AKM
Effect of type of load - page 1

Load factor: (C_L: strength reduction factor for nature of load)

Type of load	C_L
Rotating Bending	1.0
Torsion	0.58
Axial	1.0
Reversed Bending	1.0

The failure in case of torsional fatigue is due to shear endurance limit which is $0.58\,\sigma_e$. S is replaced torsional stress amplitude. The factor 0.58 is take based on maximum distortion energy theory of failure.

Gradient factor: (C_g: strength reduction factor for gradient of stress distribution)

The gradient of stress distribution significantly influence the ~~rusmer~~ number of expected critical point of failure.

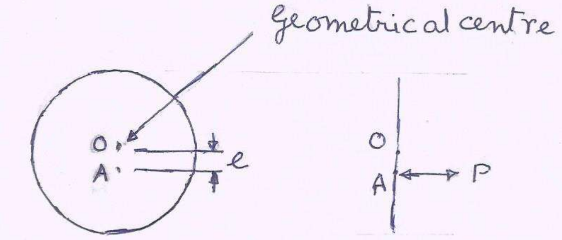
Type of load	C_g
Rotating Bending	1.0
Torsion	1.0
Axial	0.7 to 0.9
Reversed Bending	1.0

The stress distribution pattern over cross-section for rotating bending and torsional fatigue are same. For reversed bending, number of critical point is much less. So for reversed bending, C_g is more than 1.0. But it is taken as 1.0.

For axial push-pull type of fatigue loading, all points on the cross-section are equally stressed and are equally critical. So ~~likelyb~~ likelihood of failure increases statistically and C_g is low.

If in case of axial fatigue, the component is not accurately measured and manufactured, the ~~like~~ line of action for the resultant axial force does not passes through geometrical centre. The stress distribution pattern is resultant effect of axial force P through geometrical centre and moment Pe, where e is

eccentricity of load from geometrical centre. In this case, ~~cg = 0.7~~-
$c_g = 0.7$ to 0.85. It is not possible to measure the value of e. So in this situation, value of c_g is estimated on the basis of experience



Some designer combine C_L and c_g together to form a single factor C_L. Here effect c_g is included into it.

$$S_e' = C_L c_g S_e \quad \text{and} \quad S_{10^3}' = C_L c_g S_{10^3}$$

Note: For torsional fatigue load, $C_L = 0.58$.

$S_e' = 0.58 S_e = \tau_e$ $\quad$ τ_e sometimes written as S_{es} also.

Other effect (C_0 - Strength reduction factor for other effect)

There are various other effects which reduces fatigue strength like plating of metal, forging process, hot-rolling process or, some heat treatment processes. Exposition of the surface of steel to air at high temperature causes decarburization of metal surface. This changes the chemical composition of metal. This decarburization is local change but it reduces fatigue strength because it develops residual tensile stress.

But case cardening induces residual compressive stress and increase fatigue strength.

Temperature over $70°$ is observed to reduce fatigue life. Various corrosion processes are found to reduce fatigue strength and thereby reduces fatigue life.

The effects are observed but it is very difficult to estimate through analysis. So C_0, which represents the effects of various such factor, are decided on the basis of experience. It is observe that these effects are insignificant for S_{10^3} but the effects can not be neglected for S_{10^6} i.e. S_e.

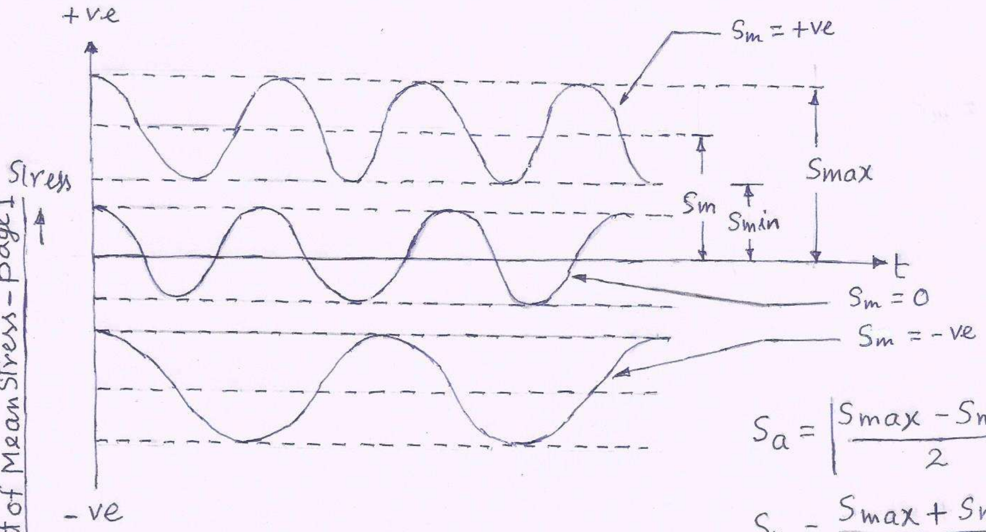
$S_e' = C_0 S_e'$

Overall

$S_{10^3}' = C_R C_L S_{10^3}$ $\qquad$ $S_{10^6} = S_e$

$S_{10^6}' = C_R C_S C_F C_c C_L C_g C_0 S_{10^6}$ $\quad$ & $S_{10^6}' = S_e'$

Effect of Mean Stress (C_M - Strength Reduction Factor for mean stress)

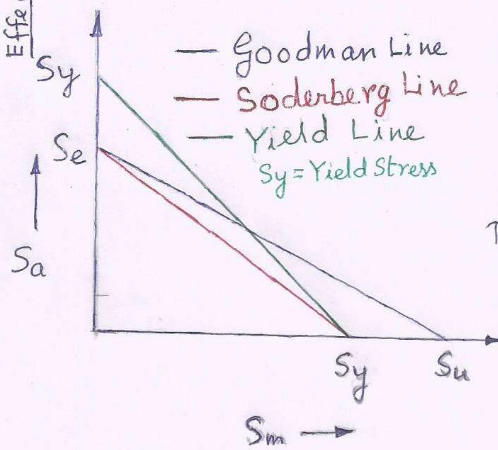


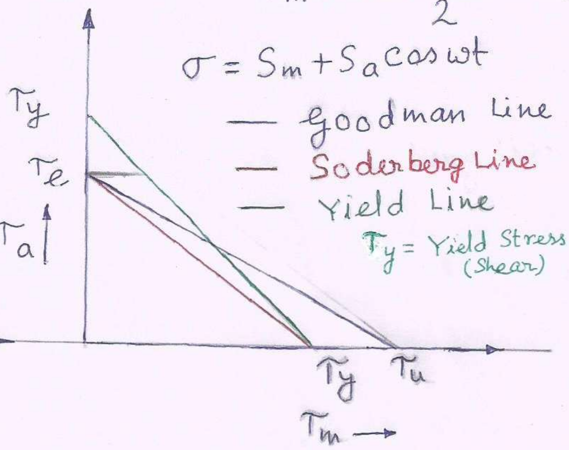
AKM
Effect of Mean Stress - page 1

$$S_a = \left| \frac{S_{max} - S_{min}}{2} \right|$$

$$S_m = \frac{S_{max} + S_{min}}{2} = S_{av}$$

$$\sigma = S_m + S_a \cos \omega t$$





Soderberg Equation:
(Straight line)

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yield}} = 1$$

$$S_a = \left(1 - \frac{S_m}{S_{yield}}\right) S_e$$

$$= S_M S_e$$

If we apply factor of safety F.S. to S_e and S_{yield}

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yield}} = \frac{1}{F.S.}$$

Goodman Equation:
(Straight line)

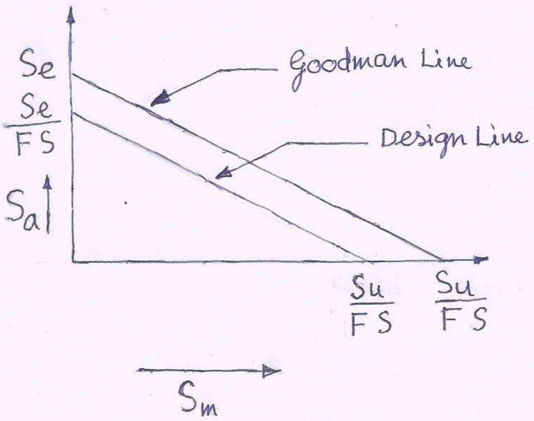
$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

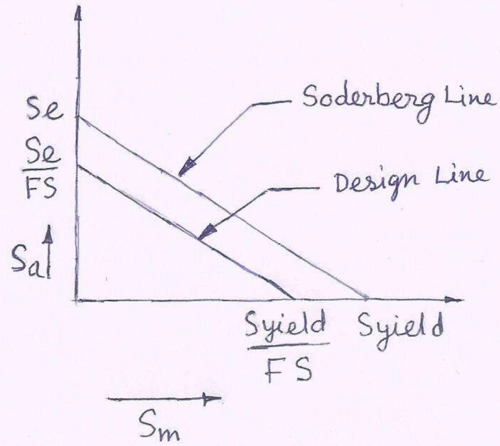
$$S_a = \left(1 - \frac{S_m}{S_u}\right) S_u$$

$$S_a = S_M S_u$$

If we apply factor of safety F.S. to S_e and S_{yield},

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = \frac{1}{F.S.}$$





$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

$$\frac{S_a}{\left(\frac{S_e}{FS}\right)} + \frac{S_m}{\left(\frac{S_u}{FS}\right)} = 1$$

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = \frac{1}{FS}$$

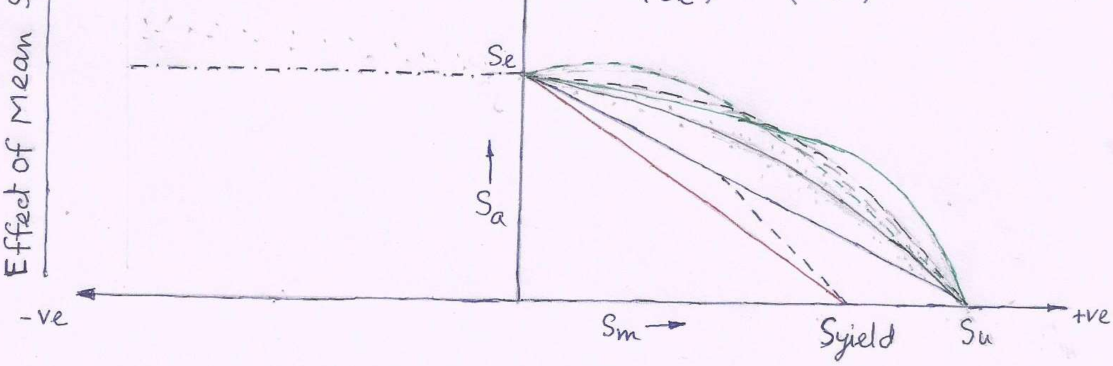
$$\frac{S_a}{S_e} + \frac{S_m}{S_{yield}} = 1$$

$$\frac{S_a}{(S_e/FS)} + \frac{S_m}{\frac{S_{yield}}{FS}} = 1$$

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yield}} = \frac{1}{FS}$$

AKM
Effect of Mean stress - page 2

$$\left(\frac{S_a}{S_e}\right)^m + \left(\frac{K S_m}{S_u}\right)^P = 1$$



Failure Line/curve	K	m	P
Soderberg	Su/Syield	1	1
Goodman	1	1	1
Gerber	1	1	2
Marin	1	2	2
Kececioglu	1	a	2
Bachi	Su/Syield	1	4
Modified Goodman			

$$\frac{S_a}{S_N} + \frac{S_m}{S_{yield}} = 1 \; ; \Rightarrow S_a = \left(1 - \frac{S_m}{S_{yield}}\right) S_N = C_M S_N$$

using factor of safety FS,

$$\frac{S_a}{\left(\frac{S_N}{FS}\right)} + \frac{S_m}{\left(\frac{S_{yield}}{FS}\right)} = 1 \; ; \quad \frac{S_a}{[S_N]} + \frac{S_m}{[S_{yield}]} = 1 \; ; \; S_a = \left(1 - \frac{S_m}{[S_{yield}]}\right)[S_N]$$

$$S_a = \left(\frac{1}{FS} - \frac{S_m}{S_{yield}}\right) S_N \qquad \text{Here } [S_N] = \frac{S_N}{FS} \text{ and } [S_{yield}] = \frac{S_{yield}}{FS}$$

If $N = 10^3$, $S_N = S_{10^3}$

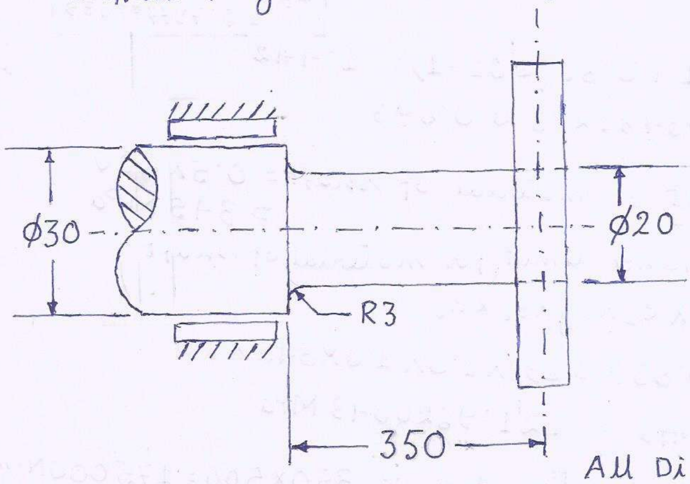
If $N = 10^6$, $S_N = S_{10^6} = S_e$

Note: S_{10^3} and S_{10^6} are referred as R.R. Moore's S_{10^3} and R.R. Moore's S_{10^6} respectively.

S'_{10^3} and S'_{10^6} are referred as corrected values for S_{10^3} and S_{10^6} ~~respect~~ respectively. The values for S'_{10^3} and S'_{10^6} can be obtained from S_{10^3} and S_{10^6} respectively as mentioned earlier for any value of N.

From S'_{10^3} and S'_{10^6}, S'_N is obtained from the formula derived earlier. Thereafter S_a is obtained from S'_N as discussed above.

It is decided to attach an overhung flywheel of weight 500N to a ~~a~~ machine as shown below. Check the design for infinite fatigue life and suggest any modification, if necessary.



Material of shaft

$\sigma_{ultimate} = 690$ MPa

$\sigma_{yield} = 580$ MPa

surface is cold drawn and machined.

99.9% Reliability may be assumed.

Fillet radius = 3 mm

Overhung length = 350 mm

All Dimensions are in mm.

Fatigue Design Problem – 1

AKM

Symbols used

C_R – Strength reduction factor for reliability

C_F – Strength reduction factor for surface finish

C_S – Strength reduction ~~for size~~ factor for size

C_e – Strength reduction factor for stress concentration

C_L – Strength reduction factor for load

C_g – Stress reduction factor for stress gradient

q – Notch Sensitivity

K_{th} – Theoretical stress concentration factor.

C_o – Strength reduction for several other miscellaneous effects.

~~Table A~~

~~Rotating Bending Reversed Bend~~

Table A

	Rotating Bending	Reversed Bending	Axial	Torsion
C_L	1.0	1.0	1.0	0.58
C_g	1.0	1.0	0.7-0.9	1.0

Reference ~~from~~ of graphs and tables from text book:

C_R – Table 5.3 K_{th} – Fig. 5.5. C_L — Table A (given above)

C_S – Table 5.2 q – Fig. 5.22 C_g — Table A (given above)

C_F – Fig. 5.24 and Table 5.1

Answer: (Use of references are made to get the values)

$C_R = 0.753$; ~~$C_F = 0.789$~~ $C_F = 0.797$; $C_S = 0.85$; $C_L = 1.0$

$C_g = 1.0$; $C_O = 1.0$; $K_{th} = 1.52$; $q = 0.85$

$\boxed{C_F \text{ calculation: } C_F = 4.51(690)^{-0.265} = 0.7977770394}$

Calculation of C_C:

$K_f = 1 + q(K_{th} - 1) = 1 + 0.85(1.52 - 1) = 1.442$

$C_C = \frac{1}{K_f} = \frac{1}{1.442} = 0.693481276 \approx 0.693$

S_e = Endurance limit of material of shaft $= 0.5 \times 690 = 345$ MPa

S_e' = Modified endurance limit for material of shaft

$= C_R \times C_F \times C_S \times C_C \times C_L \times C_g \times S_e \times C_o$

$= 0.753 \times 0.797 \times 0.85 \times 0.693 \times 1.0 \times 1.0 \times 345 \times 1.0$

~~= 120.8308199 MPa~~ $= 121.9620043$ MPa.

~~Ben~~ Bending moment at section A-A $= 350 \times 500 = 175000$ N-mm

σ_b = Bending stress $= \frac{32 \times 175000}{\pi \times 20^3} = \frac{5600000}{\pi \times 20^3} = 222.8169203$ MPa

~~$S_e' < \sigma$~~ $S_e' < \sigma_b$ – Infinite fatigue life is not possible.

Suggestion: leverage arm may be reduced. Let leverage arm for bending moment be x. Overhung length is leverge arm for BM.

$BM = 500 \times x = 500x$; $\sigma_b = \frac{32 \times 500x}{\pi \times 20^3}$

$\sigma_b \leq S_e'$; $\frac{32 \times 500x}{\pi \times 20^3} \leq 121.9620043$; $x \leq 191.5774684$ mm

we take $x = 190$ mm.

Suggestion: the learage arm may be reduced to 190 mm from 350 mm for infinite fatigue life.

Derivation: $K_f = 1 + q(K_{th} - 1)$

$K_{eff} = 1.0$ for ductile material under static load (due to local yielding)

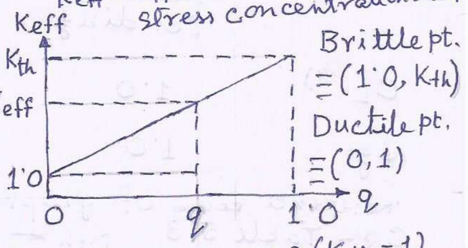
$K_{eff} = K_{th}$ for brittle material under static load.

K_{eff} = Effective value for stress concentration factor

Notch sensitivety indicates the tendency of ductile material in close neighbourhood of notch to behave towards brittle material

For ductile material, $q = 0$

~~$q = 0$~~ For brittle material, $q = 1$



~~For~~ From figure, $\frac{K_{eff} - 1}{K_{th} - 1} = \frac{q}{1}$, $K_{eff} - 1 = q(K_{th} - 1)$; $K_{eff} = 1 + q(K_{th} - 1)$

$K_{eff} = K_f$ = Fatigue stress concentration Factor

Fatigue Design Problem - 2

AKM