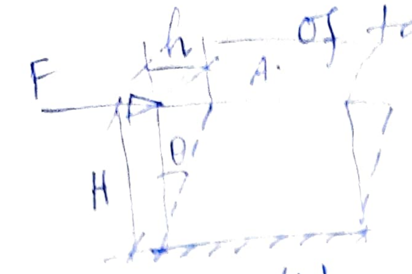
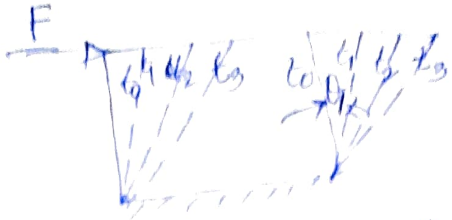
Def^n of Fluid: Fluid is ~~can~~ defined as a substance that deforms continuosly under the application of tangential force whatever small it may be.





Solid

For solid, $\tau = F/A$

Shear strain, $\theta \approx \tan\theta = \frac{h}{H}$

For Fluid bet^n two plates of same area, A

$\tau = F/A$

Using dye marker, we get dashed lines at t_0, t_1, t_2, t_3 times

$\therefore$ Shear strain is not measurable instead, strain rate in fluid is measure.

$\therefore$ $\boxed{\text{Strain rate} = \frac{\theta_1}{t_1}}$ $\Rightarrow$ $\tau_{xy} = \mu \underbrace{\frac{du}{dy}}_{\text{strain rate}}$

Properties of Fluid:- There are several properties of fluid e.g. ① Density (point function), ② Thermal conductivity (k), ③ Viscosity (μ) ④ specific heat constants (C_p & C_v)

⑤ characteristic gas constant $\left(R = \frac{R_u}{M} = C_p - C_v\right)$

⑥ Compressibility $\left(\beta = \frac{1}{K} = -\frac{1}{v}\frac{dv}{dP} = \frac{1}{\rho}\frac{d\rho}{dP}\right)$, where K = Bulk Modulous.

etc.

Newton's law of viscosity:- It is applied to Newtonian fluid e.g. air and water. $\tau_{xy} = \mu \frac{du}{dy}$

where μ = Dynamic viscosity $= \frac{\tau_{xy}}{du/dy} = \frac{N/m^2}{\frac{m/s}{m}}$

$\frac{du}{dy}$ = strain rate $\left(\frac{1}{s}\right)$ $\quad \boxed{\mu \equiv N.S/m^2}$

Kinematic viscosity, $\nu = \frac{\mu}{\rho} = \frac{N.S/m^2}{kg/m^3} = \frac{kg.\frac{m}{s^2}.S/m^2}{kg/m^3}$

$\boxed{\nu \equiv m^2/s}$

Isothermal compressibility of any fluid

β_T (water) $= \frac{1}{K} = 5 \times 10^{-10}$ m²/N.

β_T (air) $= 1.0 \times 10^{-5}$ m²/N.

Compressibility of air is about 10^5 times more than wate

$\therefore$ Water at STP (i.e. $T = 298$ K & $P_{atm} = 1.013 \times 10^5$ N/m²) is considered as incompressible Fluid. and air is Compressible fluid

Incompressible flow and compressible flow:-

If $a =$ velocity of sound through a fluid

$V =$ " of flow of that fluid.

Then, Mach No., $M = \frac{\text{Velocity of flow of any fluid}}{\text{Velocity of sound through that fluid}}$

$$M = \frac{V}{a}.$$

Now, Bulk Modulous, $K = -\frac{dp}{dv/v}$, when v = specific Volume.

$$\therefore K = \frac{dp}{d\rho/\rho}.$$

When dp = Dynamic pressure $= \frac{1}{2}\rho V^2$

Pressure → Static, stagnation, Dynamic

Dynamic press = Stagnat^n pres − Stat^c prese

And velocity of sound, $a = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\frac{dp}{d\rho/\rho}}{\rho}} = \sqrt{\frac{\frac{1}{2}\rho V^2 \cdot \rho}{\rho\, d\rho}}$

$$\therefore a^2 = \frac{\frac{1}{2}V^2}{d\rho/\rho} \Rightarrow 2 \cdot \frac{d\rho}{\rho} = \frac{V^2}{a^2} = M^2$$

$\boxed{\text{Any Fluid remains incompressible when density variation (i.e. } \frac{d\rho}{\rho}\text{) } < 5\%}$

$\therefore M^2 = 2 \times \frac{5}{100} = \frac{1}{10}$

$\therefore M = 0.3$

Continuum :- Continuous distribution of matter. i.e there is no vacant space between the molecules. In other words, if Knudsen No, $K_n = \frac{\lambda}{L} < 0.01$, the continuum assumption is valid. λ = Mean Free Path & L = Characteristic length.

$$= \frac{\text{Distance bet}^n \text{ any two successive collision}}{\text{No. of collision}}$$

For Molecular flow, $K_n > 10$

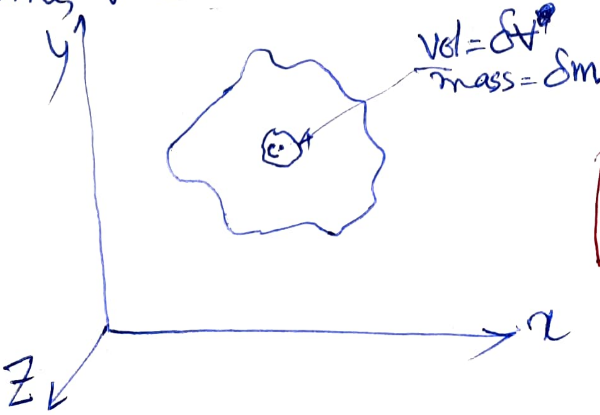
If fluid is considered to be continuum, then every property of the fluid can be defined at a point. Therefore property of fluid is a continuous function of space & time. e.g. $\rho = \rho(x, y, z, t)$ is a continuous function.

(i) **Def^n of property at a point :-** Any physical property at a point is defined as an average value of of that property over a smallest possible region of volume, δV^* around that point.

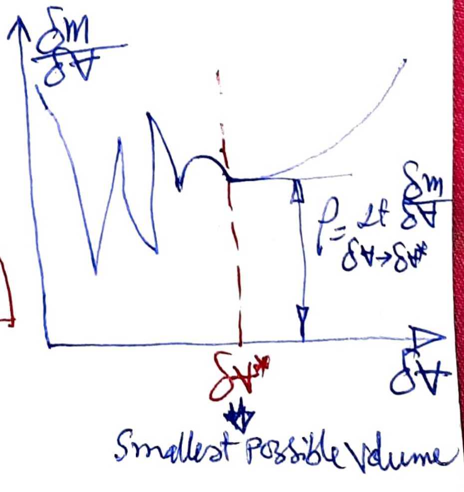
(ii) Smallest possible volume, δV^* is defined as that volume when the number of molecules entering into and no. of molecule leaving actually matter.

(iii) Fluid particle : Fluid particle is defined as a lump of fluid having smallest possible volume, δV^* under continuum assumption.

To illustrate the concept of a property at a point, a region of volume, $\forall$ with mass, m is considered.



See Page-17 in Fox & McDonald



$\therefore \rho = \frac{m}{\forall}$ is mean density which is not same as density at point C as shown in figure. To determine, density, ρ at point C, we must select a small volume, $\delta\forall$ (not smallest) surrounding the point C. Now if go on shrinking the volume $\delta\forall$ beyond a volume, $\delta\forall^*$ (small possible), the graph of $\frac{\delta m}{\delta \forall}$ starts flickering (discontinuous) i.e. no. of molecules entering & leaving the region actually matter. So $\delta\forall^*$ is smallest possible volume.

$\therefore$ Density at a point, $\boxed{\rho = \lim_{\delta\forall \to \delta\forall^*} \frac{\delta m}{\delta \forall}} = \rho(x,y,z,t)$

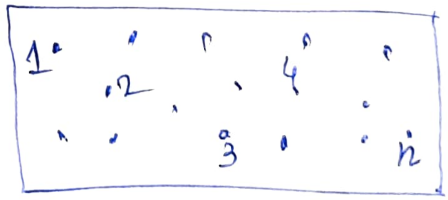
is a continuous function.

⓪ And velocity at point 'C' is called velocity of fluid particle (of Vol $= \delta\forall^*$)

And stress at point 'C' is called stress at a point of that fluid particle.

(iv) Concept of Field :- There are ~~numerous~~ numberous points (say n) in a flow field. At each point, a property (Density, Velocity etc) can be calculated and properties at those points ~~are different~~ vary due to work done by/on the fluid and/or ~~due to~~ heat transfer to the fluid with time.

$\therefore \rho_1 = \rho_1(x_1, y_1, z_1, t),\ \rho_2 = \rho_2(x_2, y_2, z_2, t)$ --- $\rho_n(x_n, y_n, z_n, t)$



so, $\vec{V}_1$ (velocity) $= \vec{V}_1(x_1, y_1, z_1, t)$, $\vec{V}_2 = \vec{V}_2(x_2, y_2, z_2, t)$ --- $V_n = V_n(x_n, y_n, z_n, t)$

$\therefore$ Field can be divided into 3 types a) scalar b) vector & c) Tensor field

a) scalar field $\Rightarrow \rho = [\rho_1, \rho_2, \cdots \rho_n]$

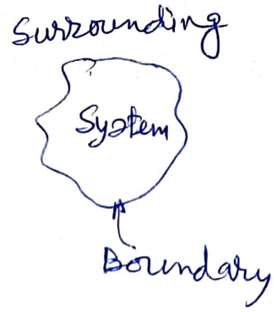
b) Vector Field $\Rightarrow \vec{V} = [V_1, V_2, \cdots V_n]$

c) Tensor Field, $\vec{\vec{\tau}} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$ --- i direct^n --- j-direct^n --- k-direct^n

τ_{xy} → Plane, direct^n

See Page-25 Fox & McDonald for Tensor.

(V) System



① Closed System (control mass)

⇓

~~Both~~ Only energy can cross system boundary

Explaination:—
If a cubical volume moves with same velocity that of flow velocity, then fluid inside the volume is fixed so mass is constant. Thus it is control mass.

e.g. – Used in particle Mechanics where "Fixed & identifiable" mass in any matter exist

② Open system (control volume)

Both energy and mass can cross boundary

Explaination:
If a cubical volume is fixed in a flow field then some fluid is entering and some old fluid is going out. So mass is not constant rather volume is constant. Thus it is control volume

e.g. – All fluid m/c

③ Isolated system

Neither energy nor mass can cross boundary

Used in combustion modelling.

(VI) Reynolds Transport Theorem (RTT):-

$$\underbrace{\left.\frac{dN}{dt}\right)_{\substack{System\\ or\\ CM}}}_{\text{control mass}} = \underbrace{\frac{\partial}{\partial t}\iiint_{CV} \eta\rho\, d\forall + \iint_{CS} \eta\rho\, \vec{V}\cdot d\vec{A}}_{\text{Control Volum}}$$

RTT is used to convert control mass formulatⁿ to control volume formulation.

N = Extensive property e.g. mass (M), momentum ($P = m\vec{V}$) energy (E) etc.

η = Intensive property = Extensive property per unit mass

$$\eta = \frac{N}{m}.$$

6

We can derive conservation equ^n's as follows: →

1) Continuity Equ^n: Put $N = M$, $\eta = \frac{M}{M} = 1$ in RTT
2) Momentum Equ^n: Put $N = M\vec{V}$, $\eta = \frac{N}{M} = \frac{M\vec{V}}{M} = \vec{V}$ in RTT
3) Angular momentum: Put $N = \vec{r} \times M\vec{V}$, $\eta = \frac{\vec{r} \times M\vec{V}}{M} = \vec{r} \times \vec{V}$ in RTT
4) Energy equ^n: Put $N = E$ and, $\eta = \frac{E}{m} = e$ in RTT

VII ⇒ Methods of Description:-

Lagrangian method of Descrip.

i) Related to "Fixed and identifiable mass i.e. molecule

Example:- To get idea of Traffic in Durgapur

ii) Say 10 students are identified at 10 different known location $(x_1, y_1, t), (x_2, y_2, t) \cdots (x_{10}, y_{10}, t)$ at a particular time, t and they are making a round over the Durgapur town and each reports separately regarding the Traffic condition.

iii) It gives us the trajectory of particle

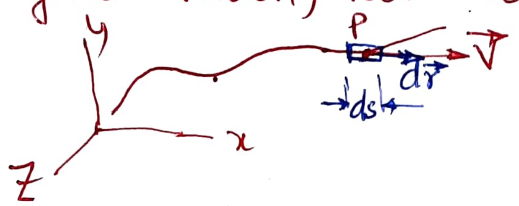
Eulerian method of desc.

i) Related to control volum

ii) The Same 10 students are identified at 10 different location of Durgapur town at they are not moving but they are watching the trafic at those points (of Flow field of traffic). i.e. Traffic at a point is noted as defined in CV.

iii) It gives us the Streamlines of the flow field

Streamline is defined as an imaginary line drawn in the flow field such that the tangent to it gives velocity vector at that point and at that instant.



∴ Equ^n of Streamline, $\vec{V} \times d\vec{r} = 0$

or, $(\hat{i}u + v\hat{j} + w\hat{k}) \times (\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz) = 0$

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

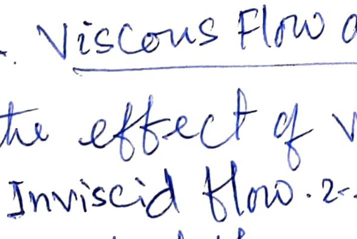
Lagrangian	Eulerian
(iv) A fluid particle is identified at $t = t_0$ as $\vec{r}_0(x_0, y_0, z_0)$. At subsequent time instants, t the position of the same particle is given as $\vec{r} = \vec{r}(\vec{r}_0, t)$ or, $x = x(x_0, y_0, z_0, t)$, $y = y(x_0, y_0, z_0, t)$ & $z = z(x_0, y_0, z_0, t)$, the path lines. $\vec{V} = \vec{V}(\vec{r}_0, t)$, $\vec{a} = \vec{a}(\vec{r}_0, t)$	(iv) $\vec{V} = \vec{V}(x, y, z, t)$ i.e. a point (x, y, z) is located in flow field at $t = t_0$. Velocity at that point (x,y,z) at any instant, t is $\vec{V} = \vec{V}(x, y, z, t)$, $\vec{a} = \vec{a}(x, y, z, t)$ } All are continuous function of x, y, z & t.

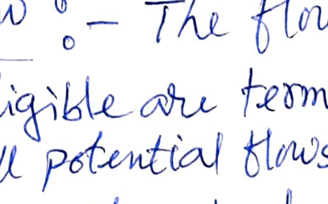
VIII: Steady and Unsteady :- Any property which does not depend on time ~~is steady or~~ for a flow is called steady flow and In a flow if property is time dependant, the flow is unsteady flow.

$$\frac{\partial}{\partial t}(\text{property}) = 0 \quad \text{for a steady flow}$$

IX : Laminar and Turbullent flow.

Laminar	Turbullent
i) Fluid motion occurs in layers	i) Flow field exhibits fluctuation both in space and time.



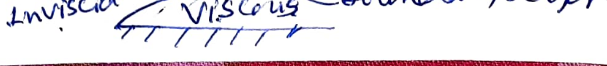


$\bar{u}$ = mean velo
u' = Fluctuation
u = Total Velocity.

$\therefore u = \bar{u} + u'$

X. Viscous Flow and Inviscid Flow :- The flows in which the effect of viscosity ~~are~~ is negligible are termed as Inviscid flow i.e. ideal flow. All potential flows are inviscid flow. In ideal flows, head lost due to roughness, bends etc is always zero. Mathematically, if $\vec{\nabla} \times \vec{V} = 0$ i.e. Curl of velo = zero, the flow is Inviscid. For viscous flow, $\vec{\nabla} \times \vec{V} \neq 0$ i.e Curl of velo exists

By Prandtl theory, flow near the solid wall is viscous and beyond the boundary layer is Inviscid



X) Time averaged, mass averaged and phase averaged property:-

a) Time averaged: $\overline{u_i} = \frac{1}{T}\int_{t_0}^{t_0+T} u_i\, dt$ where $u_i = \overline{u_i}(x,y,z) + u_i'(x,y,z,t)$

$u_i(x,y,z,t) \equiv$ Total velocity.

Rules for Time averaging

1) $\overline{u_i'} = \frac{1}{T}\int_{t_0}^{t_0+T} u_i'\, dt = \frac{1}{T}\int_{t_0}^{t_0+T}(u_i - \overline{u_i})\, dt$

$= \frac{1}{T}\left[\int_{t_0}^{t_0+T} u_i\, dt - \int_{t_0}^{t_0+T}\overline{u_i}\, dt\right]$

$= \frac{1}{T}\left[T\overline{u_i} - \overline{u_i}\int_{t_0}^{t_0+T} dt\right]$

$= \frac{1}{T}\left[\cancel{T\overline{u_i}} - \cancel{\overline{u_i} T}\right]$

$\therefore \overline{u_i'} = 0$

2) $\overline{u_i u_j} = \frac{1}{T}\int_0^T u_i u_j\, dt = \frac{1}{T}\int_0^T (\overline{u_i} + u_i')(\overline{u_j} + u_j')\, dt$

$= \frac{1}{T}\int_0^T (\overline{u_i}\,\overline{u_j} + \overline{u_i} u_j' + \overline{u_j} u_i' + u_i' u_j')\, dt$

$= \frac{1}{T}\int_0^T \overline{u_i}\,\overline{u_j}\, dt + \frac{1}{T}\int_0^T \overline{u_i} u_j'\, dt + \frac{1}{T}\int_0^T \overline{u_j} u_i'\, dt + \frac{1}{T}\int u_i' u_j'\, dt$

$= \frac{\overline{u_i}\,\overline{u_j}}{\cancel{T}}.\cancel{T} + \frac{\overline{u_i}}{T}\cancelto{0}{\int_0^T u_j'\, dt} + \overline{u_j}\cdot\frac{1}{T}\cancelto{0}{\int_0^T u_i'\, dt} + \overline{u_i' u_j'}$

$\overline{u_i u_j} = \overline{u_i}\,\overline{u_j} + \overline{u_i' u_j'}$

3) $\overline{u_i + u_j} = \overline{u_i} + \overline{u_j}$

4) $\overline{u_i' \overline{u_j}} = \overline{u_j' \overline{u_i}} = 0$

5) $\overline{\overline{u_i}} = \overline{u_i}$

b) Mass averaged :- $\tilde{u}_i = \frac{1}{\bar{\rho}} \int_{t_0}^{t_0+T} \rho(x,t)\, u_i(x,t)\, dt$

$\bar{\rho}$ = conventional Reynolds' average density $\left(\frac{m_{total}}{V_{total}}\right) = \frac{1}{T}\int_0^T \rho\, dt$.

Rule :

1) $\bar{\rho}\tilde{u}_i = \int_0^T \rho u_i\, dt = \int_0^T (\bar{\rho}+\rho')(\bar{u}_i + u_i')\, dt$

$$\bar{\rho}\tilde{u}_i = \bar{\rho}\bar{u}_i + \overline{\rho' u_i'}$$

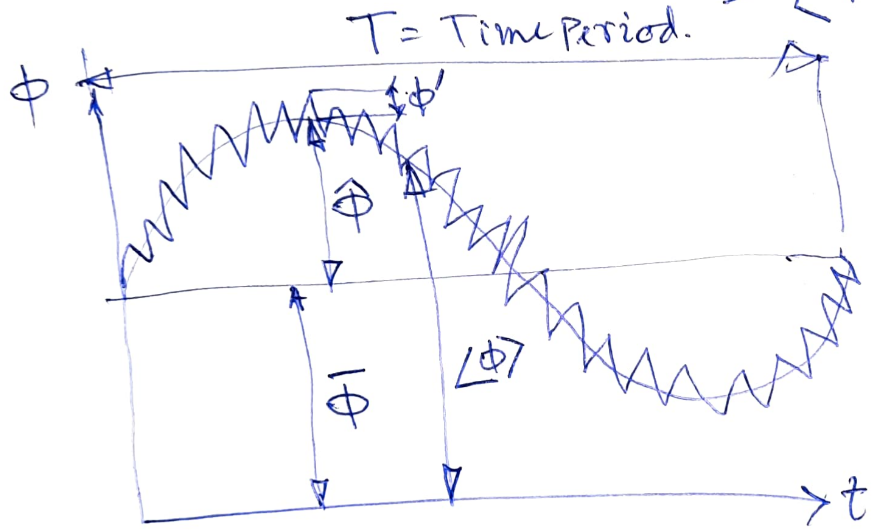
$\tilde{u}_i$ = mass averaged and u

c) 2) $\overline{\rho Q} = \bar{\rho}\tilde{Q} + \overline{\rho Q''}$ where $Q = \tilde{Q}_i + Q''$

$\tilde{Q}_i$ = mass averaged & Q'' = Fluctuating component of Q.

c) Phase averaged :- $\phi(x,t) = \bar{\Phi}(x) + \hat{\Phi}(x,t) + \phi'(x,t)$

$= \langle\phi\rangle(x,t) + \phi'(x,t)$



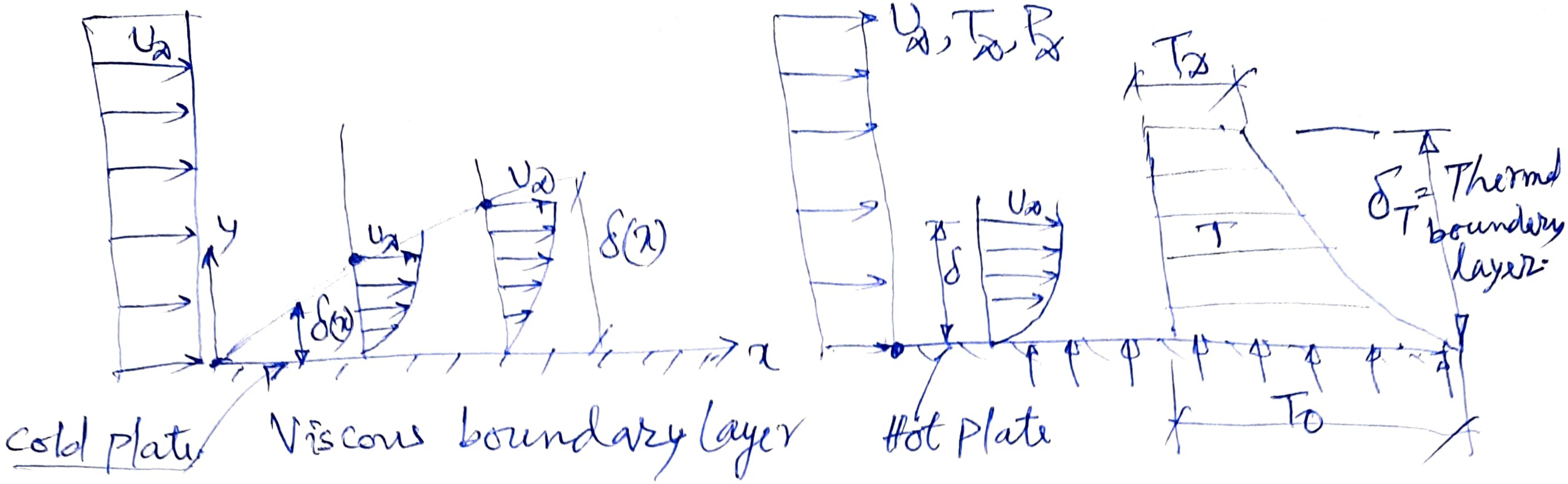
$\bar{\Phi}$ = Time averaged

$\hat{\Phi}$ = Periodic displacement at any time. i.e. fluctuating part above $\langle\Phi\rangle$.

ϕ' = Random components

XI - Viscous and Thermal boundary layer

XI Viscous & Thermal Boundary layer.



Cold plate, Viscous boundary layer

Hot plate

Prandtl No, $Pr = \dfrac{\nu \text{ (Kinematic viscosity)}}{\alpha \text{ (Thermal diffusivity)}} = \dfrac{\text{Viscous effect}}{\text{Thermal effect}}$

If $Pr > 1$, Viscous boundary layer, $\delta > \delta_T$, Thermal boundary layer.

&

If $Pr < 1$, $\delta < \delta_T$