

LECTURE 7

➤ Magnetic Flux

- ✓ Magnetic Flux is the number of magnetic field lines that exist in a particular space. The symbol for magnetic flux is the Greek capital letter phi ' Φ '. The unit of measurement in the SI system is the weber symbolized Wb.
- \checkmark A unit *N*-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a *N*-pole of *m* weber is given by Φ = m Wb

Magnetic Flux Density

✓ It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter².

If Φ Wb is the total magnetic flux passing normally through an area of A m², then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

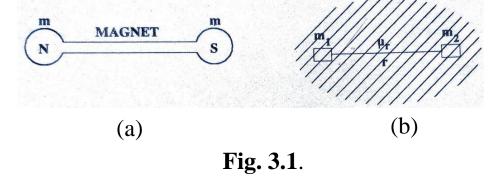
✓ Let us find an expression for the flux density at a point distant r metres from a unit N-pole (*i.e.* a pole of strength 1 Wb). Imagine a sphere of radius r metres drawn round the unit pole. The flux of 1 Wb radiated out by the unit pole falls normally on a surface of $4\pi r^2 m^2$. Hence

$$B = \frac{\Phi}{A} = \frac{1}{4\pi r^2} \text{ Wb/m}^2$$

➤ Laws of Magnetic Force

- ✓ Coulomb determined the expression for the magnetic force between two isolated point poles experimentally.
- ✓ The concept of an isolated pole is purely theoretical. Poles of a thin but long magnet may be assumed to be point poles for all practical purposes.
- ✓ He found that the force between two magnetic poles placed in a medium is
 - i) directly proportional to the pole strengths
 - ii) inversely proportional to the square of the distance between them and
 - iii) inversely proportional to the absolute permeability of the surrounding medium
- For example, if m_1 and m_2 represent the magnetic strength of the two poles, r represent the distance between them and μ represent the absolute permeability of the surrounding medium, then the force F is given by

$$F = \frac{\mu \, m_1 m_2}{4\pi r^2}$$



Magnetic Field Strength

- ✓ Magnetic field strength at any point within a magnetic field is numerically equal to the force experienced by a *N*-pole of one weber placed at that point.
- \checkmark The symbol for magnetic field strength is H and the unit of H is N/Wb.
- ✓ The magnetic field strength is also known as field intensity, magnetising force, strength of field, magnetic intensity and intensity of magnetic field.
- ✓ A pole of one weber is placed at r metre from a similar pole of m weber in the medium with absolute permeability μ . The force experienced by the pole is

$$H = \mu \frac{m \times 1}{4\pi r^2} \text{ N/Wb}$$

✓ The force represents the magnetic field strength of that point in the magnetic field

- Relation between Magnetic Flux Density, Magnetic Field Strength and permeability of the medium
 - ✓ A bar of a magnetic material, say, iron placed in a uniform field of strength H N/Wb as shown in Fig. 3.2. Suppose, a flux density of B Wb/m² is developed in the rod.
 - ✓ The absolute permeability of the material of the rod is defined as

$$\mu = B/H$$
 henry/metre

or,
$$B = \mu H = \mu_0 \,\mu_r H \, \text{Wb/m}^2$$

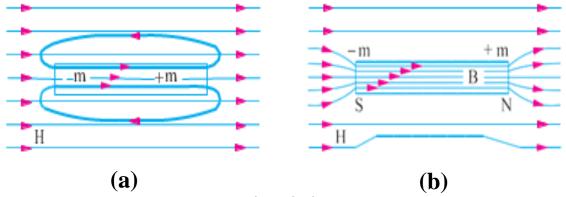


Fig. 3.2

 \checkmark When H is established in air (or vacuum), then corresponding flux density developed in air is

$$B_0 = \mu_0 H$$

When iron rod is placed in the field, it gets magnetised by induction. If induced pole strength in the rod is m Wb, then a flux of m Wb emanates from its N-pole, re-enters its S-pole and continues from S to N-pole within the magnet. If A is the pole area of the magentised iron bar, the induction flux density in the rod is

$$B_i = m/A \text{ Wb/m}^2$$

- > Relation between Magnetic Flux Density, Magnetic Field Strength and permeability of the medium
 - ✓ Hence, total flux density in the iron rod consists of two parts
 - 1. B_0 —flux density in air even when rod is not present
 - 2. B_i –induction flux density in the rod
 - ✓ Total flux density

Also,
$$B = B_0 + B_i = \mu_0 H + m/A$$

$$B = \mu_r \cdot \mu_0 H = \mu_r B_0$$

$$\therefore \mu_r = \frac{B}{B_0}$$

✓ Hence, relative permeability of a material is equal to the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetising force.

> Ampere's Work Law or Ampere's Circuital Law

✓ The law states that m.m.f. (magnetomotive force) around a closed path is equal to the current enclosed by the path.

Mathematically,
$$\oint \overrightarrow{H} \cdot \overrightarrow{dl} = I$$
 amperes

where \vec{H} is the vector representing magnetic field strength and \vec{dl} is the vector representing line element of the enclosing path l around current I ampere.

- ✓ The law is applicable to all magnetic fields whatever the shape of enclosing path.
- ✓ It is seen from **Fig. 3.3** that the m.m.f. exist around the paths a and b but it is zero around the path c because current is enclosed only paths of a and b, not the path c.
- ✓ **Magnetomotive force (m.m.f.):** It drives or tends to drive flux through a magnetic circuit. M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

(a)

Fig. 3.3

> Ampere's Work Law or Ampere's Circuital Law

- ✓ The work Law is used to determine the value of the magnetomotive force of
 - (i) a long straight current-carrying conductor and
 - (ii) a long solenoid.

✓ Magnetomotive Force around a Long Straight Conductor

- A straight conductor which is assumed to extend to infinity in either direction is shown in **Fig. 3.4**.
- Let it carry a current of *I* amperes upwards.
- The magnetic field consists of circular lines of force having their plane perpendicular to the conductor and their centres at the centre of the conductor.
- The field strength at pint c, distant r metres from the centre of the conductor is H as shown in **Fig. 3.5**.

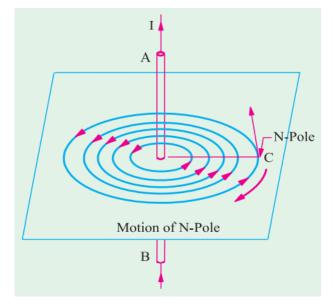
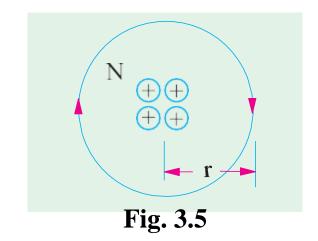


Fig. 3.4



> Ampere's Work Law or Ampere's Circuital Law

- ✓ Magnetomotive Force around a Long Straight Conductor
 - A unit *N*-pole at c will experience a force of *H* Newtons. The direction of this force would be tangential to the circular line of force passing through c.
 - If this unit *N*-pole is moved once round the conductor against this force, then work done *i.e.*

m.m.f. = force × distance =
$$I$$

 $i.e. I = H \times 2\pi r$ joules = Amperes

$$H = \frac{I}{2\pi r}$$

• If there are *N* conductors, then

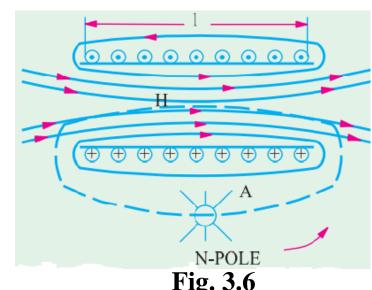
$$H = \frac{NI}{2\pi r}$$
 A/m or Oersted

$$B = \mu_0 \frac{NI}{2\pi r}$$
 Wb/m² or Tesla ———————————in air

$$B = \mu_0 \mu_r \frac{NI}{2\pi r}$$
 Wb/m² or Tesla ————————————in medium

Ampere's Work Law or Ampere's Circuital Law

- **✓** Magnetic Field Strength of a Long Solenoid
 - Let the Magnetic Field Strength along the axis of the solenoid be *H*. Let us assume that
 - 1. the value of H remains constant throughout the length l of the solenoid and
 - 2. the value of *H* outside the solenoid is negligible
 - Suppose, a unit N-pole is placed at point A outside the solenoid and is taken once round the completed path as shown in **Fig. 3.6** in a direction opposite to that of H.
 - The force of *H* newton acts on the *N*-pole only over the length *l*, and it is negligible elsewhere.
 - So, the work done in one round is $(H \times l)$ joules
 - The 'ampere-turns' linked with this path are NI where N is the number of turns of the solenoid and I the current in amperes passing through it.



- > Ampere's Work Law or Ampere's Circuital Law
 - ✓ Magnetic Field Strength of a Long Solenoid
 - According to Work Law

$$H \times l = NI$$

or,
$$H = \frac{NI}{l}$$
 A/m or Oersted

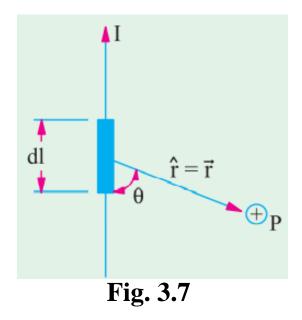
$$B = \mu_0 \frac{NI}{l}$$
 Wb/m² or Tesla ------in air

$$B = \mu_0 \mu_r \frac{NI}{l}$$
 Wb/m² or Tesla -------in medium

> Ampere's Work Law or Ampere's Circuital Law

✓ Biot-Savart Law

The expression for the magnetic field strength dH produced at point P by a vanishingly small length dl of a conductor carrying a current of I amperes as shown in **Fig. 3.7** is given by



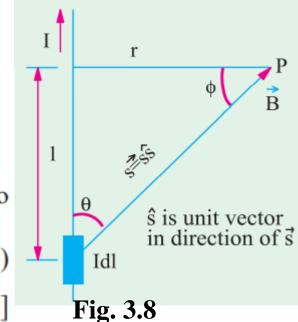
$$dH = \frac{Idl \sin \theta}{4\pi r^2}$$

- Ampere's Work Law or Ampere's Circuital Law
 - **✓ Applications of Biot-Savart Law**
 - **❖** Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

Consider a straight wire of length l carrying a steady current I. It is needed to find magnetic field strength, H at a point P which is at a distance r from the wire as shown in **Fig. 3.8.**

The magnetic field strength $d\vec{H}$ due to a small element dl of the wire is

$$d\vec{H} = \frac{\vec{l} d \vec{l} \times \hat{s}}{4\pi s^2} \text{ (By Biot-Savart Law)}$$
or, $d\vec{H} = \frac{Idl \sin \theta}{4\pi \times s^2} \hat{u}$ (where \hat{u} is unit vector perpendicular to plane containing $d\vec{l}$ and \hat{s} and into the plane.)
or, $d\vec{H} = \frac{Idl \cos \phi}{4\pi \times s^2} \hat{u}$...[: θ and ϕ are complementary angles]



- ➤ Ampere's Work Law or Ampere's Circuital Law
 - **✓ Applications of Biot-Savart Law**
 - **❖** Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

The magnetic field strength due to entire length l of the conductor as shown in **Fig. 3.9**.

$$\vec{H} = \frac{I}{4\pi} \left[\int_{0}^{l} \frac{\cos\phi \, dl}{s^2} \right] \hat{u}$$

$$= \frac{I}{4\pi} \left[\int_{0}^{l} \frac{r/s}{s^2} \, dl \right] \hat{u} \quad \left[\because \cos\phi = \frac{r}{s} \right]$$

$$= \frac{Ir}{4\pi} \left[\int_{0}^{l} \frac{dl}{s^3} \right] \hat{u} = \frac{Ir}{4\pi} \left[\int_{0}^{l} \frac{dl}{(r^2 + l^2)^{3/2}} \right] \hat{u} \quad \left[\because s = \sqrt{r^2 + l^2} \right]$$

$$= \frac{Ir}{4\pi r^3} \left[\int_{0}^{l} \frac{dl}{[1 + (r/l)^2]^{3/2}} \right] \hat{u}$$

Fig. 3.9

- > Ampere's Work Law or Ampere's Circuital Law
 - **✓ Applications of Biot-Savart Law**
 - **❖** Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

We know,
$$\frac{l}{r} = \tan \emptyset$$
 $\therefore l = r \tan \emptyset$

$$\therefore dl = r \sec^2 \phi d\phi \text{ and } 1 + (r/l)^2 = 1 + \tan^2 \phi = \sec^2 \phi$$

$$\vec{H} = \frac{Ir}{4\pi r^3} \left[\int_0^{\emptyset} \frac{r(\sec \emptyset)^2}{(\sec \emptyset)^3} d\emptyset \right] \hat{u} = \frac{Ir}{4\pi r^3} \left[\int_0^{\emptyset} \cos \emptyset \, d\emptyset \right] \hat{u} = \frac{I}{4\pi r} [\sin \emptyset]_0^{\emptyset} \hat{u}$$

$$\therefore \vec{H} = \frac{I}{4\pi r} \sin \phi \,\hat{u}$$

The wire of infinite length (i.e. extending it at both ends from $-\infty$ to $+\infty$) the limits of integration would be $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

$$\therefore \vec{H} = \frac{I}{4\pi r} \times 2 \,\hat{u} = \frac{I}{2\pi r} \hat{u}$$

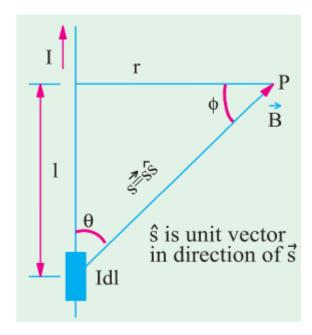


Fig. 3.9

➤ Force Between Two Parallel Conductors

- **✓** Currents in the same direction.
 - Two parallel conductors P and Q carrying currents I₁ and I₂ amperes in the same direction i.e. upwards as shown in Fig. 3.10 (a).
 - The field strength in the space between the two conductors is decreased because two fields are in opposition to each other in the space as shown in **Fig. 3.10** (b).
 - The conductors are attracted towards each other.

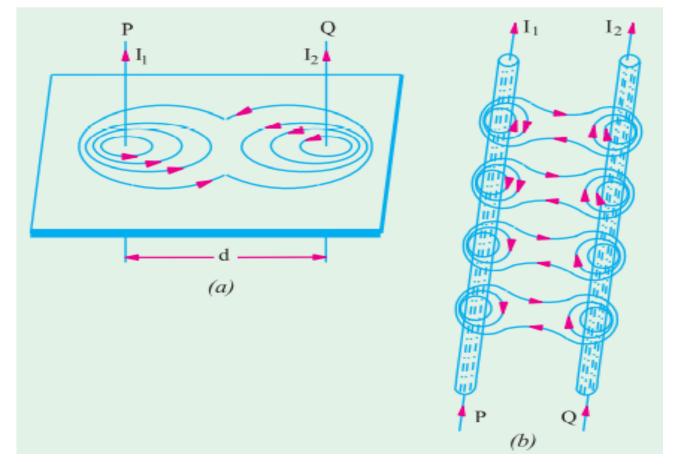


Fig. 3.10

Force Between Two Parallel Conductors

✓ Currents in opposite directions

- Two parallel conductors P and Q carrying currents I_1 and I_2 amperes in the opposite direction as shown in **Fig. 3.11** (a).
- The field strength in the space between the two conductors is increased because two fields are in the same direction in the space as shown in **Fig. 3.11** (*b*).
- The conductors experience a mutual force of repulsion.

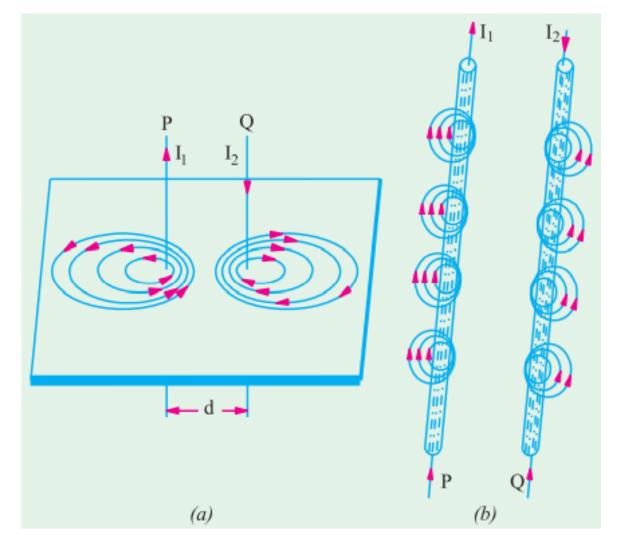


Fig. 3.11

Magnitude of Mutual Force

- ✓ In **Fig. 3.10** and **Fig. 3.11**, conductor P lies in the magnetic field of Q and Q lies in the field of P.
- \checkmark If 'd' metres is the distance between them, then flux density at Q due to P is

$$B = \frac{\mu_0 I_1}{2\pi d} \text{ Wb/m}^2$$

 \checkmark If l is the length of conductor Q lying in this flux density, then force on P (either of attraction or repulsion)

is
$$F = BI_2 l$$
 N or, $F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$ N or, $F = \frac{4\pi \times 10^{-7} I_1 I_2 l}{2\pi d}$ N or, $F = \frac{2 \times 10^{-7} I_1 I_2 l}{d}$ N

- ✓ Conductor *P* will experience an equal force in the opposite direction. The fact is known as 'Laws of Parallel Currents'
- ✓ If $I_1 = I_2 = 1$ amp and d = 1 metre, then $F = 2 \times 10^{-7}$ N
- ✓ One ampere current is defined as that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of 2×10^{-7} N per metre length.

Magnetic Circuit

- ✓ It is defined as the route or path which is followed by magnetic flux.
- ✓ Consider a solenoid on a toroidal iron ring having a magnetic path of l metre, area of cross section A m² and a coil of N turns carrying I amperes wound anywhere on it as shown in **Fig. 3.12**.
- ✓ Field strength inside the solenoid is

$$H = \frac{NI}{l}$$
 AT/m

✓ Flux density inside the solenoid is

$$B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \quad \text{Wb/m}^2$$

✓ Total flux produced

$$\emptyset = B \times A = \frac{\mu_0 \mu_r NAI}{l}$$
 Wb

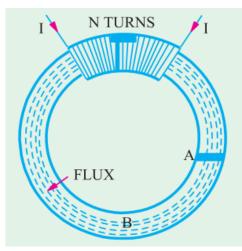


Fig. 3.12

Magnetic Circuit

✓ Total flux produced

$$\emptyset = B \times A = \frac{NI}{l/\mu_0 \mu_r A}$$
 Wb

- ✓ The numerator 'Nl' which produces magnetization in the magnetic circuit is known as magnetomotive force (m.m.f.). It's unit is ampere-turn (AT). It is analogous to e.m.f. in an electric circuit.
- ✓ The denominator $\frac{l}{\mu_0 \mu_r A}$ is called the reluctance of the circuit and is analogous to resistance in electric circuits.

$$\therefore \text{ flux} = \frac{\text{m.m.f.}}{\text{reluctance}} \quad \text{or, } \emptyset = \frac{F}{S}$$

✓ This equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuits *i.e.*

current =
$$\frac{\text{e.m.f.}}{\text{resistance}}$$
 or, $I = \frac{V}{R}$

Magnetic Circuit

✓ **Reluctance:** It is the property of a material which opposes the creation of magnetic flux in it. It measures the opposition offered to the passage of magnetic flux through a material.

It is analogous to resistance in an electric circuit. It's unit is AT/Wb.

Reluctance =
$$\frac{l}{\mu_0 \mu_r A}$$
 Resistance = $\rho \frac{l}{A} = \frac{l}{\sigma A}$

- ✓ The reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since 1 AT/Wb = 1/henry, the unit of reluctance is "reciprocal henry."
- ✓ **Permeance:** It is reciprocal of reluctance and implies the case or readiness with which magnetic flux is developed.

It is analogous to conductance in electric circuits. It's unit is Wb/AT or henry.

> Comparison Between Magnetic and Electric Circuits.

SIMILARITIES

Magnetic Circuit	Electric Circuit
FLUX Φ	EMF SAMMAN
$Flux = \frac{m.m.f.}{reluctance}$	$Current = \frac{e.m.f.}{resistance}$
M.M.F. (ampere-turns)	E.M.F. (volts)
Flux Φ (webers)	Current I (amperes)
Flux density B (Wb/m ²)	Current density (A/m ²)
Reluctance $S = \frac{l}{\mu A} \left(= \frac{l}{\mu_0 \mu_r A} \right)$	resistance $R = \rho \frac{l}{A} = \frac{l}{\rho A}$
Permeance (= 1/reluctance)	Conductance (= 1/resistance)
Reluctivity	Resistivity
Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
Total m.m.f. = $\Phi S_1 + \Phi S_2 + \Phi S_3 +$	Total e.m.f. = $IR_1 + IR_2 + IR_3 +$

Comparison Between Magnetic and Electric Circuits.

DIFFERENCES

- 1. Flux does not actually flow but electric current flows.
- 2. If temperature is kept constant, then resistance of an electric circuit is constant and is independent of the current strength (or current density).
 - The reluctance of a magnetic circuit does depend on flux (and hence flux density) established in it.
 - It is so because μ (which equals the slope of B/H curve) is not constant even for a given material as it depends on the flux density B.
 - Value of μ is large for low value of B and vice-versa. Hence, reluctance is small $(S = l/\mu A)$ for small values of B and large for large values of B.
- 3. Flow of current in an electric circuit involves continuous expenditure of energy but in a magnetic circuit, energy is needed only creating the flux initially but not for maintaining it.

LECTURE 8

➤ Composite Series Magnetic Circuit

- ✓ Fig. 3.13 shows a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths as well as one air gap ($\mu_r = 1$).
- ✓ According to Ampere Circuital Law,

$$NI = Hl = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$
or,
$$\frac{B}{\mu} l = \frac{B_1}{\mu_1} l_1 + \frac{B_2}{\mu_2} l_2 + \frac{B_3}{\mu_3} l_3 + \frac{B_g}{\mu_g} l_g$$
or,
$$\frac{\emptyset}{\mu A} l = \frac{\emptyset}{\mu_1 A_1} l_1 + \frac{\emptyset}{\mu_2 A_2} l_2 + \frac{\emptyset}{\mu_3 A_3} l_3 + \frac{\emptyset}{\mu_g A_g} l_g$$

$$\therefore \frac{l}{\mu A} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \frac{l_g}{\mu_g A_g}$$

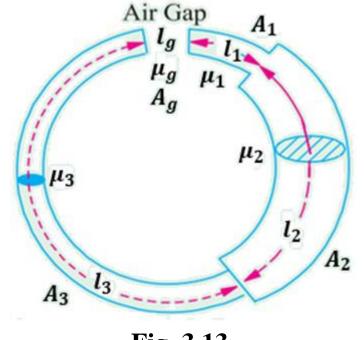
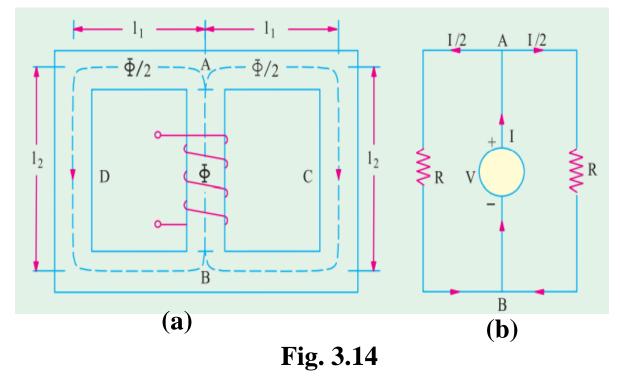


Fig. 3.13

✓ Each path will have its own reluctance and the total reluctance is the sum of individual reluctances as they are joined in series.

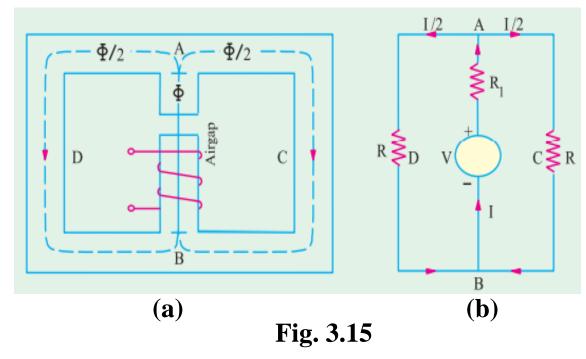
> Parallel Magnetic Circuits

- ✓ Parallel magnetic circuit as shown in **Fig. 3.14(a)** consists of two parallel magnetic paths *ACB* and *ADB* acted upon by the same m.m.f.
- ✓ Each magnetic path has an average length of $2(l_1 + l_2)$.
- ✓ The flux produced by the coil wound on the central core is divided equally at point *A* between the two outer parallel paths.
- ✓ The reluctance offered by the two parallel paths is half of the reluctance of each path.
- ✓ The equivalent electrical circuit as shown in **Fig.** 3.14 (b) has resistance $R \parallel R = R/2$.



Series-Parallel Magnetic Circuits

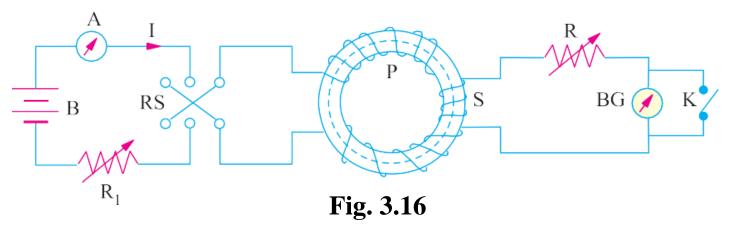
- ✓ Two parallel magnetic circuits ACB and ADB are connected across the common magnetic path AB which contains an air-gap of length l_g as shown in **Fig. 3.15**.
- ✓ The flux \emptyset in the common core is divided equally at point A between the two parallel paths which have equal reluctance.
- \checkmark The reluctance of the path AB consists of (i) air gap reluctance and (ii) the reluctance of the central core which comparatively negligible.
- ✓ The m.m.f. required for this circuit would be the sum of (i) that required for the air-gap and (ii) that required for either of two paths.
- The equivalent electrical circuit as shown in Fig. 3.15 (b) has the total resistance offered to the voltage source is $R_1 + R \parallel R = R_1 + R/2$.



Magnetisation Curves

- ✓ The magnetization curves of a magnetic material is determined by the following methods.
 - (a) by means of a ballistic galvanometer and
 - (b) by means of a fluxmeter.

✓ Magnetisation Curves by Ballistic Galvanometer



- A specimen ring of uniform cross-section wound uniformly with a coil P which is connected to a battery B through a reversing switch RS, a variable resistance R_1 and an ammeter A as shown in **Fig. 3.16**.
- Secondary coil S wound over a small portion of the ring and is connected through a resistance R to a ballistic galvanometer BG.

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➤ Magnetisation Curves

- ✓ Magnetisation Curves by Ballistic Galvanometer
 - The current through the primary P is adjusted with the help of R_1 . Suppose the primary current is I.
 - When the primary current is reversed by means of *RS*, then flux is reversed through *S*.
 - Hence an induced e.m.f. is produced in it which sends a current through *BG*. This current is of very short duration.
 - The first deflection or 'throw' of the BG is proportional to the quantity of electricity or charge passing through it. The time taken for this charge to flow is short as compared with the time of one oscillation

 θ = first deflection or 'throw' of the galvanometer when primary current I is reversed.

k = ballistic constant of the galvanometer i.e. charge per unit deflection.

 \therefore Charge passing through BG is $k \theta$ coulombs

Let $\Phi = \text{flux in Wb produced by primary current of } I \text{ amperes}$ t = time of reversal of flux

 \therefore Rate of change of flux = $\frac{2\Phi}{t}$ Wb/s

Magnetisation Curves

- ✓ Magnetisation Curves by Ballistic Galvanometer
 - If N_2 is the number of turns in secondary coil S, then average e.m.f. induces in it N_2 . $\frac{2\Phi}{t}$ volt.
 - Secondary current or current through $BG = \frac{2N_2\Phi}{R_St}$ Amperes, where, R_S is the total resistance of the secondary circuit.
 - Charge flowing through BG = average current × time = $\frac{2N_2\Phi}{R_St}$ × $t = \frac{2N_2\Phi}{R_S}$ coulomb

$$k \theta = \frac{2N_2\Phi}{R_s}$$
 $\therefore \Phi = \frac{k \theta R_s}{2N_2}$ Wb

• If $A \text{ m}^2$ is the cross-sectional area of the ring, then flux density is

$$B = \frac{\Phi}{A} = \frac{k\theta R_s}{2N_2 A} \text{ Wb/m}^2$$

• If N_1 is the number of primary turns and l metres the mean circumference of the ring, then, magnetising force $H = N_1 I/l$ AT/m.

> Magnetisation Curves

✓ Magnetisation Curves by Ballistic Galvanometer

The above experiment is repeated with different values of primary current and form the data so obtained, the B/H curves or magnetisation curves is drawn and shown in **Fig. 3.17**.

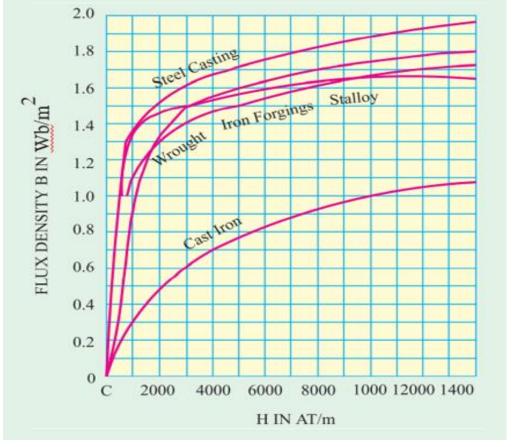


Fig. 3.17

Magnetisation Curves

- **✓** Magnetisation Curves by Fluxmeter
 - The BG of Fig. 3.16 is replaced by a fluxmeter which is just a special type of ballistic galvanometer.
 - When current through *P* is reversed, the flux is also reversed.
 - The deflection of the fluxmeter is proportional to the change in flux-linkages of the secondary coil.
 - If the flux is reversed from $+\Phi$ to $-\Phi$, the change in flux-linkages in secondary S is ΦN_2 .
 - If θ = corresponding deflection of the fluxmeter
 - C = fluxmeter constant *i.e.* weber-turns per unit deflection.
 - \therefore Change of flux-linkages in S is $C \theta$

$$2 \Phi N_2 = C \theta$$

or, $\Phi = \frac{C \theta}{N_2}$ Wb $B = \frac{\Phi}{A} = \frac{C \theta}{N_2 A}$ Wb/m²

Example – P3.1

A ring has a diameter of 21 cm and a cross-sectional area of 10 cm^2 . The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of $8 \times 10^{-4} \text{ Wb}$. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.

Solution of Example – P3.1

$$\Phi = 8 \times 10^{-4} \text{Wb}; A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2; B = 8 \times 10^{-4} / 10^{-3} = 0.8 \text{ Wb/m}^2$$

Air gap

Total air-gap length = $2 \times 0.2 = 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m}$

:. AT required = $H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$

Cast Steel Path

 $H = B/\mu_0 \mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796 \text{ AT/m}$

path = $\pi D/2 = 21 \pi/2 = 33 \text{ cm} = 0.33 \text{ m}$

 \therefore AT required = $H \times l = 796 \times 0.33 = 263$

Cast Iron Path

 $H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835 \text{ AT/m}$; path = 0.33 m

 \therefore AT required = 3,835 × 0.33 = 1265

 \therefore Total AT required = 255 + 263 + 1265 = 1783

Example – P3.2

A magnetic core in the form of a closed circular ring has a mean length of 30 cm and a cross-sectional area of 1 cm². The relative permeability of iron is 2400. What direct-current will be needed in the coil of 2000 turns uniformly wound around the ring to create a flux of 0.20 mWb in iorn? If an air-gap of 1 mm is cut through the core perpendicualr to the direction of this flux, what current will now be needed to maintain the same flux in the air gap?

Solution of Example – P3.2

Reluctance of core =
$$\frac{1}{\mu_0 \mu_r} \frac{L}{a} = \frac{1}{10\pi \times 10^{-7} \times 2400} \times \frac{30 \times 10^2}{1 \times 10^{-4}}$$
$$= \frac{30 \times 10^{-9}}{4\pi \times 2400 \times 1} = 995223 \text{ MKS units}$$
$$\phi = 0.2 \times 10^{-3} \text{Wb}$$

MMF required = $\phi \times \text{Reluctance of core}$

$$= 0.2 \times 10^{-3} \times 995223 = 199$$
 amp-tunrs

Direct current required through the 2000 turn coil = $\frac{199}{2000}$ = **0.0995** amp

Reluctance of 1 mm air gap =
$$\frac{1}{4\pi \times 10^{-7}} \times \frac{1 \times 10^{-3}}{1 \times 10^{-4}} = \frac{10^8}{4\pi} = 7961783$$
 MKS units

Addition of two reluctances = 995223 + 7961783 = 8957006 MKS units

Solution of Example – P3.2

MMF required to establish the given flux = $0.2 \times 10^{-3} \times 8957006 = 1791$ amp turns

Current required through the coil
$$=\frac{1791}{2000} = 0.8995$$
 amp

Note: Due to the high permeability of iron, which is given here as 2400, 1 mm of air-gap length is equivalent magnetically to 2400 mm of length through the core, for comparison of mmf required.

Example – P3.3

A fluxmeter is connected to a search-coil having 600 turns and mean area of 4 cm². The search coil is placed at the centre of an air-cored solenoid 1 metre long and wound with 1000 turns. When a current of 4 A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.

Solution of Example – P3.3

Magnetising force of the solenoid is H = Nl/l AT/m

:
$$B = \mu H = \mu NI/l = 4\pi \times 10^{-7} \times 1000 \times 4/1 = 16\pi \times 10^{-4} \text{Wb/m}^2$$

Flux linked with the search coil is $\Phi = BA = 64\pi \times 10^{-8}$ Wb

Total change of flux-linkages on reversal =
$$2 \times 64\pi \times 10^{-8} \times 600$$
 Wb-turns = $7.68\pi \times 10^{-4}$ Wb-turns

Fluxmeter constant
$$C$$
 is given by = $\frac{\text{Change in flux-linkages}}{\text{Deflection produced}}$

$$= 7.68\pi \times 10^{-4}/20 = 1.206 \times 10^{-4}$$
 Wb-turns/division

Example – P3.4

A ballistic galvanometer, connected to a search coil for measuring flux density in a core, gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180 ohm. The galvanometer constant is 100 μ C per scale division. The search coil has an area of 50 cm², wound with 1000 turns having a resistance of 20 ohm. Calculate the flux density in the core.

Solution of Example – P3.4

We know,

$$K \theta = 2 N_2 \Phi / R_s$$

or, $\Phi = k \theta R_s / 2 N_2$ Wb

$$\therefore B = k \theta R_s / 2 N_2 A$$

$$k = 100 \mu \text{C/division} = 100 \times 10^{-6} = 10^{-4} \text{C/division}$$

$$\theta = 100;$$
 $A = 50 \text{ cm}^2 = 5 \times 10^{-3} \text{ m}^2$

$$R_s = 180 + 20 = 200 \Omega$$

$$B = 10^{-4} \times 100 \times 200/2 \times 1000 \times 5 \times 10^{-3} = 0.2 \text{ Wb/m}^2$$

