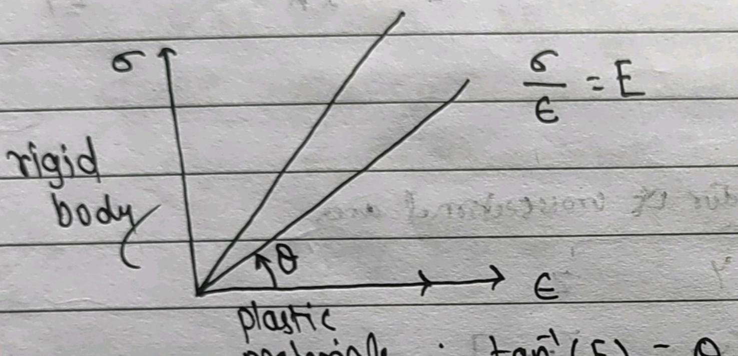
Strain causes Stress

stress is internal force while load is external one

$\sigma \propto \epsilon$ $\qquad$ ϵ is strain

σ is stress

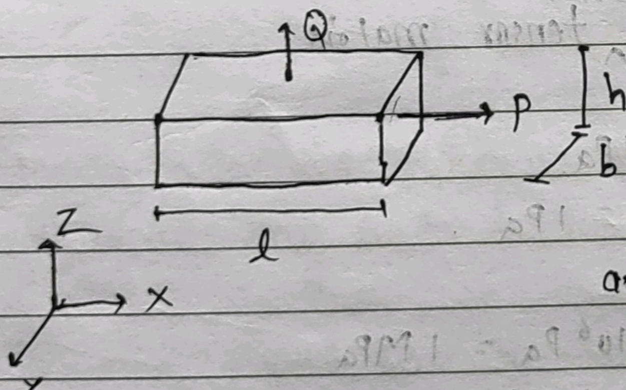
$\sigma = E\epsilon$



$\frac{\sigma}{\epsilon} = E$

plastic materials : $\tan^{-1}(E) = \theta$

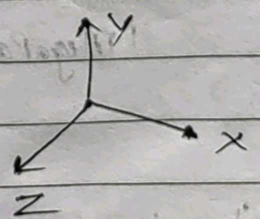
* Normal stress (σ) $\longrightarrow$



$\sigma_{xx} = \frac{P}{b \cdot h}$

another normal stress $= \sigma_{yy} = \frac{Q}{l \cdot b}$

anticlockwise rot^n is +ve

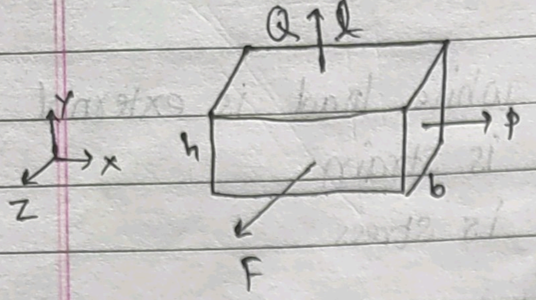


first subscript is direction of load

second subscript is the perpendicular direction of considered cross sectional area

Both subscripts in Normal stress are same.

* Shear Stress (τ) ⟶



$$\tau_{xy} = \frac{P}{lb} \qquad \tau_{zx} = \frac{F}{bh}$$

$$\tau_{yx} = \frac{Q}{bh} \qquad \tau_{yz} = \frac{Q}{lh}$$

$$\tau_{xz} = \frac{P}{lh} \qquad \tau_{zy} = \frac{F}{lb}$$

Creating a matrix,

perp. dir of crosssectional area

$$\begin{array}{c} \\ \text{dir of load} \left\{ \begin{array}{c} X \\ Y \\ Z \end{array} \right. \end{array} \begin{array}{c} \overbrace{\begin{array}{ccc} X & Y & Z \end{array}} \\ \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \end{array}$$

This is called stress tensor matrix

unit: $1\ N/m^2 = 1\ Pa$

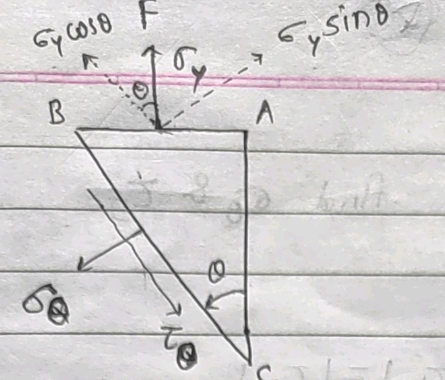
$1\ N/10^6\ mm^2 = 1\ Pa$

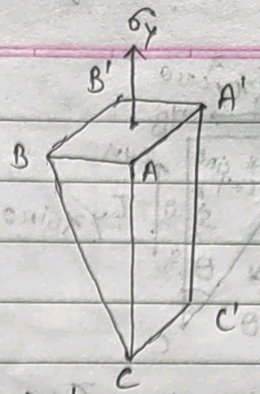
$1\ N/mm^2 = 10^6\ Pa = 1\ MPa$

1 MegaPascle $= 1\ N/mm^2$

Tensile stresses cause increment in length ∴ considered positive
Compressive stress causes decreament ∴ considered negative.

Normal stress causes longitudinal change in body.
Shear stress causes rotational change in body.





We'll equate forces here. $BB' = 1 \quad AA' = 1$

$$\sigma_\theta \cdot BC \cdot BB' = \sigma_y \sin\theta \cdot AB \cdot AA'$$

$$\sigma_\theta \, BC = \sigma_y \sin\theta \cdot BC \sin\theta \qquad \because AB = BC\sin\theta$$

$$\sigma_\theta = \sigma_y \sin^2\theta$$

$$= \cancel{\frac{1}{2}\sigma_y(1-\cos 2\theta)} = \frac{1}{2}\sigma_y(1-\cos 2\theta)$$

resolving forces parallel to BC,

$$\tau_\theta \cdot BC \cdot 1 = \sigma_y \cdot AB \cdot 1 \cdot \cos\theta$$

$$= \sigma_y \cos\theta \cdot BC \sin\theta \qquad \because AB = BC \cdot \sin\theta$$

$$\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta$$

Maximum normal force is called major principle & denoted by σ_1 & corresponding θ to it is denoted by θ_{1p}

Minimum normal force is called minor principle & denoted by σ_2 & corresponding angle to it is ~~call~~ denoted by θ_{2p}

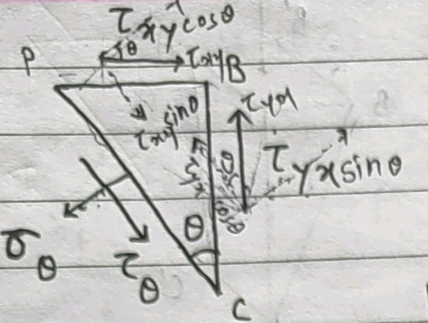
$$(\tau_\theta)_{max} = \frac{\sigma_y}{2} \Rightarrow \theta_{1s} = \pi/4 \qquad |\theta_{1s} - \theta_{2s}| = \pi/2$$

$$(\tau_\theta)_{min} = -\frac{\sigma_y}{2} \Rightarrow \theta_{2s} = 3\pi/4$$

$$(\sigma_\theta)_{max} = \sigma_y \Rightarrow \theta_{1p} = \pi/2 \qquad |\theta_{1p} - \theta_{2p}| = \pi/2$$

$$(\sigma_\theta)_{min} = 0 \Rightarrow \theta_{2p} = 0$$

$$|\theta_{1s} - \theta_{1p}| = \pi/4 \qquad |\theta_{2s} - \theta_{2p}| = \pi/4$$



find σ_θ & τ_θ

$|\tau_{xy}| = |\tau_{yx}|$

we know, $\tau_{xy} = -\tau_{yx}$

resolving forces perp. to PC

$\sigma_\theta \cdot PC \cdot 1 = \tau_{xy}\cos\theta \cdot PB \cdot 1 + \tau_{yx}\sin\theta \cdot BC \cdot 1$

Numerically τ_{xy} & τ_{yx} are equal.

	amplitude	dimension
σ_1	τ_{xy}	$\theta_{1p} = 45^\circ$
σ_2	$-\tau_{xy}$	$\theta_{2p} = 135^\circ$
τ_{max}	τ_{xy}	$\theta_{1s} = 90^\circ$
τ_{min}	$-\tau_{xy}$	$\theta_{2s} = 0^\circ$

$\therefore \sigma_\theta \cdot PC = \tau_{xy}\cos\theta \cdot PC\sin\theta + \tau_{xy}\sin\theta\, PC\cos\theta$

$\therefore \sigma_\theta = 2\tau_{xy}\sin\theta\cos\theta$

$\therefore \boxed{\sigma_\theta = \tau_{xy}\sin 2\theta}$

resolving forces parallel to PC,

$\tau_\theta \cdot PC \cdot 1 + \tau_{xy}\sin\theta \cdot PB = \tau_{xy}\cos\theta\, BC$

$\therefore \tau_\theta \cdot PC = \tau_{xy} \cdot PC \cdot \cos^2\theta - \tau_{xy} \cdot PC \cdot \sin^2\theta$

$\therefore \tau_\theta = \tau_{xy}\cos 2\theta$

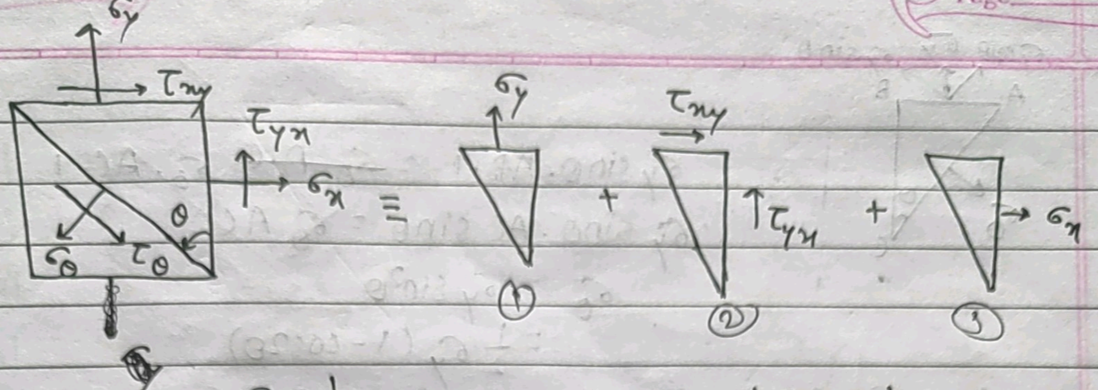
$\sigma_1 = (\sigma_\theta)_{max} = \tau_{xy}$ for $\theta_{1p} = 45^\circ$

$\sigma_2 = (\sigma_\theta)_{min} = -\tau_{xy}$ for $\theta_{2p} = 135^\circ$

$(\tau_\theta)_{max} = \tau_{xy}$ for $\theta_{1s} = 90^\circ$

$(\tau_\theta)_{min} = -\tau_{xy}$ for $\theta_{2s} = 0^\circ$

~~TMP~~



$$\sigma_\theta = \frac{1}{2}\sigma_y(1-\cos 2\theta) + \tau_{xy}\sin 2\theta + \frac{1}{2}\sigma_x(1+\cos 2\theta) \quad \text{--- (1)}$$

$$= \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta + \tau_{xy}\cos 2\theta - \frac{1}{2}\sigma_x \sin 2\theta \quad \text{--- (2)}$$

$$= -\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

lets find $(\sigma_\theta)_{max}$ & $(\sigma_\theta)_{min}$

for extremum, the condition is —

$$\frac{d\sigma_\theta}{d\theta} = 0 \Rightarrow \tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)} \quad \text{--- (3)}$$

$$\theta_{2p} \text{ or } \theta_{1p} = \frac{1}{2}\tan^{-1}\left[\frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}\right] \pm 90^\circ$$

Similarly, if $\frac{d\tau_\theta}{d\theta} = 0 \Rightarrow \frac{\frac{\sigma_x-\sigma_y}{2}}{\tau_{xy}} = \tan 2\theta_s$ — (4)

We see from (3) & (4),

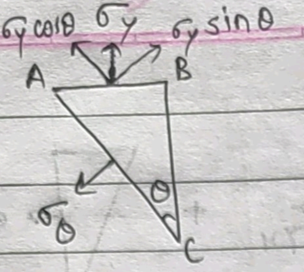
$$1 - \tan 2\theta_p \tan 2\theta_s = 0$$

we know, $\tan(2\theta_p + 2\theta_s) = \frac{\tan 2\theta_p + \tan 2\theta_s}{1 - \tan 2\theta_p \tan 2\theta_s}$

$\because$ denom is 0, $\Rightarrow 2\theta_p + 2\theta_s = 90^\circ$

$$\therefore \theta_p + \theta_s = 45^\circ$$

xplanation ①



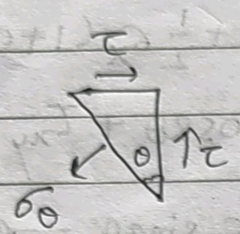
$$\sigma_y \sin\theta \cdot AB \cdot 1 = \cancel{\sigma_\theta A \cdot C}\ \sigma_\theta \cdot AC \cdot 1$$

$$\sigma_y \sin\theta \cdot AC \sin\theta = \sigma_\theta AC$$

$$\sigma_\theta = \sigma_y \sin^2\theta$$

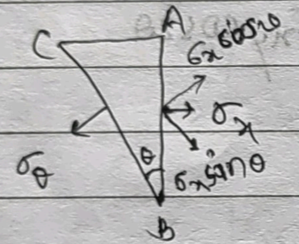
$$= \frac{1}{2}\sigma_y (1-\cos 2\theta)$$

②



we know, $\sigma_\theta = \tau \sin 2\theta$

③



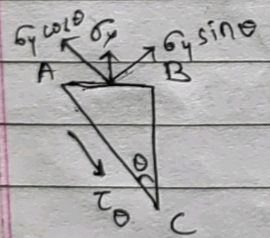
$$\sigma_x \cos\theta \cdot AB \cdot 1 = \sigma_\theta \cdot BC \cdot 1$$

$$\sigma_x \cos\theta \cdot BC \cos\theta = \sigma_\theta BC$$

$$\sigma_\theta = \sigma_x \cos^2\theta$$

$$= \frac{1}{2}\sigma_x (1+\cos 2\theta)$$

①

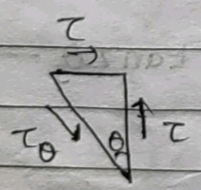


$$\sigma_y \cos\theta \cdot AB = \tau_\theta \cdot AC$$

$$\sigma_y \cos\theta \sin\theta \cdot AC = \tau_\theta AC$$

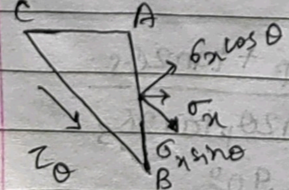
$$\therefore \tau_\theta = \frac{1}{2}\sigma_y \sin 2\theta$$

②



we know, $\tau_\theta = \tau \cos 2\theta$

③



$$\sigma_x \sin\theta \cdot AB + \tau_\theta BC = 0$$

$$\sigma_x \sin\theta \cdot BC \cos\theta = -\tau_\theta BC$$

$$\therefore \tau_\theta = -\frac{1}{2}\sigma_x \sin 2\theta$$

	σ_x	σ_y	τ_{xy}
1]	10	-10	0
2]	0	10	-10
3]	5	7.5	2.5
4]	-5	-5	5

1] $\tan 2\theta_p = \dfrac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \dfrac{0}{10}$ $\theta_{1p} = 0°$ $\theta_{2p} = 90°$

$\theta_{1s} = 45°$ $\theta_{2s} = 135°$

$\tan 2\theta_s = \dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \infty$ $\theta_{1s} = 45°$ $\theta_{2s} = 135°$

2]

2] $\tan 2\theta_p = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \dfrac{-10}{-5}$ $\theta_{1p} = 121.71°$ $\theta_{2p} = 211.71°$

$\tan 2\theta_s = \dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \dfrac{-1}{-2}$ $\theta_{1s} = 166.71°$ $\theta_{2s} = 256.71°$

3] $\tan 2\theta_p = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \cancel{-2}\ \dfrac{2.5}{-1.25}$ $\theta_{1p} = \cancel{46.56}\ 58.28°$ $\theta_{2p} = \cancel{206}\ 148.28°$

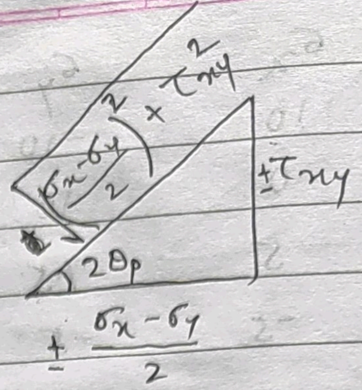
$\tan 2\theta_s = \dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \dfrac{1}{-2}$ $\theta_{1s} = 103.28°$ $\theta_{2s} = 193.28°$

4] $\tan 2\theta_p = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \infty = \dfrac{5}{0}$ $\theta_{1p} = \cancel{90}\ 45°$ $\theta_{2p} = 135°$

$\tan 2\theta_s = \dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = 0 = \dfrac{0}{5}$ $\theta_{1s} = 90°$ $\theta_{2s} = 180°$

for maximum or minimum case,

$$\tan 2\theta_p = \frac{\pm \tau_{xy}}{\pm \frac{\sigma_x - \sigma_y}{2}}$$



let $\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = R$

$\therefore \cos 2\theta_p = \pm(\sigma_x - \sigma_y)/2 \,/\, R \qquad \sin 2\theta_p = \pm \tau_{xy}/R$

$$\sigma_\theta = \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(\sigma_x+\sigma_y) \pm \frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2}{R} \pm \frac{\tau_{xy}^2}{R}$$

$$= \frac{1}{2}(\sigma_x+\sigma_y) \pm \frac{(\quad)^2 + (\quad)^2}{R}$$

$$= \frac{1}{2}(\sigma_x+\sigma_y) \pm \frac{R^2}{R}$$

$$\therefore \sigma_\theta = \sigma_{avg} \pm R$$

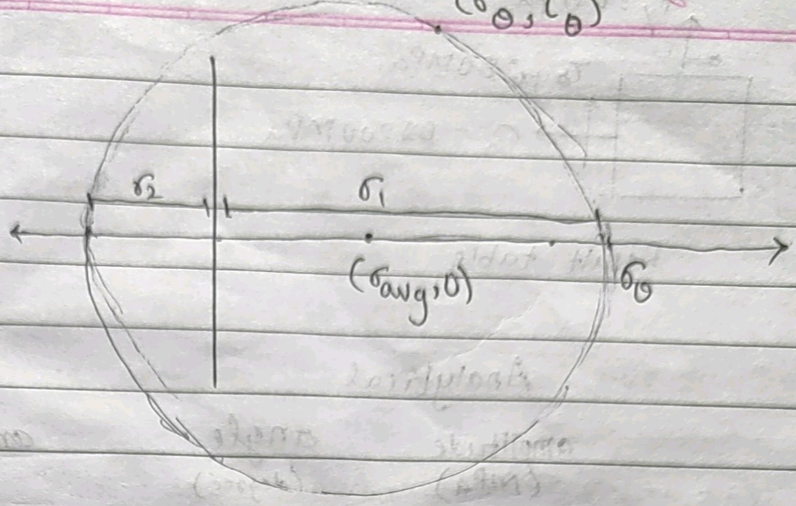
Graphical method

$$\sigma_\theta - \sigma_{avg} = \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = -\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

imp $$\boxed{(\sigma_\theta - \sigma_{avg})^2 + \tau_\theta^2 = \left[\frac{1}{2}(\sigma_x-\sigma_y)\right]^2 + \tau_{xy}^2 = R^2}$$

This is eqn of circle called as Mohr's Stress Circle

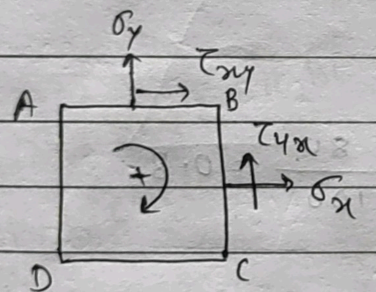


$\sigma_1 = \sigma_{avg} + R$

$\tau_{max} = R$

$\sigma_2 = \sigma_{avg} - R$

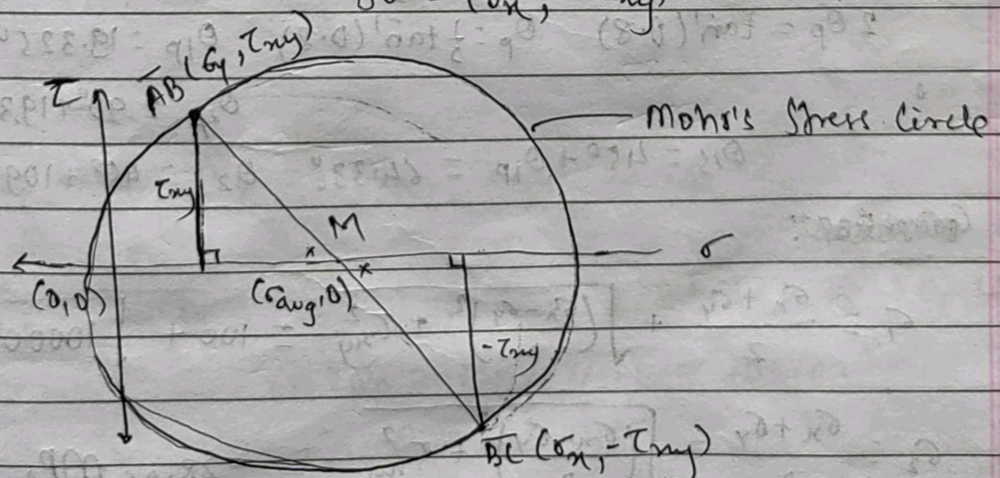
$\tau_{min} = -R$



$\tau_{xy} = -\tau_{yx}$

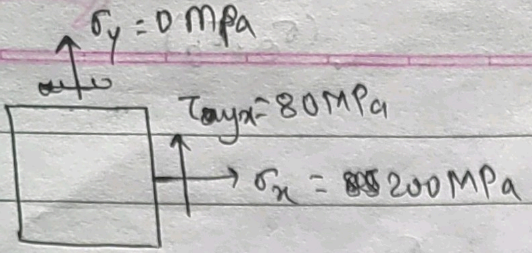
$\overline{AB} \equiv (\sigma_y, \tau_{xy})$

$\overline{BC} \equiv (\sigma_x, -\tau_{xy})$ $\quad \sigma_y < \sigma_x$



wherever $\overline{AB}$ & $\overline{BC}$ will intersect x axis will be $M \equiv (\sigma_{avg}, 0)$

1]



Result table

	Analytical		Graphical	
	amplitude (MPa)	angle (degree)	amplitude	angle
σ_1	228.06	19.33°	we'll write these things by actually measuring from the Mohr's circle	
σ_2	-28.06	109.33°		
τ_{max}	128.06	64.33°		
τ_{min}	-128.06	154.33°		

analytical:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{80}{100} = 0.8$$

$$\tan 2\theta_s = \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = 1.25$$

$$2\theta_p = \tan^{-1}(0.8) \quad \theta_p = \frac{1}{2}\tan^{-1}(0.8) \quad \theta_{1p} = 19.325^\circ$$

$$\theta_{2p} = 90^\circ + 19.325^\circ = 109.33^\circ$$

$$\theta_{1s} = 45^\circ + \theta_{1p} = 64.33^\circ \quad \theta_{2s} = 45^\circ + 109.33^\circ = 154.33^\circ$$

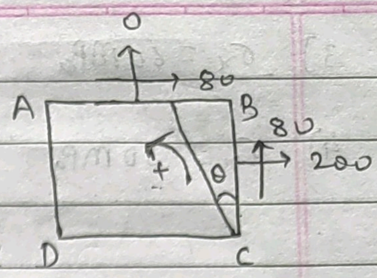
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 + \sqrt{10000 + 6400} = 228.062 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -28.06 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 128.06 \text{ MPa}$$

$$\tau_{min} = -\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -128.06 \text{ MPa}$$

Graphical method



200 normal pressure is tensile ∴ +ve

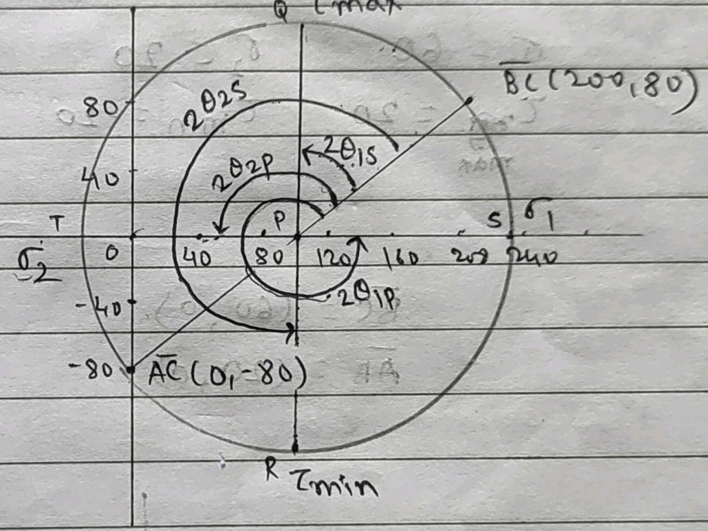
tensile is +ve & compressive is -ve

On AB, 80 will cause motⁿ in CW dir ∴ -ve

On BC, 80 —— " —— ACW dir ∴ +ve

$$\therefore \overline{AB} = (0,-80) \qquad \overline{BC} = (200,80)$$

Scale : 1cm = 40MPa



∵ θ is wrt BC, we are measuring angle wrt $\overline{BC}$ in Mohr's circle

$\tau_{max} = PQ = R \qquad \tau_{min} = PR = -R \qquad R$ - radius

$\sigma_1 = OS = \sigma_{avg} + R \qquad \sigma_2 = OT = \sigma_{avg} - R$

2] $\sigma_x = 20$ MPa $\quad \sigma_y = 30$MPa $\quad \tau_{xy} = 20$ MPa

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{20}{-5} = \frac{4}{-1} \Rightarrow y > 0 \;\&\; x < 0 \Rightarrow 2\theta_p \text{ is in } 2^{nd} \text{ quad}$$

$$\therefore 2\theta_p = \tan^{-1}(4/-1) = 104.03$$

$$\therefore \theta_{1P} = 52.015$$

$$\theta_{2P} = 142.02$$

$$\theta_{1S} = 97.02$$

$$\theta_{2S} = 187.02$$

$\sigma_1 = 45.61$ mPA $\qquad \tau_{max} = 20.61$ mPA

$\sigma_2 = 4.38$ mPA $\qquad \tau_{min} = -20.61$ mPA

3] $\sigma_x = 60\,MPa$ $\sigma_y = 20\,MPa$ $\tau_{xy} = 0$

4) $\sigma_x = -30\,MPa$ $\sigma_y = 50\,MPa$ $\tau_{xy} = -40$

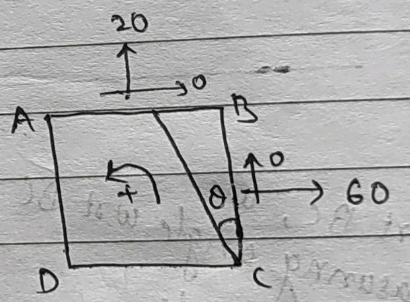
3]

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{0}{20} \qquad \theta_{1p} = 0 \qquad \theta_{2p} = 90$$

$$\tan 2\theta_s = \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{20}{0} \qquad \theta_{1s} = 45^\circ \qquad \theta_{2s} = 135^\circ$$

$\sigma_1 = 60$ $\sigma_2 = 20$

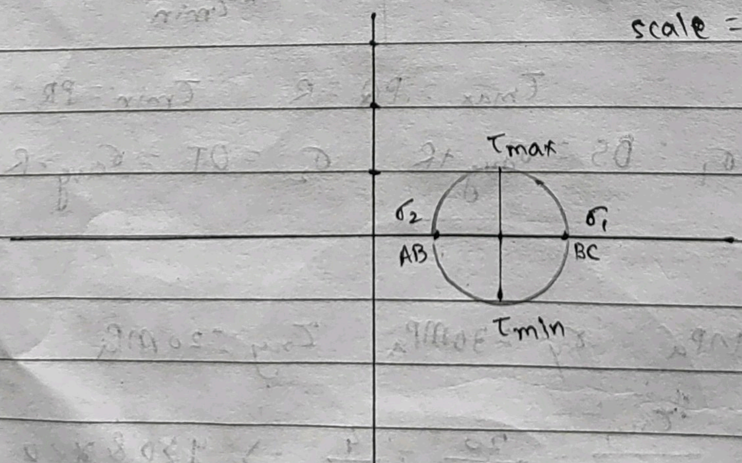
$\tau_{max} = 20$ $\tau_{min} = -20$



$\overline{BC} = (60, 0)$

$\overline{AB} = (20, 0)$

scale = 1cm = 20 mPA



τ values are measured from centre of. circle while σ values are measured from origin

clearly $\sigma_1 = 60$ $\sigma_2 = 20$

$\tau_{max} = 20$ $\tau_{min} = -20$

BC makes 0° with σ_1 $\therefore 2\theta_{1p} = 0$ $\therefore \theta_{1p} = 0$

BC makes 180° with σ_2 $\therefore 2\theta_{2p} = 180^\circ$ $\therefore \theta_{2p} = 90^\circ$

BC makes 90° with τ_{max} $\therefore 2\theta_{1s} = 90^\circ$ $\therefore \theta_{1s} = 45^\circ$

BC makes 270° with τ_{min} $\therefore 2\theta_{2s} = 270^\circ$ $\therefore \theta_{2s} = 135^\circ$

4] $\tan 2\theta_p = \dfrac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \dfrac{-40}{-40}$ $\theta_{1p} = \cancel{112.5}° \; \cancel{225°} \; 112.5°$

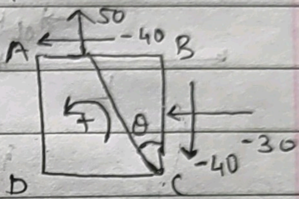
$\theta_{2p} = \cancel{202.5} \; \cancel{315°} \; 202.5°$

$\tan 2\theta_s = \dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -40/-40$ $\theta_{1s} = \cancel{157.5°} \; \cancel{270°} \; 157.5°$

$\theta_{2s} = \cancel{247.5°} \; \cancel{36} \; 247.5°$

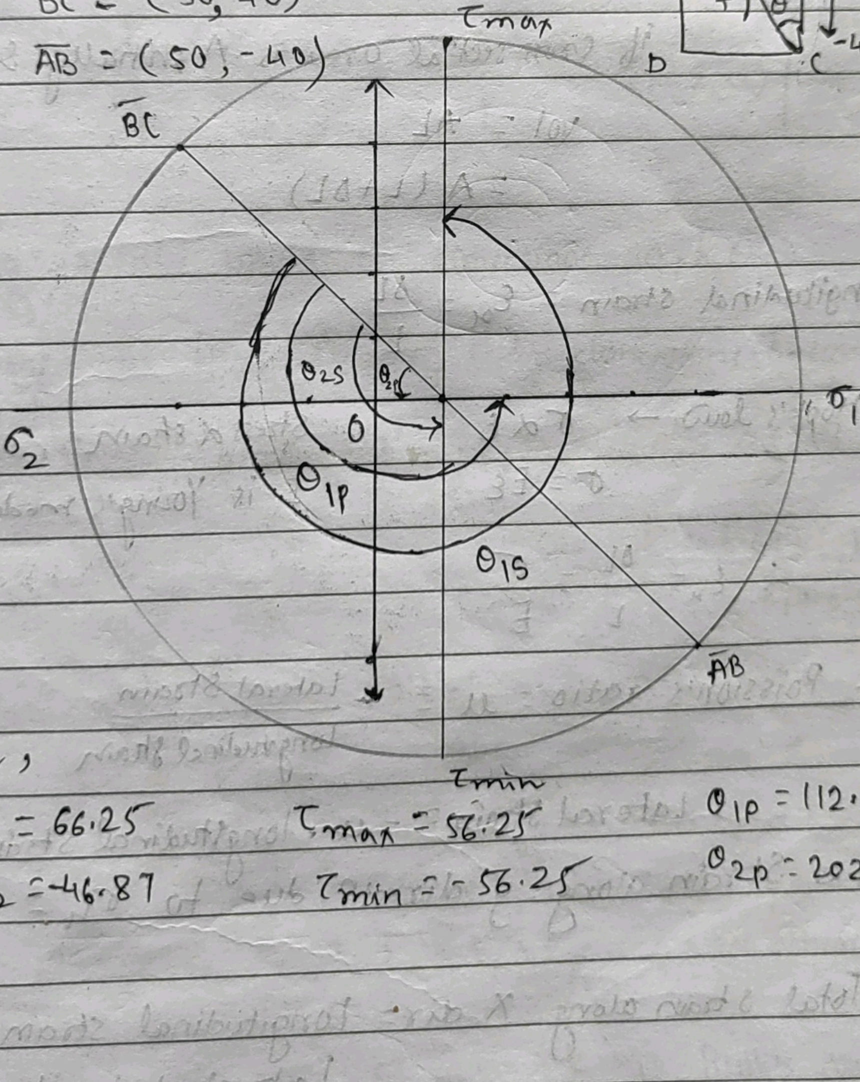
$\sigma_1 = 10 + 56.56 = 66.56$ $\tau_{max} = 56.66$

$\sigma_2 = 10 - 56.56 = -46.56$ $\tau_{min} = -56.66$



$\overline{BC} = (-30, 40)$

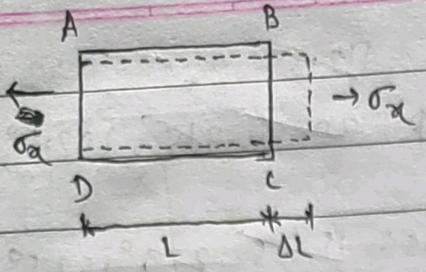
$\overline{AB} = (50, -40)$



Graphically,

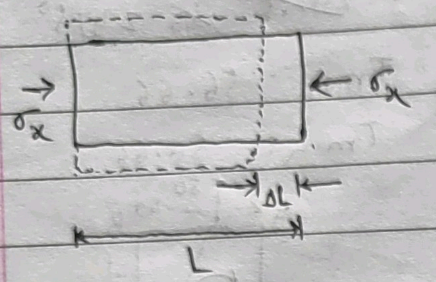
$\sigma_1 = 66.25$ $\tau_{max} = 56.25$ $\theta_{1p} = 112.5°$ $\theta_{1s} = 157.5°$

$\sigma_2 = -46.87$ $\tau_{min} = -56.25$ $\theta_{2p} = 202.5°$ $\theta_{2s} = 247.5°$



increment = +ve

i.e $\Delta L > 0$



decrement = -ve

here, $\Delta L < 0$

if cross sect^{n}al area is A initially & A' afterwards,

$$vol = AL$$
$$= A'(L + \Delta L)$$

Longitudinal strain $= \varepsilon_x = \frac{\Delta L}{L}$

Hook's law $\rightarrow$ $\sigma \propto \varepsilon$ Stress $\propto$ strain

$\sigma = E\varepsilon$ E is Young's modulus/Modulus of elasticity

$\therefore \varepsilon_x = \frac{\Delta L}{L} = \frac{\sigma}{E}$

Poission's ratio $= \mu = -\frac{\text{Lateral Strain}}{\text{Longitudinal strain}}$

Lateral Strain $= -\mu \cdot$ longitudinal strain

i.e Strain along y direct^n due to $\sigma_x = -\mu \frac{\sigma_x}{E}$

Total strain along x dir = longitudinal strain due to σ_x + lateral strain due to σ_y

$$= \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

Extending the same concept to 3 dimensions,

Date
Page

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x+\sigma_y+\sigma_z}{E} - 2\mu\left(\frac{\sigma_x+\sigma_y+\sigma_z}{E}\right)$$

V

$$\therefore \boxed{\varepsilon_x+\varepsilon_y+\varepsilon_z = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)} \quad \text{--- (1)}$$

Considering a ~~cub~~ cuboid of dimensions a, b, c
after changes in their lengths, their dimensions become a', b', c'

I

$$\therefore \frac{a'-a}{a} = \varepsilon_x \qquad \frac{a'}{a} = 1+\varepsilon_x \qquad \therefore a' = a(1+\varepsilon_x)$$

$$b' = b(1+\varepsilon_y)$$

$$c' = c(1+\varepsilon_z)$$

M

Now, volumetric strain $= \dfrac{a'b'c' - abc}{abc} = \dfrac{a(1+\varepsilon_x)\cdot b(1+\varepsilon_y)\cdot c(1+\varepsilon_z) - abc}{abc}$

$$\varepsilon_V = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1$$

$$\varepsilon_V = 1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_x\varepsilon_z+\varepsilon_x\varepsilon_y\varepsilon_z - 1$$

$\varepsilon_V = \varepsilon_x+\varepsilon_y+\varepsilon_z$ .... all others are very small

P

$\sigma_V \propto \varepsilon_V \qquad \sigma_V = K\varepsilon_V \qquad$ K is Bulk's modulus.

$$K = \sigma_V/\varepsilon_V$$

$$\sigma_V = \frac{\sigma_x+\sigma_y+\sigma_z}{3}$$

Modifying eq^n ①,

$$\boxed{\begin{gathered}\varepsilon_V = \frac{\sigma_V}{k} = \frac{3\sigma_V}{E}(1-2\mu) \\ \therefore \quad E = 3K(1-2\mu)\end{gathered}}$$

imp

$E \geq 0 \quad k \geq 0 \quad \therefore 1-2\mu \geq 0$

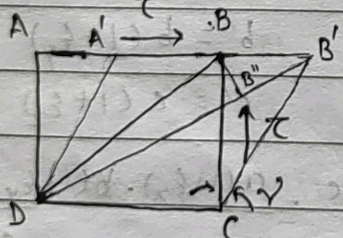
$\therefore \quad \mu \leq 0.5$

when $\mu = 0$, $E = k = 0 \Rightarrow$ There is enormous strain

Shear stress $\propto$ Shear strain

$\tau \propto \gamma$

$\tau = G\gamma \qquad$ G - modulus of Rigidity

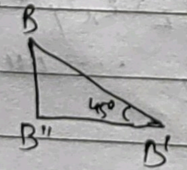


It is square of length a

$\therefore BD = a\sqrt{2} \approx B''D$

$B'D = B''D + B'B''$

now, $AB = a$ Shear strain $= \gamma \quad \therefore BB' = a\gamma$

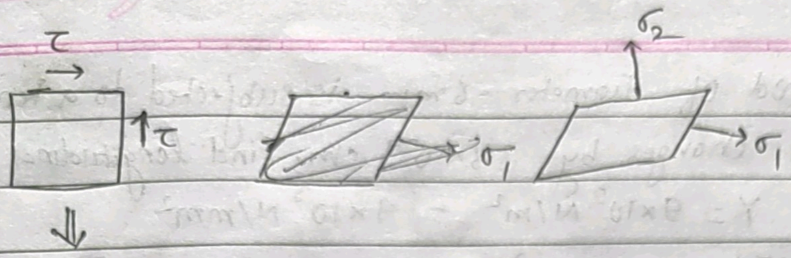


$B'B'' = BB'\cos 45^\circ = \frac{a\gamma}{\sqrt{2}}$

strain on diagonal $= B'B''/BD = a\gamma/\sqrt{2} \,/\, a\sqrt{2} = \frac{\gamma}{2}$

$= \frac{1}{2}\gamma = \frac{1}{2}\frac{\tau}{G}$

(relate with length) E - Young's modulus / Modulus of elasticity
(relate with volume) K - Bulk's modulus
(relate with angle) G - Modulus of rigidity / Shear modulus



$\Downarrow$

$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = \tau$

$\sigma_1 = 0 + \sqrt{0^2 + \tau^2} = \tau$

$\sigma_2 = 0 - \sqrt{0^2 + \tau^2} = -\tau$

Total strain on diagonal $= \dfrac{\sigma_1}{E} - \mu \dfrac{\sigma_2}{E}$

$= \dfrac{\tau}{E} - \mu \dfrac{(-\tau)}{E} = \dfrac{\tau}{E}(1+\mu)$

$\dfrac{\tau}{2G} = \dfrac{\tau}{E}(1+\mu)$

$\therefore \boxed{E = 2G(1+\mu)}$

IMP $\quad \therefore \boxed{2G(1+\mu) = E = 3K(1-2\mu)}$ $\quad \boxed{E = \dfrac{9KG}{3K+G} \quad \mu = \dfrac{3K-2G}{6K+2G}}$

again from $E = 2G(1+\mu)$, $\quad E \geq 0 \quad G \geq 0$

$\therefore 1+\mu \geq 0 \quad \therefore \mu \geq -1$

$\therefore \boxed{-1 \leq \mu \leq \frac{1}{2}}$

Or $\quad \boxed{-1 \leq \mu < 0}$ & $\boxed{0 \leq \mu \leq \frac{1}{2}}$

1] When a brass rod of diameter 6mm is subjected to a tention of 5 kN, diameter changes by 3.6×10^{-4} cm. Find longitudinal strain & Poisson's ratio. $Y = 9\times10^{9}\ N/m^2 = 9\times10^{3}\ N/mm^2$

$$\varepsilon = \frac{\sigma}{E} = \frac{(F/A)}{E} = \frac{5\times10^{3}/\frac{\pi}{4}\times6\times6}{9\times10^{3}} = 1.96\times10^{-3}\ mm$$

$$\mu = \frac{-\text{lateral strain}}{\text{longitudinal strain}} = \frac{-(-3.6\times10^{-3})/6}{1.96\times10^{-3}}$$

$$= 0.31$$

2] A metal wire of length 1.5m is loaded & elong^n of 2mm is produced. If diameter is 1mm, find change in diameter. $\mu = 0.24$

$$\mu = -\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta d/d}{\Delta L/L} = \frac{-\Delta d/1}{2/1.5\times10^{3}} = 0.24$$

$L = 1.5$ m $\quad \Delta L = 2$mm $\quad d = 1$mm $\quad \Delta d = ?$

$$\therefore \Delta d = -3.2\times10^{-4}\ mm$$

-ve means size is reduced

3] If $k = G$, find μ

$$\mu = \frac{3k-2G}{6k+2G} = \frac{k}{8k} = 0.125 \qquad E = \frac{9G}{4}$$

4] if $E = k$, find μ

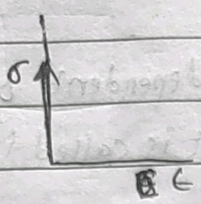
$$\mu = 1/3 \qquad G = \frac{3}{8}E$$

5] if $E = G$, find μ

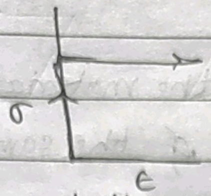
$$\mu = -1/2 \qquad k = \frac{1}{6}E$$

imp

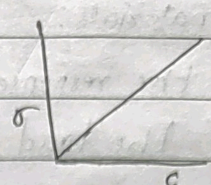
from above 3 qst, 3 elastic constants E, G, K can never be equal at the same time.



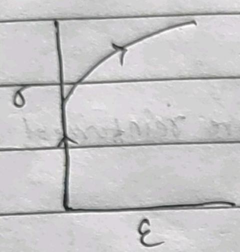
rigid material



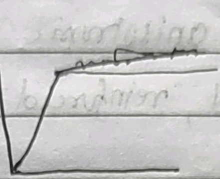
perfectly plastic



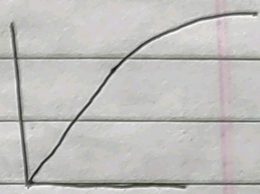
linear elastic



rigid plastic material



elastic perfectly plastic



elastic plastic

$\mu = 0.5$

$\Rightarrow E = 0 \Rightarrow$ perfectly plastic material

gold has $\mu = 0.44$ rubber has $\mu = .499999$

$\mu = 0$

They are real materials

$\mu = -1$

1

$\mu = 0$ 3K K to E ratio decreases

$\mu = -1$ 9K

They are foam like materials.

Isotropic materials:

If the response of the material is independent of the orientatⁿ of the load axis of the sample, it is called isotropic material. ex. glass, material

Anisotropic materials:

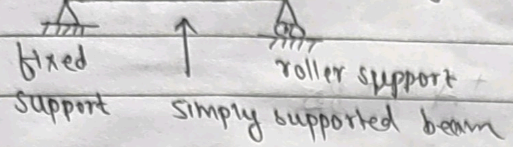
If not, anisotropic materials

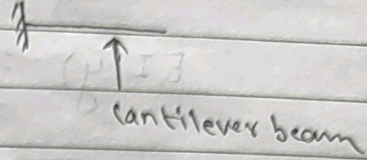
ex. wood, reinforced concrete, fibre reinforced plastic

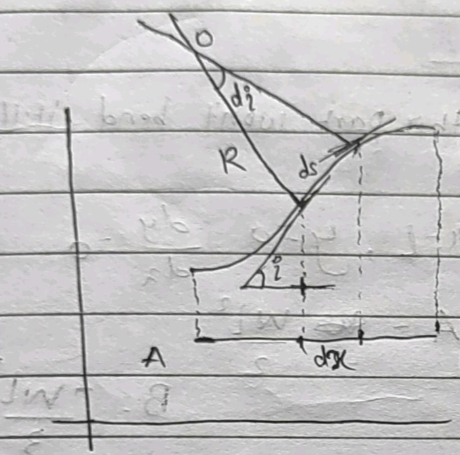
upward forces cause +ve BM
downward — " — -ve BM

Deflection of Beams

* Cantilever beam subjected to concentrated load







on applying force to beam AB, AB is deflected as shown

dx is ~~dele~~ component of ds

for very small i, $ds = dx$

$ds = R\,di$

$\tan i \approx i = \frac{dy}{dx}$

$\therefore dx = R\,di$

$\therefore \frac{di}{dx} = \frac{1}{R}$

i.e. $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{R}$ $\quad\therefore \frac{d^2y}{dx^2} = \frac{1}{R}$

we know $\frac{E}{R} = \frac{M}{I}$

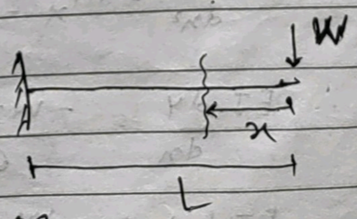
$\therefore \frac{d^2y}{dx^2} = \frac{1}{R} = \frac{M}{EI}$

M - moment
I - moment of area
E - Young's modulus.

$$\text{slope} = \frac{dy}{dx} = \int \frac{M}{EI}\,dx + C_1$$

$$\text{deflect}^n = y = \int \left[\int \frac{M}{EI}\right] dx + Ax + B$$

for cantilever beam,



W is force

$M = -Wx$

$\therefore \frac{d^2y}{dx^2} = \frac{M}{EI}$

$$\therefore EI\frac{d^2y}{dx^2} = -Wx$$

$$\therefore EI\frac{dy}{dx} = \frac{-Wx^2}{2} + A$$

$$EI(y) = \frac{-Wx^3}{6} + Ax + B$$

for a cantilever beam, this part won't bend it'll always have slope = 0

i.e. at $x = L$, $y = 0$, $\frac{dy}{dx} = 0$

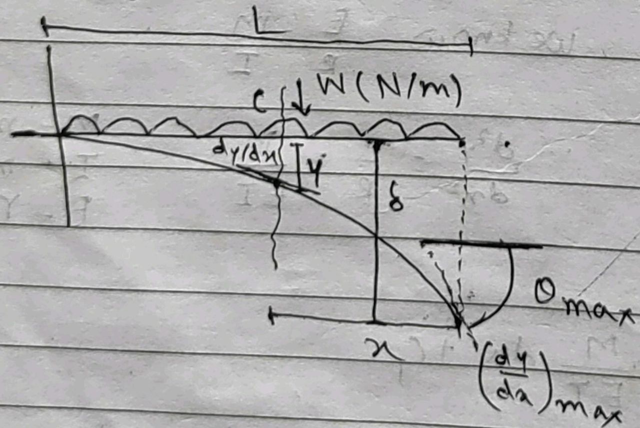
$$A = \frac{WL^2}{2} \qquad B = \frac{-WL^3}{3}$$

$$y_{max} = \frac{-WL^3}{3EI}$$

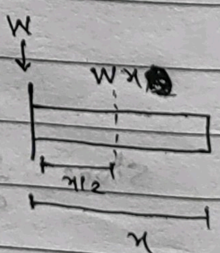
$$(slope)_{max} = \frac{WL^2}{2EI}$$

(Cantilever)

* Beam subjected to distributed load



w - force per length
W - force
M - moment of force unit Nm



$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad M_x = -wx \cdot \frac{x}{2}$$

$$\therefore EI\frac{d^2y}{dx^2} = M_x = \frac{-wx^2}{2}$$

$$\therefore EI\frac{dy}{dx} = \frac{-wx^3}{6} + C_1$$

$$EI(y) = \frac{-wx^4}{24} + C_1x + C_2$$

End / Support / Boundary conditions.

at $x = L$, $y = 0$ $\qquad C_1 = \frac{wL^3}{6}$

$$EI(0) = \frac{-wL^4}{24} + \frac{wL^3}{6} \cdot L + C_2$$

$$\therefore C_2 = \frac{wL^4}{24} - \frac{wL^4}{6} = \frac{-wL^4}{8}$$

at $x = L$, $\frac{dy}{dx} = 0$

$$EI(0) = \frac{-wL^3}{6} + C_1$$

$$\therefore C_1 = \frac{wL^3}{6}$$

$$EI\left(\frac{dy}{dx}\right) = \frac{-wx^3}{6} + \frac{wL^3}{6} \cancel{- \frac{wL^4}{8}}$$

$$\therefore EIy = \frac{-wx^4}{24} + \frac{wL^3}{6}x - \frac{wL^4}{8}$$

for y_{max}, $\frac{dy}{dx} = 0$

$$\Downarrow$$

$$y_{max} = \frac{-wL^4}{8EI}$$

$$\& \left(\frac{dy}{dx}\right)_{max} = \frac{wL^3}{6EI}$$

* Beam (Simply supported) subjected to ~~concentrated~~ distributed load

$$EI\frac{d^2y}{dx^2} = M_x = \frac{-wx^2}{2} + \frac{wL}{2}x$$

$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wL}{2} \cdot \frac{x^2}{2} + C_1$$

$$EIy = \frac{-wx^4}{24} + \frac{wL}{2} \cdot \frac{x^3}{6} + C_1x + C_2$$

at boundary con^n,

at $x=0$, $y=0$

at $x=L$, $y=0$

$$E.I(0) = 0+0+0+C_2 \qquad C_2 = 0$$

$$EI(0) = \frac{-WL^4}{24} + \frac{WL^4}{12} + C_1 L$$

$$C_1 = \frac{WL^3}{24} - \frac{WL^3}{12} = \frac{-WL^3}{24}$$

when $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-Wx^3}{6} + \frac{WLx^2}{4} - \frac{WL^3}{24} = 0$$

$$\therefore 4x^3 - 6Lx^2 + L^3 = 0$$

$$\therefore \left(x - \frac{L}{2}\right)(\quad) = 0$$

when $x = \frac{L}{2}$,

$$Y_{max} = \delta = Y_{at\, x = L/2} = \frac{WL^4}{EI}\left[\frac{-1}{384} + \frac{1}{96} - \frac{1}{48}\right]$$

$$= \frac{-5WL^4}{384EI}$$

for maximum slope i.e. for finding $\left(\frac{dy}{dx}\right)_{max}$, $\frac{d^2y}{dx^2} = 0$

$$\therefore \frac{-Wx^2}{2} + \frac{WL}{2}x = 0$$

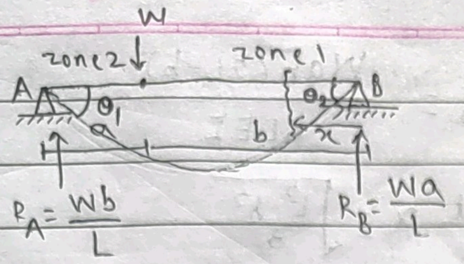
$$\Downarrow$$

$$\frac{Wx}{2}(x-L) = 0 \quad \therefore x=0 \ \& \ x=L$$

that means one will be $\left(\frac{dy}{dx}\right)_{max}$ & one will be $\left(\frac{dy}{dx}\right)_{min}$

at $x=0$, $\left(\frac{dy}{dx}\right)_{min} = \frac{-WL^3}{24}$ at $x=L$, $\left(\frac{dy}{dx}\right)_{max} = \frac{WL^3}{24}$

* Beam (Simply supported) subjected to ~~distributed~~ concentrated load



zone 1 $\quad 0 \le x \le b$

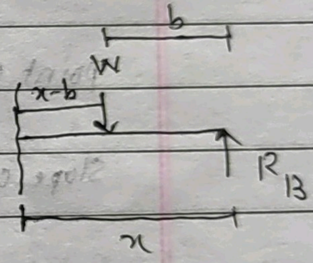
$$M_x = R_B x$$
$$= \frac{Wa}{L} x$$

$$\therefore EI \frac{d^2y}{dx^2} = \frac{Wax}{L}$$

$$EI \frac{dy}{dx} = \frac{Wax^2}{2L} + C_1$$

$$EI\,y = \frac{Wax^3}{6L} + C_1 x + C_2$$

Zone 2 $\quad b \le x \le L$



$$M_x = R_B x - W(x-b)$$

$$EI \frac{d^2y}{dx^2} = \frac{Wax}{L} - W(x-b)$$

$$EI \frac{dy}{dx} = \frac{Wax^2}{2L} - \frac{W(x-b)^2}{2} + C_3$$

$$EI\,Y = \frac{Wax^3}{6L} - \frac{W(x-b)^3}{6} + C_3 x + C_4$$

boundry con^n,

at $x=0$, $y=0$

at $x=L$, $y=0$

$$Y_{\text{zone 1 at } b} = Y_{\text{zone 2 at } b}$$

$$\frac{dy}{dx}_{\text{zone 1 at } b} = \frac{dy}{dx}_{\text{zone 2 at } b}$$

$$\theta_1 = \frac{Wb(L^2-b^2)}{6LEI} \qquad \theta_2 = \frac{Wab(2L-b)}{6LEI}$$

$$y_{max} = \frac{Wb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI} \quad \text{at } x = \sqrt{\frac{L^2-b^2}{3}}$$

If $a=b$, $\theta_1 = \theta_2 = \dfrac{WL^2}{16EI}$

$$y_{max} = \frac{WL^3}{48EI}$$

End ~~for~~ conditions :

at $x=0$, $y=0$

at $x=L$, $y=0$

point conditions at $x=b$, $y|_{zone1} = y|_{zone2}$

Slope conditions at $x=b$, $\left.\frac{dy}{dx}\right|_{zone1} = \left.\frac{dy}{dx}\right|_{zone2}$

* Macalay's Non-negativing funⁿ

Imp — It says write BM eqⁿ corresponding to last zone.

$$f(x-a) = \langle x-a \rangle^b$$
$$= (x-a)^b \quad \text{for } x \geq a$$

∴ Its value is always either +ve or 0

∴ for above case,

generalized $M_x = R_B x - W\langle x-b \rangle$

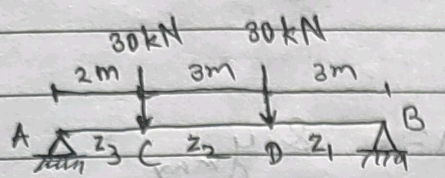
$$EI\frac{d^2y}{dx^2} = \text{—"—}$$

$$EI\frac{dy}{dx} = R_B\frac{x^2}{2} - \frac{W\langle x-b\rangle^2}{2} + C_1$$

$$EIy = R_B\frac{x^3}{6} - \frac{W\langle x-b\rangle^3}{6} + C_1 x + C_2$$

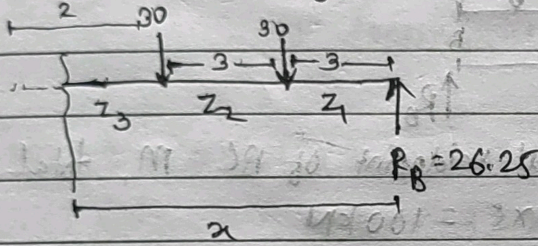
for whichever value of x, if $x<b$, put $W\langle x-b\rangle$ value as 0.

1]



$R_B \times 8 = 30\times 2 + 30\times 5$ $\quad$ $R_A \times 8 = 30\times 3 + 30\times 6$

$\therefore R_B = 26.25\ kN$ $\quad$ $= 33.75\ kN$

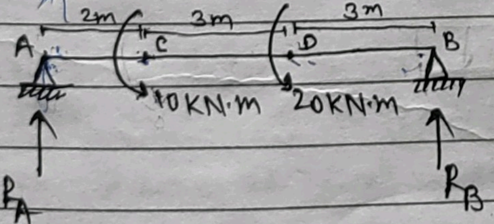


$$M_x = R_B x - 30\langle x-3\rangle - 30\langle x-6\rangle$$

$$EI\frac{dy}{dx} = R_B\frac{x^2}{2} - \frac{30\langle x-3\rangle^2}{2} - \frac{30\langle x-6\rangle^2}{2} + C_1$$

$$EIy = R_B\frac{x^3}{6} - \frac{30\langle x-3\rangle^3}{6} - \frac{30\langle x-6\rangle^3}{6} + C_1x + C_2$$

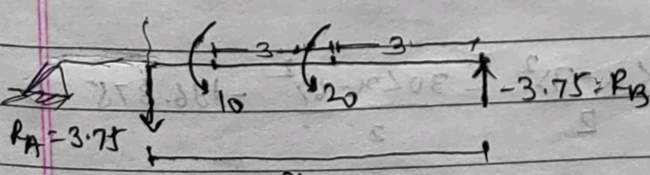
2]



R_B will cause ACW moment $\therefore R_B \times 8 + 10 + 20 = 0$

$R_B = -3.75\ kN$

$-R_A \times 8 + 10 + 20 = 0$ $\quad$ $R_A = 3.75\ kN$ ... $\because R_A$ will cause CW moment



imp

$$M_x = EI\frac{d^2y}{dx^2} = -3.75x + 20\langle x-3\rangle^0 + 10\langle x-6\rangle^0$$

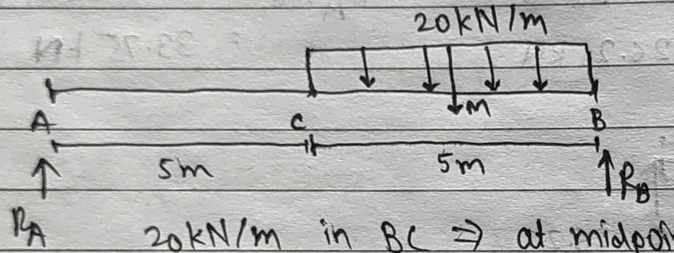
$$EI\frac{dy}{dx} = -3.75\frac{x^2}{2} + 20\langle x-3\rangle + 10\langle x-6\rangle + C_1$$

$$EIy = -3.75\frac{x^3}{6} + 20\frac{\langle x-3\rangle^2}{2} + 10\frac{\langle x-6\rangle^2}{2} + C_1 x + C_2$$

$EI = 10^6\ Nm^2$

find y at $x=3$, $x=6$ & y_{max}

3]

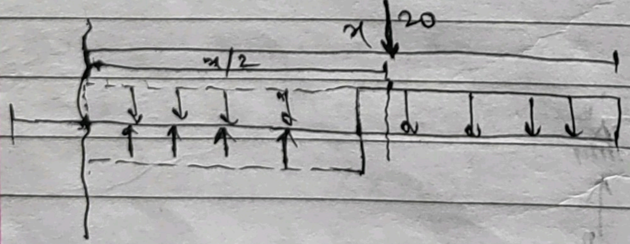


20kN/m in BC ⇒ at midpoint of BC M, total force acting will be $20 \times 5 = 100\ kN$

$\therefore R_B \times 10 = 100 \times 7.5 \qquad R_B = 75\ kN$

$\& \ R_A \times 10 = 100 \times 2.5 \qquad R_A = 25\ kN$

Now further using Macalay's funn, we've to assume uniformly distributed load all over the beam & then subtract it.



1] at $x=0$, $y=0 \Rightarrow C_2 = 0$

at $x=8$; $y=0 \Rightarrow C_1 = -196.875$

for y_{max}, $\frac{dy}{dx} = 0$

$$\therefore 26.25\frac{x^2}{2} - 30\frac{\langle x-3\rangle^2}{2} - 30\frac{\langle x-6\rangle^2}{2} - 196.875 = 0$$

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Case 1 : sol^n lies in zone 1

i.e. $0 \leq x \leq 3$

$$26.25\frac{x^2}{2} - 196.875 = 0 \Rightarrow x = 3.873$$

This value of x doesn't lie in (0,3)

∴ Not possible

Note * : As $C_1 < 0$ real sol^n of quadratic exists

tmp → if $C_1 > 0$ we can say without calculation that sol^n doesn't lie within zone 1

Case 2 : sol^n lies in zone 2

i.e. $3 \leq x \leq 6$

$$26.25\frac{x^2}{2} - 30\frac{(x-3)^2}{2} - 196.875 = 0$$

$$\Downarrow$$

$$x_1 = 4.025 \quad x_2 = 43.925$$

4.025 is physible

∴ putting in EI(y) = --- eq^n,

$$y_{max} = \frac{+512.523}{EI}$$

Case 3 : Sol^n lies in zone 3 :

$0 \leq x \leq 8$

i.e. $$26.25\frac{x^2}{2} - 30\frac{(x-3)^2}{2} - 30\frac{(x-6)^2}{2} - 196.875 = 0$$

$$\Downarrow$$

$$x_1 = 4.127 \quad \& \quad x_2 = 11.873 \text{ m}$$

not possible

2]
$$10^6 y = -3.75\frac{x^3}{6} + 20\frac{\langle x-3\rangle^2}{2} + 10\frac{\langle x-6\rangle^2}{2} + C_1x + C_2$$

at $x=0, y=0$ $\therefore C_2 = 0$

at $x=8, y=0$ $\therefore C_1 = 6.25$

putting $x=3$,

$10^6 y = -16.875 + 3C_1 + C_2$

$y = 1.875 \times 10^{-3}$

putting $x=6$,

$10^6 y = -7.5$

$y = -7.5 \times 10^{-6}$

for y_{max}

case 1: consider zone 3 $6 \leq x \leq 8$

$\therefore \frac{dy}{dx} = 0$

$$-3.75\frac{x^2}{2} + 20(x-3) + 10(x-6) + 6.25 = 0$$

$\therefore x_1 = 6.176$ & $x_2 = 9.824$

lies in zone 3 — out of zone

That means soln lies in zone 3 at $x = 6.176\,m$

$$EI\,y_{max} = -3.75\frac{(6.176)^3}{2} + \frac{20}{2}(6.176-3)^2 + \frac{10}{2}(6.176-6)^2 + 6.125 \times 6.176$$

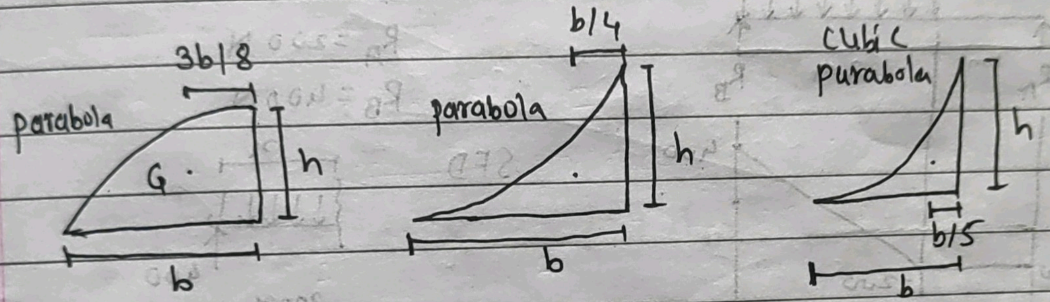
$$\therefore y_{max} = \frac{-39.97}{EI}$$

* Moment Area Method

Theorem I :

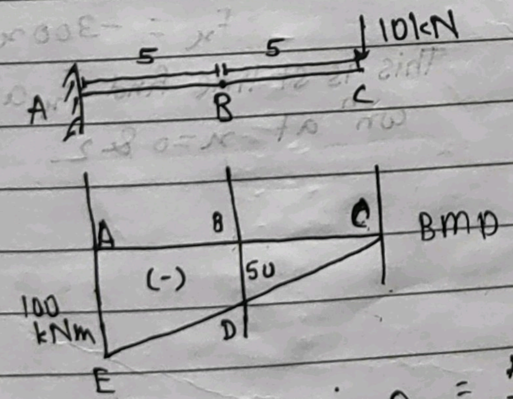
The angle θ bet^n tangents at any 2 points A & B on the elastic line is equal to the total area of corresponding portion of the bending moment diagram divided by EI

~~spr~~ slope or θ bet^n any two points $= \frac{1}{EI} \times$ area of BM diag. bet^n corresponding port^n of BMD



$A = 2bh/3$ $\quad$ $A = bh/3$ $\quad$ $A = bh/4$

1] Determine the slope at pts B & C of beam shown below



at A slope will be 0 $\quad \theta_A = 0$

$\theta_{B/A} = \theta_B - \theta_A$

$= \theta_B$

$\theta_{B/C} \neq \theta_B$

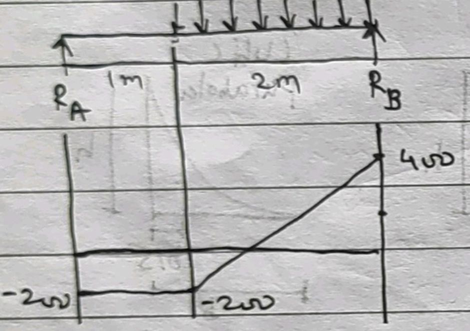
$\therefore \theta_B = \frac{\text{Area of ABED}}{EI} = \frac{-\frac{1}{2} \times 150 \times 5}{EI} = \frac{\cancel{-375}\ -375}{EI}$

$\theta_{C/A} = \theta_C = \frac{\text{Area of } \Delta ACE}{EI}$

2) Theorem 2:

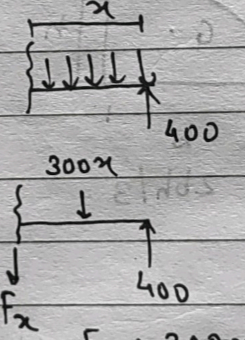
Deflectⁿ of B away from tangent at A is equal to the statical moment wrt B of the bending moment area between A & B divided by EI

$$y = \int_A^B x\,d\theta = \frac{1}{EI} \times \text{first M.O.A. wrt pt B of total BMD}$$



$R_A = 200\text{ N}$

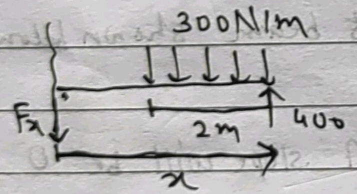
$R_B = 400\text{ N}$



$F_x + 300x - 400 = 0$

$F_x = -300x + 400$

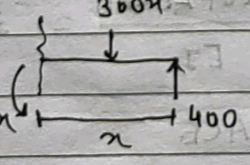
This is st line find F_x at boundary conⁿ at $x = 0$ & 2



$F_x + 600 - 400 = 0$

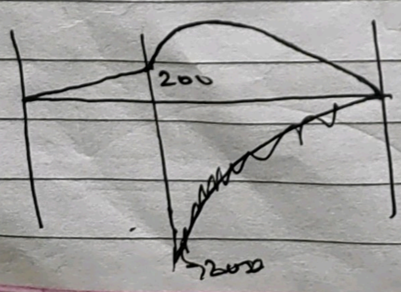
$F_x = -200$

BMD



$+M_x + 300x \cdot \frac{x}{2} - 400x = 0$

$M_x = \frac{-300x^2}{2} + 400x$



This is parabola find at boundaries

at $x = 0$, $M_x = 0$

at $x = 2$ $M_x = +200$

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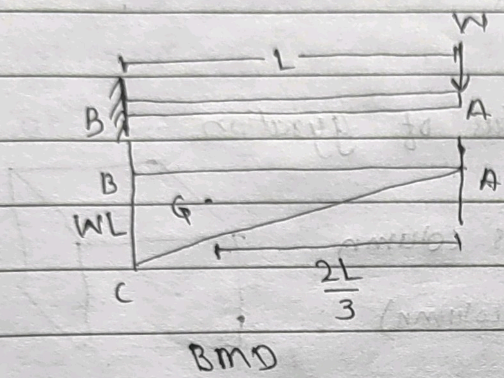
for UVL, SFD is quadratic & BMD is cubic
for conc. load, SFD is like rectangular block like & BMD is like triangular shapes

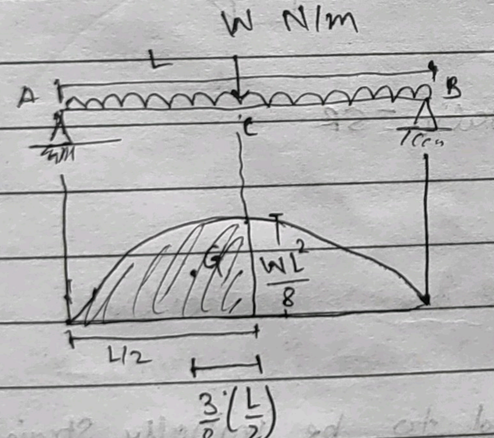
1] 

find deflect^n of B wrt A

$\delta = y = \frac{1}{EI} \times area(ABC) \times$ dist. of G from A

$= \frac{1}{EI} \times \frac{1}{2} \times WL.L \times \frac{2L}{3}$

$= \frac{WL^3}{3EI}$

2] 

find δ_{CA}

$\delta = \frac{1}{EI} \times$ area of shaded part $\times$ dist. of G from A

$= \frac{1}{EI} \times \frac{2bh}{3} \times \frac{5b}{8}$

$= \frac{1}{EI} \times \frac{2}{3} \cdot \frac{L}{2} \cdot \frac{WL^2}{8} \times \frac{5}{8} \frac{L}{2}$

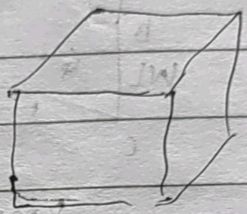
$= \frac{5WL^4}{384EI}$

Columns

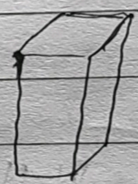
They are of 2 types : columns
struts

If l is length & k is radius of gyration

if $\frac{l}{k} \leq \left(\frac{l}{k}\right)_{CR}$ ⇒ it is column
(short column)



if $\frac{l}{k} > \left(\frac{l}{k}\right)_{CR}$ ⇒ it is strut
(long column)



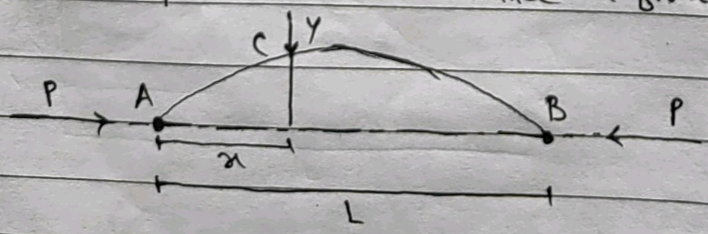
$\frac{\text{length of member}}{\text{least radius of gyration}} = \frac{l}{k}$ = slenderness ratio = SR

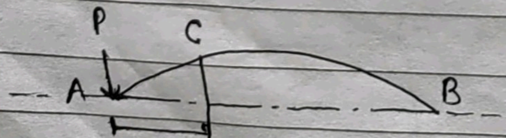
* Euler's theory
assumption:

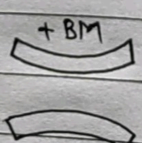
The strut is assumed to be initially straight, the end load being applied axially through centroid.

Case A : Struct with pinned ends :

Axially loaded strut subjected to an axial load P. The load P produces 'y' deflectⁿ at a distance x from one end.







BM@C = $-Py$

we know, $EI \frac{d^2y}{dx^2} = M = -Py$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0 \qquad D = d/dx$$

$$\therefore (D^2 + n^2)y = 0 \qquad n^2 = P/EI$$

let $y = A\cos\omega x + B\sin\omega x$ ..... $n = \omega$

at $x = 0$, $y = 0 \Rightarrow A = 0$

at $x = L$, $y = 0 \Rightarrow \omega L = m\pi \qquad \omega = m\pi/L$

$$\therefore \sqrt{\frac{P}{EI}}\, L = m\pi$$

$$\therefore \frac{P}{EI} L^2 = m^2\pi^2$$

$$P_{CR} = \frac{m^2\pi^2 EI}{L^2} \qquad I = \text{moment of area} = Ak^2$$

for $m = \pm 1$,

$$P_{CR} = \frac{\pi^2 EI}{L^2}$$

The least value of P which will cause the strut to buckle & it is called the Euler Crippling Load P_e

$$\boxed{P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA}{L^2/k^2} = \frac{\pi^2 EA}{(SR)^2}}$$

$\therefore$ SR is slenderness ratio

$$\boxed{\frac{P_{CR}}{A} = \sigma_{CR} = \frac{\pi^2 E}{(SR)^2}}$$

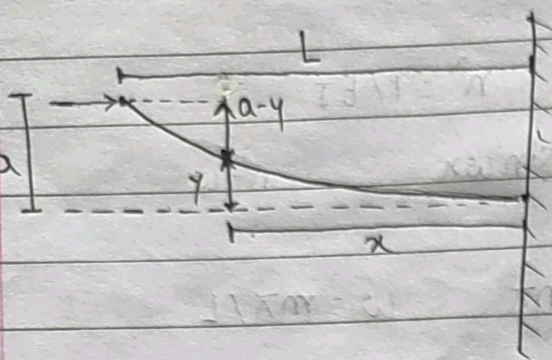
Imp — **Deflection is not proportional to load**

for $m = 1$, $L\sqrt{P/EI} = \pi$ i.e. $P_1 = \pi^2 EI/L^2$

for $m = 2$, $L\sqrt{P/EI} = 2\pi$ i.e. $P_2 = 4\pi^2 EI/L^2 = 4P_1$ → produces buckling in 2 halves

for $m = 3$, $L\sqrt{P/EI} = 3\pi$ i.e. $P_3 = 9\pi^2 EI/L^2 = 9P_1$ → produces buckling in 3 halves

Case B : One end fixed & other end free



BM = P(a-y)

i.e. $EI \frac{d^2y}{dx^2} = P(a-y)$

$\therefore \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}$

let $P/EI = n^2$

let solⁿ be $y = A\cos(nx) + B\sin(nx) + PI$

PI is particular value of y which satisfies differential eqⁿ let PI = a

at $x=0,\ y=0 \Rightarrow A = -a$

at $x=0,\ dy/dx = 0 \Rightarrow B = 0$

$\therefore y = -a\cos(nx) + a$

at $x = L,\ y = a$ $\therefore a = -a\cos(nL) + a$

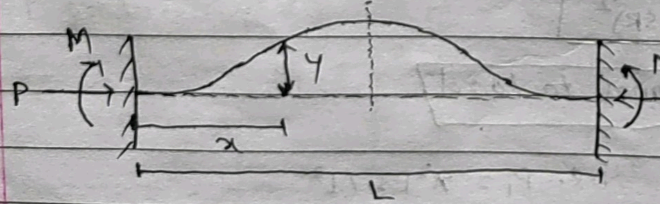
$\therefore \cos(nL) = 0$

$\therefore nL = \pi/2$

i.e $\sqrt{\frac{P}{EI}}\, L = \frac{\pi}{2}$ i.e. Euler's crippling load is given as

$$\boxed{P_e = \frac{\pi^2 EI}{4L^2}}$$

Case C : Strut with fixed ends



here, $EI\frac{d^2y}{dx^2} = M - Py$

$\therefore \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M}{EI}$

$n = \sqrt{P/EI}$ $\therefore (D^2+n^2)y = M/EI$

$y = CF + PI$

$PI = \frac{1}{D^2+n^2}\frac{M}{EI} = \frac{M}{EI} \times \frac{1}{2ni}\left[\frac{1}{D-ni} - \frac{1}{D+ni}\right]$

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let $\frac{1}{D-ni} = Y \quad \therefore DY - niY = 1 \quad \therefore e^{-in\int dx} = I.F. = e^{-inx}$

$\therefore e^{-inx} Y = \int e^{-inx} dx$

$e^{-inx} Y = \frac{e^{-inx}}{-in} \qquad \therefore Y = i/n$

Similarly $\frac{1}{D+ni} = Z \Rightarrow Z = -i/n$

$\therefore PI = \frac{M}{EI} \times \frac{1}{2ni}\left[\frac{i}{n} - \frac{-i}{n}\right] = \frac{M}{EI} \times \frac{1}{2ni}\left[\frac{2i}{n}\right]$

$= M/EIn^2 \qquad \because n^2 = P/EI$

$\therefore P = EIn^2$

$PI = M/P$

$\therefore \; y = CF + \frac{M}{P} = A\cos(nx) + B\sin(nx) + \frac{M}{P}$

at $x=0, y=0 \; \therefore B = -M/P$

at $x=0, dy/dx = 0 \; \therefore A = 0$

$\therefore y = \frac{-M}{P}\cos(nx) + \frac{M}{P}$

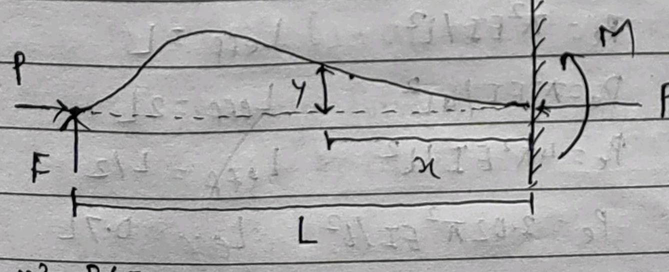
$\therefore y = \frac{M}{P}(1-\cos nx)$

at $x=L, y=0$

$\therefore \cos nL = 1 \qquad \therefore nL = 2\pi$

$\frac{P}{EI} L^2 = 4\pi^2 \qquad \therefore \boxed{P_e = \frac{4\pi^2 EI}{L^2}}$

Case D: One end fixed other end ~~free~~ pinned



$EI\frac{d^2y}{dx^2} = -Py + F(L-x)$

$\therefore \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{F}{EI}(L-x)$

$n^2 = P/EI$

$\therefore (D^2+n^2)y = \frac{F}{EI}(L-x)$

$\therefore y = CF + PI$

by solving with traditional method, $PI = \dfrac{F(L-x)}{n^2 EI} = \dfrac{F(L-x)}{P}$

$CF = A\cos nx + B\sin nx$

$y = A\cos nx + B\sin nx + \dfrac{F}{P}(L-x)$

at $x=0, y=0$

$\therefore A = -FL/P$

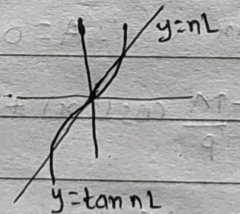
also at $x=0$ $\quad dy/dx = 0 \quad \therefore B = F/nP$

$\therefore y = \dfrac{-FL}{P}\cos nx + \dfrac{F}{nP}\sin nx + \dfrac{F}{P}(L-x)$

$\therefore y = \dfrac{F}{nP}[\sin nx - nL\cos nx + n(L-x)]$

now, $x = L \Rightarrow y = 0$

$\therefore nL\cos nL = \sin nL \Rightarrow \tan nL = nL$



$nL = 4.49$ radian

$\therefore \dfrac{PL^2}{EI} = (4.49)^2$

$\therefore \dfrac{P_e L^2}{EI} = 20.2$

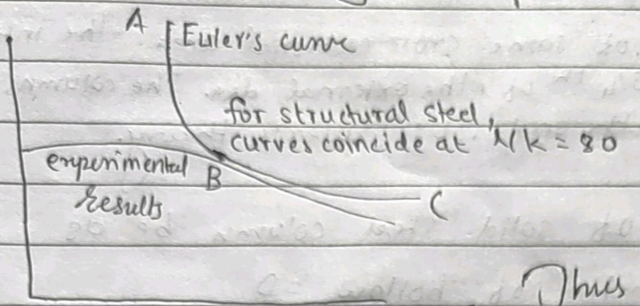
$$\boxed{P_e = \dfrac{2.05\pi^2 EI}{L^2}}$$

Crippling load can be generally written as $P_e = \dfrac{C\pi^2 EI}{L^2} = \dfrac{\pi^2 EI}{L_{eff}^2}$

L_{eff} or L_e is equivalent or effective length

Strut with pinned ends	$P_e = \pi^2 EI/L^2$	$L_{eff} = L$
One end fixed other end free	$P_e = \pi^2 EI/4L^2$	$L_{eff} = 2L$
Strut with fixed ends	$P_e = 4\pi^2 EI/L^2$	$L_{eff} = L/2$
One end fixed other pinned	$P_e = 2.02\pi^2 EI/L^2$	$L_{eff} = 0.7L$

* Comparison of Euler's theory with experimental results



Thus Euler's theory is only applicable for long columns (struts)

Rankine Gordon formula:

$$\boxed{\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}}$$

P_e - Euler crippling load
P_R - actual load to cause failure or Rankine load
P_c - crushing load or yield point load in compression

generally it is applicable for columns of all ranges.

for a very short strut, P_e is very large ∴ $1/P_e$ is neglected

∴ $P_R = P_c$

for very long struts, P_e is very small but P_c is very large ∴ $1/P_c$ is neglected

∴ $P_R = P_e$

$$P_R = \frac{P_c P_E}{P_c + P_E} \qquad \cancel{\frac{1}{P_R}} = \frac{\cancel{P_c + P_E}}{\cancel{P_c P_E}} = \frac{P_c}{1 + \frac{P_c}{P_E}} = \frac{\sigma_c A}{1 + \frac{\sigma_c A L^2}{\pi^2 E I}} = \frac{\sigma_c A}{1 + \frac{\sigma_c A L^2}{\pi^2 E A k^2}}$$

$$P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L}{k}\right)^2}$$

σ_c is crushing stress

$$\therefore \boxed{P_R = \frac{\sigma_c A}{1 + a(l/k)^2}}$$

$a = \sigma_c / \pi^2 E$

a is called Rankine const

$$\therefore P_R = \sigma_c \cdot A\left(1 + a(l/k)^2\right)^{-1} = \sigma_c A\left(1 - b(l/k)^2\right) \quad \ldots \text{higher terms neglected}$$

Johnson Parabolic formula: $\boxed{P = \sigma_c A\left(1 - b(l/k)^2\right)}$

$a \approx b$

σ_c & σ_y are same

a is of the order 10^{-4} hence order of higher terms would be lesser & hence neglected

1] Compare the ratio of the strength of solid steel column to that of the hollow steel column of same cross sectional area. The internal dia of hollow column is 3/4 th of the external dia. The columns have same length & are pinned at both ends. Use Euler's theory.

let diameter of solid steel column be ds

External diameter of hollow = D

$\therefore$ internal " " $= \frac{3}{4}D$

cross sect^n al area is same

$$\therefore \frac{\pi d_s^2}{4} = \frac{\pi}{4}\left[D^2 - \frac{9}{16}D^2\right]$$

$$d_s^2 = \frac{7}{16}D^2$$

$$(P_c)_{solid} = \frac{\pi^2 E I_s}{L^2} \qquad (P_c)_{hollow} = \frac{\pi^2 E I_h}{L^2}$$

$$\therefore \frac{(P_c)_{solid}}{(P_c)_{hollow}} = \frac{I_s}{I_h} = \frac{\frac{\pi}{64}[D^4 - (3D/4)^4]}{\frac{\pi}{64} d_s^4} = \left(1 - \frac{81}{256}\right)\left(\frac{D}{d_s}\right)^4$$

$$= \frac{175}{256} \times \frac{256}{49} = \frac{175}{49} = 3.571$$

This is indicating that for same cross sect^n al area & length, vol. of the solid & hollow is same but hollow one

2] Find Euler crushing load for a hollow cylindrical column of external dia 120mm & thickness = 20mm if it is 4.2 m long & is hinged at both ends. Compare this load with a crushing load as given by Rankine's formula. Consider $E = 80\ kN/mm^2$, $\sigma_c = 550\ MPa$, $a = 1/1600$

$$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}(120^2 - 80^4) = 8168140.89\ mm^4$$

$$\text{crippling load} = P_e = \frac{\pi^2 E I}{L_{eff}^2} = \frac{\pi^2 \times 80\times10^3 \times 8168140.89}{(4200\ mm)^2} = 365.60689\ kN$$

$$A = \frac{\pi}{4}(120^2 - 80^2) = 6283.18\ mm^2$$

$$k = \sqrt{I/A} = 36.05\ mm$$

$$P_R = \frac{\sigma_c A}{1+a(L/k)^2} = \frac{550\ (N/mm^2) \times 6283.18\ mm^2}{1+\frac{1}{1600}\left(\frac{4200}{36.05}\right)^2}$$

$$= 364.41516\ kN$$

now, $\frac{P_e}{P_R} = \frac{365.60689}{364.41516} = 1.003$

3] In above problem for what length of the column does the Euler's formulae cease to apply?

$$P_{CR} = P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA}{(L/k)^2}$$

$$\therefore \sigma_{CR} = \frac{P_{CR}}{A} = \frac{\pi^2 E}{(L/k)^2}$$

$\sigma_{CR} \leq [\sigma_c]$ $\therefore$ $[\sigma_c]$ is allowed compressive stress

$[\sigma_c] = \sigma_c / \text{factor of safety}$

$$\therefore \frac{\pi^2 E}{(L/k)^2} \leq [\sigma_c]$$

$$\therefore \left(\frac{L}{k}\right)^2 \geq \frac{\pi^2 E}{[\sigma_c]} \quad \therefore L \geq k\pi\sqrt{E/\sigma_c}$$

$\because$ f.o.s is not given, consider it as 1

$$\therefore L \geq 36.05 \times \pi \sqrt{\frac{80 \times 10^3\ N/mm^2}{550\ N/mm^2}}$$

$$L \geq 1366.108\ mm$$

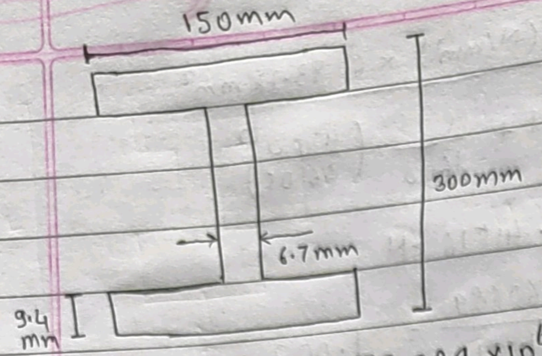
That means for L greater than 1366.108 mm it'll be considered as long column & Euler's theory is applicable. Below the value, it'll be treated as short column. We'd have to consider Johnson Parabolic formula for that

4] For an 'I' section with one end fixed & other end hinged,

i. find safe ~~crippling~~ critical load its length is 4m & f.o.s. = 2

ii. find the length of column for which the crippling load given by Euler's theory & Rankine's theory will be same.

Consider $E = 210\ GPa$, $\sigma_c = 330\ MPa$, $a = 1/7500$ for pinned ends



i. $I_{xx} = 73.329 \times 10^6 \ mm^4$

$I_{yy} = 3.762 \times 10^6 \ mm^4$

$A = 4808 \ mm^2$

whichever I is lesser we've to use it

$$k = \sqrt{I_{min}/A} = \sqrt{3.762 \times 10^6 / 4808}$$

$$= 27.97 \ mm$$

$$\left(\frac{L}{k}\right)_{CR} = \sqrt{\frac{\pi^2 E}{[\sigma_c]}} = \sqrt{\frac{\pi^2 \times 210 \times 10^3}{\sigma_c / F.O.S.}} = \sqrt{\frac{\pi^2 \times 210 \times 10^3}{330/2}}$$

$$\frac{L}{k} = \frac{4000 \ mm}{27.97 mm} = 143.010$$

$\therefore \frac{L}{k} > (L/k)_{CR}$, it is long column

$\therefore$ Euler's formula is applicable

$$P_{CR} = \frac{2.05 \pi^2 E I}{L} = \frac{2.05 \times \pi^2 \times 210 \times 10^3 \times 3.762 \times 10^6}{4000}$$

ii. for $P_e = P_R$ find L is the qst

$$\therefore \frac{\pi^2 E I}{L_{eff}^2} = \frac{\sigma_c A}{1 + a(L/k)^2}$$

$$\therefore \frac{\pi^2 \times 210 \times 10^3 \times 3.762 \times 10^3}{L_{eff}^2} = \frac{330 \times 4808}{1 + \frac{1}{7500}(L_{eff}/k)^2}$$

$\therefore L_{eff} = 5500.172 \ mm$

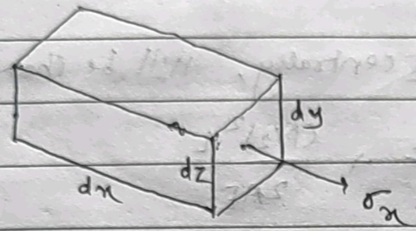
for one end fixed & other end hinged, $L_{eff} = \frac{L}{2.05}$

$\therefore L = 2.05 \times 5500.172$

$= 11.275 \ m$

Strain Energy

Internal workdone in deforming the body by the actⁿ of externally applied forces
This energy in elastic bodies is known as elastic energy



avg. force $= \frac{1}{2}\sigma_x\, dy\, dz$

elongation $= \varepsilon_x dx$

workdone $= \frac{1}{2}\sigma_x\, dy\, dz\, \varepsilon_x\, dx$

$dU = \frac{1}{2}\sigma_x \varepsilon_x\, dV$

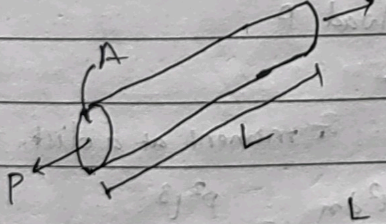
$U_0 = \frac{dU}{dV} = \frac{1}{2}\sigma_x \varepsilon_x$

$\sigma_x = E\varepsilon_x \qquad U_0 = \frac{1}{2}E\varepsilon_x^2 = \frac{1}{2}\frac{\sigma_x^2}{E}$

σ_y is yield stress

will depend on stress ← Brittle modulus of resiliance $= U_Y = \frac{\sigma_y^2}{2E}$ U_y is ability of material to store ~~or absorb energy without~~ permanent deformation

will depend on strain ← ductile " " toughness $= \frac{E\varepsilon_R^2}{2}$ ε_R is strain at rupture
It is ability to absorb energy upto fracture



rod of uniform cross sectⁿ subjected at its ends an equal & oppo. force of mag. 'P'

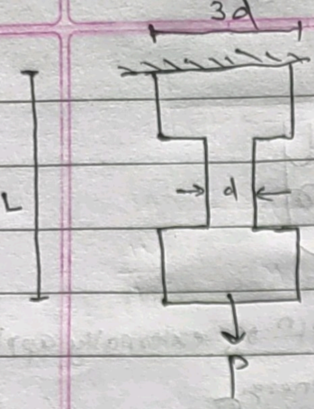
$$U = \int_0^L \frac{\sigma_x^2}{2E} dV = \int_0^L \frac{\sigma_x^2}{2E} A\, dx = \int_0^L \frac{(P/A)^2}{2E} A\, dx$$

$$\therefore U = \frac{P^2 L}{2AE}$$

$\sigma = \frac{P}{A} \qquad \varepsilon = \frac{P}{AE} \qquad \therefore \Delta l = \varepsilon L \qquad \therefore \Delta l = \frac{PL}{AE}$

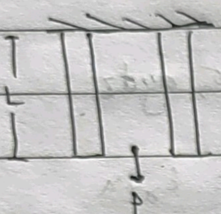
$$\frac{dU}{dP} = \frac{PL}{AE} = \Delta L$$

... Castigliano theorem I



$$U = \underbrace{\frac{P^2 (L/4)}{2AE}}_{\text{slender part}} + \underbrace{\frac{P^2(3L/4)}{2(9A)E}}_{\text{rest part}}$$

$$U = \frac{P^2 L}{6AE}$$



$\because$ P is applied centrally, it'll be shared

$$U = \frac{(P/2)^2 L}{2AE} + \frac{(P/2)^2 L}{2AE}$$

$$= \frac{P^2 L}{4AE}$$

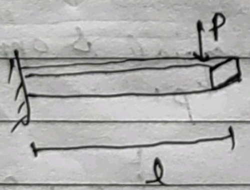
* Strain energy in bending

$$U = \int \frac{\sigma^2}{2E} dV \qquad \sigma = \frac{My}{I}$$

$$U = \int \frac{M^2 y^2}{2EI^2} dV = \int \frac{M^2 y^2}{2EI^2} dA\, dx = \int \frac{M^2}{2EI} dx \quad \ldots \quad y^2 dA = I$$

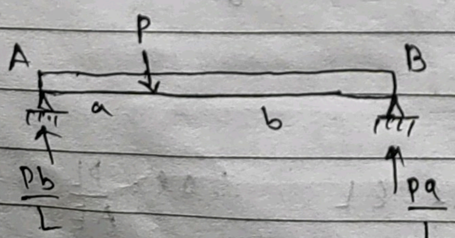
$$U = \int \frac{M^2 dx}{2EI}$$

Cantilever beam subjected to load P



$M = -Px$ .... moment at a dist. x from end

$$U = \int_0^l \frac{P^2 x^2 dx}{2EI} = \frac{P^2 l^3}{6EI}$$



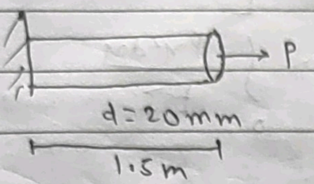
$a + b = l$

$$M_1 = \frac{Pbx}{L} \qquad M_2 = \frac{Pax}{L}$$

$$U = \int_0^a \frac{(Pbx/L)^2 dx}{2EI} + \int_0^b \frac{(Pax/L)^2 dx}{2EI}$$

$$= \frac{P^2 b^2 a^3}{6L^2EI} + \frac{P^2 a^2 b^3}{6L^2EI} = \frac{P^2 a^2 b^2 (a+b)}{6L^2EI} = \frac{P^2 a^2 b^2}{6LEI}$$

1) A rod AB must acquire strain energy of 13.6 N using E = 200 GPa. Find the required yield strength of steel if the factor of safety wrt permanent deform^n is 5



$$V = \frac{\pi d^2 L}{4} = \frac{\pi}{4} \times 400 \times 1500 = 471 \times 10^3 \text{ mm}^3$$

fos = 5

$\therefore$ Strain energy of rod = 5 [13.6] = 68 Nm = 68×10^3 Nmm

modulus of resilience is equal to strain energy density when maximum stress is σ_y

$$\frac{U}{V} = \frac{68 \times 10^3}{471 \times 10^3} = \frac{\sigma_y^2}{2E} = \frac{\sigma_y^2}{2 \times 200 \times 10^3} \qquad \therefore \sigma_y = 200 \text{ MPa}$$

imp: Since energy loads are not linearly related to the loads they produce factor of safety associated with strain energy loads should be applied to the energy loads & not to the stresses

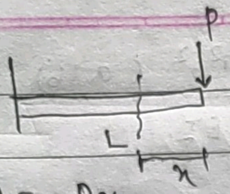
* Castigliano th^m 1 :

If strain energy of a linearly elastic structure is expressed in terms of the system of external loads. The partial derivative of strain energy wrt concentrated external load is the deflect^n of structure at the point of ~~deflect^n~~ applic^n & in the direct^n of that load

$$\frac{\partial U}{\partial P} = y$$

* Castigliano th^m 2 :

Suppose displ. of structure are changed by small amount $d\delta_i$ while all other displ. are held const, the increase in strain energy can be expressed as $dU = \frac{\partial U}{\partial \delta_i} d\delta_i \qquad \frac{\partial U}{\partial M_i} = \theta_i$



$$U = \int \frac{M^2 dx}{2EI}$$

$$M = -Px$$

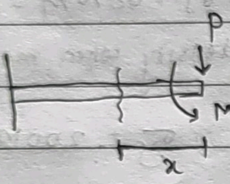
$$\frac{dM}{dP} = -x$$

$$\delta_1 = \frac{dU}{dP}$$

$$\delta_1 = \int \frac{2M \frac{dM}{dP} dx}{2EI}$$

$$= \int \frac{-Mx\,dx}{EI} = \int_0^L \frac{Px^2 dx}{EI}$$

$$\delta_1 = \frac{PL^3}{3EI}$$



hypothetical

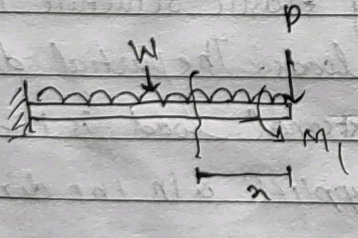
M_1 is a concentrated load equal to 0

$$\theta_1 = \frac{dU}{dM_1} = \int_0^L \frac{M}{EI}\frac{dM}{dM_1} dx$$

$$M = -Px + M_1$$

$$\theta_1 = \int_0^L \frac{(-Px + M_1)}{EI} \frac{d(-Px + M_1)}{dM_1} dx$$

$$= \int_0^L \frac{-Px(1)\,dx}{EI} = \frac{-PL^2}{2EI}$$



$$M_x = -Px - \frac{Wx^2}{2}$$

P is hypothetical
$P = 0$

$$\frac{dM_x}{dP} = -x$$

$$\delta_i = \int_0^L \frac{M_x}{EI} \cdot \frac{\partial M_x}{\partial P} dx = \int_0^L \frac{\left(-Px - \frac{Wx^2}{2}\right)}{EI} \cdot (-x)\,dx$$

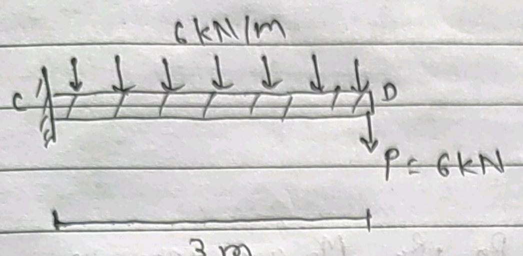
$$= \frac{1}{EI}\int_0^L \frac{Wx^3}{2} dx = \frac{WL^4}{8EI}$$

M_1 is hypothetical
$M_1 = 0$

$$\theta = \partial U / \partial M_1 = \int_0^L \left[\frac{M_1 - \frac{Wx^2}{2}}{EI}\right] \frac{\partial}{\partial M_1}\left(M_1 - \frac{Wx^2}{2}\right) dx$$

$$= \frac{-WL^3}{6EI}$$

A cantilever beam CD supports a uniformly distributed load 6 kN/m = w & a concentrated load P = 6 kN as shown find deflectⁿ at D. EI = 5 MN×m²



$$M_x = -Px - \frac{wx^2}{2}$$

$$\frac{\partial M_x}{\partial P} = -x$$

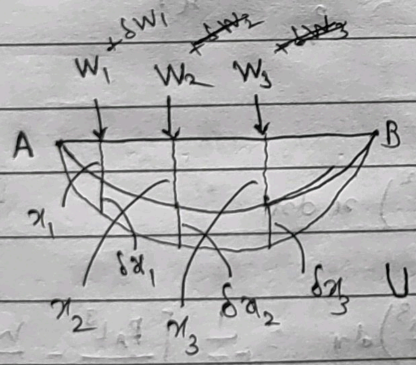
$$\delta_1 = \frac{\partial U}{\partial P} = \int_0^L \frac{M_x \partial M_x}{EI\,\partial P}\,dx = \frac{1}{EI}\int_0^L \frac{\left(-Px - \frac{wx^2}{2}\right)(-x)}{EI}\,dx$$

$$= \frac{1}{EI}\int_0^L \left(Px^2 + \frac{wx^3}{2}\right)dx = \frac{PL^3}{3} + \frac{wL^4}{8}$$

$$= \left[\frac{6\times 3^3}{3} + \frac{6\times 3^4}{8}\right]\times\frac{1}{5\times 10^6}$$

$$= 22.95\times 10^{-6}\ m$$

* proof for Castigliano's first thᵐ



$$\frac{\delta U}{\delta W_1} = x_1 \qquad \frac{\delta U}{\delta W_2} = x_2 \qquad \frac{\delta U}{\delta W_3} = x_3$$

$$U = \frac{1}{2}W_1x_1 + \frac{1}{2}W_2x_2 + \frac{1}{2}W_3x_3 \quad \text{--- (1)}$$

$$U + \delta U = \frac{1}{2}(W_1+\delta W_1)(x_1+\delta x_1) + \frac{1}{2}W_2(x_2+\delta x_2) + \frac{1}{2}W_3(x_3+\delta x_3) \quad \text{--- (2)}$$

$$\delta U = \left(W_1 + \frac{1}{2}\delta W_1\right)\delta x_1 + W_2\delta x_2 + W_3\delta x_3 + \dots$$

$$= W_1\delta x_1 + W_2\delta x_2 + W_3\delta x_3 + \dots + \frac{1}{2}\cancel{\delta W_1 \delta x_1} + \dots$$

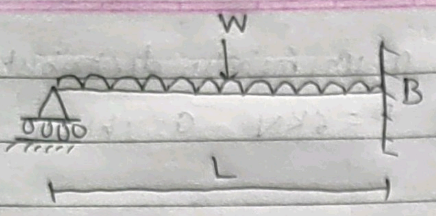
$$\therefore \delta U = W_1\delta x_1 + W_2\delta x_2 + W_3\delta x_3 + \dots$$

from eqⁿ (2),

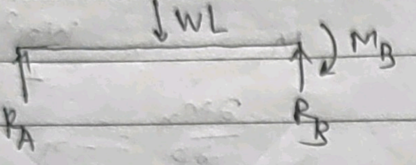
$$U + \delta U = \left[\frac{1}{2}W_1x_1 + \frac{1}{2}W_2x_2 + \dots\right] + \left[\frac{1}{2}W_1\delta x_1 + \frac{1}{2}W_2\delta x_2 + \dots\right] + \frac{1}{2}\delta W_1 x_1$$

$$\therefore U + \delta U = U + \frac{1}{2}\delta U + \frac{1}{2}\delta W_1 x_1$$

$$\therefore \frac{1}{2}\delta U = \frac{1}{2}\delta W_1 x_1 \qquad \therefore x_1 = \frac{\delta U}{\delta W_1}$$



we can find React^n forces here easily
∴ It is statically determinate beam



$R_A + R_B = WL$

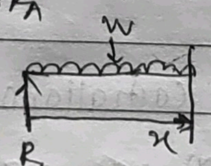
$R_B L - M_B - \frac{WL^2}{2} = 0$

R_A, R_B, M_B are unknown
we can't find them with 2 eq^{n}s only
This is statically indeterminate beam

∴ we'll use Castigliano thm here

$$U = \frac{1}{2EI}\int_0^L M_x^2\,dx$$

$$Y_A = \frac{\delta U}{\delta R_A} = \frac{1}{EI}\int_0^L M_x \frac{\delta M_x}{\delta R_A}\,dx$$



$$M_x = R_A x - \frac{Wx^2}{2}$$

$$\frac{\delta M_x}{\delta R_A} = x$$

$$\therefore Y_A = \frac{1}{EI}\int_0^L \left(R_A x - \frac{Wx^2}{2}\right) x\,dx$$

$$= \frac{1}{EI}\int_0^L \left(R_A x^2 - \frac{Wx^3}{2}\right) dx = \frac{1}{EI}\left[\frac{R_A L^3}{3} - \frac{WL^4}{8}\right]$$

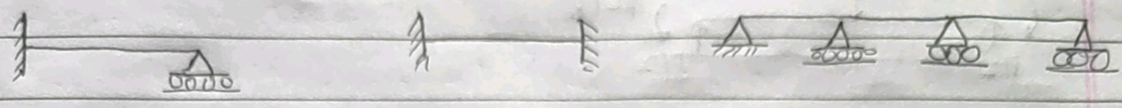
but $Y_A = 0$

$$0 = \frac{1}{EI}\left[\frac{R_A L^3}{3} - \frac{WL^4}{8}\right]$$

$$\therefore R_A = \frac{3WL}{8}$$

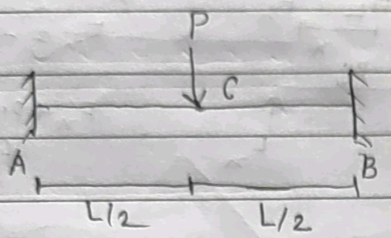
$$\therefore R_B = \frac{5WL}{8} \quad \cdots\; R_A + R_B = WL$$

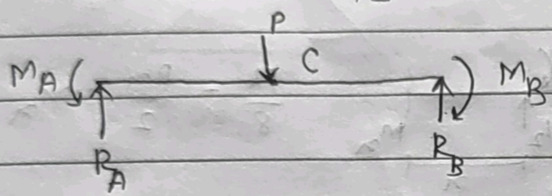
Statically Indeterminate Beams



propped cantilever   fixed beams   continuous beams

1)





$\because$ P is at midpoint of AB,

$R_A = R_B = P/2$

$M_x = M_A - R_A x$ $M'_x = R_B x - M_B$

$$U = \int_0^{L/2} \frac{M_x^2\, dx}{2EI} + \int_0^{L/2} \frac{M_x^2\, dx}{2EI}$$

$$\theta_A = \frac{\delta U}{\delta M_A} = 2\int_0^{L/2} \frac{M_x}{EI}\left(\frac{\delta M_x}{\delta M_A}\right) dx = 2\int_0^{L/2} \frac{\left(\frac{P}{2}x - M_A\right)(-1)\, dx}{EI}$$

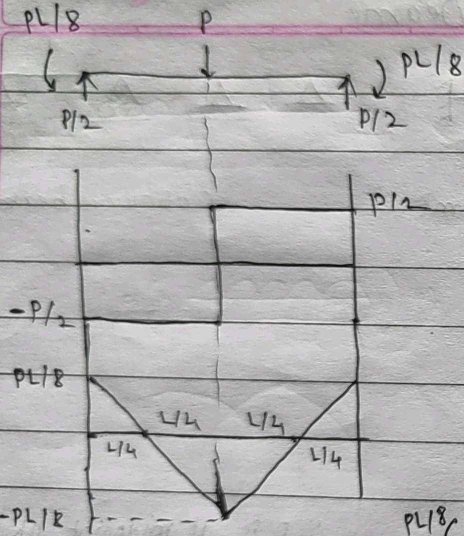
$$= \frac{-2}{EI}\left[\frac{P}{2}\frac{x^2}{2} - M_A x\right]_0^{L/2} = \frac{-2}{EI}\left[\frac{P}{4}\cdot\frac{L^2}{4} - \frac{M_A L}{2}\right]$$

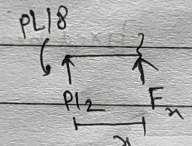
$$\theta_A = \frac{-L}{EI}\left[\frac{PL}{8} - M_A\right]$$

$\because$ both A and B are fixity supported, $\theta_A = \theta_B = 0$

$$\therefore M_A = M_B = \frac{PL}{8}$$

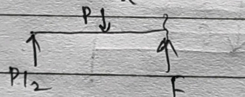
lets draw SFD & BMD for this,





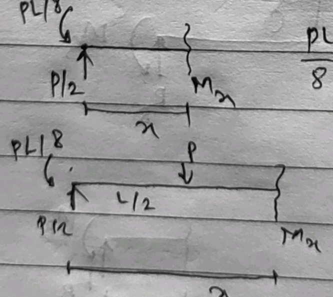
$F_x + P/2 = 0$

$F_x = -P/2$



$F_x + P/2 = P$

$F_x = P/2$



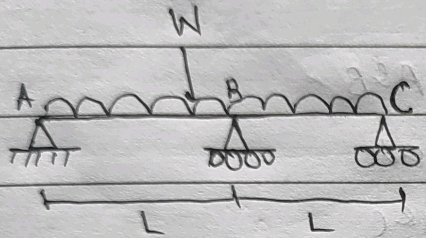
$\frac{PL}{8} - \frac{Px}{2} = M_x$ at $x = L/4$, $M_x = 0$

$\frac{PL}{8} - \frac{Px}{2} + P\left(x - \frac{L}{2}\right) = M_x$

$\frac{Px}{2} - \frac{3PL}{8} = M_x$

3]

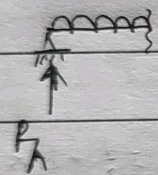
4]



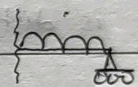
moment balancing at B,

$R_A L = R_C L$

$R_A = R_C$



$M_x = R_A x - \frac{Wx^2}{2}$



$M'_x = R_C x - \frac{Wx^2}{2}$

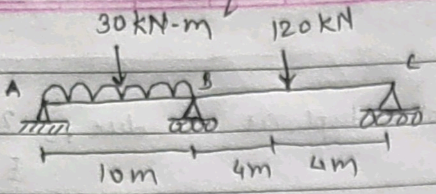
$$\frac{\delta U}{\delta R_A} = \delta_A = 2\int_0^L \frac{M_x}{EI}\frac{\delta M_x}{\delta R_A}\,dx = \frac{2}{EI}\int_0^L \left(R_A x^2 - \frac{Wx^3}{2}\right)dx$$

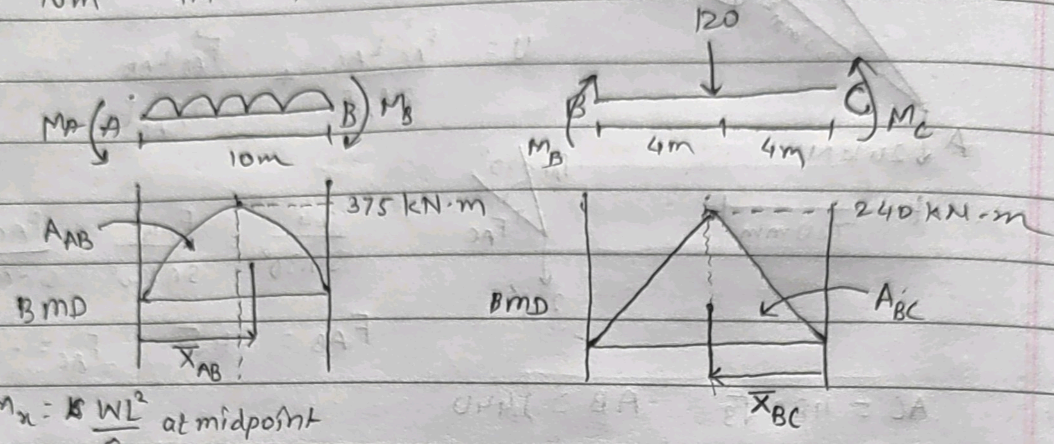
$$\delta_A = \frac{2}{EI}\left[\frac{R_A L^3}{3} - \frac{WL^4}{8}\right]$$

but $\delta_A = 0$ $\quad \therefore \frac{R_A L^3}{3} = \frac{WL^4}{8}$

$$R_A = R_C = \frac{3WL}{8}$$

* 3 moment equation





$M_x = \frac{WL^2}{8}$ at midpoint

$= 375$

$\because$ simply supported at ends, $M_A = M_C = 0$

Imp

$$L_{AB}M_A + 2(L_{AB}+L_{BC})M_B + L_{AC}M_{AC} = -\left[\frac{6A_{AB}\bar{x}_{AB}}{L_{AB}} + \frac{6A_{BC}\bar{x}_{BC}}{L_{BC}}\right]$$

A_{AB} is area under BMD b/w points A&B

$\bar{x}_{AB}$ is dist of centroid from left hand side

A_{BC} is area under BMD b/w points B&C

$\bar{x}_{BC}$ is dist of centroid from righ handside

$A_{AB} = \frac{WL^3}{12} = \frac{30\times 10^3}{12} = 2500$ $\quad A_{BC} = \frac{1}{2}\times 8\times 240 = 960$

$\bar{x}_{AB} = 5$ $\quad \bar{x}_{BC} = 4m$ ... since both are symmetric

$M_A - M_C = 0$

$$\therefore 2(18)\times M_B = -\left[\frac{2500\times 30}{10} + \frac{960\times 24}{8}\right]$$

$$\therefore M_B = -288.34 \text{ kN-m}$$

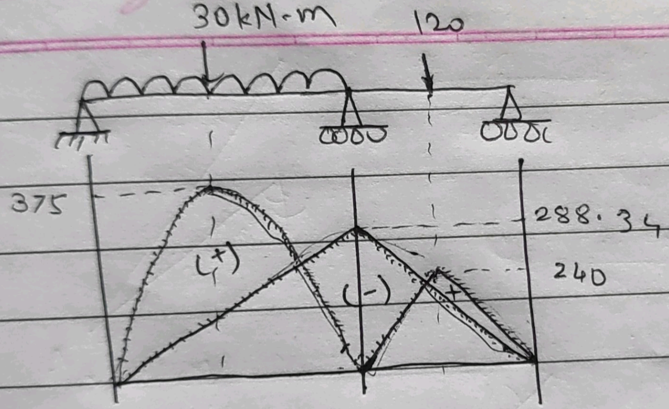
analysis of stress & strain
Mohr circle
E G K
Poisson's ratio

Beam
deflectⁿ
Macaulay's principle
Area moment method

columns
Euler
Rankine
* Euler - limitation
Short note
(graph)

Strain energy
use castigliano
find deflectⁿ
theorem proof
3 moment eqⁿ
castigliano

num. on 3M eqⁿ



This is equivalent BMD