Radiation Heat Transfer

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Lecture 3 Blackbody, Planck distribution, Wien's displacement law, Stefan-Boltzmann law

Blackbody

Context To evaluate the emissive power, irradiation, radiosity and net radiative heat flux from a real opaque surface, it is useful to first introduce the concept of an ideal surface, known as blackbody, having the following properties.

1. *A blackbody absorbs all incident radiation, regardless of wavelength and direction.*
2. *For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.*
3. *Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a diffuse emitter.*

As the perfect absorber and emitter, the blackbody serves as a *standard* against which the radiative properties of actual surfaces may be compared.

The blackbody spectral intensity is well known, having first been determined by Planck

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_o^2}{\lambda^5[\exp(hc_o/\lambda k_B T) - 1]}$$

Max Planck

Max Planck, German theoretical physicist who originated quantum theory, which won him the Nobel Prize for Physics in 1918. Planck made many contributions to theoretical physics, but his fame...

where $h = 6.626 \times 10^{-34}$ J·s and $k_B = 1.381 \times 10^{-23}$ J/K are the universal Planck and Boltzmann constants, respectively, $c_o = 2.998 \times 10^8$ m/s is the speed of light in vacuum, and T is the *absolute* temperature of the blackbody (K). Since the blackbody is a diffuse emitter, its spectral emissive power is

The Planck Distribution

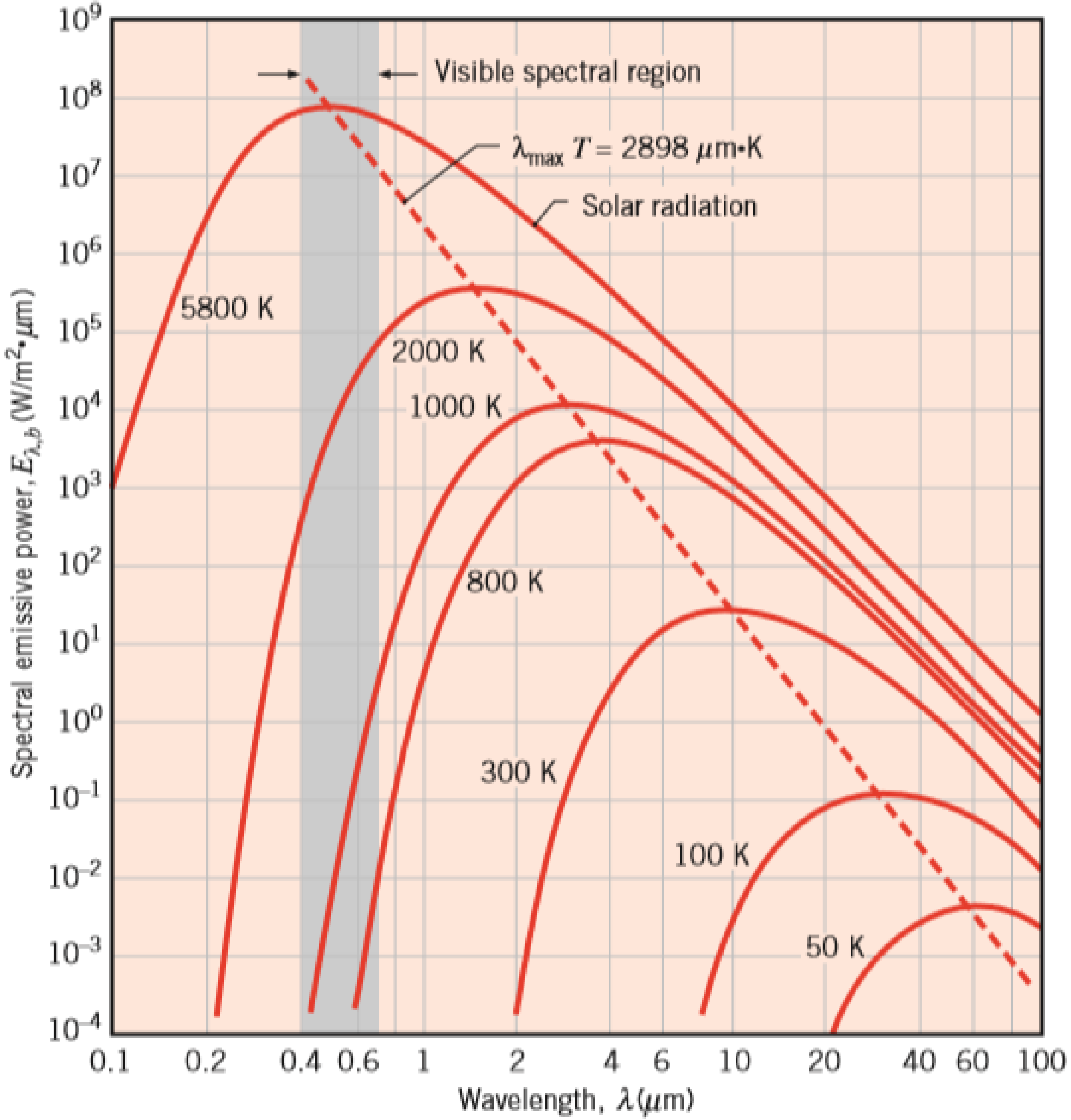
$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

where the first and second radiation constants are $C_1 = 2\pi h c_o^2 = 3.742 \times 10^8 \ \mathrm{W \cdot \mu m^4/m^2}$ and $C_2 = (hc_o/k_B) = 1.439 \times 10^4 \ \mu\mathrm{m \cdot K}$.

Physical Reflections

1. The emitted radiation varies *continuously* with wavelength.
2. At any wavelength the magnitude of the emitted radiation increases with increasing temperature.
3. The spectral region in which the radiation is concentrated depends on temperature, with *comparatively* more radiation appearing at shorter wavelengths as the temperature increases.
4. A significant fraction of the radiation emitted by the sun, which may be approximated as a blackbody at 5800 K, is in the visible region of the spectrum. In contrast, for $T \lesssim 800$ K, emission is predominantly in the infrared region of the spectrum and is not visible to the eye.

Spectral Blackbody Emissive Power



Wien's Displacement Law

From previous figure we see that the blackbody spectral distribution has a maximum and that the corresponding wavelength λ_{max} depends on temperature. The nature of this dependence may be obtained by differentiating Planck distribution with respect to λ and setting the result equal to zero. In so doing, we obtain

$$\boxed{\lambda_{max} T = C_3}$$

Wien's Displacement Law

HT: Derive

where the third radiation constant is $C_3 = 2898\ \mu m \cdot K$.

the locus of points described by the law is plotted as the dashed line

Physical Reflections

According to this result, the maximum spectral emissive power is displaced to shorter wavelengths with increasing temperature.

Significance of Wein's Displacement Law : the maximum spectral emissive power emission is in the middle of the visible spectrum ($\lambda \approx 0.50\ \mu m$) for solar radiation, since the sun emits approximately as a blackbody at 5800 K. For a blackbody at 1000 K, peak emission occurs at 2.90 μm, with some of the emitted radiation appearing visible as red light. With increasing temperature, shorter wavelengths become more prominent, until eventually significant emission occurs over the entire visible spectrum. For example, a tungsten filament lamp operating at 2900 K ($\lambda_{max} = 1\ \mu m$) emits white light, although most of the emission remains in the IR region.

The Stefan–Boltzmann Law

$$E = \int_0^\infty E_\lambda(\lambda)\, d\lambda$$

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

the total emissive power of a blackbody E_b may be expressed as

$$E_b = \int_0^\infty \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}\, d\lambda$$

HT: Derive the Stefan-Boltzmann Law

Performing the integration, it may be shown that

$$\boxed{E_b = \sigma T^4}$$

The Stefan–Boltzmann Law

where the *Stefan–Boltzmann* constant, which depends on C_1 and C_2, has the numerical value

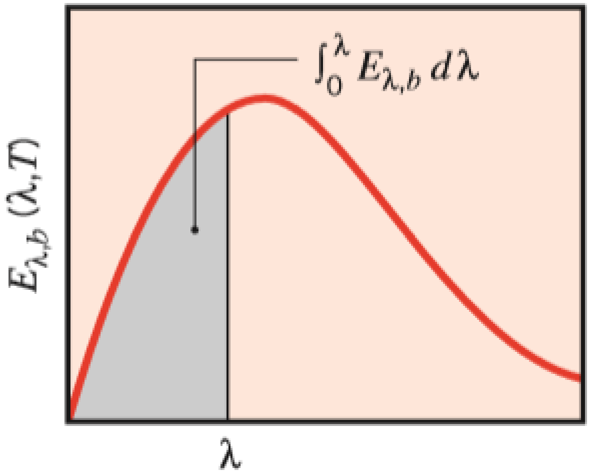
$$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$$

Physical Significance : the *Stefan–Boltzmann law* enables calculation of the amount of radiation emitted in all directions and over all wavelengths simply from knowledge of the temperature of the blackbody.

$$I_b = \frac{E_b}{\pi}$$

Because this emission is diffuse, the total intensity associated with blackbody emission is

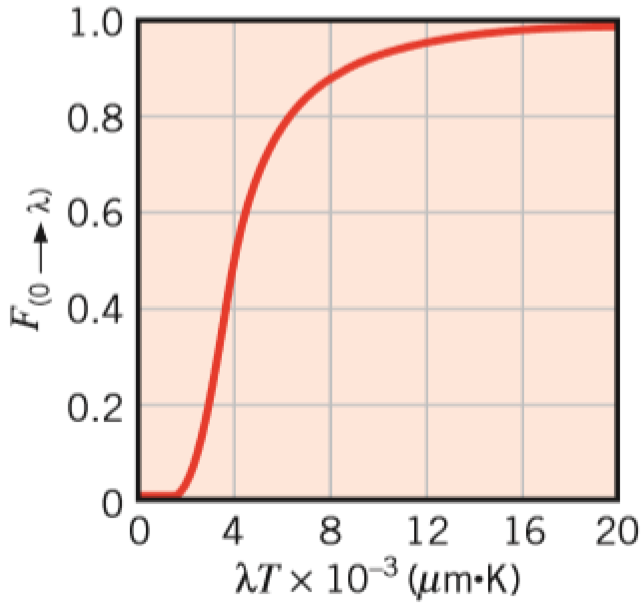
Band Emission



emission from a blackbody in the spectral band 0 to λ.

HT: Derive

$$F_{(0\to\lambda)} \equiv \frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\int_0^\infty E_{\lambda,b}\,d\lambda} = \boxed{\frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5}\,d(\lambda T) = f(\lambda T)}$$



Fraction of the total blackbody emission in the spectral band from 0 to λ as a function of λT.

- The integration shown in the previous slide cannot be solved analytically and has been done numerically. The results are presented in a Table given in the next slide, where the fraction ranging from 0 to 1 is tabulated as a function of the parameter λT. Note the advantage of introducing the combination parameter λT. The integration need not to be done for different values of λ and different values of T. Thus effort involved in doing the integration is reduced and the results are also presented in a compact form
- The third column of the table facilitates calculation of spectral intensity for a prescribed wavelength and temperature.

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_o^2}{\lambda^5[\exp(hc_o/\lambda\kappa T)-1]}$$

Units: W/m^2-μm-sr

Hence,

$$\boxed{\frac{I_{\lambda,b}(\lambda,T)}{\sigma T^5} = \frac{2hc_o^2}{(\lambda T)^5[\exp(hc_o/\kappa\lambda T)-1]}}$$

- The fourth column is used to obtain a quick estimate of the ratio of spectral intensity at any wavelength to that at λ_{max}.

TABLE Blackbody Radiation Functions

λT $(\mu m \cdot K)$	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291

= 0.24913 / 0.722318

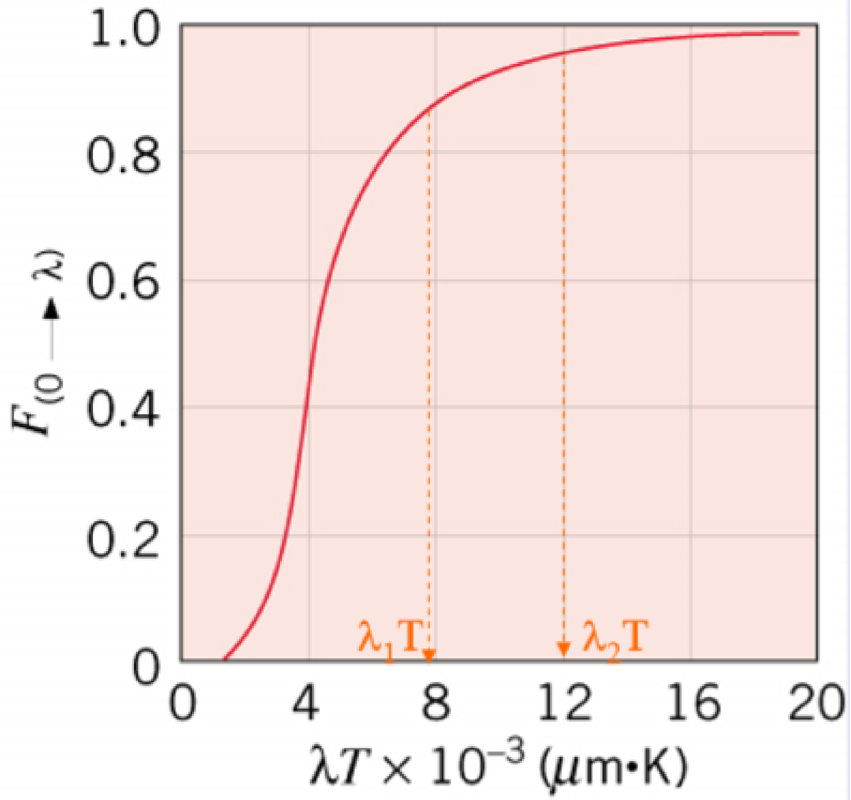
Maximum $I_{\lambda,\beta}$

- The fraction of total blackbody emission that is in a prescribed wavelength interval or band $(\lambda_1 < \lambda < \lambda_2)$ is

$$F_{(\lambda_1-\lambda_2)} = F_{(0-\lambda_2)} - F_{(0-\lambda_1)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}\,d\lambda - \int_o^{\lambda_1} E_{\lambda,b}\,d\lambda}{\sigma T^4}$$

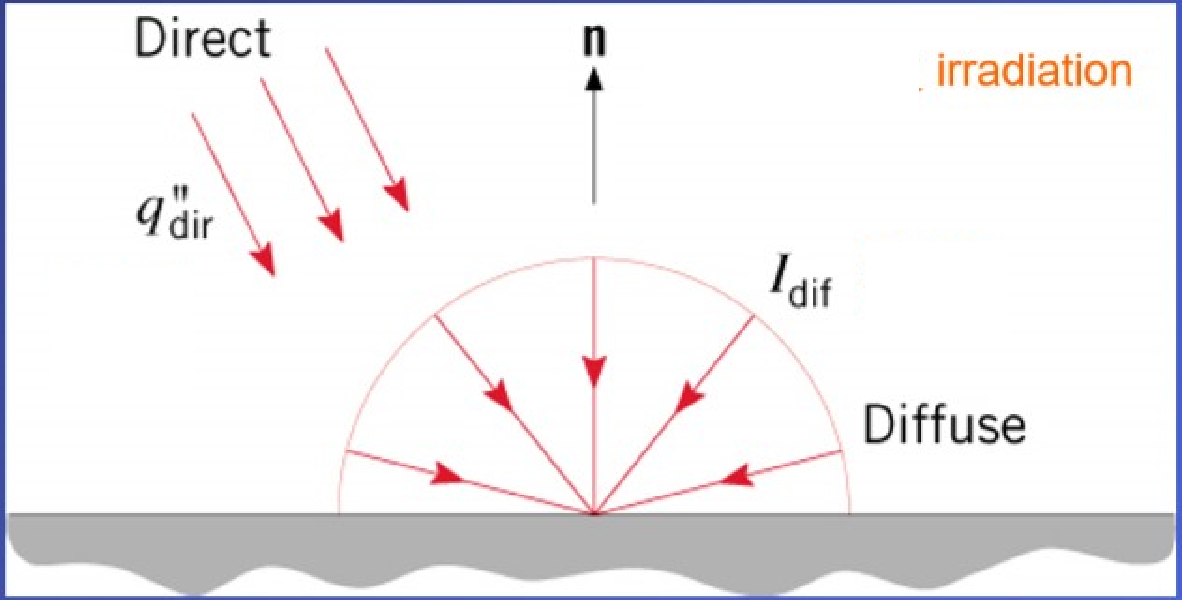
where, in general,

$$\boxed{F_{(0-\lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b}\,d\lambda}{\sigma T^4} = f(\lambda T)}$$



Numerical Exp 1

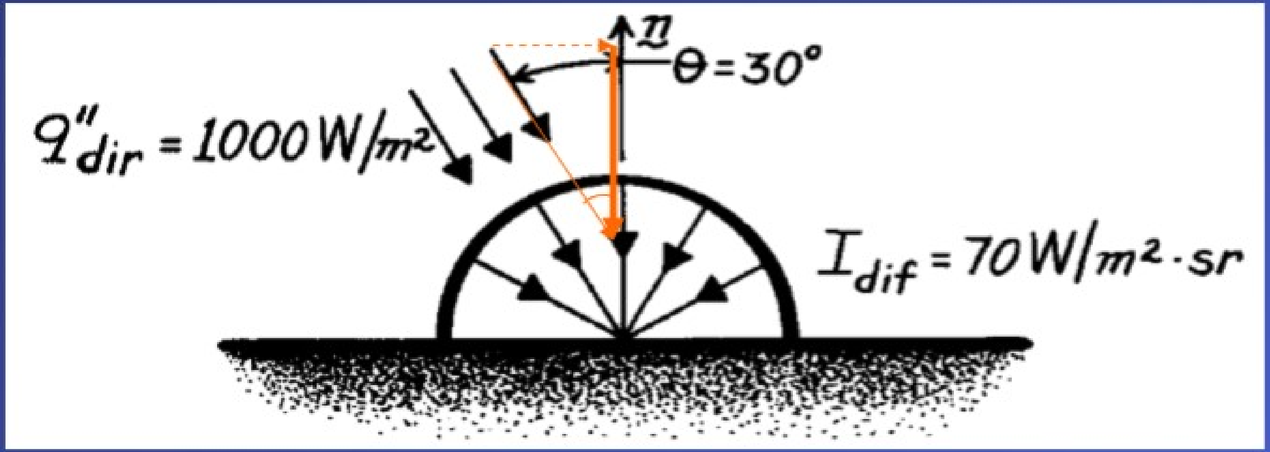
Evaluation of total solar irradiation at the earth's surface from knowledge of the direct and diffuse components of the incident radiation.



KNOWN: Flux and intensity of direct and diffuse components, respectively, of solar irradiation.

FIND: Total irradiation.

SCHEMATIC:



ANALYSIS: Since the irradiation is based on the actual surface area, the contribution due to the direct solar radiation is

$$\boxed{G_{dir} = q''_{dir} \cdot \cos\theta.}$$

For the contribution due to the diffuse radiation

$$\boxed{G_{dif} = \pi I_{dif}.}$$

Hence

$$\underline{G = G_{dir} + G_{dif}} = q''_{dir} \cdot \cos\theta + \pi I_{dif}$$

or

$$G = 1000\ \mathrm{W/m^2} \times 0.866\ +\ \pi \mathrm{sr} \times 70\ \mathrm{W/m^2 \cdot sr}$$

$$G = (866 + 220)\ \mathrm{W/m^2}$$

$$\boxed{G = 1086\ \mathrm{W/m^2}}$$

COMMENTS: Although a diffuse approximation is often made for the non-direct component of solar radiation, the actual directional distribution deviates from this condition, providing larger intensities at angles close to the direct beam.

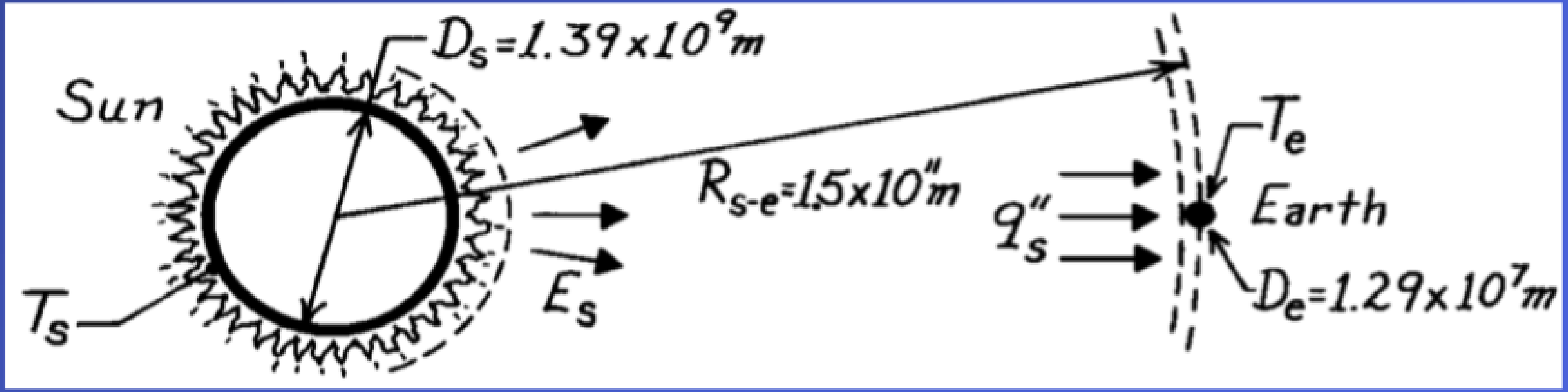
Numerical Exp 2

Determination of the sun's emissive power, temperature and wavelength of maximum emission, as well as the earth's temperature, from knowledge of the sun/earth geometry and the solar flux at the outer edge of the earth's atmosphere.

KNOWN: Solar flux at outer edge of earth's atmosphere, 1353 W/m^2.

FIND: (a) Emissive power of sun, (b) Surface temperature of sun, (c) Wavelength of maximum solar emission, (d) Earth equilibrium temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar radiation enroute to earth, (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: (a) Applying conservation of energy to the solar energy crossing two concentric spheres, one having the radius of the sun and the other having the radial distance from the edge of the earth's atmosphere to the center of the sun, it follows that

$$\boxed{E_S\left(\pi D_S^2\right) = 4\pi\left(R_{S-e} - \frac{D_e}{2}\right)^2 q_S''}$$

Hence

$$E_S = \frac{4\left(1.5\times10^{11}\text{ m} - 0.65\times10^7\text{ m}\right)^2 \times 1353\text{ W/m}^2}{\left(1.39\times10^9\text{ m}\right)^2} = 6.302\times10^7\text{ W/m}^2.$$

(b) From the Stefan-Boltzmann law, the temperature of the sun is

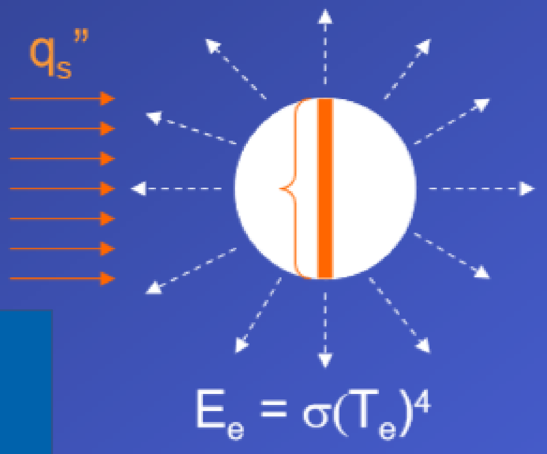
$$T_S = \left(\frac{E_S}{\sigma}\right)^{1/4} = \left(\frac{6.302\times10^7\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right)^{1/4} = 5774\text{ K}.$$

(c) From Wien's displacement law, the wavelength of maximum emission is

$$\lambda_{\max} = \frac{C_3}{T} = \frac{2898\ \mu\text{m}\cdot\text{K}}{5774\text{ K}} = 0.50\ \mu\text{m}.$$

(d) From an energy balance on the earth's surface

$$E_e\left(\pi D_e^2\right) = q_S''\left(\pi D_e^2/4\right).$$



Hence, from the Stefan-Boltzmann law,

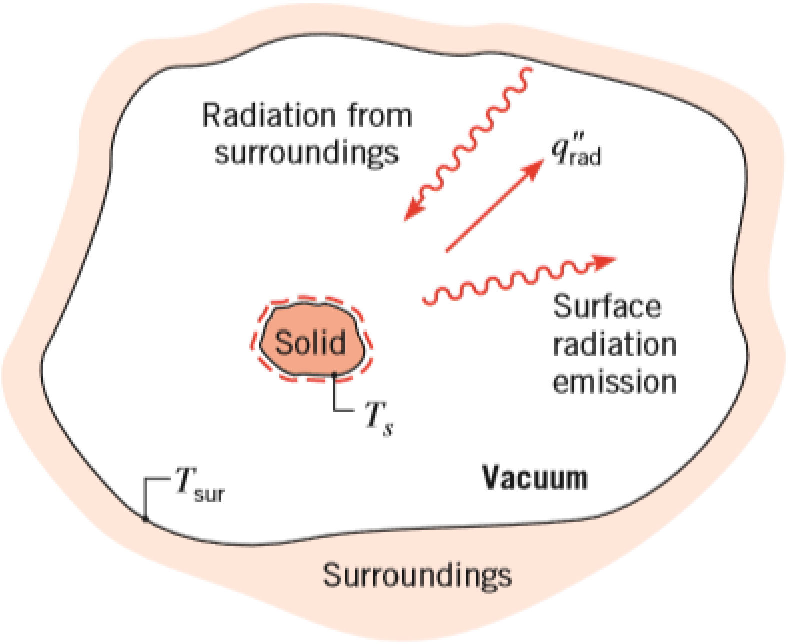
$$T_e = \left(\frac{q_S''}{4\sigma}\right)^{1/4} = \left(\frac{1353\ \mathrm{W/m^2}}{4\times 5.67\times 10^{-8}\ \mathrm{W/m^2\cdot K^4}}\right)^{1/4} = 278\ \mathrm{K}.$$

COMMENTS:

The average earth temperature is higher than 278 K due to the shielding effect of the earth's atmosphere (transparent to solar radiation but not to longer wavelength earth emission).

Problems

Q1. Determine an expression for the net radiative heat flux at the surface of the small solid object of the figure in terms of the surface and surroundings temperatures and the Stefan–Boltzmann constant. The small object is a blackbody.



Q2. Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000 K. Calculate the emissive power of the radiation that emerges from a small aperture on the enclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated? What is the wavelength λ_2 above which 10% of the emission is concentrated? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure?

Q3. A surface emits as a blackbody at 1500 K. What is the rate per unit area (W/m^2) at which it emits radiation over all directions corresponding to $0° \leq \theta \leq 60°$ and over the wavelength interval $2\ \mu m \leq \lambda \leq 4\ \mu m$?

Thank you