3 Jan

S. K. Sir

Heat transfer — The heat or thermal energy in transit due to temperature difference.

$q_x \Rightarrow$ Rate of heat transfer along 'x' direction (J/s or watt)

$q'_x \Rightarrow$ Rate " " " " " per unit length (W/m)

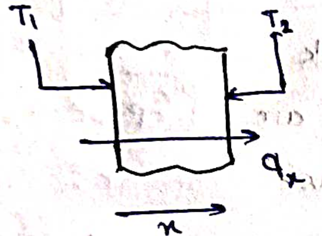
$q''_x \Rightarrow$ " " " " " " per unit area (W/m²)

$q'''_x \Rightarrow$ " " " " " " per unit vol. (W/m³)

→ Conduction — Diffusion

→ Convection — Advection + Diffusion (Bulk movement of fluid particles)

Conduction
[conduction through]



$T_1 > T_2$

$$q_x = -kA \frac{dT}{dx}$$

[fourier's law of heat conduction]

rate of heat transfer

cross sectional area

Thermal cond.

Temp. gradient

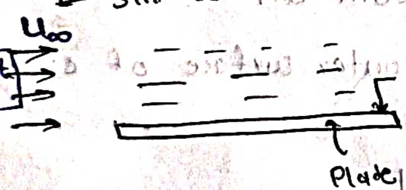
Diffusion.

Convection
[from... to...]

→ natural convection (density difference)

→ Force convection

← still velocity condition



[Bulk movement of fluid]

Newton's law of cooling

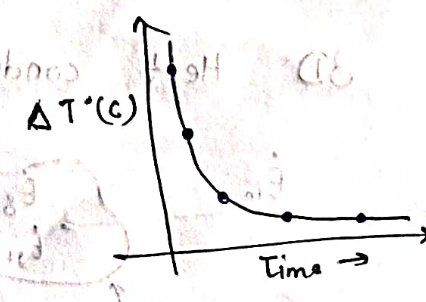
$$q_{conv} = hA(T_s - T_\infty) \qquad T_s > T_\infty$$

convective heat transfer coefficient (W/m² K)

temp. of body

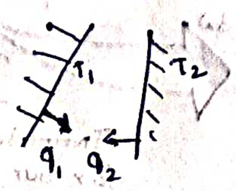
temp. of environment

surface area of hot plate



Radiation [No need of medium]

• Stefan - Boltzmann eqn.



$$\sigma = 5.67 \times 10^{-8}\ W/m^2$$

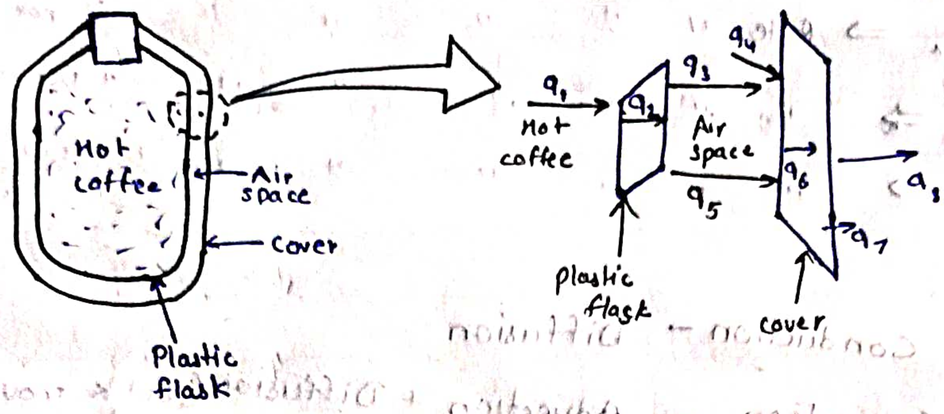
$$q_{rad} = q_1 - q_2$$

Net radiation exchange between surface 1 & surface 2

$$q_{rad} = \sigma A (T_1^4 - T_2^4)$$

12 Jan

Identify the different modes of Heat Transfer



q_1 = Free convection from hot coffee to plastic flask

q_2 = conduction through plastic flask

q_3 = Free convection from flask to air

q_4 = Force convection from air to cover

q_5 = Net radiation exchange between outer surface of plastic flask and inner surface of cover

q_6 = Conduction through cover

q_7 = Free convection from cover to room air

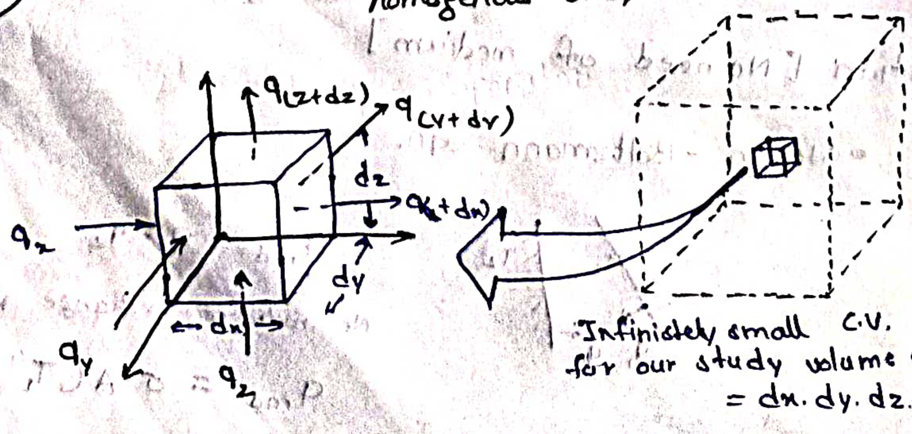
q_8 = Net radiation exchange between outer surface of cover and room air.

$$\boxed{q_{loss} = \sum_{i=1}^{8} q_i}$$

3D Heat conduction Eqn in Cartesian coordinate system

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

Homogenous body & no advection



Infinitely small C.V. is selected for our study volume of C.V. $= dx.dy.dz.$

$$\boxed{\dot{E}_{in} = q_x + q_y + q_z}$$

Taylor series expansion & neglect the higher order terms

$$q_{x+dx} = q_x + \frac{\partial}{\partial x} q_x \cdot dx$$

$$q_{y+dy} = q_y + \frac{\partial}{\partial y} \cdot q_y \cdot dy$$

$$q_{z+dz} = q_z + \frac{\partial}{\partial z} \cdot q_z \, dz$$

$$\boxed{\dot{E}_g = q''' (dx \cdot dy \cdot dz)}$$

$$\boxed{\dot{E}_{out} = q_{x+dx} + q_{y+dy} + q_{z+dz}}$$

$$\boxed{\dot{E}_{st} = \rho C_p \frac{\partial T}{\partial x} (dx \cdot dy \cdot dz)}$$

$$q_x + q_y + q_z + q''' (dx \cdot dy \cdot dz) - (q_{x+dx} + q_{y+dy} + q_{z+dz}) = \rho C_p \frac{dT}{dx} (dx \cdot dy \cdot dz)$$

$$-k (dy \cdot dz) \frac{\partial T}{\partial x} - k (dx \cdot dz) \cdot \frac{dT}{\partial y} - k (dx \cdot dy) \frac{dT}{\partial z} + q''' (dx \cdot dy \cdot dz)$$

$$- q_x - \frac{\partial}{\partial x} q_x \cdot dx - q_y - \frac{\partial}{\partial y} \cdot q_y \cdot dy - q_z - \frac{\partial}{\partial z} q_z dz = \rho C_p \frac{dT}{dx} (dx dy dz)$$

$$\boxed{\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + q''' = \rho C_p \frac{\partial T}{\partial t}}$$

→ 3-D heat conduction eq. in ~~cat~~ cartesian coordinate system.

→ 3D, variable K, uniform heat generation & unsteady state conduction eqn.

If k = const. in heat eqn.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Since $k = f(T)$

In solids :- $T \uparrow$, $k \downarrow$

In gases :- $T \uparrow$, $k \uparrow$

In liquids :-

Let α = thermal diffusibility $= \frac{k}{\rho C_p} = \frac{\text{thermal conductivity}}{\text{Heat capacity}}$

larger α means ~~to~~ faster heat transfer.

for solids → α value is large.

for gases → α value is small.

Q - If it is a ~~θ~~ 2-D heat eqn, with variable k and steady state internal heat generation.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + q''' = 0$$

Q - 3D heat conduction eq with const. k, no internal heat generation and stead state.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (\because \text{Laplace eqn})$$

$$\boxed{\nabla^2 T = 0}$$

∇^2 → Laplace operator

1-D Laplace

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Solution by integration :-

$T = C_1 x + C_2$ } 2 boundarly conditions.

For 3D - 6 boundary conditions are required.

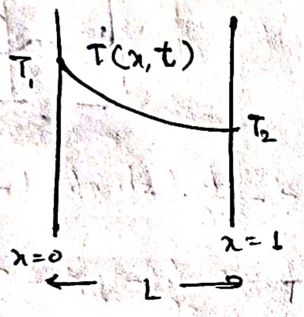
For unsteady state another 1 condition is required known as Initial condition.

Constant Surface B.C. /

- B.C. of 1st Kind / Dritchlet B.C.

$T(0,t) = T_1$

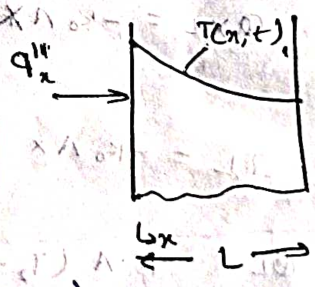
$T(L,t) = T_2$



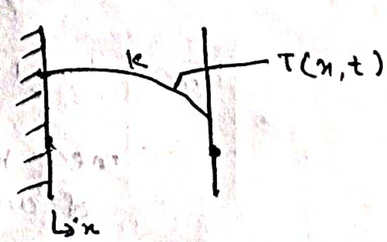
~~Constan~~

- Constant heat flux / B.C. of 2nd Kind / Neumann B.C

$$-k \left.\frac{dT}{dx}\right|_{x=0} = q_x''$$



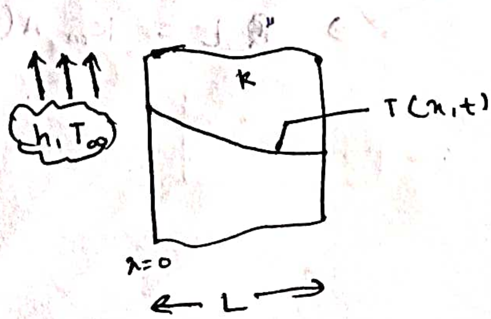
- special B.C. (Insulated B.C / 2nd B.C.)



$t=0 \rightarrow T = T_1$

- Convective B.C.

$$-k \left.\frac{dT}{dx}\right|_{x=0} = h\left[T(0,t) - T_\infty\right]$$



2 - Feb

$K = f(T)$

$K = K_0(1 + \alpha T)$

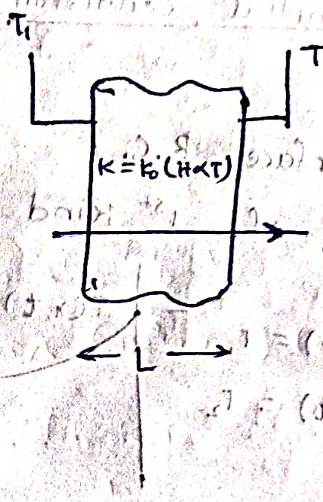
$K \Rightarrow$ Thermal conductivity

$\alpha \rightarrow$ +ve (Non-metal)
-ve (metals)

$$Q = -KA\frac{dT}{dx}$$

$$= -K_0(1+\alpha T)A\frac{dT}{dx}$$

$$= K_0(1+\alpha T)A\frac{(T_1 - T_2)}{L}$$



$T_1 > T_2$

$$\boxed{QL = K_0(1+\alpha T)A(T_1 - T_2)}$$

$$\int_0^L Q\,dx = -K_0 A \int_{T_1}^{T_2} (1+\alpha T)\,dT$$

$$Q \times L = -K_0 A \times \left[T + \frac{\alpha T^2}{2}\right]_{T_1}^{T_2}$$

$$QL = -K_0 A \times \left[(T_2 - T_1) + \frac{\alpha}{2}(T_2^2 - T_1^2)\right]$$

$$QL = -K_0 A(T_2 - T_1)\left[1 + \frac{\alpha}{2}(T_2 + T_1)\right]$$

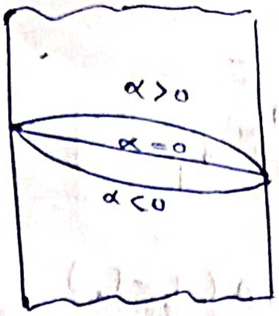
$$K_m = K_0\left[1 + \alpha\left(\frac{T_1 + T_2}{2}\right)\right]$$

= mean value of thermal conductivity of arithematic mean temp.

$$QL = K_m A(T_1 - T_2)$$

$$Q = -K_0 (1 + \alpha T) A \frac{dT}{dx}$$

$$\frac{dT}{dx} = \left[\frac{-Q}{K_0 (1+\alpha T) A} \right]$$



If $\alpha = 0$

$k = k_0$, slope will be constant

If $\alpha > 0$

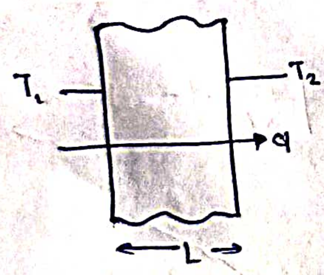
Slope follows a positive line ~~at~~ along the material direction.

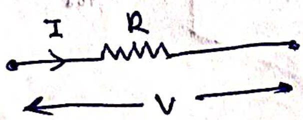
If $\alpha < 0$

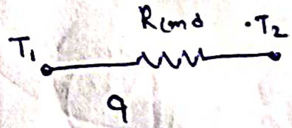
slope follows negative line along the material direction.

Electrical analogy for heat transfer problems

Conduction :-







$$I = \frac{V}{R}$$

$$R_{cond} = \frac{L}{KA}$$

$$\dot{q} = KA \frac{(T_1 - T_2)}{L}$$

$$\dot{q} = \frac{(T_1 - T_2)}{L/KA}$$

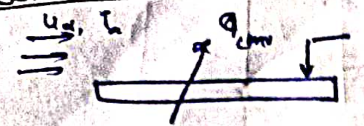
$q \sim I$

$T_1 - T_2 \sim V$

$\frac{L}{KA} \sim R$

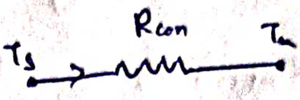
$q \rightarrow \frac{dQ}{dt}$

Convection :-



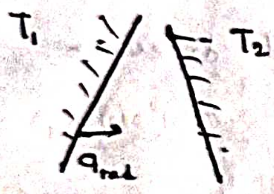
$$q_{conv} = hA(T_s - T_\infty)$$

$$= \frac{T_s - T_\infty}{1/hA}$$



$$\boxed{R_{conv} = \frac{1}{hA}}$$

Radiation



$$q_{rad} = \sigma A(T_1^4 - T_2^4)$$

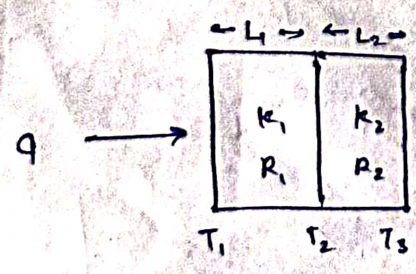
$$q_{rad} = \sigma A\,(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$$

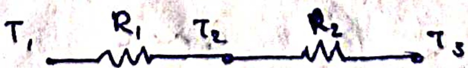
$$q_{rad} = \frac{(T_1 - T_2)}{\frac{1}{\sigma A(T_1+T_2)(T_1^2+T_2^2)}}$$

$$R_{rad} = \frac{1}{\sigma A(T_1+T_2)(T_1^2+T_2^2)}$$

Composite Wall

Series



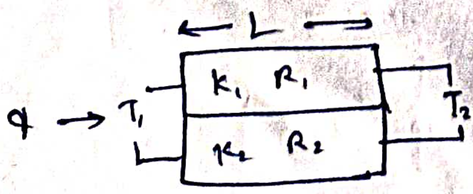


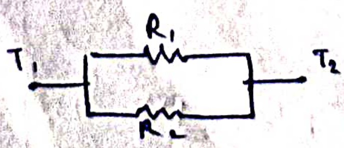
$$R_1 = \frac{L_1}{K_1 A}$$

$$R_2 = \frac{L_2}{K_2 A}$$

$$q_{cond} = \frac{T_1 - T_3}{R_1 + R_2}$$

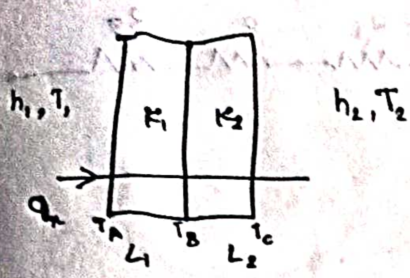
$$\& \quad \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

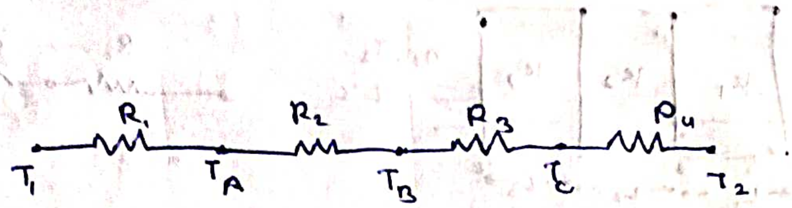




$$q_{cond} = \frac{T_1 - T_2}{R}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$





$R_1 = \frac{1}{h_1 A}$ $\qquad R_4 = \frac{1}{h_2 A}$

$R_2 = \frac{L_1}{K_1 A}$

$R_3 = \frac{L_2}{K_2 A}$

$$q_x = \frac{T_1 - T_2}{R_1 + R_2 + R_3 + R_4}$$

$$= \frac{T_1 - T_2}{\frac{1}{h_1 A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{1}{h_2 A}}$$

U = overall heat transfer coeff. (W/m².K)

$$q_x = UA\,(T_1 - T_2)$$

$$= \frac{T_1 - T_2}{\frac{1}{UA}}$$

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{1}{h_2 A}$$

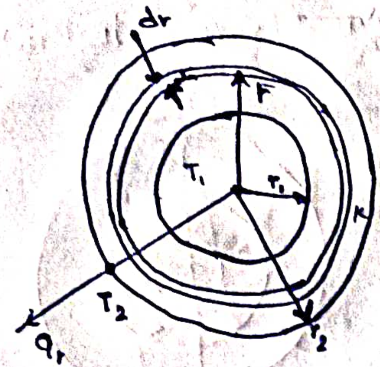
$$\frac{1}{U} = \frac{1}{h_1} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{1}{h_2}$$

Q- A cold storage room has walls made of 0.23 m brick on the outside, 0.08 m of plastic foam and finally 1.5 cm of wood on the inside. The outside and inside air temper. are 22 °C and -2 °C respec. If the inside and outside head transfer coeffi are 29 and 12 W/m²K and thermal conductivity of brick, foam wood are 0.98, 0.02, 0.17 W/m K. determine —

① The rate of heat removed by the refrig. if the total wall area is 90 m².

② The temp. of the inside surface of the brick.

Heat transfer through hollow sphere



$$q_r = -kA\frac{dT}{dr}$$

$$q_r = -k\,4\pi r^2 \frac{dT}{dr}$$

$$\int_{T_1}^{T_2} dt = -\frac{q_r}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2}$$

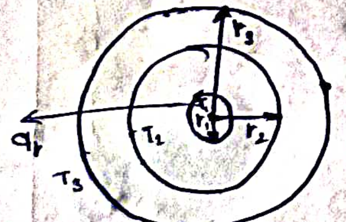
$$T_2 - T_1 = -\frac{q_r}{4\pi k}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$q_r = \frac{T_1 - T_2}{\frac{1}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

$$\boxed{q_r = \frac{T_1 - T_2}{R_{sphere}}}$$

$$\boxed{R_{sphere} = \frac{1}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

Heat transfer through composite walled sphere :-



T_1 — T_2 — T_3

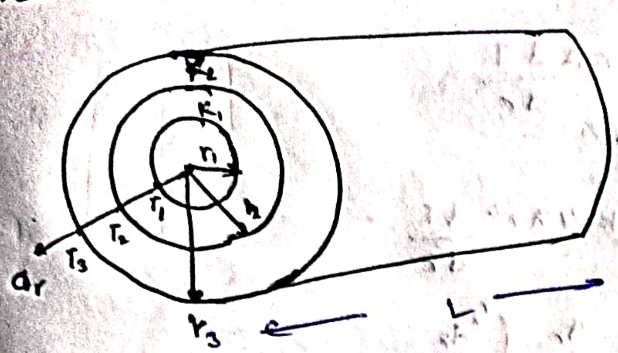
$$R_1 = \frac{1}{4\pi k_1}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

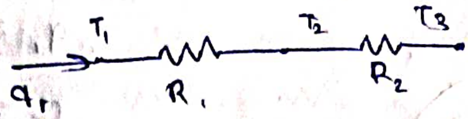
$$R_2 = \frac{1}{4\pi k_2}\left(\frac{1}{r_2}-\frac{1}{r_3}\right)$$

$$q_r = \frac{T_1 - T_3}{}$$

Q- A hollow aluminium sphere of therm. conduc. 230 w/mk with a electric heater at in the centre is used in test to determine the thermal conductivity of insulating materials. The inner and outer radii of sphere are 0.15 m and 0.18 m respectively, and testing is done under steady state conditions with the inner surface of the aluminium maintained at 230°C in a particular test a spherical shell of insulation is cast on the outersurface of the sphere to a thickness of 0.12 m. The system is in a room for which air temp. is 20°C and

Heat conduction through Composite Walls cylinder



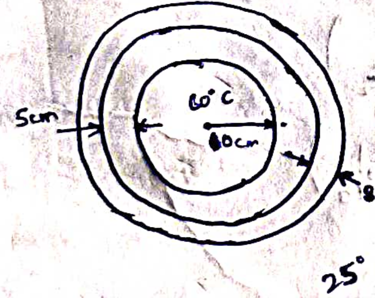


$$R_1 = \frac{\ln \frac{r_2}{r_1}}{2\pi K_1 L}$$

$$R_2 = \frac{\ln \frac{r_3}{r_2}}{2\pi K_2 L}$$

$$\boxed{q_r = \frac{T_1 - T_3}{R_1 + R_2}}$$

Q- Hot air at a temp. 60° C is flowing through a steel pipe ~~of~~ of 10 cm dia. The pipe is covered with two diff. layers of insul. material of thickness 5 cm and 3 cm and their ther. condu. are 0.23 and ~~0.27~~ 0.37 w/mK and ~~5~~ 58 and 12 w/mk The The inside and outside are atms. a is at 25° C. The rate of heat loss from a 50 m length of pipe. Neglect the resis. of heat loss.



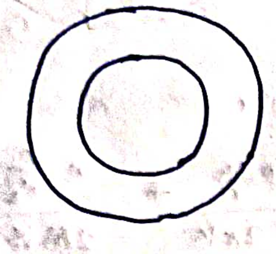
$$q_r = \frac{60° - 25°}{R_1 + R_2}$$

$$R_1 = \frac{\ln \frac{15}{10}}{2\pi \times 0.23 \times L}$$

$$R_2 = \frac{\ln \frac{18}{15}}{2\pi \times 0.37 \times L}$$

the convective coeffic. of outer surface insula. is 30 w/ mK

At the 80 w are dissipated ~~cond.~~ by heater under steady state condition. what is thermal ~~condiw~~ conductivity of the insulation.



R_1

230° 20°

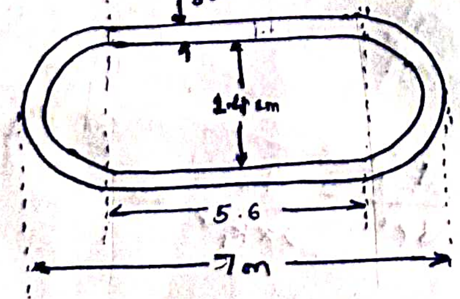
$R_1 = \frac{1}{4\pi \times 230}\left(\frac{1}{0.15} - \frac{1}{0.18}\right)$

=

$80 = \frac{230 - 20}{3.84\times10^{-4} + \frac{0.177}{k_2} + 0.023}$

$k_2 = 0.062$ w/mK

Q- A cylindrical liquid oxygen tank has dia. ~~1.4m~~ 1.4 m and 7m long and has spherical ends. The boiling point of liquid oxgen is $-182°c$ and its ~~lab~~. Latent head of evapo. is 214 kJ/kg The tank is insulated in order to reducet heat transfer to the tank in a such a way that, in steady rate the rate of oxygen boiler should not exceed 14 kg/hr calcuate thermal condu. of insulating material. If its 8 cm thick layer of insulation is applied ~~and~~, and its outside surface is maintain at 30° C.



The rate of heat transfer to oxygen

$q = \dot{m} \times h_{fg}$

$= 14 \frac{kg}{hr} \times 214 \frac{kJ}{kg} = 2996 \frac{kJ}{hr}$

$q = q_{cyln.} + q_{spere}$

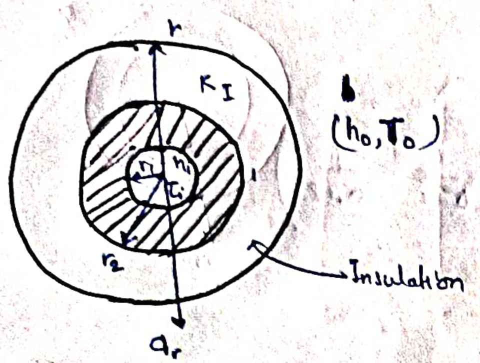
$= \frac{T_2 - T_1}{\frac{1}{2\pi k L}\ln\frac{r_2}{r_1}} + \frac{T_2 - T_1}{\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$

$= \frac{212}{}$

Ans $= 0.0095$ w/mK

2nd March

Critical thickness of a cylindrical insulation



$$\frac{q_r}{L} = \frac{T_i - T_o}{R}$$

$$R = R_1 + R_2 + R_3 + R_4$$

$$R = \frac{1}{h_i\, 2\pi r_1} + \frac{\ln \frac{r_2}{r_1}}{2\pi K} + \frac{\ln \frac{r}{r_2}}{2\pi K_I} + \frac{1}{h_o \cdots}$$

Now,

$$\frac{dR}{dr} = 0$$

$$\frac{1}{2\pi K_1} \times \frac{r_2}{r} \cdot \frac{1}{r_2} + \frac{1}{h_o \times 2\pi r^2} = 0$$

$$\frac{1}{h_o r} = \frac{1}{K_1}$$

critical radius $r_{cr} = \frac{K_I}{h_o}$

critical thickness of insulation $= r_{cr} - r_2$

$$r = \frac{K}{h_o}$$

$$\boxed{r = \frac{K_I}{h_o}} = r_{cr}$$

$$-\frac{1}{2\pi K_1 r^2} + \frac{2}{h_o \times 2\pi r^3} = \frac{d^2R}{dr^2}$$

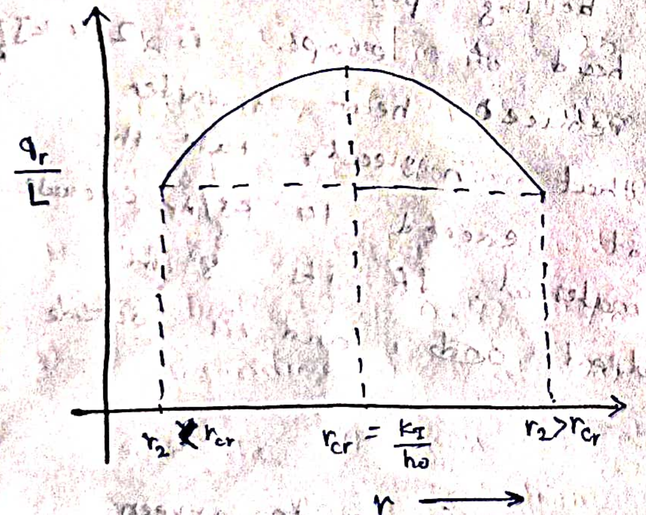
$$\frac{2}{h_o r}$$

$$\frac{2K_I}{h_o}$$

$$\frac{d^2R}{dr^2} = \frac{2}{h_o \times 2\pi r^3} - \frac{1}{2\pi K_1 r^2}$$

$$=$$

$$\frac{d^2R}{dr^2} = \frac{h_o^2}{2\pi K_1^3}$$



- In small pipe if you add insulation, the heat transfer rate is going to increase instead of reducing till it reaches critical radius, there if we add insulation the heat transfer rate is going to reduce. Previous statement said that addition of insulation is has diinsulating effect.

Q- Dose heat loss from 2 inch outer diameter pipe decrease if asbestos of thermal cond. K of 0.151 W/m K is added to insulate it assume that the $h_o = 5\ W/m^2 k$

Ans — deinsulating effect

Q- Calculate the critical radius of insulation for asbestos of $k = 0.17$ w/m surrounding a pipe and expose to a room air at 20°C with ~~convection heat~~ $h = 3\ W/m^2 K$. Calculate the heat loss from a ~~tube~~ 200°C and 5 cm dia. meter when covered with the critical radius of insulation and without insulation.

$$r_{cr} = \frac{0.17}{3} = 5.6\ cm$$

$$\frac{q_r}{L} = \frac{200 - 20°}{R_t}$$

$$R_t = \frac{1}{h_o 2\pi r_2} + \frac{\ln \frac{r_2}{r_1}}{2\pi K}$$

with insulation,

$$\frac{q_r}{L} = \frac{T_1 - T_2}{R_1 + R_2} = 105.7\ w/m$$

without insulation.

$$\frac{q_r}{L} = h_o 2\pi r_1 T_1 - T_2$$

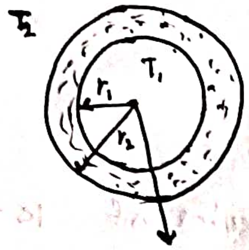
$$= 3 \times 2\pi \times 0.025 \times (200 - 20)$$

$$= 84.78\ w/m$$

The addition of 0.0317 m of heat insulation increases rise ~~in the~~ the heat transfer rate by 26.4%

16th March

Critical thickness of Insulation over a sphere

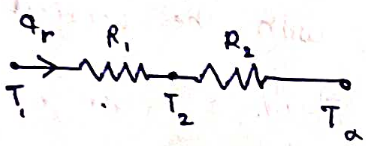


$$q_r = \frac{T_1 - T_\alpha}{R_1 + R_2} = \frac{T_1 - T_\alpha}{R} = \frac{T_1 - T_\alpha}{\frac{1}{4\pi k_I}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h 4\pi r_2^2}}$$

$$\frac{dR}{dr_2} = 0$$

$$\frac{1}{4\pi k_I} \times \frac{1}{r_2^2} = \frac{2}{h 4\pi r_2^3}$$

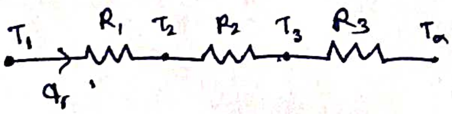
$$\boxed{r_2 = \frac{2K}{H}}$$



Critical radius $\boxed{r_2 = \frac{2K_I}{h}}$

critical thickness of insulation $= r_{cr} - r_1$

Q- The current of thousand Amps through a long copper conductor $k = 390$ W/m K, 25 mm in diameter having its electric resistivity of 1.08 $\mu\Omega$cm. This rod is insulated to a radius of 17.5 mm with fibrous cotton of $K = 0.058$ W/m which is furter covered by a layer of plastic of thermal conductivity 0.42 W/mK and then it is exposed to surrounding air at 20 °C with a heat transfer coefficient of 20.5 W/ Calculate thickness of plastic layer which gives minimum temp. in a cotton insulation. The temperature of the copper rod and max. temp in the plastic layer for the above condition.



$$q_r = \frac{T_1 - T_\alpha}{R_1 + R_2 + R_3} = \frac{T_1 - T_\alpha}{\frac{1}{2\pi k_c L} \times \ln\frac{r_2}{r_1} + \frac{1}{2\pi k_p L} \times \ln\frac{r_3}{r_2} + \frac{1}{h \times 2\pi r_3 L}}$$

$$\frac{dR}{dr_3} = 0$$

$$r_3 = 20.48 \text{ mm}$$

$$r_3 - r_2 = 20.48 - 17.5 = 2.98 \text{ m}$$

$$\frac{1}{\cancel{2\pi} k_p \cancel{L}} \times \frac{\cancel{r_2}}{\cancel{r_3}} \cdot \frac{1}{\cancel{r_2}} = \frac{1}{h \times \cancel{2\pi} r_3^{\cancel{2}} \cancel{L}}$$

$$r_3 = \frac{k_p}{h}$$

$$R_e = \frac{\rho l}{A} = \frac{1.08 \times 10^{-8} \times 1}{\pi r_1^2} = 2.2 \times 10^{-5} \ \Omega/m$$

$$q_r = I^2 \times R_e = (1000)^2 \times 2.2 \times 10^{-5} = 22 \ W/m$$

$$R_1 + R_2 + R_3 = \frac{1}{2\pi k_c L} \times \ln\frac{r_2}{r_1} + \frac{1}{2\pi k_p L} \ln\frac{r_3}{r_2} + \frac{1}{h \times 2\pi r_3 L}$$

$$R = \frac{1}{2\pi \times 0.058 \times L} \times \ln\frac{17.5}{12.5} + \frac{1}{2\pi \times 0.42 L} \times \ln\frac{20.48}{17.5} + \frac{1}{20.5 \times 2 \times \pi \times 20.48 \times L}$$

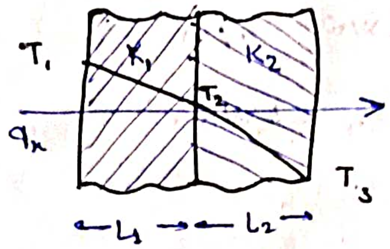
$$R = \frac{0.923}{L} + \frac{0.06}{L} + \frac{0.00037}{L}$$

$$R = 0.98 \ \Omega$$

$T_1 = 49.95°C$, $T_2 = 29.6°C$

Contact Resistance

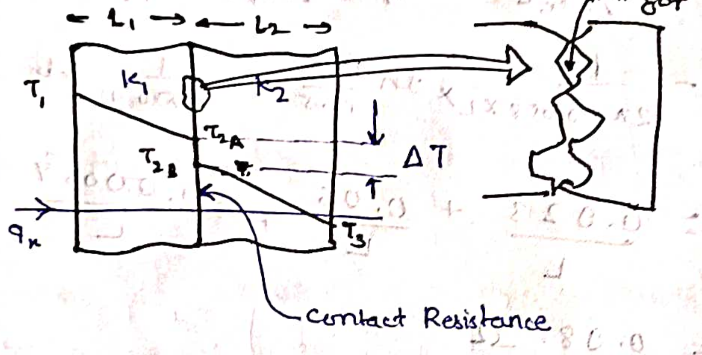
- Without Contact Resistance



$$q''_x = \frac{T_1 - T_3}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

$$q''_x = \frac{T_1 - T_2}{\frac{L_1}{K_1}} = \frac{T_2 - T_3}{\frac{L_2}{K_2}}$$

- With Contact Resistance



h_c = contact heat transfer coefficient

$$q''_x = \frac{T_1 - T_3}{\frac{L_1}{K_1} + \frac{1}{h_c} + \frac{L_2}{K_2}}$$

$$= \frac{T_1 - T_{2A}}{\frac{L_1}{K}}$$

$$= \frac{T_{2A} - T_{2B}}{\frac{1}{h_c}}$$

$$q''_x = \frac{T_{2B} - T_3}{\frac{L_2}{K_2}}$$

Ways to minimise contact resistance :—

1. Contact Resistance can be minimised if surfaces are polished.
2. Application of highly thermal conductive silicon based gels over the surfaces.
3. Do experiments / operations in vacuum space
4. By increasing the surface area.
5. Applications of external pressure over the surfaces can minimise the contact resistance.
6. If very highly thermal conductive material is used.

Q- The termal contact ~~Resistance~~ conductance at the interface of two aluminium plates is measured to be 11000 w/ m^2 k. Determine the thickness of aluminium plate whose thermal resistance is equal to thermal resistance of the interface between plates. Consider ther. Konductivity of 237 W / m^2k.
Aluminium

Sol)

$$\frac{L}{K} = \frac{1}{h_c} = \quad L = \frac{237}{11000} = 0.0215 \text{ m}$$

Heat conduction with internal heat generation :-

3-D heat conduction eqn with internal heat generation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

1-D steady state condition

$$\frac{\partial^2 T}{\partial x^2} + \frac{q'''}{k} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = -\frac{q'''}{K}$$

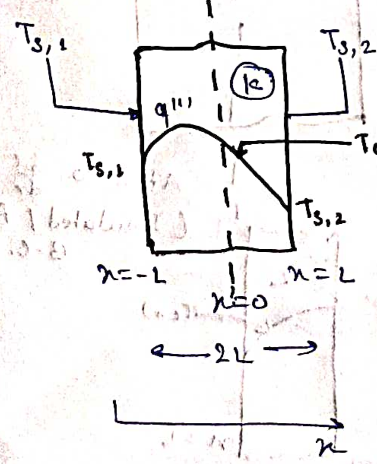
Integrating :

$$\frac{\partial T}{\partial x} = \frac{-q'''}{k} x + C_1$$

Integrating :-

$$T(x) = \frac{-q'''}{k} \frac{x^2}{2} + C_1 x + C_2$$

Heat conduction with heat generation in plane wall



Assymetrical B.C. (surface temperature are not same)

$$\frac{\partial^2 T}{\partial x^2} = \frac{-q'''}{k}$$

$$\frac{\partial T}{\partial x} = \frac{-q'''}{k} x + C_1$$

$$T = \frac{-q'''}{k}\frac{x^2}{2} + C_1 x + C_2$$

$T_{s,1} \neq T_{s,2}$

B.C. ① $x = -L$, $T = T_{s,1}$

② $x = L$, $T = T_{s,2}$

$$T_{s,1} = -\frac{q''' L^2}{2k} - C_1 L + C_2$$

$$T_{s,2} = \frac{-q''' L^2}{2k} + C_1 L + C_2$$

$$C_2 = \frac{q''' L^2}{2k} + \frac{T_{s,1} + T_{s,2}}{2}$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L}$$

$$T_{(x)} = \frac{-q'''}{k}\frac{x^2}{2} + \frac{T_{s,2} - T_{s,1}}{2} \cdot \frac{x}{L} + \frac{q''' L^2}{2k} + \frac{T_{s,1} + T_{s,2}}{2}$$

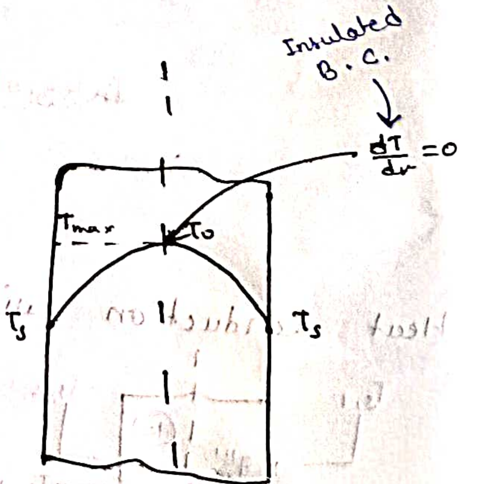
$$T_{(x)} = \frac{q''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\,\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2}$$

To get the max temp (T_0)

$$\frac{dT}{dx} = 0 \Rightarrow x \Rightarrow T_0$$

Symmetrical Boundary Condition :–

$$T_{s,1} = T_{s,2} = T_s$$



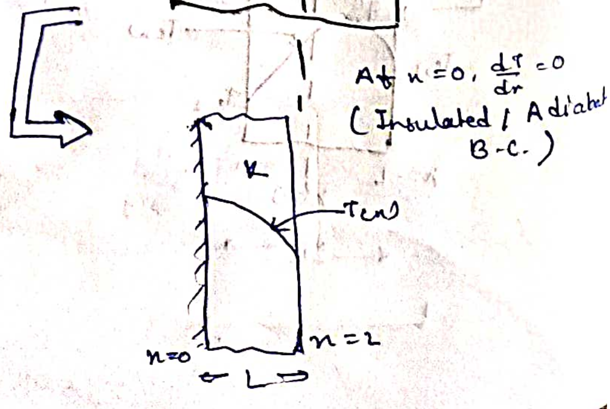
Now $T_{(x)} = \frac{q''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$

max temp.

$$\frac{dT}{dx} = 0$$

$$\boxed{x = 0}$$

$$\boxed{T_0 = \frac{q''' L^2}{2k} + T_s}$$



$$T_x - T_o = -\frac{q''' L^2}{2K}\left(\frac{x}{L}\right)^2$$

$$T_s - T_o = \frac{-q''' L^2}{2K}$$

$$\boxed{\frac{T_{(x)} - T_o}{T_s - T_o} = \left(\frac{x}{L}\right)^2}$$

Q- A plane wall of thermal conduct. .45 W/m K. 10 cm thick generated at uniform rate of 8×10^6 w/m3. The two sides of wall are maintained at 180°C and 120°C. Neglect the end effects. Calculate :-

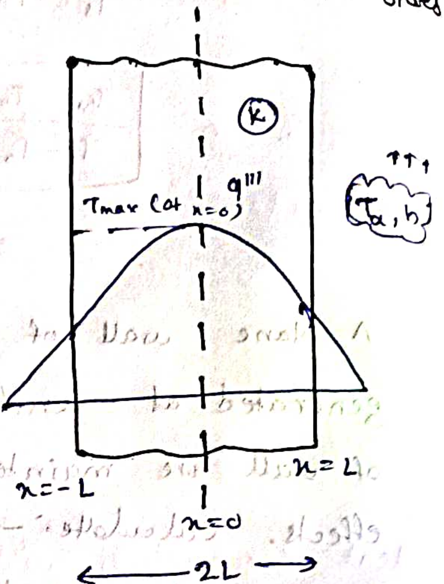
① Temperature distribution across the plate

② Position and magnitude of the max. Temp.

③ The heat flow rate from each surface of the plate.

21 March

Plane wall with convective boundary conditions at both the sides



$$\frac{d^2T}{dx^2} + \frac{q'''}{k} = 0$$

$$\frac{dT}{dx} = \frac{-q'''}{k} \cdot x + C_1$$

$$T(x) = \frac{-q'''}{2k} x^2 + C_1 x + C_2$$

At $x = -L$

$$-kA \left.\frac{dT}{dx}\right|_{x=-L} = hA \left[T|_{x=-L} - T_\alpha \right]$$

At $x = L$

$$-kA \left.\frac{dT}{dx}\right|_{x=L} = hA \left[T|_{x=L} - T_\alpha \right]$$

At $x = 0$

$$\frac{dT}{dx} = 0 \Rightarrow C_1 = 0$$

~~Now $\frac{k}{h} [T|_{x=L} - T_\alpha] + \frac{q'''}{k} L$~~

$$q''' L =$$

$$T|_{x=L} = -\frac{q'''}{2k} L^2 + C_2$$

$$\frac{-k}{h} \left.\frac{dT}{dx}\right|_{x=L} + T_\alpha = -\frac{q'''}{2k} L^2 + C_2$$

$$-\frac{\cancel{k}}{h} \times \frac{-q'''}{\cancel{k}} L + T_\alpha = -\frac{q'''}{2k} L^2 + C_2$$

$$C_2 = \frac{q''' \cdot L}{h} + \frac{q''' L^2}{2k} + T_\alpha$$

$\therefore$

$$\boxed{T(x) = \frac{-q'''}{2k} x^2 + \frac{q''' L}{h} + \frac{q''' L^2}{2k} + T_\alpha}$$

$$\frac{dT}{dx} = 0$$

$$2x \cdot -\frac{q'''}{2k} = 0$$

$$\boxed{x = 0}$$

Q- A large 3 cm thick steel plate of $k = 15.1$ W/mK is generating heat uniformally at the rate 5×10^8 W/m3. Its both side are exposed to convection to an ambient ~~of~~ at 30°C with a $h = 600$ W/m²K. Explain where in the the highest and lowest temp. occur, and the calculate the values.

Sol) $2L = 3\ cm = \cancel{3\times10}\ 0.03\ m$

$$T_{max} = \frac{5\times10^6 \times \frac{0.03}{2}}{600} + \frac{5\times10^6 \times 0.03^2}{2\times 15.1 \times 4} + 30°$$

$$= 125 + 37.25 + 30$$

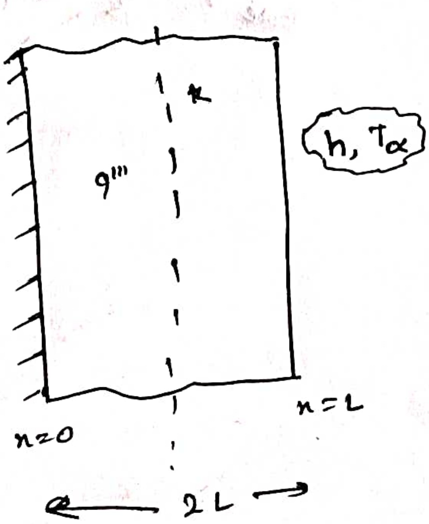
$$T_{max} = 192.25\ °C$$

$$T|_{x=L} = \cancel{-\frac{5\times10^6}{2\times15.1} \times \left(\frac{0.03}{2}\right)^2}\ \ 125 + 30 = 155\ °C$$

Plane wall with insulated and convective B.C.

$T_{cnd} = ??$

$T_{max} = ?$ & x at which $T = T_{max}$



B.C.

① At $x = 0$, $\frac{dT}{dx} = 0$

② At $x = L$

$$-k\frac{dT}{dx}\Big|_{x=L} = h\left[T|_{x=L} - T_\infty\right]$$

6th April

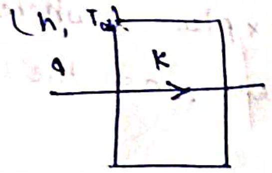
Extended Surface (fin)

$$q = hA\Delta T$$

$$q\uparrow \longrightarrow \begin{cases} h\uparrow \\ A\uparrow \end{cases}$$

Bi (Biot Number) ⇒ Non-dimensional number

$$B_i = \frac{\text{Internal Resistance}}{\text{External Resistance}}$$



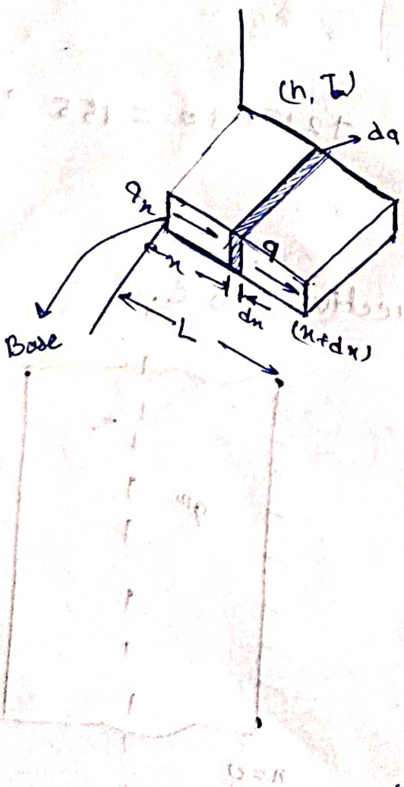
$$B_i = \frac{L/k}{\frac{1}{h}} = \frac{hL}{k}$$

$$B_i \text{ should be very low} \longrightarrow \begin{cases} h\downarrow \\ L\downarrow \\ K\uparrow \end{cases}$$

$$\boxed{(B_i < 0.1)}$$

→ Both conduction & convection are working

→ The partial differential eqn is converted into ODE



$$dq_{conv} = hA_s(T - T_\infty)$$

Energy in LHS = Energy in RHS

$$q_x = q_{(x+dx)} + dq_{conv.}$$

surface area

$$\text{or, } q_x = q_x + \frac{d}{dx}(q_x)dx + hA_s(T - T_\infty)$$

cross sectional area

$$\frac{d}{dx}\left(-kA_c\frac{dT}{dx}\right) + hP.dx(T - T_\infty) = 0$$

$$\text{or} \quad \frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

$$\text{consider,} \quad \frac{hP}{kA} = m^2$$

then,

$$\frac{d^2T}{dx^2} - m^2(T - T_\alpha) = 0$$

consider,

$$T - T_\alpha = \theta$$

$$\frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

$$\therefore \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

Also, $\theta = C_3 \sinh mx + C_4 \cosh mx$

$$\sinh mx = \frac{e^{mx} - e^{-mx}}{2}$$

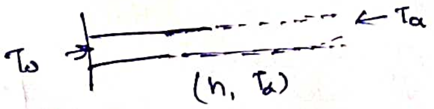
$$\cosh mx = \frac{e^{mx} + e^{-mx}}{2}$$

B.C. (1) At $x = 0$, $T = T_0$

$$T_0 - T_\alpha = \theta_0$$

$$C_1 + C_2 = \theta_2$$

B.C. (2) consider infinitely long fin



At $x = \infty$

$$T = T_a$$

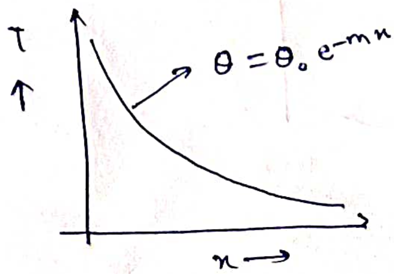
$$\therefore C_1 = 0, \quad C_2 = \theta_2$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx} = \theta_0 e^{-mx}$$

$$\frac{\theta}{\theta_0} = e^{-m} \Rightarrow \text{(non-dimensional temp)}$$

Temp. profile :

$$\frac{T - T_a}{T_0 - T_a} = e^{-mx}$$



Heat transfer from fin -

$$q = -KA \left.\frac{dT}{dx}\right|_{x=0}$$

$$= -KA \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$= -KA\,(-m\theta_0)$$

$$= -KA\, m\,\theta_0$$

$$= KA \sqrt{\frac{hP}{KA}}\,\theta_2$$

$$\boxed{q = \sqrt{hPKA}\;\theta_0}$$

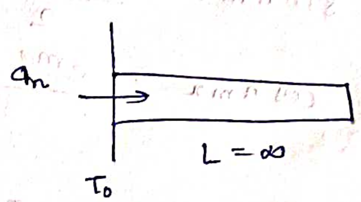
$$\theta = \theta_0 e^{-mx}$$

$$\frac{d\theta}{dx} = -m\theta_0 e^{-mx}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = -m\,\theta_0$$

since $m^2 = \frac{hP}{KA}$

$$\theta = \theta_0 e^{-mx}$$



$$q = \int_0^\infty hA\,(T - T_\infty)$$

$$\int_0^L hA\,(T - T_\infty)$$

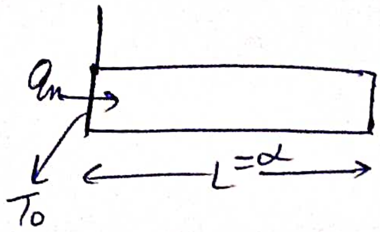
$$= \int_0^\infty hP\, dx\,(T - T_\infty)$$

$$= hP \int_0^\infty \theta \, dx$$

$$= hP\theta_0 \int_0^\infty e^{-mx} dx$$

$$= \frac{hP\theta_0}{m}$$

$$q = \frac{hP\theta_0}{m} =$$



$$q_x = \int_0^{\alpha} hA(T-T_\alpha)$$

$$q = \int_0^{\alpha} h\,P\,dx\,(T-T_\alpha)$$

$$q = hP\int_0^{\alpha}\theta\,dx \Rightarrow q = hP\theta_0\int_0^{\alpha} e^{-mx}\,dx$$

$$\Rightarrow q = \frac{hP\theta_0}{(-m)}(-1) = \frac{hP\theta_0}{m} = \frac{hP\theta_0}{\sqrt{\frac{hP}{kA}}} = \sqrt{hPkA}\,\theta_0$$

Extended Surface :

Case I: Infinitely long fin

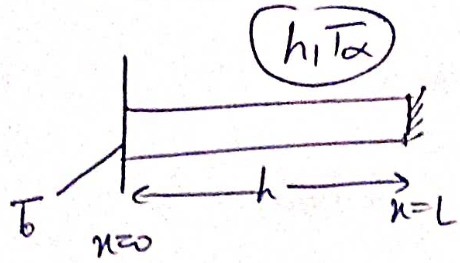
$$\theta = \theta_0 e^{-mx}$$
$$q = \sqrt{hPKA}\,\theta_0$$

$$\theta = T - T_\alpha$$
$$\theta_0 = T_0 - T_\alpha$$

$$\frac{d^2T}{dx^2} - m^2\theta = 0$$

BC $\rightarrow$ (i) At $x=0$, $T = T_0$, $\theta = \theta_0$

Case 2. Infinite length, Insulated tip.



At $x=L \Rightarrow \frac{dT}{dx} = 0$

i.e. $\frac{d\theta}{dx} = 0$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0 \Rightarrow \theta = C_3 \sinh mx + C_4 \cosh mx$$

At $x=0$, $T = T_0$, $\theta = \theta_0$

$$\boxed{C_4 = \theta_0}$$

$$\Rightarrow C_3 = \frac{-\theta_0 \sinh mL}{\cosh mL};$$

$$\Rightarrow \theta = \frac{-\theta_0 \sinh mL}{\cosh mL} \sinh mx + \theta_0 \cosh mx$$

$$\theta = \frac{\theta_0}{\cosh mL} \cosh m(L-x)$$

$$\Rightarrow \boxed{\frac{\theta}{\theta_0} = \frac{\cosh m(L-x)}{\cosh mL}}$$ (Non-dim. form of Temp.)

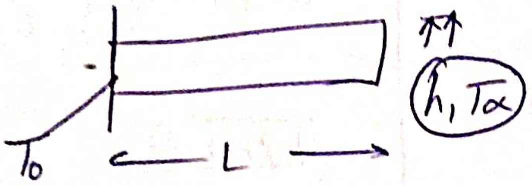
$\Rightarrow$ At $x=L$, $\theta = \theta_L \Rightarrow \boxed{\theta_L = \frac{\theta_0}{\cosh mL}}$

$$\Rightarrow q = -KA \left.\frac{dT}{dx}\right)_{x=0} = -KA \left.\frac{d\theta}{dx}\right)_{x=0};$$

$$\Rightarrow \quad \theta = \frac{\theta_0 \cosh m(L-x)}{\cosh mL}$$

$$\Rightarrow \quad \boxed{q = \sqrt{hPKA}\,(\tanh mL)\,\theta_0}$$

Case 3 : Finite length and Convective tip.



B.C. $\rightarrow$ 1) At $x=0 \Rightarrow \theta = \theta_0$

2) At $x = L$,

$$-k\frac{dT}{dx} = h\left[T - T_\alpha\right]$$

$$\Rightarrow \theta = C_3 \sinh mx + C_4 \cosh mx \qquad \Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\Rightarrow \frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL} \qquad \left\{-k\frac{d\theta}{dx} = h\theta\right\}$$

$$\Rightarrow q = -KA\left.\frac{d\theta}{dx}\right)_{x=0}$$

$$= \sqrt{hPKA}\,\theta_0\,\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}$$

or

$$q = \sqrt{hPKA}\,\theta_0 \frac{\frac{h}{mk} + \tanh mL}{1 + \frac{h}{mk}\tanh mL}$$

$\rightarrow$ $\frac{d^2\theta}{dx^2} - m^2\theta = 0$, $m = \sqrt{\frac{hP}{KA}}$, 1^{st} BC : At $x=0$, $T = T_0$ i.e. $\theta = \theta_0$

Type of Fin	2^{nd} B.C.	θ/θ_0	$q = KA \left.\frac{d\theta}{dx}\right)_{x=0}$
1) Infinitely long fin	$\theta(x=\infty) = 0$	e^{-mx}	$\sqrt{hPKA}\,\theta_0$
2). Finite length, Insulated tip	$\left.\frac{d\theta}{dx}\right)_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$\sqrt{hPKA}\,\theta_0 \tanh mL$
3) Finite length Convective tip	$-k\left.\frac{d\theta}{dx}\right)_{x=L} = h[T(x=L) - T_\infty]$	$\frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh m(L-x)}$	$\sqrt{hPKA}\,\theta_0 \frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}$

$\Rightarrow$ If fin is circular $\equiv$ Pin fin

$\Rightarrow$ for circular fin $\rightarrow$ $P = \pi d$, $A = \frac{\pi d^2}{4}$ ** [we have to write this 2]

1.Q. A very long 25 mm dia. Cu of th. Cond. 380 $W/m°K$ rod extends from a surface at 120°C. The temp. of surr. air is 25°C. & heat tr. Coeff. over the rod is 10 $W/m^2°K$. Calculate,

1) Heat loss from the rod.

2). How long the rod should be in order to be consider infinite.

$[T = T_\infty \ , \ \theta = 0 = T - T_\infty]$

Ass.

Q. A very long rod 5 mm in dia. has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with conv. heat tr. Coeff. of 100 $W/m^2°K$

Determine,

1) Temp. distr. along rod. Constructed from pure Cu, 2024 Al alloy, AISI 316 stainless steel. what are the corresponding heat losses from the end.

2). Estimate how long the rods must be for assumption of ∞ length to yield an accurate estimate of heat loss.

Ans 1. An infinitely long fin has the tip temp. = T_∞, i.e. very similar to insulated tipped fin.

$q_{\text{infinite fin}} = q_{\text{insulated tip}}$

$\sqrt{hPKA}\ \theta_0 = \sqrt{hPKA}\ \theta_0 \tanh mL$

$\tanh mL \geq 0.99 \Rightarrow mL \geq \tanh^{-1}(0.99)$

≥ 2.646

$\geq \frac{2.646}{m} \geq \frac{2.646}{2.052} \geq 1.29$

Q. An electric motor is to be connected by hor. steel shaft of the cond. $42.56\ W/m^{\circ}K$, 25mm in dia. to an impeller of a pump circulation liq. metal at a temp. of 540°C. If temp. of electr. motor is lim. to a max. of 52°C with the ambient air at 27°C & heat tr. coeff. of $40.7\ W/m^2{}^{\circ}K$. What length of the shaft should be specified b/w the motor & pump.

$\Rightarrow m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{40.7 \times \pi \times 25 \times 10^{-3}}{42.56 \times \pi/4 \times 625 \times 10^{-6}}} =$

$\Rightarrow \frac{\theta}{\theta_0} = \frac{\cosh m(L-x)}{\cosh mL} =$

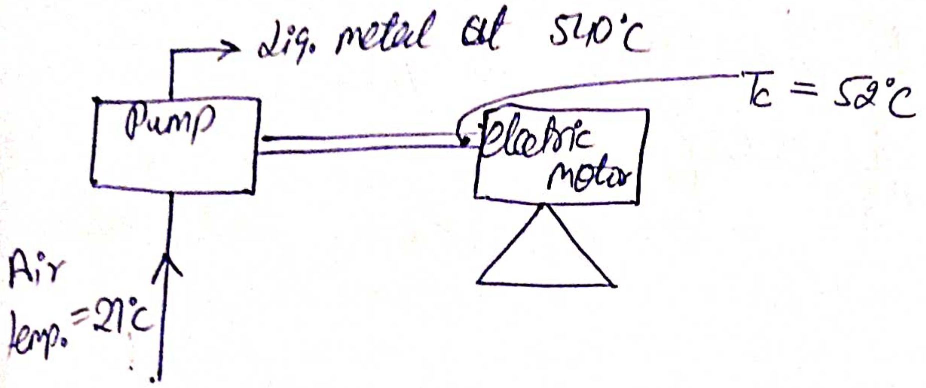
Ans.

1) steady state cond.

2) Const. pr.

3) Since one end of shaft is connected to electric motor

das assuming no heat loss from fin tip.



$P = \pi d = 0.025\pi$ m

$A = \pi/_4 d^2 = 4.90 \times 10^{-4}$ m^2

$\Rightarrow$ At $x = L$ fin is insulated tip fin,

$$\frac{\theta}{\theta_0} = \frac{1}{\cosh mL} \Rightarrow \frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh mL}$$

$$\Rightarrow \frac{52-27}{540-27} = \frac{1}{\cosh mL} \Rightarrow \cosh mL = \frac{573}{25}$$

$$\Rightarrow mL = \cosh^{-1}\left(\frac{573}{25}\right) \Rightarrow \boxed{L = 0.30\ m} \quad , m = 12.37\ m^{-1}$$

Q. A 1.25 cm dia., 15 cm long Iron rod of th. cond. 40 W/m°K foot protrud out from a heat source at 130°C into an ambient at 20°C with Convection Coeff.

of 20 $W/m^2 K$. Determine, $L = 15cm$

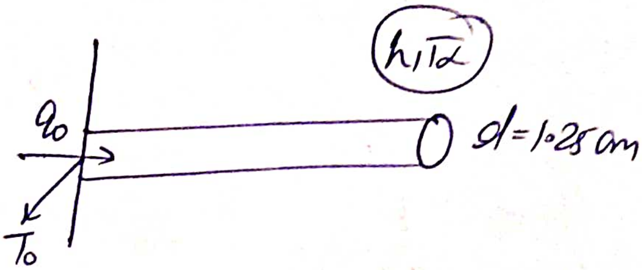
(i) Temp. at free end.

(ii) Heat flow out the source

(iii) Heat flow rate at free end.

$$\Rightarrow \frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + \frac{h}{mK}\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh m(L-x)} = \frac{T_L - T_\alpha}{T_0 - T_\alpha}$$

$\Rightarrow$



$\Rightarrow$ At $x = L$. $\Rightarrow T = T_\alpha$

$$\frac{1}{\cosh mL} = \frac{T_L - 20}{110}$$

$$m = \sqrt{\frac{20 \times \pi \times 1.25 \times 10^{-2}}{40 \times \pi/4 \times (1.25 \times 10^{-2})^2}}$$

$$\Rightarrow T_L = \frac{110}{\cosh mL} + 20$$

$$m = \sqrt{\frac{2.5}{0.490}} = 2.256$$

$$\Rightarrow T_L = 57.1^\circ C$$

(ii)

$$\rightarrow q = \sqrt{hPKA}\,\theta_0 \frac{\sinh mL + \frac{h}{mK}\cosh mL}{\cosh mL + \frac{h}{mK}\sinh mL} = 6.54 W$$

$$m = \sqrt{\frac{hP}{KA}} = 1.897 \; m^{-1}$$

For Circular, $m = \sqrt{\frac{4h}{Kd}}$

(iii) At free end, $q_L = hA(T_L - T_\infty)$

$$q_L = 20 \times \frac{\pi}{4} d^2 (51.1 - 20)$$

$$\boxed{q_L = 0.0763 \, W}$$

Rem. i.e. $(6.54 - 0.0763)\, W$ is at the surface.

Q. One end of long rod 3cm dia. is inserted into a furnance with outer end projecting into the outside the air. once the steady state is reached. Temp. of rod is measured at 2 points 15cm apart ~~at~~ & ~~atm. air~~ found to be 140°C & 100°C when the atm. air is at 30°C with conv. coeff. of 20 W/m²K. Calculate the th. cond. of rod mat.

$\Rightarrow$ $\frac{\theta}{\theta_0} = e^{-mx}$ $\Rightarrow$ $\frac{T_L - T_\infty}{T_0 - T_\infty} = e^{-mL}$, $\boxed{K = 293.74}$

$\Rightarrow$ $\frac{140 - 30}{100 - 30} = e^{-m(0.15)}$

$\Rightarrow$ $\frac{110}{70} = e^{-0.15m}$ $\Rightarrow$ $e^{0.15}$

Fin performance Parameter :

1) **Fin Effectiveness** (ϵ_f) $= \dfrac{\text{Heat tr. with fin}}{\text{Heat tr. without fin}}$

$$= \frac{q_f}{hA\theta_0}$$

→ Insulated tip,

$$\epsilon_f = \frac{\sqrt{hPKA}\,\theta_0 \tanh mL}{hA\theta_0} = \sqrt{\frac{PK}{hA}}\tanh mL = \frac{\tanh mL}{m}$$

⇒ Effectiveness is always > 1 $[\epsilon_f > 1]$

If $\epsilon_f \geq 2$ ⇒ fin is

2) **Fin Efficiency** (η_f)

$$\Rightarrow \quad \eta_f = \frac{\text{Actual heat tr.}}{\text{Heat tr. if the fin is at base temp.}}$$

⇒ For Insulated tip,

$$\eta_f = \frac{\sqrt{hPKA}\;\theta_0 \tanh mL}{h(PL)\times\theta_0} = \frac{\tanh mL}{mL}$$

→ Area of fin

Relationship b/w η_f & ϵ_f :

$$\Rightarrow \epsilon_f = \frac{q_f}{h \times A_c \times \theta_0} = \frac{q_f}{h \times A_f \times \theta_0} \times \frac{A_f}{A_c}$$

A_f → Area of fin

A_c → Cross section area of fin

$$\Rightarrow \boxed{\epsilon_f = \eta_f \times \frac{A_f}{A_c}}$$