01/08/2022

Material

Neutral axis ??

$\tau_{yx} = \tau_{xy}$

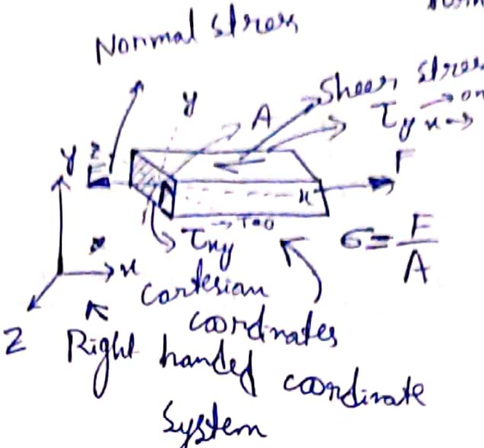
$\tau_{xy} \rightarrow$ complementary of τ_{yx}

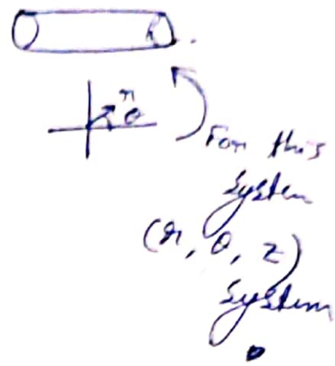
τ_{xy} stops the (turning) turning action of τ_{yx}

- Sheet/plate and Section

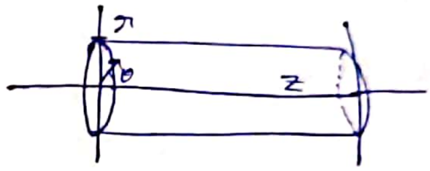
Foldable — Not foldable

Thicker than plate $\rightarrow$ slab





- Deformable body $\rightarrow$ Takes deformation till the external force is present (Will show retention of shape once the applied force is removed)



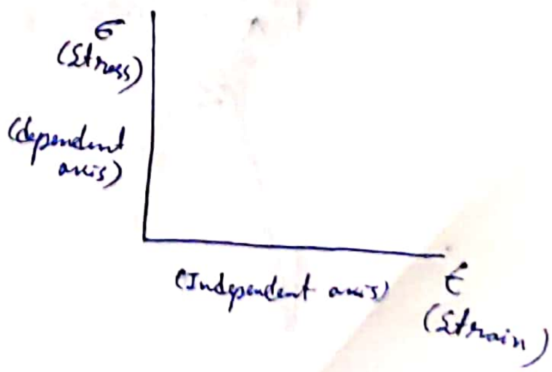
- Deformed body

Mechanics of deformable body

- The strain causes the stress

The strain is the external mechanism of deformation (Chilling, magnetic field, stress, etc)

The stresses are the internal reactive forces to counter the deformation. For elastic ~~bodies~~ materials they are related by to Hooke's law and represented by stress strain diagram



Hooke's law $\rightarrow Y = \frac{\sigma}{\epsilon}$ (Young's modulus)

- Shear stress $\rightarrow$ Force $\rightarrow \sigma_{xx} \rightarrow$ Force

x surface $\rightarrow$ ⊥ to x direction

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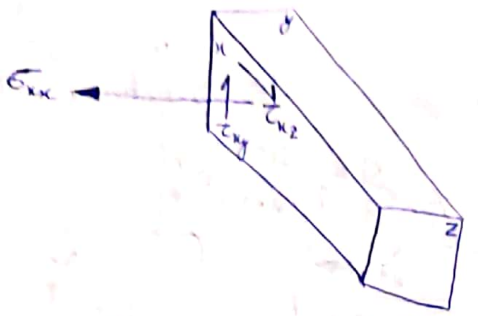


Element is a small piece that I have taken from a member (from truss)

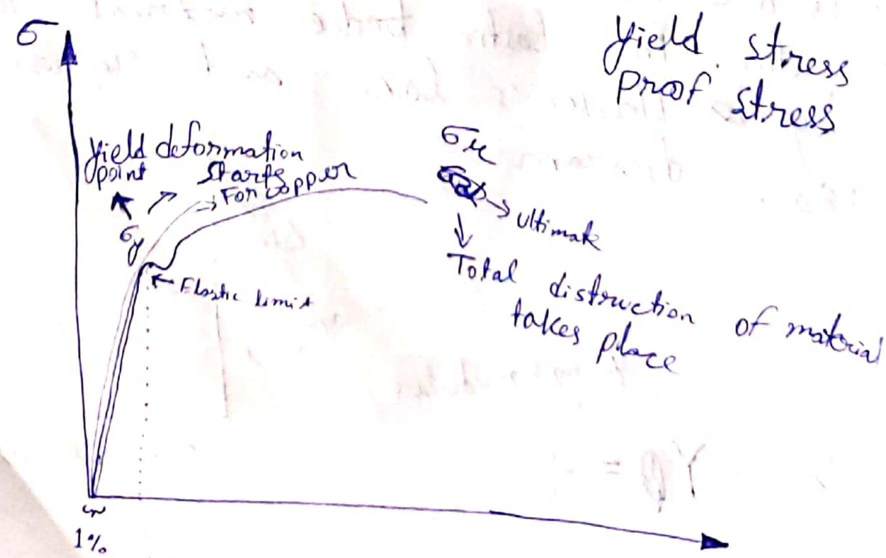
$\tau_{ab} \rightarrow$ Towards
$\downarrow$
on the plane

3 stress components in each plane $\rightarrow$ Any element is subjected to max 9 stress components in any direction
$\rightarrow$ 3 Normal stress, 6 sheer stress } Stress tensor

Simplification and Assumptions

I) Elastic deformable elements

II) Loaded upto elastic limit



Yield stress
Prof stress

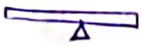
III) The material is homogeneous and isotropis
Exibit same mechanical pro...

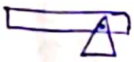
• Bamboo → Example of anisotropic material
Steel also ↗

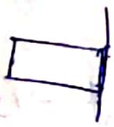
For anisotropic materials the stress-strain graph will change for x, y and z direction

• ~~For copper →~~

Support

Pin support → 

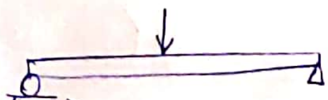
Hinge support → 

Grouting → ~~F~~ Fixed → 

• Beam →  → Subjected to Bending load

Column →  → subjected to compressive load

Difference in loading conditions

• Beam 

*** → To neutralise bending

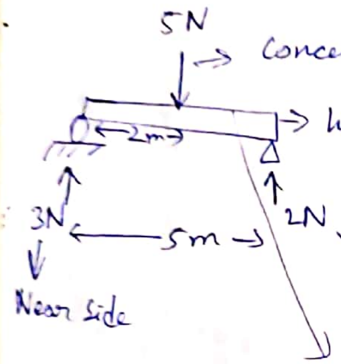
Min. no of supports → Stable | less than min → ~~Sta~~ unstable
More th...

Bridge → min 6 Inch diameter ~~do~~ roller of hardened steel.

- The beams are the horizontal members to withstand the transverse load

  transverse load: Load acting across the cross section.

- Axial load → another type of load



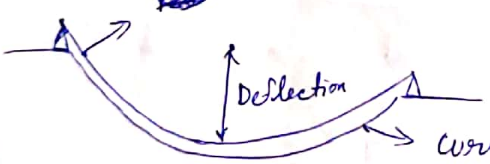
→ concentrated load / point load (unit → N)

→ weight of the beam is uniformly distributed load (UDL)
(Unit → N/m)

IV) The self weight of the member is ~~no~~ neglected

curvature

curvature is uniform throughout



→ curvature is same with upper one (but radius is different)

- Relative displacement → shearing stress

Thus bending load also introduces ~~some shear~~ transverse shear force

- UDL → full UDL → Shared by the support
- / point load → Shared by the support

} Write ~~the~~ like this in exams

02/08/2022

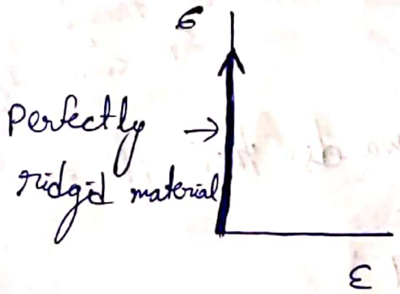
Stress and Strain
(Teacher → NBH Sir)

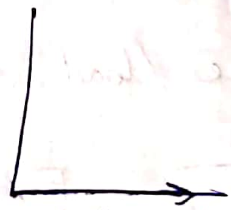
- GATE → Go through topics
- Non-deformable bodies are ~~ridg~~ rigid bodies (Ex→ cast iron)
- Mechanics of materials / Strength of materials
- External Force → LOAD
- Load can never be internal, Internal force is a resistive thing

- $\sigma \propto \varepsilon$ (σ → Stress, Dependent variable; ε → Strain, Independent variable) } i.e. Stress does not cause strain, Strain causes stress

Stress is non-measurable
Strain is measurable
Stress is calculated from measured Strain

$\therefore$ $\sigma = E\varepsilon$ (E → Proportionality constant (Young's modulus))

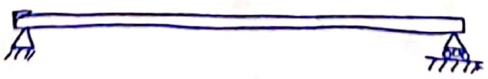




$\theta = \tan^{-1} E$

03/08/2022

Transverse loading of beam



} Simple supported beam

} overhang beam (beam extends beyond support)
(lower deflection characteristic)

Simple ...

Contilever beam (Very high deflection characteristic)

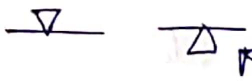
↳ Indetermunant beam

} Propped Contilever beam

↑ Redundant support

- Span → distance between two supports of beam

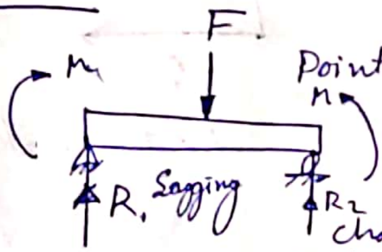
→ positive support



↖ negative support

- Distributed load → Uniform
  → Non uniform (But not arbitrary it can be triangular, sine curve, etc)

- When the force /load is downwards then it is taken as positive



Point load

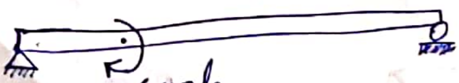
Bending moment (complementary to each other)

changes with x (For const. we write just w)

w(x) uniformly

→ Distrubuted load

Three types of couple



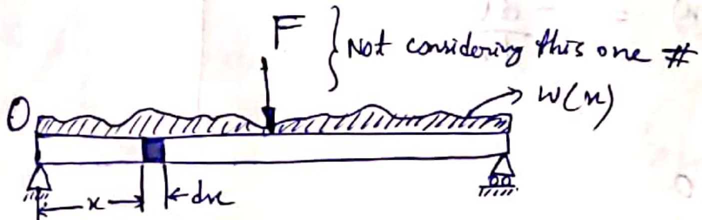
Ex → turbo prop on the wings of an aeroplane (pure couple)

In the analysis of the beam all the forces are considered as Sheer force and denoted by V. Right Side down is a positive sheer force. The Bending moment which deflects the beam downwards due to its action is Known as positive ~~der~~ bending moment. The effect on the beam is Sagging

Note: When the bending moment is upwards (negative) then it is ~~hog~~ hogging

~~The ant~~ The anti-clockwise moment is taken as positive.

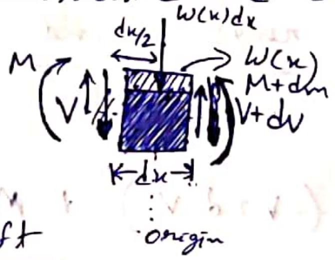
(The beam is Subjected to UDL only)



The beam is subjected to uniformly distributed load defined by $w(x)$

A

The dx element →



~~HW~~ Redo

Starting from left

Now, $\sum$ Forces $= 0$; $-V + w(x)dx + (V + dv) = 0$

$$\Rightarrow w(x)dx + dv = 0$$

$$\Rightarrow w(x)dx = -dv$$

$$\Rightarrow w(x) = \frac{-dv}{dx}$$

The changing pattern of the Shear force and the bending moment along the span of the beam is given by shear force and bending moment diagram (SF and BM diagram)

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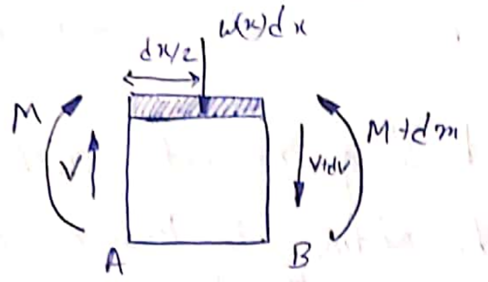
HW



Summing the moments around B

$$-M-(Vdx)+(wdx)\left(\frac{dx}{2}\right)+M+dM=0$$

Now, $\Sigma F = 0$

$$\Rightarrow -V + w(x)dx + V+dV = 0$$

$$\Rightarrow w(x)dx = -dV$$

$$\Rightarrow w(x) = -\frac{dV}{dx}$$

Now, $\Sigma M = 0$

Summing the moments around the free end A we have

$$dx(V+dV) + M + dm = 0$$

$$\Rightarrow Vdx + dxdV + M + dm = 0 \quad \text{— (I)}$$

Summing the moments around the free end B we have

Taking origin at B

$$(-dx)(-V) + (-M) = 0$$

$$\Rightarrow Vdx = M \quad \text{— (II)}$$

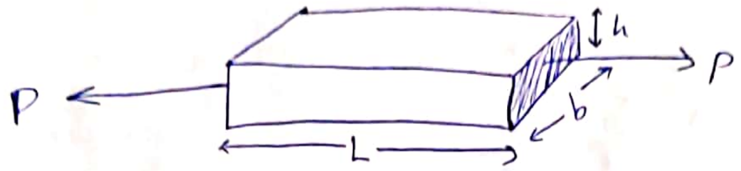
Substituting this value in (I) we get

$$2M + dxdV + dm = 0$$

05/08/20

Different types of Stresses

Normal Stresses



$$\text{Stress} = \frac{P}{b \cdot h}$$ → cross sectional area

• Z → ~~direc~~ direction of axis of motion
(direction of motion)

Gravity along y-direction

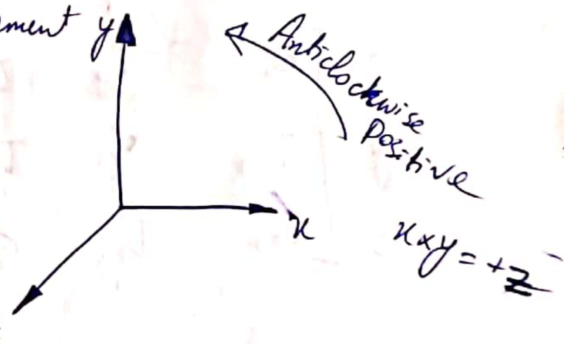
We do not allow movement along y-direction.

***

~~Move Ze~~ move Z by 90° to get

x in place of y to get movement

in y axis



$\sigma_{\square\square}$ → denotes the $\perp$ direction of the load

Denotes the direction in which load is applied (i.e. direction of load)

Ex

$\sigma_{xx} = \sigma_x$; $\sigma_{yy} = \sigma_y$

$$\tau_{xy} = \frac{P}{b \cdot L}$$

• Shear Stress (→ Tearing) → Cross sectional area ∥ to the application of load

$$\tau_{yx} = \frac{Q}{b \cdot h}$$

$$\tau_{xz} = \frac{P}{L \cdot h}$$

$$\tau_{zx} = \frac{F}{h \cdot b}$$

$$\tau_{xy} = \frac{P}{Lb}$$

$$\cancel{\tau_{xy}} = \cancel{\tau_{yz}}\ \tau_{yz} = \frac{Q}{L \cdot h}$$

$$\cancel{P}\ \tau_{zy} = \frac{F}{L \cdot b}$$

Matrix Form

magnitude is same, only direction are different (thus known as complementary forces)

$$\underbrace{\begin{array}{c|ccc} & x & y & z \\ \hline x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ z & \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{array}}_{\text{Stress Tensor}}$$

Unit of Stress

$$1\,N/m^2 = Pa$$

$$1\,N/mm^2 = 10^6\,N/m^2$$

$$= 10^6\,Pa$$

$$= 1\,MPa$$

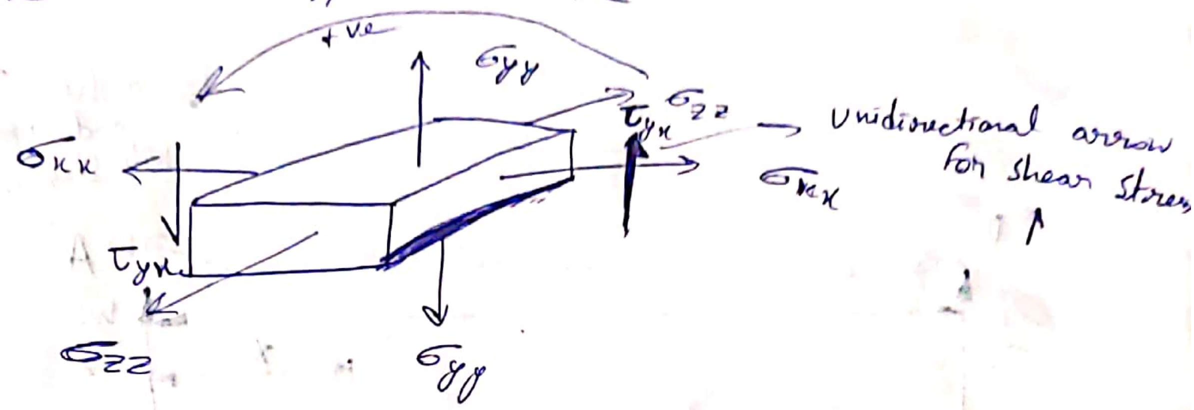
- Sign-conventions

Normal Stress

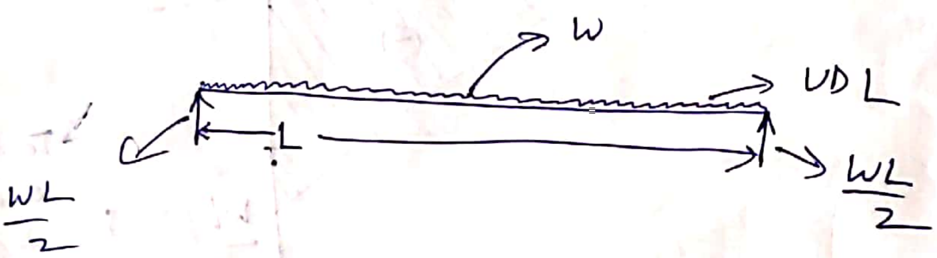
Increment always considered +ve

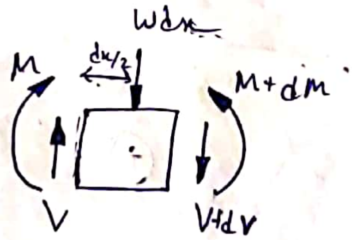
Tensile stress " " "

Compressive " " " -ve



08/08/2022





$$\Sigma F = 0 \Rightarrow -w = \frac{dV}{dx} \longrightarrow \text{I}$$

$$\Sigma M = 0 \Rightarrow V = \frac{dM}{dx} \longrightarrow \text{II}$$

combining I and II

$$\boxed{-w = \frac{dV}{dx} = \frac{d^2M}{dx^2}}$$

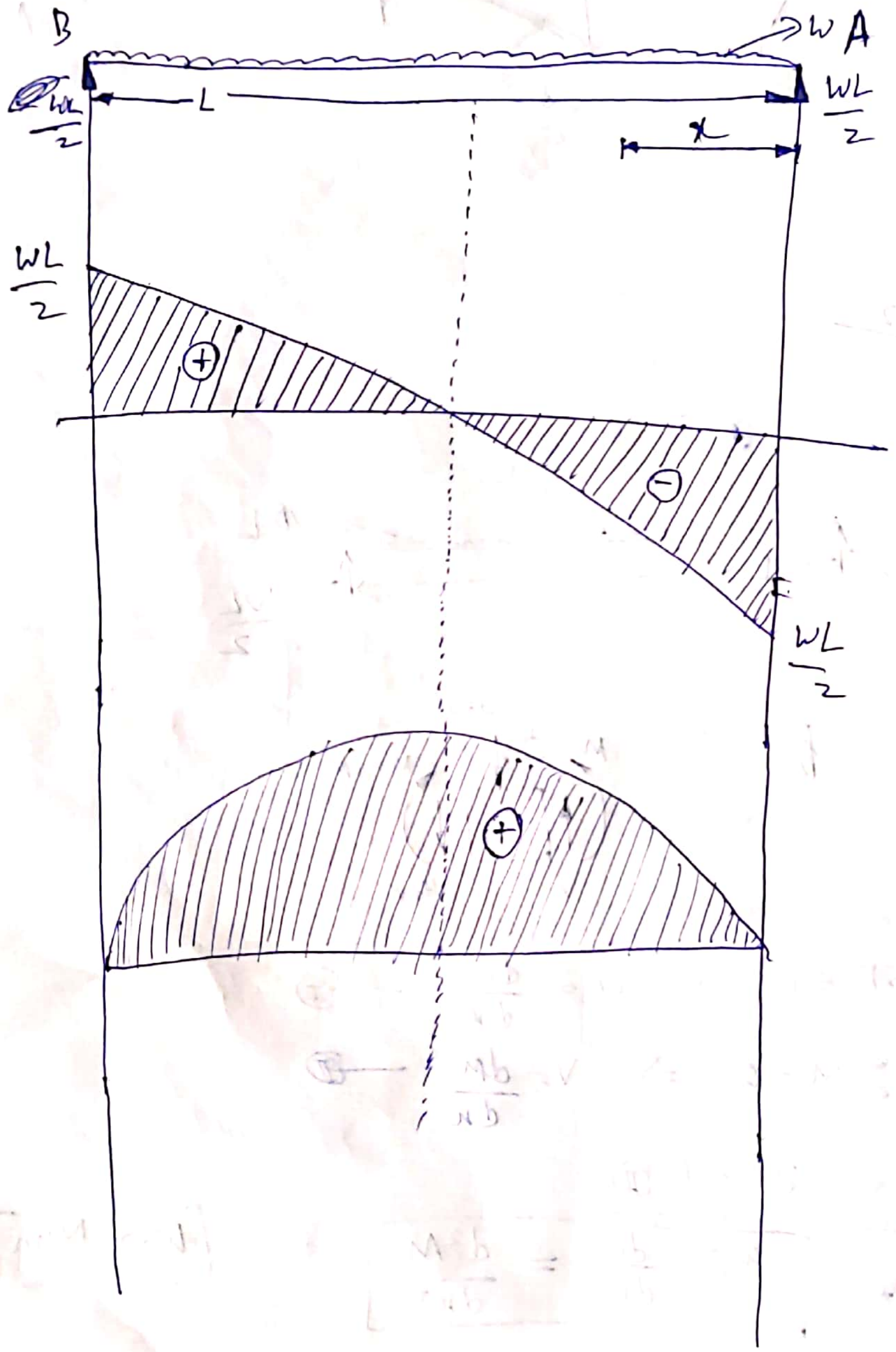
[w ⇒ N/m]

Fundamental differential equation of Bending of ~~Beam~~ Beam

To draw to SF and BM diagram

Shear force comes first and BM second (As we know W and ~~from there~~ from there we can find M)

→ The SF, BM and drawing diagram will stay on the same page



→ To find where the BM + SF will be max or min

At a distance x from the right hand point $\underset{\substack{\downarrow \\ \text{Shear force}}}{V} = \frac{-WL}{2} + \underset{\substack{\downarrow \\ x = 0, L/2, L, \text{etc}}}{\cancel{Wx}}$

$\underbrace{\qquad\qquad\qquad}_{\text{For point load}}$

$\boxed{\text{• Unit of Bending moment} \rightarrow \text{Nm}}$

• Bending moment at ~~A~~ a distance x from A ~~equal~~

$$(M) = \cancel{+\frac{WL}{2}x} \quad \left(+\frac{WL}{2}\right)x - \left(\frac{Wx^2}{2}\right) \rightarrow ??.$$

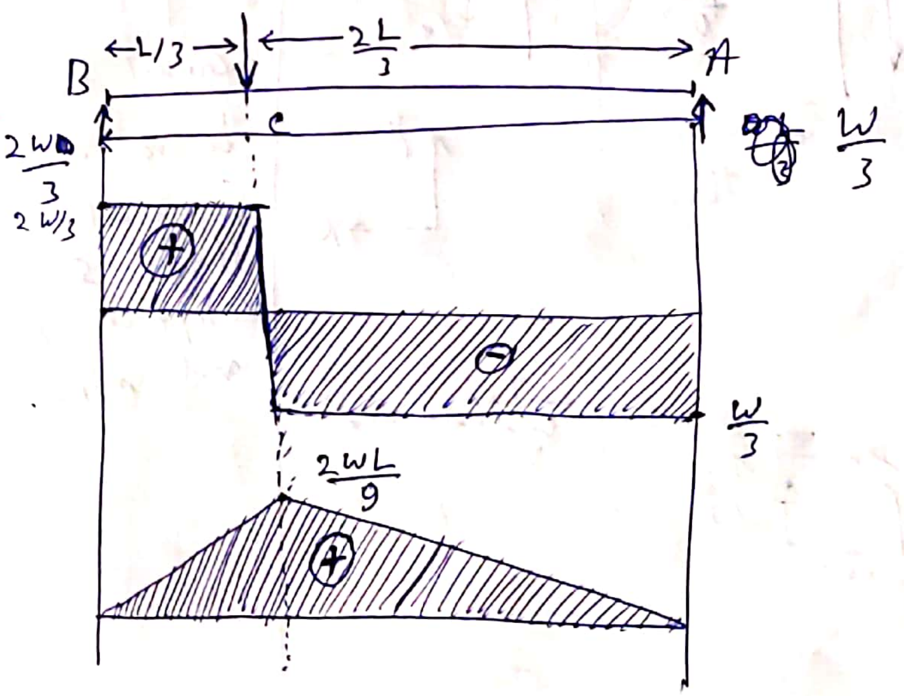
$$\Rightarrow M = \frac{Wx}{2}(L - x)$$

• At the free end of the beam the bending moment is zero (i.e. at $x=0$ and $x=L$)

In cantilever we will only have ~~bend~~ bending moment at the grouted / fixed point

• The bending moment at the mid-point (i.e. $x = L/2$)

$$= \cancel{WL^2} \; \cancel{WL^2} \; \frac{WL^2}{8}$$



12/08/2022 :

State of Stress at a Point

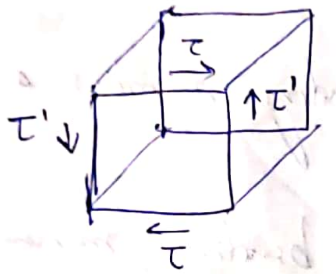
(Teacher :- NBH Sir)

- Complementary → ~~Nu~~ Numerically equal

ex → ~~a+ib~~ a+ib has complementary a−ib

$A = \sqrt{a^2+b^2}$ (Amplitude) $\qquad A = \sqrt{a^2+b^2}$

direction $\tan^{-1}\left(\frac{b}{a}\right)$ ← different in sign → direction $\tan^{-1}\left(-\frac{b}{a}\right)$



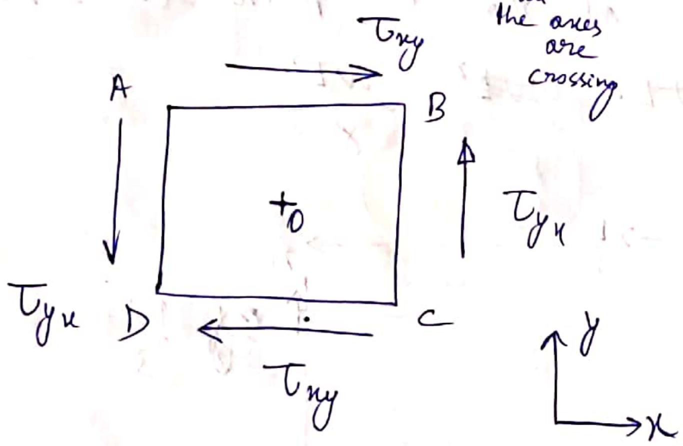
$\tau = \tau'$ for equillibrium (No rotational motion)

Note

Locate equillibrium point on Symmetricity i.e. everything around the ~~eq~~ equillibrium point will be symmetric

Symmetric point ↘ where all the axes are crossing

- Proof



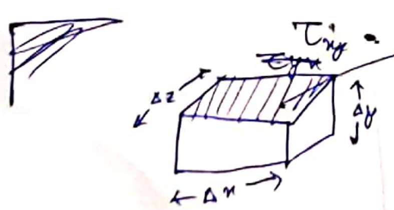
Moment ↗ Force

Static balance equation

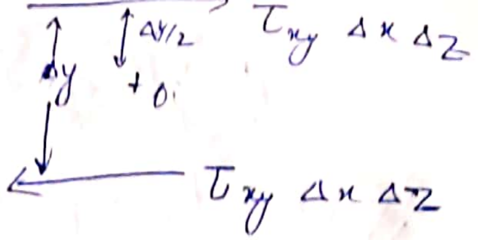
Dynamic " "

∴ Cannot have stress balance equation

So cannot compare stress so we compare forces and moments



$\tau_{xy}\,\cdot \rightarrow$ $\underbrace{\tau_{xy}(\Delta x\,\Delta z)}_{\text{Force}}$



~~com~~ Couple amount → Force × distance between members

$$= \tau_{xy}\,\Delta x\,\Delta y\,\Delta z$$

Similarly,

Couple amount → Force × distance between members

$$= (\tau_{yx} \times \Delta y\,\Delta z) \times \Delta x$$

$$= \tau_{yx}\,\Delta x\,\Delta y\,\Delta z$$

Now, since there is no net moment of the member

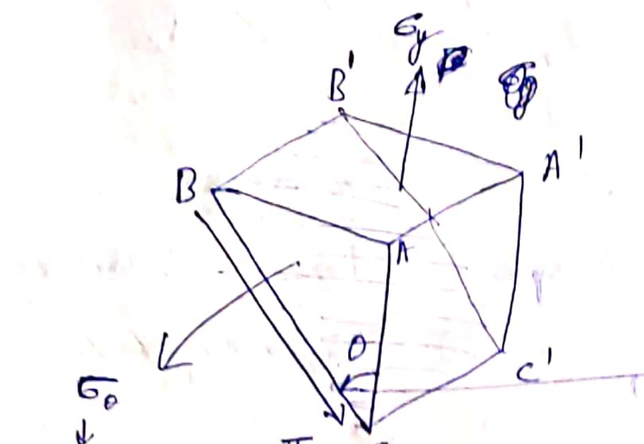
So, $\tau_{xy}\,\Delta x\,\Delta y\,\Delta z = \tau_{yx}\,\Delta x\,\Delta y\,\Delta z$

$$\Rightarrow \boxed{\tau_{xy} = \tau_{yx}}$$

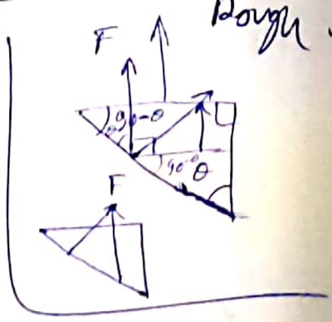
- But as τ_{xy} is clockwise and τ_{yx} is anti-clockwise
- So we take $|\tau_{xy}| = \cancel{|\tau_{yx}|}\ |\tau_{yx}|$

Analysis of Stresses

next page →



→ Aθ Anti clockwise positive



σ_θ ↓ Normal stress

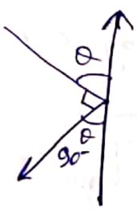
τ_θ on BB'C'C

~~Tf~~ To find the stresses on BB'C'C

To find σ_θ

To find the forces acting along the σ_θ direction

$$\sigma_\theta\, BC \times CC' = \sin\theta\, \underbrace{\sigma_y\, AB \times AA'}_{F}$$



$F_y = \sigma_y A$

$\sigma_y\, AB \times \underset{\sim}{1}$ taking $AA' = 1$

taking $CC' = 1$

$\sigma_\theta\, BC \times 1 = F_\theta$

σ_θ max → denoted by σ_1	Amplitudes	θ
		θ_{1p} → major principle angle
σ_θ min ↓ denoted by σ_2		θ_{2p} ↓ minor principle angle

$\therefore F_\theta\, \cancel{\cos\theta} = F_y \sin\theta$

$\Rightarrow \sigma_\theta = \sigma_y \frac{AB}{BC}$

$\Rightarrow \sigma_\theta = \sigma_y \sin\theta \sin\theta$

$\Rightarrow \sigma_\theta = \sigma_y \sin^2\theta$

$\Rightarrow \sigma_\theta = \frac{\sigma_y}{2}(1 - \cos 2\theta)$

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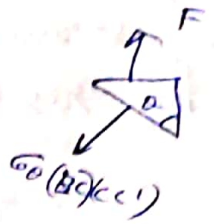
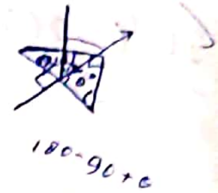
The major and minor principle stresses act at mutually ~~⊥ di~~ perpendicular directions

Also angular difference between τ_{max} and τ_{min} is 90°

$$\sigma_\theta BC \times CC' = \sin\theta \cdot \underbrace{\sigma_y AB \times AA'}_{F}$$



$$\therefore \sigma_\theta \times BC \times 1 = F_\theta$$ [Taking $CC' = 1$ and $AA' = 1$]



and $F_y = \sigma_y AB \times 1$

$$\therefore F_\theta = F_y \sin\theta$$

$$\Rightarrow \sigma_\theta = \frac{\sigma_y AB}{BC} \sin\theta$$

Now, $\frac{AB}{BC} = \cancel{\sec\theta}\ \sin\theta$

$$\therefore \sigma_\theta = \sigma_y \sin^2\theta$$

$$\Rightarrow \sigma_\theta = \frac{\sigma_y}{2}(1-\cos 2\theta)$$

Now, ~~rest~~ resolving the forces parallel to BC,

$$\tau_\theta BC \cdot 1 = \cancel{\sigma_y \cos\theta \Sigma AB \sin\theta = 1}\ \cos\theta\, \sigma_y AB \times AA'$$

~~Again $AC = BC\cos\theta$~~

~~$\tau_\theta BC \cdot 1 = \sigma_y \cos\theta \cdot BC \sin\theta \cdot 1$~~

$$\Rightarrow \tau_\theta = \frac{\sigma_y AB \cancel{AA'}^{1} \cos\theta}{BC}$$

$$\therefore \tau_\theta = \cancel{2}\ \sigma_y \sin\theta\cos\theta$$

$$\Rightarrow \tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta$$

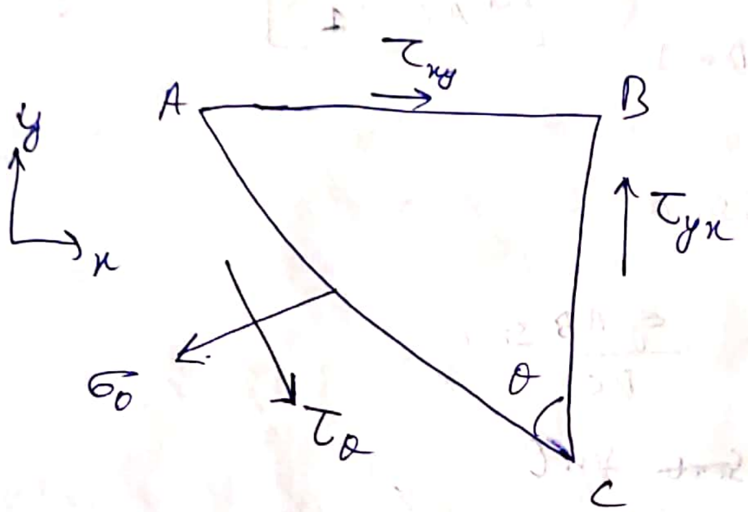
$\therefore$ The ~~vo~~ value of direct stress σ_θ is maximum and is equal to σ_y when $\theta = \cancel{90}$ $90°$

The ~~stea~~ shear stress τ_θ has a maximum value of ~~0.5σy~~ $0.5\,\sigma_y$ when $\theta = 45°$

The stresses σ_θ ~~and~~ and τ_θ are not simply ~~the~~ the resolution of ~~σy~~ σ_y

---

16/08/2022 Teacher → NBH Sir



$$\sigma_\theta\, AC \times 1 = \tau_{xy} \times AB \times \cos\theta \times 1 + \tau_{yx} \times BC \times \sin\theta \times 1$$

Also, $$\tau_\theta\, AC \times 1 = -\tau_{xy} \times AB \times \sin\theta \times 1 + \tau_{yx} \times BC \times \cos\theta \times 1$$

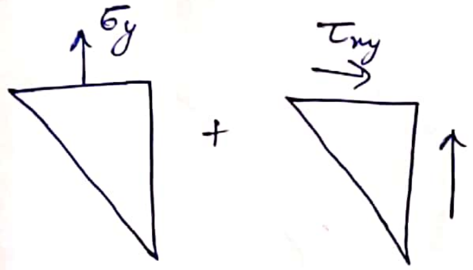
Now, $$\sigma_\theta = \tau_{xy} \sin\theta\cos\theta + \tau_{yx} \sin\theta\cos\theta$$
$$= \tau_{xy} \sin 2\theta$$

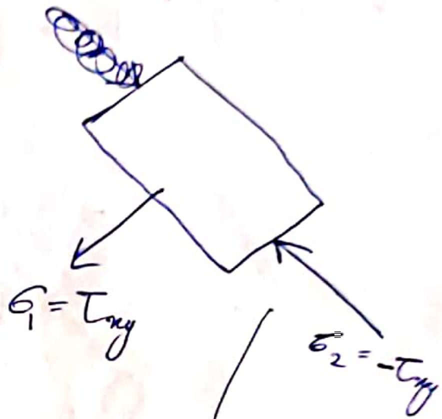
$$\tau_\theta = -\tau_{xy} \sin^2\theta + \tau_{yx} \cos^2\theta$$
$$= -\left[\tau_{xy}\left(\sin^2\theta + \cos^2\theta\right)\right] = -\tau_{xy} \cos 2\theta$$

σ_1 σ_2

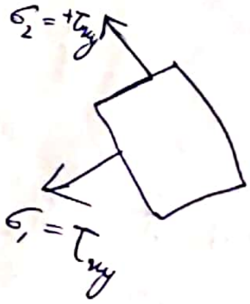
θ_{1p} θ_{2p}

	magnitude	Direction
σ_1	τ_{xy}	$\theta_{1p} = 45^\circ$
σ_2	$-\tau_{xy}$	$\theta_{2p} = 135^\circ$
τ_{max}	τ_{xy}	$\theta_{1s} = 90^\circ$
τ_{min}	$-\tau_{xy}$	$\theta_{2s} = 0^\circ$ or 135°



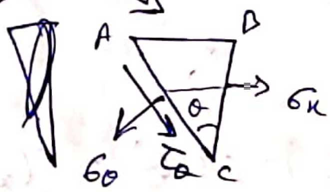


can also be drawn as



$$\Rightarrow \sigma_\theta = \sigma_y \frac{(1-\cos 2\theta)}{2} + \tau_{xy}\sin 2\theta + \frac{\sigma_x}{2}(1+\cos 2\theta) \rightarrow \text{I}$$

$$\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta - \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta \rightarrow \text{II}$$



$$\rightarrow \sigma_\theta = \frac{1}{2}\sigma_x(1+\cos 2\theta)$$

$$\tau_\theta = -\frac{1}{2}\sigma_x \sin 2\theta$$

$$\sigma_\theta \, AC \cdot 1 = \sigma_x \cos\theta \, BC$$

$$= \sigma_x \cos^2\theta$$

$$= \frac{\sigma_x}{2}(1+\cos 2\theta)$$

Also, $\tau_\theta \cdot AC = -\sigma_x \sin\theta \; BC$

$$= -\frac{\sigma_x \sin 2\theta}{2}$$

$\rightarrow$ For $\rightarrow$

Now, to find where σ_θ will be maximum

$$\frac{d\sigma_\theta}{d\theta} = 0 = \frac{1}{2}(\sigma_x - \sigma_y)(-2\sin 2\theta) + \tau_{xy}(2\cos 2\theta) = 0$$

$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

$$= \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y/2\right)}$$

$$\therefore 2\theta_{1p} = \tan^{-1}\left[\frac{\tau_{xy}}{\left[\frac{\sigma_x - \sigma_y}{2}\right]}\right] \pm 90^\circ \quad \left[\text{As } |\theta_{1p} - \theta_{2p}| = 90^\circ\right] \quad \text{— III}$$

Now

$$\frac{d\tau_\theta}{d\theta} = 0 = -\frac{1}{2}(\sigma_x - \sigma_y)(2\cos 2\theta) - \tau_{xy}(-2\sin 2\theta) = 0$$

$$\therefore \tan 2\theta_s = \frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}} \quad \text{— IV}$$

Multiplying III and IV we get

$$1 - \tan 2\theta_p \tan 2\theta_s = 0$$

$$\therefore \tan(2\theta_p + 2\theta_s) = \frac{\tan 2\theta_p + \tan 2\theta_s}{1 - \tan 2\theta_p \tan 2\theta_s}$$

$\therefore \tan(2\theta_p + 2\theta_s) = \infty$

$\Rightarrow 2\theta_p + 2\theta_s = n \times 90^\circ$

$\Rightarrow \theta_p + \cancel{3}\,\theta_s = \pm n 45^\circ$

HW

	σ_x	σ_y	τ_{xy}
1)	10	−10	0
2)	0	+10	−10
3)	5	75	25
4)	−5	−5	5

Find

σ_1
σ_2
τ_{max}
τ_{min}
θ_{1s}
θ_{2s}
θ_{2p}
θ_{1p}

Answers

Sl.No	σ_1	σ_2	τ_{max}	τ_{min}	θ_{1s}	θ_{2s}	θ_{1p}	θ_{2p}
1	~~0~~ 10	−10	10	−10	135° ~~270~~	90°	0°	90°
2	~~10~~ 15	~~−10~~ −5	~~10~~ 5	~~−10~~ −5	~~90~~ 45°	~~0~~ 90°	~~90~~ ~~45~~ 135°	~~0~~ 45°
3	75	5	35	−25	45°	0°	90°	0°
4	0	−10	5	−5	45° or 90°	0°	45°	~~90~~ 135°

1) $\sigma_\theta = \sigma_y \dfrac{(1-\cos 2\theta)}{2} + \tau_{xy} \sin 2\theta + \dfrac{\sigma_x}{2}(1+\cos 2\theta)$

$= -5(1-\cos 2\theta) + 0 + 5(1+\cos 2\theta)$

$\therefore \sigma_\theta$ is max when $1-\cos 2\theta = 0 \Rightarrow \cos 2\theta = 1$

$\Rightarrow \cos 2\theta = \cos 0^\circ$

$\theta = 0^\circ$

$\therefore$ ~~$\sigma_{max} = 5$~~ $\sigma_1 = \cancel{5}\ 10$

Similarly σ_θ is min when $(1+\cos 2\theta) = 0 \Rightarrow \theta = 90^\circ$

Now, $\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta - \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta$

$\therefore \tau_\theta = -5\sin 2\theta + 0 - 5\sin 2\theta$

$= -10\sin 2\theta$

$\therefore \tau_\theta$ is maximum when $\sin\theta = -1$

$\therefore 2\theta = 270^\circ \Rightarrow \theta = 135^\circ$

$\Rightarrow \tau_1 = 10$

and τ_θ is minimum when $\sin\theta = 1$

$\Rightarrow \theta = 90^\circ$

$\Rightarrow \tau_2 = -10$

2) $\sigma_\theta = 0 + 10\sin 2\theta - 5(1+\cos 2\theta)$

$\tau_\theta = 0 - 10\cos 2\theta + 5\sin 2\theta$

σ_θ max $= 5$ at $\theta = 45^\circ$

σ_θ min $= -10$ at $\theta = 0^\circ$

τ_θ max $=$ ~~5 at θ = 45~~ 10 at $\theta = 90^\circ$

τ_θ min $= -10$ at $\theta = 0^\circ$

~~3) σ_θ =~~

2) $\sigma_\theta = 5(1-\cos 2\theta) - 10\sin 2\theta \Rightarrow 5 - 5\cos 2\theta - 10\sin 2\theta$

$\tau_\theta = 5\sin 2\theta + 10\cos 2\theta$

3) $\sigma_\theta = \frac{75}{2}(1-\cos 2\theta) + 25\sin 2\theta + \frac{5}{2}(1+\cos 2\theta) = -35\cos 2\theta + 25\sin 2\theta + 40$

$\tau_\theta = \frac{75}{2}\sin 2\theta - 25\cos 2\theta - \frac{5}{2}\,\cancel{(1+\cos 2\theta)}\,\sin 2\theta$

$= 35\sin 2\theta - 25\cos 2\theta$

4) $\sigma_\theta = -\frac{5}{2}(1-\cos 2\theta) + 5\sin 2\theta - \frac{5}{2}(1+\cos 2\theta) = 5\sin 2\theta - 5$

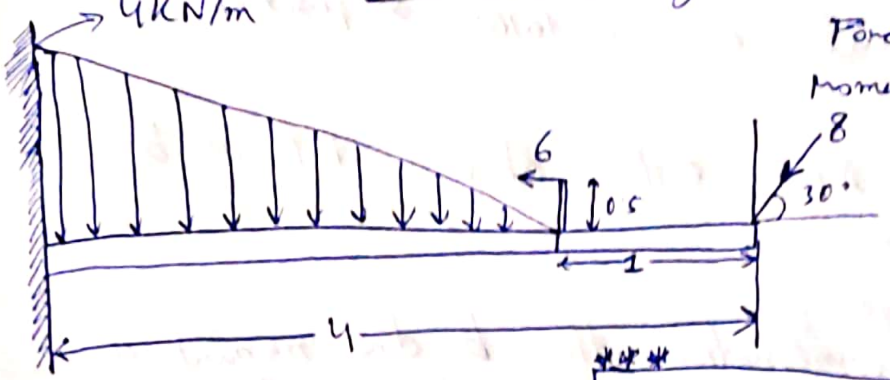
$\tau_\theta = \frac{5}{2}\sin 2\theta - \overset{5\cos 2\theta}{\cancel{5\sin 2\theta}} + \frac{5}{2}\sin 2\theta$

$= 5\sin 2\theta - 5\cos 2\theta$

17/08/2022

SF - BM diagrams

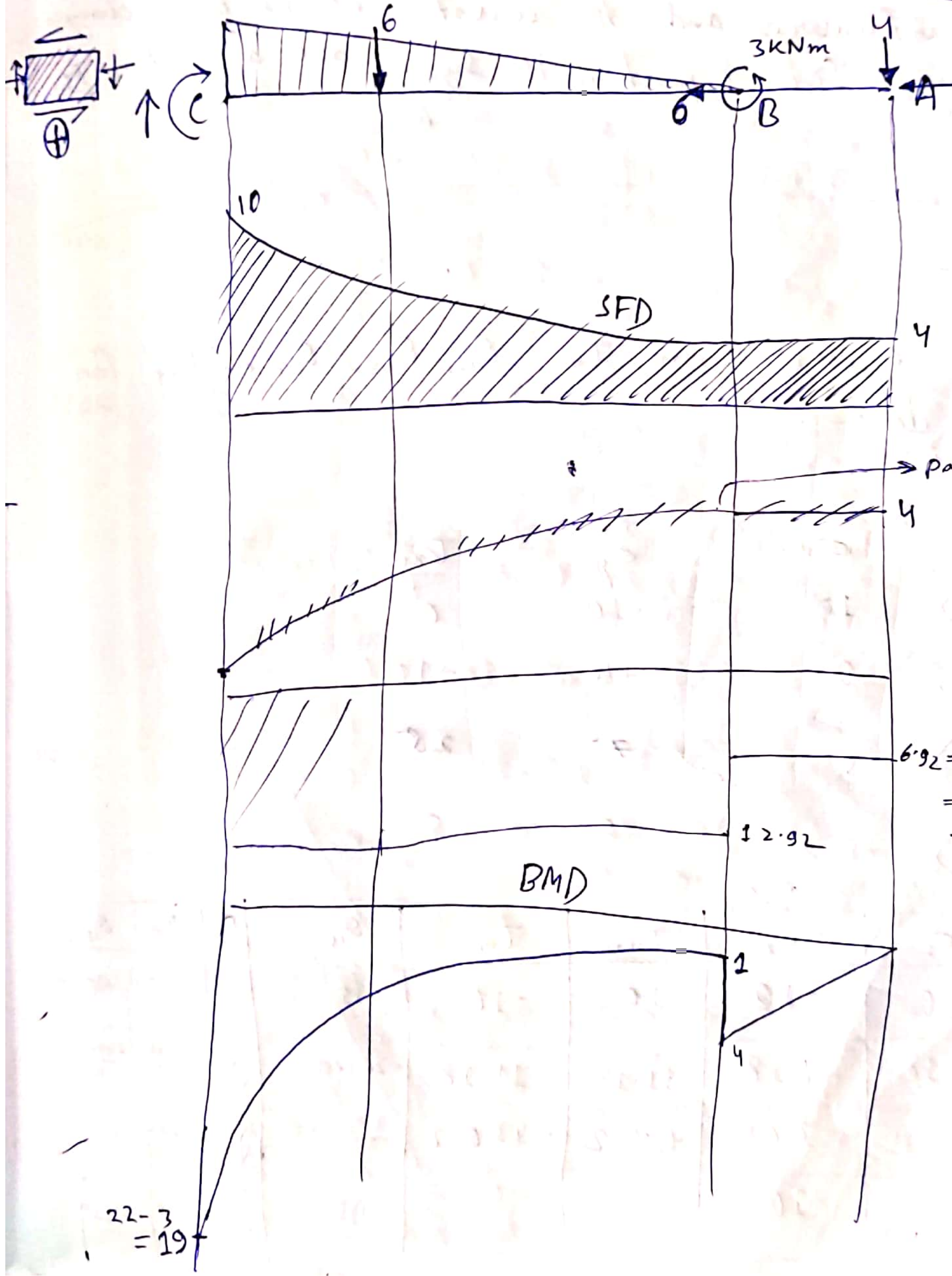
Units
Length = m
Force/Load = KN
Moment/~~couple~~ couple → KN-m



Load diagram

+ve direction couple → clockwise
-ve " " → Anticlockwise

Degree of BM is always 1 higher than the degree of SF



BM at C

= (4×4) + (6×2)

= 22 - couple

= 22 - 3

= 19

The axial forces are represented by axial force diagram (AFD). The tensile force is taken as positive

Note

: Always Draw AFD either above SFD or below BMD

In some force combination the bending moment diagram changes sign i.e. crosses the axis from positive to negative or negative to positive. These points are known as point of contraflexure and the curvature of the beam changes at that point (i.e. a portion is sagging and a portion is hogging)

HW
16/08/2022

Q) Find σ_1, σ_2, τ_{max}, τ_{min}, θ_{1s}, θ_{2s}, θ_{1p} and θ_{2p} for the given data :-

	σ_x	σ_y	τ_{xy}
1)	10	-10	0
2)	0	+10	-10
3)	5	75	25
4)	-5	-5	5

Ans)

Sl.No	σ_1	σ_2	τ_{max}	τ_{min}	θ_{1s}	θ_{2s}	θ_{1p}	θ_{2p}
	10	-10	10	-10	135°	90°	0°	90°
	16.18	-6.18	11.18	-11.18	-26.5650°	153.4349°	$\frac{63.434}{2}$	$\frac{243.435}{2}$
	83.011	-3.011	43.012	-43.012	$\frac{234.462}{2}$	$\frac{54.462}{2}$	72.231	-35.53
	0	-10	5	-5	90°	0°	45°	-45°

1) $\sigma_\theta = \sigma_y \frac{(1-\cos 2\theta)}{2} + \tau_{xy} \sin 2\theta + \frac{\sigma_x}{2}(1+\cos 2\theta)$

$= -5(1-\cos 2\theta) + 0 + 5(1+\cos 2\theta)$

$= -5 + 5\cos 2\theta + 5 + 5\cos 2\theta$

$= 10\cos 2\theta$

Now, $2\theta_{1P} = \tan^{-1}\left[\frac{\tau_{xy}}{\left[\frac{\sigma_x - \sigma_y}{2}\right]}\right] \pm 90^\circ$

$= \tan^{-1}\left[\frac{0}{\left[\frac{10-(-10)}{2}\right]}\right] \pm 90^\circ$

$= \tan^{-1}(0^\circ)$

$= 0^\circ \Rightarrow \theta_{1P} = 0^\circ$

Now, $\sigma_1 = 10\cos 0^\circ$

$= 10$

Now, σ_θ is minimum when $\cos 2\theta = -1 \Rightarrow \cos 2\theta = \cos 180^\circ$

$\Rightarrow \theta = 90^\circ$

$\therefore \sigma_2 = 10\cos 180^\circ$

$= -10$

and $\theta_{2P} = 90^\circ$

Now, $\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta - \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta$

$\therefore = -5\sin 2\theta + 0 - 5\sin 2\theta$

$= -10\sin 2\theta$

~~Now, $2\theta_{1S} = \tan^{-1}\left[\frac{\left[\frac{\sigma_x - \sigma_y}{2}\right]}{\tau_{xy}}\right]$~~

~~$= \tan^{-1}\left[\frac{\frac{10-(-10)}{2}}{0}\right]$~~

$= \tan^{-1}(\infty) \Rightarrow 2\theta_{1S} = 90^\circ$

$\therefore \cancel{\theta_{1s} = 45^\circ}$

$\cancel{\text{Now, } \tau_{max} = -10 \sin 90^\circ}$

$\cancel{= -1}$

~~and~~

Now, τ_θ is maximum when $\sin 2\theta = -1$

$= \sin 270^\circ$

$\Rightarrow 2\theta_{1s} = 270^\circ$

$\Rightarrow \theta_{1s} = 135^\circ$

and $\tau_{max} = 10$

and τ_θ is minimum when $\sin 2\theta_{2s} = 1$

$= \sin 90^\circ$

$\Rightarrow 2\theta_{2s} = 90^\circ$

$\Rightarrow \theta_{2s} = 45^\circ$

$\therefore \tau_{min} = -10$

2) $\sigma_\theta = \sigma_y \left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \sin 2\theta + \frac{\sigma_x}{2}(1+\cos 2\theta)$

$= 5(1-\cos 2\theta) + (-10 \sin 2\theta) + 0$

$\Rightarrow 5 - 5\cos 2\theta - 10 \sin 2\theta$

~~Now, For maximum σ_θ, $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$~~

$\cancel{= \frac{2(-10)}{0-10}}$

$\cancel{= 2}$

$\therefore \cancel{2\theta = \tan^{-1}(2)}$

$\cancel{= 63.43^\circ}$

$\cancel{\therefore \theta = 31.717^\circ}$

$\cancel{\text{and } \sigma_1 = 5 - 2.236 - 8.944 = -6.180}$

Now, the maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$ and the minimum value is $-\sqrt{a^2+b^2}$

$\therefore$ The maximum value of σ_θ is $\sigma_1 = 5 + \sqrt{100+25}$
$= 16.1801$

and The minimum value of σ_θ is $\sigma_2 = 5 - \sqrt{100+25}$
$= -6.180$

Now, $2\theta_{2p} = \tan^{-1}\left(\frac{-10}{-5}\right)$ [As $\tan\theta = \frac{A}{B}$ from $A\sin\theta + B\cos\theta$]

$= \tan^{-1} 2$

$\therefore \theta_{2p} = 63.4349^\circ/2 = 31.717^\circ$

$\theta_{1p} = \theta_{2p} + 90^\circ$

$= 121.717^\circ$

Now, $\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta - \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta$

$= -5\sin 2\theta + 10\cos 2\theta$

$\therefore \tau_{max} = \sqrt{25+100}$ at $2\theta_{1p} = \tan^{-1}\left(\frac{1}{2}\right)$

$= 11.180$

$= 26.56^\circ$

$\Rightarrow \theta_{1s} = 13.28^\circ$

and, $\tau_{min} = -6.70$ at $2\theta_{2s} = \cos^{-1}\left(\frac{-10}{\sqrt{25+100}}\right)$

$= 153.43^\circ$

$\Rightarrow \theta_{2s} = 76.717^\circ$

3) $\sigma_\theta = \frac{\sigma_y(1-\cos 2\theta)}{2} + \tau_{xy}\sin 2\theta + \frac{\sigma_x}{2}(1+\cos 2\theta)$

$= \frac{75}{2}(1-\cos 2\theta) + 25\sin 2\theta + \frac{5}{2}(1+\cos 2\theta)$

$= \frac{75}{2} + \frac{5}{2} - \frac{75}{2}\cos 2\theta + \frac{5}{2}\cos 2\theta + \cancel{25 \sin} \; 25\sin 2\theta$

$= 40 - 35\cos 2\theta + 25\sin 2\theta$

Now, maximum value of $a\cos\theta + b\sin\theta = \cancel{\tan^{-1}\left(\frac{b}{a}\right)}$ $\sqrt{a^2+b^2}$ at $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore$ $\cancel{\sigma_{max}}$. $\sigma_2 = 40 - \sqrt{(25)^2+(35)^2}$

$= -3.011$

and, $2\theta_{2p} = \tan^{-1}\left(-\frac{25}{35}\right)$

$= -35.53^\circ$

$\Rightarrow \theta_{2p} = -17.76^\circ$

$= 360^\circ - 17.76^\circ$

$= 342.24^\circ$

Now,

$\sigma_1 = 40 + \sqrt{(25)^2+(35)^2}$

$= 83.011$ at $\theta_{1p} = 342.24 + 90^\circ$

$= 432.24^\circ$

$= 72.24^\circ$

Now, $\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta - \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta$

$= \frac{75}{2}\sin 2\theta - 25\cos 2\theta - \frac{5}{2}\sin 2\theta$

$= 35\sin 2\theta - 25\cos 2\theta$

$\therefore$ $\tau_{\cancel{max}\,min} = -43.011$ at $2\theta_{1s} = \tan^{-1}\left(\frac{35}{-25}\right)$

$= -54.46$

$\Rightarrow \theta_{2s} = \cancel{?}\ -27.23$

$\Rightarrow \theta_{2s} = 332.76°$

$70\cos 2\theta + 50\sin 2\theta = 0$

$\Rightarrow \tan 2\theta = \frac{-70}{50}$

$2\theta = -54.46°$

$\theta = -27.23°$

$-140\sin 2\theta + 100\cos 2\theta$

---

2) $\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{yx}\sin 2\theta$

$= 0 + \frac{20}{2}\cos 2\theta + 0$

$= 10\cos 2\theta$

max $\Rightarrow \sigma_1 = 10$ at $2\theta = 0°$ $\Rightarrow \theta = 0°$ | ~~max~~ min $\Rightarrow -10$ at $2\theta = 180°$ $\Rightarrow \theta = 90°$

$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$= \frac{20}{2}\sin 2\theta - 0$

$= 10\sin 2\theta$

max $= 10$ at $2\theta = 90°$

$\theta = 45°$

min $= -10$ at $2\theta = -90°$

$\Rightarrow \theta = -45°$

3) $\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{yx}\sin 2\theta$

$= \left(\frac{80}{2}\right) + \left(\frac{-70}{2}\right)\cos 2\theta + 25\sin 2\theta$

$= 40 - 35\cos 2\theta + 25\sin 2\theta$

Now, $\sigma_{max/min} = 0 + 70\sin 2\theta + 50\cos 2\theta = 0$

$\Rightarrow \tan 2\theta = -\frac{5}{7}$

$\Rightarrow 2\theta = -35.53^\circ$

$\Rightarrow$ ~~θ = ...~~

$= 324.4623^\circ$

$\Rightarrow \theta = 162.231^\circ$

$\therefore \sigma_{min} = \cancel{...}\ -3.011$ at $\theta_{2p} = \cancel{...}\ -35.53^\circ$

and $\sigma_{max} = 83.011$ at $\theta_{1p} = 252.231^\circ$

$= 72.231^\circ$

$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \cancel{\tau_{xy}}\ \tau_{yx}\cos 2\theta$

$= \left(-\frac{70}{2}\right)\sin 2\theta - 25\cos 2\theta$

$= -35\sin 2\theta - 25\cos 2\theta$

Differentiating w.r.t. θ

Now, $-70\cos 2\theta + 50\sin 2\theta = 0$

$\Rightarrow \tan 2\theta = 7/5$

$\Rightarrow 2\theta = \tan^{-1}(7/5)$

~~= 35.53~~

$= 54.462$

$\therefore \tau_{min} = -43.012$ at $\theta_{2s} = \frac{54.462^\circ}{2}$

$\therefore\ \tau_{max} = 43.012$ at $\theta_{1s} = \dfrac{234.462}{2}$

4) $\sigma_\theta = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) + \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{yx}\sin 2\theta$

$\Rightarrow \left(\dfrac{-10}{2}\right) + 0 + 5\sin 2\theta$

$= -5 + 5\sin 2\theta$

$\Rightarrow \sigma_1 = 0$ at $2\theta = 90°$

$\Rightarrow \theta_{1P} = 45°$

$\sigma_2 = \cancel{5}\ -10$ at $2\theta = -90°$

$\Rightarrow \theta_{2P} = -45°$

$\tau_\theta = \dfrac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{yx}\cos 2\theta$

$= -5\cos 2\theta$

$\Rightarrow \tau_{max} = \cancel{0}\ 5$ at $2\theta = 180°$

$\Rightarrow \theta_{1s} = 90°$

$\Rightarrow \tau_{min} = -5$ at $2\theta = 0°$

$\Rightarrow \theta_{2s} = 0°$

2) $\sigma_\theta = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) + \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{yx}\sin 2\theta$

$= \left(\dfrac{10}{2}\right) + \left(\dfrac{-10}{2}\right)\cos 2\theta - 10\sin 2\theta$

$= 5 - 5\cos 2\theta - 10\sin 2\theta$

Now, Differentiating w.r.t. θ

$10\sin 2\theta - 20\cos 2\theta = 0 \Rightarrow \tan 2\theta = 2$

$\Rightarrow 2\theta = \tan^{-1}(2)$

$\Rightarrow \theta = 63.434$

$\therefore \sigma_2 = -6.18$ at $2\theta_{2p} = 63.434° \Rightarrow \theta_{2p} = \frac{63.434°}{2}$

and, $\sigma_1 = 16.18$ at $2\theta_{1p} = 243.435° \Rightarrow \theta_{1p} = \frac{243.435°}{2}$

Now, $\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$

$= -\frac{10}{2}\sin 2\theta + 10\cos 2\theta$

$= -5\sin 2\theta + 10\cos 2\theta$

Differentiating w.r.t. x

$-10\cos 2\theta - 20\sin 2\theta$

$\Rightarrow \tan 2\theta = -1/2$

$\Rightarrow 2\theta = \cancel{-45} -26.565°$

$\therefore \cancel{\tau_{max}}\ \tau_{max} = 11.18$ at $\theta_{1s} = (-26.5650°)$

and $\tau_{min} = -11.18$ at $\theta_{2s} = 153.4349°$

Scanned with CamScanner