Fluid Statics

* Properties of fluid

1] Mass density (ρ) $= \frac{m}{V}$ for water, $1 gm/cc = 1000\ kg/m^3$

2] Specific weight (w) $= \frac{\text{wt. of fluid}}{\text{Vol. of fluid}} = \frac{mg}{V} = \rho g$

for water, $9810\ N/m^3$

3] Specific volume = vol. per unit mass $= \frac{V}{m} = \frac{1}{\rho} = \nu$

4] Specific gravity (Relative density) $(S)_{liq} = \dfrac{\text{specific wt. (or density) of fluid}}{\text{specific wt. (or density) of water}}$

$S_{mercury} = 13.6$ density of mercury $= 13600\ kg/m^3$

specific wt of mercury $= 13.6 \times 9810 = 133416\ N/m^3$

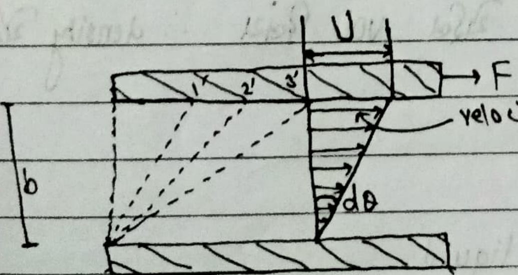
$S_{gas} = \dfrac{\text{Specific wt (or density) of fluid}}{\text{Specific wt (or density) of air}}$

5] Viscosity (μ)

$$\tau \propto \frac{du}{dy} \qquad \therefore \tau = \mu \frac{du}{dy}$$

τ - sheer stress μ - absolute / dynamic viscosity

$\frac{du}{dy}$ - velocity gradient, rate of shear strain



conditions:
Laminar flow
Bottom layer should be flat

the rate of deformation $= \frac{U}{b}$

$\frac{d\theta}{dt}$ = rate of angular deformation

$$\frac{d\theta}{dt} = \frac{U}{b} \quad \text{i.e} \quad \boxed{\frac{d\theta}{dt} b = U}$$

if area of upper solid surface is A,

$$\tau = \frac{F}{A}$$

$$T = \mu \frac{du}{dy}$$

$$\therefore \frac{F}{A} = \mu \frac{du}{dy} = \mu \frac{d\theta}{dt}$$

SI unit of μ is $N \cdot s/m^2$ or $kg/m \cdot s$

MKS unit of μ is $kg/m \cdot s$

CGS unit of μ is $dyne \cdot s/cm^2$

$1 \, dyne \cdot s/cm^2 = 1 \, poise$

Kinamatic viscosity (ν)

$$\nu = \frac{\mu}{\rho}$$

SI unit : m^2/s

CGS unit : cm^2/s

$1 \, cm^2/s = 1$ stoke

1] find sp. wt, density, sp. gravity for 5l liq. of weight 10N

$$sp \cdot wt = \frac{wt}{vol} = \frac{10 \, N}{5000 \times 10^{-6} \, m^2} = 2000 \, N/m^2$$

weight તથા mass ચેન્જ vol. ફિક્સ $\therefore$ density ચેન્જ.

* Concept of continuum

Continuous flow of liquid

At STP, No. of molecules : $N = 6.023 \times 10^{23} / 22.4$ lit

i.e. $\rho = 2.68 \times 10^{25}$ molecules$/m^3$

$\approx 3 \times 10^{25}$ molecules$/m^3$

* Knudsen number

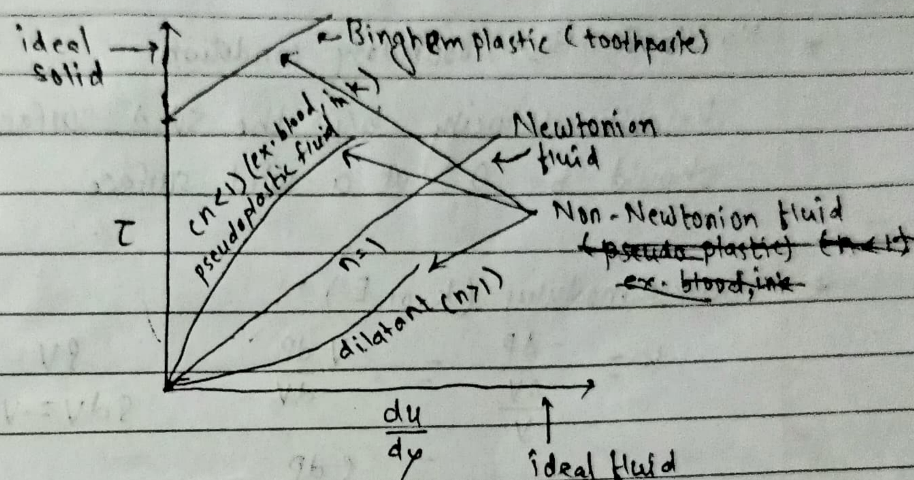
$$k_n = \frac{\lambda}{L}$$

λ - mean free path
L - characteristic length

$\frac{\lambda}{L}$ i.e. $k_n < 0.01 \Rightarrow$ Continuum flow

$0.01 < k_n \leq 0.1 \Rightarrow$ Slip flow
$0.1 < k_n < 10 \Rightarrow$ transition flow
$k_n \geq 10 \Rightarrow$ free molecular path

for ideal solid, $\mu = \infty$
for ideal liquid, $\mu = 0$



$\tau = \mu \frac{du}{dy}$ →Newton's law of viscosity

Thouse who don't follow are referred to as Non-Newtonian fluids

A fluid with no dynamic viscosity is called as ideal fluid i.e. $\mu = 0$ i.e. line through origin with slope 0 i.e. X axis
Much deformation can be seen for ideal fluid

Ideal solid: Technically it is solid with infinite viscosity i.e. it is Y axis. No deformation is seen in ideal solid at all

* Power law of ~~liquid~~ fluid

$$\tau = m\left(\frac{du}{dy}\right)^n$$

n - flow consistency index
m - flow behaviour index

$$\boxed{\tau = m\left(\frac{du}{dy}\right)^n}$$

$$\frac{\tau}{\frac{du}{dy}} = m\left(\frac{du}{dy}\right)^{n-1}$$

$$\mu_{apparent} = \boxed{\mu_{app} = m\left(du/dy\right)^{n-1}}$$

* No slip Condition / Boundary condition

Relative velocity b/w the solid surface & the fluid should be 0 at a solid surface

* Bulk's modulus (k or E)

$$k = \frac{-\Delta P}{\frac{\Delta V}{V}} = \frac{-V\,dP}{dV} = \frac{\rho\,dP}{d\rho}$$

$\rho V = m$ — m is const

$\rho\,dV = -V\,d\rho$

compressibility $= \beta = \frac{1}{k}$

$E_{water} = 2\times10^6\ kN/m^3$

$E_{air} = 101\ kN/m^3$

air is more easily compressible than liquid

$\beta_{air} > \beta_{water}$

$K_{air} < K_{water}$

* Bernaulli's eq^n

$$O(\Delta P) \approx O\left(\tfrac{1}{2}\rho v^2\right)$$

order of change in pressure should be that of $\frac{1}{2}\rho v^2$

ρ is density of fluid

v is velocity of fluid

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* Mach number (M_a)

$$\frac{d\rho}{\rho} = \frac{dP}{E}$$

$$\Rightarrow O\left(\frac{d\rho}{\rho}\right) = O\left(\frac{dP}{E}\right) \qquad dP = \frac{1}{2}\rho V^2$$

$$O\left(\frac{d\rho}{\rho}\right) \approx O\left(\frac{\frac{1}{2}\rho V^2}{E}\right)$$

$$O\left(\frac{d\rho}{\rho}\right) \approx O\left(\frac{V^2}{2\,E/\rho}\right)$$

$$O\left(\frac{d\rho}{\rho}\right) \approx O\left(\frac{V^2}{2a^2}\right)$$

we know, $\sqrt{E/\rho} = a$ = accoustic velocity
i.e. velocity of sound

$$O\left(\frac{d\rho}{\rho}\right) \approx O\left(\frac{M_a^2}{2}\right) \qquad \frac{V}{a} = \text{Mach no.} = M_a$$

for $M_a < 0.33 \Rightarrow \frac{d\rho}{\rho}$ is very less $\Rightarrow d\rho$ is very less
i.e density isn't changing much i.e fluid is incompressible

$$M_a = \frac{V}{a} \qquad a = 335$$

that means for $V \leq 110$ m/s, fluid is incompressible

$0.33 < M_a < 0.8 \Rightarrow$ Subsonic flow
$0.8 < M_a < 1.2 \Rightarrow$ Transonic flow
$M_a = 1 \Rightarrow$ Sonic velocity $\quad V = a$
$1.2 < M_a < 3 \Rightarrow$ Supersonic flow
$3 < M_a < 10 \Rightarrow$ hypersonic flow
$10 < M_a < 25 \Rightarrow$ high hypersonic flow
$25 < M_e \Rightarrow$ Re-entry speed

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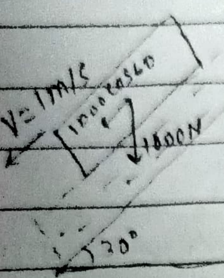
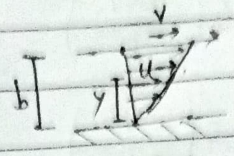
A body weighing 1000N slides down at a uniform speed of 1 m/s along an lubricated inclined plane making 30° angle with horizontal. Viscosity of lubricant is 0.1 kg.s/m². Contact area is 0.25 m². Find thickness of lubricant assuming uniform velocity distribution.





$$\frac{V}{b} = \frac{u}{y}$$

$\therefore f = 1000\cos 60 = 500 \qquad \therefore u = \frac{Vy}{b}$

$\tau = \mu \frac{du}{dy}$ i.e. $\frac{F}{A} = \mu \frac{du}{dy} \qquad \frac{du}{dy} = V/b$

$$\frac{500}{0.25} = \frac{0.1 \times 1}{b}$$

$$b = \frac{25 \times 10^{-3}}{500} = \frac{10^{-3}}{20} = 0.5 \times 10^{-4}$$

$$= 5 \times 10^{-5} \text{ m}$$

$$= 0.05 \text{ mm}$$

Oil viscosity is given 0.1 N.s/m² used for lubricating the clearance of 1mm betn a shaft of diameter 15cm & its general bearing. If shaft rotates with 200 rpm. find τ

$\mu = 0.1 \text{ N.s/m}^2 \qquad D = 0.15\text{m} \qquad N = 200 \text{ rpm}$

$u = \frac{\pi D N}{60} \qquad du = u - 0$

$dy = 1 \times 10^{-3}$

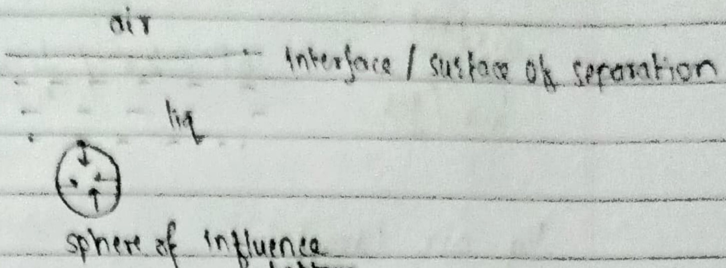
$$\tau = \mu \frac{du}{dy}$$

$u = \frac{\pi \times 0.15 \times 200}{60} \qquad \therefore du = u$

$dy = 10^{-3}$

$$\tau = \frac{0.1 \times \pi \times 0.15 \times 200}{60 \times 10^{-3}} = 157 \text{ N/m}^2$$

Surface Tension (σ)

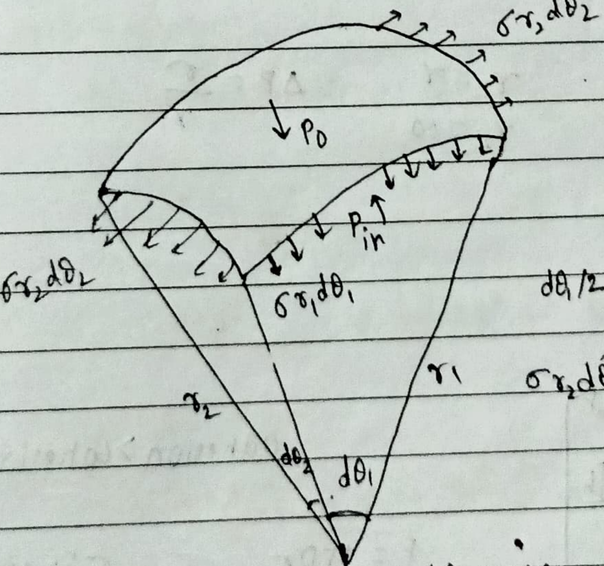


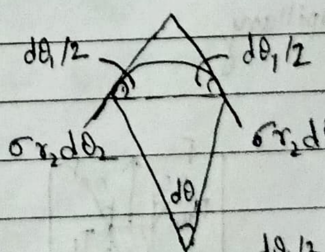
When molecule is brought from ~~top~~ bottom to the surface, the energy req is called surface energy.

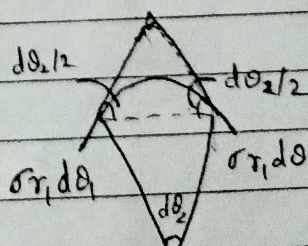
$\sigma_{water,vap} = 0.073\ N/m$

$\sigma = \frac{F}{L}$

It is because of surface tension, a curved liq surface seperating a liq from gas, creates high pressure on concave side rather than convex side.







$\sin\frac{d\theta_1}{2} = \frac{d\theta_1}{2}$ $\qquad \sin\frac{d\theta_2}{2} = \frac{d\theta_2}{2}$

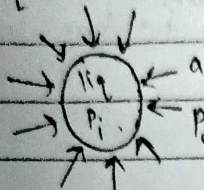
$$2\sigma r_1 \frac{d\theta_2}{2} d\theta_1 + 2\sigma r_2 d\theta_2 \frac{d\theta_1}{2} = \underset{\Delta P}{(P_i - P_o)} \underbrace{r_1 d\theta_1 r_2 d\theta_2}_{\text{area of upper surface}}$$

$$\therefore \Delta P = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Thin film

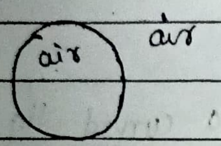
$\frac{air/gas}{air/gas}$ $\Delta P = 2\sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

liq-air (drop)



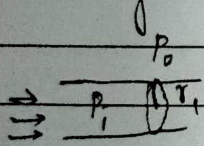
$r_1 = r_2 = r$ $\therefore \Delta P = \frac{2\sigma}{r}$

liq bubble (soap)



$\Delta P = \frac{4\sigma}{r}$

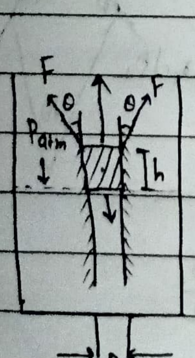
Water-jet



$r_1 = r$
$r_2 = \infty$

$\Delta P = \frac{\sigma}{r}$

* Capillary

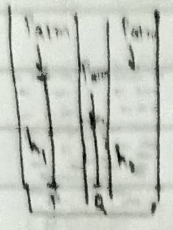


adhesion > cohesion

$F = \pi D \sigma$ σ is surface tension
$F\cos\theta = mg$ * r is radius $r = D/2$

$\pi D \sigma \cos\theta = \frac{\pi D^2 h \rho}{4} g$

$\therefore h = \frac{4\sigma\cos\theta}{\rho g D} = \frac{2\sigma\cos\theta}{\rho g r}$



$h_1 > h_2$

at point P, pressure $= P_{atm} + \rho g h_1$

at point Q, pressure $= P_{atm} + \rho g h_2$

$P_P > P_Q$ as $h_1 > h_2$

1) A spherical soap bubble of diameter d_1 coalesces with another one of diameter d_2 to form a new one that of d_3 carrying same amount of air. Assuming an isothermal process derive an expression for d_3 as a fun^n^ of d_1 & d_2. The ambiant pressure is P_0 & surface tension of soap is σ.

Same amount of air indicates $n_1 + n_2 = n_3$

$P_1V_1 = n_1RT \qquad P_3V_3 = (n_1+n_2)RT$

$P_2V_2 = n_2RT$

for soap bubble, $\Delta P = \frac{4T}{R} = \frac{8\sigma}{D}$

$\therefore \quad P_1 = P_0 + \frac{8\sigma}{d_1} \qquad P_2 = P_0 + \frac{8\sigma}{d_2} \qquad P_3 = P_0 + \frac{8\sigma}{d_3}$

$V_1 = \frac{1}{6}\pi d_1^3 \qquad V_2 = \frac{1}{6}\pi d_2^3 \qquad V_3 = \frac{1}{6}\pi d_3^3$

$P_1V_1 + P_2V_2 = P_3V_3$

$\therefore d_1^3\left(P_0 + \frac{8\sigma}{d_1}\right) + \left(P_0 + \frac{8\sigma}{d_2}\right)d_2^3 = \left(P_0 + \frac{8\sigma}{d_3}\right)d_3^3$

2] If the capillary rise of water is 15 mm in a 2 mm diameter tube. How much will be the rise in capillary tube when diameter is 0.5 mm?

$h = \frac{4\sigma\cos\theta}{\rho g D}$ i.e. $hD = $ const. $\quad h_1d_1 = h_2d_2$

$\therefore 30 = 0.5 \times h_2$

$\therefore h_2 = 60 \text{ mm}$

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3) What is the pressure within a 1mm diameter spherical droplet of water relative to the atmospheric pressure.

$\sigma_{water} = 0.073$ N/m

$$\Delta P = \frac{2\sigma}{r} = \frac{4\sigma}{D} = \frac{4 \times 0.073}{10^{-3}} = 292 \text{ atm}$$

Compressible :

~~Pressure~~ Density changes with time ~~as well as~~ or distance

Incompressible :

Density is idependent of time & distance (i.e. density is constant)

Fluid ~~Statics~~ Kinematics

Steady flow

$\vec{V} \neq V(t)$

$\vec{V} = V(\vec{s})$

$\vec{a} \neq a(t)$

$\vec{a} = a(\vec{s})$

Unsteady flow

$\vec{V} = V(t)$

Uniform flow

$\cancel{\vec{V} = V(t)}$

$\vec{V} \neq V(s)$

Non uniform flow

$\vec{V} = V(s)$

steady non uniform

$\vec{V} = V(\vec{s})$

$V \neq V(t)$

unsteady uniform

$\vec{V} = V(t)$

$V \neq V(s)$

when flow is uniform & steady,

velocity $= u = u(x, y, z, t)$

$$\therefore u + \Delta u = u(x, y, z, t) + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z + \frac{\partial u}{\partial t} + \ldots$$

higher order terms

$$\lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} = \lim_{\Delta t \to 0}\left[\frac{\partial u}{\partial x} \cdot \frac{\Delta x}{\Delta t} + \frac{\partial u}{\partial y}\frac{\Delta y}{\Delta t} + \frac{\partial u}{\partial z}\frac{\Delta z}{\Delta t}\right] + \frac{\partial u}{\partial t}$$

$$a_x = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{Dw}{Dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

if last term of each a_x, a_y, a_z is 0, flow will be ~~uniform~~ steady

if 1st 3 terms of each a_x, a_y, a_z is 0, flow will be ~~steady~~

if $a_x = a_y = a_z = 0$, flow is steady & uniform

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local acceln : $\frac{\partial(\text{velocity})}{\partial t}$

convective acceln : $v \frac{\partial(\text{velocity})}{\partial x}$ or $v \frac{\sim}{\partial y}$ or $v \frac{\sim}{\partial z}$

	convective	local
steady non uniform	Exist	0
steady uniform	0	0
unsteady uniform	0	Exist
~~unsteady non uniform~~	~~Exist~~	~~Exist~~

1) $u = 2x^2 + 3y$ $\quad V = -2xy + 3y^2 + 3yz$

$W = -\frac{3}{2} z^2 + 2xz - 9y^2 z$

flow is steady non uniform find acceln at (1,1,1)

$\Downarrow$

$\frac{\partial \text{ velocity}}{\partial t} = 0$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

$= 32$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 7.5$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 23$

$\therefore a_{(1,1,1)} = \sqrt{32^2 + 7.5^2 + 23^2}$

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2] $\vec{V} = (4 + xy + 2t)\hat{i} + 6x^3\hat{j} + (3xt^2 + z)\hat{k}$ find $a_{(2,4,-4)}$ when $t = 3$

$u = 4 + xy + 2t$ $\quad V = 6x^3$ $\quad W = 3xt^2 + z$

$\frac{\partial u}{\partial x} = y \quad \frac{\partial u}{\partial y} = x \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = 2$

$\frac{\partial v}{\partial x} = 18x^2 \quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} = 0$

$\frac{\partial w}{\partial x} = 3t^2 \quad \frac{\partial w}{\partial y} = 0 \quad \frac{\partial w}{\partial z} = 1 \quad \frac{\partial w}{\partial t} = 6xt$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$= 18(4) + 2(48) + 0 + 2 = 170$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

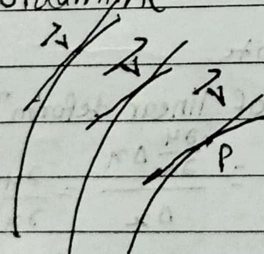
$= 1296$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

$= 488$

$\therefore a = \sqrt{(170)^2 + (1296)^2 + (488)^2}$

* Streamline



The tangent drawn to streamline are velocity vectors

Streamlines never cross each other.

for very small element $\vec{ds}$ at P, making θ angle with V,

$\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ $\quad$ let $V = u\hat{i} + v\hat{j} + w\hat{k}$

$V \times ds = 0 \quad \therefore \begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$

$u\,dy - v\,dx = 0$

$v\,dz - w\,dy = 0$

$u\,dz - w\,dx = 0$

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i.e. $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$ Streamline eqⁿ

* Pathline

It is trajectory of a fluid particle of a given identity

$\vec{V} = V(s,t)$ $\qquad \frac{ds}{dt} = V(s,t)$

$\int ds = \int V(s,t)\, dt$

$s = s(s,t)$

* Streak line:

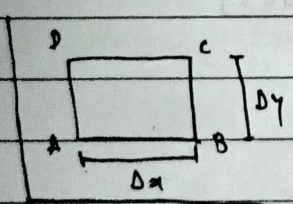
Streakline at any instant of time is the locus of temporary locations of all particles that have passed through a fixed point in the flow

* Types of motion of a fluid

1] Fluid - linear deformⁿ

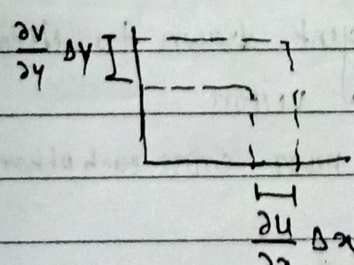
$u = u(x) \qquad \partial u / \partial y = 0$

$v = v(y) \qquad \partial v / \partial x = 0$



linear strain rate

/ Rate of linear deformⁿ :



$$\dot{\epsilon}_x = \frac{\frac{\partial u}{\partial x}\Delta x}{\Delta x} = \frac{\partial u}{\partial x}$$

$$\dot{\epsilon}_y = \frac{\frac{\partial v}{\partial y}\Delta y}{\Delta y} = \frac{\partial v}{\partial y}$$

$$\dot{\epsilon}_z = \frac{\partial w}{\partial z}$$

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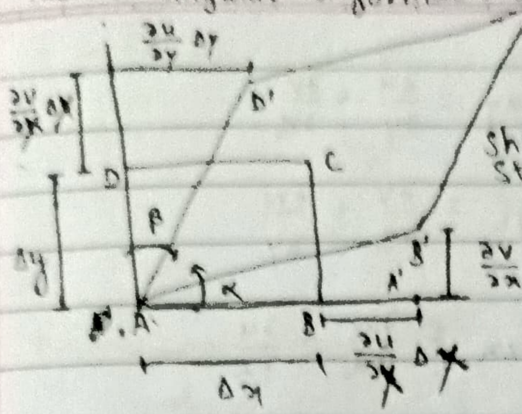
8) * Fluid - angular deformation



$u = u(x,y)$
$v = v(x,y)$

Shear Strain rate / Rate of angular deform^n

$\dot{\gamma}_{xy} = \alpha + \beta$

In $\triangle AA'B$

$$\tan\alpha = \frac{A'B}{AA'} = \frac{\frac{\partial v}{\partial x}\Delta x}{\Delta x\left(1+\frac{\partial u}{\partial x}\right)}$$

$$\therefore \alpha = \frac{\partial v}{\partial x}\left(1+\frac{\partial u}{\partial x}\right)^{-1}$$

$$\tan\beta = \frac{\frac{\partial u}{\partial y}\Delta y}{\Delta y\left(1+\frac{\partial v}{\partial y}\right)}$$

$$\beta = \frac{\partial u}{\partial y}\left(1+\frac{\partial v}{\partial y}\right)^{-1}$$

$$\dot{\gamma}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

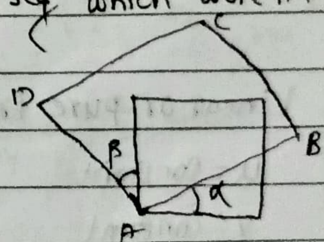
Angular rot^n (ω_z) (about Z axis)

Avg angular rot^n of 2 linear seg which were initially $\perp$

$$\omega_z = \frac{1}{2}(\alpha + (-\beta))$$

$$= \frac{1}{2}(\alpha - \beta)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$



case 1: $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow |\alpha| = |\beta|$

$$\dot{\gamma}_{xy} = 0$$

$$\omega_z = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

case 2: $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \Rightarrow \cancel{\omega_z = \frac{2\partial u}{\partial y} - \frac{2\partial v}{\partial x}}$

$$\omega_z = 0$$

$u = u(x, y, z)$
$v = v(x, y, z)$
$w = w(x, y, z)$

$$\dot{\gamma}_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\dot{\gamma}_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\dot{\gamma}_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

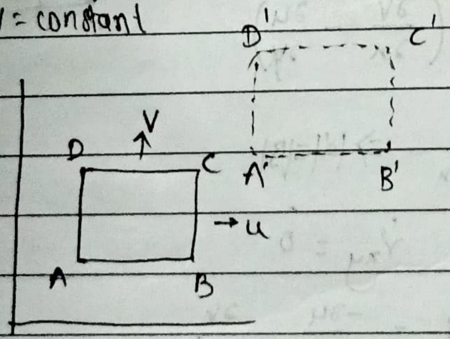
$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\vec{W} = \frac{1}{2}\ \vec{\nabla} \times \vec{V} \qquad \vec{V} = \partial u\,\hat{i} + \partial v\,\hat{j} + \partial w\,\hat{k}$$

5] * Linear or pure translation

$u = \text{Constant}$
$v = \text{constant}$



* Bernauli's eq^n

2-D steady, incompressible flow

$$\frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \quad \text{....along streamline} \quad -(1)$$

Irrotational flow

condition is $\nabla \times V = 0$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad \text{i.e.} \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{dv}{dy} = -\frac{1}{\rho}\frac{dP}{dy} - g \quad -(2)$$

$(1) \times dx + (2) \times dy$

$$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial v}{\partial x}dy\right) + v\left(\frac{\partial u}{\partial y}dx + \frac{\partial v}{\partial y}dy\right)$$

$$= -\frac{1}{\rho}\left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy\right) - g\,dy$$

$$\therefore u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right) + v\left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy\right) = \text{---}$$

$$\therefore u\,du + v\,dv = -\frac{1}{\rho}dP - g\,dy$$

$$\therefore \int u\,du + \int v\,dv = -\frac{1}{\rho}\int dP - g\int dy$$

$$\therefore \frac{u^2}{2} + \frac{v^2}{2} = \frac{-P}{\rho} - gy + c$$

$$\therefore \frac{P}{\rho} + \frac{V^2}{2} + gy = c \qquad V = u\hat{i} + v\hat{j}$$

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A 2D flow field is defined as $\vec{V} = x\hat{i} - y\hat{j}$. find streamline eq^n which passes through point (1,1)

$u = x \qquad v = -y$

$\frac{dx}{u} = \frac{dy}{v} \qquad -y\,dx - x\,dy = 0 \qquad \therefore d(xy) = 0$

$\therefore -xy - xy + c = 0$

passing through (1,1) $\qquad \therefore xy = c$

$\Rightarrow 1 \times 1 = c$

$\therefore c = 1$

$\therefore$ eq^n is $xy = 1$

$T = xy + z + 3t$

$\vec{V} = xy\,\hat{i} + z\,\hat{j} + 5t\,\hat{k}$ $\quad$ T is temp. find rate of change in T at P(2,-2,1) after 2 sec

$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{u\,\partial T}{\partial x} + \frac{v\,\partial T}{\partial y} + \frac{w\,\partial T}{\partial z}$

$u = xy \qquad v = z \qquad w = 5t$

$= 3 + xy \cdot y + z \cdot x + 5t \cdot 1$

$= 3 + 2(-2)(-2) + 1(2) + 5(2)(1)$

$= 3 + 8 + 2 + 10 = 23$ K/s

$u = e^x \cosh(y) \qquad v = -e^x \cosh(x)$

$v\,dx - u\,dy = 0$

$-e^x \cosh(x)\,dx - e^x \cosh(y)\,dy = 0$

$\therefore \int \cosh(x)\,dx + \int \cosh(y)\,dy = 0$

$\therefore \sinh x + \sinh y = c$

$u = cx + 2W_o y + u_o$

$v = cy + v_o$

$w = -2cz + w_o$ $\qquad$ c, u_o, v_o, w_o are const.

find linear strain rate, angular deform^n & angular rot^n

$$\underbrace{\dot{\varepsilon}_x = \frac{\partial u}{\partial x} = c \qquad \dot{\varepsilon}_y = \frac{\partial v}{\partial y} = c \qquad \dot{\varepsilon}_z = \frac{\partial w}{\partial z} = -2c}_{\text{linear strain rates}}$$

$$\left.\begin{aligned} \dot{\gamma}_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2W_o \\ \dot{\gamma}_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \\ \dot{\gamma}_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \end{aligned}\right\} \text{angular deform}^n$$

$$\left.\begin{aligned} W_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}(0) = 0 \\ W_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0 \\ W_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -W_o \end{aligned}\right\} \text{angular rot}^n$$

Potential flow

Incompressible

Irrotational

$\psi = \psi(x, y)$

$u = \dfrac{\partial \psi}{\partial y}$ $\qquad$ $v = \dfrac{-\partial \psi}{\partial x}$

$\dfrac{dy}{dx} = \dfrac{v}{u}$ $\qquad$ $u\,dy - v\,dx = 0$ ..... streamline eq^n

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = -v\,dx + u\,dy$$

irrotation con^n : $w_z = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0$

& $\vec{\nabla} \times \vec{V} = 0$

$\vec{V} = -\nabla \cdot \phi$ velocity potential gradient

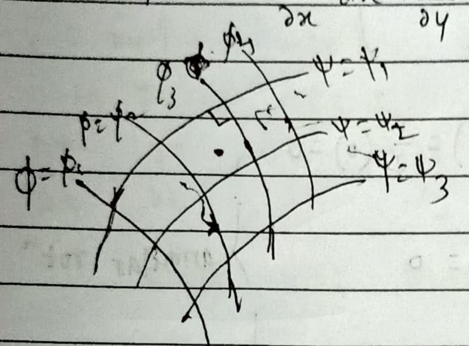
$\Downarrow$

$V = u\hat{i} + v\hat{j} + w\hat{k}$

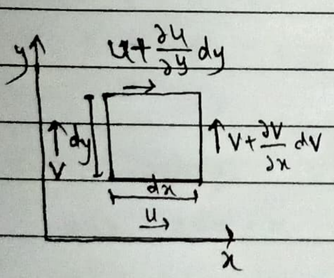
$\therefore u = \frac{\partial \phi}{\partial x}$ $\quad v = \frac{\partial \phi}{\partial y}$ $\quad w = \frac{\partial \phi}{\partial z}$

$\phi = C$

$d\phi = 0 = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$



Circulation: $\Gamma = \oint \vec{V} \cdot \vec{n}\, ds$



$\Gamma = (u\,dx) + \left(v + \frac{\partial v}{\partial x} dx\right) dy - \left(u + \frac{\partial u}{\partial y} dy\right) dx - v\,dy$

$\Gamma = \frac{\partial v}{\partial x} dx dy - \frac{\partial u}{\partial y} dx dy$

$\Gamma = dx dy \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$= 2 w_z\, dx dy$ $\qquad dx dy = \text{area}$

let $\Omega_z = 2 w_z$ $\qquad \Omega$ is called as vorticity

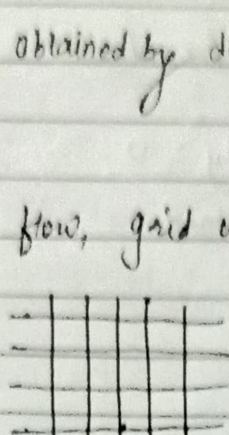
$\therefore \frac{\Gamma}{\text{area}} = \Omega_z$

$$\iint_A (\nabla \times \vec{F}) \cdot n_2 \, dA = \oint_s \vec{F} \cdot \hat{n}_1 \, ds \quad \therefore \vec{F} = \vec{V}$$

Flow net

A grid obtained by drawing a series of equipotential line & streamline.

for very high flow, grid collapses to become a perfect square



$\psi = c$

$\phi = c$

Flow is irrotational

if flow is not incompressible, ψ funⁿ doesn't exist
" " " irrotational, ϕ " "

if ψ exists, It'll satisfy continuity eqⁿ
ϕ exist, irrotational conⁿ satisfy

if ψ exist, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad \therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ i.e. $\nabla^2 \psi = 0$

if ϕ exist, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \therefore \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ i.e. $\nabla^2 \phi = 0$

1] $\phi = -\frac{xy^3}{3} - x^2 + y^2 + \frac{x^3 y}{3}$ check whether flow is possible & find u & v

$u = \frac{\partial \phi}{\partial x} = -\frac{y^3}{3} - 2x + x^2 y$

$v = \frac{\partial \phi}{\partial y} = -xy^2 + 2y + \frac{x^3}{3}$

$\frac{\partial^2 \phi}{\partial x^2} = -2 \quad \frac{\partial^2 \phi}{\partial y^2} = 2 \quad \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \nabla^2 \phi = 0 \quad \therefore$

for a solid plane

value of ψ will be---?

ψ = constant

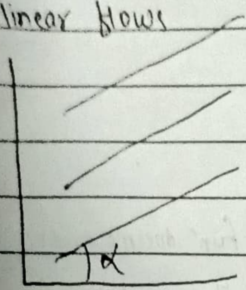
Boundary con^n for fluid is velocity = 0 at boundary

$d\phi = u\,dx + v\,dy = 0 \quad \cdots \because u = v = 0$

$\therefore \phi$ = constant

* Basic flows

1) Rectilinear flows



$V = u\hat{i} + v\hat{j}$

$u = V\cos\alpha$

$v = V\sin\alpha$

ψ is stream fun^n & ϕ is velocity potential fun^n

$u = \dfrac{\partial \psi}{\partial y} \qquad v = \dfrac{-\partial \psi}{\partial x}$

$\therefore \psi = uy + f(x) + C_1$

$\psi = -vx + f(y) + C_2$

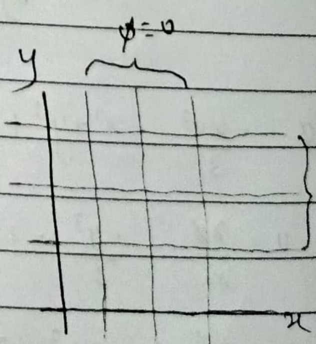
$\therefore \psi = uy - vx + C$

$\dfrac{\partial \phi}{\partial x} = -u \qquad \dfrac{\partial \phi}{\partial y} = -v$

$\therefore \phi = -ux - vy + C$

If $\psi = 0 \Rightarrow y$ = const

If $\phi = 0 \Rightarrow x$ = const



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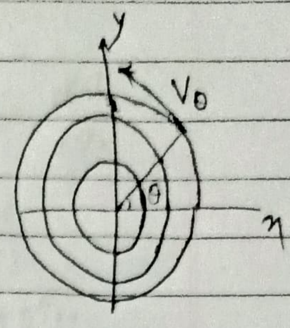
2] Plane circular vortex flow / 2D vortex flow

polar co-ordinate system

$V_r = 0$, $V_\theta \neq 0$



Continuity eq in r & θ is given by

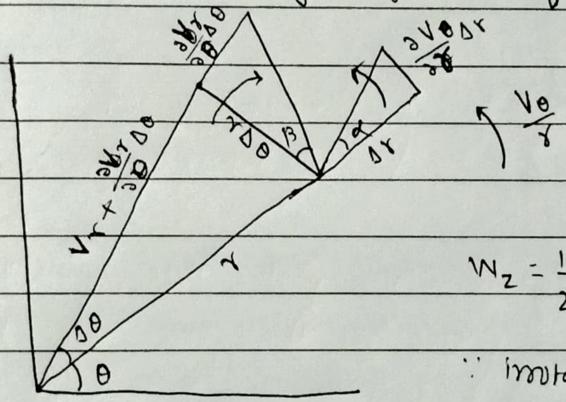
$$\frac{1}{r}\frac{\partial (r V_r)}{\partial r}^{0} + \frac{1}{r}\frac{\partial (V_\theta)}{\partial \theta} = 0$$

$$\therefore V_\theta = V_\theta(r) + c$$

V_θ is a funⁿ of r

3] Free vortex flow

- It is an irrotational flow
- Constant mechanical energy throughout the field



$$W_z = \frac{1}{2}\left(\cancel{\frac{-\partial V_r}{r\,\partial \theta}}^{0} + \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right)$$

$\because$ irrotational $\therefore W_z = 0$

$$\therefore \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(V_\theta r) = 0$$

$$\therefore V_\theta r = C$$

$$\boxed{V_\theta r = \text{const}}$$

by Bernoulli's eqⁿ,

$$h = \frac{P}{\rho} + \frac{V_\theta^2}{2} + gz$$

$$\frac{dh}{dr} = \frac{\partial P}{\rho\,\partial r} + V_\theta \frac{\partial V_\theta}{\partial r} + g\frac{\partial z}{\partial r} = 0$$

$$= \frac{\partial P}{\rho\,\partial r} + V_\theta \frac{\partial V_\theta}{\partial r} + g\sin\theta = 0 \quad \text{--- (1)}$$

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r momentum eq^n

$$\frac{DV_r}{Dt}^{\to 0} - \frac{V_\theta^2}{r} = -\frac{\partial P}{\rho \partial r} - g\sin\theta$$

$$\frac{\partial P}{\rho \partial r} + g\sin\theta = \frac{V_\theta^2}{r} \quad -②$$

eq^n ① & ② gives $\quad V_\theta \frac{\partial V_\theta}{\partial r} + \frac{V_\theta^2}{r} = 0$

from r momentum eq^n.

$$\frac{\partial P}{\rho \partial r} = \frac{V_\theta^2}{r} - g\sin\theta$$

θ dir momentum eq^n

$$\frac{DV_\theta}{Dt} - \frac{V_\theta V_r}{r} = -\frac{\partial P}{\rho \partial \theta} - g\sin\theta$$