Bubble :-

$$\Delta P = \frac{4\sigma}{r}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Water Jet

P_0

P_1

$r_1 = r$

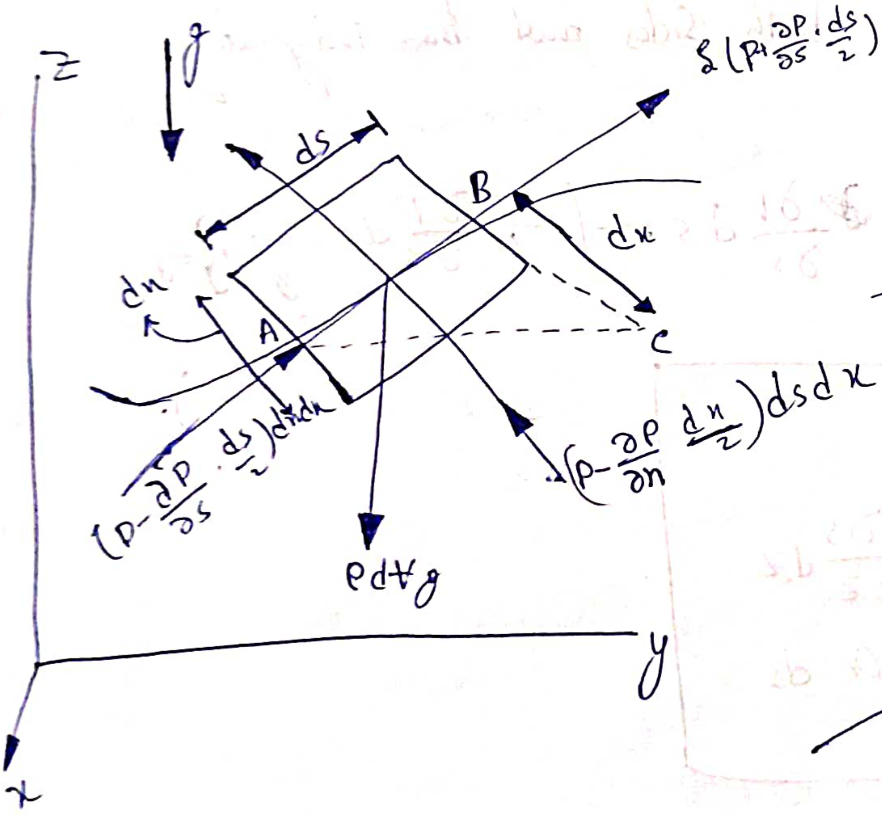
$r_2 = \infty$

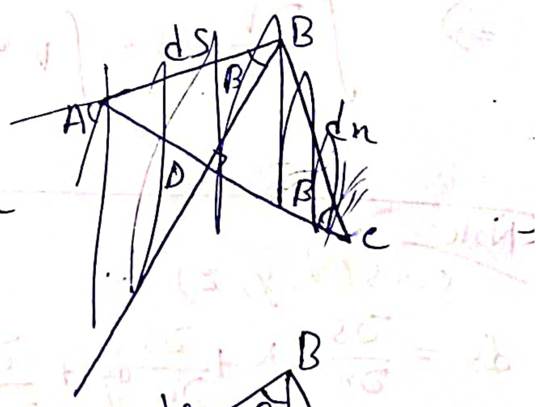
$$\Rightarrow \Delta p = \frac{\sigma}{r}$$

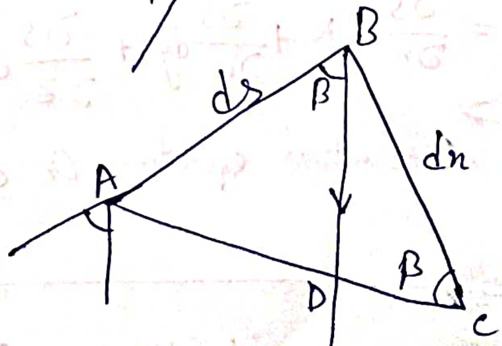
25/08/2022 **Euler's equation along streamline and normal to streamline**

- Class Test from 9 am to 11 am on 06/09/2022

In the Graphics department







$$\cos\beta = \frac{dz}{ds}, \quad \sin\beta = \frac{dz}{dn}$$

Along the streamline $\Rightarrow \left[\left(P - \frac{\partial P}{\partial s} \cdot \frac{ds}{2}\right) - \left(P + \frac{\partial P}{\partial s} \cdot \frac{ds}{2}\right)\right] dn\,dx$

$$- \rho(d\forall g \cos\beta)$$

$$= \rho d\forall \frac{DV_s}{Dt}$$

$$\Rightarrow -\frac{\partial P}{\partial s} d\forall - \rho g d\forall \frac{dz}{ds} = \rho d\forall \frac{DV}{\cancel{D}Dt}$$

As the normal component is zero so we can write $V = V_s$

V → velocity, V_s → velocity along streamline

$$\Rightarrow \boxed{\frac{DV}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial s} - g\frac{dz}{\cancel{ds}\,ds}}$$

→ Euler's equation ~~along~~ along streamline

Now

$$\frac{\partial V}{\partial t} + V\frac{dV}{ds} - \frac{1}{S}\frac{dP}{ds} - g\frac{dz}{ds}$$

Multiplying ~~b~~ ds on on both sides and then integrating

$$\Rightarrow \int \frac{\partial V}{\partial t} ds + \int V \cancel{\partial}\frac{\partial V}{\partial s} ds = -\int \frac{1}{\rho}\frac{\partial P}{\partial s} ds - \cancel{g}\int g\frac{dz}{ds} ds$$

Note
$s = s(x, y, z)$

$$ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy + \frac{\partial s}{\partial z}dz$$

Exact differential equation of ds

$$\Rightarrow \Rightarrow d\left(\frac{V^2}{2}\right) = \frac{\partial (V^2/2)}{\partial s} ds$$

$$d(V^2/2)$$

$$= \int \frac{\partial V}{\partial t} ds + \int d\left(\frac{V^2}{2}\right) = -\int \frac{dP}{\rho} - \int d(gz)$$

$$\Rightarrow \int \frac{dV}{dt} ds + \frac{V^2}{2} + \int \frac{\partial P}{\rho} + gz = \text{Constant}$$

For incompressible, ρ = constant, so bernoulli's equation

$$\Rightarrow \underbrace{\int \frac{\partial V}{\partial t} ds}_{=0 \text{ for Steady State}} + \frac{V^2}{2} + \frac{P}{\rho} + gz = \text{Constant}$$

Assumptions for Bernoulli's equation

1) Applicable along a particular streamline (As we have derived it from Euler's equation for a particular streamline)
2) Steady ~~state~~ flow
3) Incompressible flow
4) Inviscous (Shear force not considered)

• Pressure force Shear force } Two types of surface forces

Boundary layer theory → Flow away from Solid Surface can be inviscous but flow near the solid surface is ~~viscous~~ viscous.

For real fluid the 4th assumption is removed

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

h_f → ~~head~~ head loss ↓ All loses

minor loss, major loss

Darcy Weisbach → $h_f = \dfrac{fLV^2}{2gd}$; f = coefficient of friction

d → diameter of duct (pipe)

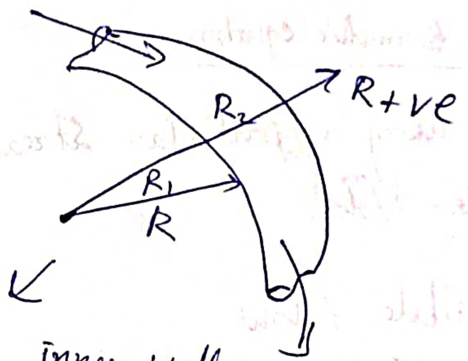
1 Along normal to Streamline

$$-\frac{\partial P}{\partial n}\, d\forall - \rho g\, d\forall \sin\beta = \rho\, a_n\, d\forall$$

$$= \rho\left(\frac{-V^2}{R}\right) d\forall$$

$$\Rightarrow -\frac{\partial P}{\partial n} - \rho g \frac{dz}{dn} = \rho\left(\frac{-V^2}{R}\right)$$

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{\partial P}{\partial n} + g\frac{\partial z}{\partial n} = \frac{V^2}{R}}$$



In the inner wall pressure is less,
In the outer ~~to~~ wall pressure is more
because as R ~~increases~~ ↑ increases pressure ↑ ~~inc~~ increases