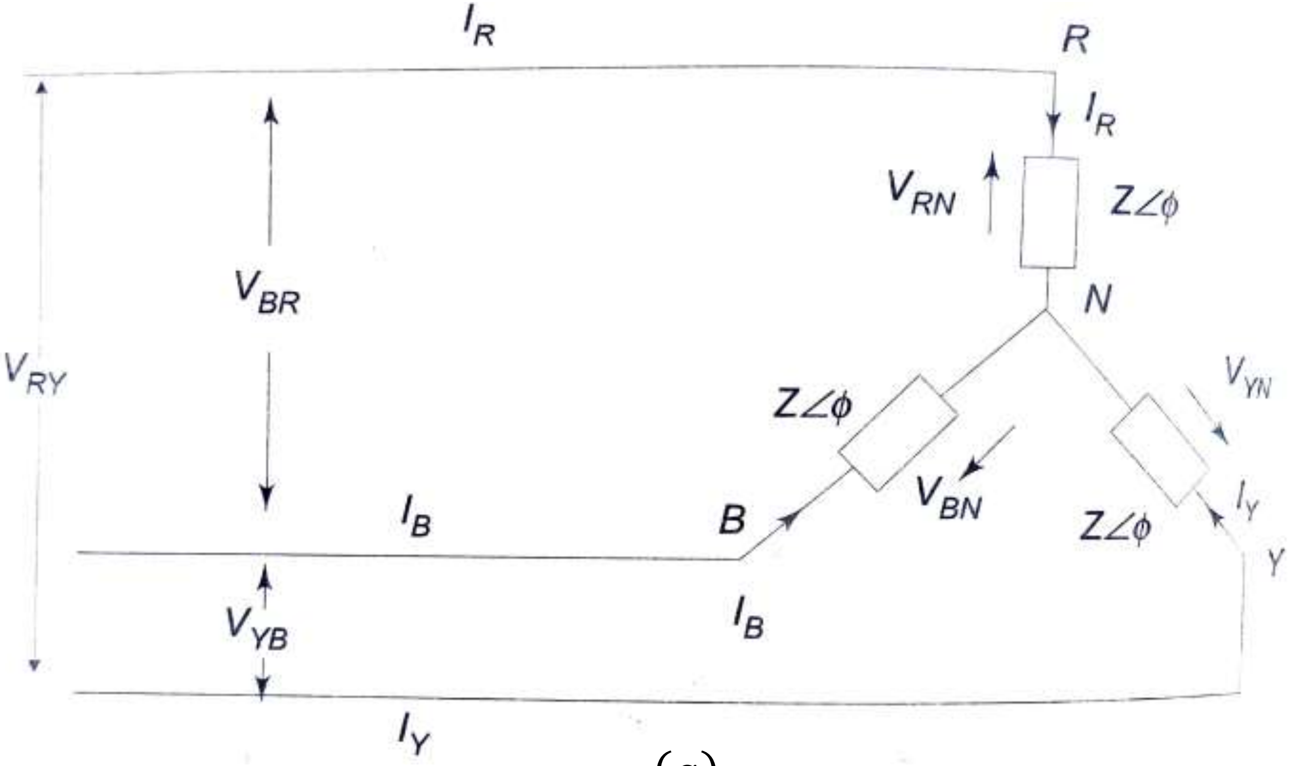
$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle(-\emptyset - 270^0)$$

Polyphase Circuits

- **Three-phase Balanced Circuits**

  - **Balanced Three-Phase System — Star Connected Load**

    - A three-phase, three wire system supply power to a balanced three phase star connected load as shown in **Fig. 7.17** (a). The phase sequence RYB is assumed.
    - In star connection, the current in the phase is also flowing in the line.
    - The three line (phase) currents are I_R, I_Y and I_B.
    - V_{RN}, V_{YN} and V_{BN} represent three phase voltages of the network, i.e. the voltage between any line and neutral.



(a)

Fig. 7.17

Polyphase Circuits

- **Three-phase Balanced Circuits**
  - **Balanced Three-Phase System — Star Connected Load**
    - Let us assume the voltage $V_{RN} = V\angle 0^0$ as the reference phasor. The phase voltages are

$$V_{RN} = V\angle 0^0$$

$$V_{YN} = V\angle -120^0$$

$$V_{BN} = V\angle -240^0$$

    - Hence

$$I_R = \frac{V_{RN}\angle 0}{Z\angle \emptyset} = \frac{V}{Z}\angle -\emptyset$$

$$I_Y = \frac{V_{YN}\angle -120}{Z\angle \emptyset} = \frac{V}{Z}\angle -120-\emptyset$$

$$I_B = \frac{V_{BN}\angle -240}{Z\angle \emptyset} = \frac{V}{Z}\angle -240-\emptyset$$

Polyphase Circuits

- **Three-phase Balanced Circuits**

  - **Balanced Three-Phase System — Star Connected Load**

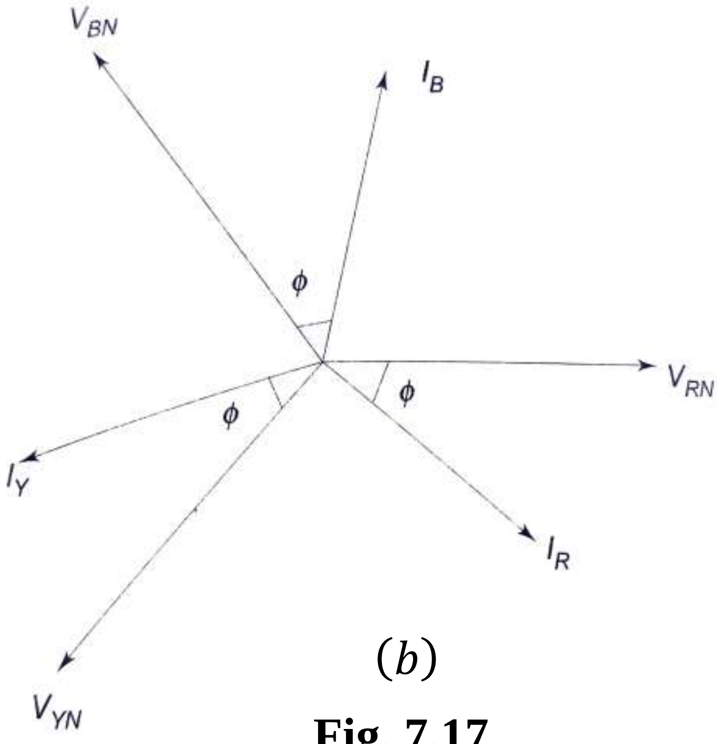
    - The currents, I_R, I_Y and I_B, are equal in magnitude and have a 120^0 phase difference. The disposition of these vectors is shown in **Fig. 7**.17 (b).

    - Sometimes, a 4th wire, called neutral wire is run from the neutral point, if the source is star-connected. This gives three-phase, four-wire star-connected system.

    - If the three line currents are balanced, the current in the fourth wire is zero.

    - The availability of the neutral wire makes it possible to use all the three phase voltages, as well as the three line voltages.

    - Usually, the neutral is grounded for safety and for the design of insulation.



(b)

Fig. 7.17

    - The sources is connected either in star or in delta. The voltage across each phase of the delta connected source is $\sqrt{3}$ times the voltage across each phase of the star-connected source.

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Types of Unbalanced Load**
    - An unbalance exists in a circuit when the impedances in one or more phases differ from the impedances of the other phases.
    - In unbalanced condition, line or phase currents are different and are displaced from one another by unequal angles.
    - However, the source voltages are assumed to be balanced. If the system is a three-wire system, the currents flowing towards the load in the three lines must add to zero at any given instant.
    - If the system is a four-wire system, the sum of the three outgoing line currents is equal to the return current in the neutral wire.
    - In practice, there are following unbalanced loads:
      1. Unbalanced delta-connected load
      2. Unbalanced three-wire star-connected load, and
      3. Unbalanced four-wire star-connected load.

Polyphase Circuits

- **Three-phase Unbalanced Circuits**

  - **Unbalanced Delta-connected Load**

    - An unbalanced delta-load is connected to a balanced three-phase supply as shown in **Fig. 7.18**.
    - The voltage across the load phase is fixed since there is no unbalanced in three-phase supply.
    - It is independent of the nature of the load and so it is equal to the line voltage of the supply.
    - The current in each load phase is equal to the line voltage divided by the impedance of that phase.
    - The line current is the phasor difference of the corresponding phase currents.
    - Taking V_{RY} as the reference phasor and assuming RYB phase sequence.
    - The voltages are as follows:

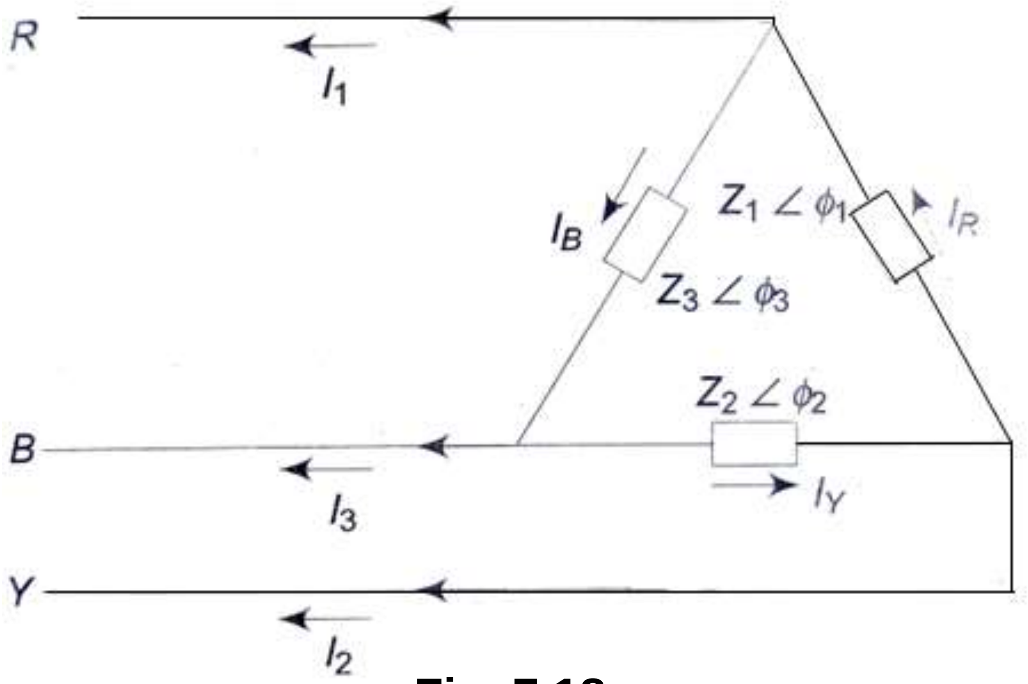


Fig. 7.18

$V_{RY} = V\angle 0^0$ V $\qquad V_{YB} = V\angle -120^0$ V $\qquad V_{BR} = V\angle - 240^0$ V

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Unbalanced Delta-connected Load**
    - Phase currents are

$$I_R = \frac{V_{RY}\angle 0}{Z_1\angle\emptyset_1} = \frac{V\angle 0}{Z_1\angle\emptyset_1} = \left|\frac{V}{Z_1}\right|\angle-\emptyset_1 \quad A$$

$$I_Y = \frac{V_{YB}\angle-120}{Z_2\angle\emptyset_2} = \frac{V\angle-120}{Z_2\angle\emptyset_2} = \left|\frac{V}{Z_2}\right|\angle-120-\emptyset_2 \quad A$$

$$I_B = \frac{V_{BR}\angle-240}{Z_3\angle\emptyset_3} = \frac{V\angle-240}{Z_3\angle\emptyset_3} = \left|\frac{V}{Z_3}\right|\angle-240-\emptyset_3 \quad A$$

    - The three line currents are

$I_1 = I_R - I_B$ phasor difference

$I_2 = I_Y - I_R$ phasor difference

$I_3 = I_B - I_Y$ phasor difference

Polyphase Circuits

- **Three-phase Unbalanced Circuits**

  - **Unbalanced Four Wire Star-Connected Load**

    - An unbalanced star load is connected to a balanced 3-phase, 4-wire supply as shown in **Fig. 7.19**.
    - The star point N_L, of the load is connected to the star point, N_S of the supply.
    - The star points of the supply N_S (generator) and the load N_L are at the same potential.
    - The voltage across each load impedance is equal to the phase voltage of the supply (generator).

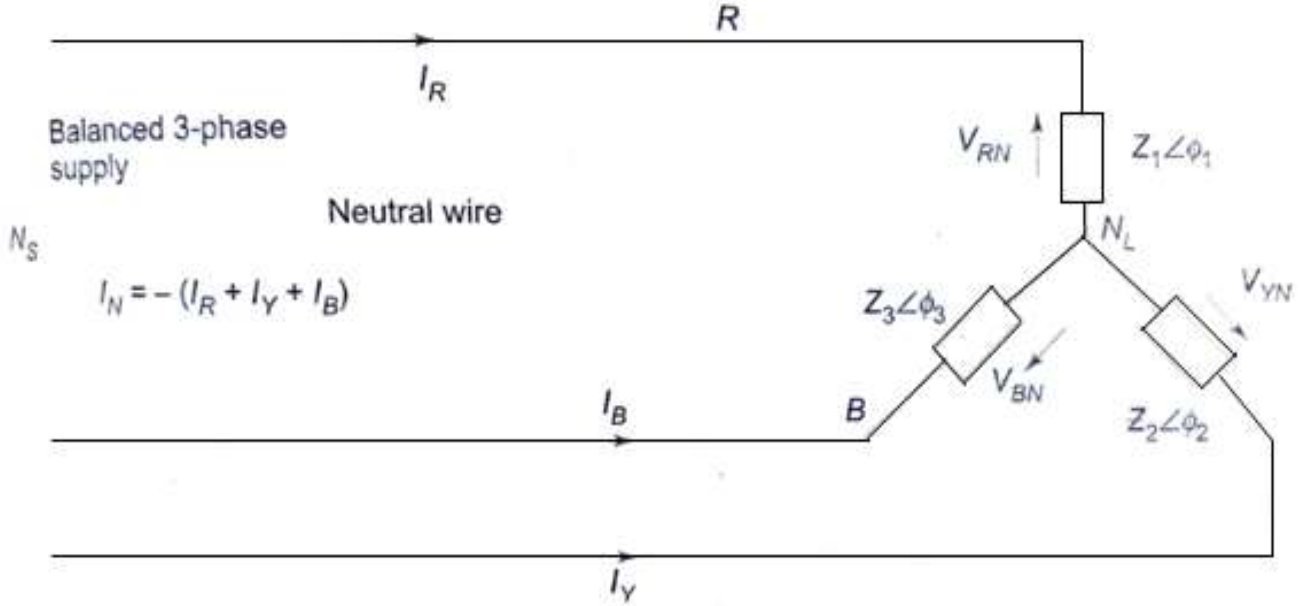


Fig. 7.19.

    - The voltages across three load impedances are equal even though load impedances are unequal.
    - The current in each phase (or line) will be different. The vector sum of the currents in the three lines is not zero, but is equal to neutral current.
    - Phase currents are calculated in the similar way of an unbalanced delta-connected load.

Polyphase Circuits

Three-phase Unbalanced Circuits

- Taking the phase voltage $V_{RN} = V\angle 0^0$ V as reference and assuming RYB phase sequences, three phase voltages are as follows:

  $V_{RN} = V\angle 0^0$ V, $V_{YN} = V\angle -120^0$ V, $V_{BN} = V\angle -240^0$ V

- The phase currents are

$$I_R = \frac{V_{RN}}{Z_1} = \frac{V\angle 0^0}{Z_1\angle\emptyset_1} = \left|\frac{V}{Z_1}\right|\angle -\emptyset_1 \quad A$$

$$I_Y = \frac{V_{YN}}{Z_2} = \frac{V\angle -120}{Z_2\angle\emptyset_2} = \left|\frac{V}{Z_2}\right|\angle -120-\emptyset_2 \quad A$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{V\angle -240}{Z_3\angle\emptyset_3} = \left|\frac{V}{Z_3}\right|\angle -240-\emptyset_3 \quad A$$

- Incidently, I_R, I_Y and I_B are also the line currents; the current in the neutral wire is the vector sum of the three-line currents.

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - ✓ **Unbalanced Three Wire Star-Connected Load**
    - In a three-phase, four-wire system if the connection between supply neutral and load neutral is broken, it would result in an unbalanced three-wire star-load.
    - This type of load is rarely found in practice, because all the three wire star loads are balanced. Such a system is shown in **Fig. 7.20**.
    - The supply star point (N_S) is isolated from the load star point (N_L).
    - The potential of the load star point is different from that of the supply star point.
    - The load phase voltages is not equal to the supply phase voltage.
    - They are not only unequal in magnitude, but also subtend angles other than 120^0 with one another.

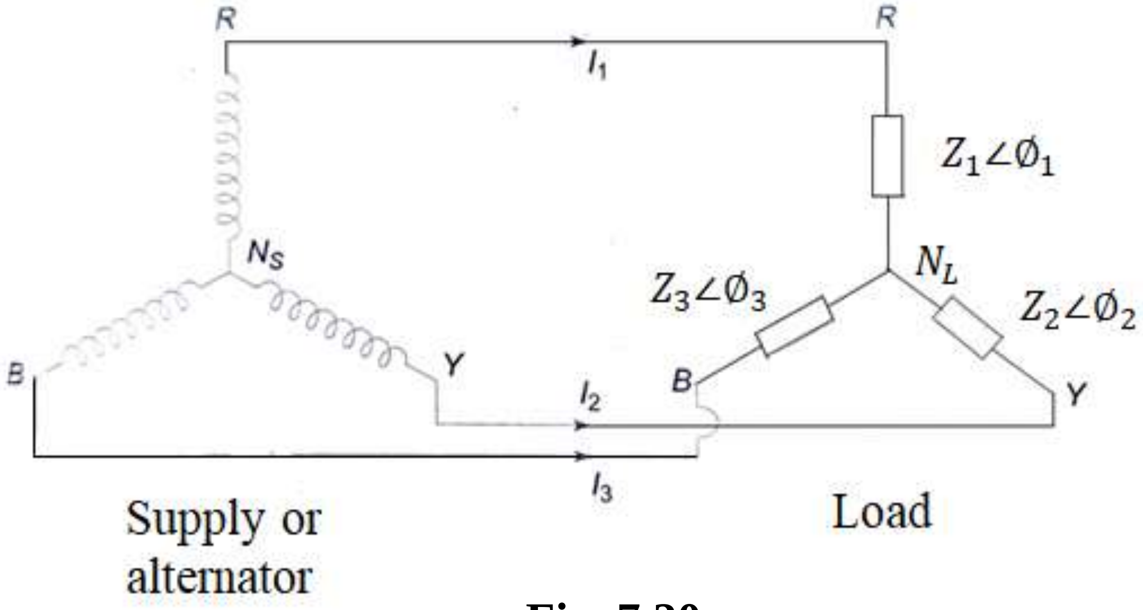


Fig. 7.20.

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Unbalanced Three Wire Star-Connected Load**
    - The magnitude of each phase voltage depends upon the individual phase loads.
    - The potential of the load neutral point changes according to changes in the impedances of the phases, that is why sometimes the load neutral is also called a floating neutral point.
    - All star-connected, unbalanced loads supplied from polyphase systems without a neutral wire have floating neutral point.
    - The phasor sum of the three unbalanced line currents is zero.
    - The phase voltage of the load is not $1/\sqrt{3}$ of the line voltage.
    - Load phase voltages cannot be determined directly from the given supply line voltages.
    - Two methods are generally used to solve unbalanced Y-connected loads. They are
      1. Star-delta conversion method, and
      2. The application of Millman’s theorem

Polyphase Circuits

- **Unbalanced Three Wire Star-Connected Load**
  - **Millman's Method of Solving Unbalanced Load**
    - The star-delta conversion method is laborious and involves lengthy calculations. This type of problems are solved by using Millman's theorem in a much easier way.
    - Consider an unbalanced wye (Y) load connected to a balanced three-phase supply in **Fig. 7.21**.
    - $V_{RO,}$ V_{YO} and V_{BO} are the phase voltages of the supply.
    - They are equal in magnitude, but displaced by 120^0 from one another.
    - $V_{RO'}$, $V_{YO'}$ and $V_{BO'}$ are the load phase voltages and they are unequal in magnitude and differ in phase by unequal angles.

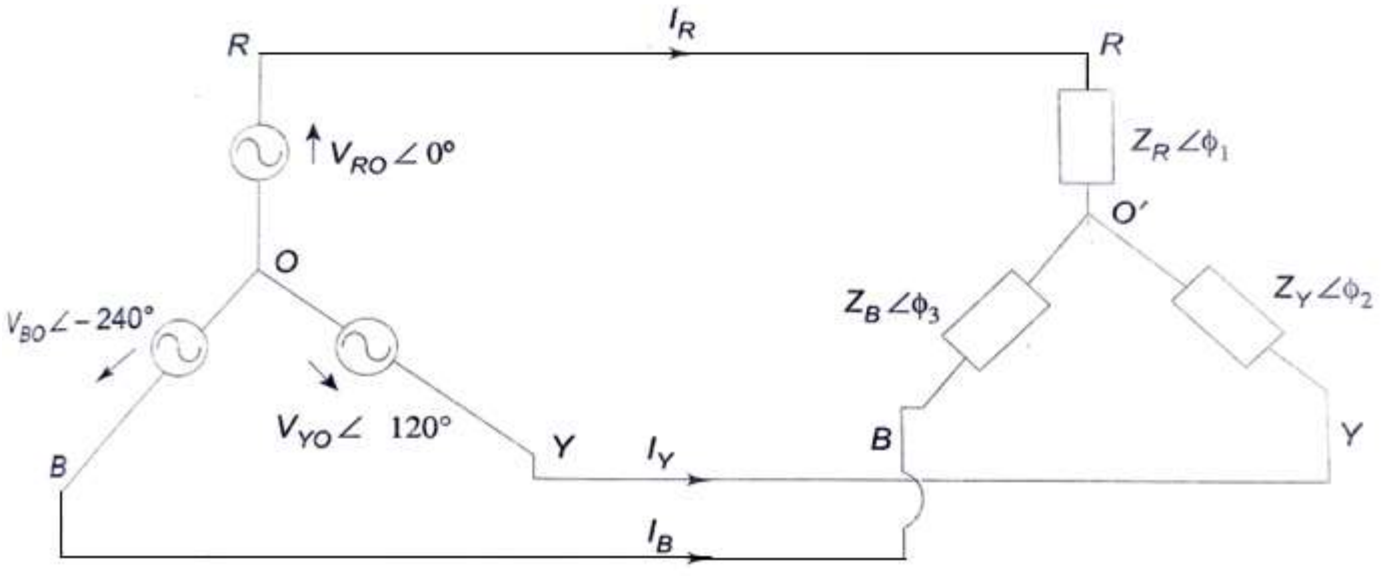


Fig. 7.21

- Z_R, Z_Y and Z_B are the impedances of the branches of the unbalanced wye (Y) connected load.

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Unbalanced Three Wire Star-Connected Load**
    - The triangular phasor diagram of the complete system is shown in **Fig. 7.22**
    - Distances RY, YB and BR represent the line voltages of the supply as well as load. They are equal in magnitude, but displaced by 120^0.
    - Here O is the star-point of the supply and is located at the centre of the equilateral triangle RYB.
    - O' is the load star point. The star point of the supply which is at the zero potential is different from that of the star point at the load, due to the load being unbalanced.
    - O' has some potential with respect to O and is shifted away from the centre of the triangle.
    - Distance $O'O$ represents the voltage of the load star point with respect to the star point of the supply $V_{O'O.}$

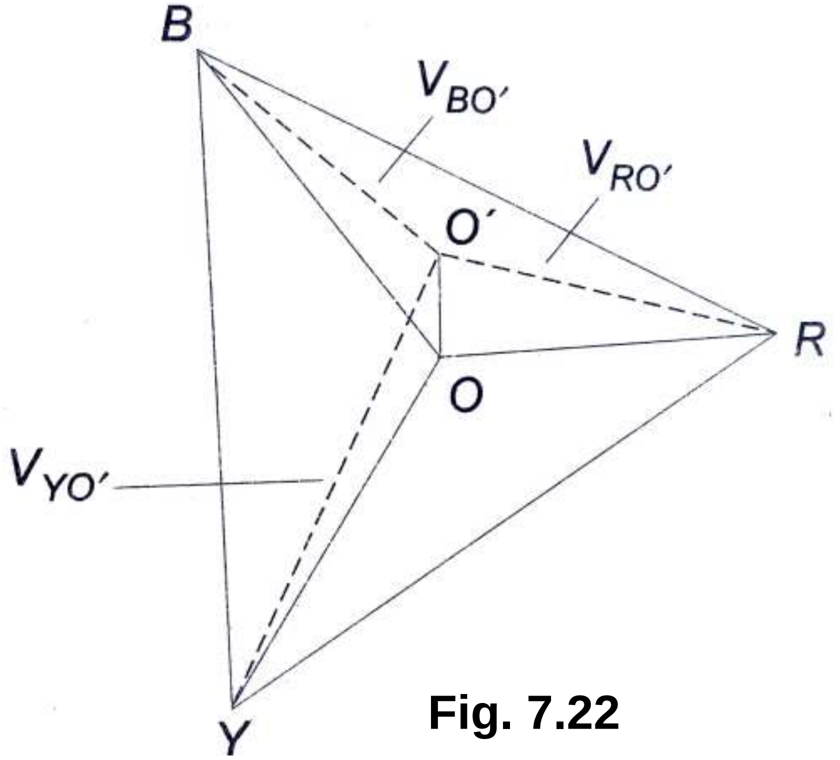


Fig. 7.22

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Unbalanced Three Wire Star-Connected Load**
    - According to Millman's theorem, $V_{O'O}$ is given by

$$V_{O'O} = \frac{V_{RO}Y_R + V_{YO}Y_Y + V_{BO}Y_B}{Y_R + Y_Y + Y_B}$$

where the parameters Y_R, Y_Y and Y_B are the admittances of the branches of the unbalanced wye connected load.

- From **Fig. 7.21**, we get

$$V_{RO} = V_{RO'} + V_{O'O}$$

or the load phase voltage

$$V_{RO'} = V_{RO} - V_{O'O}$$

Similarly,

$$V_{YO'} = V_{YO} - V_{O'O} \text{ and } V_{BO'} = V_{BO} - V_{O'O}$$

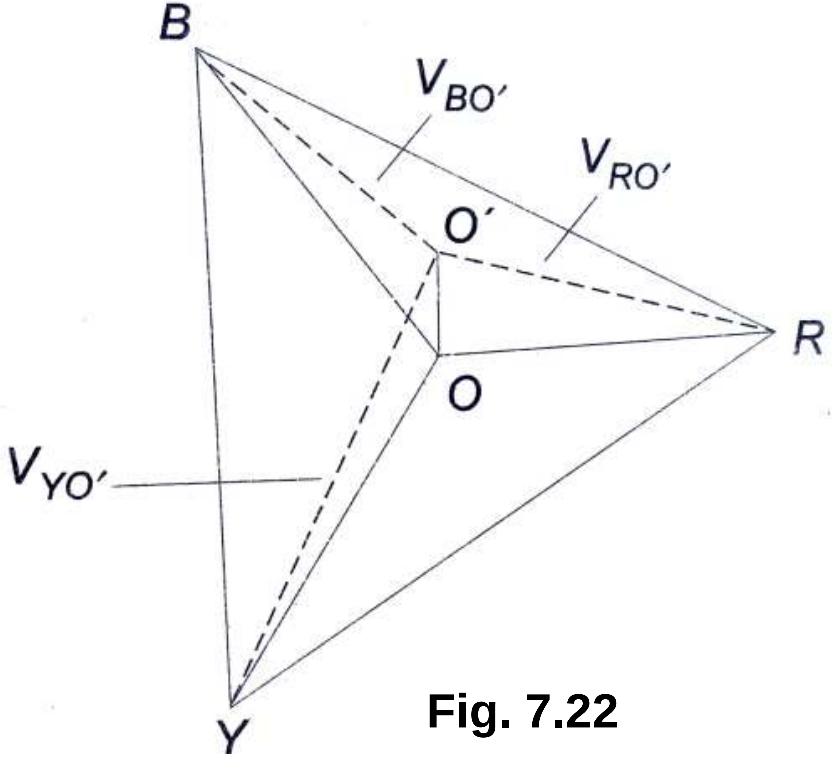


Fig. 7.22

Polyphase Circuits

- **Three-phase Unbalanced Circuits**
  - **Unbalanced Three Wire Star-Connected Load**
    - The line currents in the load are

$$I_R = \frac{V_{RO'}}{Z_R} = (V_{RO} - V_{O'O})Y_R$$

$$I_Y = \frac{V_{YO'}}{Z_Y} = (V_{YO} - V_{O'O})Y_Y$$

$$I_B = \frac{V_{BO'}}{Z_B} = (V_{BO} - V_{O'O})Y_B$$

Polyphase Circuits

Example – P 7.4

An unbalanced four-wire, star-connected load has a balanced voltage of 400 V, the loads are

$$Z_1 = (4 + j8)\ \Omega;\ Z_2 = (3 + j4)\ \Omega;\ Z_3 = (15 + j20)\ \Omega;$$

Calculate the (i) line currents (ii) current in the neutral wire and (iii) the total power.

Polyphase Circuits

Solution of Example – P 7.4

$$Z_1 = (4 + j8)\ \Omega = 8.94\angle 63.40^0\ \Omega$$
$$Z_2 = (3 + j4)\ \Omega = 5\angle 53.1^0\ \Omega$$
$$Z_3 = (15 + j20)\ \Omega = 25\angle 53.13^0\ \Omega$$

Let us assume RYB phase sequence,

The phase voltage $V_{RN} = 400/\sqrt{3} = 230.94\, V$

Taking V_{RN} as the reference phasor, we have

$$V_{RN} = 230.94\angle 0^0\ V,$$
$$V_{YN} = 230.94\angle -120^0\ V$$
$$V_{BN} = 230.94\angle -240^0\ V$$

Solution of Example – P 7.4

The three line currents are

(i)
$$I_R = \frac{V_{RN}}{Z_1} = \frac{230.94\angle 0^0}{8.94\angle 63.40^0}\ \text{A} = 25.83\angle -63.40^0\ \text{A}$$

$$I_Y = \frac{V_{YN}}{Z_2} = \frac{230.94\angle -120^0}{5\angle 53.1^0}\ \text{A} = 46.188\angle -173.1^0\ \text{A}$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{230.94\angle -240^0}{25\angle 53.13^0}\ \text{A} = 9.23\angle -293.13^0\ \text{A}$$

Solution of Example – P 7.4

(ii) To find the neutral current, the three line currents are to be added. The neutral current is equal and opposite to this sum.

Thus, $$I_N = -(I_R + I_Y + I_B)$$

$$= -(25.83\angle -63.40^0 + 46.188\angle -173.1^0 + 9.23\angle -293.13^0) \text{ A}$$

$$= -[(11.56 - j23.09) + (-45.85 - j5.54) + (3.62 + j8.48)] \text{ A}$$

$$= -[(-30.67 - j20.15)] \text{ A}$$

$$= (30.67 + j20.15) \text{ A}$$

$$= 36.69\angle 33.30^0 \text{ A}$$

(iii) Power in R phase $= I_R^2 \times R_R = (25.83)^2 \times 4 = 2668.75$ W

Power in Y phase $= I_Y^2 \times R_Y = (46.18)^2 \times 3 = 6397.77$ W

Power in B phase $= I_B^2 \times R_B = (9.23)^2 \times 15 = 1277.89$ W

$\therefore$ Total power absorbed by the load $= 2668.75 + 6397.77 + 1277.89 = 10344.41$ W

LECTURE 18

Polyphase Circuits

- **Power Measurement in Three-Phase Circuits**
  - ✓ Wattmeters are generally used to measure power in the circuits. The diagram of a wattmeter used to measure power in a single phase circuit is shown in **Fig. 7.23**.
  - ✓ A wattmeter consists of two coils, one coil is called the current coil and the other is called the pressure or voltage coil.
  - ✓ The coil with less number of turns between M and L is the current coil, which carries the current in the load and has very low impedance.
  - ✓ The coil with more number of turns between the common terminal (comm) and V is the pressure coil, which is connected across the load and has high impedance.

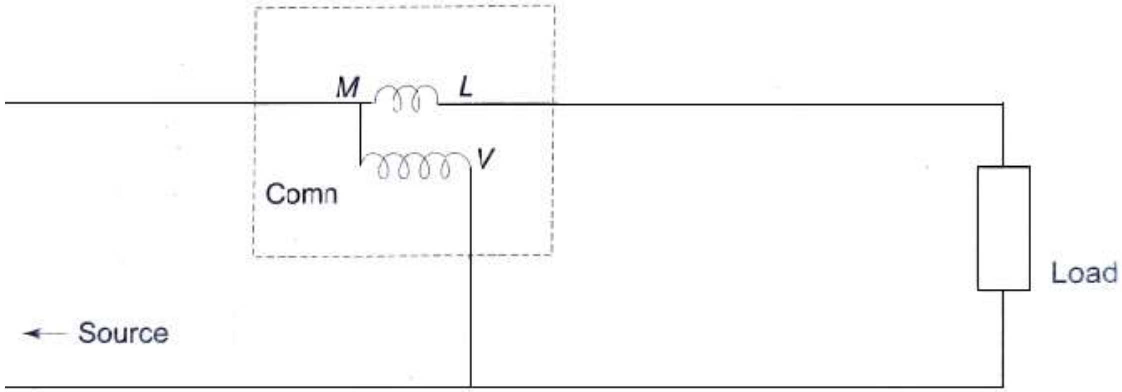


Fig. 7.23

  - ✓ The load voltage is impressed across the pressure coil.
  - ✓ The terminal M denotes the main side, L denotes load side, $common$ denotes the common point of current coil and pressure coil, and V denotes the second terminal of the pressure coil, usually selected as per the range of the load voltage in the circuit.

Polyphase Circuits

Power Measurement in Three-Phase Circuits

- A wattmeter has four terminals, two for current coil and two for potential coil.
- When the current flow through the two coils, they set up magnetic fields in space. An electromagnetic torque is produced by the interaction of the two magnetic fields.
- Under the influence of the torque, one of the coils (which is movable) moves on a calibrated scale against the action of a spring.
- The instantaneous torque produced by electromagnetic action is proportional to the product of the instantaneous values of the currents in the two coils.
- The small current in the pressure coil is equal to the input voltage divided by the impedance of the pressure coil.
- The inertia of the moving system does not permit it to follow the instantaneous fluctuations in torque.
- The wattmeter deflection is therefore, proportional to the average power ($VI\cos\emptyset$) delivered to the circuit.

Polyphase Circuits

➢ **Power Measurement in Three-Phase Circuits**

- Sometimes, a wattmeter connected in the circuit to measure power gives downscale reading or backward deflection. This is due to improper connection of the current coil and pressure coil.

- To obtain up scale reading, the terminal marked as ‘Comm’ of the pressure coil is connected to opposite terminal of the current coil as shown in **Fig. 7.24.**

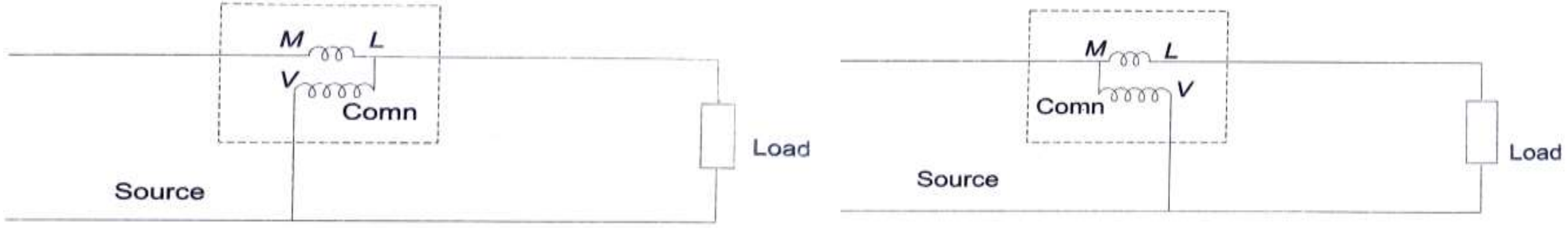


Fig. 7.24

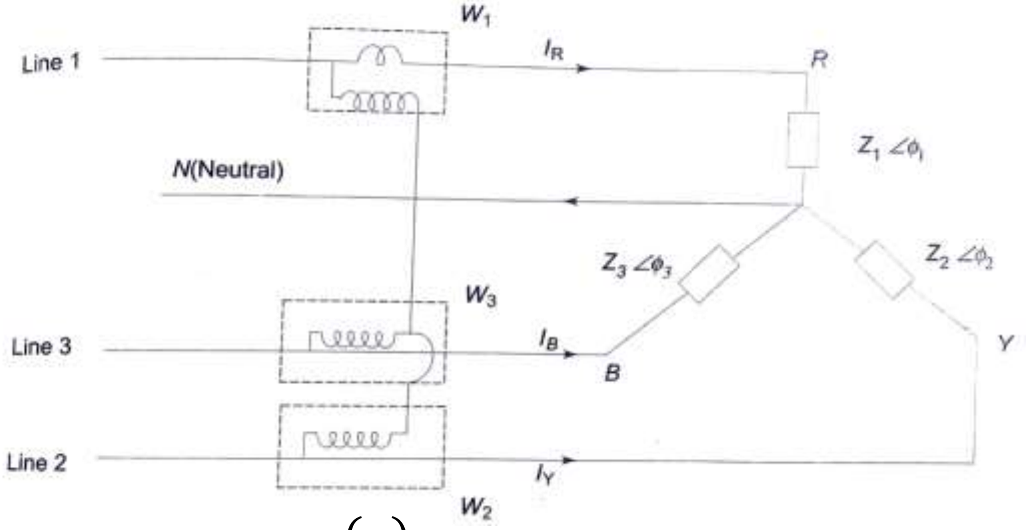
- The connection between the current coil terminal and pressure coil terminal is not inherent, but is to be made externally.

- Even with proper connections, sometimes the wattmeter give downscale reading whenever the phase angle between the voltage across the pressure coil and the current through the current coil is more than 90^0.

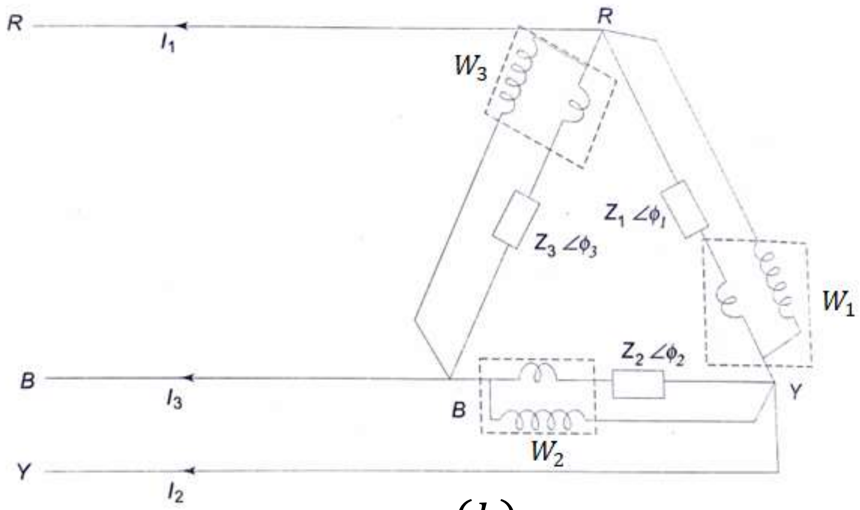
- In such a case, connection of either the current coil or the pressure coil must be reversed.

Polyphase Circuits

- **Power in Three-Phase Circuits**
  - ✓ Three wattmeters, one in each phase are used to measure the power consumed in a three-phase system.
  - ✓ Total power consumption in either balanced or unbalanced three-phase loads is the algebraic sum of the readings of the three wattmeters.
  - ✓ In a balanced three-phase system, it is necessary to measure power only in one phase and the reading is multiplied by three to get the total power in the three phases.
  - ✓ The neutral must be available for connecting the pressure coil terminals for a star-connected load and the current coils must be inserted in each phase for a delta-connected load as shown in **Fig. 7.25** (a) and (b).



(a)



(b)

Fig. 7.25

Polyphase Circuits

- **Power in Three-Phase Circuits**
  - ✓ Such connections sometimes may not be practicable because the neutral terminal is not available all the time in a star-connected load and the phases of the delta-connected load are not accessible for connecting the current coils of the wattmeter.
  - ✓ In most of the commercially available practical three-phase loads, only three line terminals are available.
  - ✓ So, it is required to measure power in the three-phases with an access to the three lines connecting the source to the load.
  - ✓ Three wattmeter and two wattmeter methods are used to measure three-phase power.

Polyphase Circuits

- **Three Wattmeter Method**
  - ✓ The connection of three wattmeters is shown in **Fig. 7.26**.
  - ✓ The three wattmeters are connected in the three lines.
  - ✓ The current coils of the three wattmeters are connected in the three lines.
  - ✓ One terminal of each potential coil is connected to one terminal of the current coil.
  - ✓ The other three terminals of the potential coils are connected to some common point which forms an neutral n.

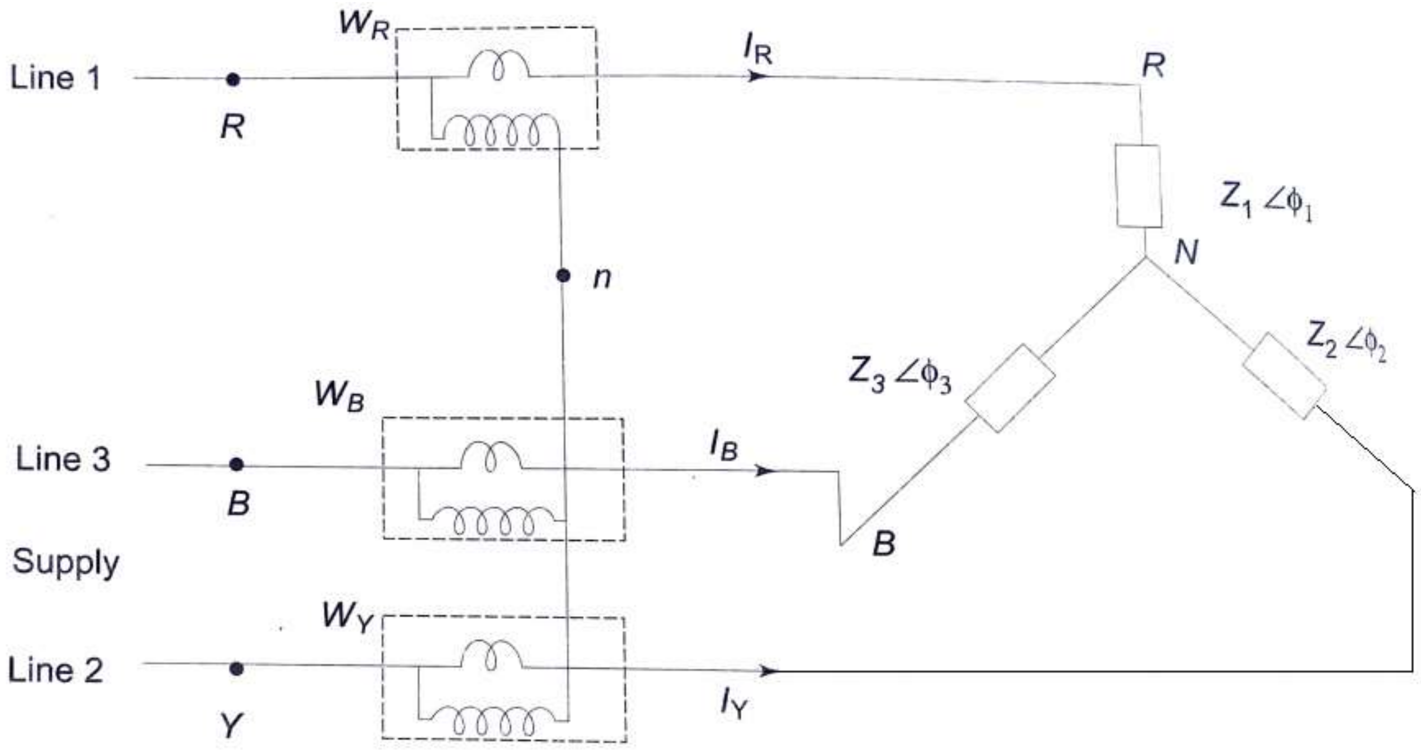


Fig. 7.26

Polyphase Circuits

- **Three Wattmeter Method**

  - A star-connected load is considered and the neutral of this load is denoted by N.
  - The reading on the wattmeter W_R correspond to the average value of the product of the instantaneous value of the current I_R flowing in line 1, with the voltage drop V_{Rn}, where V_{Rn} is the voltage between points R and n.

$$V_{Rn} = V_{RN} + V_{Nn}$$

where V_{RN} is the load phase voltage and V_{Nn} is the voltage between load terminal, N, and the common point, n.

  - Similarly, $V_{Yn} = V_{YN} + V_{Nn,}$ and $V_{Bn} = V_{BN} + V_{Nn}$.
  - Therefore, the average power, W_R indicated by the wattmeter is given by

$$W_R = \frac{1}{T}\int_0^T V_{Rn} I_R dt$$

where T is the time period of the voltage wave

$$W_R = \frac{1}{T}\int_0^T (V_{RN} + V_{Nn}) I_R dt$$

Polyphase Circuits

➢ **Three Wattmeter Method**

Similarly, $W_Y = \dfrac{1}{T}\displaystyle\int_0^T V_{Yn} I_Y dt$

$$= \frac{1}{T}\int_0^T (V_{YN} + V_{Nn})\, I_Y\, dt$$

and $W_B = \dfrac{1}{T}\displaystyle\int_0^T V_{Bn} I_B\, dt$

$$= \frac{1}{T}\int_0^T (V_{BN} + V_{Nn}) I_B dt$$

Total average power = $W_R + W_Y + W_B$

$$= \frac{1}{T}\int_0^T (V_{RN} I_R + V_{YN} I_Y + V_{BN} I_B) I_B dt + \frac{1}{T}\int_0^T V_{Nn}(I_R + I_Y + I_B)\, dt$$

$$= \frac{1}{T}\int_0^T (V_{RN} I_R + V_{YN} I_Y + V_{BN} I_B) dt$$ [Since, $I_R + I_Y + I_B = 0$ at any given instant]

Polyphase Circuits

- **Three Wattmeter Method**

  - ✓ If the system has a fourth wire, i.e. if the neutral wire is available, then the common point, n is to be connected to the system neutral, N. In that case, V_{Nn} would be zero, and the equation for power is still valid.
  - ✓ The algebraic sum of the three currents I_R, I_Y and I_B is zero, whatever be the value of V_{Nn}. Hence, the sum $V_{Nn}(I_R + I_Y + I_B)$ would be zero.

- **Two Wattmeter Method**

  - ✓ Now, the common point, n, in **Fig. 7.26** is connected to line B. So, $V_{Nn} = V_{NB}$.
  - ✓ The voltage across the potential coil of wattmeter W_B is zero. So, this wattmeter read zero. Hence, this can be removed from the circuit.
  - ✓ So, the total power read by the remaining two wattmeters, W_R and W_Y.

$$\therefore \qquad \text{Total power} = W_R + W_Y$$

➢ Two Wattmeter Method

- ✓ The connection of two wattmeters is shown in **Fig. 7.27**.
- ✓ The average power indicated by wattmeter W_R and W_Y are

$$W_R = \frac{1}{T}\int_0^T V_{RB}\,.\,I_R\,dt$$

and
$$W_Y = \frac{1}{T}\int_0^T V_{YB}\,.\,I_Y\,dt$$

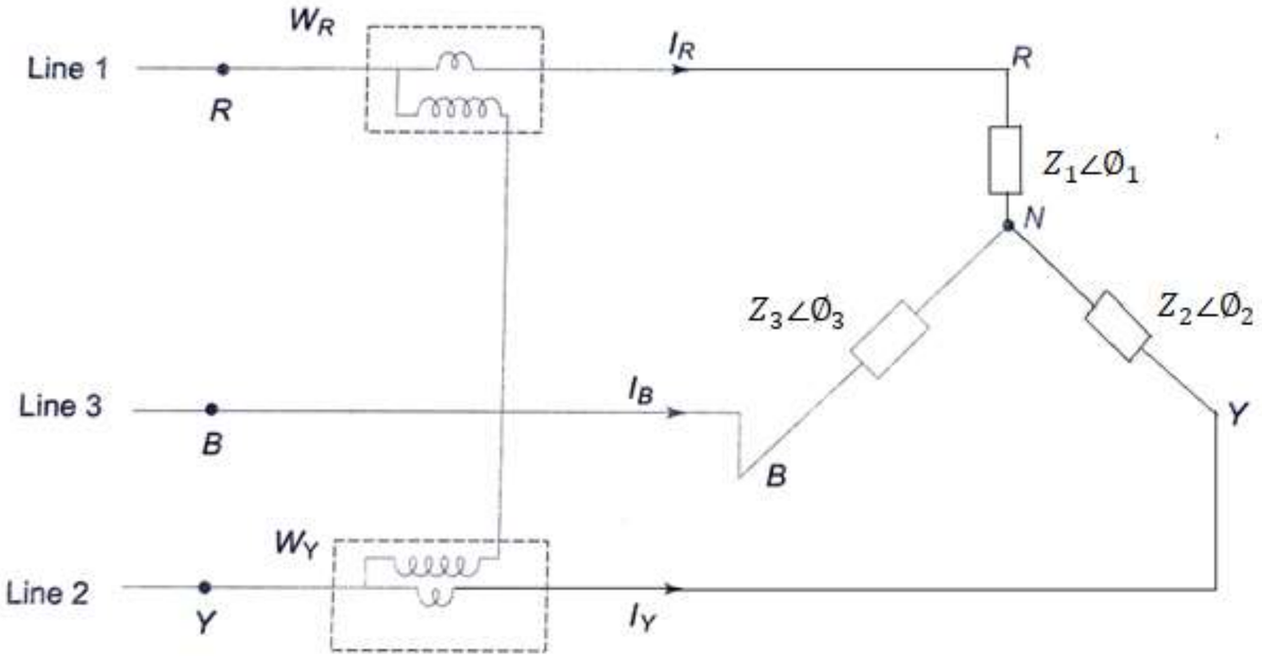


Fig. 7.27

Also
$$V_{RB} = V_{RN} + V_{NB}$$
$$V_{YB} = V_{YN} + V_{NB}$$

$$\therefore \quad W_R + W_Y = \frac{1}{T}\int_0^T (V_{RB}\,.\,I_R + V_{YB}\,.\,I_Y)dt = \frac{1}{T}\int_0^T \{(V_{RN} + V_{NB})\ I_R + (V_{YN} + V_{NB})\ I_Y\}\,dt$$

$$= \frac{1}{T}\int_0^T \{(V_{RN}\ I_R + V_{YN}\ I_Y) + (I_R + I_Y)V_{NB}\}\,dt$$

Polyphase Circuits

- **Two Wattmeter Method**

  - We know that $$I_R + I_Y + I_B = 0$$

$$\therefore\ I_R + I_Y = -I_B$$

  - Substituting this value, we get

$$W_R + W_Y = \frac{1}{T}\int_0^T \{(V_{RN}\ I_R + V_{YN}\ I_Y) + (-I_B)V_{NB}\}\,dt$$

$$= \frac{1}{T}\int_0^T (V_{RN}.I_R + V_{YN}.\ I_Y + V_{BN}.I_B)\,dt$$

$$[\because\ \ V_{NB} = -V_{BN}]$$

  - Which indicates the total power in the load.

Polyphase Circuits

- **Two Wattmeter Method**

  - ✓ The power in a three-phase load, whether balanced or unbalanced, star-connected or delta-connected, three-wire or four wire, can be measured with only two wattmeters as shown in **Fig. 7.27**.
  - ✓ The two wattmeter method of measuring power in three-phase loads is a universal method.
  - ✓ The current flowing through the current coil of each wattmeter is the line current, and the voltage across the pressure coil is the line voltage.
  - ✓ In case the phase angle between line voltage and current is greater than 90^0, the corresponding wattmeter would indicate downscale reading.
  - ✓ To obtain upscale reading, the connections of either the current coil, or the pressure coil has to be interchanged.
  - ✓ Reading obtained after reversal of coil connection should be taken as negative.
  - ✓ Then, the algebraic sum of the two wattmeter readings gives the total power.

Polyphase Circuits

- **Power Factor by Two Wattmeter Method**
  - ✓ The power factor in a balanced load is the power factor of any phase.
  - ✓ A separate power factor for every phase is to be determined in three-phase unbalanced circuits.
  - ✓ The two wattmeter method can be used to calculate the power factor of the load.
  - ✓ The vector diagram of the circuit in **Fig. 7.27** is shown in **Fig. 7.28**.
  - ✓ Since the load is assumed to be balanced for the star-connected load,

$$Z_1\angle\emptyset_1 = Z_2\angle\emptyset_2 = Z_3\angle\emptyset_3 = Z\angle\emptyset$$

  - ✓ RYB phase sequence is assumed.
  - ✓ The three rms load phase voltages are V_{RN}, V_{YN} and V_{BN} respectively.
  - ✓ I_R, I_Y and I_B are the rms line (phase) currents.
  - ✓ The currents lag behind their respective phase voltages by an angle $\emptyset$ as the inductive load is considered.

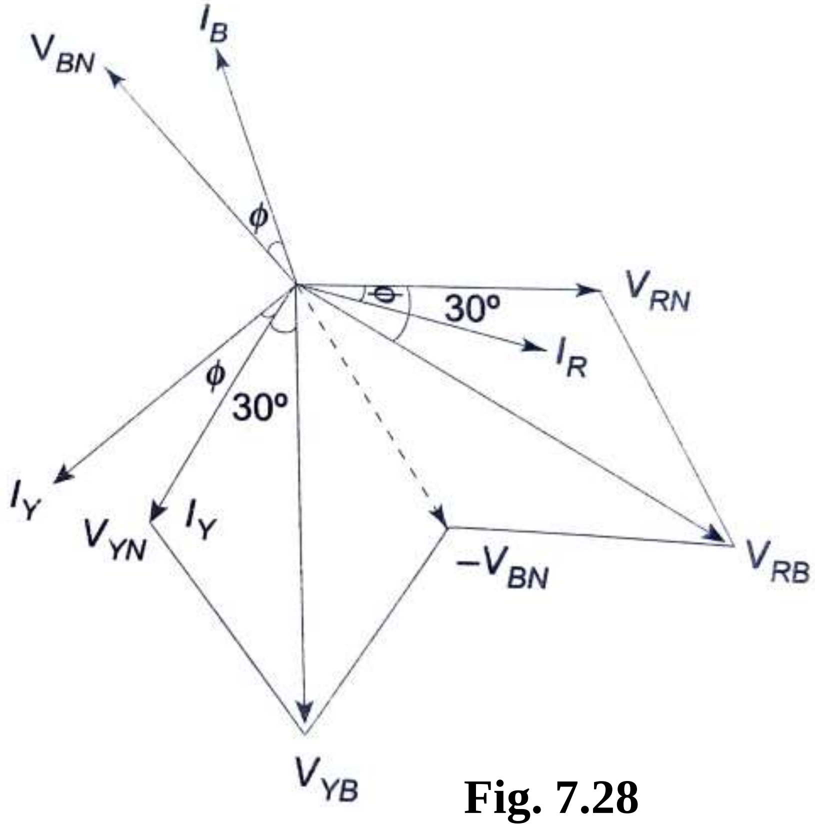


Fig. 7.28

Polyphase Circuits

- **Power Factor by Two Wattmeter Method**

  - ✓ It is clear from the **Fig. 7.27** and **Fig. 7.28** that
  - ✓ W_R measures the product of effective value of the current through its current coil I_R, effective value of the voltage across its pressure coil V_{RB} and the cosine of the angle between the phasors I_R and V_{RB}.
  - ✓ The voltage across the pressure coil of W_R is $V_{RB} = V_{RN} - V_{BN}$ phasor difference
  - ✓ The phase angle between V_{RB} and I_R is $(30^0 - \emptyset)$

$$\therefore \qquad W_R = V_{RB}.I_R \cos(30 - \emptyset)$$

  - ✓ Similarly, W_Y measures the product of effective value of the current through its current coil I_Y, the effective value of the voltage across its pressure coil, V_{YB} and the cosine of the angle between the phasors V_{YB} and I_Y.
  - ✓ The voltage across the pressure coil of W_Y is $V_{YB} = V_{YN} - V_{BN}$ phasor difference
  - ✓ The phase angle between V_{YB} and I_Y is $(30^0 + \emptyset)$.

$$\therefore \qquad W_Y = V_{YB}.I_Y \cos(30^0 + \emptyset)$$

Polyphase Circuits

➢ Power Factor by Two Wattmeter Method

- ✓ Since the load is balanced, the line voltage $V_{RB} = V_{YB} = V_L$ and the line current $I_R = I_Y = I_{L.}$

$$W_R = V_L \,.\, I_L \,\cos(30^0 - \emptyset)$$
$$W_Y = V_L \,.\, I_L \,\cos(30^0 + \emptyset)$$

- ✓ Total power in the circuit is given by adding W_R and W_Y .

$$W_R + W_Y = \sqrt{3} V_L \, I_L \, \cos \emptyset$$

- ✓ It is clear from the two wattmeter readings that the wattmeter W_R registers more power for the same load angle $\emptyset$ when the load is inductive. Therefore, W_R is higher reading wattmeter in the circuit of **Fig. 7.28**.
- ✓ It is also clear that the wattmeter W_R is connected in the leading phase as the phase sequence is RYB.

$$W_R = V_L \, I_L \, \cos(30^0 - \emptyset) \qquad \text{(Higher reading)}$$
$$W_Y = V_L \, I_L \, \cos(30^0 + \emptyset) \qquad \text{(Loewer reading)}$$

- ✓ In other words, the wattmeter connected in the leading phase reads less for the same load angle if the load is capacitive.
- ✓ So, the phase sequence of the system is easily identified if the nature of the load is known.

Polyphase Circuits

➢ Power Factor by Two Wattmeter Method

- ✓ The higher reading wattmeter always reads positive. The power factor of the load is obtained by using two wattmeter readings.

$$W_R + W_Y = \sqrt{3} V_L \, I_L \, \cos\emptyset$$

$$W_R - W_Y = V_L \, I_L \, \sin\emptyset$$

- ✓ Taking the ratio of the two, we get

$$\frac{W_R - W_Y}{W_R + W_Y} = \frac{\tan\emptyset}{\sqrt{3}}$$

or, $$\tan\emptyset = \sqrt{3}\left[\frac{W_R - W_Y}{W_R + W_Y}\right]$$

$$\therefore \quad \emptyset = \tan^{-1}\sqrt{3}\left[\frac{W_R - W_Y}{W_R + W_Y}\right]$$

- ✓ Therefore, the power factor $\cos\emptyset$ can be found from the value of $\emptyset$.

Polyphase Circuits

- **Variation in Wattmeter Readings with Load Power Factor**

  - ✓ The readings of the two wattmeters depend on the load power factor angle $\emptyset$, such that

$$W_R = V_L\, I_L\, \cos(30^0 - \emptyset)$$

$$W_Y = V_L\, I_L\, \cos(30^0 + \emptyset)$$

It is concluded from the above equations that

1. When $\emptyset$ is zero, i.e. power factor is unity, two wattmeters indicate equal and positive values.
2. When $\emptyset$ rises from 0 to 60^0, i.e. upto power factor 0.5, wattmeter W_R reads positive as it is connected in the leading phase, whereas wattmeter W_Y reads positive, but less than W_R.

   when $\emptyset = 60^0$, $W_Y = 0$ and the total power is being measured only by wattmeter W_R.
3. If the power factor is further reduced from 0.5, i.e. when $\emptyset$ is greater than 60^0, W_R indicates positive value, whereas W_Y reads down scale reading

Polyphase Circuits

➢ **Variation in Wattmeter Readings with Load Power Factor**

3. Wattmeter W_Y reads downscale for the phase angle between 60^0 and 90^0.

   The connections of either the current coil, or the pressure coil of the wattmeter W_Y is to be interchanged to obtain an upscale reading, and the reading thus obtained must be given a negative sign.

   Thus, total power in the circuit would be $W_R + (-W_Y) = W_R - W_{Y.}$

   When the power factor is zero (i.e. $\emptyset = 90^0$), the two wattmeters read equal and opposite values.

   i.e. $$W_R = V_L\, I_L\, \cos(30^0 - 90^0) = 0.5 V_L\, I_L$$

   $$W_Y = V_L\, I_L\, \cos(30^0 + 90^0) = -0.5\, V_L\, I_L$$

➢ Leading Power Factor Load

- ✓ Now, the load in **Fig. 7**.27 is considered capacitive. So, the wattmeter connected in the leading phase would read less value.
- ✓ Thus, W_R is the lower reading wattmeter, and W_Y is the higher reading wattmeter.
- ✓ The phasor diagram for the leading power factor is shown in **Fig. 7.29**.
- ✓ The phase currents, I_R, I_Y and I_B are leading their respective phase voltage by an angle $\emptyset$.
- ✓ The reading of the wattmeter connected in the leading phase is given by

$$W_R \text{ (lower reading wattmeter)} = V_{RB}\,.\,I_R\,\cos(30+\emptyset)^0$$

$$= V_L I_L \cos(30+\emptyset)^0$$

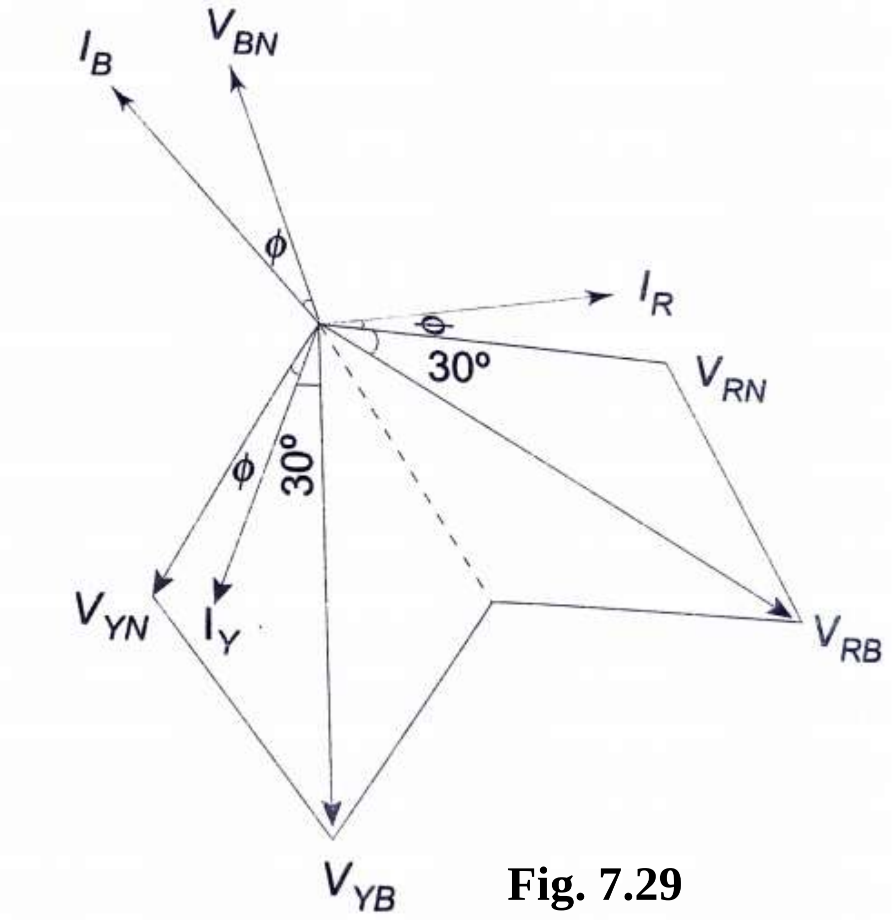


Fig. 7.29

Polyphase Circuits

- **Leading Power Factor Load**

  - Similarly, the reading of the wattmeter connected in the lagging phase is given by

$$W_Y \text{ (higher reading wattmeter)} = V_{YB}\,.\,I_Y\ \cos(30-\emptyset)^0$$
$$= V_L I_L \cos(30-\emptyset)^0$$

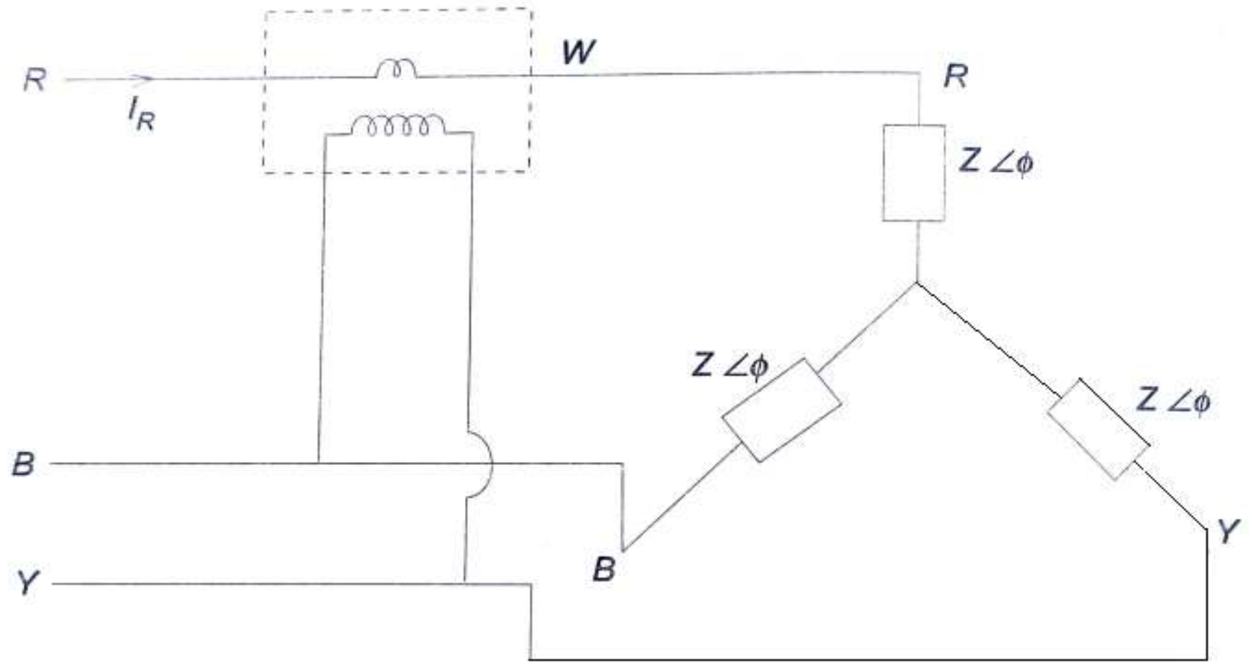
  - Total power is given by $W_R + W_Y = \sqrt{3}\,V_L\ I_L\ \cos\emptyset$
  - and $W_Y - W_R = V_L\ I_L\ \sin\emptyset$
  - Hence,

$$\tan\emptyset = \sqrt{3}\left[\frac{W_Y - W_R}{W_Y + W_R}\right]$$

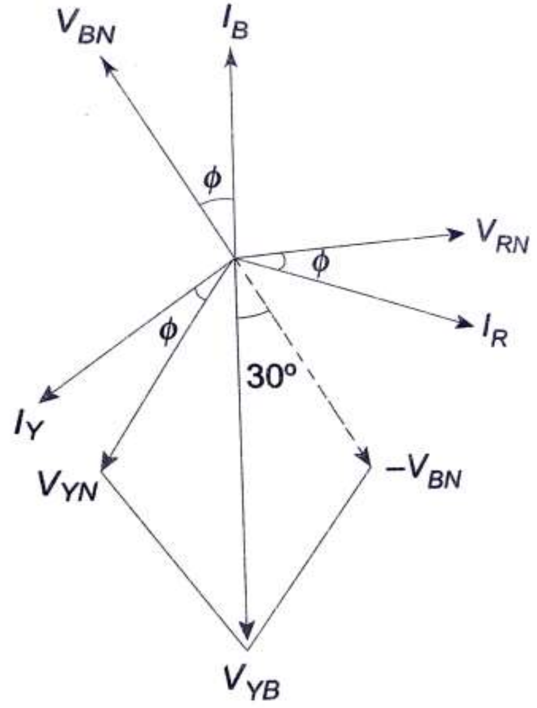
  - The lagging power factor reveals that W_R is the higher reading wattmeter and W_Y is the lower reading wattmeter.
  - Whereas leading power factor reveals that W_R is the lower reading wattmeter and W_Y is the higher reading wattmeter.

Reactive Power with Wattmeter

- Reactive power in a balanced three-phase load can be calculated by using a single wattmeter. The connection of single wattmeter is shown in **Fig. 7.30** (a) and the phasor diagram is shown in **Fig. 7.30** (b) considering RYB phase sequence and inductive load of phase angle $\emptyset$.
- The current coil of the wattmeter is connected in the line of R phase and the pressure coil of the wattmeter is connected across the lines between Y and B phases.



(a)



(b)

Fig. 7.30

Polyphase Circuits

- **Reactive Power with Wattmeter**

  - ✓ The power of the wattmeter is proportional to the product of current through its current coil, I_R, voltage across its pressure coil, V_{YB}, and cosine of the angle between V_{YB} and I_R.

The voltage $V_{YB} = V_{YN} - V_{BN} = V_L$

The angle between V_{YB} and I_R is $(90 - \emptyset)^0$

$\therefore$ Wattmeter reading$= V_{YB} I_R \cos(90 - \emptyset)^0 = V_L\ I_L\ \sin\emptyset$ VAr

Thus, total reactive power in the load $= \sqrt{3} V_L\ I_L\ \sin\emptyset = \sqrt{3} \times$ Wattmeter reading

Polyphase Circuits

Example – P 7.5

The two wattmeter method is used to measure power in a three-phase load. The wattmeter readings are 400 W and – 35 W. Calculate (i)total active power (ii) power factor, and (iii) reactive power.

Solution of Example – P 7.5

The two wattmeter readings are $W_R = 400$ W (higher reading wattmeter) and $W_Y = -35$ W (lower reading wattmeter).

(i) Total active power$= W_R + W_Y = 400 + (-35) = 365$ W

$$(ii) \quad \tan \emptyset = \sqrt{3}\,\frac{W_R - W_Y}{W_R + W_Y} = \sqrt{3}\,\frac{400-(-35)}{400+(-35)} = \sqrt{3} \times \frac{435}{365} = 2.064$$

$$\therefore \quad \emptyset = \tan^{-1}(2.064) = 64.15^0$$

$$\therefore \quad p.f. = \cos\emptyset = 0.43$$

(iii) Reactive power $= \sqrt{3} V_L I_L \sin\emptyset$

We know, $W_R - W_Y = V_L I_L \sin\emptyset$

$$W_R - W_Y = 400 - (-35) = 435$$

Reactive power $= \sqrt{3} \times 435 = 753.44$ VAr

Polyphase Circuits

Example – P 7.6

The input power to a three-phase load is 10 kW at 0.8 pf. Two wattmeters are connected to measure the power, find the individual readings of the wattmeters.

Polyphase Circuits

Solution of Example – P 7.6

Let W_R be the higher reading wattmeter and W_Y be the lower reading wattmeter.

$$W_R + W_Y = 10 \text{ kW}$$

$$\emptyset = \cos^{-1}(0.8) = 36.8°$$

$$\tan\emptyset = 0.75 = \sqrt{3}\frac{W_R - W_Y}{W_R + W_Y}$$

$$W_R - W_Y = \frac{(0.75)}{\sqrt{3}}(W_R + W_Y) = \frac{(0.75)}{\sqrt{3}} \times 10 = 4.33 \text{ kW}$$

$$W_R + W_Y = 10 \text{ kW}$$

$$W_R - W_Y = 4.33 \text{ kW}$$

$$2W_R = 14.33 \text{ kW}$$

$$\therefore \quad W_R = 7.165 \text{ kW}$$

and $\quad W_Y = 2.835 \text{ kW}$

Polyphase Circuits

Example – P 7.7

The readings of the two wattmeters used to measure power in a capacitive load are -3000 W and 8000 W, respectively. Calculate (i) the input power, and (ii) the power factor at the load. Assume RYB sequence.

Solution of Example – P 7.7

(i) Total power $= W_R + W_Y = -3000 + 8000 = 5000$ W

(ii) As the load is capacitive, the wattmeter connected in the leading phase gives less value.

$\therefore$ $W_R = -3000$

Consequently $W_Y = 8000$

$$\tan\emptyset = \sqrt{3}\frac{W_Y - W_R}{W_Y + W_R} = \sqrt{3}\frac{8000-(-3000)}{5000} = 3.81$$

$\therefore$ $\emptyset = 75.29^0$ (lead);

$\cos\emptyset = 0.25$

Polyphase Circuits

Example – P 7.8

A single wattmeter is connected to measure reactive power of a three-phase, three-wire balanced load as shown in **Fig. 7.30** (a). The line current is 17 A and the line voltage is 440 V. Calculate the power factor of the load if the reading of the wattmeter is 4488 VAr.

Polyphase Circuits

Solution of Example – P 7.8

We know,

Wattmeter reading is equal to $V_L I_L \sin\emptyset$

$$\therefore \quad 4488 = 440 \times 17 \sin\emptyset$$

$$\sin\emptyset = 0.6$$

Power factor $= \cos\emptyset = 0.8$


Thank you