Superposition of a harmonic oscillations

- collinear
  - diff Frequency → Beats
  - Same Frequency → Beats
- $\perp r$ present in Syll
  - Same Frequency
  - diff Frequency

$x_1 = A_1 \cos(\underline{\omega t} + \phi)$

$x_2 = A_2 \cos(\underline{\omega t} + \phi)$

$x = x_1 + x_2 \ldots$

$x = A \cos(\underline{\omega t} + \delta)$ $\qquad A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)$

$\tan\delta = (A_1 \sin\phi_1 + A_2 \sin\phi_2) / (A_1\cos\phi_1 + A_2\cos\phi_2)$

Superposition of two perpendicular oscillation of same frequency

$x = A_1 \cos\omega t \quad \text{---- } (1)$

$y = A_2 \cos(\omega t + \delta) \quad \text{---- } (2)$

$\cos\omega t = \frac{x}{A_1} \Rightarrow \sin\omega t = \left(1 - \frac{x^2}{A_1^2}\right)^{1/2} \quad \text{---- } (3)$

$\cos(\omega t + \delta) = y/A_2 \quad \cancel{\Rightarrow \sin\omega t}$

$\cos\omega t \cos\delta - \sin\omega t \sin\delta = y/A_2$

$x/A_1 \cos\delta - \left(1 - \frac{x^2}{A_1^2}\right)^{1/2} \sin\delta = y/A_2$

$x/A_1 \cos\delta - y/A_2 = \left(1 - \frac{x^2}{A_1^2}\right)^{1/2} \sin\delta$

$\frac{x^2}{A_1^2}\cos^2\delta + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}\cos\delta = \sin^2\delta \quad \text{---- } (4)$

Ellipse

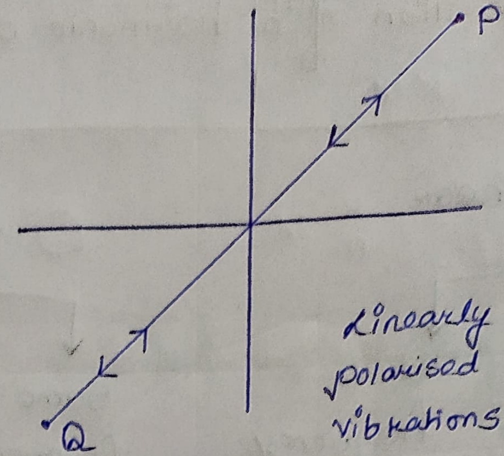
i) $\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2} = 0 \quad \text{---- } (5)$

$\Rightarrow \left(y - x\frac{A_2}{A_1}\right) = 0$ $\qquad$ coincident Straight lines $\Rightarrow y = \frac{A_2}{A_1} x$

$x = A_1 \cos \omega t$

$y = A_2 \cos \omega t$

ωt	x	y
0	A_1	A_2
$\pi/2$	0	0
π	$-A_1$	$-A_2$



Linearly polarised vibrations

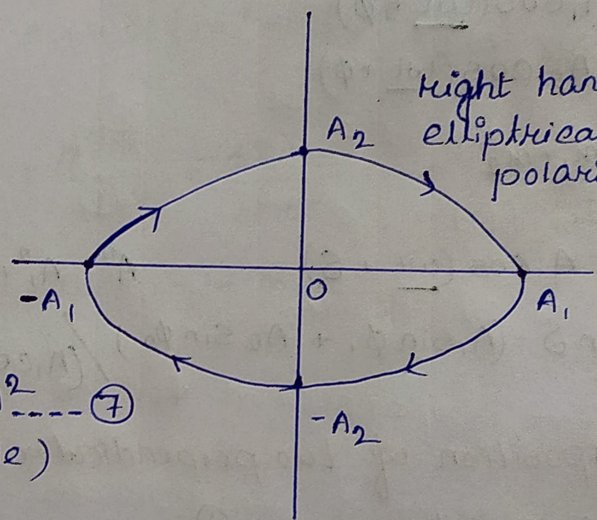
ii) $\delta = \pi/2$

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1 \text{ ...... } (6)$$

$x = A_1 \cos \omega t$

$y = A_2 \cos(\omega t + \pi/2)$

$y = -A_2 \sin(\omega t)$



Right handed elliptrically polarirized

if $A_1 = A_2 = A \Rightarrow x^2 + y^2 = A^2$ ---- (7)
(eq of circle)

→ Right angle
→ Same frequency
→ Same amplitude

iii) $\delta = \pi$

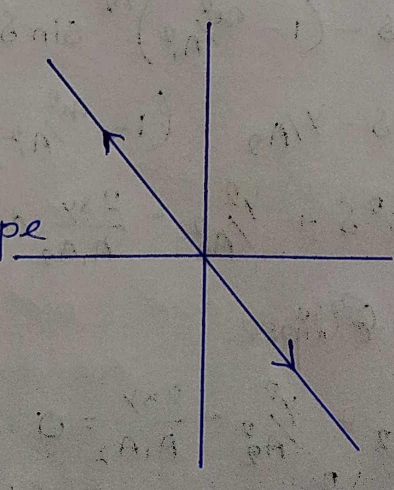
$x = \cancel{y} A_1 \cos \omega t$

$y = A_2 \cos(\omega t + \pi)$

$= -A_2 \cos \omega t$

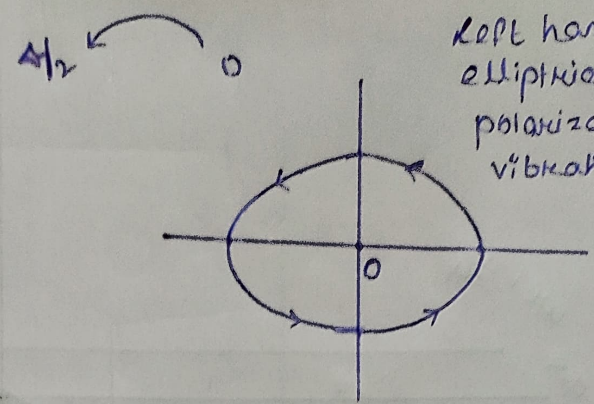
$$\left(y + \frac{A_2 x}{A_1}\right)^2 = 0$$

$y = -\frac{A_2}{A_1} x$ -ve slope



iv) $\delta = 3\pi/2$

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$



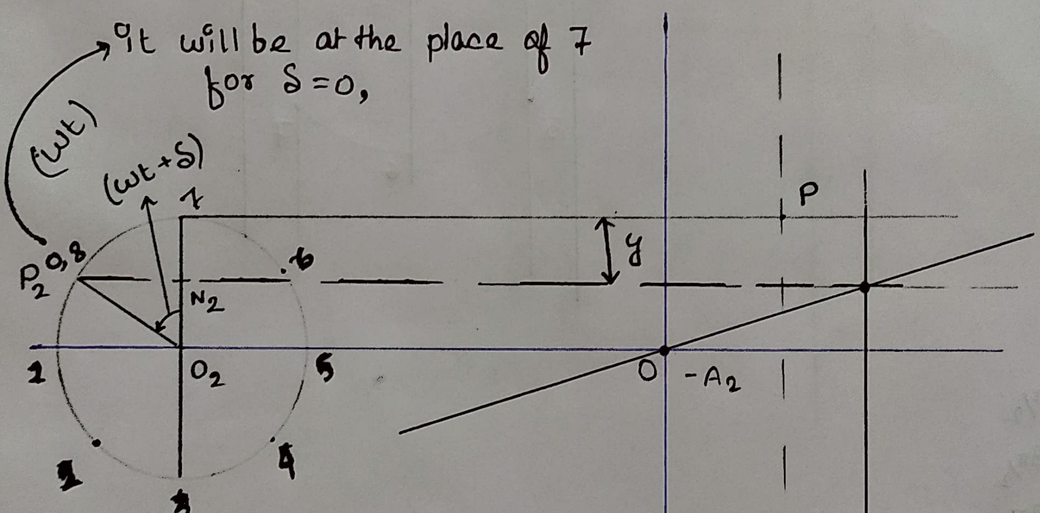
Left handed elliptically polarized vibration

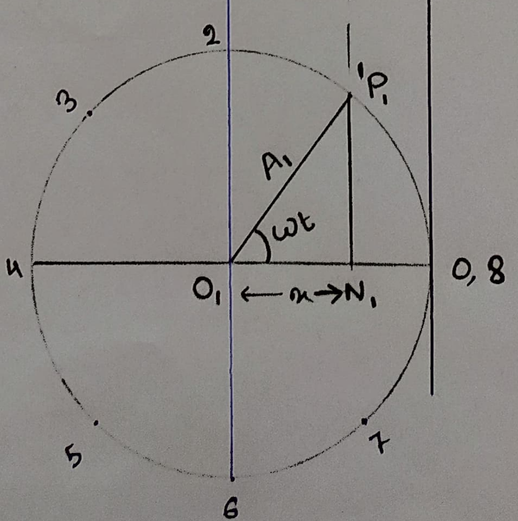
ⓑ Graphical Method

$x = A_1 \cos\omega t$

$y = A_2 \cos(\omega t + \delta)$

Resultant → by double application of the rotating vector technique.





$$\frac{O_1N_1}{O_1P_1} = \frac{x}{A_1}$$

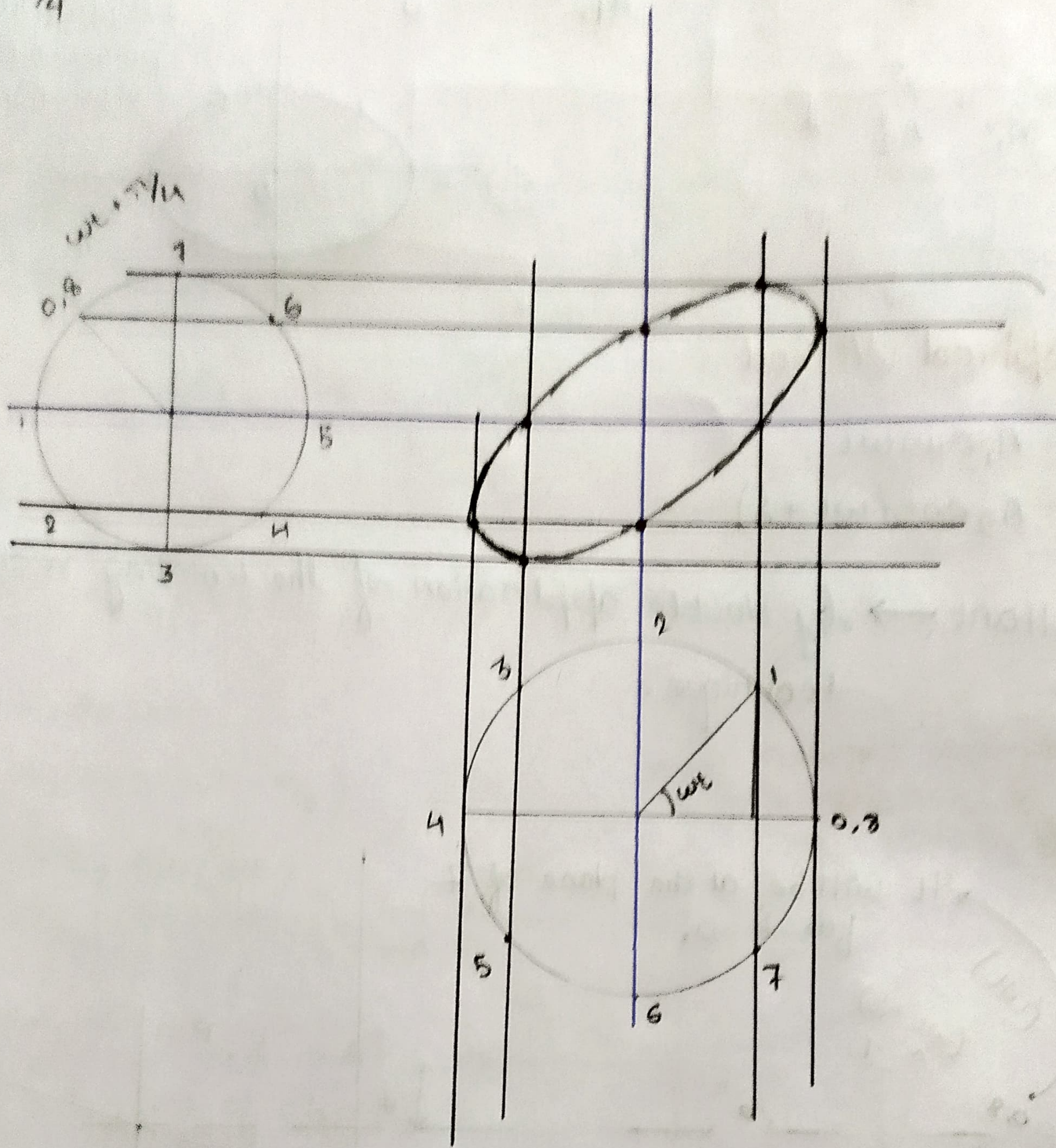
$x = A_1 \cos\omega t$

$\delta = 0$

$x = A_1 \cos\omega t$

$y = A_2 \cos(\omega t + 0)$

S. π/4



H.W π/2

3π/2

2π

$\delta = \pi/2$