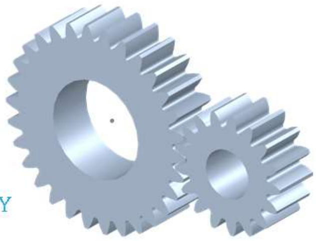
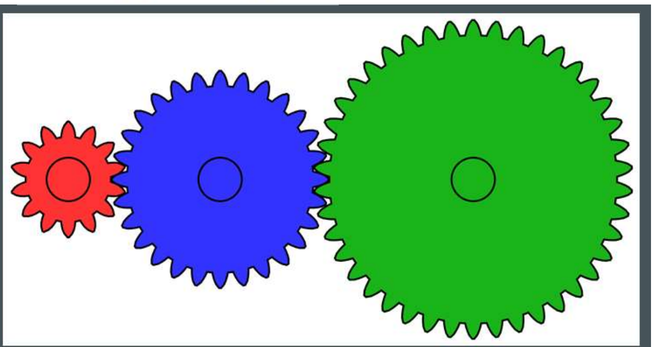
GEARS

TYPES, GEAR GEOMETRY & GEAR TERMINOLOGY

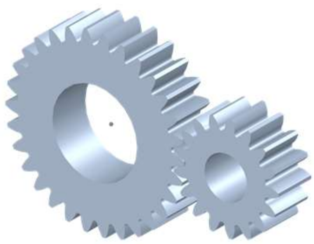




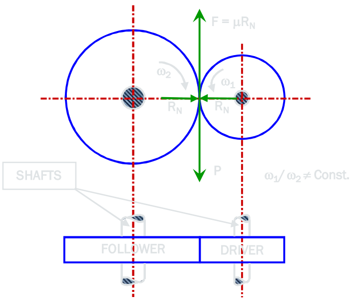
Department of Mechanical Engineering
NATIONAL INSTITUTE OF TECHNOLOGY DURGAPUR

GEARS or TOOTHED WHEEL

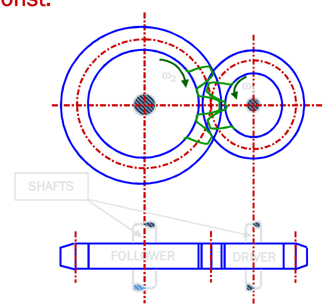
- a toothed element & higher pair mechanism
- Commonly used for transmitting rotary motions from one shaft to another by successively engaging teeth
- Pair of gears are used for transmission of constant angular velocity ratio between two shafts



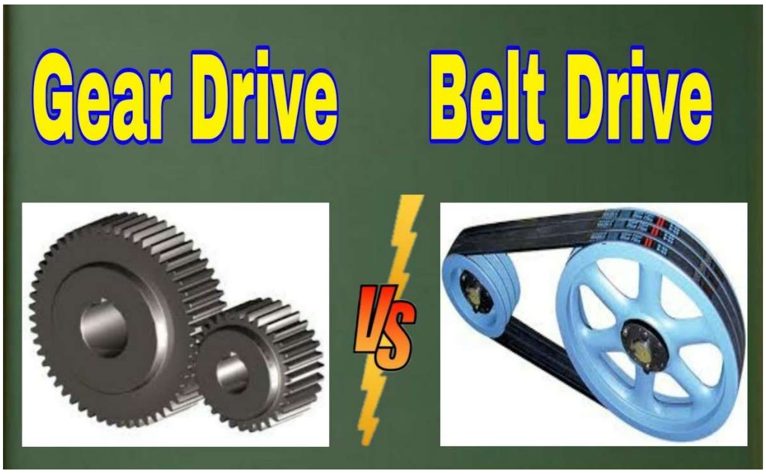
$\omega_1 / \omega_2 = \text{Const.}$



FRICTION WHEEL



GEAR or TOOTHED WHEEL



✓ **Positive Drive**

✓ **Lower Centre Distance**

Types of Gears

- **Spur gear**
- **Helical gear**
- **Herringbone gear**
- **Bevel gear**
- **Spiral bevel gear**
- **Worm & Worm wheel**
- **Rack & Pinion …**

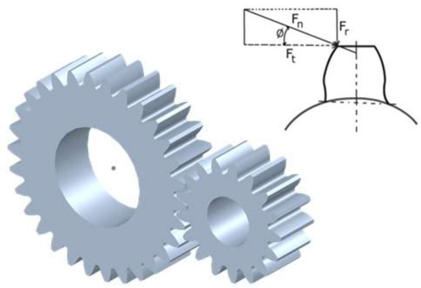
Pinion: *the smaller of two mating gears is called pinion*

Gear or Wheel: *the bigger or larger gear in gear pair is called Gear or wheel*

Types of Gears

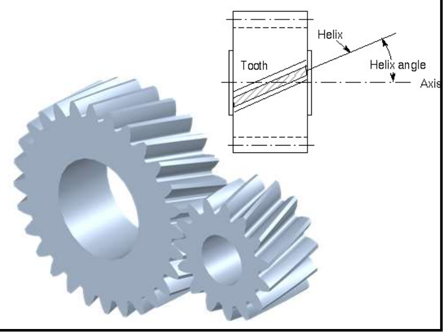
Spur gear

- Spur gears have straight teeth parallel to the axis of rotation
- Mounted on two parallel shafts to transmit motion
- At the time of engagement, entire face width (on straight line parallel to axis of rotation) of the tooth comes in contact with mating tooth. This results in sudden application of the load, high impact stresses & excessive noise at high speeds.
- As teeth are parallel to axis, spur gears are not subjected to axial thrust due to teeth load



Helical gear

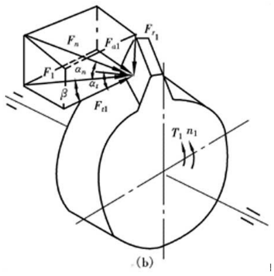
- In helical gear, teeth are inclined at an angle (called helix angle) with the axis of rotation (gear axis) [Helix angle=15° to 25°]
- Contact occurs at the point of leading edge of the teeth during the beginning of engagement. As the gears rotate, the contact extends along a line across the face of the teeth. Thus, the load applied is gradual resulting in low impact stresses & less noise
- The helical gear can be used in higher velocities than spur gears & have greater load carrying capacity

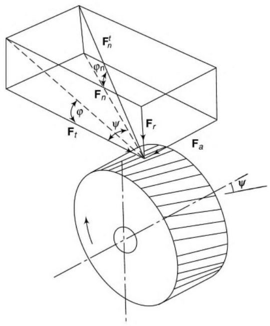
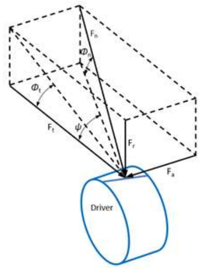


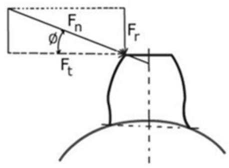
Helical gears

Drawbacks

- As teeth are inclined to gear axis, helical gears are subjected to axial thrust due to force component along the gear axis
- The bearings & the assemblies mounting the helical gears must be able to withstand this axial force.





Double helical gear & Herringbone gear

- Axial thrust which occurs in case of helical gear is eliminated by the use of double helical gears or Herringbone gears
- A double helical gear is equivalent to a pair of helical gears secured together, one having a right-hand helix & the other a left-hand helix. The teeth of the two rows are separated by a groove used for tool run out.
- If the left & the right inclinations of a double helical gear meet at a common apex & there is no groove in between, the gear is known as Herringbone gear.
- The axial thrust of the two rows (left & right inclinations) of teeth cancel each other

Teeth are cut on cylindrical blank or cylinder

Types of Gears

Bevel gear or Straight Bevel gear

- When teeth formed on the truncated cones are straight, the gears are known as Bevel gear or Straight bevel gear
- The teeth are straight, radial to the point of intersection of the shaft axes & vary in cross section throughout their length.
- At the beginning of engagement, straight bevel gears make the line contact similar to spur gears
- Kinematically, bevel gears are equivalent to rolling cones

Mitre gear

- Bevel gears of the same size & connecting two shafts at right angles to each other are known as Mitre gear

Spiral bevel gear

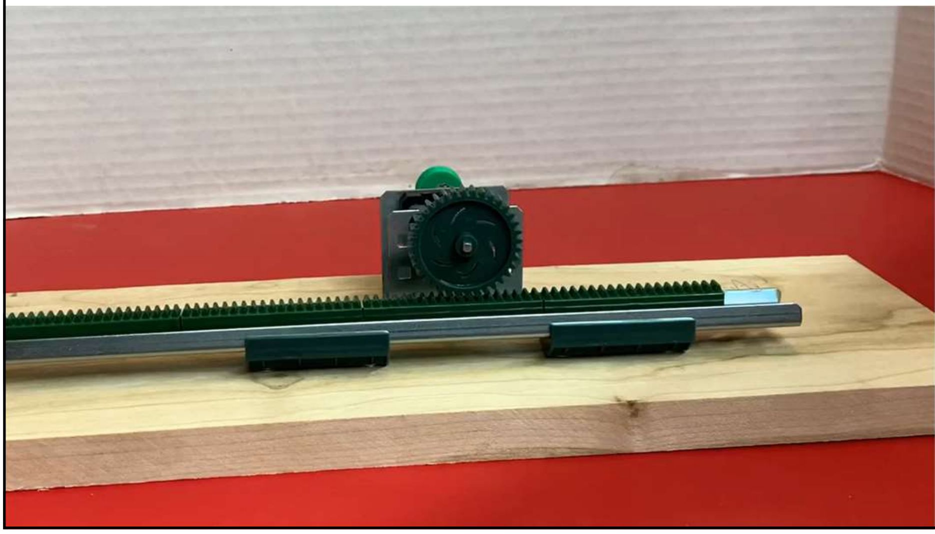
- When teeth formed on the truncated cones are inclined at an angle to the face of the bevel, they are known as Spiral Bevel gear
- They are smoother in action & quieter than straight bevel gears as there is gradual load application.

Worm & Worm wheel

- Used to transmit rotary motion between non-parallel & non-intersecting shafts
- There are screw threads on the worm & teeth on the worm wheel

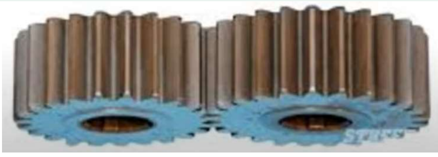
Rack & Pinion

- Convert rotary motion to linear motion and vice-versa



Spur Gear Vs. helical Gear

Spur Gear	Helical Gear
Teeth are parallel to axis of gear or motion axis	Teeth are inclined to axis of gear or motion axis
Imposes only radial load on bearings	Imposes radial load & axial thrust load on bearings
At the time of engagement, entire face width (on straight line parallel to axis of rotation) of the tooth comes in contact with mating tooth. (Sudden engagement of teeth)	Contact occurs at the point of leading edge of the teeth during the beginning of engagement. As the gears rotate, the contact extends along a line across the face of the teeth. (Gradual engagement of teeth)
This results in sudden application of the load, high impact stresses & vibration & excessive noise at high speeds	Thus, the load applied is gradual resulting in low impact stresses & less noise, smooth & quite operation
Low velocity application & less load carrying capacity	can be used in higher velocities than spur gears & have greater load carrying capacity
Low to medium speed application	Relatively high speed application



Classification of Gears

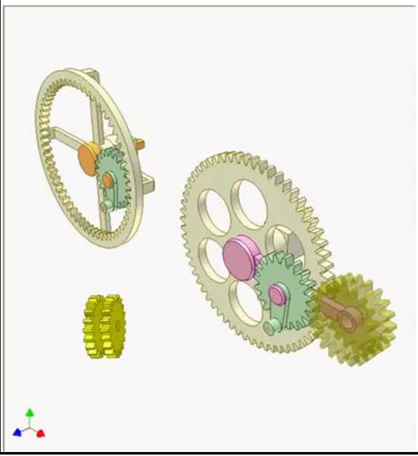
According to the relative position of axes of shafts

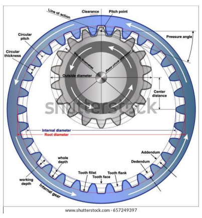
- **Parallel**
  - Spur gear
  - Helical gear
  - Double helical gear & Herringbone gear
- **Intersecting**
  - Straight Bevel gears
  - Spiral Bevel gear
- **Non-parallel & non-intersecting**
  - Worm & Worm wheel

Classification of Gears

According to the type of gearing:

- **External gearing**
- **Internal gearing**
- **Rack & Pinion**





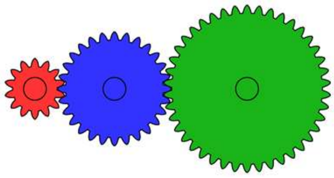
Classification of Gears

According to the peripheral velocity of the gears:

- Low velocity : < 3 m/s
- Medium velocity : 3 m/s to 15 m/s
- High velocity : > 15 m/s

ADVANTAGES and DISADVANTAGES of GEAR drive

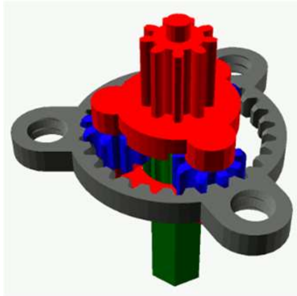
ADVANTAGES	DISADVANTAGES
1. Transmits exact velocity ratio	1. Costly drive
2. Transmit heavy torque *(wide range of power transmission)*	2. Special manufacturing tools
3. Has high efficiency	3. Noise for error tooth cutting
4. Has reliable & simple service	4. Vibration for incorrect tooth profile
5. Compact Design	5. Need suitable lubrication



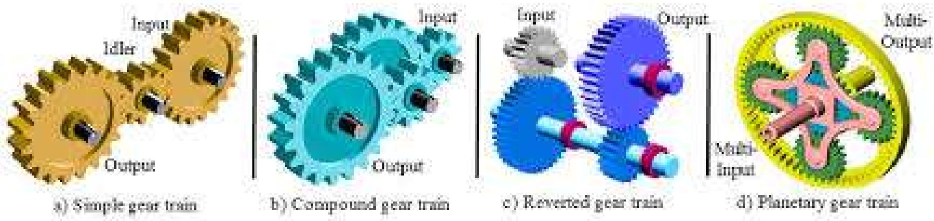
Gear Train : A combination of gears that are used for transmitting motion from one shaft to another shaft

Types of Gear Train

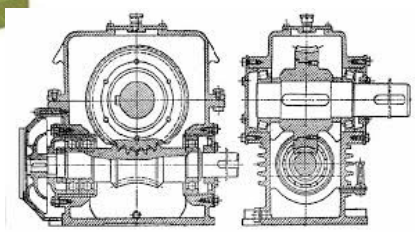
- Gear Train
  - Simple Gear Train
  - Compound Gear Train
  - Reverted Gear Train
  - Epicyclic Gear Train



Epicyclic : axes of rotation of the gears are not fixed in space



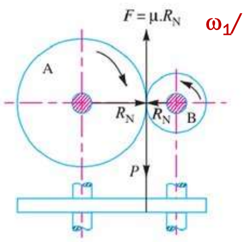
Applications



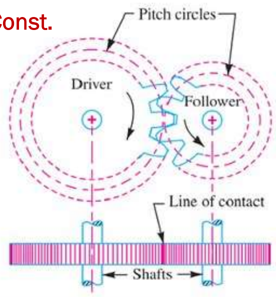
Applications

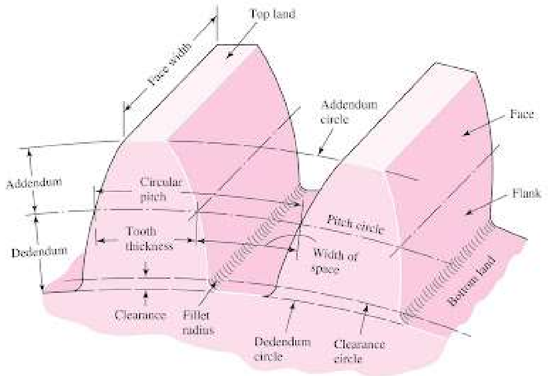
Spur Gear Geometry
&
Spur Gear Terminology

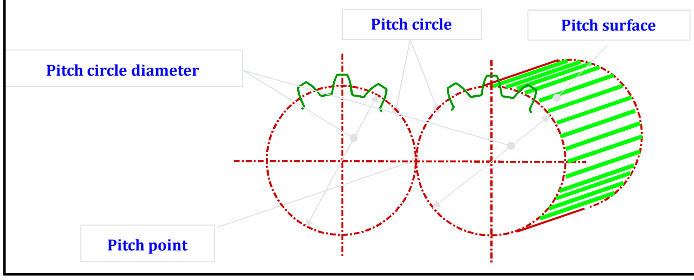
Gear Terminology



ω_1 / ω_2 = Const.







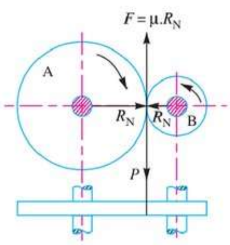
Pitch cylinders

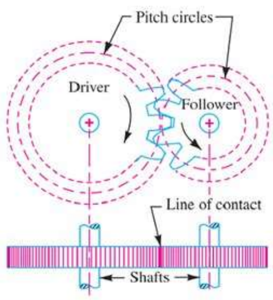
- an imaginary friction cylinders, which by pure rolling together, transmit the same motion as the pairs of gears.

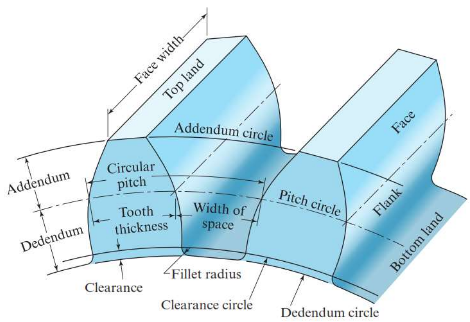
Pitch circle :

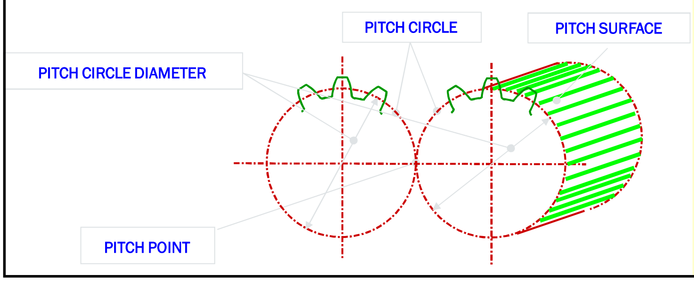
- an imaginary circle gives same motion of actual gear on pure rolling action.

Gear Terminology









Pitch circle diameter :

- diameter of pitch circle or pitch cylinder, also known as pitch diameter.

Pitch point :

- point of contact of two pitch circles.

Pitch surface :

- surface of pitch cylinder.

Line of centres:

- A line through the centre of rotation of a pair of mating gears.

Gear Terminology

Top Land : Surface of the top of the tooth

Bottom Land : Surface of the bottom of the tooth between adjacent fillets

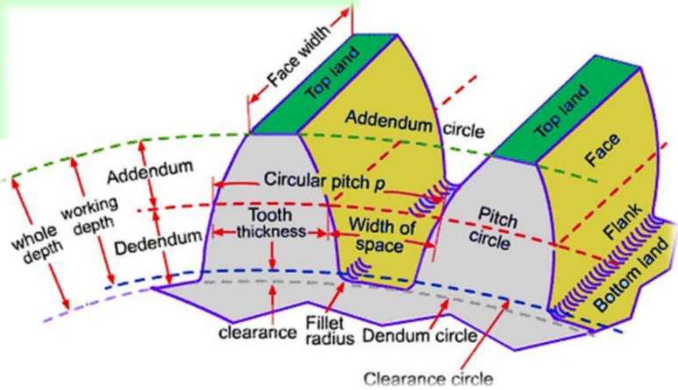
Face : Tooth surface above pitch surface.

Flank : Tooth surface between pitch surface & bottom land.

Face Width : Tooth width along gear axis

Profile : Curve formed by the flank and face.

Fillet Radius : Radius connects dedendum circle to tooth profile.



Gear Terminology

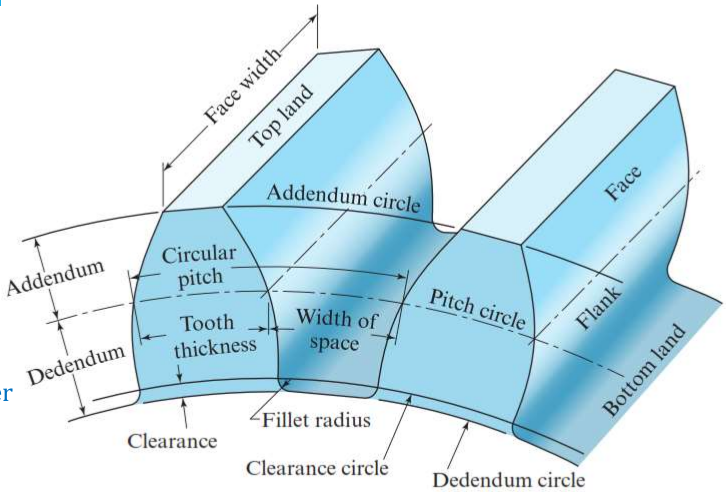
Pitch
- Circular pitch (p)
- Diametral pitch (P)

Circular Pitch

The distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point of the adjacent tooth

$$p = \frac{\pi d}{T}$$

Where,
d=Pitch circle diameter
T=number of teeth
p=Circular pitch

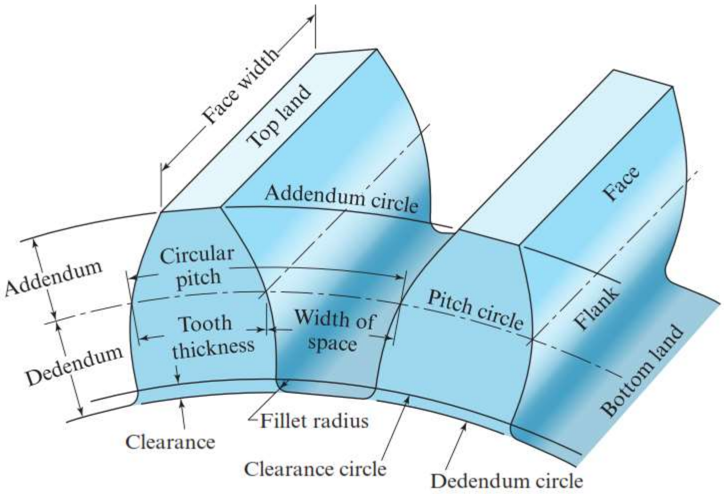


Pitch angle

The angle subtended by the circular pitch at the centre of the pitch circle.

Gear Terminology

Pitch
- Circular pitch (p)
- Diametral pitch (P)



Circular Pitch (p)

$$p = \frac{\pi d}{T}$$

Where,
d=Pitch circle diameter
T=number of teeth
p=Circular pitch

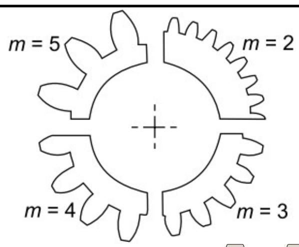
Diametral pitch (P)

The number teeth per unit length of the pitch circle diameter

$$P = \frac{T}{d}$$

$$pP = \frac{\pi d}{T}\frac{T}{d} = \pi$$

Gear Terminology



Diametral pitch (P)

The number teeth per unit length of the pitch circle diameter

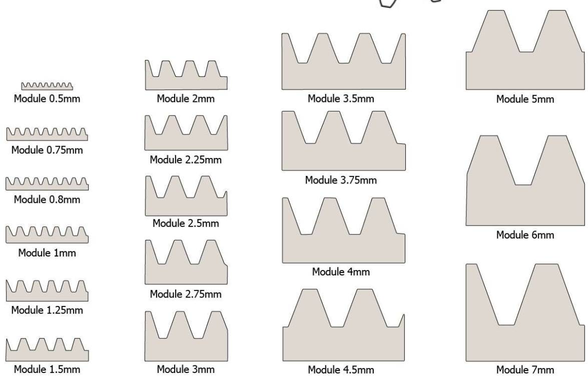
$$P = \frac{T}{d}$$

Module (m)

The ratio of pitch circle diameter (in mm) of gear to the number teeth
The reciprocal of the diametral pitch

$$m = \frac{d}{T}$$

Where,
d=Pitch circle diameter (mm)
T=number of teeth
m=module (mm)



m=module (mm)

Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Gear Terminology

Addendum circle : circle through top land of the teeth, concentric to pitch circle

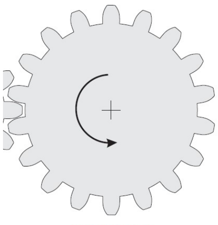
Dedendum circle : circle through bottom land of the teeth, concentric to pitch circle.

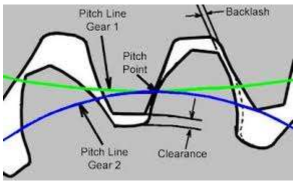
Clearance circle : circle touching addendum circle of meshing gear, concentric to pitch circle.

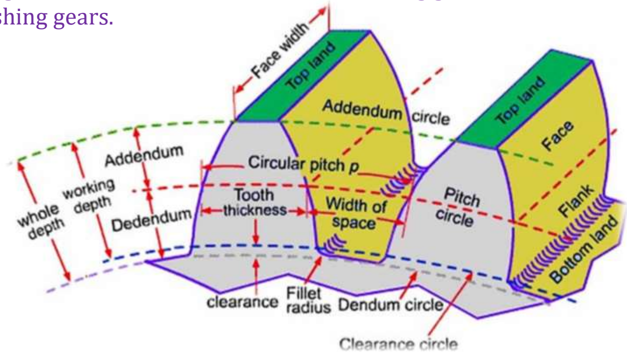
Addendum : tooth height from pitch circle to top of tooth i.e radial dist. between pitch circle & addendum circle

Dedendum : tooth depth from pitch circle to bottom of tooth i.e radial dist. between pitch circle & dedendum circle

Clearance : the amount by which the dedendum of a gear exceeds the addendum of the mating gear or radial distance between dedendum circle and addendum circle of meshing gears.





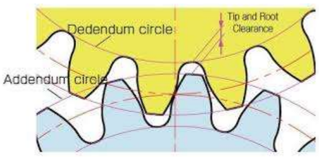


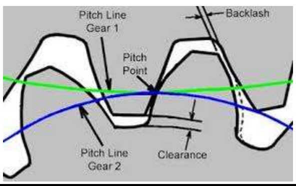
Gear Terminology

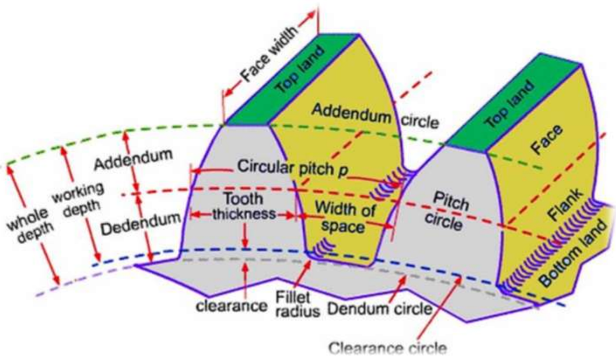
Full depth of teeth : total radial depth of the tooth space
Full depth=Addendum+Dedendum

Working depth of teeth : the maximum depth to which a tooth penetrates into the tooth space of the mating gear
Working depth=Sum of addendums of the two mating gears

	Standard value (mm) (20° full depth Involute)
Addendum	m
Dedendum	1.25×m
Addendum circle diameter	d+2×m
Dedendum circle diameter	d-2×1.25×m
Clearance	0.25×m







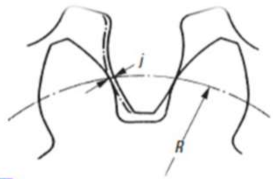
Gear Terminology

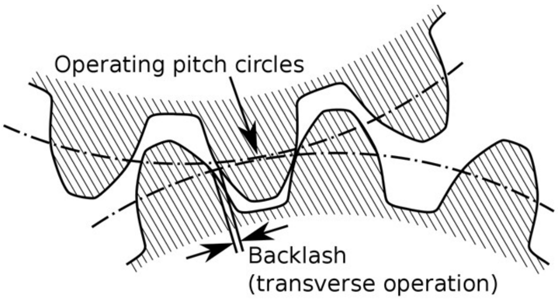
Tooth thickness : thickness of the tooth measured along the pitch circle

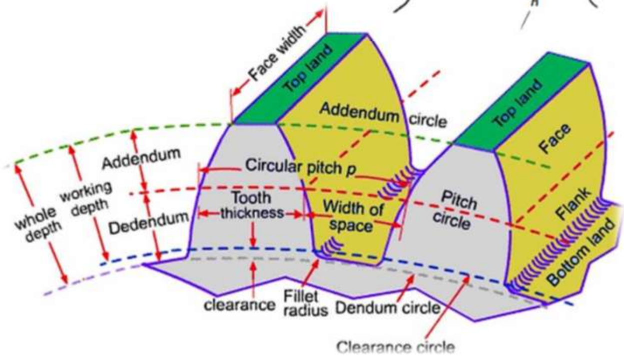
Space width : width of the tooth space along the pitch circle

Backlash : the difference between the space width & the tooth thickness along the pitch circle
Backlash=Space width – Tooth thickness

Face width : the length or width of the tooth parallel to the gear axis







Velocity ratio (VR)

It is the ratio of angular velocity of the driving gear to the angular velocity of driven gear

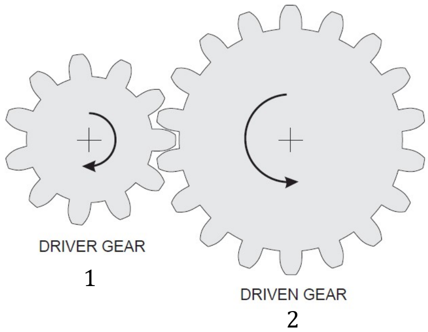
Velocity ratio= Angular velocity of driver gear (ω_1)/ Angular velocity of driven gear (ω_2)

Velocity ratio= ω_1/ω_2

Velocity ratio= N_1/N_2 $\omega_1=2\pi N_1/60$; $\omega_2=2\pi N_2/60$

Velocity ratio= d_2/d_1 $\pi d_1 N_1=\pi d_2 N_2$

Velocity ratio= T_2/T_1 $m=d_1/T_1=d_2/T_2$



	Max. Velocity Ratio
Spur gear	6:1
Helical gear	10:1
Worm gear	100:1

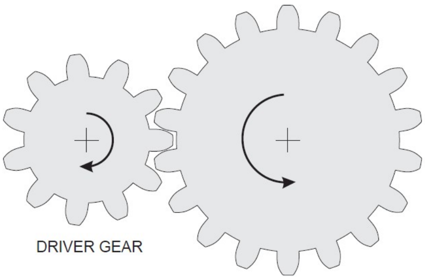
Gear ratio (G)

It is the ratio of number of teeth on the gear (T) to number of teeth on the pinion (t)

$$G = \frac{T}{t}$$

Example 1

Two spur gears have a velocity ratio 4:1. the driven gear has 80 teeth of 8 mm module & rotates at 240 rpm. Calculate: (i) the number of teeth, (ii) the speed of the driver, (iii) the pitch line velocities



Solution:

Number of teeth on driven gear (T_2)=80 teeth
RPM of driven gear (N_2)=240 rpm.

Velocity ratio= N_1/N_2 $4=N_1/240$ $N_1=240\times4=960$ rpm

Velocity ratio= T_2/T_1 $4=80/T_1$ $T_1=80/4=20$

Module $m=d_1/T_1=d_2/T_2$ $8=d_1/20$ $d_1=160$ mm

$\omega_1=2\pi N_1/60$; $\omega_2=2\pi N_2/60$

Pitch line velocity (v_p)= $\omega_1 r_1=\omega_1 d_1/2=2\pi N_1 d_1/(60\times2)$

Pitch line velocity (v_p)= $2\pi\times960\times160/(60\times2)=8038.4$ mm/s

Example 2

The number of teeth of a spur gear is 40 & it rotates at 200 rpm. What will be its pitch line velocity if it has a module of 2 mm?

Solution:

Number of teeth on gear (T)=40
RPM of gear (N)=200 rpm.
Module (m)=2 mm

Module $m=d/T$

$2=d/40$ $d=80$ mm

$\omega=2\pi N/60$

Pitch line velocity (v_p)= $\omega r=\omega d/2=2\pi Nd/(60\times2)$

Pitch line velocity (v_p)= $2\pi\times200\times80/(60\times2)=837.3$ mm/s

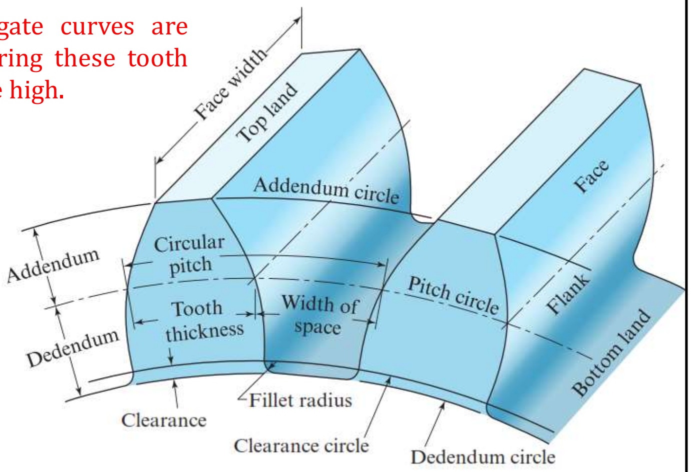
Forms of Gear Tooth

- Two curves of any shape that fulfil the law of gearing can be used as the profiles of teeth.
- An arbitrary shape of one of the mating teeth can be taken & applying the law of gearing the shape of the other can be determined. Such gears are said to have Conjugate Teeth.
- Even though a large number of conjugate curves are possible, real problem lies in manufacturing these tooth profiles in large quantities. The cost will be high.
- Difficulties in interchangeability

Thus, there arises the need to standardize forms of gear tooth

Common forms of gear teeth

- **Involute profile tooth**
- **Cycloidal profile tooth**

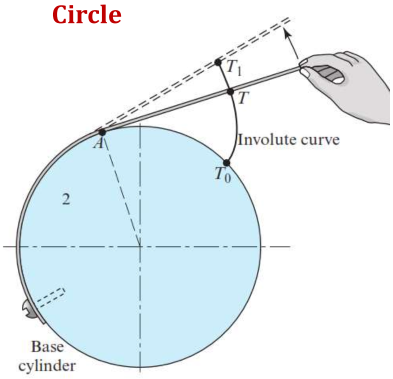


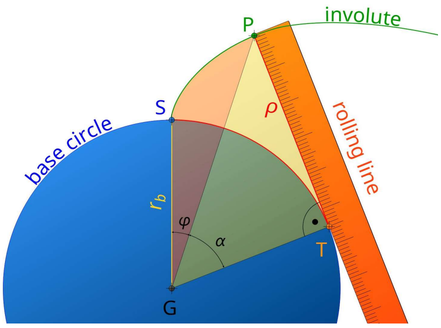
Involute profile tooth

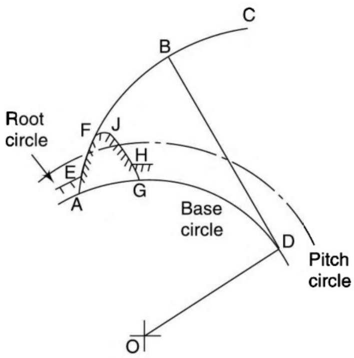
An involute is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle

It is the path traced by the end of a taut cord (string/thread) as it is unwound from a stationary cylinder or circle. The string is always tangent to the cylinder or circle.

The circle on which the straight line rolls or from which the string is unwound is known as **Base Circle**

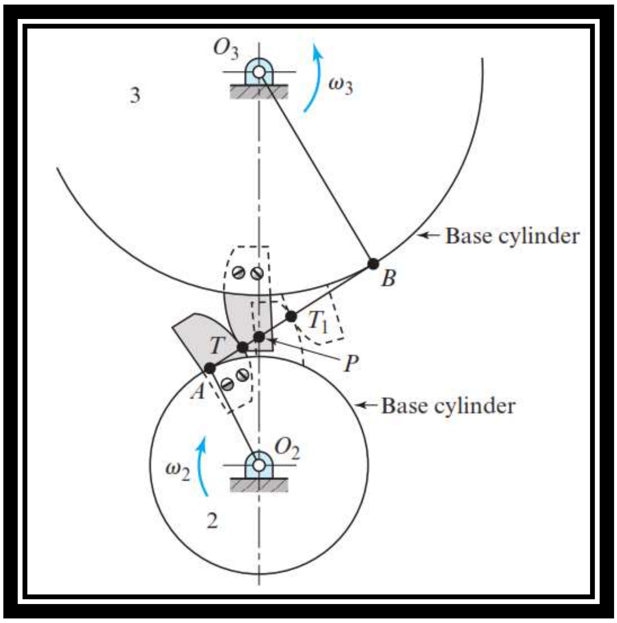






Forms of Gear Tooth

Involute profile tooth



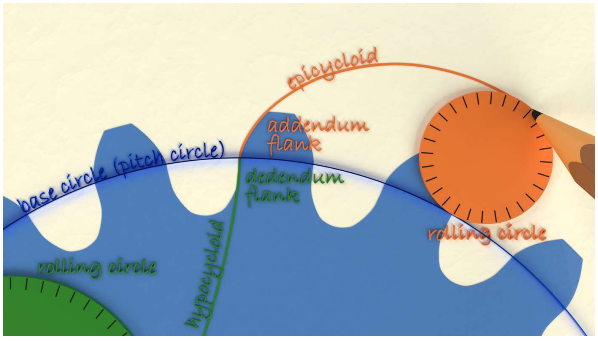
Forms of Gear Tooth

Cycloidal profile tooth

The cycloidal profile is made of two curves:

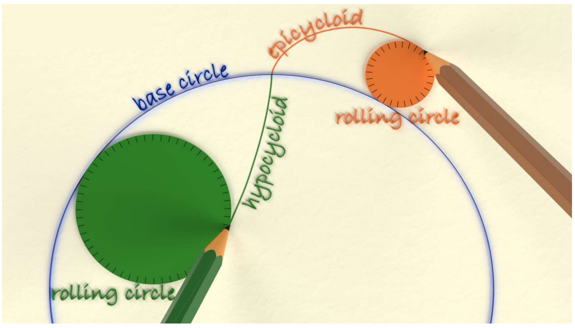
Hypocycloid : Hypocycloid curve (Concave curve) below the pitch circle

Epicycloid : Epicycloid curve (Convex curve) above the pitch circle



Hypocycloid :
curve traced by a point on the circumference of a generating circle which rolls inside of a fixed circle without slipping.

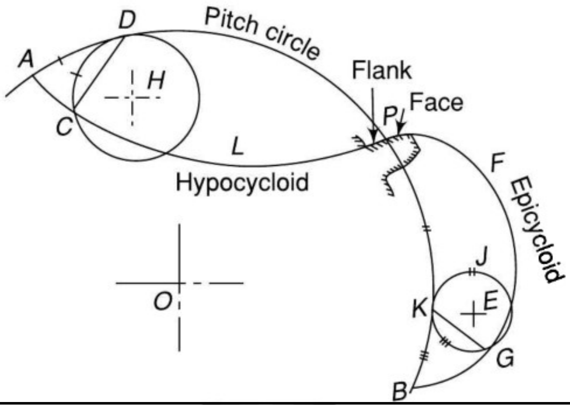
Epicycloid :
curve traced by a point on the circumference of a generating circle which rolls outside of a fixed circle without slipping.



Cycloidal teeth

Epicycloid :
curve traced by a point on the circumference of a circle which rolls outside of pitch circle without slipping.

Hypocycloid :
curve traced by a point on the circumference of a circle which rolls inside of pitch circle without slipping.



Involute profile Vs. Cycloidal profile

Involute profile teeth	Cycloidal profile teeth
Center distance can be varied within limit without changing velocity ratio.	Exact center distance to be maintained to transmit a constant velocity ratio
Pressure angle remains constant from start to end of engagement. This results in smooth running of the gears	Pressure angle is max. at start, zero at pitch point and again max. at end of engagement. This results in less smooth running of the gears
Face and flank are generated by single curve. It involves single curve for teeth resulting in simplicity of manufacturing & of tools. These are cheaper	Face is generated by epicycloid and flank is generated by hypocycloid. This complicates the manufacturing. These are costlier
Interference can occur if the condition of minimum number of teeth on a gear is not followed. **Undercut is necessary to remove interference.**	Interference does not occur at all
Thinner flank for same pitch & thus are weaker as compared to the cycloidal form for the same pitch	Wider flank for same pitch & thus are stronger
Two convex surfaces are in contact & thus there is more wear	A convex flank of one gear always has contact with a concave face of mating gear resulting in less wear

Line of action or Pressure line

The force which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth. This line is also the common normal at the point of contact of the mating gears & is known as the **Line of action** or the **Pressure line.**

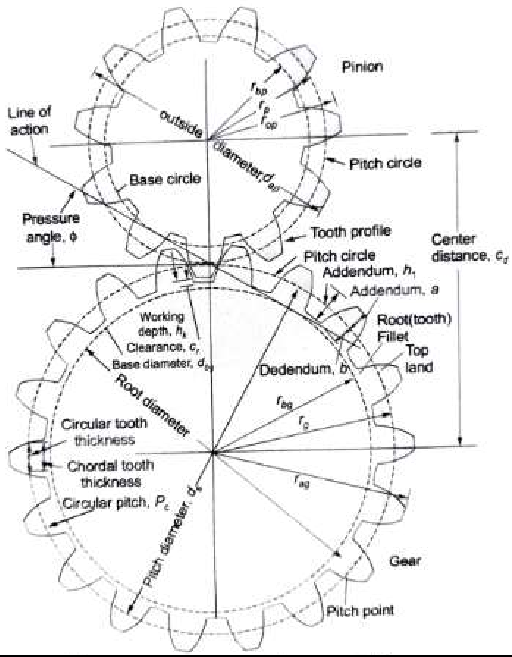
The line of action is the common tangent to the base circles of mating gears.

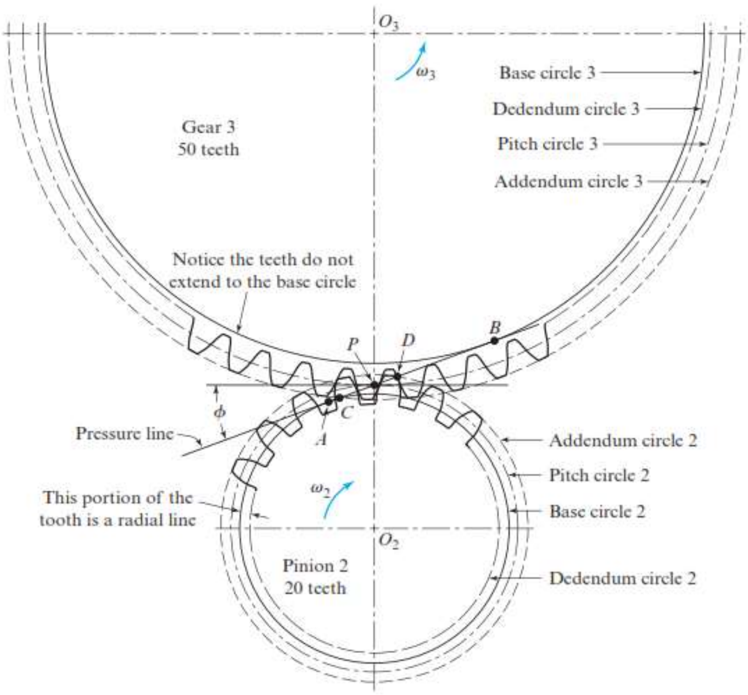
Pressure angle or Angle of obliquity

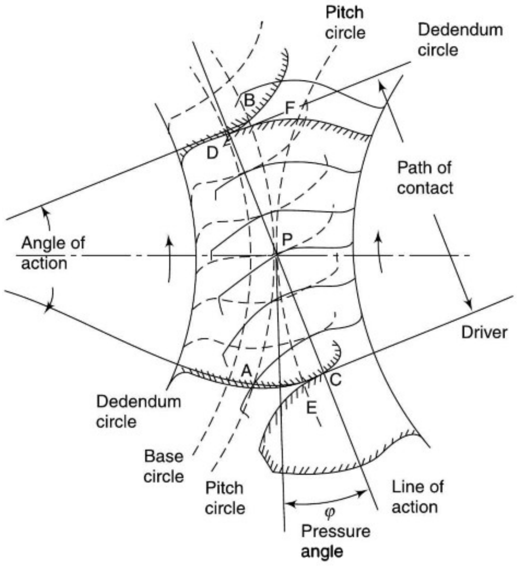
The angle between the pressure line & the common tangent to the pitch circles is known as pressure angle

The pressure angle must be kept small for more power transmission & lesser thrust on the bearings.

Standard pressure angles are 20° and 25°. Gears with 14.5° pressure angle have become almost obsolete.







Path of contact or Contact length

The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the **Path of contact** or the **Contact length.**
The pitch point **P** is always one point on the path of contact.
CD is the path of contact = **CP+PD**

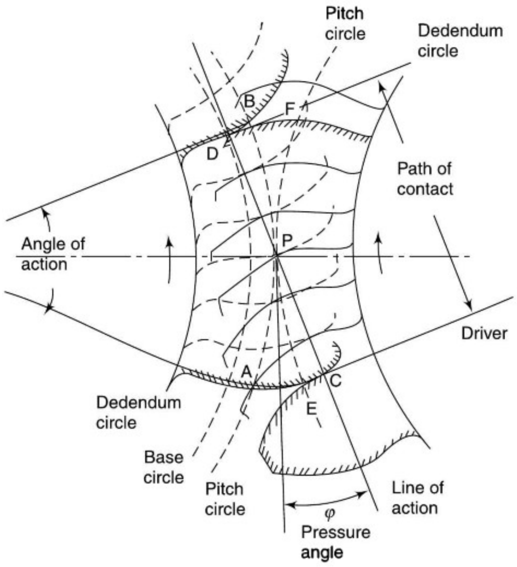
Path of contact (CD)=Path of approach (CP)+Path of recess (PD)

Path of approach (CP)

Portion of the path of contact from the beginning of engagement to the pitch point i.e **CP**

Path of recess (PD)

Portion of the path of contact from the pitch point to the end of engagement i.e **PD**



Arc of contact

The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the **Arc of contact.**
Arc APB *or* Arc EPF.
APB = AP+PB
EPF = EP+PF

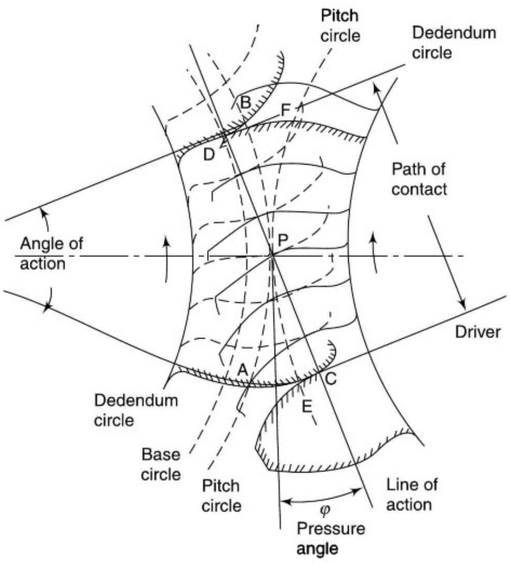
Arc of contact =Arc of approach + Arc of recess

Arc of approach

The portion of the arc of contact from the beginning of engagement to the pitch point i.e **AP** *or* **EP**

Arc of recess

The portion of the arc of contact from the pitch point to the end of engagement i.e **PB** *or* **PF**



Angle of action

It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e., **the angle turned by arcs of contact of respective gears.**

The angle will have different values for the driving & the driven gears

$$\text{Angle of action} = \frac{\text{Arc of contact}}{\text{Pitch circle radius}}$$

Angle of action (δ)=Angle of approach (α) + Angle of recess (β)

Contact ratio

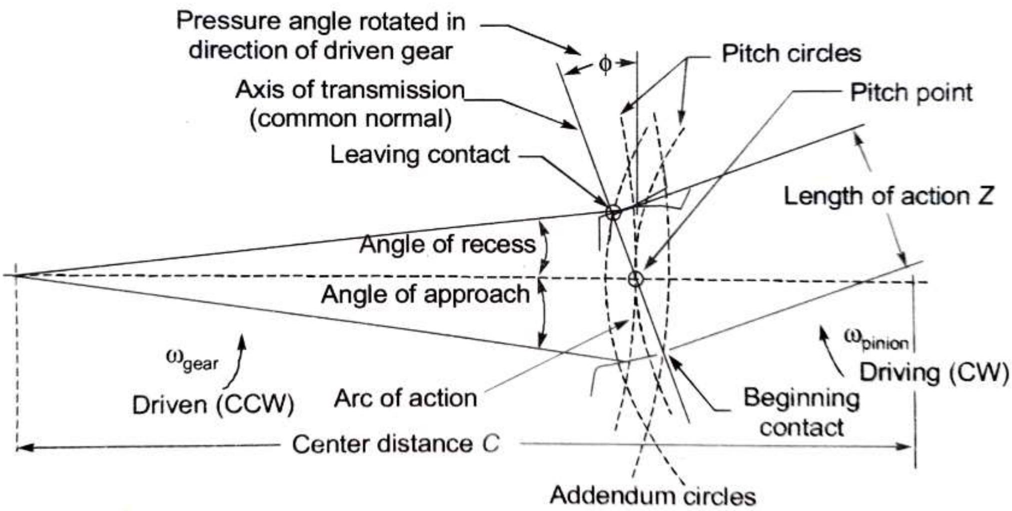
It is the ratio of angle of action & pitch angle

$$\text{Contact Ratio} = \frac{\delta}{\gamma} = \frac{\alpha + \beta}{\gamma}$$

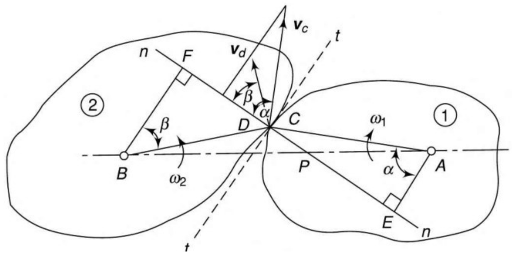
Alternatively,

$$\text{Contact Ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

As the angle of action is the angle subtended by arc of contact & the pitch angle is the angle subtended by the circular pitch at the centre of the pitch circle, contact ratio is the ratio of the arc of contact to the circular pitch.



Law of Gearing

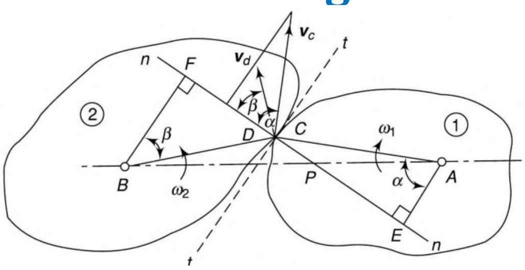


The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears.

The law of gearing provides a basis to decide conjugate tooth surfaces

The law of gearing states "the common normal to the tooth profile at the point of contact should always pass through a fixed point, called pitch point, in order to obtain constant angular velocity ratio between two gears.

Law of Gearing



Two bodies 1 & 2 representing a portion of the two gears in mesh

A point C on the tooth profile of the gear 1 is in contact with a point D on the tooth profile of gear 2.

The two curves in contact at points C or D must have a common normal (n-n) at the point.

ω_1= instantaneous angular velocity of gear 1 (CW)
ω_2= instantaneous angular velocity of gear 2 (CCW)

Linear velocity of C: $v_c=\omega_1.AC$ in a direction perpendicular to AC or at an angle α to n-n

Linear velocity of D: $v_d=\omega_2.BD$ in a direction perpendicular to BD or at an angle β to n-n

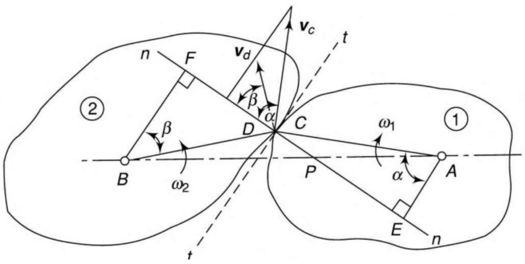
If the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent t-t.
The relative motion between the surfaces along the common normal n-n must be zero to avoid the separation or the penetration of the two teeth into each other.

Component of v_c along n-n= $v_c \cos\alpha$

Component of v_d along n-n= $v_d \cos\beta$

Relative motion along n-n= $v_c \cos\alpha - v_d \cos\beta$

Law of Gearing



AE is perpendicular from point *A* on n-n
BF is perpendicular from point *B* on n-n

$\angle ACE=(180°-90°-\alpha) = 90°-\alpha$; $\angle CAE = \alpha$
$\angle BDF= 90°-\beta$; $\angle FBD = \beta$

For proper contact,

Relative motion along n-n= $v_c \cos\alpha - v_d \cos\beta = 0$

$v_c \cos\alpha - v_d \cos\beta = 0$

$\omega_1.AC \cos\alpha - \omega_2.BD \cos\beta = 0$

$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

$$\omega_1 AE - \omega_2 BF = 0$$

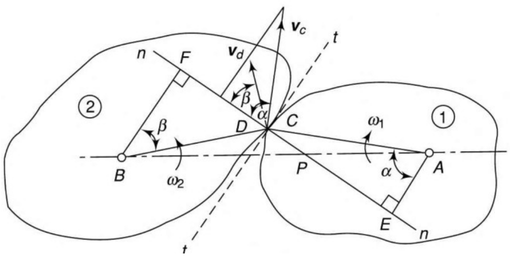
$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$$\frac{\omega_1}{\omega_2} = \frac{BP}{AP}$$

$\because$ ΔAEP and BEP are similar

It is seen that the centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears.
If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears.

Velocity of Sliding



If the curved surfaces of the two teeth of the gears 1 & 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent *t-t* at C or D.

Component of v_c along *t-t* = $v_c \sin\alpha$
Component of v_d along *t-t* = $v_d \sin\beta$

Velocity of sliding= $v_c \sin\alpha - v_d \sin\beta$

$$= \omega_1 AC \frac{EC}{AC} - \omega_2 BD \frac{FD}{BD}$$
$$= \omega_1 EC - \omega_2 FD$$
$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$
$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

$$= (\omega_1 + \omega_2)\, PC + \omega_1 EP - \omega_2 FP$$
$$= (\omega_1 + \omega_2)\, PC$$

$\omega_1 EP = \omega_2 FP$

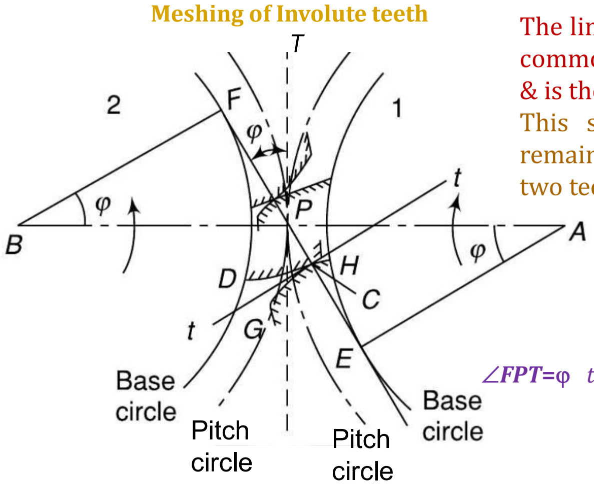
$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$\because$ ΔAEP and BEP are similar

$\omega_1 EP = \omega_2 FP$

Relationship between Base circle diameter & Pitch circle diameter

Meshing of Involute teeth



The line of action in case of involute teeth is along the common normal at the point of contact, which is fixed & is the common tangent to the two base circles.
This shows that the pressure angle in this case remains constant throughout the engagement of the two teeth.

EF i.e., line of action is the common tangent to the two base circles 1 & 2.
AE is perpendicular to *EF*
AEP is a right angled triangle

$\angle FPT=\varphi$ *then* $\angle EAP=\varphi$ **where φ=pressure angle**

$AE=AP\cos(\varphi)$ *Similarly,* $\angle FBP=\varphi$, $BF=BP\cos(\varphi)$

$2\times AE=2\times AP\cos(\varphi)$

Base circle diameter (d_b)=Pitch circle diameter (d) × cosφ

Example 7.1 Select the correct statements pertaining to spur gears:

1. Module of a spur gear is defined as ratio of pitch circle diameter (PCD) to the numbe teeth.
2. Circular pitch is defined as sum of thickness of a tooth and the space between t adjacent teeth measured on base circle.
3. The depth of a tooth in a spur gear is difference between the radii of addendum and pit circles.

(a) 1, 2 and 3 (b) 1 only (c) 1 and 3 (d) 3 only

Example 7.2 Radii of base circle is a product of

(a) Radius of addendum circle and cosine of pressure angle
(b) Radius of dedendum circle and cosine of pressure angle
(c) clearance and cosine of pressure angle
(d) Radius of pitch circle and cosine of pressure angle

Example 7.3 Line of action or pressure line of spur gears is

(a) Tangent of tooth surface and normal to base circle.
(b) Normal of tooth surface and tangent to pitch circle.
(c) Normal of tooth surface and tangent to base circle.
(d) Tangent of tooth surface and normal to pitch circle.

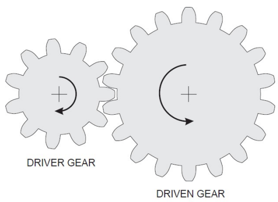
Example 7.4 Which of the following gears are used to transmit power between non-intersecting shafts that are at right angles to each other?

(a) Racks (b) Worm gears (c) Spur gears (d) Bevel gears

Example 3

The following data relate to two meshing gears : velocity ratio=3:1, module=4 mm. Pressure angle = 20°, centre distance = 200 mm. Determine the number of teeth & the base circle diameter of the gear.

Solution:



Velocity ratio= N_1/N_2

Velocity ratio= $T_2/T_1=3$ $\quad$ $\mathbf{T_2=3T_1}$

Module m=4

Module m=$d_1/T_1=d_2/T_2=4$ $\quad$ d_1=Pitch Circle Diameter of gear 1 (PCD1)

$d_1=4T_1$ $\quad$ $d_2=4T_2$ $\quad$ d_2=Pitch Circle Diameter of gear 2 (PCD2)

Centre distance = (PCD1+PCD2)/2=$(d_1+d_2)/2$

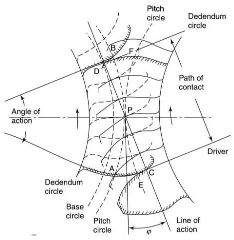
$200=(d_1+d_2)/2$; $\quad$ $200=4(T_1+T_2)/2$; $\quad$ $200=4(T_1+3T_1)/2$

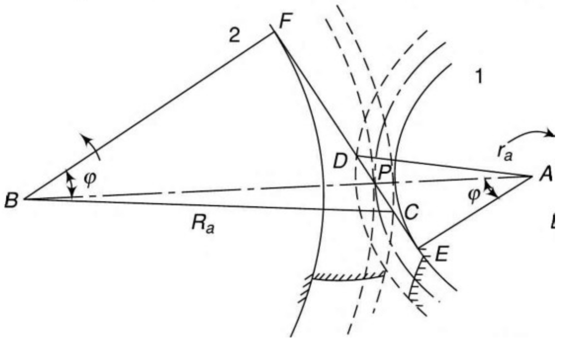
$4(T_1+3T_1)/2=200$; $\quad$ $T_1=25$; $\mathbf{T_2=3T_1=75}$ $\quad$ Nos. of teeth in gear=75

$d_2=4T_2=4\times75=300$ mm $\quad$ **where φ=pressure angle=20°**

Base circle diameter (d_b)=Pitch circle diameter (d_2) × cosφ = 300× cos(20°)=282 mm

Path of contact or Contact length

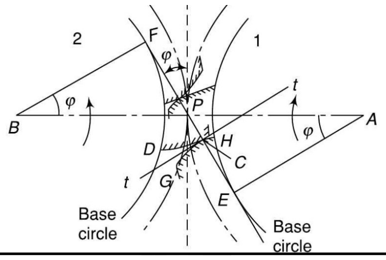




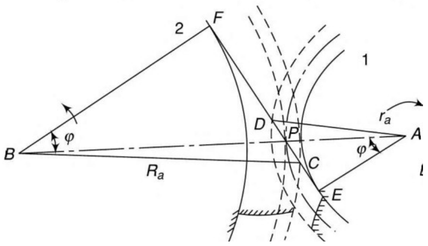
Two gears with centres A & B are in contact at P.
The pinion 1 is the driver & is rotating clockwise. The gear 2 is driven in the counter-clockwise direction.
EF is their common tangent to the base circles

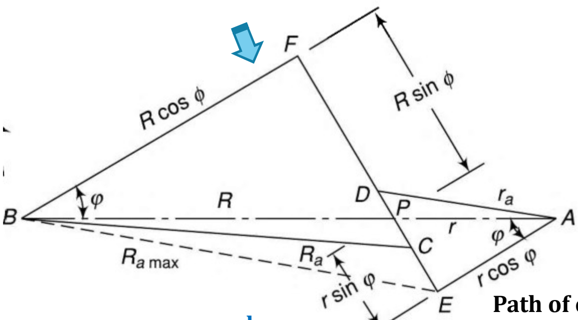
Contact of the two teeth is made where the addendum circle of the wheel meets the line of action EF i.e., at C and is detached where the addendum circle of the pinion meets the line of action i.e., at D.

CD is the path of contact



Path of contact or Contact length





φ=pressure angle

r = Pitch circle radius of pinion=AP
R = Pitch circle radius of gear=BP

r_a = Addendum circle radius of pinion=AD
R_a= Addendum circle radius of gear=BC

Path of contact (CD)=Path of approach (CP)+Path of recess (PD)

CD=CP + PD
CD=(CF-PF) + (DE-PE)

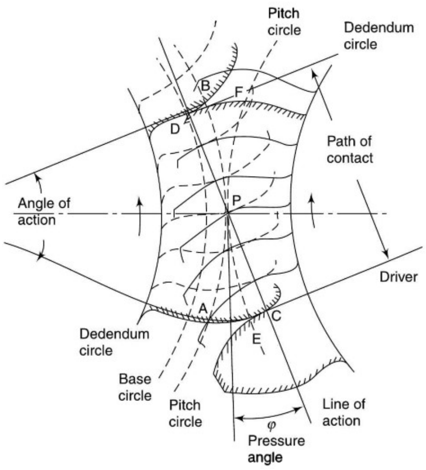
Path of approach: CP = CF - PF

$$=\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi$$

Path of recess: PD = DE - PE

$$=\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi$$

Path of contact (CD)=$\left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$



Arc of contact

The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the **Arc of contact.**

The arc of contact is the distance travelled by a point on either pitch circle of the two gears during the period of contact of a pair of teeth

Arc of contact = Arc of approach + Arc of recess

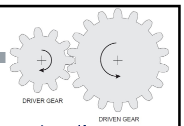
$$\text{Arc of approach} = \frac{\text{Path of approach}}{\cos\phi}$$

$$\text{Arc of recess} = \frac{\text{Path of recess}}{\cos\phi}$$

$$\text{Arc of contact} = \frac{\text{Path of approach} + \text{Path of recess}}{\cos\phi}$$

Path of contact $= \left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}}$$



Contact Ratio : Number of pairs of teeth in contact

The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth

Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other gear

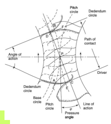
$$\boxed{\text{Therefore, the number of teeth in contact, } n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{CD}{\cos\varphi}\frac{1}{p}}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

Contact ratio is the ratio of the arc of contact to the circular pitch (p)

Circular Pitch (p)

$$p = \frac{\pi d}{T}; \; p = \pi.m$$



Thus, number of teeth in contact is also expressed in terms of contact ratio

For continuous transmission of motion, atleast one tooth of one gear must be in contact with another tooth of the second gear. Therefore **contact ratio must be greater than one**

If n lies between 1 and 2, the number of teeth in contact at any time will not be less than one and never more than two. If n is between 2 and 3, it is never less than two pairs of teeth and not more than three pairs, and so on. If n is 1.6, one pair of teeth are always in contact whereas two pairs of teeth are in contact for 60% of the time.

Example 4

A pinion having 40 teeth drives a gear having 90 teeth. The profile of the gears is involute with 20 pressure angle, 10 mm module and the addendum equal to one module. Find the path of contact, arc of contact and contact ratio.

Solution:

No. of teeth in pinion=T_1=40 No. of teeth in gear=T_2=90

Module m=10

Module m=d/T_1=D/T_2=10 d=Pitch Circle Diameter of pinion, d=10T_1=400 mm D=Pitch Circle Diameter of gear D=10T_2=900 mm

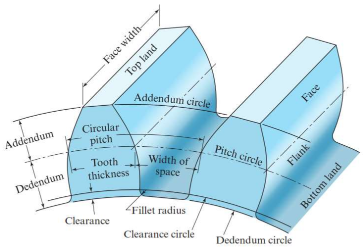
r=Pitch Circle Radius of pinion=d/2=200 mm; R=Pitch Circle Radius of gear=D/2=450 mm

Addendum= 1 module=10 mm

Addendum circle radius of pinion: r_a=r+Addendum=200+10=210 mm;

Addendum circle radius of Gear: R_a=R+Addendum=450+10=460 mm;

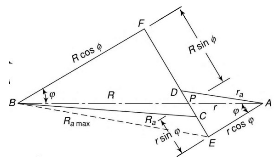
Pressure angle φ=20°



Example 4

A pinion having 40 teeth drives a gear having 90 teeth. The profile of the gears is involute with 20 pressure angle., 10 mm module and the addendum equal to one module. Find the path of contact, arc of contact and contact ratio.

Path of contact (CD)=Path of approach (CP)+Path of recess (PD)



CD=CP + PD

CD=(CF-PF) + (DE-PE)

Path of approach: $CP = CF - PF$

$$= \sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi$$

Path of recess: $PD = DE - PE$

$$= \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi$$

Pitch Circle Radius of gear: R=450 mm

Addendum circle radius of Gear: R_a=460 mm

Pitch Circle Radius of pinion: r=200 mm

Addendum circle radius of pinion: r_a=210 mm

Pressure angle: φ=20°

Path of contact (CD)=$\left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$

Path of contact (CD)=52.465 mm

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

Arc of contact=52.465/cos20° =55.832 mm

Circular Pitch (p)

$$p = \frac{\pi d}{T}; \; p=\pi.m=31.4 \text{ mm}$$

Module m=d/T=10

Where,
d=Pitch circle diameter
T=number of teeth

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Contact ratio=55.832/31.4=1.778

Contact ratio is 1.778 which means that one pair of teeth are always in contact whereas two pairs of teeth are in contact for 77.8% of the time

Example 5

Each of two gears in a mesh has 48 teeth & a module of 8 mm. the teeth are of 20 degree involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Solution:

Given data: $\varphi = 20°;\ t = T = 48;\ m = 8$ mm

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192 \text{ mm};\quad R_a = r_a$$

$$\text{Arc of contact} = 2.25 \times \text{Circular pitch} = 2.25\pi\,\text{m} = 2.25\pi \times 8 = 56.55 \text{ mm}$$

$$\text{Path of contact} = 56.55 \times \cos 20° = 53.14 \text{ mm}$$

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}}$$

$$\text{Path of contact (CD)} = \left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$$

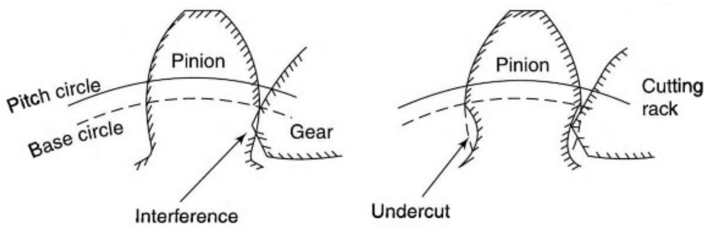
$$2\left(\sqrt{R_a^2 - 192^2\cos^2 20°} - 192\sin 20°\right) = 53.14$$

$$R_a = 202.6 \text{ mm}$$

$$\text{Addendum} = R_a - R = 202.6 - 192 = 10.6 \text{ mm}$$

Interference in Involute Gears

A gear tooth has involute profile only outside the base circle. In fact, the involute profile begins at the base circle. In some cases, the dedendum is so large that it extends below this base circle. In such situations, the portion of the tooth below the base circle is not involute. The tip of the tooth on the mating gear, which is involute, interferes with this non-involute portion of the dedendum. This phenomenon of tooth profiles overlapping and cutting into each other is called **Interference**. In this case, the tip of the tooth overlaps and digs into the root section of its mating gear. Interference is non-conjugate action and results in excessive wear, vibrations and jamming.



When the gears are generated by involute rack cutters, this interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This is called **undercutting**.
Undercutting solves the problem of interference.
However, an undercut tooth is considerably weaker. Undercutting not only weakens the tooth, but also removes a small involute portion adjacent to the base circle. This loss of involute profile may cause a serious reduction in the length of the contact

Interference is the main disadvantage of involute gears

Minimum no. of teeth to avoid interference of involute gears

Minimum number of teeth to avoid interference of involute gears

Let t = number of teeth on the pinion
T = number of teeth on the wheel

$$R = \frac{mT}{2}, r = \frac{mt}{2} \quad \text{Gear ratio } G = \frac{T}{t}$$

$$T \geq \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$; a_w=Addendum co-efficient

$$Min.\ T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

For $a_w = 1$, i.e., when the addendum is equal to one module

$$Min.\ T = \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

Example 6

Two 20 degree involute spur gears mesh externally & give a gear ratio of 3. The module is 3 mm & the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine the: (i) minimum number of teeth on each wheel to avoid interference, (ii) contact ratio.

Solution:

$\varphi = 20°$ $\quad N_p = 120$ rpm
$G = 3$ $\quad$ Addendum = 1.1 m
$m = 3$ mm $\quad a_w = 1.1$

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio,

i.e., $T = 51$ and $t = \frac{51}{3} = \underline{17}$

Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \left(\frac{\text{Path of contact}}{\cos\varphi}\right) \times \frac{1}{\pi m}$$

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi + \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi}{\cos\varphi \times \pi m}$$

$$n = \frac{\left[\begin{array}{l}\sqrt{(79.8)^2 - (76.5\cos 20°)^2} - 76.5\sin 20° \\ +\sqrt{(28.8)^2 - (25.5\cos 20°)^2} - 25.5\sin 20°\end{array}\right]}{\cos 20° \times \pi \times 3}$$

$$= \frac{34.646 - 26.165 + 15.977 - 8.720}{\cos 20° \times \pi \times 3}$$

$$= 1.78$$

We have, $R = \frac{mT}{2} = \frac{3 \times 51}{2} = 76.5$ mm

$R_a = R + 1.1\,m = 76.5 + 1.1 \times 3 = 79.8$ mm

$r = \frac{mt}{2} = \frac{3 \times 17}{2} = 25.5$ mm

$r_a = 25.5 + 1.1 \times 3 = 28.8$ mm

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact.

Example 7

Two involute spur gears having pressure angle 20° and module 6 mm are in mesh. The gear ratio is 3 & the nos. of teeth on pinion is 24. The pitch line velocity is 1.5 m/s & the addendum equal to one module. Determine the (i) path of contact, (ii) arc of contact, (iii) angle of action of the pinion, & (iv) the max. velocity of sliding.

Solution:

No. of teeth in pinion=t=24

Gear ratio G=3 $\quad G = \frac{T}{t}$ $\quad$ No. of teeth in gear=T=3×24=72

Module m=6

Module m=d/t=D/T=6

d=Pitch Circle Diameter of pinion, d=6t=144 mm

D=Pitch Circle Diameter of gear D=6T=432 mm

r=Pitch Circle Radius of pinion=d/2=72 mm;

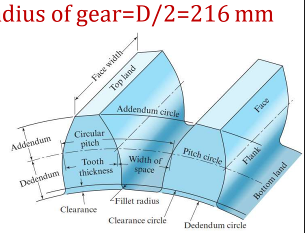
R=Pitch Circle Radius of gear=D/2=216 mm

Addendum= 1 module=6 mm

Addendum circle radius of pinion: r_a=r+Addendum=72+6=78 mm;

Addendum circle radius of Gear: R_a=R+Addendum=216+6=222 mm;

Pressure angle φ=20°



Example 7

Path of contact (CD)=Path of approach (CP)+Path of recess (PD)

CD=CP + PD

CD=(CF-PF) + (DE-PE)

Path of approach: CP = CF - PF

$$= \sqrt{R_a^2 - R^2 \cos^2 \varphi} - R \sin \varphi$$

Path of recess: PD = DE - PE

$$= \sqrt{r_a^2 - r^2 \cos^2 \varphi} - r \sin \varphi$$

Pitch Circle Radius of gear: R=216 mm

Addendum circle radius of Gear: R_a=222 mm

Pitch Circle Radius of pinion: r=72 mm

Addendum circle radius of pinion: r_a=78 mm

Pressure angle: φ=20°

Path of contact (CD)=$\left[\sqrt{R_a^2 - R^2 \cos^2 \varphi} - R \sin \varphi\right] + \left[\sqrt{r_a^2 - r^2 \cos^2 \varphi} - r \sin \varphi\right]$

Path of contact (CD)=30.22 mm

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

Arc of contact=30.22/cos20° =32.16 mm

$$\text{Angle of actionof pinion} = \frac{\text{Arc of contact}}{\text{Pitch circle radius (r)}} = \frac{32.16}{72} = 0.4467\ rad = 25.59$$

Velocity of sliding=(ω_1+ ω_2)×Path of approach

Velocity of sliding=(v_p/r+ v_g/R)×CP=(1500/72+1500/216)×16.04=**445.6 mm/s**

Pitch line velocity at pinion (v_p)= ω_1r

Pitch line velocity at gear (v_g)= ω_2R

Pitch line velocity v_p= v_g=1500 mm/s

Thanking You