Induction Motor :-

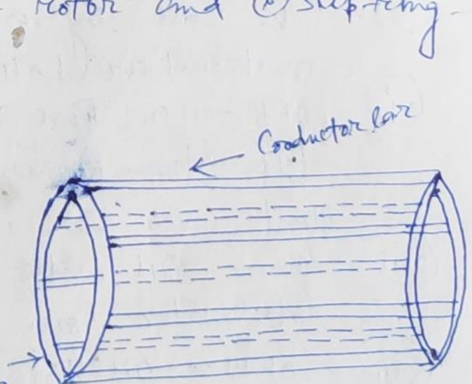
Conversion of electrical power into mechanical power takes place in the rotating part of an electric motor. In d.c. motors, electrical power is conducted directly to the armature (i.e. rotating part) through brushes and commutator. Hence, a d.c. motor can be called a conduction motor. However, in a.c. motors, the rotor does not receive electric power by conduction, but by induction, in exactly the same way as the secondary of a two winding transformer receives its power from the primary. That is why such motors are known as induction motors.

Construction:- An induction motor consists of two main parts — (i) Stator and (ii) Rotor.

(i) **Stator:-** The stator carries a 3-phase winding and is fed from a 3-phase supply. It is made up of a number of stampings which are slotted to receive the windings. It is wound for a definite number of poles. The stator windings, when supplied with 3-phase currents, produce a magnetic flux which is of constant magnitude, but revolves at synchronous speed. This revolving magnetic flux induces an emf. in the rotor by mutual induction.

(ii) **Rotor:-** There are two types of rotors in an induction motor — (a) cage rotor and (b) Slip-ring rotor.

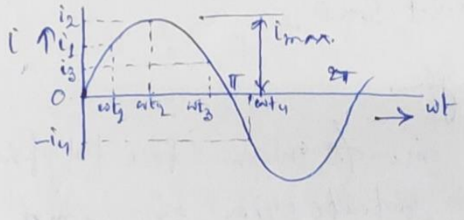
(a) **Cage rotor:-** The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors. The conductors are shorted at both ends by means of thick end-rings. Since, the rotor conductors' arrangement looks like a squirrel cage, it is called a cage rotor.

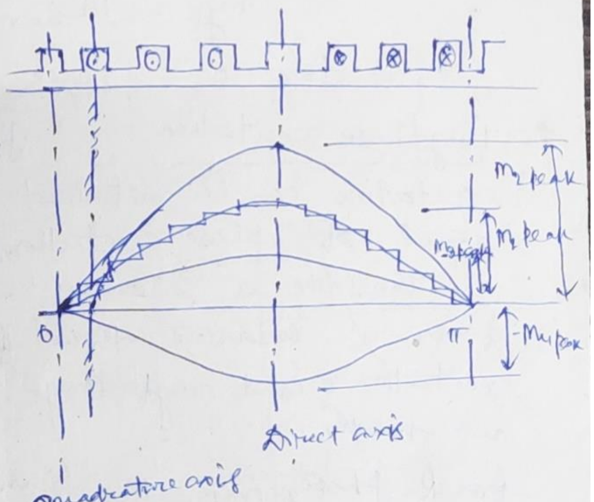


(ii) Slip-ring rotor:- A slip-ring or wound rotor carries a normal three or six-phase winding, connected in star or delta and terminated on three slip-rings, which are short-circuited when the motor is running.

Production of Rotating Field:-

Single phase ac excitation:-





When a uniformly distributed winding is excited from single phase a.c supply, the instantaneous mmf waveform is stepped one and the resulting field form is approximately sinusoidal in space. For sinusoidally varying exciting current, the field form is pulsating in nature. At any instant t_1 exciting current is i_1 and the m.m.f. wave is stepped. Neglecting space harmonics in the m.m.f. waveform, the m.m.f. and flux density waveforms are sinusoidal in space and the equation for the instantaneous mmf. is given by,

$$mf = M_{1peak} \sin\theta$$

where, M_{1peak} corresponds to i_1.

At another instant $t = t_2$, exciting current is i_2 and $mf = M_{2peak} \sin\theta$.

Similarly at instants t_3 and t_4, currents are respectively i_3 and $-i_4$ and mmf. equations are

$$mf = M_{3peak} \sin\theta, \quad mf = -M_{4peak} \sin\theta.$$

Since, m_1 peak, m_2 peak, m_3 peak and $-m_4$ peak are respectively proportional to i_1, i_2, i_3 and $-i_4$, which lie on a sinusoid,

$$m_{peak} = M_{max} \sin w_s t,$$

where M_{max} corresponds to the maximum value of exciting current I_{max}, the m.m.f. wave for a single phase sinusoidal excitation is,

$$m_f = M_{max} \sin w_s t \sin\theta$$

where θ = space angle.

<u>Polyphase excitation:</u>— The m.m.f. wave for polyphase excitation can be obtained by superposition of m.m.f. waves of phase excitations.

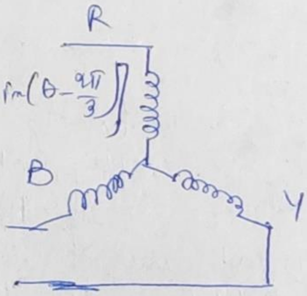
Consider a balanced three phase winding excited from a balanced three phase supply. Assuming sinusoidal excitation and neglecting space harmonics in m.m.f. waves, we have,

for R-phase, $m_R = M_{max} \sin w_s t \sin\theta$

for Y-phase, $m_Y = M_{max} \left[\sin\left(w_s t - \frac{2\pi}{3}\right) \sin\left(\theta - \frac{2\pi}{3}\right)\right]$

for B phase

$$m_B = M_{max}\left[\sin\left(w_s t - \frac{4\pi}{3}\right)\sin\left(\theta - \frac{4\pi}{3}\right)\right]$$



Hence, the resultant m.m.f,

$$m_f = m_R + m_Y + m_B$$

$$= M_{max}\left[\sin w_s t \sin\theta + \sin\left(w_s t - \frac{2\pi}{3}\right)\sin\left(\theta - \frac{2\pi}{3}\right) + \sin\left(w_s t - \frac{4\pi}{3}\right)\sin\left(\theta - \frac{4\pi}{3}\right)\right]$$

$$= \frac{M_{max}}{2}\left[2\sin w_s t \sin\theta + 2\sin\left(w_s t - \frac{2\pi}{3}\right)\sin\left(\theta - \frac{2\pi}{3}\right) + 2\sin\left(w_s t - \frac{4\pi}{3}\right)\sin\left(\theta - \frac{4\pi}{3}\right)\right]$$

$$= \frac{M_{max}}{2}\left[\cos(w_s t - \theta) - \cos(w_s t + \theta) + \cos(w_s t - \theta) - \cos\left(w_s t + \theta - \frac{4\pi}{3}\right) + \cos(w_s t - \theta) - \cos\left(w_s t + \theta - \frac{8\pi}{3}\right)\right]$$

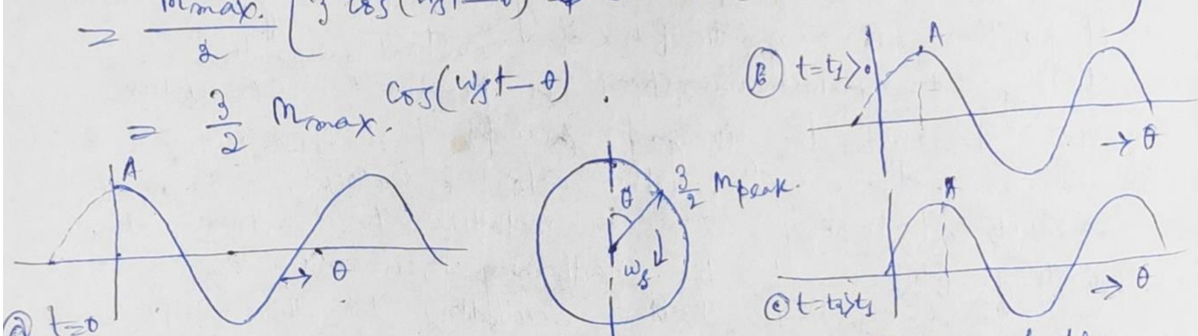
$$= \frac{M_{max}}{2}\Big[3\cos(\omega_s t-\theta) - \cos(\omega_s t+\theta) - \cos\Big\{\pi+\frac{\pi}{3}-(\omega_s t+\theta)\Big\} - \cos\Big\{3\pi-\frac{\pi}{3}-(\omega_s t+\theta)\Big\}\Big]$$

$$= \frac{M_{max}}{2}\Big[3\cos(\omega_s t-\theta) - \cos(\omega_s t+\theta) + \cos\Big\{\frac{\pi}{3}-(\omega_s t+\theta)\Big\} + \cos\Big\{\frac{\pi}{3}+(\omega_s t+\theta)\Big\}\Big]$$

$$= \frac{M_{max}}{2}\Big[3\cos(\omega_s t-\theta) - \cos(\omega_s t+\theta) + 2\cdot\cos\frac{\pi}{3}\cos(\omega_s t+\theta)\Big]$$

$$= \frac{M_{max}}{2}\Big[3\cos(\omega_s t-\theta) - \cos(\omega_s t+\theta) + \cos(\omega_s t+\theta)\Big]$$

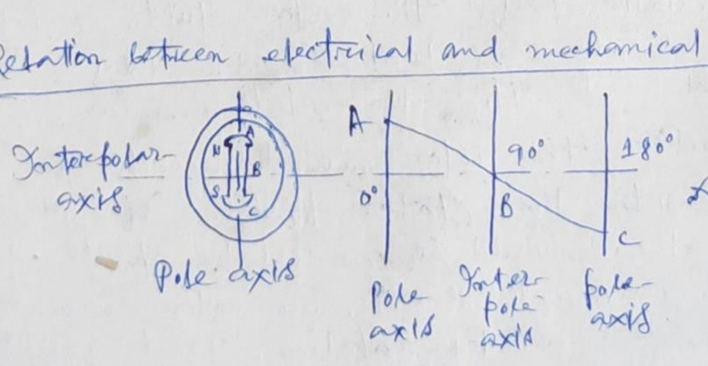
$$= \frac{3}{2} M_{max} \cos(\omega_s t-\theta).$$



So, the resultant m.m.f. is rotating in space at the same angular frequency ω_s rad./sec. as that of the exciting current and has a maximum amplitude of $\frac{3}{2}$ times that of each phase. [It may be noted that with time, the point A moves to the right in the positive direction of θ. Thus, $\cos(\omega_s t-\theta)$ represents a travelling i.e., rotating wave.]

* The direction of rotation of the rotating field can be reversed by interchanging the supply of any two phases.

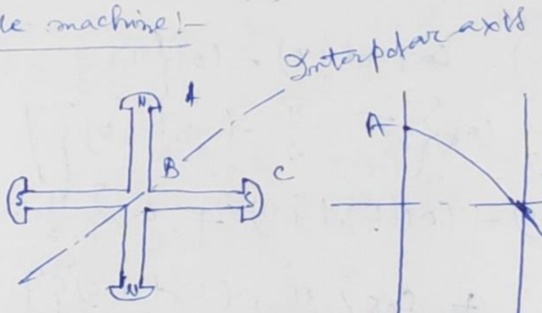
* <u>Relation between electrical and mechanical degrees:-</u>

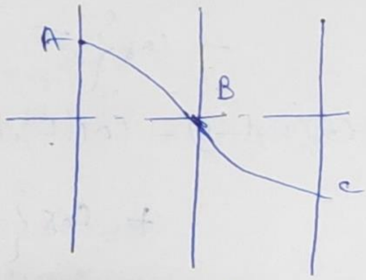


Distribution of flux

$\therefore\ \theta_m = \theta_e = 180°$

For a 4-pole machine:-





$\theta_m = 90^\circ$, $\theta_e = 180^\circ$

$\therefore$ For P-pole machine, $\theta_e = \frac{P}{2}\theta_m$.

Why does rotor rotate?

When the 3 phase stator windings are fed by a 3 phase supply, then a magnetic flux of constant magnitude but rotating at synchronous speed is produced. The flux passes through the air gap, sweeps past the rotor surface and so cuts the rotor conductors, which are, as yet, stationary. Due to relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter according to Faraday's laws of electromagnetic induction. The frequency of induced e.m.f. is same as the supply frequency. Since, the rotor conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts rotating in the same direction as that of the flux and tries to catch up with the rotating flux.

Slip:- In practice, the rotor never succeeds in catching up with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why, the rotor runs at a speed which is always less than the speed of the stator field. The difference between the synchronous speed N_s and the actual speed N of the rotor is known as slip.

$$\%\ \text{slip} = \frac{N_s - N}{N_s} \times 100.$$

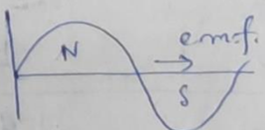
Synchronous speed:- It is the speed of the rotating magnetic field.

$$N_s = \frac{2f}{P} \quad \text{where, } P = \text{No. of poles.}$$

If speed is expressed in r.p.m.,

$$N_s = \frac{120f}{P} \text{ r.p.m.}$$

For a 2 pole machine, in one revolution $\frac{2}{2} = 1$ - cycles of e.m.f. is produced.



For a two pole machine.

For a 4- pole " , " " " $\frac{4}{2} = 2$ " " " " "

For a P pole " , " " " $\frac{P}{2}$ " " " " " .

$$\therefore \frac{P}{2} \cdot N_s = f$$

$$\Rightarrow N_s = \frac{2f}{P}.$$

Frequency of rotor current :-

~~Letting speed~~

Let, at any slip speed, the frequency of the rotor current be f'. Then, $N_s - N = \frac{120f'}{P}$

Also $N_s = \frac{120f}{P}$

$$\therefore \frac{N_s - N}{N_s} = \frac{f'}{f} = s \quad \Rightarrow f' = sf.$$

Equivalent circuit of an Induction Motor :-

$$f' = sf$$

Per phase-stator winding induced e.m.f. is given by

$$E_1 = 4.44 f N_1 \phi_m \times K_{w1} \text{ Volts.}$$

where, ϕ_m = Mutual flux or resultant air gap flux/pole.

N_1 = No. of turns per phase in the winding

K_{w1} = Stator winding factor.

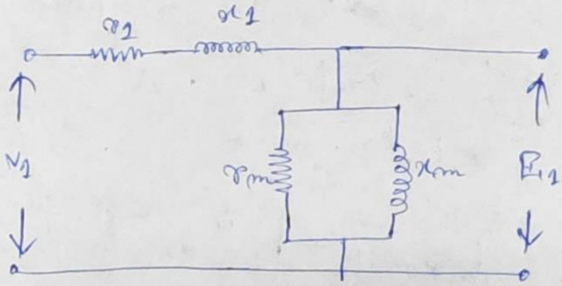
Per phase rotor induced e.m.f. at $s = 1$ is given by

$$E_2 = 4.44 f N_2 \phi_m \times K_{w2} \text{ Volts.}$$

where, N_2 = No of turns in rotor / phase
K_{w2} = Rotor winding factor.

At any slip s,

$$E_2 = 4.44\, s f N_2 \phi_m K_{w2} \text{ Volts.}$$



Stator equivalent circuit

r_1 = per phase stator resistance
x_1 = per phase stator leakage reactance
$r_m + jx_m$ = magnetizing branch
V_1 = supply voltage
E_1 = e.m.f. induced per phase in the stator winding.

Let, E_2 = standstill rotor e.m.f..

$$s = \frac{n_s - n_r}{n_s}$$

At standstill, $n_r = 0$.

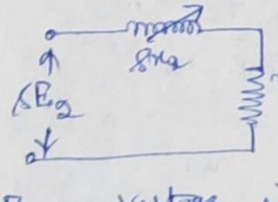
$\therefore$ $s = 1$.

$E_2 \propto sf$, at standstill, $E_2 = 4.44 f N_2 \phi_m K_{w2}$.

At any slip s, $E_{2s} = s E_2$.

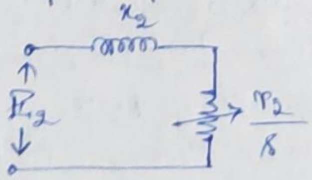
$x_2 = 2\pi f L$

$\therefore$ $x_2 \propto f' \propto sf$.



$\rightarrow$ Rotor equivalent circuit at frequency sf.

sE_2 = Voltage induced in the rotor at any slip s.
x_2 = per phase rotor leakage reactance
r_2 = per phase rotor resistance.



$\rightarrow$ Rotor equivalent circuit at frequency f.

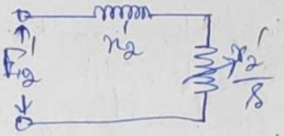
Let, $E_2' =$ e.m.f. induced in the rotor referred to Stator

$$\therefore E_2' = E_2 \times \frac{N_1}{N_2}$$

where, $N_1 =$ No. of turns per phase in Stator
$N_2 =$ No. of turns per phase in rotor.

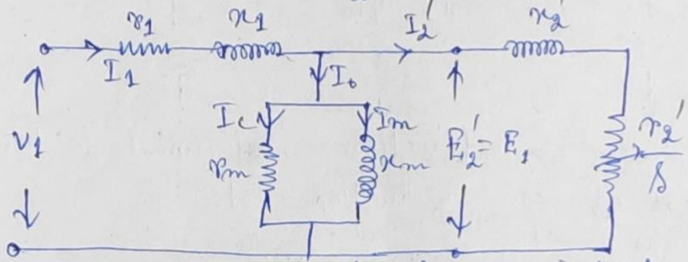
$x_2' =$ per phase rotor leakage reactance referred to Stator

$r_2' \rightarrow$ per phase rotor resistance referred to Stator.

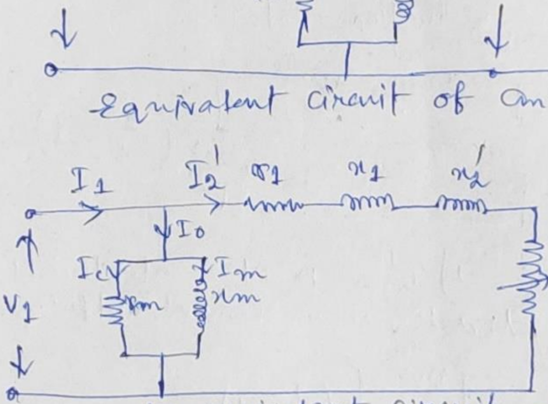


$\rightarrow$ Rotor equivalent circuit referred to Stator.

Now, $E_1 = E_2'$



Equivalent circuit of an induction motor.



$\rightarrow$ This simplification is not permissible in case of an induction motor, since no-load c.t. is 30 to 50% of full load c.t. and per unit stator leakage reactances are comparatively higher.

Approximate equivalent circuit of an induction motor.

In a transformer, no-load current varies from 2% to 6% of full-load current, where as in an induction motor, it varies from 30% to 50%.

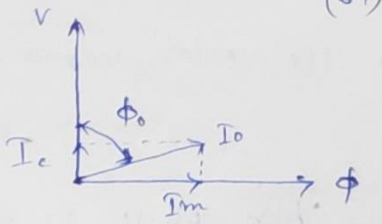
$$\text{Flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

Since, flux = constant, $\therefore$ m.m.f. $\propto$ reluctance.

In case of induction motor, mutual flux passes through air. Air has much reluctance.

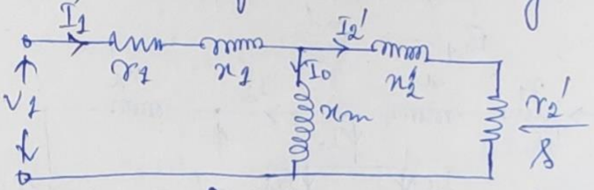
So, m.m.f. is high.

So, magnetizing current becomes high.



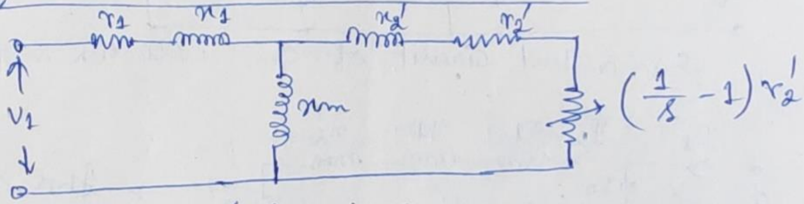
Power factor angle $\phi_0 \simeq 90°$.
$\therefore \cos 90° = 0$.
So, an induction motor at no-load operates at very low power factor.

Under normal operating conditions of constant voltage and frequency, core loss in induction motors is usually constant. In view of this fact, core loss resistance r_m - representing the motor core loss, can be omitted from the equivalent circuit. But for determining the shaft power or shaft torque, the constant core loss must be taken in to consideration along with the friction, windage and stray load losses.

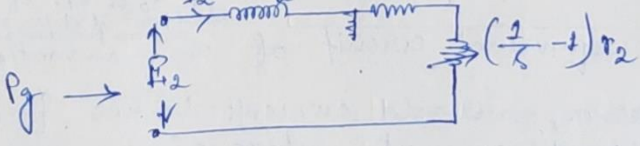


Constant losses $\rightarrow$ friction and windage losses + Core loss.

<u>Power equations of an induction motor :-</u>



Variable resistance $\left(\frac{1}{s}-1\right) r_2'$ - is the electrical analogue of mechanical load or mechanical output in electrical form.



Per phase rotor current at standstill $= \dfrac{E_2}{\sqrt{r_2^2+x_2^2}}$.

Per phase rotor current at any slip s is given by,

$$I_2 = \frac{sE_2}{\sqrt{r_2^2+(sx_2)^2}} = \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2+x_2^2}}.$$

P_g = Per phase power input to rotor
= per phase power transferred from stator to rotor.
= Air gap power.

$\therefore P_g = I_2^2 \cdot \frac{r_2}{s}$

Rotor copper loss $= I_2^2 \cdot r_2 = s P_g$.

Internal mechanical power developed,

$P_m =$ power input to rotor − rotor copper loss

$= P_g - s P_g = (1-s) P_g$.

Internal (or gross) electromagnetic torque developed per phase,

$$T_e = \frac{\text{Internal mechanical power developed in rotor}}{\text{Rotor speed}}$$

$$= \frac{(1-s) P_g}{\omega_r} = \frac{(1-s) P_g}{(1-s)\,\omega_s} \quad \left[\begin{array}{l} s = \frac{n_s - n_r}{n_s} \\ \therefore \omega_r = (1-s)\,\omega_s \end{array} \right]$$

$$= \frac{P_g}{\omega_s} \text{ N-m.}$$

$\omega_s \rightarrow$ Synchronous speed in mechanical radian/sec.

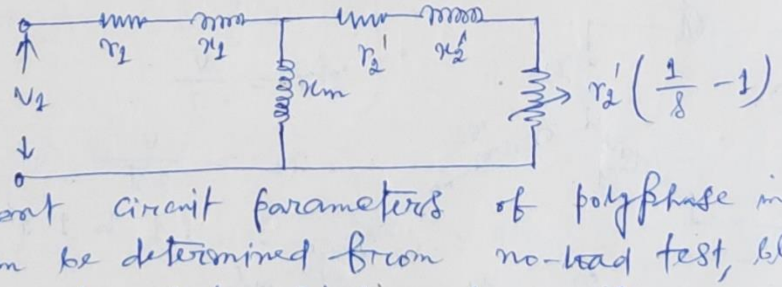
Output or shaft power, $P_{sh} = P_m -$ Fixed losses (friction, windage and core losses)

Shaft torque, $T_{sh} = \frac{P_{sh}}{\omega_r}$ N-m.

$\therefore T_{sh} < T_e$.

If stator input is known, the air gap power P_g is given by, $P_g =$ stator power input − stator I^2R loss − stator core loss.

Determination of equivalent circuit parameters:-

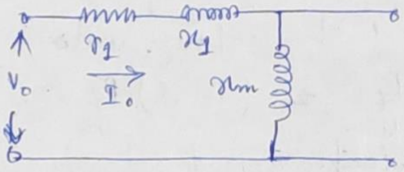


The equivalent circuit parameters of polyphase induction motors can be determined from no-load test, blocked-rotor test and stator winding d.c. resistance.

① No-load test :- The induction motor is made to run at no-load at rated voltage and frequency. Per phase values of applied stator voltage V_0, input current I_0 and input power W_0 are recorded.

No-load slip S_{nl} is very small. So, $\frac{r_2'}{S_{nl}}$ is very large as compared to x_m.

So, this is similar to a transformer on open-circuit.



→ Equivalent circuit at no load.

$$R_0 = \frac{W_0}{I_0^2}, \quad Z_0 = \frac{V_0}{I_0}$$

$$\therefore X_0 = \sqrt{Z_0^2 - R_0^2} = x_1 + x_m \quad \text{—— (1)}$$

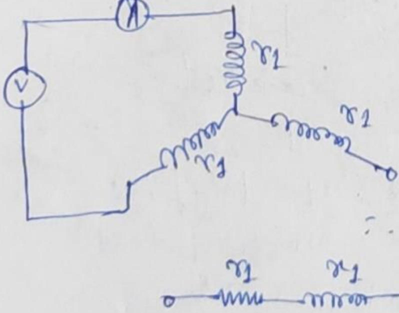
Rotational losses are assumed constant and ~~is~~ are given by, $P_R = m(W_0 - I_0^2 r_1)$

where, m = No. of stator phases.

(ii) <u>Blocked Rotor test</u> :— Blocked rotor test is similar to short circuit test on a transformer.

For performing this test, the rotor shaft is blocked by external means. Now, balanced polyphase voltages at rated frequency are applied to the stator terminals through a polyphase variac. This applied voltage is adjusted till rated current flows in the stator winding. Per phase values of applied voltage V_b, input current I_b and input power W_b are recorded.

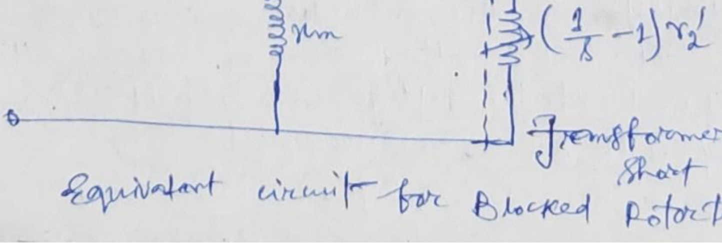
Now d.c. resistance per phase of the stator winding is measured.



$$2r_1 = \frac{V}{I}$$

$$\therefore r_1 = \frac{V}{2I}$$

$$\therefore r_{1\,a.c.} = 1.1 \text{ to } 1.5 \text{ times } r_{1\,d.c.}$$



Transformer on Short circuit.

Equivalent circuit for Blocked Rotor test.

$$Z_{b} = \frac{V_b}{I_b}, \quad R_b = \frac{W_b}{I_b^2}$$

$$\therefore X_b = \sqrt{Z_b^2 - R_b^2}$$

$$Z_b = r_1 + jx_1 + \frac{jx_m(r_2' + jx_2')}{r_2' + j(x_2' + x_m)}$$

$$= r_1 + jx_1 + \frac{jx_m(r_2' + jx_2')\left[r_2' - j(x_2' + x_m)\right]}{\left[r_2' + j(x_2' + x_m)\right]\left[r_2' - j(x_2' + x_m)\right]}$$

$$= r_1 + jx_1 + \frac{jx_m\left[r_2'^2 + jr_2'x_2' - jr_2'(x_2' + x_m) + x_2'(x_2' + x_m)\right]}{r_2'^2 + (x_2' + x_m)^2}$$

$$\Rightarrow \quad R_b + jX_b = r_1 + jx_1 + \frac{jx_m\left[r_2'^2 + x_2'X_2 - jr_2'x_m\right]}{r_2'^2 + X_2^2}$$

where, $X_2 = x_2' + x_m$ = rotor self reactance.

$$\therefore R_b = r_1 + \frac{r_2' x_m^2}{r_2'^2 + X_2^2}$$

$$X_2 >> r_2'$$

$$\therefore R_b = r_1 + r_2'\left(\frac{x_m}{X_2}\right)^2$$

$$\therefore r_2' = (R_b - r_1)\left(\frac{X_2}{x_m}\right)^2 \quad \text{—— (2)}$$

$$X_b = x_1 + \frac{x_m\left[r_2'^2 + x_2'X_2\right]}{\left[r_2'^2 + X_2^2\right]}$$

$$= x_1 + \frac{x_m\left[\frac{r_2'^2}{X_2} + x_2'\right]}{\left[\frac{r_2'^2}{X_2} + X_2\right]}$$

$$X_2 >> r_2',$$

$$\therefore X_b = x_1 + \frac{x_m x_2'}{X_2} = x_1 + \frac{x_m x_2'}{x_2' + x_m}$$

$$= x_1 + \frac{x_2'}{1 + \frac{x_2'}{x_m}}$$

$x_m >> x_2'$, $\therefore X_b = x_1 + x_2'$ —— (3)

There is no practical method of separating x_1 and x_2'.
For wound rotor machines, x_1 is assumed equal to x_2'.
$\therefore x_1 = x_2' = \frac{1}{2}X_b$.

Once x_1 is known, $x_m = X_0 - x_1$.

Hence, x_m – from eqn. ①, r_2' – from eqn. – ②, r_1 – from d.c. resistance per phase of stator winding and x_1, x_2' – from eqn. – ③ – can be determined from three tests.

Power flow diagram of an induction motor:–

Power input to stator
↓
- Stator core loss and Stator copper loss
- Power input to the rotor OR Power transferred across the airgap (P_g)
  ↓
  - Rotor copper loss
  - Mechanical power developed (P_m) or Rotor gross output
    ↓
    - Friction and windage losses (Mechanical losses)
    - Shaft power ($P_{sh.}$) or, B.H.P.

Problems :-

① The power input to a 3 phase induction motor is 60 KW. The total stator loss is 1 KW. Find the total mechanical power developed and the rotor copper loss per phase, if the motor is running with a slip of 3%.

Soln:- Here, slip $s = 0.03$

Power input to the rotor, $P_g = (60 - 1)$ KW $= 59$ KW.

Mechanical power developed,

$$P_m = (1-s)P_g = (1 - 0.03) \times 59 \text{ KW} = 57.2 \text{ K}$$

Total rotor copper loss $= sP_g$

$$= (0.03 \times 59) \text{ KW}$$

Rotor copper loss per phase $= \dfrac{0.03 \times 59}{3}$ KW.

$$= 590 \text{ W.}$$

② The power input to a 500 V., 50 Hz., 6 pole, 3-phase induction motor running at 975 r.p.m. is 40 KW. The stator losses are 1 KW and the friction and windage losses total 2 KW. Calculate (i) the slip (ii) rotor copper loss (iii) the brake horse power (shaft power) -

and (iv) efficiency.

Soln. :- Synchronous speed of the machine,

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

$n_r = 975$ r.p.m.

(i) ∴ slip, $s = \frac{1000 - 975}{1000} = 0.025$

(ii) Power input to the rotor,

$P_g = (40 - 1)$ kw. $= 39$ kw.

∴ Rotor copper loss $= sP_g$

$= (0.025 \times 39)$ kw. $= 975$ watt.

(iii) Mechanical power developed,

$P_m = (1-s)P_g = (1 - 0.025) \times 39$ kw.

$= 38.025$ kw.

$= 38025$ watt.

∴ Brake horse power (shaft power),

$P_{sh} = (38025 - 2000)$ w.

$= \frac{36025}{746}$ H.P. $= 48.35$ H.P.

(iv) Efficiency, $\eta = \frac{P_{sh}}{\text{Power input}}$

$= \frac{36,025}{40,000} \times 100\% = 90.06\%$