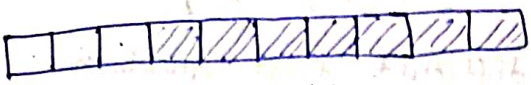
Probability Assignment

Q1)



Considering 3 available space as 1 block we get total 8 blocks in which 7 blocks are same i.e. filled spaces.

$\therefore$ Required Probability $= \frac{{}^{8}C_{1}}{\frac{10!}{3!7!}} = \frac{8 \times 3 \times 2!}{5 \times 4 \times 3} = \frac{1}{15}$

Q2) Let, A be probability of buying a second hand car and B be the probability of buying a japanese car.

Given, $P(A' \cap B') = 0.55 = 1 - P(A \cup B)$

and $P(A) = 0.25$, $P(B) = 0.3$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.25 + 0.3 + 0.45$$
$$= 0.55 - 0.45$$
$$P(A \cap B) = 0.10$$

Q3) Let, $P(S)$ be the probability that students take spanish language and $P(F)$ " " " " French ".

Given, $P(S) = 0.35$ $P(F) = 0.15$

$P(S \cup F) = 0.4$

Now, $P(S \cap F) = P(S) + P(F) - P(S \cup F)$

$= 0.35 + 0.15 - 0.4$

$P(S \cap F) = 0.1$

$$P(F/S) = \frac{P(F \cap S)}{P(S)} = \frac{0.1}{0.35} = \frac{2}{7}$$

Q4) Let $P(H)$ be the probability of hard exam and $P(E)$ be the probability of easy exam.

Given, $P(H) = 0.8$, $P(E) = 0.2$

$P(F)$ be probability of difficult first question.

and, $P(F/H) = 0.9$ and $P(F/E) = 0.15$

$$P(F/H) = \frac{P(F \cap H)}{P(H)} \quad \text{and} \quad P(F/E) = \frac{P(F \cap E)}{P(E)}$$

$$P(F \cap H) = 0.72 \quad \text{and} \quad P(F \cap E) = 0.03$$

$$P(F) = P(F \cap H) + P(F \cap E)$$
$$= 0.72 + 0.03$$
$$= 0.75$$

$$P(H/F) = \frac{P(H) \times P(F/H)}{P(H) \times P(F/H) + P(E) \times P(F/E)}$$

$$= \frac{0.8 \times 0.9}{0.72 + 0.03}$$

$$= \frac{24}{25}$$

Q5) Sample space with one boy and two girls.

A.T.Q.

Probability of given event $= \frac{{}^3C_1}{{}^3C_1 + {}^3C_2 + {}^3C_3} = \frac{3}{7}$

Q6) p : even number
q : odd number

A.T.Q.

$$^{10}C_5\, p^5 q^5 = 2 \times {}^{10}C_4\, p^4 q^6$$

$$252p = 420q$$

$$252p = 420(1-p)$$

$$p = 5/8$$

$$q = 1 - \frac{5}{8} = \frac{3}{8}$$

For no. even no. $\rightarrow$ $^{10}C_0\, p^0 q^{10} = 1 \times 1 \times \left(\frac{3}{8}\right)^{10}$

$$= \left(\frac{3}{8}\right)^{10}$$

Q7) ~~P~~

E : Passing Examination then, $P(E) = 0.6$
F : Failing Examination $P(F) = 0.4$

Probability that out of 6 candidates, at least 4 passed examination is =

$$^6C_4 (0.6)^4 (0.4)^2 + {}^6C_5 (0.6)^5 (0.4) + {}^6C_6 (0.6)^6 (0.4)^0$$

$$= 0.824256$$

Q8) E : Hitting target successfully $P(E) = \frac{1}{5}$

F : Failure in hitting target $P(F) = \frac{4}{5}$

Probability of hitting target at least twice $= 1 - \left[{}^5C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^5 + {}^5C_1 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4 \right]$

$$= 0.26272$$

Q9> E : Patient recovers from blood disease $P(E) = 0.4$

F : Do not recover from disease $P(F) = 0.6$

a> Probability at least 10 survive $= {}^{15}C_{10}(0.4)^{10}(0.6)^5$
$+ {}^{15}C_{11}(0.4)^{11}(0.6)^4$
$+ {}^{15}C_{12}(0.4)^{12}(0.6)^3$
$+ {}^{15}C_{13}(0.4)^{13}(0.6)^2$
$+ {}^{15}C_{14}(0.4)^{14}(0.6)^1$
$+ {}^{15}C_{15}(0.4)^{15}(0.6)^0 = 0.0338$

c> Probability that exactly 5 survive $= {}^{15}C_5(0.4)^5(0.6)^{10} = 0.1859$

Q10> mean resistance $\mu = 40\ \Omega$

$\sigma = 2\ \Omega$

x (resistance of resistor) $= 43\ \Omega$

$$z = \frac{x-\mu}{\sigma} = \frac{43-40}{2} = 1.5$$

$P(x > 43) = P(z > 1.5) = 1 - P(z \leq 1.5) = 1 - 0.9332$
$= 0.0668$

Required percentange $= 0.0668 \times 100 = 6.68$

Q11> Diameter limits is $x_1 = 2.99$ and $x_2 = 3.01$

No. of ball bearing having dimeter between x_1 & x_2 is

$P(2.99 < x < 3.01) =$

Now, ~~$z = \frac{3 - 2.99}{}$~~ $z = \frac{2.99 - 3}{0.005} = -2$

and $z = \frac{3.01 - 3}{0.005} = 2$

$P(2.99 < x < 3.01) = P(-2 < z < 2) = P(z < 2) - P(z < -2)$
$= 1 - P(z \geq 2) - P(z < -2)$
$= 1 - 0.0228 - 0.0228$
$= 0.9544$

Scrapped ball bearings $= 1 - 0.9544 = 0.0456$

Required percentage is $0.0456 \times 100 = \boxed{4.56\%}$

Q 12) $\mu = 40$, $\sigma = 6$ and $P(z < p) = 0.45$

From table it is found that $p = -0.13$

$$\therefore \quad p = \frac{x - \mu}{\sigma}$$

$$-0.13 = \frac{x - 40}{6}$$

$$\boxed{x = 39.22}$$

Q 13) T : Getting true $P(T) = \frac{1}{2}$

F : Getting False and $P(F) = \frac{1}{2}$

Probability of getting all correct questions $= \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

Q 14) T : Getting true , $P(T) = \frac{1}{2}$

F : Getting false $P(F) = \frac{1}{2}$

Probability of getting at least 7 out of 10 correct ~~$= {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7$~~

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$$

$$\boxed{= \frac{11}{64} \text{ Ans}}$$

Q 15) A.T.Q. Required probability $= \frac{{}^5C_2}{{}^{30}C_2} = \frac{\frac{5!}{3!\,2!}}{\frac{30!}{2!\,28!}} = \frac{10}{15 \times 29} = \boxed{\frac{2}{87}}$

Q 16) A.T.Q Required probability $= 1 - \frac{25}{30} \times \frac{24}{29} = \boxed{0.31}$

Q 17>

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

Since

$\Sigma P(x) = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$4k = 0.4$

$\boxed{k = 0.1}$

$\mu = \frac{-0.2 - k + 2k + 0.6 + 3k}{1} = \boxed{0.8}$

$\sigma^2 = \Sigma p_i x_i^2 - (\mu)^2$

$= 2.8 - 0.64$

$\boxed{= 2.16}$

Q 18> A.T.Q.,

$N = 3$

$p = \frac{2}{6} = \frac{1}{3}$

$q = \frac{2}{3}$

$\mu = \text{mean} = np = 3 \times \frac{1}{3} = 1$

$\sigma = npq = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3}$

Q 19> $\mu = \frac{8 \times \frac{1}{8} + 12 \times \frac{1}{6} + 16 \times \frac{3}{8} + 20 \times \frac{1}{4} + 24 \times \frac{1}{12}}{1}$

$= 16$

$\sigma^2 = 276 - 256 = 20$

$\sigma = \sqrt{20} = 4.47$

Q20) $f(x) = 6x(1-x)$, $0 \le x \le 1$

$\therefore \int_0^1 6x(1-x)\,dx = [3x^2 - 2x^3]_0^1$

$= 1$

Hence f is a probability density function

Now,

$\mu = \int_0^1 x f(x)\,dx$

$= \int_0^1 x(6x(1-x))\,dx$

$= \int_0^1 (6x^2 - 6x^3)\,dx$

$= \frac{1}{2}$

$\sigma^2 = \int_0^1 (x - \frac{1}{2})^2 [6x(1-x)]\,dx$

$= \int_0^1 (x^2 + \frac{1}{4} - x)[6x - 6x^2]\,dx$

$= \int_0^1 (6x^3 - 6x^4 + \frac{3}{2}x - \frac{3}{2}x^2 - 6x^2 + 6x^3)\,dx$

$= \frac{1}{20}$

Q21)

x :	0	1	2
P(x) :	$\frac{{}^{48}C_2}{{}^{52}C_2} = \frac{188}{221}$	$\frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{32}{221}$	$\frac{{}^4C_2}{{}^{52}C_2} = \frac{1}{221}$

Q22)

x:	0	1	2
P(x):	$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$	$3 \times \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{15}{28}$	$3 \times \frac{5}{8} \times \frac{2}{7} \times \frac{5}{6} = \frac{15}{56}$

Q23) E : Chance of suffering $P(E) = 0.20$

F : Chance of not suffering $P(F) = 0.80$

A. T. Q.

Required Probability $= {}^6C_4 (\frac{1}{5})^4 (\frac{4}{5})^2 + {}^6C_5 (\frac{1}{5})^5 (\frac{4}{5})^1 + {}^6C_6 (\frac{1}{5})^6 (\frac{4}{5})^0$

$= \frac{53}{3125}$ Ans

Q25 E : defective pen $P(E) = 0.1$

F : Non-defective pen $P(F) = 0.9$

i) Required probability $= {}^6C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 = 0.24576$

i) Required Probability $= {}^{12}C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{10}$

ii) Required Probability $= 1 - {}^{12}C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{12} - {}^{12}C_1\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{11}$

iii) Required Probability $= {}^{12}C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{12}$

Q24) E : Strike the target $P(E) = \frac{1}{5}$

F : didn't strike the target $P(F) = \frac{4}{5}$

i) Required probability $= {}^6C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 = 0.24576$

ii) Required Probability $= 1 - {}^6C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6 - {}^6C_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^5$

$= 0.34464$

Q26) $\mu = 400$, $\sigma = 80$

350 are desired to pass

$\therefore$ Area under curve $= \frac{350}{10000} = 0.035$

from table, $z = -1.81$

Now, $+1.81 = \frac{x - 400}{80}$

$\boxed{x = 544.8}$

Q27 a) Given Area $= 0.3770$

$\therefore z = \pm 1.16$

b) Area $= 0.8621$

$z = 1.09$

Q 28) $\mu = 1, \quad \sigma = 3$

a) Now, $z_1 = \frac{3.43-1}{3} = 0.81$

$z_2 = \frac{6.19-1}{3} = 1.73$

$P(0.81 < z < 1.73) = 0.4582 - 0.2910$

$= 0.1672$

b) Here,

$z_1 = -\frac{1.43-1}{3} = -0.81$

$z_2 = \frac{6.19-1}{3} = 1.73$

$P(-0.81 < z < 1.93) = 0.2910 + 0.4582$

$= 0.7492$

Now $\mu = 60, \sigma = 5, n = 60$

$z = \frac{n-\mu}{\sigma} = 0$

$\therefore$ percentage = 50%.

Q29) $\mu = 14, \quad \sigma = 2.5$

a) $z_1 = \frac{12-14}{2.5} = -0.8$

$z_2 = \frac{15-14}{2.5} = 0.4$

$P(-0.8 < z < 0.4) = 0.2881 + 0.1554$

≈ 0.444

$\therefore$ required no. of students $= 1000 \times 0.444 = 444$

b) $z = \frac{18-14}{2.5} = 1.6$

$P(z > 1.6) = 0.0543 \approx 0.055$

required students $= 1000 \times 0.055 = 55$

c) $z = \frac{8-14}{2.5} = -2.4$

$P(z < -2.4) = 0.008$

required students $= 1000 \times 0.008 = 8$

d> $z = \frac{16-14}{2.5} = 0.8$

$P(z = 0.8) = 0.116$

required students $= 0.116 \times 1000 = 116$

Q30> $\mu = 1.9$ gm , $\sigma^2 = 0.01$ gm $\Rightarrow \sigma = 0.1$

i> $z = \frac{2 - 1.9}{0.1} = 1$

$P(z > 1) = 0.5 - 0.3413 \approx 0.159$

Hence required enevelops $= 1000 \times 0.159 = 159$

ii> $z = \frac{2.1 - 1.9}{0.1} = 2$

$P(z > 2) = 0.5 - 0.4772 \approx 0.023$

required enevelops $= 1000 \times 0.023 = 23$

Q31> $\mu = 8$, $\sigma = 2$

$z = \frac{12-8}{2} = 2$

$P(z < 2) = 0.9772$

required no. of pairs $= 0.9772 \times 5000 = 4886$

Q32> $\mu = 151$, $\sigma = 15$, $z_1 = -2.06$, $z_2 = 0.266$

$P(-2.06 < z < 0.266) = 0.4803 + 0.1026$

$= 0.5829$

Hence $\Rightarrow 0.5829 \times 500 \approx 294$ (Ans)

Q33) $\mu = 40\%$, $\sigma = 10\%$

a) $z = \frac{50-40}{10} = 1$

$P(z>1) = 0.5 - 0.3413$

$= 0.1587$

$\Rightarrow 500 \times 0.1587 = 79.35 \approx 79$

b) Area under curve $= \frac{350}{500} = 0.7$

corresponding z value $= -0.5$

$-0.5 = \frac{x-40}{10}$

$x = 35\%$

c) $z = \frac{60-40}{10} = 2$

$P(z>2) = 0.5 - 0.4772 = 0.0228$

Now, $0.228 \times 500 = 11.4 \approx 11$

Q34) $\mu = 65$, $\sigma = 5$

Now, $np = 65$

$\sqrt{npq} = 5 \Rightarrow npq = 25$

$q = \frac{25}{65} = \frac{5}{13}$

$p = \frac{8}{13}$

Here,

p = got more than 70 = 0.1587

q = got less than 70 = 0.8413

$P(x=2) = {}^3C_2 \times (0.1587)^2 (0.8413)$

$= 0.06357$

Q35) $p = 1/2$, $q = 1/2$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$= \frac{11}{64}$

Q36> According to question →

P (Both Bulbs chosen are defective) $= \frac{5}{30} \times \frac{4}{29} = \boxed{\frac{2}{87}}$ Ans

Q37> E_1 = selecting urn I $\Rightarrow P(E_1) = \frac{1}{2}$

E_2 = selecting urn II $\Rightarrow P(E_2) = \frac{1}{2}$

P (getting white ball from urn I) $= P(A/E_1) = \frac{3}{7}$

P (getting white ball from urn II) $= P(A/E_1) = 5/11$

P (it was drawn from urn I) $= P(E_1/A) = \frac{1/2 \times 3/7}{1/2 \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}}$

$= \frac{33}{68}$ Ans

Q38> E_1 = selecting urn I , A = ball drawn is red.

E_2 = selecting urn II

E_3 = selecting urn III

Now, $P(A/E_1) = \frac{6}{10}$, $P(A/E_2) = \frac{4}{10}$, $P(A/E_3) = \frac{5}{10}$

According to questions, $P(E_1/A) = \frac{1/3 \times 6/10}{1/3 \times 6/10 + 1/3 \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10}}$

$= \frac{6}{15} = \frac{2}{5}$ Ans

Q39> P(A) = 0.25 , D = defective bolts

P (B) = 0.35

P (C) = 0.4

P(D/A) = 0.05

P(D/B) = 0.04

P (D/C) = 0.02

A.T.Q.,

$$P(B/D) = \frac{0.04 \times 0.35}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4}$$

$$= \frac{140}{125 + 140 + 80}$$

$$= \frac{28}{69}$$

Q 40) Required Probability $= \frac{5}{30} \times \frac{25}{29} + \frac{25}{30} \times \frac{5}{29} + \frac{5}{30} \times \frac{4}{29}$

$$= \frac{9}{29} \text{ Ans}$$

Q41) $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

Required Probability $= {}^{12}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$

$$= \frac{99}{128}$$