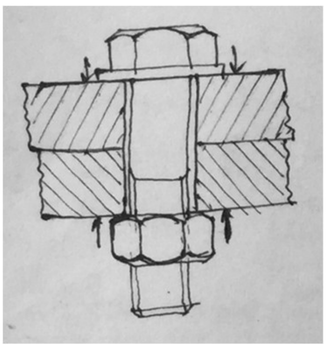
Design Analysis
of
Threaded or Bolted Joints

Purpose

The purpose of the treaded joint or threaded fasteners is to clamp the two or more parts together. Twisting the nut stretches the bolt to produce the clamping force. This clamping force is called the PRE-TENTION or BOLT-PRE-LOAD.

Clamping force which produces tension in the bolt, induces compression in the plate or members.

The bolt has been pre-loaded due to tightening of the nut.



Load on the Bolts

- Static load
- Cyclic load
- Impact load

Design Analysis of Threaded or Bolted Joints under Static Load

Aim: Determine (i) Nos. of Bolts or Screws
(ii) Required size of standard Bolts or Screws

Stresses in threaded fasteners due to Static loading

(I) Stresses due to initial tightening of the nut —
- Tensile Stress
- Torsional Shear Stress
- Shear Stress across the threads
- Crushing Stress on thread

(II) Stresses due to external load/forces. —
- Axial loading [Pure Axial tensile load]
- Eccentric loading [Eccentric load]

(I). Stresses induced due to initial tightening of the nut

(i) Tensile Stress due to Stretching of the bolt : $\sigma_t = \dfrac{P}{\frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2}$

(ii) Torsional Shear Stress caused by the frictional resistance of bet^n the threads in engagement during its tightening :

Torsional Shear stress $\tau = \dfrac{T}{J} \cdot r = \dfrac{16T}{\pi (d_c)^3}$

(iii) Shear Stress across the threads : $\tau_s = \dfrac{P}{\pi d_c \cdot b \cdot n}$ for bolt thread

(iv) Compressive Stress (Crushing stress) on threads $\tau_c = \dfrac{P}{\pi d\, b \cdot n}$ for nut thread

(v) Bending Stress if the surfaces under the bolt head & nut are not perfectly normal to the bolt axis $\sigma_c = \dfrac{P}{\pi [d^2 - d_c^2] n}$

b = width of thread
n = no. of threads in engagement

None of the stresses mentioned earlier can be determined accurately. Hence, in practice, the bolts are designed on the basis of tensile stress induced due to stretching of the bolt but with a high factor of safety.

Axial Loading

Stresses due to external forces (Axial tensile force) (Pure tensile loading)

Approximate Analysis	Exact Analysis
Weakest area = circular area corresponding to core dia. Or minor dia	Tensile stress area

The cross-section at core/minor diameter (d_c) is the weakest section.

Weakest Area $A_c = \frac{\pi}{4} d_c^2$

Tensile Stress induced

$\sigma_t = \frac{P}{A_c} \leq [\sigma_t]$ where $[\sigma_t] = \frac{S_{yt}}{f.s.}$

$\frac{P}{\frac{\pi}{4} d_c^2} \leq [\sigma_t]$

Tensile stress area $(A_t) = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$

$\sigma_t = \frac{P}{A_t} \leq [\sigma_t]$

$\frac{4P}{\pi\left(\frac{d_p+d_c}{2}\right)^2} \leq [\sigma_t]$

→ two unknown ?

[From Table, the ... to the just higher value of calculated d_c']

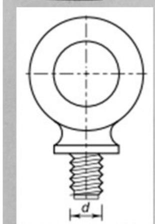
Otherwise, following approximate relationship can be used for finding 'd'.

$d_c = 0.8 d \quad \therefore d = \frac{d_c}{0.8}$ | $d = \frac{d_c}{0.84}$ for Coarse, $= d_c/0.855$ for fine

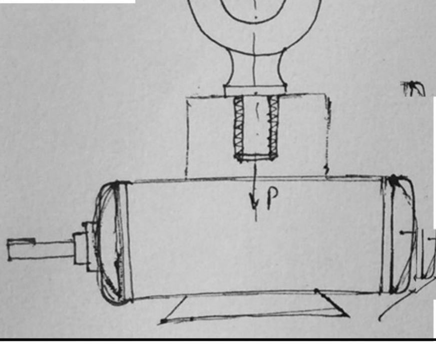
Axial Load

Stresses due to external forces (Axial tensile force) (Pure tensile loading)

Prob. 1



An electric motor weighing 10 kN is lifted by means of an eye bolt. The eye bolt is screwed into the frame of the motor. Eye bolt is made up of plain C-steel 40C8 & factor of safety '5'. Assume coarse thread.



Eye bolt or lifting eyebolt — is used for lifting & transporting heavy m/c or (called eye) consists of a ring of circular c/s at the top end & threaded portion at the lower end.

— Threaded portion is screwed inside a threaded hole on the top surface of the m/c to be lifted.

The threaded portion of eye bolt is subjected to tensile stress due to weight being lifted.

Axial Load

Prob. 1

An electric motor weighing 10 kN is lifted by means of an eye bolt. The eye bolt is screwed into the frame of the motor. Eye bolt is made up of plain C-steel 40C8 & factor of safety '5'. Assume Coarse thread

Given data

$P = 10\,kN = 10 \times 10^3\,N$

For Plain C-Steel of 40C8 : Min. Tensile yield strength $S_{yt} = 380\,N/mm^2$

Factor of Safety $= 5$ [IS : 1570]

Permissible/Allowable tensile stress $[\sigma_t] = \frac{380}{5} = 76\,N/mm^2$

Axial Load

Approximate Analysis	Exact Analysis
Weakest area = circular area corresponding to core dia. Or minor dia	Tensile stress area

$$\sigma_t = \frac{P}{\frac{\pi}{4} d_c^2} \le [\sigma_t]$$

$$\therefore d_c \ge \sqrt{\frac{4P}{\pi [\sigma_t]}}$$

where Weakest area $= A_c = \frac{\pi}{4} d_c^2$

d_c = root dia / Core dia

$$\ge \sqrt{\frac{4 \times 10^4}{\pi \times 76}}$$

$$\ge 13\,mm$$

$d = 16\,mm$ Corresponding to $d_c = 13.54$

$$\sigma_t = \frac{P}{A_t} \le [\sigma_t]$$

$$A_t \ge \frac{P}{[\sigma_t]}$$

$$\ge \frac{10^4}{76}$$

$$\ge 131.57\,mm^2$$

Otherwise, following approximate relationship can be used for finding 'd':

$d_c = 0.8\,d \quad \therefore d = \frac{d_c}{0.8}$ $\left[d = \frac{d_c}{0.84} \text{ for Coarse} \right.$

$= d_c/0.88$ for fine

Recommendation: M16

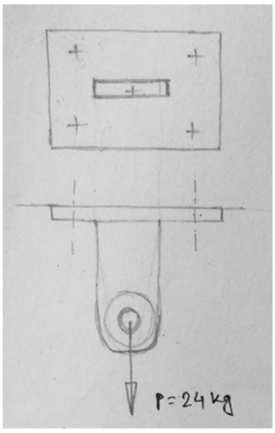
Basic Dimensions for Metric threads (Coarse series) [*IS : 4218 (Part-III)- 1996*]

Designation	*Pitch (mm)*	*Major or Nominal dia. (mm)*	*Pitch dia. (mm)*	*Minor or Core dia. (mm)*		*Tensile stress area (sq. mm)*
				Bolt	Nut	
M4	0.7	4	3.545	3.141	3.242	8.78
M6	1	6	5.35	4.773	4.917	20.1
M8	1.25	8	7.188	6.488	6.647	36.6
M10	1.5	10	9.026	8.16	8.376	58
M12	1.75	12	10.86	9.85	10.106	84.3
M16	2	16	14.7	13.54	13.83	157
M20	2.5	20	18.376	16.933	17.294	245

Basic Dimensions for Metric threads (Fine series) [*IS : 4218 (Part-III)- 1996*]

Designation	*Pitch (mm)*	*Major or Nominal dia. (mm)*	*Pitch dia. (mm)*	*Minor or Core dia. (mm)*		*Tensile stress area (sq. mm)*
				Bolt	Nut	
M8×1	1.0	8	7.35	6.773	6.917	39.2
M10 ×1.25	1.25	10	9.188	8.466	8.647	61.2
M20 ×1.5	1.5	20	19.026	18.16	18.376	272

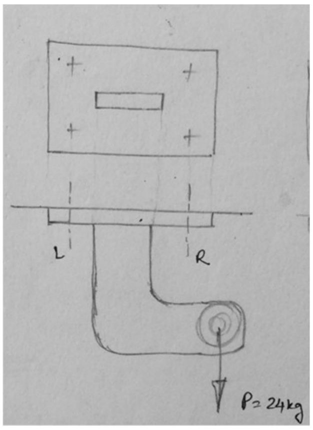
Axial Load



Bolts are equally loaded

$P_L = P_R = P_b = \frac{24}{4} kg = 6 kg$

Eccentric Load



Bolts are not equally loaded

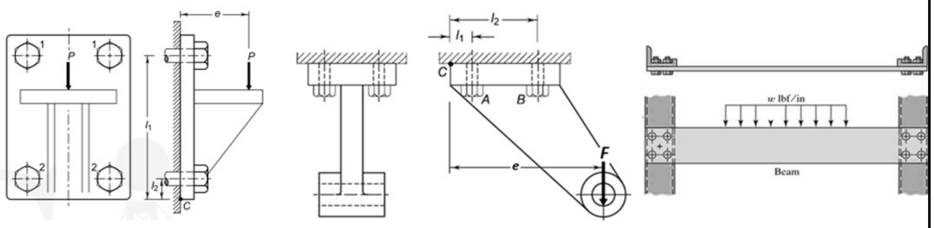
Eccentric Load

Three cases of eccentrically loaded bolted joints will be considered

Case (i) Eccentric load perpendicular to the axis of the bolt

Case (ii) Eccentric load parallel to the axis of the bolt

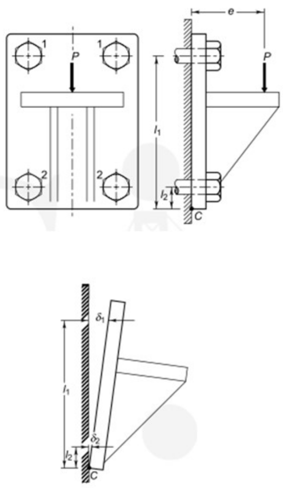
Case (iii) Eccentrically loaded bolted joints in Shear



Bolted structure subject to Eccentric Load

Case (i) Eccentric load perpendicular to the axis of the bolt

Line of action of the load is perpendicular to the bolt axis



A bracket, fixed to the steel structure by means of four bolts, is shown in Fig. 7.20(a). It is subjected to eccentric force P at a distance e from the structure. The force P is perpendicular to the axis of each bolt. The lower two bolts are denoted by 2, while the upper two bolts by 1. In this analysis, the following assumptions are made:

(i) The bracket and the steel structure are rigid.

(ii) The bolts are fitted in reamed and ground holes.

(iii) The bolts are not preloaded and there are no tensile stresses due to initial tightening.

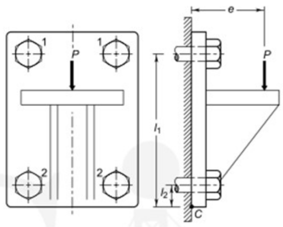
(iv) The stress concentration in threads is neglected.

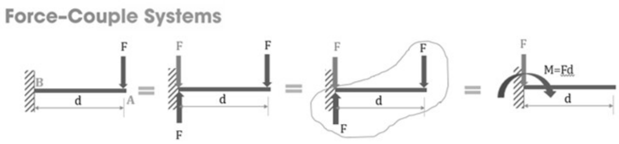
(v) All bolts are identical.

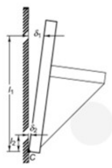
Bolted structure subject to Eccentric Load

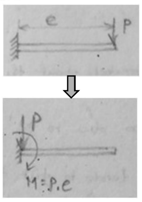
Case (i) Eccentric load perpendicular to the axis of the bolt

Line of action of the load is perpendicular to the bolt axis









Force induced Stresses

The bolts are subjected to following stresses:

(i) Direct Shear Stress due to 'P'

(ii) Tensile Stress due to tendency of the bracket to tilt in CW dirⁿ about the edge 'AA'

i.e Tensile Stress due to 'M'.

The moment ($M = P.e$), due to eccentric force 'P', tends to tilt the bracket about the edge C-C

Force induced Stresses

(i) Direct shear stress due to 'P'.

Since the bolts are identical, the shear force on each bolt is same.

$$P_{d1} = P_{d2} = \frac{P}{\text{No. of bolts}} = \frac{P}{4} \quad \left[\text{As } P_{d1} + P_{d2} + P_{d3} + P_{d4} = P \quad 4 \cdot P_d = P \Rightarrow P_d = P/4\right]$$

Direct Shear Stress in each bolt / Screw

$$\tau = \frac{P_{d1}}{A_c} = \frac{P_{d2}}{A_c}$$

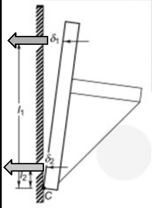
Where A_c = C/s area of the Screw at Core dia.

(ii) Tensile Stress due to 'M'

When the load tends to tilt the bracket about the edge 'C-C', each bolt is stretched (or elongated) by an amount that depends upon its distance from the tilting edge.

(ii) Tensile Stress due to 'M'

When the load tends to tilt the bracket about the edge 'C-C', each bolt is stretched (or elongated) by an amount that depends upon its distance from the tilting edge.



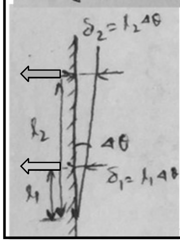
$\delta_1 = l_1 \Delta\theta$
$\delta_2 = l_2 \Delta\theta$

$\therefore$ Elongation or Stretch $\propto$ distance from tilting edge.

Strain $\propto$ elongation, because $\epsilon = \delta / l$

Stress $\propto$ strain, because $\sigma = E \cdot \epsilon$

Resisting force $\propto$ Stress, because $P = \sigma \cdot A$



$\therefore$ Elongation or Stretch $\propto$ distance from tilting edge.

$\delta_i \propto l_i$

Within proportional limit,

$\underbrace{\text{Resisting force} \propto \text{Stress}} \propto \underbrace{\text{Strain} \propto \text{Elongation} \propto \text{dist}^n}$

Resisting force $\propto$ Stress, because $P = \sigma \cdot A$

Stress $\propto$ strain, because $\sigma = E \cdot \epsilon$

Strain $\propto$ elongation, because $\epsilon = \delta / l$

Elongation $\propto$ distance from edge

$\therefore$ Resisting force $\propto$ distance from tilting edge

$\therefore$ Elongation or Stretch $\propto$ distance from tilting edge.

$\delta_i \propto l_i$

Within proportional limit,

$\underbrace{\text{Resisting force} \propto \text{Stress}} \propto \underbrace{\text{Strain} \propto \text{Elongation} \propto \text{dist}^n}$

Resisting force $\propto$ Stress, because $P = \sigma \cdot A$

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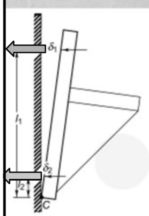
Strain $\propto$ elongation, because $\epsilon = \delta / l$

Elongation $\propto$ distance from edge

$\therefore$ Resisting force $\propto$ distance from tilting edge



$P_i \propto l_i$

$\therefore P_i = C l_i$

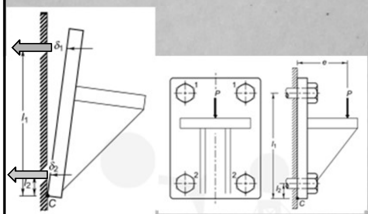
Resisting force on each bolt at a distance 'l_1' = $P_1 = C l_1$

" " " " " " " 'l_2' = $P_2 = C l_2$

Force induced Stresses

Resisting force on each bolt at a distance 'l_1' $= P_1 = c\,l_1$

" " " " " " " 'l_2' $= P_2 = c\,l_2$



where l_1 = dist^n of 1st two bolts from tilting edge

l_2 = dist^n of 2nd " " " " "

$l_2 > l_1$

Total moments of resisting forces about tilting edge

$$M' = 2(P_1 l_1 + P_2 l_2) = 2(c.l_1^2 + c.l_2^2)$$

[2 bolts at each row]

Moment due to the external force (P) about the tilting edge

$M = P.e$ where e = dist^n of force 'P' from the tilting edge

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Force induced Stresses

$$M' = 2(P_1 l_1 + P_2 l_2) = 2(c.l_1^2 + c.l_2^2)$$

[2 bolts at each row]

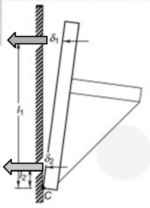
$M = P.e$

Equating moment due to external force to moment of resisting forces about tilting edge.

$$M = M'$$

$$P.e = 2.(c l_1^2 + c.l_2^2)$$

$$\therefore \; c = \frac{P.e}{2(l_1^2 + l_2^2)}$$



$$P_1 = c.l_1 = \frac{P.e.l_1}{2(l_1^2+l_2^2)}$$

$$P_2 = c.l_2 = \frac{P.e.l_2}{2(l_1^2+l_2^2)}$$

Resisting force on each bolt at a distance 'l_1' $= P_1 = c\,l_1$

" " " " " " " 'l_2' $= P_2 = c\,l_2$

Force induced Stresses

$P_1 = C \cdot l_1 = \dfrac{P \cdot e \cdot l_1}{2(l_1^2+l_2^2)}$

$P_2 = C \cdot l_2 = \dfrac{P \cdot e \cdot l_2}{2(l_1^2+l_2^2)}$

Critical bolt

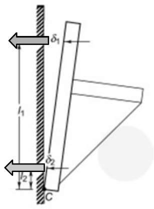
A bolt, which is located at the farthest distⁿ (or greatest distⁿ) from the tilting edge 'A-A', is subjected to maximum force.

As $l_2 > l_1$

$\therefore P_2 > P_1$

Tensile force on heavily loaded bolt (ie bolt at distⁿ l_2)

$$P_2 = \frac{P \cdot e \cdot l_2}{2(l_1^2+l_2^2)}.$$



Tensile stress in heavily loaded bolt

$$\sigma_t = \frac{P_2}{A_c}$$

Force induced Stresses

The bolts are subjected to following stresses:

(i) Direct shear stress due to 'P'

(ii) Tensile stress due to tendency of the bracket to tilt in CW dirⁿ about the edge 'AA'.

i.e Tensile stress due to 'M'.

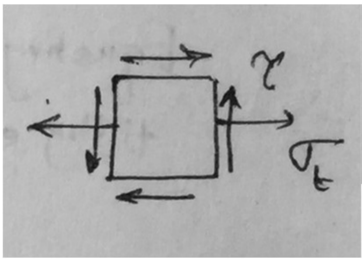
Direct shear stress in each bolt/screw

$\tau = \dfrac{P_{d1}}{A_c} = \dfrac{P_{d2}}{A_c}$ when A_c = c/s area of the screw at core dia.

Tensile stress in heavily loaded bolt

$\sigma_t = \dfrac{P_2}{A_c}$

Bolts are subjected to bi-axial stress i.e. shear stress & tensile stress.

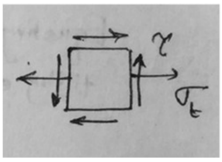


The bolts can be designed on the basis of:

- Maxᵐ principal stress theory (Rankine's theory) [1850]
- Maxᵐ shear stress theory (Coulomb, Tresca & Guest's theory)
- Distortion energy theory (Von Mises & Hencky's theory)

Draw Mohr circle

Force induced Stresses



<u>Maxm Shear Stress theory</u> (Coulomb, Tresca & Guest's theory)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2} \leq [\tau]$$

$$\sqrt{\left(\frac{P_2}{2A_c}\right)^2 + \left(\frac{P_{d2}}{A_c}\right)^2} \leq [\tau]$$

$\therefore A_c \geq$ ——— (B)

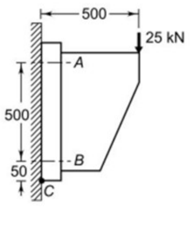
where $[\sigma_t] = \frac{S_{yt}}{F.S}$

$[\tau] = 0.5\,[\sigma_t]$

Otherwise, following approximate relationship can be used for finding 'd'.

$d_c = 0.8\,d \quad \therefore d = \frac{d_c}{0.8}$ $\left[d = \frac{d_c}{0.84} \text{ for Coarse}, = d_c/0.85 \text{ for fine} \right]$

Example 7.6 *A wall bracket is attached to the wall by means of four identical bolts, two at A and two at B, as shown in Fig. 7.21. Assuming that the bracket is held against the wall and prevented from tipping about the point C by all four bolts and using an allowable tensile stress in the bolts as 35 N/mm², determine the size of the bolts on*



Given $P = 25$ kN $\quad e = 500$ mm

$(\sigma_t)_{max.} = 35$ N/mm^2

Step I *Direct shear stress in bolt*

Two bolts at *A* are denoted by 1 and two bolts at *B* by 2. The direct shear force on each bolt is given by,

$$P_1' = P_2' = \frac{P}{\text{(No. of bolts)}}$$

$$\therefore \quad P_1' = P_2' = \frac{25 \times 10^3}{4} = 6250 \text{ N}$$

The direct shear stress in each bolt is given by,

$$\tau = \frac{6250}{A} \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Tensile stress in bolt*

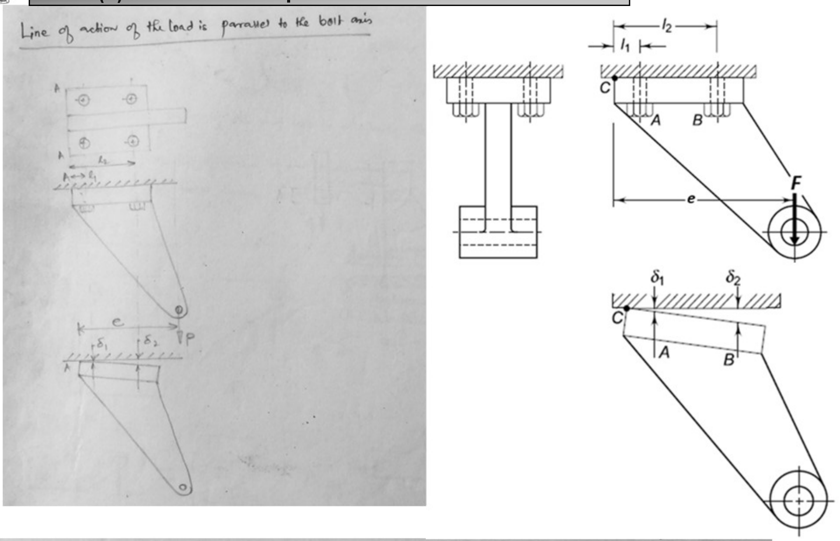
Since the tendency of the bracket is to tilt about the edge *C*, the bolts at *A* denoted by 1, are at the farthest distance from *C*. Therefore, bolts at *A* are subjected to maximum tensile force. From Eq. (7.10),

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)} = \frac{(25 \times 10^3)(500)(550)}{2(550^2 + 50^2)}$$
$$= 11\,270.49 \text{ N}$$

The tensile stress in bolts at *A* is given by,

$$\sigma_t = \frac{11270.49}{A} \qquad \text{(ii)}$$

Case (ii) Eccentric load parallel to the axis of the bolt



The bracket is subjected to an eccentric force 'P' at a distance 'e' from tilting edge 'A-A'. The force 'P' is parallel to the axis of each bolt. Load 'P' tends to tilt the bracket about the edge 'A-A'.

Force induced Stresses

The bolts are subjected to following stresses:

(i) Direct tensile stress due to load 'P' $\left[P_{d1} = P_{d2} = \frac{P}{4}\right]$; $\left[\sigma_{td} = \frac{P_{d1}}{A_c} = \frac{P_{d2}}{A_c}\right]$

(ii) Tensile stress due to tendency of the bracket to tilt in CW direction about the edge 'A-A'. $\left[P_2 = \frac{P.e.l_2}{2(l_1^2+l_2^2)}\right]$ $\left[\sigma_{tt} = \frac{P_2}{A_c}\right]$

Using Principle of Superposition:

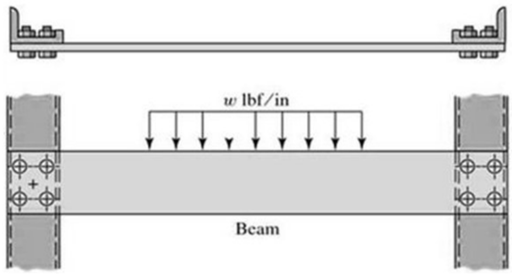
Total tensile force on most heavily loaded bolt

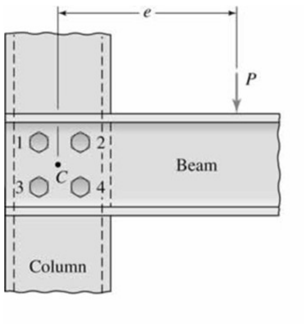
$$P_t = P_{d2} + P_2$$

$\therefore$ Tensile stress on heavily loaded bolt $\sigma_t = \frac{P_t}{A_c} = \frac{P_{d2}+P_2}{\frac{\pi}{4}d_c^2} \leq [\sigma_t]$

$$\therefore d_c \geq \sqrt{\frac{P_{d2}+P_2}{\frac{\pi}{4}[\sigma_t]}}$$

Case (iii) Eccentrically loaded bolted joints in Shear

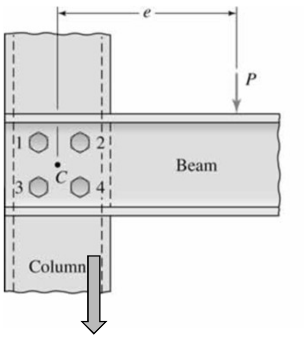




Assumption: *Same as previous*

Case (iii) Eccentrically loaded bolted joints in Shear

Calculation of Centroid of the Bolt System



In structural connections, a group of bolts is frequently employed, as shown in Fig. 7.15. Let $A_1, A_2, ..., A_5$ be the cross-sectional areas of the bolts. The co-ordinates $(x_1, y_1), (x_2, y_2), ..., (x_5, y_5)$ indicate the position of bolt-centres with respect to

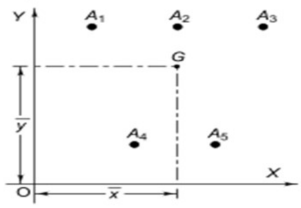


Fig. 7.15

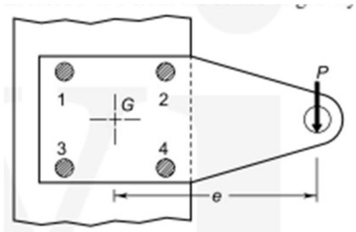
the origin. G is the centre of gravity of the group of bolts. The co-ordinates $(\bar{x}, \bar{y})$ indicate the location of the centre of gravity.

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + ... + A_5x_5}{A_1 + A_2 + ... + A_5}$$

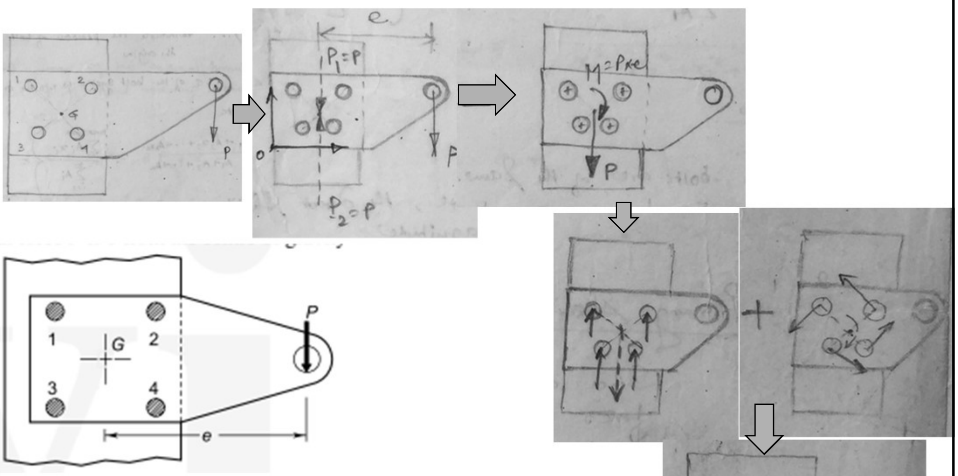
$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i}$$

Similarly,

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$



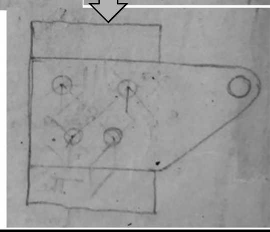
Case (iii) Eccentrically loaded bolted joints in Shear



When bolted joint subjected to an eccentric load in the plane of bolt, the following two types of forces are induced:

(a) **Primary** (or Direct) **Shear force** on each bolt acting parallel to load P

(b) **Secondary Shear force** due to turning moment (which tends to rotate the joint about the CG of the bolted system)



Primary Shear Forces

Primary Shear forces

Since all the bolts are of the same size, the direct shear forces on each bolt of equal magnitude

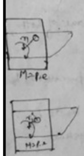
∴

$$P_{d1} = P_{d2} = P_{d3} = P_{d4} = \frac{P}{(\text{no. of bolts})}$$

Secondary Shear Forces

3. Secondary Shear forces

The moment ($P \times e$) about the C.G results in Secondary Shear forces (P_{s1}, P_{s2}, P_{s3}, P_{s4}, etc.)

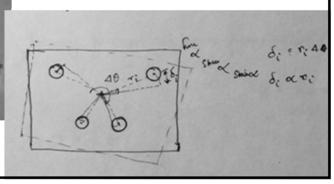


** It is assumed that (i) the magnitude Secondary Shear forces at any bolt is proportional to the radial distn from the CG.

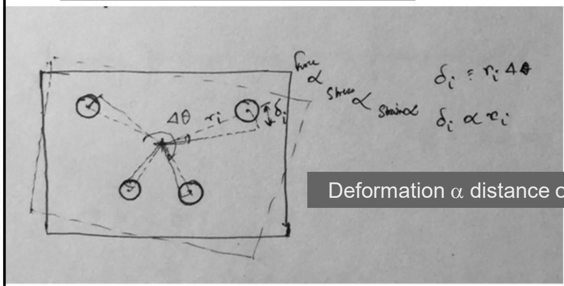
(ii) the direction of Secondary Shear forces is perpn to the line joining the centre of the bolt to CG of the bolt system.

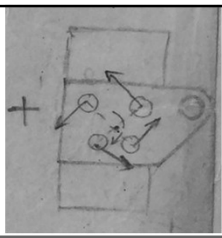
$P_{s1}, P_{s2}, P_{s3}, P_{s4}$ = Secondary Shear forces on the bolts 1, 2, 3, 4 ...

r_1, r_2, r_3, r_4 = radial distn of the bolts 1, 2, 3, 4 etc. fr C.G. of the bolt system.



Secondary Shear Forces





Deformation α distance of bolt centre from CG of bolt system

Resisting force α Stress α Strain α Deformation α distance of bolt centre from CG of bolt system

Secondary Shear force (Resisting force) α distance of bolt centre from CG of bolt system

$$P_{s1} \propto r_1 \quad \therefore P_{s1} = C r_1$$

$$P_{s2} \propto r_2 \quad \therefore P_{s2} = C r_2$$

$$P_{s3} \propto r_3 \quad \therefore P_{s3} = C r_3$$

$$P_{s4} \propto r_4 \quad \therefore P_{s4} = C r_4$$

Secondary Shear Forces

The Sum of the external turning moment due to the eccentric load & of internal resisting moment of the bolts must be equal to zero.

$$P.e = P_{s1}.r_1 + P_{s2}.r_2 + P_{s3}.r_3 + P_{s4}.r_4$$

$$P.e = C.r_1.r_1 + C.r_2.r_2 + C r_3.r_3 + C r_4.r_4$$

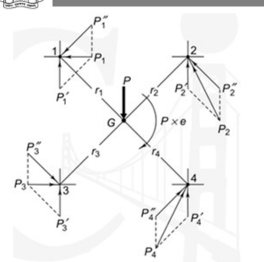
$$\therefore C = \frac{P.e}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

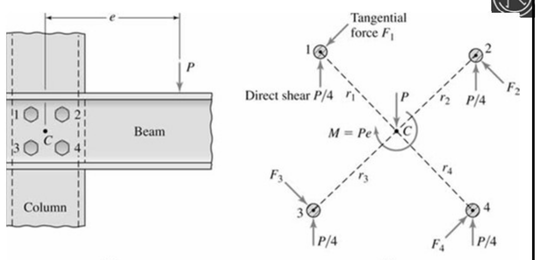
Secondary Shear Forces

$$P_{s_1} = C \cdot r_1 = \frac{P \cdot e \cdot r_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

$$P_{s_2} = \frac{P \cdot e \cdot r_2}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$ & so on.

Resultant Shear Forces





(a) (b)

The direct (Primary + Secondary) Shear forces may be added vectorially to determine the resultant Shear load (P_R) on each bolt.

Resultant Shear force of i^{th} bolt $\cancel{R}$ $P_{Ri} = \sqrt{(P_{di})^2 + (P_{si})^2 + 2 P_{di} P_{si} \cos\theta_i}$

θ_i = Angle bet^n the primary & Secondary Shear force on i^{th} bolt.

Critical bolt

- The max^m loaded bolt i.e $(P_R)_{max}$ becomes the Critical one for determining the Strength of the bolt jt.

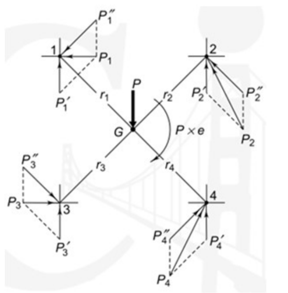
The direct /Primary & Secondary Shear forces may be added vectorially to determine the resultant Shear load (P_R) on each bolt.

Resultant Shear force of ith bolt $P_{Ri} = \sqrt{(P_{di})^2 + (P_{si})^2 + 2P_{di}P_{si}\cos\theta_i}$

θ_i = Angle betn the primary & Secondary Shear force on ith bolt.

Critical bolt

– The maxm Loaded bolt i.e $(P_R)_{max}$ becomes the Critical one for determining the Strength of the bolt jt.

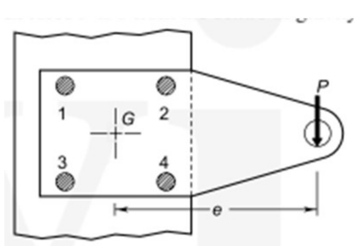


Weakest area = Core area of the bolt (A_c) = $\frac{\pi}{4}d_c^2$

$$\tau_{max} = \frac{(P_R)_{max}}{A_c} \le [\tau]$$

$$\frac{4(P_R)_{max}}{\pi d_c^2} \le [\tau]$$

The structural connection shown in Fig. is subjected to an eccentric force P of 10 kN with an eccentricity of 500 mm from the CG of the bolts. The centre distance between bolts 1 and 2 is 200 mm, and the centre distance between bolts 1 and 3 is 150 mm. All the bolts are identical. The bolts are made from plain carbon steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 2.5. Determine the size of the bolts.



Given $P = 10$ kN $\quad S_{yt} = 400$ N/mm² $\quad (fs) = 2.5$
$e = 500$ mm

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{2.5} = 80 \text{ N/mm}^2$$

Step II: Primary Shear Forces

Primary Shear forces

Since all the bolts are of the same size, the direct shear forces on each bolt of equal magnitude

$\therefore$

$$P_{d1} = P_{d2} = P_{d3} = P_{d4} = \frac{P}{(\text{no. of bolts})} = \frac{10\,000}{4} = 2500\text{ N}$$

Step III: Secondary Shear Forces

Secondary Shear force (Resisting force) α distance of bolt centre from CG of bolt system

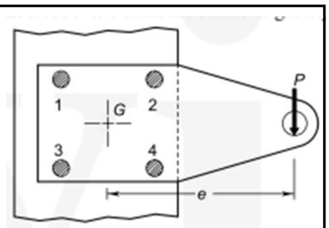
$P_{s1} \propto r_1 \quad \therefore P_{s1} = Cr_1$

$P_{s2} \propto r_2 \quad \therefore P_{s2} = Cr_2$

$P_{s3} \propto r_3 \quad \therefore P_{s3} = Cr_3$

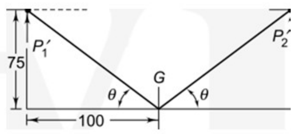
$P_{s4} \propto r_4 \quad \therefore P_{s4} = Cr_4$

The centre distance between bolts 1 and 2 is 200 mm, and the centre distance between bolts 1 and 3 is 150 mm. All the bolts are identical.



By symmetry, the centre of gravity G is located at a distance of 100 mm to the right of bolts 1 and 3 and 75 mm below bolts 1 and 2. Thus,

$$r_1 = r_2 = r_3 = r_4 = r$$



$$r = \sqrt{(100)^2 + (75)^2} = 125\text{ mm}$$

Step III: Secondary Shear Forces

The Sum of the external turning moment due to the eccentric load & of internal resisting moment of the bolts must be equal to zero.

$$P \cdot e = P_{s1} \cdot r_1 + P_{s2} \cdot r_2 + P_{s3} \cdot r_3 + P_{s4} \cdot r_4$$

$$P \cdot e = C \cdot r_1 \cdot r_1 + C \cdot r_2 \cdot r_2 + C\, r_3 \cdot r_3 + C\, r_4 \cdot r_4$$

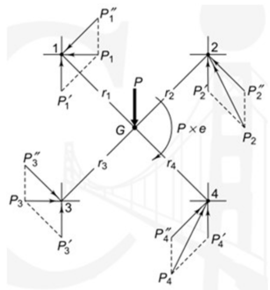
$$\therefore\ C = \frac{P \cdot e}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

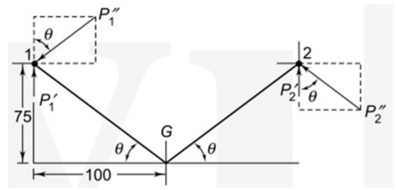
$$P_{s1} = C r_1$$
$$P_{s2} = C r_2$$
$$P_{s3} = C r_3$$
$$P_{s4} = C r_4$$

$$P_{s1} = C \cdot r_1 = \frac{P \cdot e \cdot r_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2} = 10\,000\ \text{N}$$

$$P_{s2} = \frac{P \cdot e \cdot r_2}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$ & so on.

Step IV: Resultant Shear Forces





$$\tan\theta = \frac{75}{100} = 0.75 \quad \text{or} \quad \theta = 36.87°$$

$$P_{Ri} = \sqrt{(P_{di})^2 + (P_{si})^2 + 2 P_{di} P_{si} \cos\theta_i}$$

θ_i = Angle betn the primary & Secondary Shear force on i^{th} bolt.

$P_1 = 8139.41$ N $\quad P_2 = 12093.38$ N

Critical bolt : heavily loaded bolt

Step V: Size of the bolt

$$\frac{P_2}{A} \leq \tau$$

$$\frac{12093.38}{\frac{\pi}{4} d_c^2} \leq 80$$

$d_c \geq 13.87$ mm

$$d > \frac{d_c}{0.8} = \frac{13.87}{0.8}$$

> 17.34 mm

Recommendation: M20