VA-1

(32)

Variable area Adiabetic flow :

1) $Q = 0$
2) No friction
3) A not const.

Assumptions made : steady, 1-d flow $\delta q = 0$

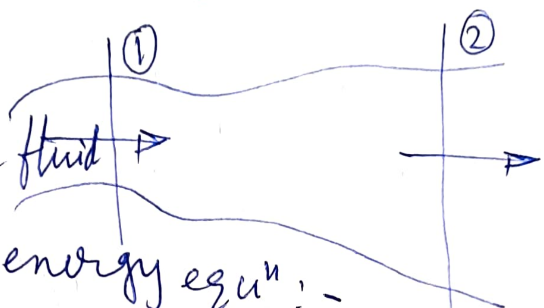
(Area, heat, friction) explain. 1. Adiabetic flow $\Rightarrow \delta q = 0,\ ds_e = 0$

2. No shaft work $\Rightarrow \delta w_s = 0$

Neglect potential $= dz = 0$

3. No ~~loss~~ friction $\Rightarrow ds_i = 0$

Relation indicating variation of fluid properties with changes in area and Mach no.



Writing general energy eqn :-

$$\cancel{\delta q}^{0} + \cancel{\phi} = dh + \frac{dv^2}{2} + \cancel{g\,dz}^{0} + \cancel{\delta w_s}^{0}$$

$$\therefore dh = -\,d\left(\frac{V^2}{2}\right)$$

$$\therefore \boxed{dh = -V\,dV} \quad \text{--- ①}$$

$$T\,ds = dh - v\,dp = dh - \frac{dp}{\rho}.$$

$$T(\cancel{ds_e}^{0} + \cancel{ds_i}^{0}) = dh - \frac{dp}{\rho}.$$

$$\therefore \boxed{dh = \frac{dp}{\rho}} \quad \text{--- ②}$$

So from ① & ②, $-V\,dV = \dfrac{dp}{\rho}$.

$$\therefore \boxed{dV = -\frac{dp}{\rho V}} \quad \text{--- ③}$$

33

Now from differential form of continuity we have

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

or $$\frac{d\rho}{\rho} + \frac{dA}{A} - \frac{dP}{\rho V^2} = 0$$

$$\therefore \frac{dP}{\rho} = + V^2\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right) \quad \cdots (4)$$

but $a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$

As the flow from (1) to (2) is isentropic

so, $a^2 = \frac{dP}{d\rho}$

$$\Rightarrow dP = a^2 d\rho$$

Now from (4) $\Rightarrow$ $$a^2 \frac{d\rho}{\rho} = V^2\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

$$\frac{d\rho}{\rho} = \frac{V^2}{a^2}\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

$$= M^2\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

or, $$(1-M^2)\frac{d\rho}{\rho} = M^2 \frac{dA}{A}$$

$$\therefore d\rho/\rho = \left(\frac{M^2}{1-M^2}\right)\frac{dA}{A}$$

or, $$-\frac{dA}{A} - \frac{dV}{V} = \frac{M^2}{1-M^2}\cdot\frac{dA}{A}$$

$$-\frac{dV}{V} = \left(\frac{\cancel{M^2} + 1 - \cancel{M^2}}{1-M^2}\right)\frac{dA}{A}$$

$$-\frac{dV}{V} = \frac{1}{1-M^2}\frac{dA}{A}$$

$$\frac{dV}{V} = -\left(\frac{1}{1-M^2}\right)\frac{dA}{A}$$

from (4) we have — putting $\frac{d\rho}{\rho}$ value in eq^n (4).

$$\frac{dp}{\rho} = V^2\left[\left(\frac{M^2}{1-M^2}\right)+1\right]\frac{dA}{A}$$

$$dp = \rho V^2\left(\frac{1}{1-M^2}\right)\frac{dA}{A}$$

$$\frac{dA}{A} = (M^2-1)\frac{dV}{V}$$

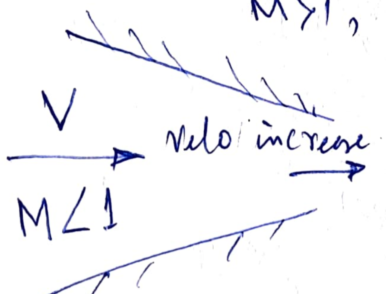
$$\frac{dA}{A} = \frac{1-M^2}{\rho V^2}\cdot dp$$

$$\frac{dA}{A} = \left(\frac{1-M^2}{M^2}\right)\frac{d\rho}{\rho}$$

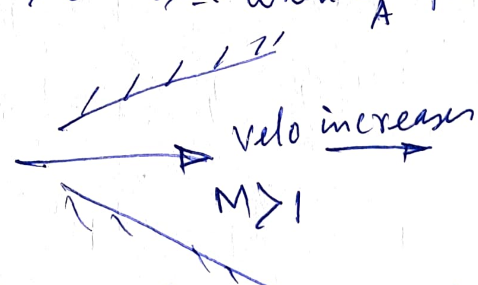
--- (A)

For $M<1$ $M^2-1<0$ ⇒ If $\frac{dA}{A}$ ↓es $\frac{dV}{V}$ ↑es.

$M>1$, $M^2-1>0$ ⇒ ∴ when $\frac{dA}{A}$ ↑es $\frac{dV}{V}$ ↑es.



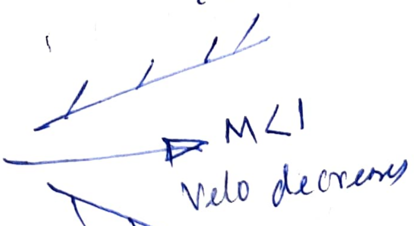
Subsonic nozzle



Supersonic nozzle.

For $M<1$, $M^2-1<0$ ⇒ if $\frac{dA}{A}$ ↑es $\frac{dV}{V}$ ↓es.

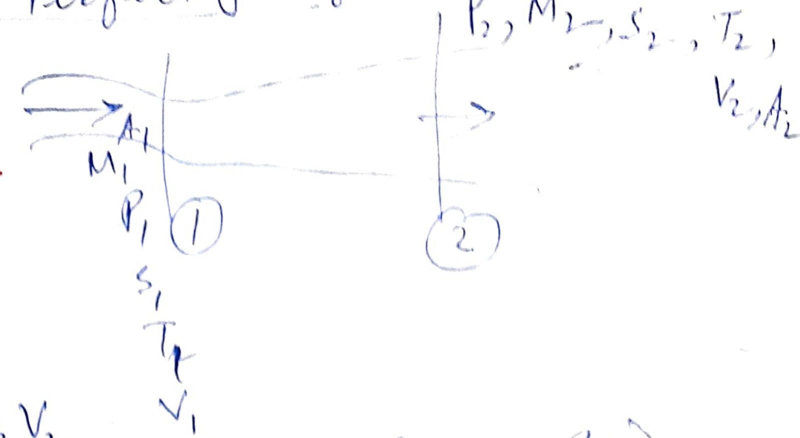
" $M>1$ $M^2-1>0$ ⇒ if $\frac{dA}{A}$ ↓ $\frac{dV}{V}$ ↓es.



Subsonic defuser

Super Sonic defuser

Working equations for perfect gas :-



Find exact exp of $\frac{P_2}{P_1} = ?$

$$\frac{P_2}{P_1} = f(M_1, M_2, \gamma, \Delta s)$$

State: $P = \rho R T$

Continuity:- $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\frac{A_2}{A_1} = \frac{\rho_1 V_1}{\rho_2 V_2} = \left(\frac{P_1/RT_1}{P_2/RT_2}\right)\left(\frac{M_1 a_1}{M_2 a_2}\right)$$

$$\therefore \frac{A_2}{A_1} = \frac{P_1 T_2 M_1 a_1}{P_2 T_1 M_2 a_2} = \frac{P_1 T_2 M_1 \sqrt{\gamma R T_1}}{P_2 T_1 M_2 \sqrt{\gamma R T_2}}$$

$$\frac{A_2}{A_1} = \frac{P_1}{P_2}\left(\frac{T_2}{T_1}\right)^{1/2} \cdot \frac{M_1}{M_2}$$

or, $\cancel{\frac{A_2}{A_1} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2} \frac{M_1}{M_2}}$

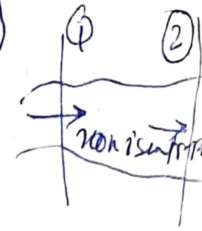
$$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1}\,\frac{P_{t2}/P_2}{P_{t1}/P_1} = \frac{P_2}{P_1}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1} = e^{-\Delta s/R}.$$

$$\therefore \boxed{\frac{P_1}{P_2} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1} \cdot e^{\Delta s/R}}$$

$$1 = \frac{T_{t2}}{T_{t1}} = \frac{T_{t2}/T_2}{T_{t1}/T_1} \cdot \frac{T_2}{T_1} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)\frac{T_2}{T_1}.$$

$$\boxed{\text{or, } \frac{T_1}{T_2} = \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}}$$

$$\boxed{\frac{A_2}{A_1} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot e^{\Delta S/R}} \quad \text{--- (B)}$$



By introducing isentropic $*$ reference state which is corresponding to $M = 1$.

Now we apply equ^n (B) bet^n 1^* & 2^* state points.

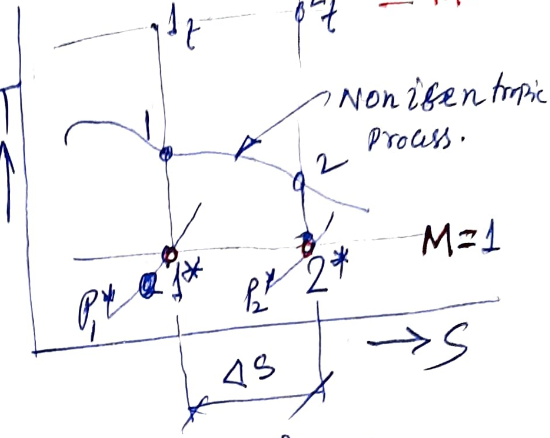


fig-1

Put $\Rightarrow$ $A_1 \equiv A_1^*$, $A_2 = A_2^*$.

$M_1 = M_1^* = 1$, $M_2 = M_2^* = 1$

$$\frac{A_2^*}{A_1^*} = \frac{1}{1}\left(\frac{1+\frac{\gamma-1}{2}\cdot 1^2}{1+\frac{\gamma-1}{2}\cdot 1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot e^{\Delta S/R}.$$

$$\left[\frac{P_{t2}}{P_{t1}} = e^{-\Delta S/R} \text{ for adiabatic flow}\right] \text{--- (D)}$$

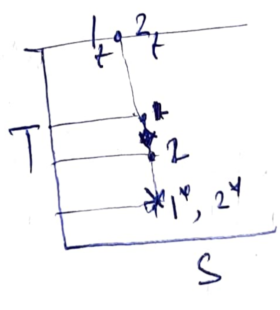
$$\boxed{\frac{A_2^*}{A_1^*} = e^{\Delta S/R}} \text{ --- (C)}$$

<u>Case-1</u> :- $\Delta S = \Delta S_{1-2} = 0 \quad \therefore \quad \frac{A_2^*}{A_1^*} = 1 \Rightarrow A_1^* = A_2^*$

1^* & 2^* coincides at one point

1_t & 2_t " " " "

$P_{1t} = P_{2t}$ & $A_1^* = A_2^*$



<u>Case-2</u> <u>$\Delta S \neq 0$</u>. & $\dot{m} = const \Rightarrow \rho_1^* A_1^* V_1^* = \rho_2^* A_2^* V_2^*$

From fig-1, $V_1^* = V_2^*$ & $T_1^* = T_2^*$, $P_1^* > P_2^*$ & $\rho_1^* > \rho_2^*$

$\therefore \rho_1^* A_1^* = \rho_2^* A_2^*$

$$\frac{A_1^*}{A_2^*} = \frac{\rho_2^*}{\rho_1^*} < 1 \Rightarrow A_1^* < A_2^*$$

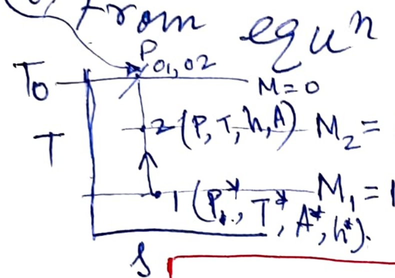
$$\frac{P_{t2}}{P_{t1}} = e^{-\Delta s/R} \quad \dots \text{(D)}$$

Multiply (C) & (D) $\Rightarrow$ $\dfrac{P_{t2}A_2^*}{P_{t1}A_1^*} = e^{-\Delta s/R}\cdot e^{\Delta s/R} = 1$.

(1) $\Leftarrow$ $\therefore$ $\boxed{P_{t2}A_2^* = P_{t1}A_1^*}$ $\Rightarrow$ For state (1) to state (2) is non-isentropic

(P_0, T_0, A_0, h_0)

Isentropic Relations:

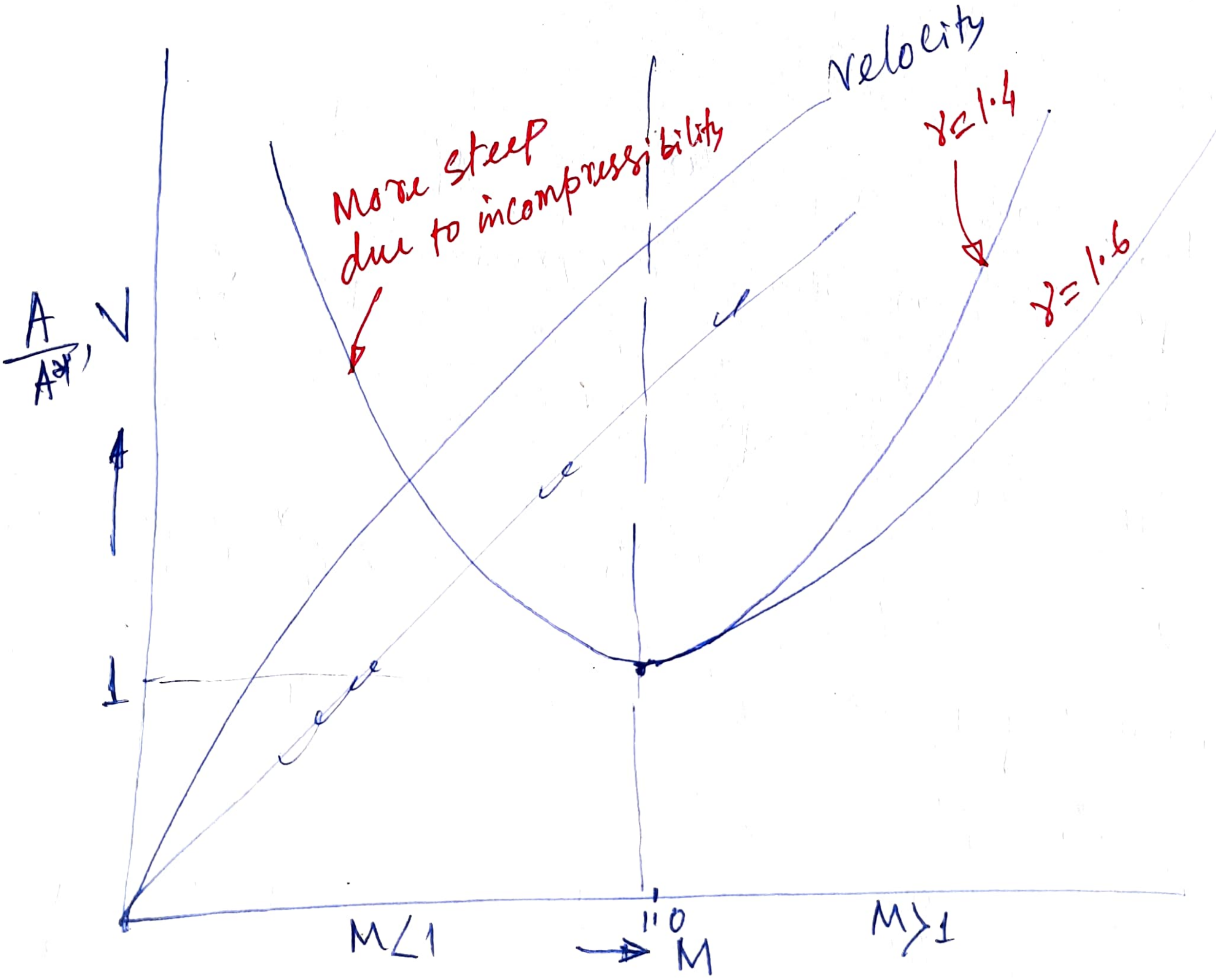


From equ^n (B), Put $A_2 = A$, $M_2 = M$ (Any value) & $A_1 = A^*$, $M_1^* = 1$

$\Delta s = 0$

(2) $\Leftarrow$

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} = f(M,\gamma).$$



VA-7

From $\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$, Put $T_2 = T$, $M_2 = M$

~~$T_1 = T^*$, $M_1 = 1$~~

$T_1 = T_t$ $M_1 = 0$.

~~we get $\frac{T}{T^*} = \frac{1+\frac{\gamma-1}{2}}{1+\frac{\gamma-1}{2}M^2} = \frac{\gamma+1}{2}$~~

(3) $$\boxed{\frac{T}{T_t} = \frac{1}{1+\left[\frac{\gamma-1}{2}\right]M^2} = f(M,\gamma)}$$

(4) $$\boxed{\frac{P}{P_t} = \left(\frac{1}{1+\frac{\gamma-1}{2}M^2}\right)^{\gamma/\gamma-1} = f(M,\gamma)}$$

(5) — $$\boxed{\frac{\rho}{\rho_t} = \left(\frac{1}{1+(\gamma-1)/2\,M^2}\right)^{1/\gamma-1} = f(M,\gamma).}$$

Using, (2), (3), (4), ~~4~~ (5) ~~& (6)~~ we can construct isentropic table for some $\underline{\gamma}$ with diff. $\underline{'M'}$

Multiplying $\frac{A}{A^*}$ & $\frac{P}{P_t}$ we get.

(6) — $$\frac{AP}{A^*P_t} = f(M,\gamma)$$

Relation between, 1) T_0 & T^*, P_0 & P^* & ρ_0 & ρ^*

$$\frac{T_0}{T^*} = 1+\frac{\gamma-1}{2}\cdot 1 = \frac{\gamma+1}{2},$$

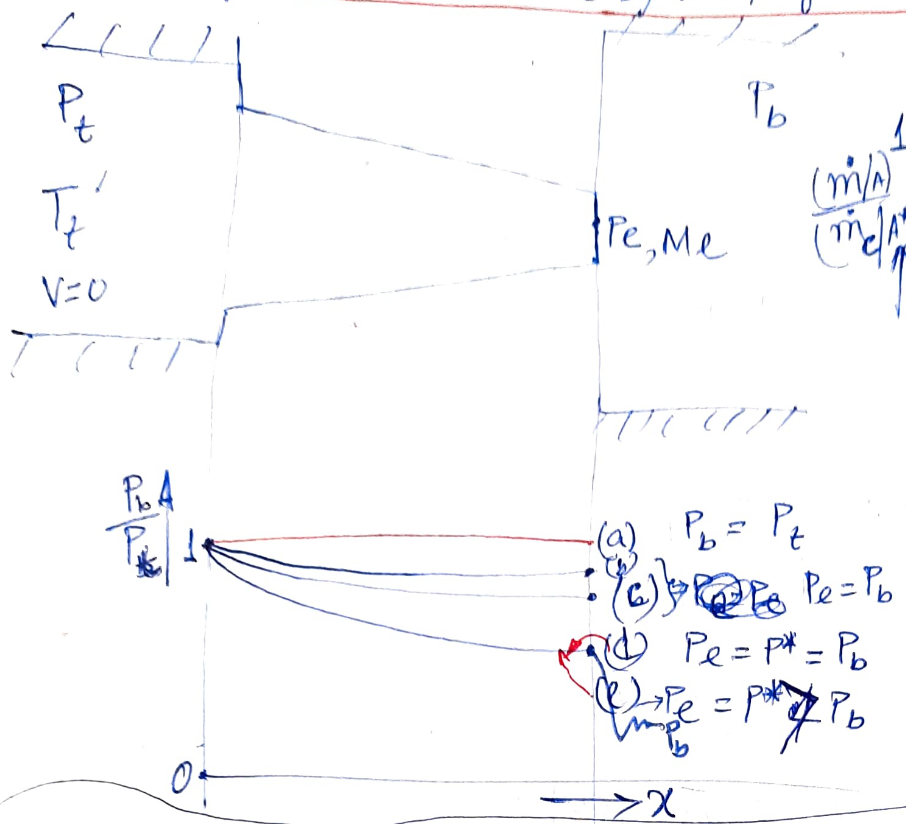
$$\frac{P_0}{P^*} = \left(1+\frac{\gamma-1}{2}\cdot 1\right)^{\gamma/\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}$$

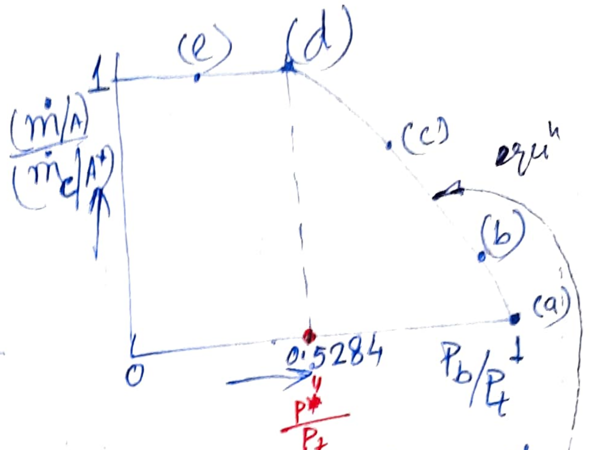
$$\frac{\rho_0}{\rho^*} = \left(1+\frac{\gamma-1}{2}\cdot 1\right)^{1/\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{1/\gamma-1}$$

Nozzle – operation

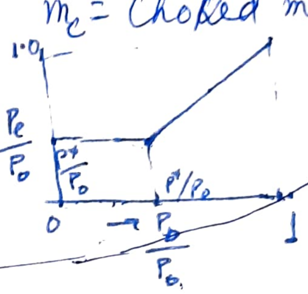
Converging only | C-D NOZZLE.

Effect of back pressure (P_b) on performance of nozzle :–





$\dot m_c$ = Choked mass flow rate



(a) $P_b = P_t \Rightarrow$ No flow takes place.

(b) $P_b = P_e \Rightarrow$ Flow takes place

(c) Reduce P_b further but not below P^*.

(d) Reduce P_b to get $M_e = 1$, Net mass flow rate is $\dot m_c$

(e) " " further mass flow rate remains same. No change in pressure distribution inside the nozzle.

Explaination :– If $\frac{P_b}{P_t} > \frac{P^*}{P_t}$ i.e. $\frac{P_b}{P_t} > 0.5284$

$$\frac{\frac{\dot m}{A}}{\frac{\dot m_c}{A^*}} = \frac{M\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$\frac{P_e}{P_t} = \frac{P_b}{P_t}$, $M_e < 1$ and function of $\frac{P_e}{P_t}$,

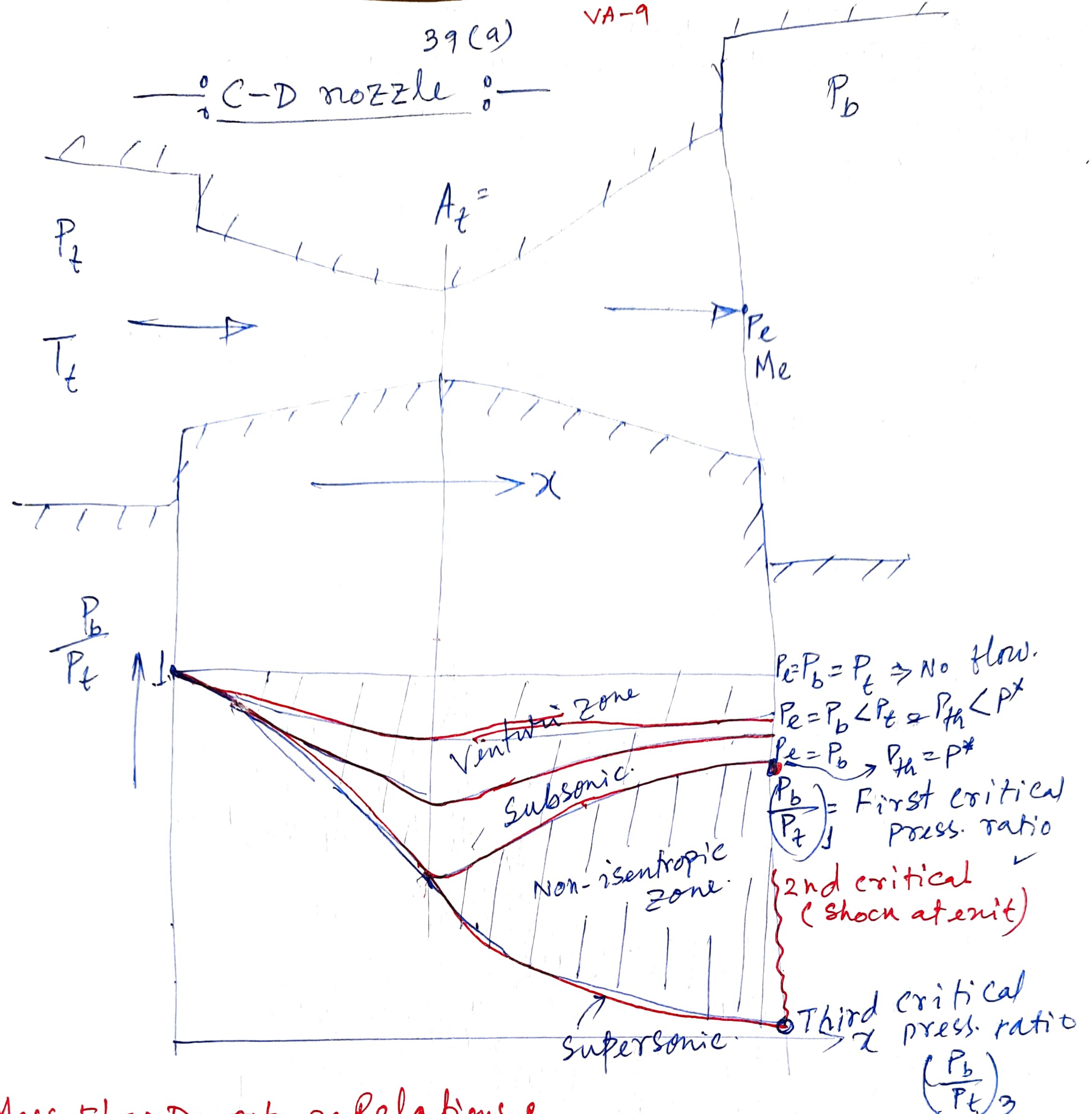
$\frac{(\dot m/A)}{(\dot m_c/A^*)} = \frac{A^*}{A_e}$ and pressure distribution is function of P_b

If $\frac{P_b}{P_t} \leq \frac{P^*}{P_t}$ or $\frac{P_b}{P_t} \leq 0.5284$

$\frac{P_e}{P_t} = \frac{P^*}{P_t} > \frac{P_b}{P_t}$, $M_e = 1$, $\dot m = \dot m_c = \text{constant} = \text{Const.}\frac{P_t}{\sqrt{T_t}}$

& pressure distribution will remain unchanged within nozzle & independent of P_b. Flow from exit plane takes place from high press. zone (P_e) to low press. zone (P_b)

C-D nozzle :-

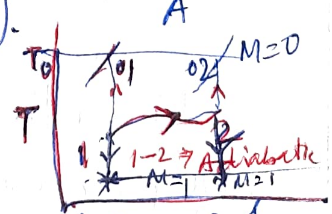


Mass Flow ~~Density &~~ Relations :-

mass flow density, $G = \frac{\dot m}{A} = \rho V$, $\frac{\dot m}{A} = \frac{\rho A V}{A^*} \cdot \frac{A^*}{A} = \frac{\rho^* A^* V^*}{\cancel{A^*}} \cdot \frac{A^*}{A}$

$p^* = \rho^* R T^*$, $a^* = \sqrt{\gamma R T^*} = V^*$ ($\because M^* = 1$). $= \frac{\rho^* A^* V^*}{A}$

$T^* = \frac{2 T_t}{\gamma+1}$, $p^* = p_t \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$, $\rho^* = \frac{p^*}{RT^*}$



$$\frac{\dot m}{A} = \frac{p^*}{RT^*} \cdot \sqrt{\gamma R T^*} \cdot \frac{A^*}{A} = \sqrt{\frac{\gamma}{RT^*}} \cdot p^* \cdot \frac{A^*}{A}$$

$$= \sqrt{\frac{\gamma}{R}} \cdot P_t \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{\left(\frac{2T_t}{\gamma+1}\right)^{1/2}} \cdot \frac{A^*}{A} = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_t}{\sqrt{T_t}} \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1} - \frac{1}{2}\right)} \cdot \frac{A^*}{A}$$

$$\frac{\dot m}{A} = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_t}{\sqrt{T_t}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{A^*}{A} = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_t}{\sqrt{T_t}} \cancel{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \left\{ M \frac{\cancel{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \right\}$$

Max^m mass flow rate must occur when $M=1$, i.e. $A=A^*$.

$$\left(\frac{\dot m}{A}\right)_{max} = \sqrt{\frac{\gamma}{R}}\,\frac{P_t}{\sqrt{T_t}}\cdot\left(1+\frac{\gamma-1}{2}\cdot 1^2\right)^{-(\gamma+1)/2(\gamma-1)} = \frac{\dot m_{max}}{A^*} = \frac{\dot m_c}{A^*}$$

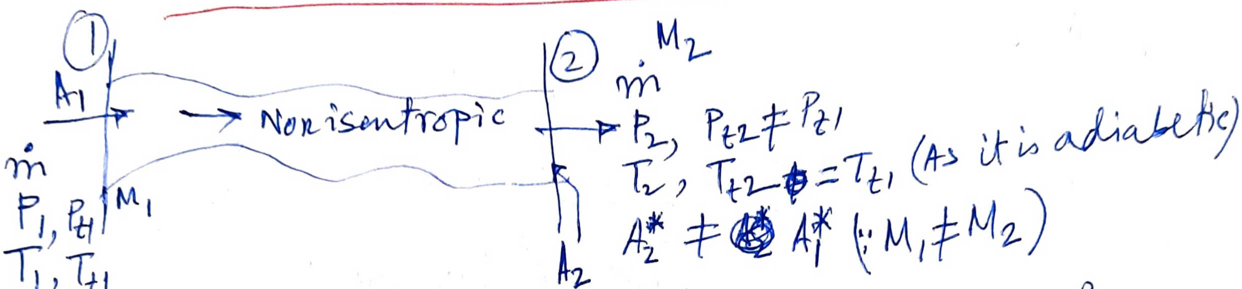
$$\left(\frac{\dot m}{A}\right)_{max} = \sqrt{\frac{\gamma}{R}}\,\frac{P_t}{\sqrt{T_t}}\left(\frac{\gamma+1}{2}\right)^{-(\gamma+1)/2(\gamma-1)} = \frac{\dot m_{max}}{A^*} = \frac{\dot m_c}{A^*}$$

$$\left(\frac{\dot m_c}{A^*}\right) = \sqrt{\frac{\gamma}{R}}\,\frac{P_t}{\sqrt{T_t}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = f(P_t, T_t) = \cancel{0.0404\,f(P_t,T_t)} = 0.0404\frac{P_0}{\sqrt{T_0}}$$

$$= \cancel{0.0525\,f(P_0,T_0)}$$

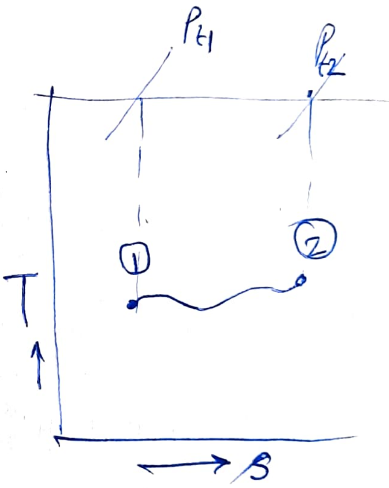
For P_t & T_t are maintained constant, $\dot m$ is max for $M=1$ & $\left(\frac{\dot m}{A}\right) = $ const.

Relation bet^n A^* & stagnation pressure in adiabatic but nonisentropic flow :—



At ①, $$\frac{\dot m}{A_1^*} = \frac{P_{t1}}{\sqrt{T_{t1}}}\sqrt{\frac{\gamma}{R}}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{--- (1)}$$

At ② $$\frac{\dot m}{A_2^*} = \frac{P_{t2}}{\sqrt{T_{t2}}}\sqrt{\frac{\gamma}{R}}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{--- (2)}$$



By (2) ÷ (1) we get, $$\frac{P_{t2}}{P_{t1}} = \frac{A_1^*}{A_2^*}$$

As the process from state (1) to state (2) adiabatic, so heat exchange i.e. $Q=0$. But the process is non-isentropic i.e. the process is adiabatic irreversible. As we know irreversibility causes only increase in entropy. Therefore state (2) must be right side of state (1) in T-s diagram. Therefore, $P_{t1} > P_{t2}$ for adiabatic irreversible process

& $P_{t1} = P_{t2}$ for reversible, adiabatic process.

$$\boxed{\therefore \frac{P_{t2}}{P_{t1}} = \frac{A_1^*}{A_2^*} \le 1}$$

P_{02} $T_e = T_e^*\ 550\,K$, $\dot m_c = 0.0\ldots$

VA-13

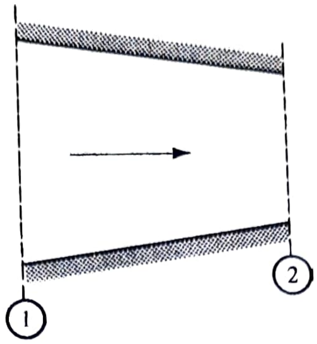


Figure E5.6

from 6 ft² to 2.5 ft² (Figure E5.6). You may assume steady, one-dimensional flow and a perfect gas. (See the table in Appendix A for gas properties.)

(a) Find M_1, p_{t1}, T_{t1}, and h_{t1}.
(b) If there are losses such that $\Delta s_{1-2} = 0.005$ Btu/1bm-°R, find M_2, p_2, and T_2.

(a) First, we determine conditions at station 1.

$$a_1 = (\gamma g_c R T_1)^{1/2} = [(1.4)(32.2)(48.3)(600)]^{1/2} = 1143 \text{ ft/sec}$$

$$M_1 = \frac{V_1}{a_1} = \frac{2960}{1143} = 2.59$$

$$p_{t1} = \frac{p_{t1}}{p_1} p_1 = \left(\frac{1}{0.0509}\right)(20) = 393 \text{ psia}$$

$$T_{t1} = \frac{T_{t1}}{T_1} T_1 = \left(\frac{1}{0.4271}\right)(600) = 1405°\text{R}$$

$$h_{t1} = c_p T_{t1} = (0.218)(1405) \doteq 306 \text{ Btu/lbm}$$

(b) For a perfect gas with $q = w_s = 0$, $T_{t1} = T_{t2}$ (from an energy equation), and also from equation (5.29):

$$\frac{A_1^*}{A_2^*} = e^{-\Delta s/R} = e^{-(0.005)(778)/48.3} = 0.9226$$

Thus

$$\frac{A_2}{A_2^*} = \frac{A_2}{A_1}\frac{A_1}{A_1^*}\frac{A_1^*}{A_2^*} = \left(\frac{2.5}{6}\right)(2.8688)(0.9226) = 1.1028$$

From the isentropic table we find that $M_2 \approx$ ______. Why is the use of the isentropic table legitimate here when there are losses in the flow? Continue and compute p_2 and T_2.

$p_2 =$ $(P_2 \approx 117 \text{ psia})$

$T_2 =$ $(T_2 \approx 1017°\text{R})$

Could you find the velocity at section 2?

5.7 NOZZLE OPERATION

We will now start a discussion of nozzle operation and at the same time gain more experience in use of the isentropic table. Two types of nozzles are considered: a converging-only nozzle and a converging–diverging nozzle. We start by examining the physical situation shown in Figure 5.6. A source of air at 100 psia and 600°R is contained in a large tank where stagnation conditions prevail. Connected to the tank is a converging-only nozzle and it exhausts into an extremely large receiver where the pressure can be regulated. We can neglect frictional effects, as they are very small in a converging section.

If the receiver pressure is set at 100 psia, no flow results. Once the receiver pressure is lowered below 100 psia, air will flow from the supply tank. Since the supply tank has a large cross section relative to the nozzle outlet area, the velocities in the tank may be neglected. Thus $T_1 \approx T_{t1}$ and $p_1 \approx p_{t1}$. There is no shaft work and we assume no heat transfer. We identify section 2 as the nozzle outlet.

Energy

$$h_{t1} + \cancel{q} = h_{t2} + \cancel{w_s} \tag{3.19}$$

$$h_{t1} = h_{t2}$$

and since we can treat this as a perfect gas,

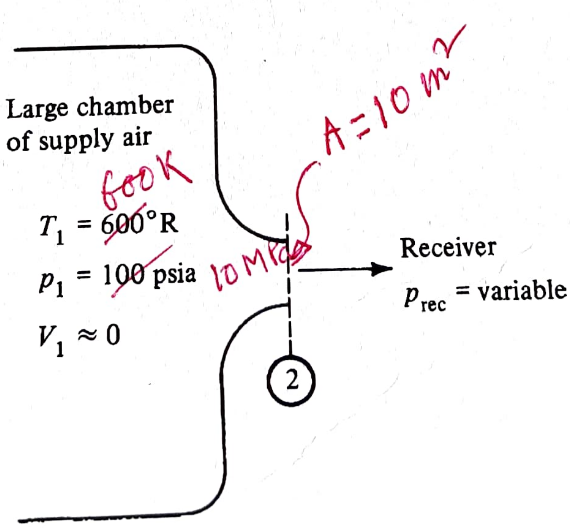


Figure 5.6 Converging-only nozzle.

$$T_{t1} = T_{t2}$$

It is important to recognize that the receiver pressure is controlling the flow. The velocity will increase and the pressure will decrease as we progress through the nozzle until the pressure at the nozzle outlet equals that of the receiver. This will always be true *as long as* the nozzle outlet can "sense" the receiver pressure. Can you think of a situation where pressure pulses from the receiver could not be "felt" inside the nozzle? (Recall Section 4.4.)

Let us assume that

Case 1 : $$p_{rec} = 80.2 \text{ psia} \quad 8.02 \text{ MPa}$$

Then

$$p_e \quad p_2 = p_{rec} = 80.2 \text{ psia} = 8.02 \text{ MPa}$$

and

check for choking $$\frac{p_2}{p_{t2}} = \frac{p_2}{p_{t1}}\frac{p_{t1}}{p_{t2}} = \left(\frac{80.2}{100}\right)(1) = 0.802$$ not choked.

Note that $p_{t1} = p_{t2}$ by equation (4.28) since we are neglecting friction.

From the isentropic table corresponding to $p/p_t = 0.802$, we see that

$$M_2 = 0.57 \quad \text{and} \quad \frac{T_2}{T_{t2}} = 0.939 \Rightarrow \frac{T_e}{T_{oe}} = 0.939$$

M_e

Thus

$$\rho_e = \frac{P_e}{RT_e} = \frac{8.02\times10^6}{287\times563} = 49.63 \text{ kg/m}^3$$

$$T_2 = \left(\frac{T_2}{T_{t2}}\right) T_{t2} = (0.939)(600) = 563°\text{R} \qquad 563 \text{ K} = T_e$$

$$a_2^2 = (1.4)(32.2)(53.3)(563) \qquad a_e = 476 \text{ m/s}$$

$$\dot{m} = \rho_e A_e V_e$$

$$a_2 = 1163 \text{ ft/sec}$$

and $$V_e = 0.57\times476 = 271.34 \text{ m/s}$$

$$\dot{m} = 49.63\times10\times271.34 = 1.35\times10^5 \text{ kg/s}$$

$$V_2 = M_2 a_2 = (0.57)(1163) = 663 \text{ ft/sec}$$

Figure 5.7 shows this process on a T–s diagram as an isentropic expansion. If the pressure in the receiver were lowered further, the air would expand to this lower pressure and the Mach number and velocity would increase. Assume that the receiver pressure is lowered to 52.83 psia. *Show* that $P_b = 5.283$ MPa.

Case 2

$$\frac{p_2}{p_{t2}} = 0.5283, \quad M = 1, \quad \frac{T_2}{T_0} = 0.834$$

and thus

$$\dot{m}_c = \sqrt{\frac{\gamma}{R}}\frac{P_0}{\sqrt{T_0}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times A^*$$

$$T_2 = 600\times0.834 \qquad T_e = 500.4 \text{ K}$$

$$= 0.0404 \frac{P_0}{\sqrt{T_0}} \quad \cancel{0.0525} \quad = 1.165\times10^5 \text{ kg/s}$$

$$a_e = \sqrt{1.4\times287.5\times500.4} = 448$$

$$\frac{\dot{m}_c}{A^*} = \dot{m}_0 = \cancel{2.14\times10^5} \text{ kg/s}$$

$$V_2 = 448 \text{ m/s} \quad M = 1$$

Case-3 $$\frac{P_2}{P_{02}} < 0.5283 \Rightarrow M = 1, \; V_2 = 448 \text{ m/s}, \; P_e = 5.283 \text{ MPa}, \; T_e = T^* = 500 \text{ K}, \; \dot{m}_c = 0.0525 \frac{P_0}{\sqrt{T_0}} \cdot A^*$$

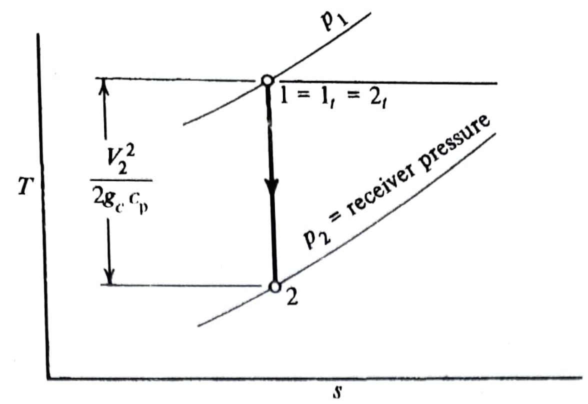


Figure 5.7 *T–s* diagram for converging-only nozzle.

$$M_2 = 1.00 \qquad \text{with} \quad V_2 = 1096 \text{ ft/sec}$$

Notice that the air velocity coming out of the nozzle is exactly sonic. If we now drop the receiver pressure below this *critical pressure* (52.83 psia), the nozzle has no way of adjusting to these conditions. Why not? Assume that the nozzle outlet pressure could continue to drop along with the receiver. This would mean that $p_2/p_{t2} < 0.5283$, which corresponds to a supersonic velocity. We know that if the flow is to go supersonic, the area must reach a minimum and then increase (see Section 5.3). Thus for a converging-*only* nozzle, the flow is governed by the receiver pressure until sonic velocity is reached at the nozzle outlet and *further reduction of the receiver pressure will have no effect on the flow conditions inside the nozzle*. Under these conditions, the nozzle is said to be *choked* and the nozzle outlet pressure remains at the *critical pressure*. Expansion to the receiver pressure takes place *outside* the nozzle.

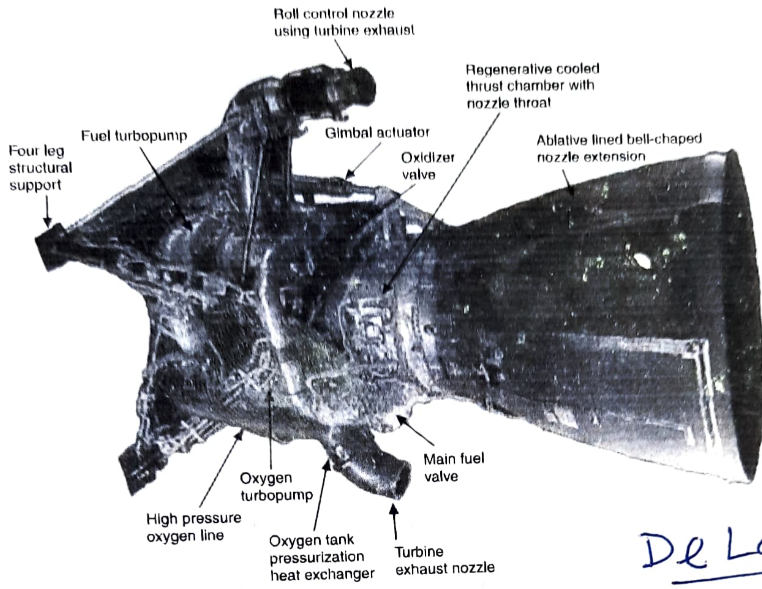
In reviewing this example you should realize that there is nothing magical about a receiver pressure of 52.83 psia. The significant item is the *ratio* of the static to total pressure at the exit plane, which for the case of no losses is the *ratio* of the receiver pressure to the inlet pressure. With sonic velocity at the exit, this *ratio* is 0.5283.

The analysis above assumes that conditions within the supply tank remain constant. One should realize that the choked flow rate can change if, for example, the supply pressure or temperature is changed or the size of the throat (exit hole) is changed. It is instructive to take an alternative view of this situation. You are asked in Problem 5.9 to develop the following equation for isentropic flow:

$$\frac{\dot{m}}{A} = M\left(1 + \frac{\gamma - 1}{2}M^2\right)^{-(\gamma+1)/2(\gamma-1)} \left(\frac{\gamma g_c}{R}\right)^{1/2} \frac{p_t}{\sqrt{T_t}} \tag{5.44a}$$

Applying this equation to the outlet and considering choked flow, $M = 1$ and $A = A^*$. Then

VA-17



De Laval Nozzle

Figure 5.9 Typical converging–diverging nozzle. (Courtesy of the Boeing Company, Rocketdyne Propulsion and Power.)

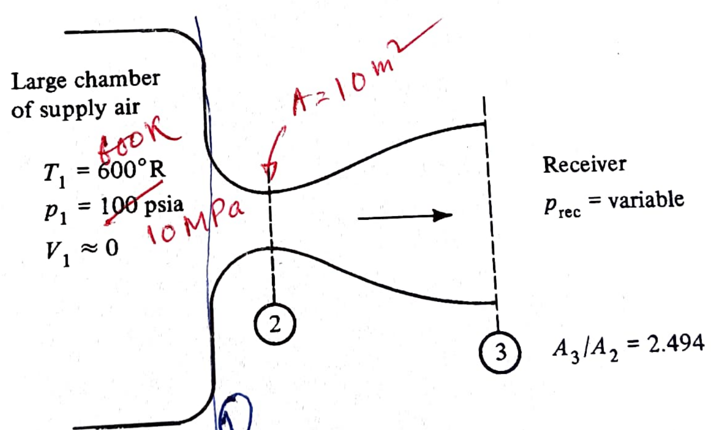


Figure 5.10 Converging–diverging nozzle.

To discover the conditions that exist at the exit (under design operation), we seek the ratio A_3/A_3^*:

$$\frac{A_3}{A_3^*} = \frac{A_3}{A_2}\frac{A_2}{A_2^*}\frac{A_2^*}{A_3^*} = (2.494)(1)(1) = 2.494$$

Note that $A_2 = A_2^*$ since $M_2 = 1$, and $A_2^* = A_3^*$ by equation (5.29), as we are still assuming isentropic operation. We look for $A/A^* = 2.494$ in the *supersonic* section of the isentropic table and see that

$1 = MPa = 10^3 \times Pa$
$= 10^3 \times N/m^2$

$$M_3 = 2.44, \qquad \frac{p_3}{p_{t3}} = 0.0643, \qquad \text{and} \qquad \frac{T_3}{T_{t3}} = 0.4565$$

Thus

$$P_e = p_3 = \frac{p_3}{p_{t3}}\frac{p_{t3}}{p_{t1}}p_{t1} = (0.0643)(1)(\cancel{100}\ 10) = 6.43 \text{ psia} \quad 0.643 \text{ MPa}$$

$P_b = P_e = 0.643$ MPa.

and to operate the nozzle at this *design condition* the receiver pressure *must be* at 6.43 psia (0.643 MPa). The pressure variation through the nozzle for this case is shown as curve "a" in Figure 5.11. This mode is sometimes referred to as *third critical*. From the temperature ratio T_3/T_{t3} we can easily compute T_3, a_3, and V_3 by the procedure shown previously. $T_3 = 0.4565 \times 600 = 273.5$ K, $a_3 = \sqrt{\gamma R T_3} = 331.5$

$V = a_3 M_3 = 2.44 \times 331.5 = 808.9$ m/s

One can also find $A/A^* = 2.494$ in the subsonic section of the isentropic table. (Recall that these two answers come from the solution of a quadratic equation.) For this case

$\frac{\dot{m}_c}{A^*} = 0.0404 \frac{P_0}{\sqrt{T_0}}$

$\dot{m}_c = 0.0404 \frac{P_0}{\sqrt{T_0}} \times A_{th}$

$= 0.0404 \times \frac{10 \times 10^6}{\sqrt{600}} \times 10 \times 10^{-4}$

$= 0.165 \times 10^3$

$= 165$ kg/s.

$$M_3 = 0.24, \qquad \frac{p_3}{p_{t3}} = 0.9607 \qquad \frac{T_3}{T_{t3}} = 0.9886$$

(7) $\frac{P_b}{P_0}$ lies in between $\left(\frac{P}{P_0}\right)_I$ & $\left(\frac{P}{P_0}\right)_{III}$, $P_e < P_b$

flow is overexpanded

Thus

$$p_3 = \frac{p_3}{p_{t3}}\frac{p_{t3}}{p_{t1}}p_{t1} = (0.9607)(1)(\cancel{100}\ 10) = \cancel{96.07 \text{ psia}} \quad 9.6 \text{ MPa}$$

and to operate at this condition the receiver pressure *must be* at 96.07 psia. With this receiver pressure the flow is subsonic from 1 to 2, sonic at 2, and *subsonic* again from

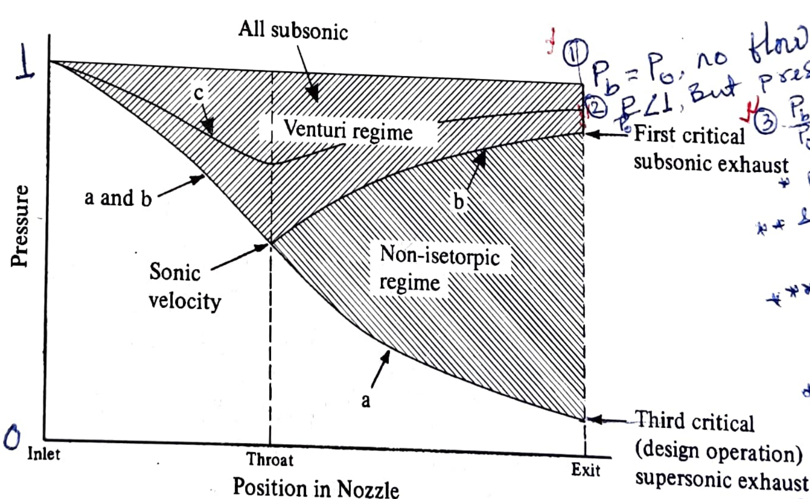


Figure 5.11 Pressure variation through converging–diverging nozzle.

(1) $P_b = P_0$, no flow

(2) $P_b < 1$, But pressure at throat $P < P^*$

(3) $\frac{P_b}{P_0} < 1$, But $P_{throat} = P^*$,

* Flow upto throat subsonic
** & " after " is also "
*** $\left(\frac{P_b}{P_0}\right)_I$ is called 1st critical pressure
**** Nozzle is choked & thus $\dot{m}_c = \dot{m}_{max}$
$P_e = P_b$

(4) $\frac{P_b}{P_0} < \left(\frac{P_m}{P_0}\right)_I$ but $\frac{P_b}{P_0} > \left(\frac{P}{P_0}\right)_{II}$

Flow is non-isentropic & shock occurs. Nozzle is choked. $\dot{m}_c = \dot{m}_{max}$, $P_e = P_b$

(5) Design condition, $\frac{P_b}{P_0} < \left(\frac{P}{P_0}\right)_{II}$ & corresponds to supersonic isentropic flow after throat. Nozzle is choked, Mass flow rate maximum. This $\frac{P_b}{P_0}$ is called third critical, $\left(\frac{P}{P_0}\right)_{III}$. $P_e = P_b$

(6) $\frac{P_b}{P_0} < \left(\frac{P}{P_0}\right)_{III}$, no change in properties upto nozzle exit, thus P_e remains same as condition (5), so $P_e > P_b$ thus expansion wave occurs, non isentropically & it is called underexpanded nozzle.

a) $\frac{P_b}{P_0} = 9.8$ MPa ✓

VA-19

b) $\frac{P_b}{P_0} = 9.6$ MPa

c) $\frac{P_b}{P_0} = 0.0643$ ~~0.05~~ MPa.

2 to 3. The device is nowhere near its design condition and is really operating as a *venturi tube*; that is, the converging section is operating as a nozzle and the diverging section is operating as a diffuser. The pressure variation through the nozzle for this case is shown as curve "b" in Figure 5.11. This mode of operation is frequently called *first critical*.

Note that at both the first and third critical points, the flow variations are identical from the inlet to the throat. Once the receiver pressure has been lowered to 96.07 psia, Mach 1.0 exists in the throat and the device is said to be *choked*. *Further lowering of the receiver pressure will not change the flow rate.* Again, realize that it is not the pressure in the receiver by itself but rather the receiver pressure *relative* to the inlet pressure that determines the mode of operation.

Example 5.7 A converging–diverging nozzle with an area ratio of 3.0 exhausts into a receiver where the pressure is 1 bar. The nozzle is supplied by air at 22°C from a large chamber. At what pressure should the air in the chamber be for the nozzle to operate at its design condition (third critical point)? What will the outlet velocity be?

With reference to Figure 5.10, $A_3/A_2 = 3.0$:

$$\frac{A_3}{A_3^*} = \frac{A_3}{A_2}\frac{A_2}{A_2^*}\frac{A_2^*}{A_3^*} = (3.0)(1)(1) = 3.0$$

From the isentropic table:

$$M_3 = 2.64 \qquad \frac{p_3}{p_{t3}} = 0.0471 \qquad \frac{T_3}{T_{t3}} = 0.4177$$

$$p_1 = p_{t1} = \frac{p_{t1}}{p_{t3}}\frac{p_{t3}}{p_3}p_3 = (1)\left(\frac{1}{0.0471}\right)(1\times 10^5) = 21.2\times 10^5\ \text{N/m}^2$$

$$T_3 = \frac{T_3}{T_{t3}}\frac{T_{t3}}{T_{t1}}T_{t1} = (0.4177)(1)(22+273) = 123.2\text{K}$$

$$V_3 = M_3 a_3 = (2.64)\,[(1.4)(1)(287)(123.2)]^{1/2} = 587\ \text{m/s}$$

We have discussed only two specific operating conditions, and one might ask what happens at other receiver pressures. We can state that the first and third critical points represent the only operating conditions that satisfy the following criteria:

1. Mach 1 in the throat
2. Isentropic flow throughout the nozzle
3. Nozzle exit pressure equal to receiver pressure

With receiver pressures above the first critical, the nozzle operates as a venturi and we never reach sonic velocity in the throat. An example of this mode of operation is shown as curve "c" in Figure 5.11. The nozzle is no longer choked and the flow rate is less than the maximum. Conditions at the exit can be determined by the procedure

5(a) when $\left(\frac{P}{P_0}\right)_{III} < \frac{P_b}{P_0} < \left(\frac{P}{P_0}\right)_{II}$ ⇒ $P_b > P_e$. This is called overexpanded ~~condition~~ nozzle. Thus, compression takes place.