Strength of Materials

Practical Problem:

In the professional field, a Designer faces one of the two following problems:

(i) For a given external load system, designer has to economically design the member (i.e. Shape & Size) so that it can safely withstand the external load without being structurally damaged or functionally unserviceable.

(ii) To findout the maximum external load which can be safely applied on a given Load carrying member beyond which it will be structurally & functionally unserviceable.

Strength of Materials

A load carrying member may be called structurally damaged or functionally Unserviceable if:

(a) the internal force created due to applied external load is more than the limiting resistance capacity of the member } Strength aspect

(b) the elastic deformation exceeds certain limiting value } Stiffness aspect

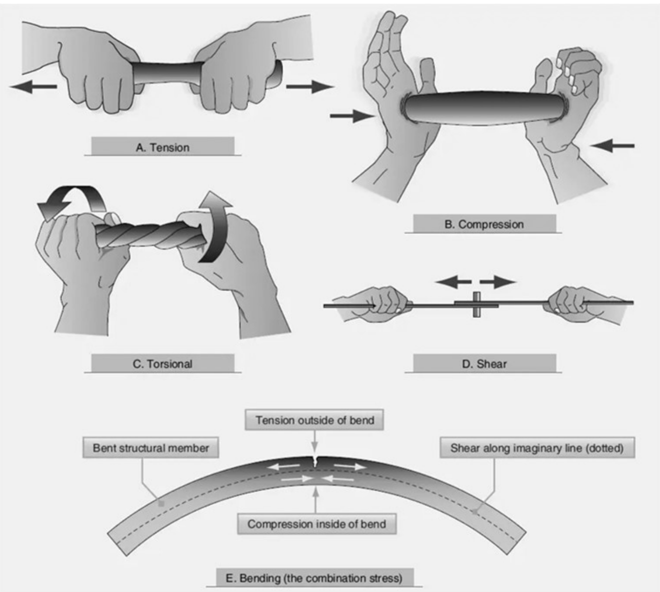
(c) the initial Configuration of the member becomes Unstable } Stability aspect

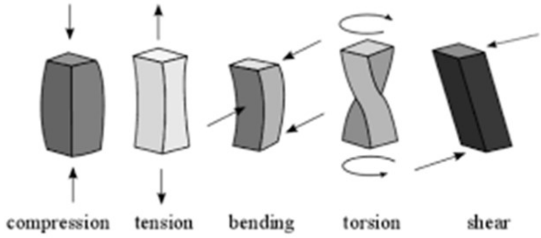
Here,

- The term External load is used in a generalized Sense. It may be Force, Moment, Torque
- The term Deformation is used in generalized Sense. It may be elongation/Compression, Deflection, Angle of Twist, change of shape

Strength of Materials

Topics		
Properties of Materials		
Stress & Strain		
Shear force & Bending moment in Beams		
Flexural or Bending Stress in Straight Beams *(Theory of pure bending)*	Strength (S)	Rigidity (R)
Shear stress in Straight Beams	Strength (S)	
Torsion of circular shafts	Strength (S)	Rigidity (R)
Combined loading (Mohr's circle for stress)	Strength (S)	
Deflection of beams		Rigidity (R)
Columns	Stability (S)	





Fundamentals of Stress

Basic Stresses

- **Normal Stresses** – stresses are normal to the area concerned
- **Shear Stresses** – subject to a distribution of forces which are parallel to the area concerned

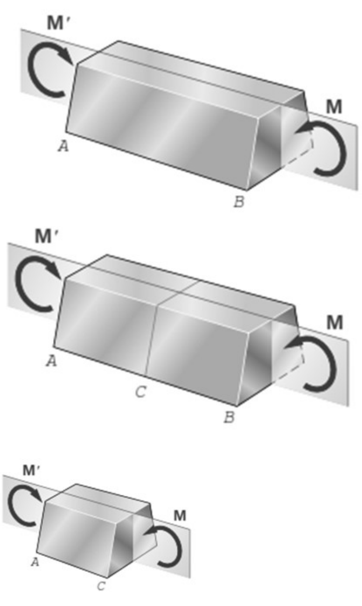
$$\sigma = \frac{dP}{dA}$$

When total force or total load (gradually applied) carried by the body is uniformly distributed over its cross section, the **average stress** (**uniformly distributed stress**) induced in the body is:

$$\sigma = \frac{P}{A}$$

In order to fulfill this assumption of uniform stress distribution, the following conditions must be satisfied

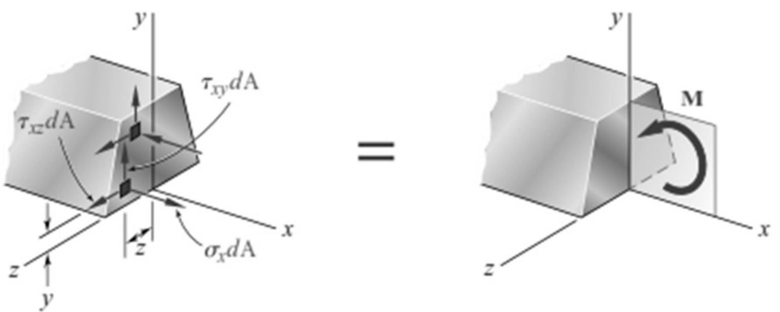
- The bar/member must be straight & made of homogeneous & isotropic material.
- The line of action of the resultant force contains the centroid of the section.
- That portion of the member being considered must have a uniform c/s & remote from discontinuity or abrupt change in c/s.
- The stress must be computed for section well remote from the section on which the load is applied since the stresses in the vicinity of the contact areas are highly concentrated.



Bending Moment

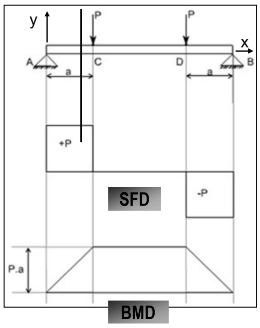
Thus, the internal forces in any cross-section of a symmetric member in pure bending are equivalent to a couple.

The Moment of the couple formed by internal forces is referred as Bending Moment



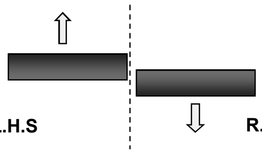
Shear Force & Bending Moment

- At any c/s of a beam, the shear force 'V' is the algebraic sum of all the lateral components of the forces acting on either side of the c/s.
- The bending moment to be simply as the algebraic sum of the moments about an c/s of all the forces acting on either side of the section.



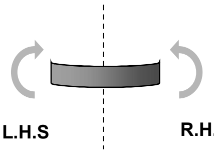
Shear Force $V = \frac{dM}{dx}$ Load intensity $q = \frac{dV}{dx} = \frac{d^2M}{dx^2}$

Sign Convention for Shear Force:

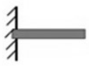
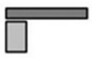


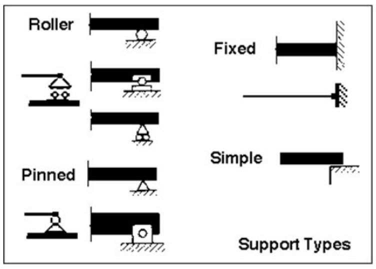
Positive S.F

Sign Convention for Bending Moment:



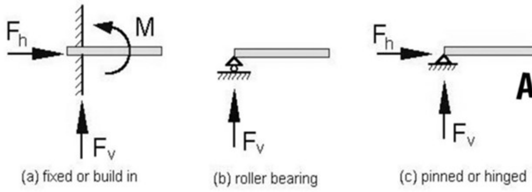
Positive B.M

S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads



Support types		reactions
Roller		
Pinned		
Fixed		

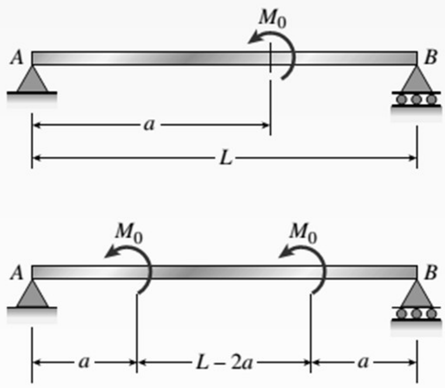
types of supports



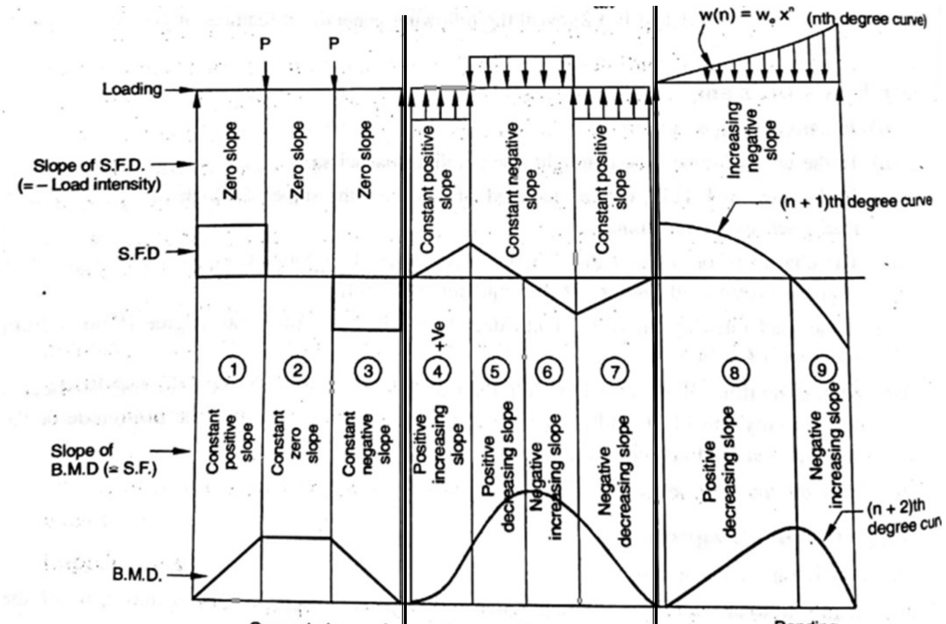
AKTU

SF and BM diagram

Load	0	0	Constant
Shear	Constant	Constant	Linear
Moment	Linear	Linear	Parabolic



PROBLEM 4.5-2

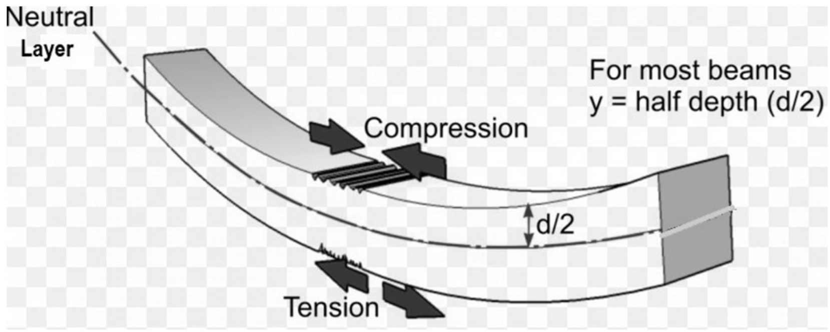


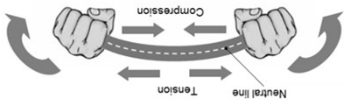
General shape of shear force and bending moment curves

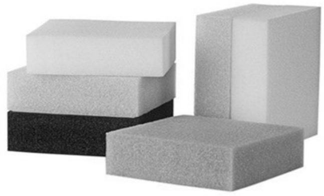
Bending

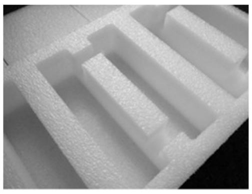
Flexural or Bending Stress in Straight Beams

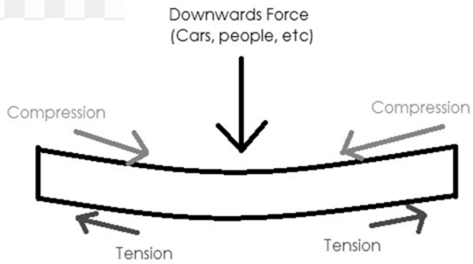
Flexural or Bending Stress in Straight Beams











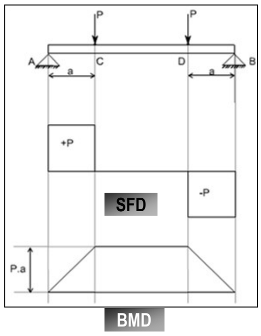
Concept of pure bending

Elastic Flexure formula : Bernoulli-Euler Flexure Formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

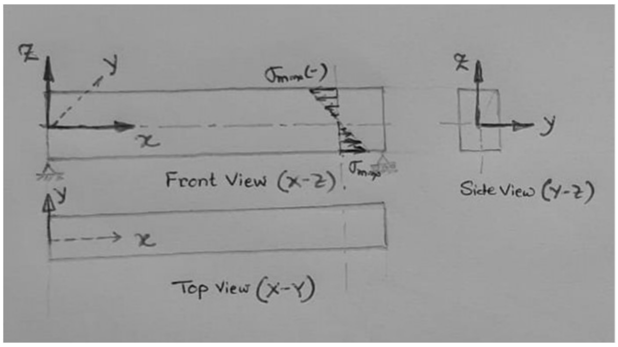
$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z}$$

Section Modulus $Z = I/y_{max}$

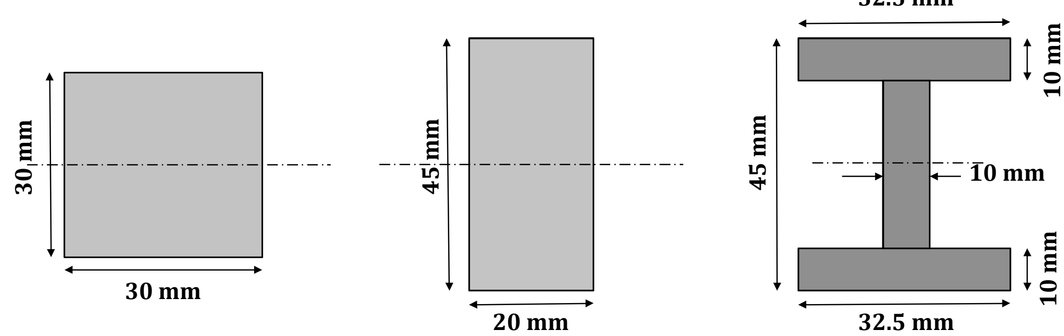


The following assumptions must be satisfied

- The beam is subjected to pure bending (shear force=0, no torsion, no axial loads).
- The beam is initially straight with a c/s that is constant throughout the length.
- The beam has an axis of symmetry in the plane of bending.
- Plane c/s of the beam remain plane during bending.
- Couples are assumed to be loaded in the plane of symmetry.
- The beam material is homogeneous, isotropic & obeys Hooke's law.

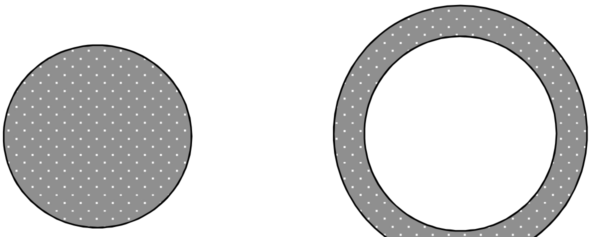


Calculation of MoI of the cross section about NA



	Square section	Rectangular section	I-section
Area (A)	900 sq. mm	900 sq. mm	900 sq. mm
MoI (I)	67500 mm4	151875 mm4	217500 mm4
Section Modulus (Z)	4500 mm3	6750 mm3	9666.6 mm3
Remarks		2.25 times	3.22 times 1.43 times

Calculation of MoI of the cross section about NA



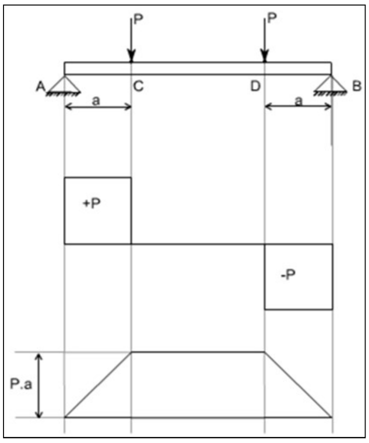
	Solid Circular	Hollow circular
Area (A)	sq. mm	sq. mm
MoI (I)		
Section Modulus (Z)		
Remarks		

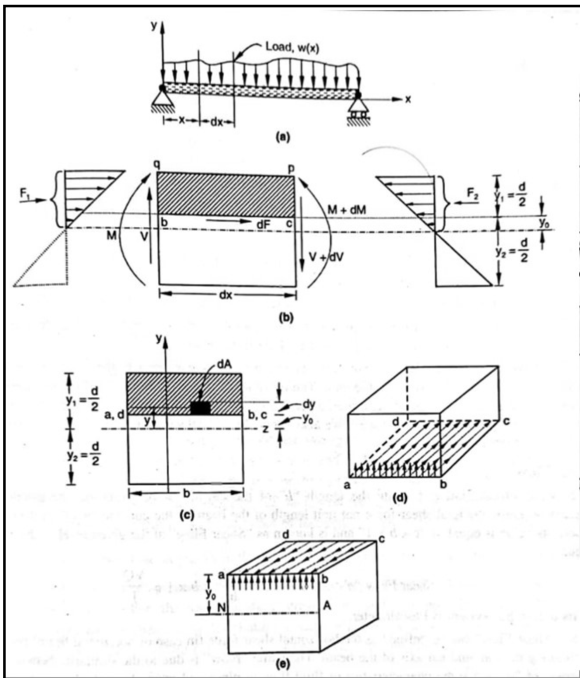
Example A beam 240 mm × 400 mm weighing 500 N/m is simply supported over a span of 4 m. It carries a concentrated load of 20,000 N at a distance 1 m from the left support. Determine (*i*) maximum stress in the beam (*ii*) maximum stress at the mid span (*iii*) stress at 50 mm below the top fibre at the section of maximum bending moment (*iv*) percentage of bending moment resisted by the central and outer half of the cross-section (*v*) nature of variation of maximum stress at a section along the length of beam (*vi*) nature of variation of curvature along the length of the beam and its maximum value. Given

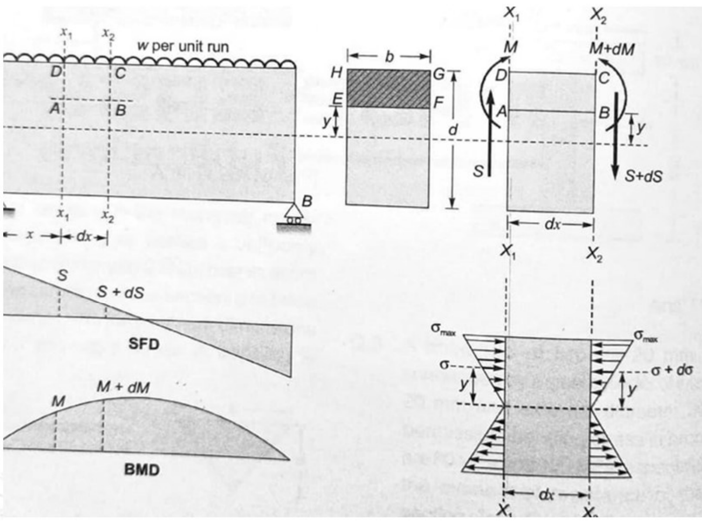
$$E = 1 \times 10^4 \text{ N/mm}^2$$

1. Determination of maximum bending moment or the bending moment at the required section by drawing S.F.D. and B.M.D.
2. Determination of centroid of the cross-section and thus fix up the position of neutral axis.
3. Calculate the moment of inertia of the cross-section about neutral axis.
4. Determine y_1 and y_2 *i.e.*, the distance of the extreme fibres from neutral axis.
5. Apply the "Flexure Formula".

Transverse Shear Stress in Straight Beams







Transverse shearing stresses due to bending

In addition to normal stresses induced by bending of a beam, transverse shearing stresses are induced between the elements, provided the bending moment varies along the length of the beam.

$$\tau = \frac{V}{Ib}\int_{z}^{c} y\,dA = \frac{VQ}{Ib}$$

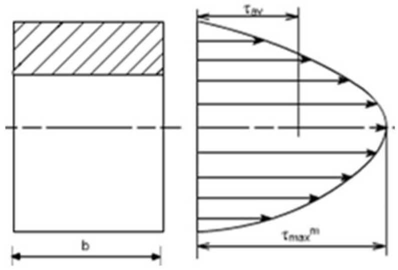
I : MOI of c/s about NA

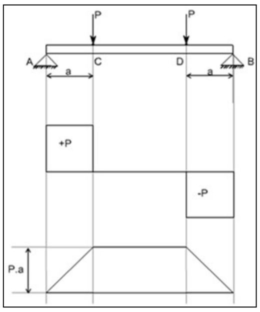
V: Shear force at beam section under consideration

b: Beam width at section

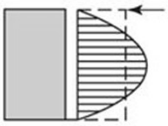
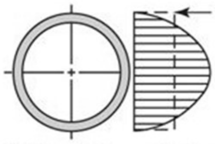
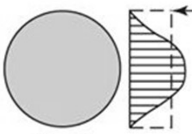
Q: Moment of area of element about NA

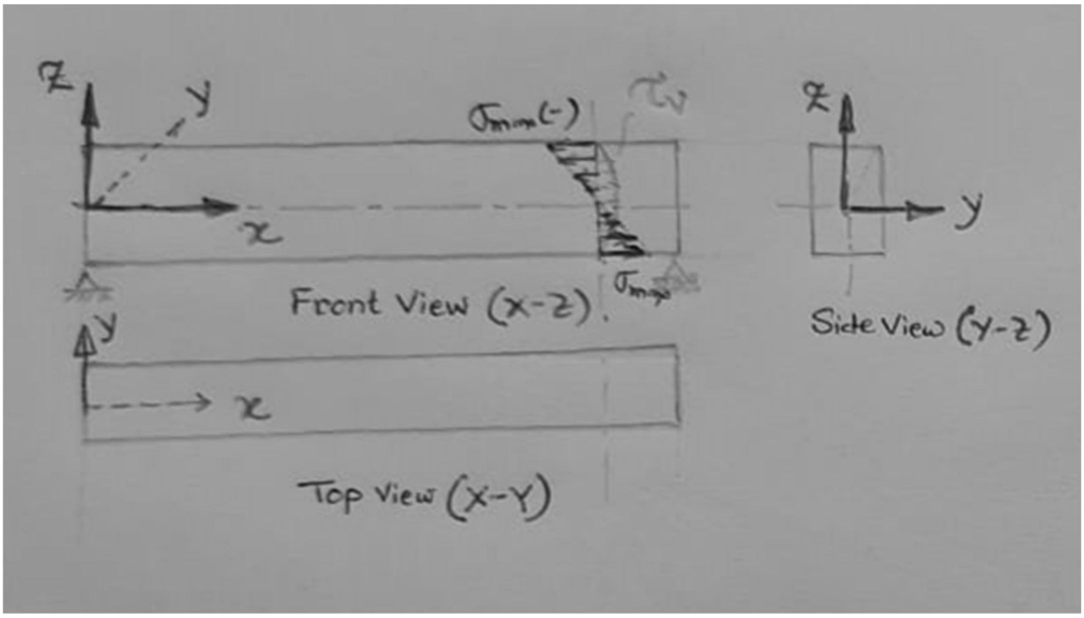
z: Location where shear stress is desired

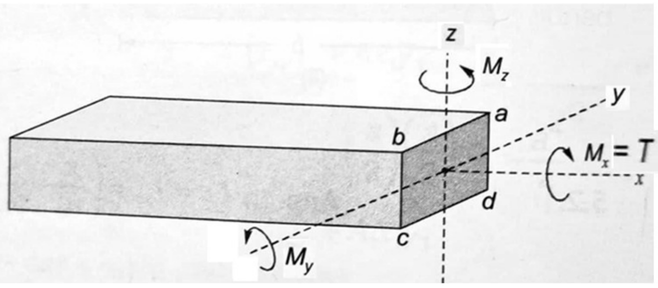




Maximum Values of Transverse Shear Stress

Beam Shape	Formula	Beam Shape	Formula
 Rectangular, $\tau_{avc} = \frac{V}{A}$	$\tau_{max} = \frac{3V}{2A}$	 Hollow, thin-walled round, $\tau_{avc} = \frac{V}{A}$	$\tau_{max} = \frac{2V}{A}$
 Circular, $\tau_{avc} = \frac{V}{A}$	$\tau_{max} = \frac{4V}{3A}$	 Structural I beam (thin-walled), A_{web}	$\tau_{max} \doteq \frac{V}{A_{web}}$





Transverse Axis: Y-axis & Z-axis

Moment about transverse axis causes Bending Moment.

Longitudinal Axis : X-axis

Moment about longitudinal axis causes rotation about that axis. This moment is called twisting moment or torsional moment or Torsion

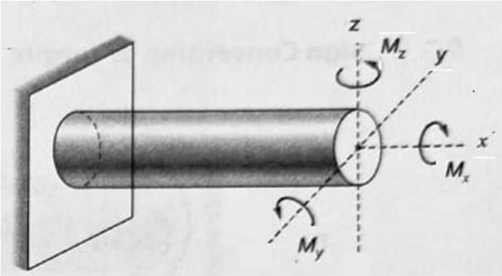
Difference between Bending Moment and Twisting Moment

Bending Moment (BM)

1. In bending, moment or couple acts about transverse axis
2. In bending, the plane of cross-section rotates about NA
3. Due to bending, normal stresses are produced which vary from zero at NA to maximum at top and bottom surface i.e., at extreme fibres.

Twisting Moment (TM)

1. In twisting, couple acts about longitudinal axis or polar axis.
2. In twisting, plane of cross-section rotates about polar axis. Hence, radii rotate about polar axis.
3. Due to twisting, only shear stresses are produced which acts in two mutually perpendicular planes and vary from zero at polar axis to maximum at the surface in circumferential directions.



A circular prismatic bar or circular shaft

My & Mz causes rotations about transverse axis. Hence My & Mz are Bending moments

Mx causes rotations about longitudinal axis or polar axis. Hence Mx is twisting moment or torsional moment or Torsion

Difference between Bending Moment and Twisting Moment

Bending Moment (BM)

1. In bending, moment or couple acts about transverse axis
2. In bending, the plane of cross-section rotates about NA
3. Due to bending, normal stresses are produced which vary from zero at NA to maximum at top and bottom surface i.e., at extreme fibres.

Twisting Moment (TM)

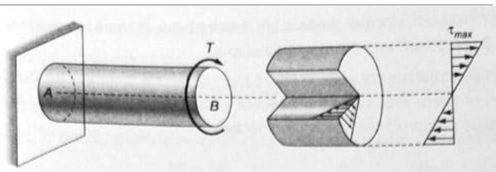
1. In twisting, couple acts about longitudinal axis or polar axis.
2. In twisting, plane of cross-section rotates about polar axis. Hence, radii rotate about polar axis.
3. Due to twisting, only shear stresses are produced which acts in two mutually perpendicular planes and vary from zero at polar axis to maximum at the surface in circumferential directions.

Torsion of circular shafts

Torsion formula :

$$\underbrace{\frac{\tau}{\rho}}_{\text{Strength Criteria}} = \overbrace{\frac{T}{J} = \frac{G\theta}{L}}^{\text{Rigidity or Stiffness Criteria}}$$

where, J = Polar moment of inertia
G = Modulus of rigidity
θ = Angle of twist in radian
L = Length of shaft
R = Radius of shaft



Assumptions

- The bar/member must be straight & circular.
- The sections under consideration are remote from the point of application of the load.
- That portion of the member being considered must be free from abrupt change in c/s.
- Adjacent c/s originally plane & parallel remain plane & parallel after twisting.
- The material is homogeneous, isotropic & obeys Hooke's law.

Torsion formula

$$\underbrace{\frac{\tau}{\rho}=\frac{T}{J}}_{\text{Strength Criteria}}=\overbrace{\frac{G\theta}{L}}^{\text{Rigidity Criteria}}$$

$$\tau=\frac{T\rho}{J}$$

$$\tau_{max}=\frac{T}{J/\rho_{max}}=\frac{T}{Z_p}$$

Z_p=Polar modulus
=Torsional Section Modulus

Where, $J=\int_A \rho^2 dA$ = Poller Moment Of Inertia $=\frac{\pi d^4}{32}$ For Dia. d.

So, $T=G\theta\frac{\pi d^4}{32}$ & $\theta=\frac{32}{\pi}\frac{T}{Gd^4}$ And Total Angle Of Twist Is, $\varphi=\theta l=\frac{TL}{GJ}$.

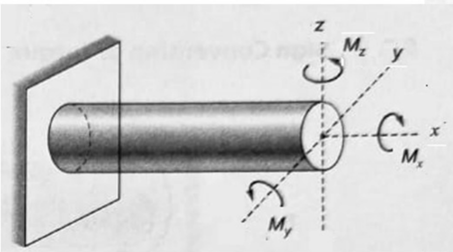
GJ=Torsional Rigidity

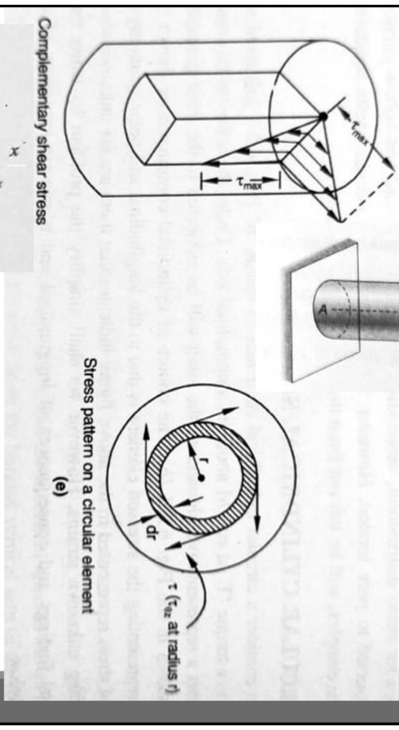
The τ_{max} Of A Solid Shaft In Twist Is, $\tau_{max}=\frac{Tr}{J}=\frac{16T}{\pi d^3}$.

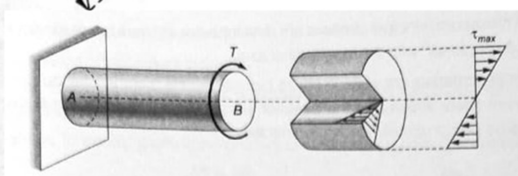
The Shaft Diameter Is, $d=\sqrt[3]{\frac{16T}{\pi\tau}}$.

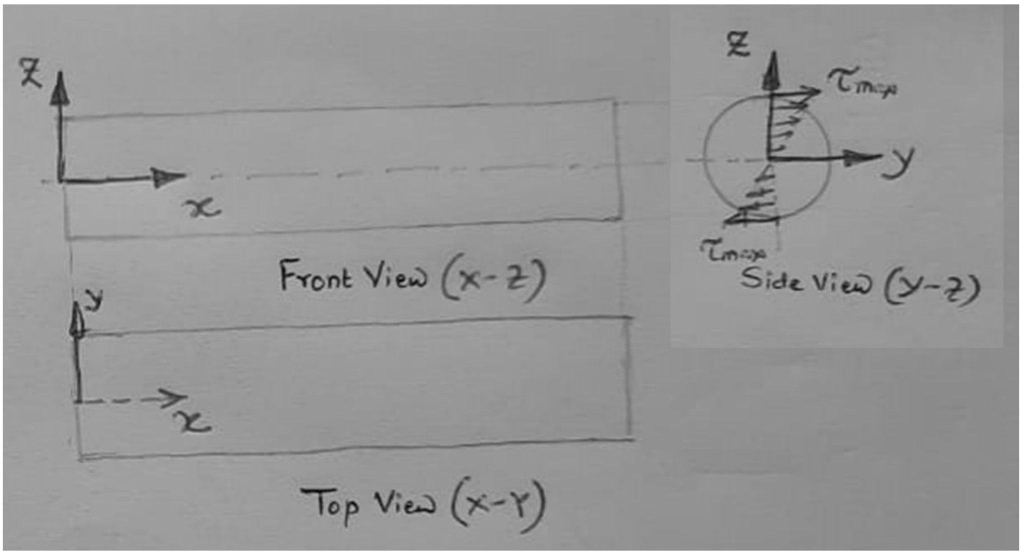
And τ At Any Point At A Distance ρ From Center Is, $\tau=\frac{T\rho}{J}$.

For Hollow Shaft, J Is Replaced By The Value, $J=\frac{\pi}{32}(d_o^4-d_i^4)$.



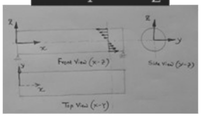




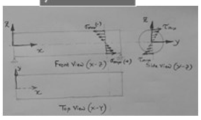


Example 6.1: A steel shaft circular in cross-section has to withstand a torque of 12×10^3 N.m. If the shearing stress is not to exceed 45 MPa and angle of twist has to remain within one degree per 5 m length of the shaft, find (*a*) the minimum diameter of the solid shaft, (*b*) minimum diameter of hollow shaft if external diameter is twice the internal diameter, (*c*) percentage of material saved in case of the above hollow shaft. Given G = 8×10^4 MPa.

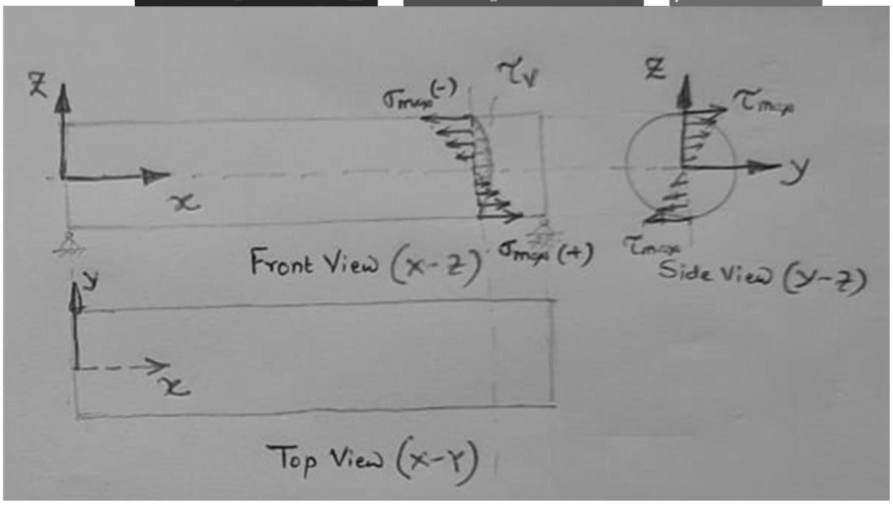
$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z}$$

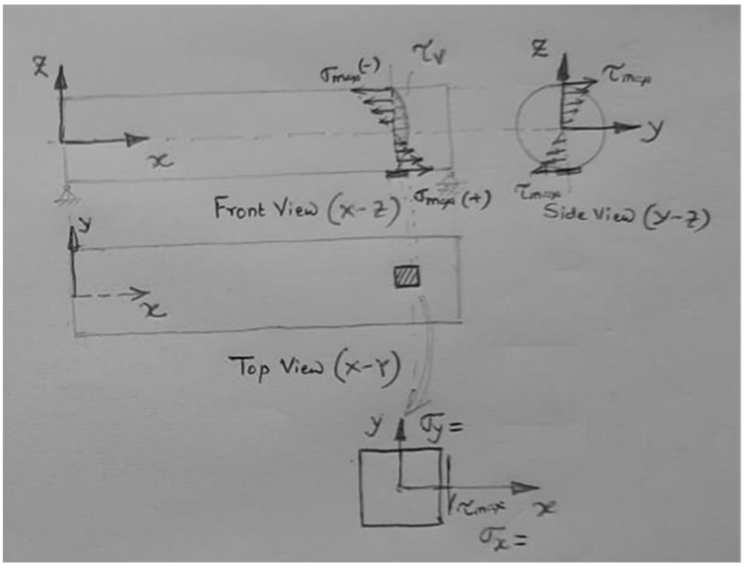


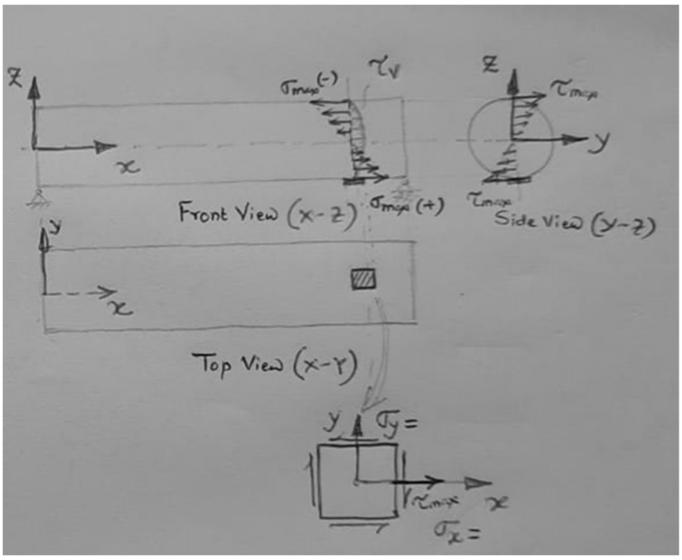
$$\frac{\tau}{\rho} = \frac{T}{J} = \frac{G\theta}{L}$$

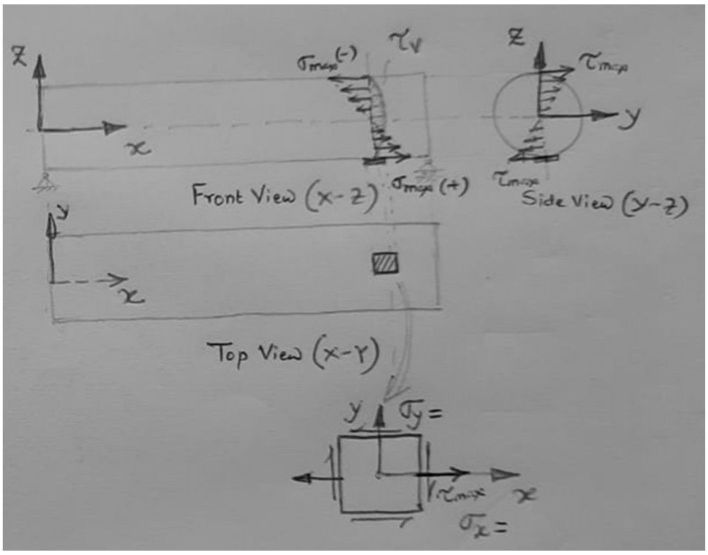


$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z} \qquad \tau = \frac{V}{Ib}\int_{z}^{c} y\,dA = \frac{VQ}{Ib} \qquad \frac{\tau}{\rho} = \frac{T}{J} = \frac{G\theta}{L}$$

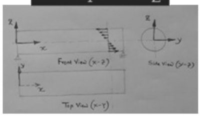




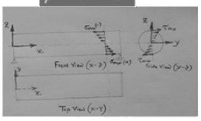

z
x
σ_{max} (-)
τ_v
σ_{max} (+)
Front View (x-z)
z
τ_{max}
y
τ_{max}
Side View (y-z)
y
x
Top View (x-y)
y
σ_y =
τ_{max}
x
σ_x =


z
x
σ_{max} (-)
τ_v
σ_{max} (+)
Front View (x-z)
z
τ_{max}
y
τ_{max}
Side View (y-z)
y
x
Top View (x-y)
y
σ_y =
τ_{max}
x
σ_x =

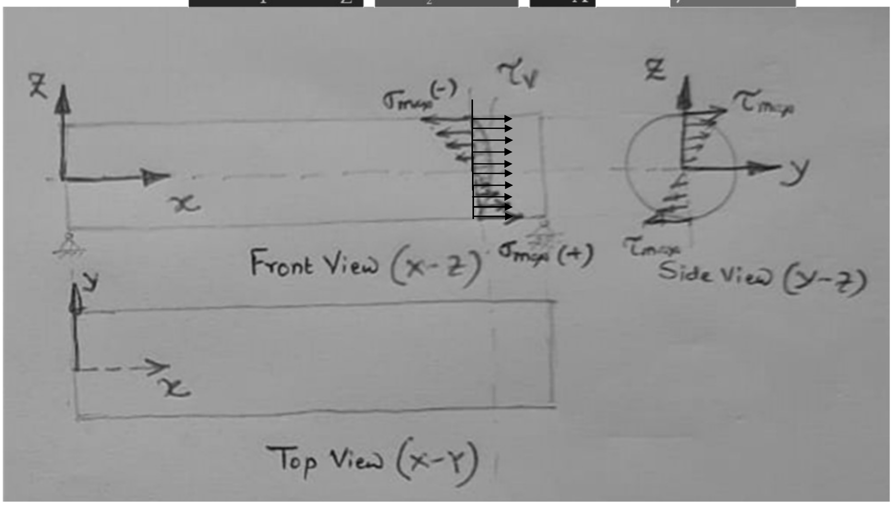
$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z}$$

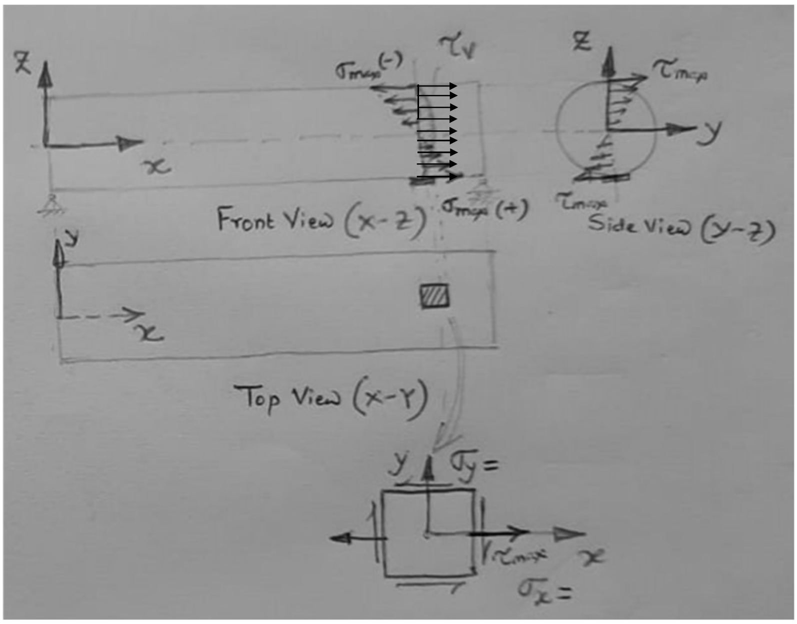


$$\frac{\tau}{\rho} = \frac{T}{J} = \frac{G\theta}{L}$$



$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z} \qquad \tau = \frac{V}{Ib}\int_z^c y\,dA = \frac{VQ}{Ib} \qquad \sigma = \frac{P}{A} \qquad \frac{\tau}{\rho} = \frac{T}{J} = \frac{G\theta}{L}$$





Deflection of beams

Deflection due to bending:

Methods ▪ Double-integration method.

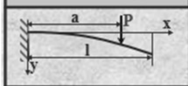
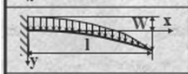
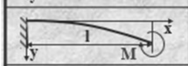
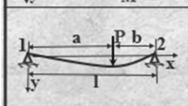
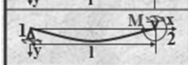
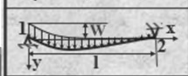
Governing equation

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$y = \iint \frac{M}{EI}\,dx + C_1x + C_2$$

Where, y is the lateral deflection of the beam at any point x along its length

Beam Deflection Formulas

Beam Type	Slope, θ	Equation Of ***Elastic Line***, y = f(x)	δ_{max}
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x), [0 < x < a]$ $y = \frac{Pa^2}{6EI}(3x - a), [a < x < l]$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
	$\theta = \frac{wl^3}{6EI}$	$y = \frac{wx^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{wl^4}{8EI}$
	$\theta = \frac{Wl^3}{24EI}$	$y = \frac{Wx^2}{120EI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{Wl^4}{30EI}$
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}(l^2 - x^2 - b^2), [0 < x < a]$ $y = \frac{Pb}{6lEI}\left[\frac{1}{b}(x - a)^3 + (l^2 - b^2)x - x^3\right],$ $[a < x < l]$	$\delta_{max} = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,lEI}$ At $x = \sqrt{\frac{l^2 - b^2}{3}}$ $\delta_{l/2} = \frac{Pb}{48EI}(3l^2 - 4b^2), [a > b]$
	$\theta_1 = \theta_2 = \frac{wl^3}{24EI}$	$y = \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \frac{5wl^4}{384EI}$
	$\theta_1 = \frac{Ml}{6EI} = \frac{1}{2}\theta_2$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}EI}$ At $x = \frac{l}{\sqrt{3}}$ $\delta_{l/2} = \frac{Ml^2}{16EI}$
	$\theta_1 = \frac{7Wl^3}{360EI}$ $\theta_2 = \frac{Wl^3}{45EI}$	$y = \frac{Wx}{360EI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = 0.00652\frac{Wl^4}{EI}$ At $x = 0.5191$ $\delta_{l/2} = 0.00651\frac{Wl^4}{EI}$