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CHAPTER 12

Q. 12.4

Fundamentals of Heat and Mass Transfer [EXP-7090]
(https://holooly.com/sources/fundamentals-of-heat-and-mass-transfer-exp-7090/)

Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000 K. Calculate the emissive power of the radiation that emerges from a small aperture on theenclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated? What is the wavelength λ_2 above which 10% of the emission is concentrated? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure?

Step-by-Step

Verified Answer

Report Solution (https://holooly.com/report-a-problem/)

Known: Large isothermal enclosure at uniform temperature.

Find:

1. Emissive power of a small aperture on the enclosure.

2. Wavelengths below which and above which 10% of the radiation is concentrated.

3. Spectral emissive power and wavelength associated with maximum emission.

4. Irradiation on a small object inside the enclosure.

Assumptions: Areas of aperture and object are very small relative to enclosure surface.

Analysis:

1. Emission from the aperture of any isothermal enclosure will have the characteristics of blackbody radiation. Hence, from Equation 12.32,

$E_b = \sigma T^4$ (12.32)

$$E = E_b(T) = \sigma T^4 = 5.670 \times 10^{-8} W/m^2 \cdot K^4 (2000K)^4$$

$$E = 9.07 \times 10^5 W/m^2$$

2. The wavelength λ_1 corresponds to the upper limit of the spectral band $(0 \rightarrow \lambda_1)$ containing 10% of the emitted radiation. With $F_{(0 \rightarrow \lambda_1)} = 0.10$ it follows from Table 12.2 that $\lambda_1 T = 2195 \mu m \cdot K$. Hence

Table 12.2 Blackbody radiation functions

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λT $(\mu m \cdot K)$	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{\max},T)}$
200	0	0.375034×10^{-27}	0

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[illegible]	0	0.490335×10^{-13}	0
600	0	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.00779	0.134411×10^{-4}	0.186082
1,600	0.019718	0.24913	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.18312	0.701292	0.970891
2,800	0.227897	0.720239	0.997123

2,[illegible]8	0.250108	0.722318×10^{-4}	1
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.69672
4,600	0.57928	0.467343	0.647004
4,800	0.607559	0.433109	0.59961
5,000	0.633747	0.400813	0.554898
5,200	0.65897	0.370580×10^{-4}	0.513043
5,400	0.68036	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671

[illegible]	0.737818	0.270121	0.373965
6,200	0.75414	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.25409
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.20336
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.92371	0.560522	0.0776

[illegible]	0.93189	0.483321	0.066913
11,500	0.939959	0.418725	0.05797
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.968933	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.98086	0.908240×10^{-6}	0.012574
20,000	0.985602	0.62331	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.99534	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

The wavelength λ_2 corresponds to the lower limit of the spectral band $(\lambda_2 \rightarrow \infty)$ containing 10% of the emitted radiation. With

$$F_{(\lambda_2 \rightarrow \infty)} = 1 - F_{(0 \rightarrow \lambda_2)} = 0.1$$

$$F_{(0 \rightarrow \lambda_2)} = 0.9$$

it follows from Table 12.2 that $\lambda_2 T = 9382 \mu m \cdot K$. Hence

$$\lambda_2 = 4.69 \mu m$$

3. From Wien's displacement law, Equation 12.31, $\lambda_{\max} T = 2898 \mu m \cdot K$ Hence

$\lambda_{\max} T = C_3$ (12.31)

$$\lambda_{\max} = 1.45 \mu m$$

The spectral emissive power associated with this wavelength may be computed from Equation 12.30 or from the third column of Table 12.2. For $\lambda_{\max}T = 2898 \mu m \cdot K$ it follows from Table 12.2 that

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T)-1]} \quad (12.30)$$

$$I_{\lambda,b}(1.45 \mu m, T) = 0.722 \times 10^{-4} \sigma T^5$$

Hence

$$I_{\lambda,b}(1.45 \mu m, 2000K) = 0.722 \times 10^{-4} (\mu m \cdot K \cdot sr)^{-1} \times 5.67$$

$$\times 10^{-8} W/m^2 \cdot K^4 (2000K)^5$$

$$I_{\lambda,b}(1.45 \mu m, 2000K) = 1.31 \times 10^5 W/m^2 \cdot sr \cdot \mu m$$

Since the emission is diffuse, it follows from Equation 12.16 that

$$E_{\lambda}(\lambda) = \pi I_{\lambda,e}(\lambda) \quad (12.16)$$

$$E_{\lambda,b} = \pi I_{\lambda,b} = 4.12 \times 10^5 W/m^2 \cdot \mu m$$

Irradiation of any small object inside the enclosure may be approximated as being equal to emission from a blackbody at the enclosure surface temperature. Hence $G = E_b(T)$, in which case

$G = 9.07 \times 10^5 W/m^2$