1) Given, $\sigma_x = 60 MPa$, $\sigma_y = -10 MPa$, $\tau_{xy} = 40 MPa$,

$\sigma_z - \tau_{yz} = \tau_{zx} = 0$

Properties of cast iron : $\sigma_{ut} = 70 MPa$, $\sigma_{uc} = -140 MPa$

To determine the factor of safety

According to maximum principle stress theory

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{60-10}{2} + \sqrt{\left(\frac{60+10}{2}\right)^2 + (40)^2}$$

$$= 25 + 53.15$$

$$= 78.15 \ MPa$$

Now, $FOS = \cancel{\frac{\text{Failure stress}}{\text{allowable stress}}}$ ~~allo~~ $\frac{\text{Failure stress}}{\text{allowable stress}}$

$$= \frac{70}{78.15}$$

$$= 0.988 \ (\text{tensile})$$

$$FOS_{(compression)} = \frac{-140}{78.15}$$

$$= 1.97$$

∴ We should take FOS = 2.

2) Given - ductile material

$\sigma_x = -90 MPa$ $\quad \sigma_y = 270 MPa$ $\quad \tau_{xy} = 240 MPa$

$\sigma_z = \tau_{yz} = \tau_{zx} = 0$

$\sigma_{yield} = \pm 460 MPa$ $\quad \sigma_{ut} = 840 MPa$

- Max shear stress theorem
- Distortion energy theory

# Machine Design

2) $\sigma_1 = \frac{-90+270}{2} + \sqrt{\left(\frac{-90 \mp 270}{2}\right)^2 + \tau_{xy}^2}$

$= 90 + \sqrt{\cancel{8100} + 57600}$ $(180)^2$

$= \cancel{90 + 256.32}$ 390

$= \cancel{346}$

~~$\sigma_2 = -166.32$~~ $\sigma_2 = -210$

$\therefore F_s = \frac{\sigma_{yield}}{\sigma_1} = \frac{460}{390} = 1.17 > 1$

$\sigma_3 = 0$

$\tau_{max} = \frac{\tau_{yield}}{FOS}$

$\frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_{yield}}{2 \cdot FOS}$

$\therefore \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{yield}}{2\,FOS}$

$FOS = 1.179$

also by Rankine theory, $FOS = \frac{\sigma_{ut}}{\sigma_1} = \frac{840}{390} = 2.25$

$\therefore$ Dimension will be such that FOS will lie between 1.179 and 2.15

3) Given, $\sigma_x = 50$ MPa $\sigma_y = 0$

$\tau_{xy} = 100$ MPa

$\sigma_{yield} = \pm 500$ MPa , $\sigma_{ut} = 925$ MPa

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \cancel{128.1128}\ 128.0778$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= -78.0776$$

$$\therefore FOS = \frac{\sigma_{yield}}{\sigma_1} = 3.90$$

$$\frac{\sigma_{yield}}{\sigma_2} = 6.40$$

$\therefore$ FOS will be in 3.90 to 6.40

4)

$\sigma_1 = 200\ MPa$

$\sigma_2 = 100\ MPa$, $\sigma_3 = 0$

To find the tensile yield strength to get an FOS of ~~2~~

2 $\therefore$

$$\tau_{max} = \max\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right\}$$

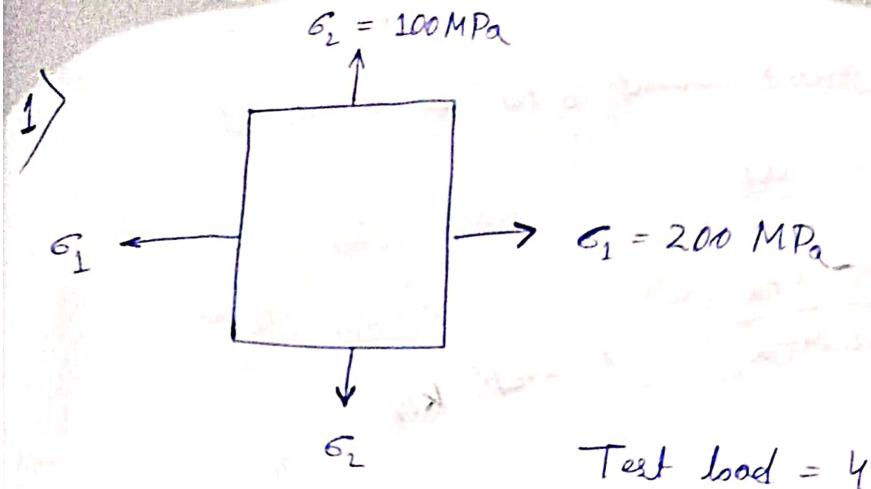
$$= 100 \leq \frac{\sigma_{yield}}{2 \cdot FOS}$$

$$\therefore 200 \leq \frac{\sigma_{yield}}{2}$$

$$\therefore \sigma_{yield} \geq 400\ MPa$$

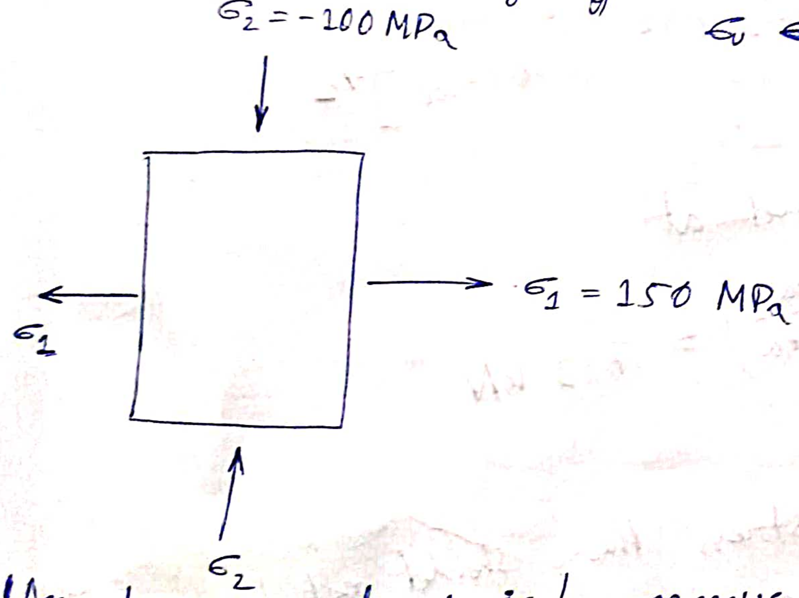
# Part II

1)




Test load = 4 kN

$\sigma_y = 400\,MPa$
$\tau_y = 250\,MPa$ } For steel
$\sigma_u = \pm 780\,MPa$




Assumption: The material is homogeneous

Analysis

1) For the maximum-normal-stress theory, the $\sigma_1 - \sigma_2$ plot shows point a to be critical. Failure is predicted at

$$\text{Load} = 4\,kN \left( \frac{400\,MPa}{200\,MPa} \right) = 8\,kN$$

2) For maximum-shear-stress theory, the $\sigma_1 - \sigma_2$ plot shows point b to be critical. Failure is predicted at

$$\text{Load} = 4\,kN \left( \frac{240\,MPa}{150\,MPa} \right) = 6.4\,kN$$

3) ~~I~~ For maximum-distortion-energy theory, the $\sigma_1 - \sigma_2$ plot shows point b to be critical. Failure is predicted at

$$\text{Load} = 4\,kN\left(\frac{275\ MPa}{150\ MPa}\right) = 7.3\ kN$$

More precisely from

$$\sigma_e = [(250)^2 + (-100)^2 - (250)(-100)]^{1/2}$$
$$= 218\ MPa$$

Thus, failure is predicted at

$$\text{Load} = 4\,kN\left(\frac{400\ MPa}{218\ MPa}\right) = 7.3\ kN$$

Comment

1) Maximum normal stress theory should not be used for this application since it gives good results only for brittle fractures.

2) Maximum shear stress theory may be used but it is not very accurate.

3) Maximum distortion energy theory will ~~give~~ give the best results for this application.

4) yelding is ~~expecte~~ ~~ex~~ expected to being at a load of 7.3 kN.

4) $\sigma = 70MPa = P/A$ $\quad \sigma_{yt} = \cancel{\pm 275\ MPa}$ $\pm 450\ MPa$

$T/J = \tau = 200\ MPa$

$Mc/I = \sigma_x = 300\ MPa$

I) Maximum Principle Stress theory (Normal stress Theory)

Maximum principle stress $= \frac{\sigma + \sigma_w}{2} + \frac{1}{2}\left[\sqrt{(\sigma+\sigma_w)^2 + 4\tau^2}\right]$

$= \frac{300 + 70}{2} + \frac{1}{2}\left[\sqrt{(370)^2 + 4(200)^2}\right]$

$= 457.443$ MPa

Minimum principle stress $(\sigma_{MPS}) \Rightarrow \frac{\sigma + \sigma_w}{2} - \frac{1}{2}\sqrt{(\sigma+\sigma_w)^2 + 4\tau^2}$

$= -87.443$ MPa

Factor of Safety (FS) $= \frac{\sigma_{yt}}{\sigma_{MPS}} = \frac{450}{457.443} = 0.9$

II) Maximum shear stress theory

Max shear stress $= \tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[457.443 - (-87.443)\right]$

$\tau_{max} = 272.443$ MPa

FOS $= \frac{\sigma_{yt}}{2\tau_{max}} = \frac{450}{2(272.443)} = 0.9$

III) Maximum Distortion Energy Theory

$\sigma_t^2 = (\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1}\sigma_{t2}$

$= (457.443)^2 + (-87.443)^2 - 2(457.443)(-87.443)$

$\therefore \sigma_{td} = \sqrt{\quad} = 544.886$

FOS $= \frac{\sigma_{yt}}{\sigma_{td}} = \frac{400}{544} = 0.9$

# Part 3

1) To determine the inside diameter and outside diameter of a hollow shaft which will replace a solid shaft made of the same material. The hollow shaft should be equally strong in torsion yet weigh half as much per meter ~~to~~ ~~b~~ length.

Ans) We know, $\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$

Solid shaft $J = \frac{\pi D^4}{32}$

Hollow shaft $J = \frac{\pi(D_o^4 - D_i^4)}{32}$

$\tau$ = ~~Shear~~ Shear stress induced due to Torsion, T

G = Modulus of Rigidity

$\theta$ = Angular deflection of the shaft

R, L = Shaft radius and length respectively

$D_s$ = Diameter of the solid shaft

$D_o$ = outer diameter of Hollow shaft

$D_i$ = Inner Diameter of Hollow shaft

## Calculation

Given

$\tau_{Hollow} = \tau_{solid}$

Volume Hollow = $\frac{1}{2}$ Volume of Solid

$$\Rightarrow \frac{\pi}{4} D_o^2 l - \frac{\pi}{4} D_i^2 l = \frac{1}{2}\left(\frac{\pi}{4} D_s^2 l\right)$$

$$\Rightarrow D_o^2 - D_i^2 = \frac{1}{2} D_s^2$$

To find $D_o$ and $D_i$:

~~Now, $\left(\frac{T}{J}\right)_{hollow} = \left(\frac{T}{J}\right)_{solid}$~~

Now, $\frac{T}{J} = \frac{G\theta}{L}$

$\therefore T \propto J$ ($\theta$, G, L Same for both or constant

$$\frac{T_H}{T_s} = \frac{J_H}{J_s} = \frac{\frac{\pi}{32}(D_o^4 - D_i^4)}{\frac{\pi}{32}(D_s)^4}$$

$$= \frac{(D_o^2 + D_i^2)\cancel{(D_o^2 - D_i^2)}}{(2)^4 (D_o^2 - D_i^2)^{\cancel{2}}}$$

$$\Rightarrow \frac{D_o^2 + D_i^2}{D_o^2 - D_i^2} = 1 \cdot (2)^4$$

$$\Rightarrow D_o^2 + D_i^2 = \cancel{10}\ 16\ (D_o^2 - D_i^2)$$

$$\Rightarrow 15 D_o^2 = 17 D_i^2$$

$$\therefore \frac{D_o}{D_i} = \sqrt{\frac{17}{15}}$$

$$= 1.06$$

$\therefore \boxed{D_o = 1.06 D_i}$ (Answer)

2) According to question

Hollow shaft Diameter, $D_o = \frac{110}{100} \times$ Solid shaft diameter (d)

$= 1.1\,d$

We have to find out

ratio of weight of the hollow shaft to that of solid shaft

We know that ~~If~~ from Torsion equation

$$\frac{T_R}{J} = \frac{\tau_{max}}{R} = \frac{G\theta}{L}$$

Now, $\tau_{max} = \dfrac{T_R}{J/R}$

$= \dfrac{T_R}{Z_p}$

Where, $T_R$ = Resisting Torque

$J$ = Polar moment of inertial about shaft axis

$\tau_{max}$ = maximum shear stress

$R$ = Radius

$G$ = Shear modulus

$\theta$ = Angle of twist

$L$ = Length of the shaft

$Z_p$ = polar section modulus of cross section of shaft

$= J/R$

As, given, the hollow and solid shaft are made of same material and equal ~~shaft~~ strength.

i.e. $\tau_{max}$ and $T_R$ are same for both shafts

$\therefore (Z_p)_{solid} = (Z_p)_{Hollow}$

or, $\left(\dfrac{J}{R}\right)_{solid} = \left(\dfrac{J}{R}\right)_{Hollow}$

or, $\dfrac{\frac{\pi}{64} d^4}{d/2} = \dfrac{\frac{\pi}{64}\left(D_o^4 - D_i^4\right)}{D_o/2}$

or, $d^3 = D_o^3 (1-k^4)$ $\left[\text{Where}, k = \frac{D_i}{D_o}\right]$

or, $1 - k^4 = \left(\frac{d}{D_o}\right)^3 = \frac{1}{(1 \cdot 1)^3}$

or, $k = \cancel{0.706}$ $0.706$

Now, we know, weight = mass $\times g$ = (volume $\times$ density) $\times g$

= (Area $\times$ Length $\times$ density) $\times g$

So, $\dfrac{\text{Weight of Hollow Shaft}}{\text{Weight of Solid shaft}}$

$= \dfrac{\text{Area of Hollow shaft}}{\text{Area of Solid Shaft}}$

$= \dfrac{\frac{\pi}{4}(D_o^2 - D_i^2)}{\frac{\pi}{4} d^2}$

$= \left(\frac{D_o}{d}\right)^2 (1 - k^2)$

$= (1 \cdot 1)^2 (1 - 0.706^2)$

$= 0.606$

So, the weight of hollow shaft is 0.606 time the weight of the Solid Shaft

3) Given that a hollow shaft, 500 mm outside diameter and 300 mm inside diameter is supported in two bearings 6 m apart. The shaft is driven by a flexible coupling at one end and drive a ship's, propeller at 100 rpm. The maximum ~~Ho~~ thrust on the propeller is 500 kN when the shaft is transmitting 6000 kW. Determine the factor of ~~Safet~~

Safety. Gives your comment on the result obtained by ~~you~~ you.

Ans) Here the bending moment is calculated as Mb :-

$$Mb(max) = \frac{WL}{8} = \frac{60000 \times 6}{8} = 45000 \ Nm$$

The torsional moment is calculated as Mt :-

$$M_t (max) = \frac{6000 \ (9550)}{100} = 573000 \ Nm$$

The moment of inertia is calculated as :-

$$I = \frac{\pi \ (\text{outer diameter}^4 - \text{Inner diameter}^4)}{64}$$

$$= \pi \left( \frac{0.5^4 - 0.3^4}{64} \right)$$

$$= 2.67 \times 10^{-3} \ m^4$$

The cross section area of shaft A is

$$A = \frac{\pi \ (O.D^2 - ID^2)}{4} = \frac{\pi \ (0.5^2 - 0.3^2)}{4}$$

$$= 0.126 \ m^2$$

The radius of gyration K is ~~cal~~ calculated as

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.67 \times 10^{-3}}{0.126}} = 0.146$$

$$\frac{l}{K} = \frac{6}{0.146} = 41.1$$ this is < 115

∴ The column action factor $\alpha$ is given by

$$\alpha = \frac{1}{1 - 0.0044 \left( \frac{L}{k} \right)} = \frac{1}{1 - 0.0044 \ (41.1)} = 1.22$$

also, $d_o = 0.5\,m$ and $d_i = 0.3\,m$

$$K = \frac{d_i}{d_o} = \frac{0.3}{0.5} = 0.6$$

The maximum shear stress is ~~given~~ is calculated as

$$S_s = \frac{16}{\cancel{\pi (0.5)^3 (1-0.6)^4}\ \pi d_o^3 (1-K^4)} \sqrt{\left[K_b M_b + \frac{\alpha F_a d_o (1+K^2)}{8}\right]^2 + (K_t M_t)^2}$$

$$S_s = \frac{16}{\pi (0.5)^3 (1-0.6^4)} \sqrt{(1.5 \times 45000) + \frac{1.22 \times 500000 \times 0.5(1+0.6}{8}} + (1 \times 573000)^2$$

$$S_s = 27.4\ MN/m^2$$

$\therefore$ The maximum shear stress in the shaft is

$$S_s = 27.4\,MN/m^2$$