handwritten annotations: P, n, V_0, $\leftarrow \mathcal{E}$, v_n, V_0, v_p, Electrostatic Potential

P-type, n-type, E_c, E_v, E_i, qV_0, E_{cn}, E_F, P, i=P, E_{vn}

Particle current, Hole diffusion, Hole drift, Electron diffusion, Electron drift, flow

created at the junction (Fig. 5–11b). If the two regions were boxes of red air molecules and green molecules (perhaps due to appropriate types of pollution), eventually there would be a homogeneous mixture of the two after the boxes were joined. This cannot occur in the case of the charged particles in a p-n junction because of the development of space charge and the electric field $\mathcal{E}$. If we consider that electrons diffusing from n to p leave behind uncompensated[3] donor ions (N_d^+) in the n material, and holes leaving the p region leave behind uncompensated acceptors (N_a^-), it is easy to visualize the development of a region of positive space charge near the n side of the junction and negative charge near the p side. The resulting electric field is directed from the positive charge toward the negative charge. Thus $\mathcal{E}$ is in the direction opposite to that of diffusion current for each type of carrier (recall electron current is opposite to the direction of electron flow). Therefore, the field creates a drift component of current from n to p, opposing the diffusion current (Fig. 5–11c).

Since we know that no *net* current can flow across the junction at equilibrium, the current due to the drift of carriers in the $\mathcal{E}$ field must exactly cancel the diffusion current. Furthermore, since there can be no net buildup of electrons or holes on either side as a function of time, the drift and diffusion currents must cancel for *each* type of carrier.

$$J_p(\text{drift}) + J_p(\text{diff.}) = 0 \qquad (5\text{-}3a)$$

$$J_n(\text{drift}) + J_n(\text{diff.}) = 0 \qquad (5\text{-}3b)$$

Therefore, the electric field $\mathcal{E}$ builds up to the point where the net current is zero at equilibrium. The electric field appears in some region W about the junction, and there is an equilibrium potential difference V_0 across W. In the electrostatic potential diagram of Fig. 5–11b, there is a gradient in potential in the direction opposite to $\mathcal{E}$, in accordance with the fundamental relation[4] $\mathcal{E}(x) = -dV(x)/dx$. We assume the electric field is zero in the neutral regions outside W. Thus there is a constant potential V_n in the neutral n material, a constant V_p in the neutral p material, and a potential difference $V_0 = V_n - V_p$ between the two. The region W is called the *transition region*,[5] and the potential difference V_0 is called the *contact potential*. The contact potential appearing across W is a *built-in* potential barrier, in that it is necessary to the maintenance

---

[3] We recall that neutrality is maintained in the bulk materials of Fig. 5–11a by the presence of one electron for each ionized donor ($n = N_d^+$) in the n material and one hole for each ionized acceptor ($p = N_a^-$) in the p material (neglecting minority carriers). Thus, if electrons leave n, some of the positive donor ions near the junction are left uncompensated, as in Fig. 5–11b. The donors and acceptors are fixed in the lattice, in contrast to the mobile electrons and holes.

[4] When we write $\mathcal{E}(x)$, we refer to the value of $\mathcal{E}$ as computed in the x-direction. This value will of course be negative, since it is directed opposite to the true direction of $\mathcal{E}$ as shown in Fig. 5–11b.

[5] Other names for this region are the space charge region, since space charge exists within W while neutrality is maintained outside this region, and the depletion region, since W is almost depleted of carriers compared with the rest of the crystal. The contact potential V_0 is also called the diffusion potential, since it represents a potential barrier which diffusing carriers must surmount in going from one side of the junction to the other.

of equilibrium at the junction: it does not imply any external potential. Indeed, the contact potential cannot be measured by placing a voltmeter across the devices, because new contact potentials are formed at each probe, just canceling V_0. By definition V_0 is an equilibrium quantity, and no net current can result from it.

The contact potential separates the bands as in Fig. 5–11b: the valence and conduction energy bands are higher on the p side of the junction than on the n side[6] by the amount qV_0. The separation of the bands at equilibrium is just that required to make the Fermi level constant throughout the device. We discussed the lack of spatial variation of the Fermi level at equilibrium in Section 3.5. Thus if we know the band diagram, including E_F, for each separate material (Fig. 5–11a), we can find the band separation for the junction at equilibrium simply by drawing a diagram such as Fig. 5–11b with the Fermi levels aligned.

To obtain a quantitative relationship between V_0 and the doping concentrations on each side of the junction, we must use the requirements for equilibrium in the drift and diffusion current equations. For example, the drift and diffusion components of the hole current just cancel at equilibrium:

$$J_p(x) = q\left[\mu_p p(x)\mathcal{E}(x) - D_p \frac{dp(x)}{dx} \right] = 0 \qquad (5\text{–}4a)$$

This equation can be rearranged to obtain

$$\frac{\mu_p}{D_p}\mathcal{E}(x) = \frac{1}{p(x)}\frac{dp(x)}{dx} \qquad (5\text{–}4b)$$

where the x-direction is arbitrarily taken from p to n. The electric field can be written in terms of the gradient in the potential, $\mathcal{E}(x) = -dV(x)/dx$, so that Eq. (5–4b) becomes

$$-\frac{q}{kT}\frac{dV(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx} \qquad (5\text{–}5)$$

with the use of the Einstein relation for μ_p/D_p. This equation can be solved by integration over the appropriate limits. In this case we are interested in the potential on either side of the junction, V_p and V_n, and the hole concentration just at the edge of the transition region on either side, p_p and p_n. For a step junction it is reasonable to take the electron and hole concentration in the neutral regions outside the transition region as their equilibrium values. Since we have assumed a one-dimensional geometry, p and V can be taken reasonably as functions of x only. Integration of Eq. (5–5) gives

[6]The electron energy diagram of Fig. 5–11b is related to the electrostatic potential diagram by $-q$, the negative charge on the electron. Since V_n is a higher potential than V_p by the amount V_0, the electron energies on the n side are lower than those on the p side by qV_0.

$$-\frac{q}{kT}\int_{V_p}^{V_n} dV = \int_{p_p}^{p_n} \frac{1}{p} dp$$

$$-\frac{q}{kT}(V_n - V_p) = \ln p_n - \ln p_p = \ln\frac{p_n}{p_p} \qquad (5\text{-}6)$$

The potential difference $V_n - V_p$ is the contact potential V_0 (Fig. 5–11b). Thus we can write V_0 in terms of the equilibrium hole concentrations on either side of the junction:

$$V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} \qquad (5\text{-}7)$$

If we consider the step junction to be made up of material with N_a acceptors/cm^3 on the p side and a concentration of N_d donors on the n side, we can write Eq. (5–7) as

$$V_0 = \frac{kT}{q}\ln\frac{N_a}{n_i^2/N_d} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} \qquad (5\text{-}8)$$

by considering the majority carrier concentration to be the doping concentration on each side.

Another useful form of Eq. (5–7) is

$$\frac{p_p}{p_n} = e^{qV_0/kT} \qquad (5\text{-}9)$$

By using the equilibrium condition $p_p n_p = n_i^2 = p_n n_n$, we can extend Eq. (5–9) to include the electron concentrations on either side of the junction:

$$\boxed{\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}} \qquad (5\text{-}10)$$

This relation will be very valuable in calculation of the I–V characteristics of the junction.

EXAMPLE 5–1

An abrupt Si p-n junction has $N_a = 10^{17}$ cm^{-3} on the p side and $N_d = 10^{16}$ cm^{-3} on the n side. At 300 K, (a) calculate the Fermi levels, draw an equilibrium band diagram and find V_0 from the diagram; (b) compare the result from (a) with V_0 calculated from Eq. (5–8).
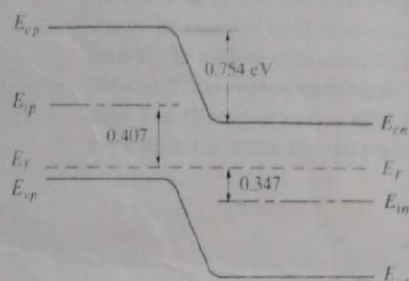
SOLUTION

(a) Find E_F on each side

$$E_{ip} - E_F = kT\ln\frac{p_p}{n_i} = 0.0259\ln\frac{10^{17}}{(1.5 \times 10^{10})} = 0.407\ \text{eV}$$

$$E_F - E_{in} = kT \ln\frac{n_n}{n_i} = 0.0259 \ln\frac{10^{16}}{(1.5 \times 10^{10})} = 0.347\ \text{eV}$$

$$qV_0 = 0.407 + 0.347 = 0.754\ \text{eV}$$

(b) Find V_0 from Eq. (5–8)

$$qV_0 = kT \ln\frac{N_a N_d}{n_i^2} = 0.0259 \ln\frac{10^{33}}{2.25 \times 10^{20}} = 0.754\ \text{eV}$$

5.2.2 Equilibrium Fermi Levels

We have observed that the Fermi level must be constant throughout the device at equilibrium. This observation can be easily related to the results of the previous section. Since we have assumed that p_n and p_p are given by their equilibrium values outside the transition region, we can write Eq. (5–9) in terms of the basic definitions of these quantities using Eq. (3–19):

$$\frac{p_p}{p_n} = e^{qV_0/kT} = \frac{N_v e^{-(E_{Fp}-E_{vp})/kT}}{N_v e^{-(E_{Fn}-E_{vn})/kT}} \tag{5–11a}$$

$$e^{qV_0/kT} = e^{(E_{Fn}-E_{Fp})/kT} e^{(E_{vp}-E_{vn})/kT} \tag{5–11b}$$

$$qV_0 = E_{vp} - E_{vn} \tag{5–12}$$

The Fermi level and valence band energies are written with subscripts to indicate the p side and the n side of the junction.

From Fig. 5–11b the energy bands on either side of the junction are separated by the contact potential V_0 times the electronic charge q; thus the energy difference $E_{vp} - E_{vn}$ is just qV_0. Equation (5–12) results from the fact that the Fermi levels on either side of the junction are equal at equilibrium ($E_{Fn} - E_{Fp} = 0$). When bias is applied to the junction, the potential barrier is

raised or lowered from the value of the contact potential, and the Fermi levels on either side of the junction are shifted with respect to each other by an energy in electron volts numerically equal to the applied voltage in volts.

5.2.3 Space Charge at a Junction

Within the transition region, electrons and holes are in transit from one side of the junction to the other. Some electrons diffuse from n to p, and some are swept by the electric field from p to n (and conversely for holes); there are, however, very few carriers within the transition region at any given time, since the electric field serves to sweep out carriers which have wandered into W. To a good approximation, we can consider the space charge within the transition region as due only to the uncompensated donor and acceptor ions. The charge density within W is plotted in Fig. 5–12b. Neglecting carriers within the space charge region, the charge density on the n side is just q
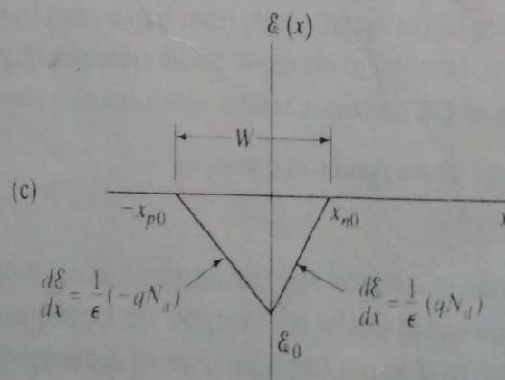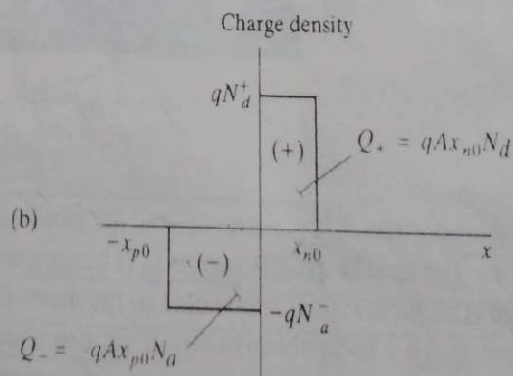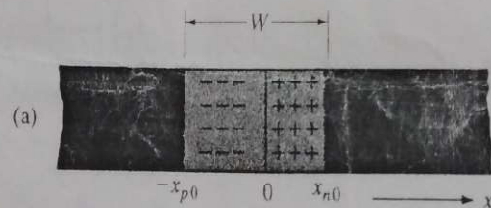

Figure 5–12
Space charge and electric field distribution within the transition region of a p-n junction with $N_d > N_a$: (a) the transition region, with $x = 0$ defined at the metallurgical junction; (b) charge density within the transition region, neglecting the free carriers; (c) the electric field distribution, where the reference direction for $\mathcal{E}$ is arbitrarily taken as the +x-direction.

times the concentration of donor ions N_d, and the negative charge density on the p side is $-q$ times the concentration of acceptors N_a. The assumption of carrier depletion within W and neutrality outside W is known as the *depletion approximation*.

Since the dipole about the junction must have an equal number of charges on either side,[7] ($Q_+ = |Q_-|$), the transition region may extend into the p and n regions unequally, depending on the relative doping of the two sides. For example, if the p side is more lightly doped than the n side ($N_a < N_d$), the space charge region must extend farther into the p material than into the n, to "uncover" an equivalent amount of charge. For a sample of cross-sectional area A, the total uncompensated charge on either side of the junction is

$$q A x_{p0} N_a = q A x_{n0} N_d \qquad (5-13)$$

where x_{p0} is the penetration of the space charge region into the p material, and x_{n0} is the penetration into n. The total width of the transition region (W) is the sum of x_{p0} and x_{n0}.

To calculate the electric field distribution within the transition region, we begin with *Poisson's equation*, which relates the gradient of the electric field to the local space charge at any point x:

$$\frac{d\mathcal{E}(x)}{dx} = \frac{q}{\epsilon}(p - n + N_d^+ - N_a^-) \qquad (5-14)$$

This equation is greatly simplified within the transition region if we neglect the contribution of the carriers $(p - n)$ to the space charge. With this approximation we have two regions of constant space charge:

$$\frac{d\mathcal{E}}{dx} = \frac{q}{\epsilon}N_d, \quad 0 < x < x_{n0} \qquad (5-15a)$$

$$\frac{d\mathcal{E}}{dx} = -\frac{q}{\epsilon}N_a, \quad -x_{p0} < x < 0 \qquad (5-15b)$$

assuming complete ionization of the impurities ($N_d^+ = N_d$, and ($N_a^- = N_a$). We can see from these two equations that a plot of $\mathcal{E}(x)$ vs. x within the transition region has two slopes, positive ($\mathcal{E}$ increasing with x) on the n side and negative ($\mathcal{E}$ becoming more negative as x increases) on the p side. There is some maximum value of the field $\mathcal{E}_0$ at $x = 0$ (the metallurgical junction between the p and n materials), and $\mathcal{E}(x)$ is everywhere negative within the transition region (Fig. 5-12c). These conclusions come from Gauss's law, but we could predict the qualitative features of Fig. 5-12 without equations. We

[7]A simple way of remembering this equal charge requirement is to note that electric flux lines must begin and end on charges of opposite sign. Therefore, if Q_+ and Q_- were not of equal magnitude, the electric field would not be contained within W but would extend farther into the p or n regions until the enclosed

expect the electric field $\mathcal{E}(x)$ to be negative throughout W, since we know that the $\mathcal{E}$ field actually points in the $-x$-direction, from n to p (i.e., from the positive charges of the transition region dipole toward the negative charges). The electric field is assumed to go to zero at the edges of the transition region, since we are neglecting any small $\mathcal{E}$ field in the neutral n or p regions. Finally, there must be a maximum $\mathcal{E}_0$ at the junction, since this point is between the charges Q_+ and Q_- on either side of the transition region. All the electric flux lines pass through the $x = 0$ plane, so this is the obvious point of maximum electric field.

The value of $\mathcal{E}_0$ can be found by integrating either part of Eq. (5-15) with appropriate limits (see Fig. 5-12c in choosing the limits of integration).

$$\int_{\mathcal{E}_0}^{0} d\mathcal{E} = \frac{q}{\epsilon}N_d\int_{0}^{x_{n0}} dx, \qquad 0 < x < x_{n0} \qquad (5\text{-}16a)$$

$$\int_{0}^{\mathcal{E}_0} d\mathcal{E} = -\frac{q}{\epsilon}N_a\int_{-x_{p0}}^{0} dx, \quad -x_{p0} < x < 0 \qquad (5\text{-}16b)$$

Therefore, the maximum value of the electric field is

$$\mathcal{E}_0 = -\frac{q}{\epsilon}N_d x_{n0} = -\frac{q}{\epsilon}N_a x_{p0} \qquad (5\text{-}17)$$

It is simple to relate the electric field to the contact potential V_0, since the $\mathcal{E}$ field at any x is the negative of the potential gradient at that point:

$$\mathcal{E}(x) = -\frac{dV(x)}{dx} \quad \text{or} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathcal{E}(x)dx \qquad (5\text{-}18)$$

Thus the negative of the contact potential is simply the area under the $\mathcal{E}(x)$ vs. x triangle. This relates the contact potential to the width of the depletion region:
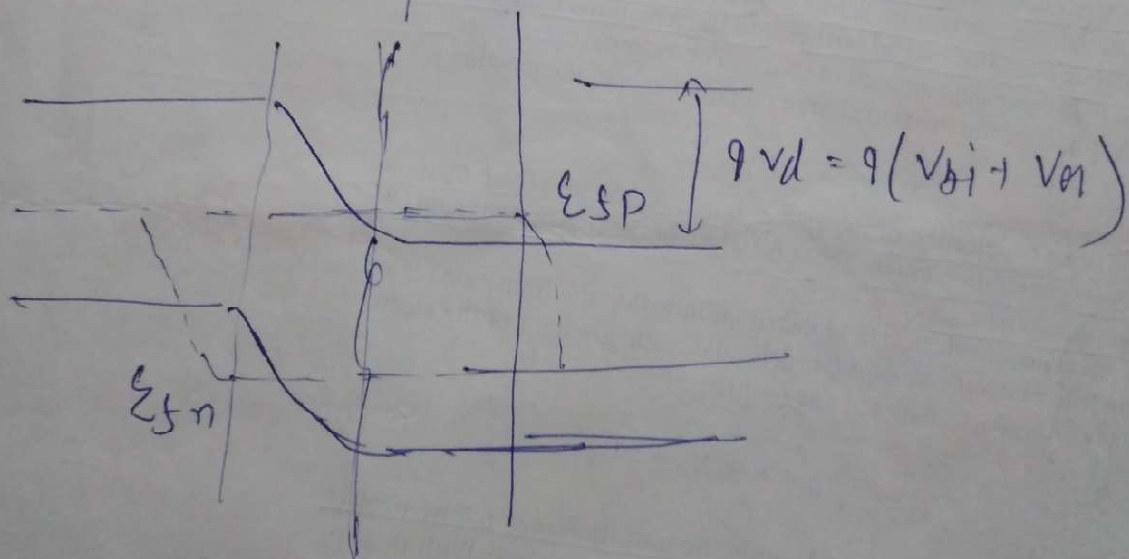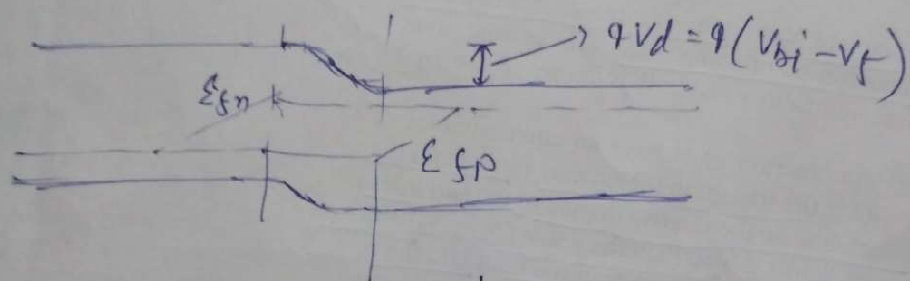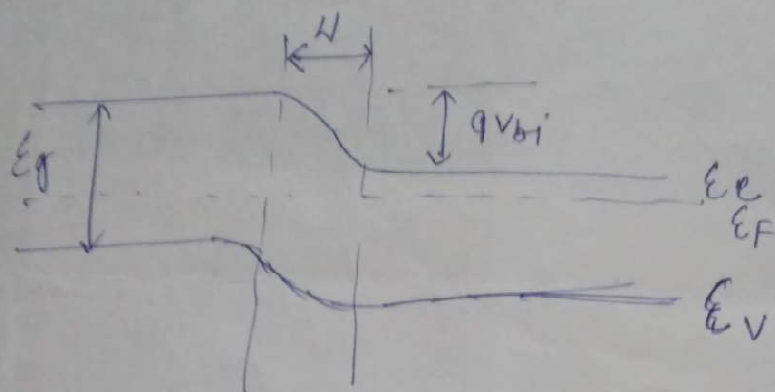
$$V_0 = -\frac{1}{2}\mathcal{E}_0 W = \frac{1}{2}\frac{q}{\epsilon}N_d x_{n0} W \qquad (5\text{-}19)$$

Since the balance of charge requirement is $x_{n0}N_d = x_{p0}N_a$, and W is simply $x_{p0} + x_{n0}$, we can write $x_{n0} = WN_a/(N_a + N_d)$ in Eq. (5-19):

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}\frac{N_a N_d}{N_a + N_d}W^2 \qquad (5\text{-}20)$$

By solving for W, we have an expression for the width of the transition region in terms of the contact potential, the doping concentrations, and known constants q and ϵ.

qV_{bi}

$\mathcal{E}_g$

$\mathcal{E}_c$
$\mathcal{E}_F$

$\mathcal{E}_v$

$qV_d = q(V_{bi} - V_f)$

$\mathcal{E}_{fn}$

$\mathcal{E}_{fp}$

$\mathcal{E}_{fp}$

$qV_d = q(V_{bi} + V_{or})$

$\mathcal{E}_{fn}$

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} \quad (5\text{-}21)$$

There are several useful variations of Eq. (5–21); for example, V_0 can be written in terms of the doping concentrations with the aid of Eq. (5–8):

$$W = \left[\frac{2\epsilon kT}{q^2}\left(\ln\frac{N_a N_d}{n_i^2}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} \quad (5\text{-}22)$$

$V_0 = \dfrac{kT}{q}\ln\dfrac{N_a N_d}{n_i^2}$

We can also calculate the penetration of the transition region into the n and p materials:

$$x_{p0} = \frac{WN_d}{N_a + N_d} = \frac{W}{1 + N_a/N_d} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_d}{N_a(N_a + N_d)}\right]\right\}^{1/2} \quad (5\text{-}23a)$$

$$x_{n0} = \frac{WN_a}{N_a + N_d} = \frac{W}{1 + N_d/N_a} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_a}{N_d(N_a + N_d)}\right]\right\}^{1/2} \quad (5\text{-}23b)$$

As expected, Eqs. (5–23) predict that the transition region extends farther into the side with the lighter doping. For example, if $N_a \ll N_d$, x_{p0} is large compared with x_{n0}. This agrees with our qualitative argument that a deep penetration is necessary in lightly doped material to "uncover" the same amount of space charge as for a short penetration into heavily doped material.

Another important result of Eq. (5–21) is that the transition width W varies as the square root of the potential across the region. In the derivation to this point, we have considered only the equilibrium contact potential V_0. In Section 5.3 we shall see that an applied voltage can increase or decrease the potential across the transition region by aiding or opposing the equilibrium electric field. Therefore, Eq. (5–21) predicts that an applied voltage will increase or decrease the width of the transition region as well.

Boron is implanted into an n-type Si sample ($N_d = 10^{16}$ cm^{-3}), forming an abrupt junction of square cross section, with area $= 2 \times 10^{-3}$ cm^2. Assume that the acceptor concentration in the p-type region is $N_a = 4 \times 10^{18}$ cm^{-3}. Calculate V_0, x_{n0}, x_{p0}, Q_+, and $\mathscr{E}_0$ for this junction at equilibrium (300 K). Sketch $\mathscr{E}(x)$ and charge density to scale, as in Fig. 5–12.
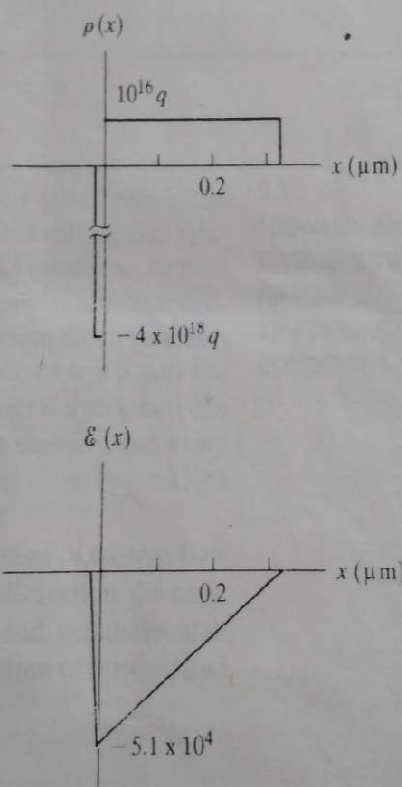
EXAMPLE 5–2

From Eq. (5–8),

SOLUTION

$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = 0.0259\ln\frac{4 \times 10^{34}}{2.25 \times 10^{20}}$$

$$= 0.0259\ln(1.78 \times 10^{14}) = 0.85\ V$$

From Eq. (5–21),

$$W = \left[\frac{2\epsilon V_0}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

$$= \left[\frac{2(11.8 \times 8.85 \times 10^{-14})(0.85)}{1.6 \times 10^{-19}} (0.25 \times 10^{-18} + 10^{-16}) \right]^{1/2}$$

$$= 3.34 \times 10^{-5} \text{ cm} = 0.334 \text{ } \mu\text{m}$$

$t = t_S t_0$

$= t_S = \times 8.85 \times 10^{-14}$

$= (11.8) \times 10^{-2} =$

$\rho(x)$

$10^{16} q$

0.2

$x \, (\mu m)$

$-4 \times 10^{18} q$

$\mathcal{E}(x)$

0.2

$x \, (\mu m)$

-5.1×10^4

From Eq. (5–23),

$$x_{n0} = \frac{3.34 \times 10^{-5}}{1 + 0.0025} \simeq 0.333 \text{ } \mu\text{m}$$

$$x_{p0} = \frac{3.34 \times 10^{-5}}{1 + 400} \simeq 8.3 \times 10^{-8} \text{ cm} = 8.3 \text{ Å}$$

Note that $x_{n0} \simeq W$.

we shall make in analyzing the bipolar junction transistor in Chapter 7.

5.5.4 Capacitance of p-n Junctions

There are basically two types of capacitance associated with a junction: (1) the *junction capacitance* due to the dipole in the transition region and (2) the *charge storage capacitance* arising from the lagging behind of voltage as current changes, due to charge storage effects.[14] Both of these capacitances are important, and they must be considered in designing p-n junction devices for use with time-varying signals. The junction capacitance (1) is dominant under reverse-bias conditions, and the charge storage capacitance (2) is dominant when the junction is forward biased. In many applications of p-n junctions, the capacitance is a limiting factor in the usefulness of the device; on the other hand, there are important applications in which the capacitance discussed here can be useful in circuit applications and in providing important information about the structure of the p-n junction.

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[14]The capacitance (1) above is also referred to as *transition region capacitance* or *depletion layer capacitance*; (2) is often called the *diffusion capacitance*.

The junction capacitance of a diode is easy to visualize from the charge distribution in the transition region (Fig. 5–12). The uncompensated acceptor ions on the p side provide a negative charge, and an equal positive charge results from the ionized donors on the n side of the transition region. The capacitance of the resulting dipole is slightly more difficult to calculate than is the usual parallel plate capacitance, but we can obtain it in a few steps.

Instead of the common expression $C = |Q/V|$, which applies to capacitors in which charge is a linear function of voltage, we must use the more general definition

$$C = \left| \frac{dQ}{dV} \right| \tag{5-55}$$

since the charge Q on each side of the transition region varies nonlinearly with the applied voltage (Fig. 5–30a). We can demonstrate this nonlinear dependence by reviewing the equations for the width of the transition region (W) and the resulting charge. The equilibrium value of W was found in Eq. (5–21) to be

$$W = \left[\frac{2\epsilon V_0}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \quad (equilibrium) \tag{5-56}$$

Since we are dealing with the nonequilibrium case with voltage V applied, we must use the altered value of the electrostatic potential barrier ($V_0 - V$), as discussed in relation to Fig. 5–13. The proper expression for the width of the transition region is then

$$W = \left[\frac{2\epsilon(V_0 - V)}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} \quad (with\ bias) \tag{5-57}$$

In this expression the applied voltage V can be either positive or negative to account for forward or reverse bias. As expected, the width of the transition region is increased for reverse bias and is decreased under forward bias. Since the uncompensated charge Q on each side of the junction varies with the transition region width, variations in the applied voltage result in corresponding variations in the charge, as required for a capacitor. The value of Q can be written in terms of the doping concentration and transition region width on each side of the junction (Fig. 5–12):

$$|Q| = qAx_{n0}N_d = qAx_{p0}N_a \tag{5-58}$$

Relating the total width of the transition region W to the individual widths x_{n0} and x_{p0} from Eqs. (5–23) we have

$$x_{n0} = \frac{N_a}{N_a + N_d} W, \quad x_{p0} = \frac{N_d}{N_a + N_d} W \tag{5-59}$$
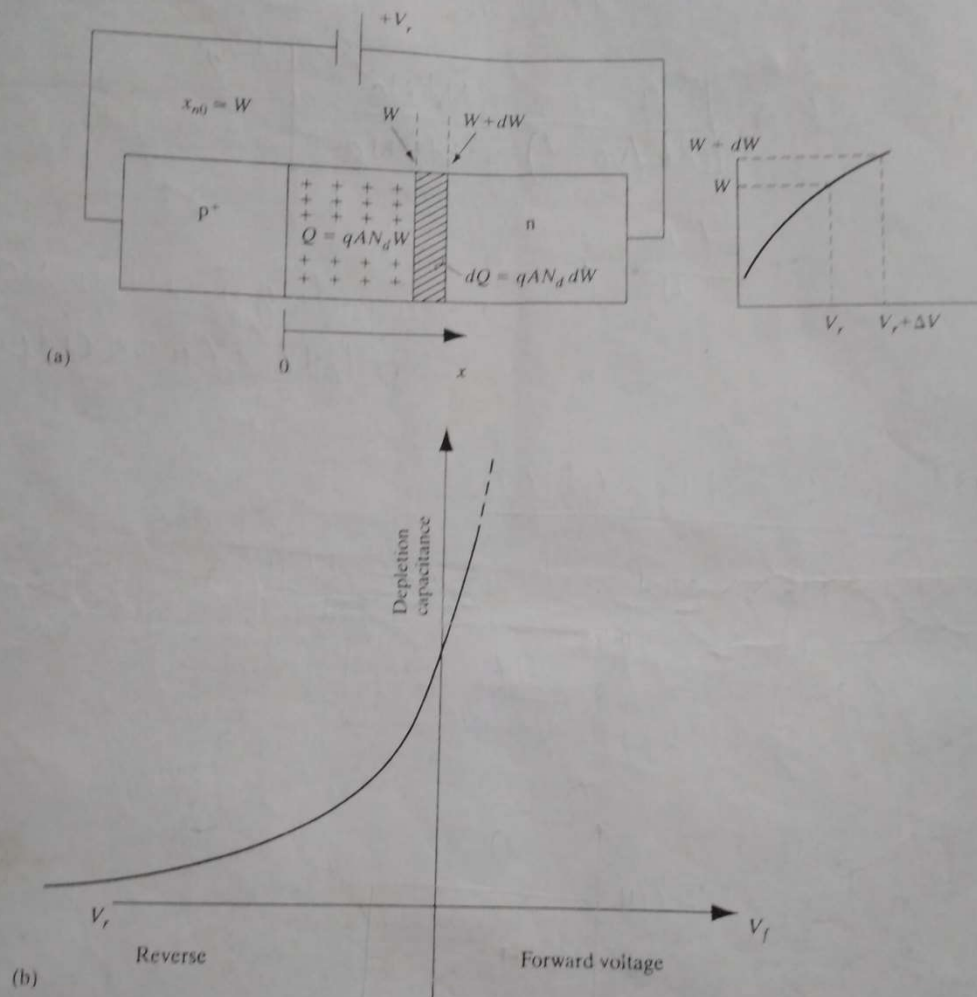
and therefore the charge on each side of the dipole is

(a)

(b)

Figure 5–30
Depletion capacitance of a junction: (a) p^+-n junction showing variation of depletion edge on n side with reverse bias. Electrically, the structure looks like a parallel plate capacitor whose dielectric is the depletion region, and the plates are the space charge neutral regions; (b) variation of depletion capacitance with reverse bias [Eq. (5–63)]. We neglect x_{p0} in the heavily-doped p^+ material.

$$|Q| = qA\frac{N_dN_a}{N_d + N_a}W = A\left[2q\epsilon(V_0 - V)\frac{N_dN_a}{N_d + N_a}\right]^{1/2} \quad (5\text{–}60)$$

Thus the charge is indeed a nonlinear function of applied voltage. From this expression and the definition of capacitance in Eq. (5–55), we can calculate the junction capacitance C_j. Since the voltage that varies the charge in

$$C_j = \left| \frac{dQ}{d(V_0 - V)} \right| = \frac{A}{2} \left[\frac{2q\varepsilon}{V_0 - V} \frac{N_d N_a}{N_d + N_a} \right]^{1/2}$$

$$\boxed{C_j} \propto (V_0 - V)^{-1/2}$$

↳ variation.

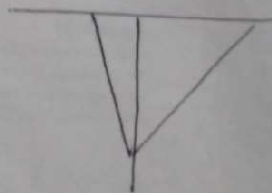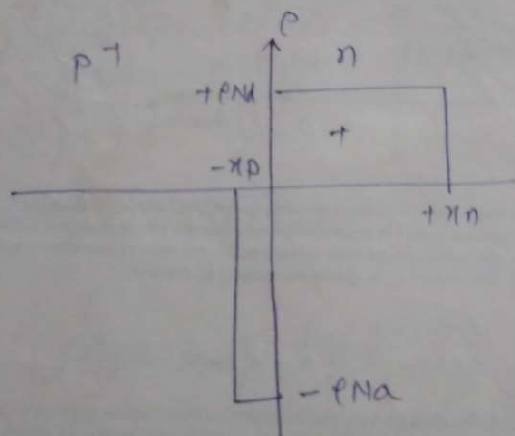$$C_j = \varepsilon A \left[\frac{q}{2\varepsilon(V_0 - V)} \frac{N_d N_a}{N_d + N_a} \right]^{1/2}$$

$$= \frac{\varepsilon A}{W} \qquad \longrightarrow \text{ analogy with parallel}$$
$$\text{plate capacitor.}$$

p^+ - n : Junction:

$$N_a \gg N_d \qquad x_{no} \simeq W.$$

$$C_j = \frac{A}{2} \left[\frac{2q\varepsilon}{V_0 - V} N_d \right]^{1/2}$$

The built in potential of the junction can be determined by extrapolating the curve to the point where

$$\left(\frac{1}{C_j} \right)^2 = 0 .$$

$$\text{slope} = \frac{2}{q\varepsilon_s N_d}$$

$$\left(\frac{y}{n} \right) = \frac{2}{q\varepsilon_s N_a}$$

Diagram labels: P | n | e | +dq'; +eNd; dxp; -xp; +xn; -eNa; e' = $\frac{dq'}{dv_R}$; e/cm²; f/cm²; dq' = eNd·dxn = eNa·dxp; -dq'; ←— With applied V_R —→; ←— With applied $V_A + dV_R$ —→

Problem:

Find out the impurity doping concentration in a p^+n junction, at $T = 300°K$, $n_i = 1.5 \times 10^{10} cm^3$ $V_{bi} = 0.855$ volt and that the slope is 1.32×10^5 $(F/cm²)^{-2} (volt)^{-1}$.

Ans:

$$slope = \frac{2}{N_d e \varepsilon_s} \quad , \quad N_d = \frac{2}{\underline{\hspace{3cm}}}$$

$$N_d = 9.15 \times 10^{15} cm^{-3}.$$

$$\therefore V_{bi} = V_i \ln\left(\frac{N_a N_d}{n_i v}\right) = \frac{KT}{e} \ln\left(\frac{N_a N_d}{n_i v}\right)$$

$$N_a = \frac{n_i v}{N_d} \exp\left(\frac{e V_{bi}}{KT}\right)$$
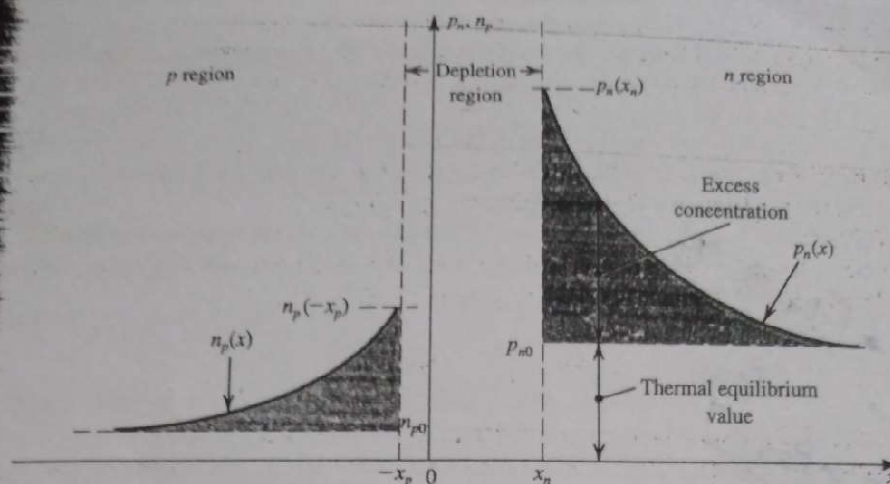
$$\therefore N_a = 5.34 \times 10^{18} cm^{-3}.$$

FIGURE 2.50 Minority-carrier distribution in a forward-biased pn-junction. It is assumed that the p region is more heavily doped than the n region; $N_A \gg N_D$.

the depletion layer and will decrease (exponentially) as one moves away from the junction, eventually reaching zero. Figure 2.50 shows such a minority-carrier distribution.

In the steady state the concentration profile of **excess minority carriers** remains constant, and indeed it is such a distribution that gives rise to the increase of diffusion current I_D above the value I_S. This is because the distribution shown causes injected minority holes to diffuse away from the junction into the n region and disappear by recombination. To maintain equilibrium, an equal number of electrons will have to be supplied by the external circuit, thus replenishing the electron supply in the n material.

Similar statements can be made about the minority electrons in the p material. The diffusion current I_D is, of course, the sum of the electron and hole components.

The Current–Voltage Relationship We now show how the diode i–v relationship of Eq. (2.1) arises. Toward that end, we consider in some detail the current component caused by the holes injected across the junction into the n region. An important result from semiconductor physics relates the concentration of minority carriers at the edge of the depletion region, denoted by $p_n(x_n)$ in Fig. 2.50, to the forward voltage V,

$$p_n(x_n) = p_{n0}e^{V/V_T} \tag{2.58}$$

This is known as the **law of the junction**; its proof is normally found in textbooks dealing with device physics.

The distribution of excess hole concentration in the n region, shown in Fig. 2.50, is an exponentially decaying function of distance and can be expressed as

$$p_n(x) = p_{n0} + [p_n(x_n) - p_{n0}]e^{-(x-x_n)/L_p} \tag{2.59}$$

where L_p is a constant that determines the steepness of the exponential decay. It is called the **diffusion length** of holes in the n-type silicon. The smaller the value of L_p, the faster the injected holes will recombine with majority electrons, resulting in a steeper decay of minority-carrier

*concentration. In fact, L_p is related to another semiconductor parameter known as the **excess-minority-carrier lifetime**, τ_p. It is the average time it takes for a hole injected into the n region to recombine with a majority electron. The relationship is

$$L_p = \sqrt{D_p \tau_p} \tag{2.60}$$

where, as mentioned before, D_p is the diffusion constant for holes in the n-type silicon. Typical values for L_p are 1 μm to 100 μm, and the corresponding values of τ_p are in the range of 1 ns to 10,000 ns.

The holes diffusing in the n region will give rise to a hole current whose density can be evaluated using Eqs. (2.37) and (2.59) with $p_n(x_n)$ obtained from Eq. (2.58),

$$J_p = q\frac{D_p}{L_p}p_{n0}(e^{V/V_T}-1)e^{-(x-x_n)/L_p}$$

Observe that J_p is largest at the edge of the depletion region ($x = x_n$) and decays exponentially with distance. The decay is, of course, due to the recombination with the majority electrons. In the steady state, the majority carriers will have to be replenished, and thus electrons will be supplied from the external circuit to the n region at a rate that will keep the current constant at the value it has at $x = x_n$. Thus the current density due to hole injection is given by

$$J_p = q\frac{D_p}{L_p}p_{n0}(e^{V/V_T}-1) \tag{2.61}$$

A similar analysis can be performed for the electrons injected across the junction into the p region resulting in the electron-current component J_n,

$$J_n = q\frac{D_n}{L_n}n_{p0}(e^{V/V_T}-1) \tag{2.62}$$

where L_n is the diffusion length of electrons in the p region. Since J_p and J_n are in the same direction, they can be added and multiplied by the junction cross-sectional area A to obtain the total current I as

$$I = A\left(\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right)(e^{V/V_T}-1)$$

Substituting for $p_{n0} = n_i^2/N_D$ and for $n_{p0} = n_i^2/N_A$, we can express I in the form

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T}-1) \tag{2.63}$$

We recognize this as the diode equation where the saturation current I_S is given by

$$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right) \tag{2.64}$$

Observe that, as expected, I_S is directly proportional to the junction area A. Furthermore, I_S is proportional to n_i^2, which is a strong function of temperature (Eq. 2.36). Also, note that the exponential in Eq. (2.63) does not include the constant n; n is a "fix-up" parameter that is included to account for nonideal effects.

Diffusion Capacitance From the description of the operation of the pn junction in the forward region, we note that in the steady state a certain amount of excess minority-carrier charge is stored in each of the p and n bulk regions. If the terminal voltage changes, this charge