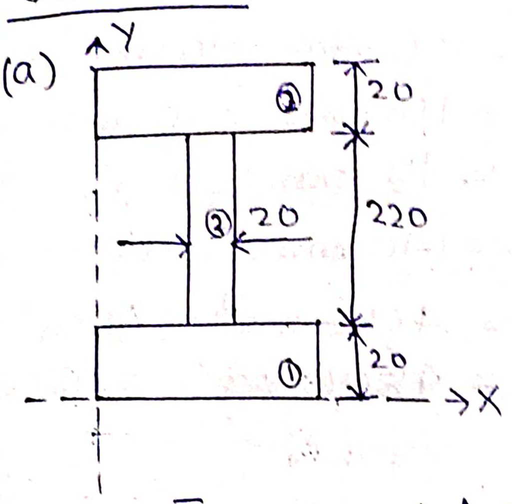
Question 3:

(a)



$x_1 = 100 mm = 10 cm \qquad y_1 = 25 cm$

$x_2 = 100 mm = 10 cm \qquad y_2 = 1 cm$

$x_3 = 100 mm = 10 cm \qquad y_3 = 13 cm$

$A_1 = 40 cm^2 ; \quad A_2 = 40 cm^2$

$A_3 = 44 cm^2 ;$

$$\therefore \bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = \frac{40 \times 10 + 40 \times 10 + 44 \times 10}{40 + 40 + 44} = 10 cm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{40 \times 25 + 40 \times 1 + 44 \times 13}{40 + 40 + 44} = 13 cm$$

$$I_x = \frac{1}{12}(bh^3 - b_1 h_1^3) = \frac{1}{12}(20 \times 26^3 - 18 \times 22^3) = 13321 cm^4$$

$$I_y = \frac{1}{12}(2b_3 h_3^3 + b_4 h_4^3) = 2 \times \frac{1}{12} \times 2 \times 20^3 + \frac{1}{12} \times 22 \times 2^3 = 2681 cm^4$$

$P_{xy} = 0$

Mohr's circle:

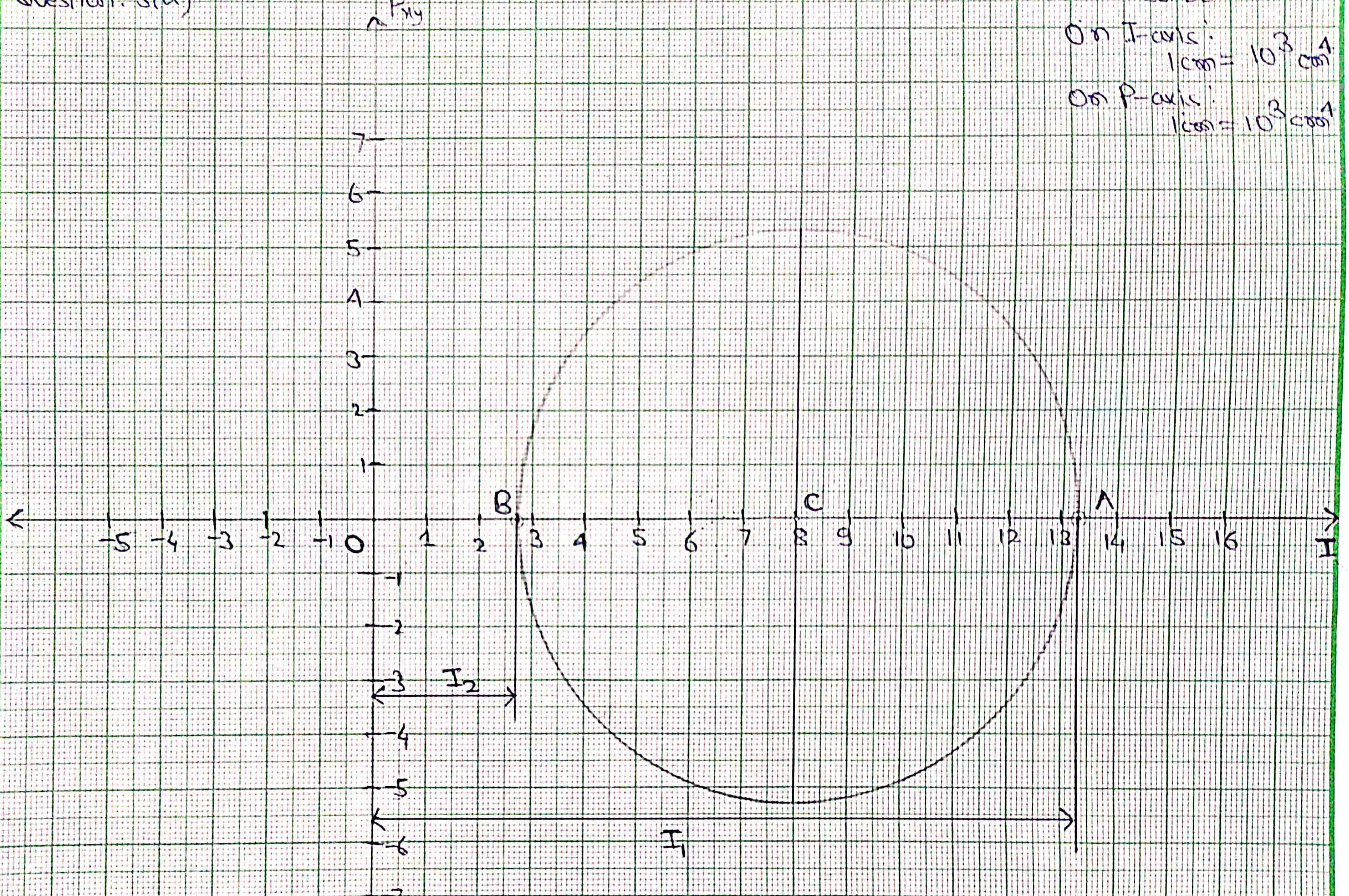
Result Table:

	$cm^4 (\times 10^3)$	Direction from x-axis (Anticlockwise)
OC	8	–
I_1	13.3	0°
I_2	2.7	90°
P_1	5.3	45°
P_2	–5.3	135°
R	5.3	–

Name: Adarsh Arya
Roll no.: 20ME8022
Reg. no.: 20U10095
Q: 3(a)
Session: 2022–23

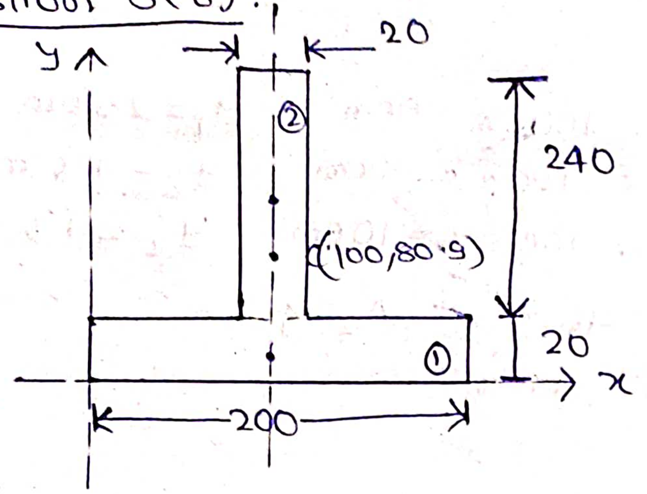
Question: 3(a)

SCALE
On I-axis:
1cm = 10^3 cm^4
On P-axis:
1cm = 10^3 cm^4



Name: Adarsh Arya
Roll no.: 20ME8022
Reg. No.: 20U10095
Q: 3(a)
Session: 2022-23

Question 3(b):



$x_1 = 100$ mm $= 10$ cm
$x_2 = 100$ mm $= 10$ cm
$y_1 = 10$ mm $= 1$ cm
$y_2 = 140$ mm $= 14$ cm
$A_1 = 4000$ mm^2 $= 40$ cm^2
$A_2 = 4800$ mm^2 $= 48$ cm^2

$$\bar{X} = \frac{A_1x_1 + A_2x_2}{A_1+A_2} = \frac{4000\cancel{00} + 4800\cancel{00}}{88\cancel{00}} = 100\text{ mm} = 10\text{ cm}$$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2} = \frac{4000\cancel{0} + 48\cancel{00}\times14\cancel{0}}{88\cancel{00}} = 80.9\text{ mm} = 8.09\text{ cm}$$

$\therefore$ Centroid $(\bar{X}, \bar{Y}) = (100, 80.9)$ mm

Moment of Inertia:

$$I_x = I_{x_1} + I_{x_2} = I_{g_1x} + A_1(y_1-\bar{Y})^2 + I_{g_2x} + A_2(y_2-\bar{Y})^2$$

$$= \frac{200\times20^3}{12} + 4000(10-80.9)^2 + \frac{20\times240^3}{12} + 4800(140-80.9)^2$$

$$= 6\times10^7\text{ mm}^4$$

$$I_y = I_{y_1} + I_{y_2} = I_{g_1y} + A_1(x_1-\bar{X})^2 + I_{g_2y} + A_2(x_2-\bar{X})^2$$

$$= \frac{20\times200^3}{12} + \cancel{4000(100-100)^2}^{\,0} + \frac{240\times20^3}{12} \cdot \cancel{(100-100)^2}^{\,0}$$

$$= 1.35\times10^7 \simeq 1.4\times10^7\text{ mm}^4$$

Product of inertia $(P_{xy}) = 0$ (as the centroids of both rectangles lie on the centroidal axis).

Name: Adarsh Arya
Roll no.: 20ME8022
Reg. no.: 20U10095
Q. 3(b)
Session: 2022-23

3(b) Mohr's Circle:

Result Table:

	mm^4 ($\times 0.5\times10^7$)	Direction with x-axis (Anticlockwise)
OC	3.7	–
R	2.3	–
I_1	6	0°
I_2	1.4	90°
P_1	2.3	45°
P_2	−2.3	135°

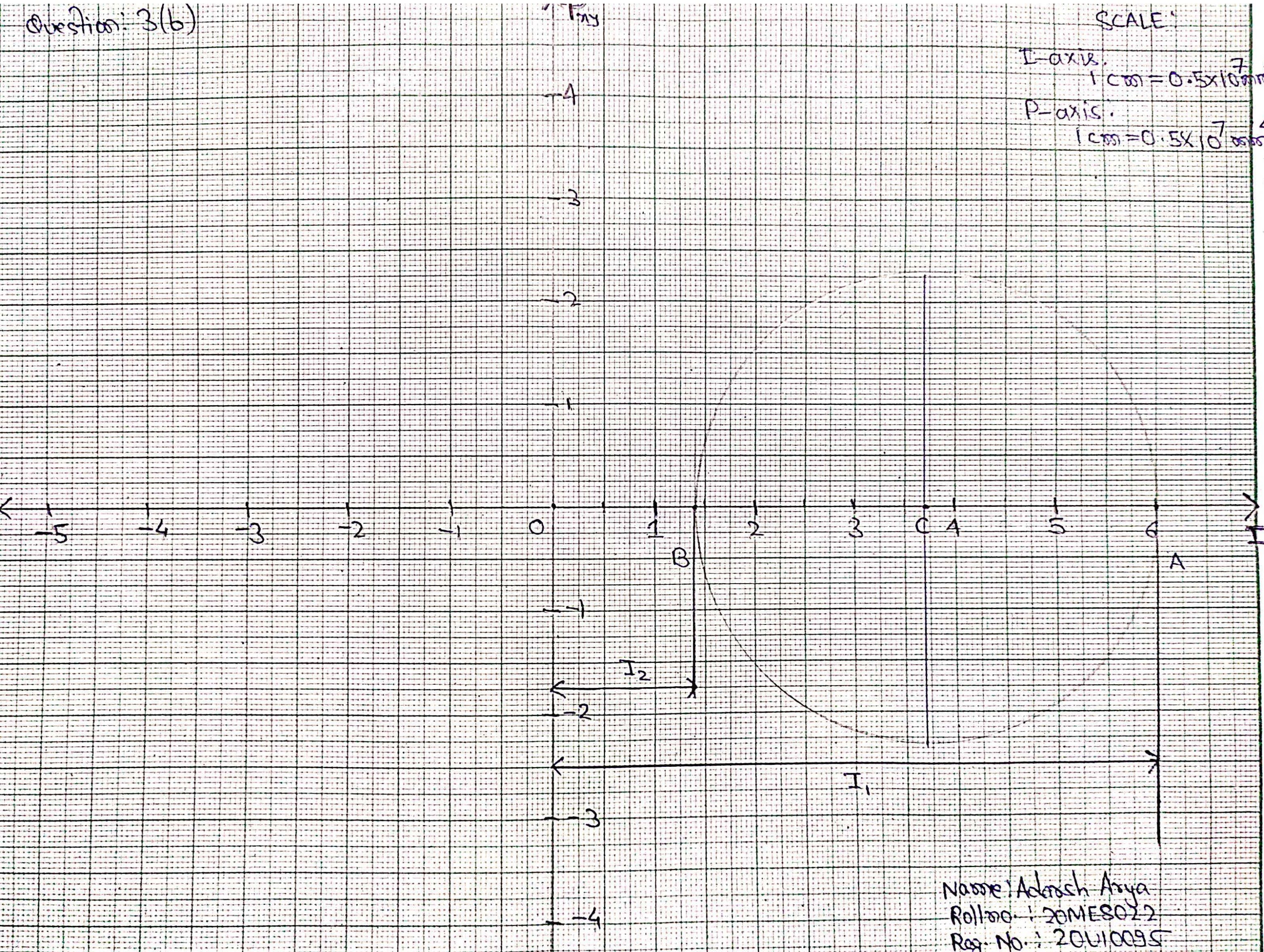
Name: Adarsh Arya
Roll: 20 MES022
Reg. No.: 20010095
Q: 3(b)
Session: 2022-23

Question: 3(b)

SCALE:

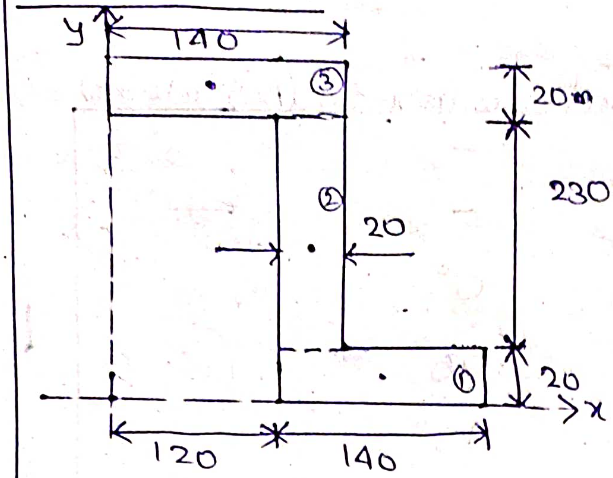
I-axis:
1 cm = 0.5×10^7 mm^4

P-axis:
1 cm = 0.5×10^7 mm^4



Name: Adarsh Arya
Roll no.: 20MES022
Reg. No.: 20U10095
Q: 3(b)
Session: 2022-23

Question : 3(c)



$x_1 = 190\ mm = 19\ cm$
$x_2 = 130\ mm = 13\ cm$
$x_3 = 70\ mm = 7\ cm$
$y_1 = 10\ mm = 1\ cm$
$y_2 = 135\ mm = 13.5\ cm$
$y_3 = 260\ mm = 26\ cm$
$A_1 = 2800\ mm^2 ; = 28\ cm^2$
$A_2 = 4600\ mm^2 ; = 46\ cm^2$
$A_3 = 2800\ mm^2 = 28\ cm^2$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{2800\times190 + 4600\times130 + 2800\times70}{10200}$$
$$= \cancel{4.384\times10^7\ mm^4 = 4.384\times10^3\ cm^4}$$
$$= 130\ mm = 13\ cm$$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{2800\times10 + 4600\times135 + 2800\times260}{10200}$$
$$= 135\ mm = 13.5\ cm$$

∴ centroid (13, 13.5) cm

Moment of Inertia:

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = \frac{140\times20^3}{12} + 2800(10-135)^2$$
$$+ \frac{20\times230^3}{12} + 4600\cancel{(135-135)}^2$$
$$+ \frac{140\times20^3}{12} + 2800(260-135)^2$$

$$I_x = 4.384\times10^7\ mm^4$$
$$= 4.384\times10^3\ cm^4$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} = \frac{20\times140^3}{12} * 2800(190-130)^2$$
$$+ \frac{230\times20^3}{12} + 4600(130-130)^2$$
$$+ \frac{20\times140^3}{12} + 2800(70-130)^2$$
$$= 2.945\times10^7\ mm^4 = 2.945\times10^3\ cm^4$$

Name : Adarsh Arya
Roll No. : 20ME8022
Reg. No. : 20U10095
Q : 3(c)

Q: 3(C) **Product of Inertia:**

$$P_{xy} = (P_{xy})_1 + \cancel{(P_{xy})_2}^{\,0} + (P_{xy})_3$$
$$= A_1 a_1 b_1 + A_3 a_3 b_3 = 2800(60)(-125) + 2800(-60)(125)$$
$$= -4.2 \times 10^7 \text{ mm}^4$$
$$= -4.2 \times 10^3 \text{ cm}^4$$

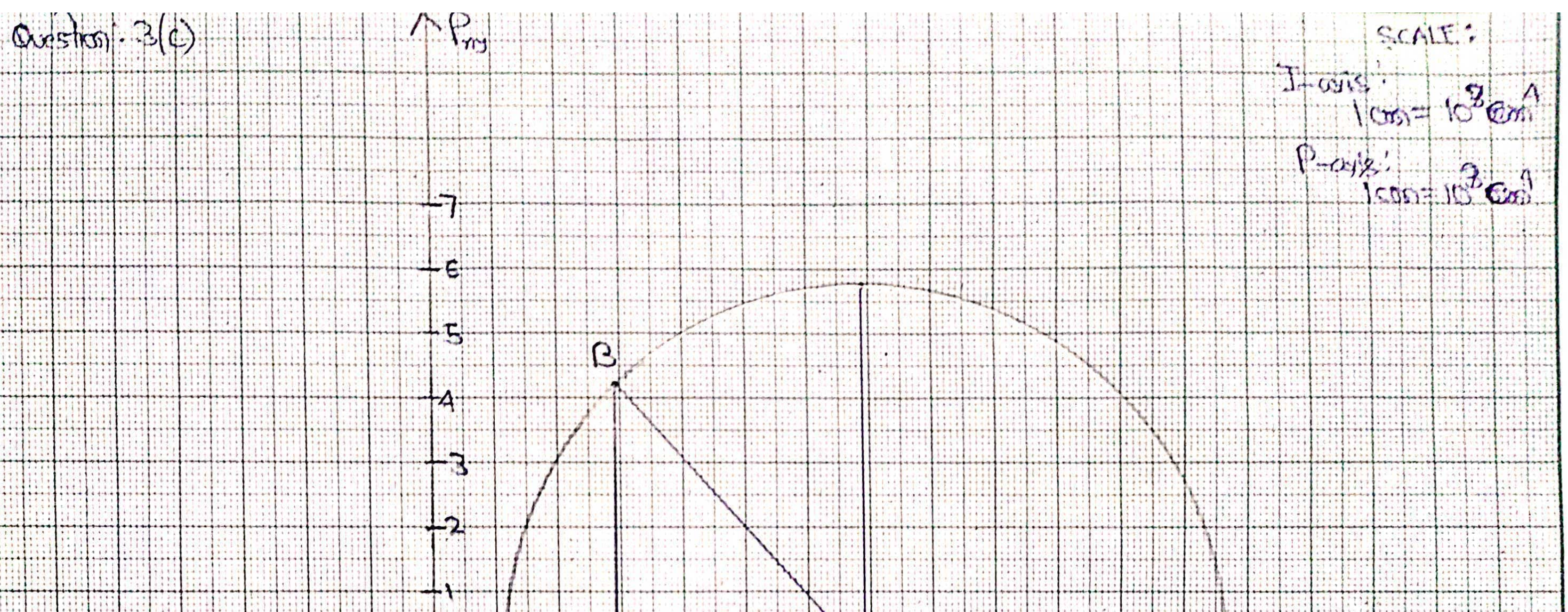
$$\tan 2\theta_p = \frac{2P_{xy}}{I_x - I_y} = \frac{-2 \times 4.2 \times \cancel{10^3}}{4.384 \times \cancel{10^3} - 2.945 \times \cancel{10^3}} \approx -47^\circ$$

Mohr's Circle:

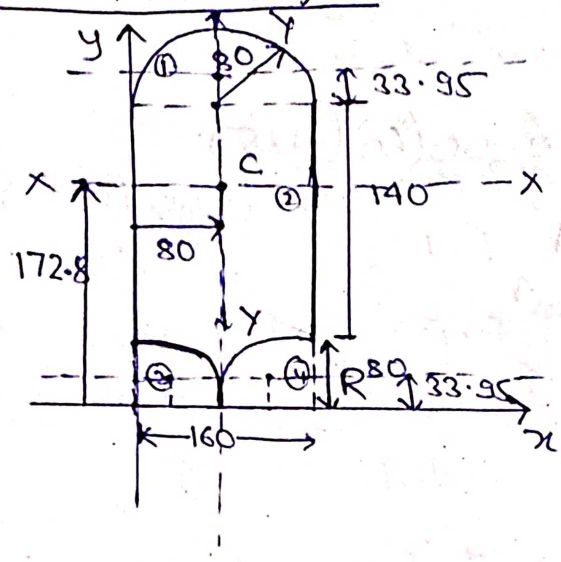
Result Table:

	$cm^4 (\times 10^3)$	Direction with x-axis (ACW)
OC	6.9	–
R	5.8	–
I_1	12.7	23.5°
I_2	1.1	113.5°
P_1	5.8	68.5°
P_2	−5.8	158.5°

Name: Adarsh Arya
Roll No.: 20ME8022
Reg. No.: 20U10095
Q: 3(C)
Session: 2022-23


Question: 3(c)
P_{xy}
SCALE:
I-axis: 1 cm = 10^{8} cm^{4}
P-axis: 1 cm = 10^{8} cm^{4}
B
C
A
47°
I_2
I_1
I
Name: Adarsh Arya
Roll No.: 20ME8022
Reg. No.: 20U10095
Q: 3(c)
session: 2022-23

Question 3(d):



$x_1 = 80\ mm = 8\ cm$
$x_2 = 80\ mm = 8\ cm$
$x_3 = 33.95\ mm = 3.395\ cm$
$x_4 = 126.05\ mm = 12.605\ cm$

$y_1 = 253.95\ mm = 25.395\ cm$
$y_2 = 110\ mm = 11\ cm$
$y_3 = 33.95\ mm = 3.395\ cm$
$y_4 = 33.95\ mm = 3.395\ cm$

$A_1 = 10053.1\ mm^2$
$A_2 = 35200\ mm^2$
$A_4 = A_3 = \frac{\pi}{4} \times 80^2 = 5026.55\ mm^2$

∴ Centroid:

$$\bar{X} = \frac{A_1x_1 + A_2x_2 - A_3x_3 - A_4x_4}{A_1 + A_2 - A_3 - A_4}$$

$$= \frac{(10053.1(80) + 35200(80) - 5026.55(33.95) - 5026.55(126.05))}{(10053.1 + 35200 - 5026.55 - 5026.55)}$$

$= 80\ mm = 8\ cm$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2 - A_3y_3 - A_4y_4}{A_1 + A_2 - A_3 - A_4}$$

$$= \frac{10053.1(253.95) + 35200(110) - 5026.55(33.95)\times 2}{10053.1 + 35200 - 2\times 5026.55}$$

$= 172.88\ mm = 17.288\ cm$

∴ Centroid: $(80, 172.88)\ mm$

Name: Adarsh Arya
Roll No.: 20ME8022
Reg. No.: 20U10095
Q: 3(d)
Session: 2022-23

Moment of Inertia:

$I_x = I_{x_1} + I_{x_2} - I_{x_3} - I_{x_4}$

$= 0.11(80)^4 + 10053.1(253.95-172.8)^2 + \frac{160\times220^3}{12}$

$+ 35200(110-172.8)^2 - 0.055\times80^4 - 5026.55(33.95-172.8)^2$

$- 0.055\times(80)^4 - 5026.55(33.95-172.8)^2$

$= 15.31\times10^7 \text{ mm}^4$

$= 15.31\times10^3 \text{ cm}^4$

$I_y = I_{y_1} + I_{y_2} - I_{y_3} - I_{y_4}$

$= 0.393\times80^4 + \cancel{10053.1(80-80)^2}^{\,0} + \frac{220\times160^3}{12} + \cancel{35200(80-80)^2}^{\,0}$

$- 0.055\times80^4 - 5026.55(33.95-80)^2$

$- 0.055\times80^4 - 5026.55(126.05-80)^2$

$= 6.54\times10^7 \text{ mm}^4 = 6.54\times10^3 \text{ cm}^4$

Product of Inertia:

$P_{xy} = \cancel{(P_{xy})_1}^{\,0} + \cancel{(P_{xy})_2}^{\,0} - (P_{xy})_3 - (P_{xy})_4$

$= -\left(5026.55\times(-46.05)\times\cancel{(-138.85)} + 5026.55\times46.05\cancel{(-138.5)}\right)$

$= 0$

$\therefore$ A(15.31, 0) ; B(6.54, 0)

$\tan 2\theta_p = \frac{2P_{xy}}{I_x - I_y} = 0 \quad \therefore \theta_p = 0°;$

Name: Adarsh Arya
Roll No.: 20ME8022
Reg. No.: 20U10095
Q: 3(d)
Session: 2022-23

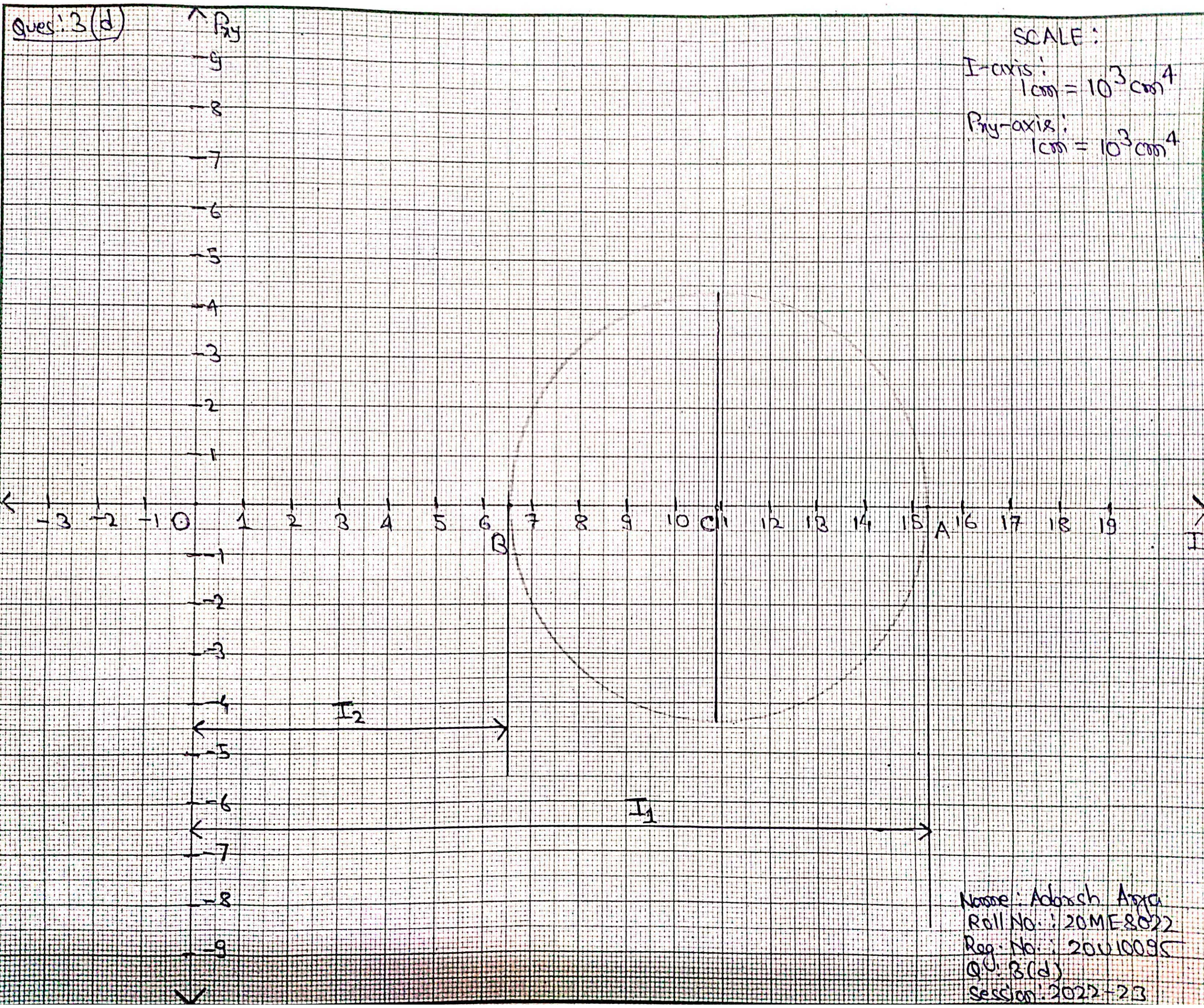
Mohr's Circle:

Result Table:

	$cm^4 (\times 10^3)$	Direction with x-axis (Anticlockwise)
OC	10.9	-
R	4.4	-
I_1	15.3	0°
I_2	6.5	90°
P_1	4.4	45°
P_2	-4.4	135°

Name: Adarsh Arya
Roll No.: 20ME8022
Reg. No.: 20U10095
Q.: 3(d)
Session: 2022-23

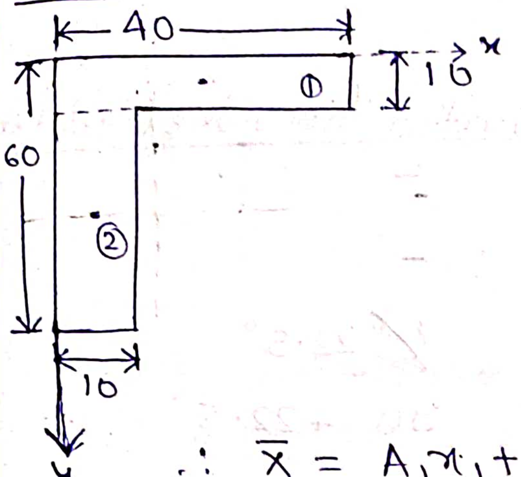
Ques: 3(d)



SCALE :
I-axis :
$1cm = 10^3 cm^4$
P_{xy}-axis :
$1cm = 10^3 cm^4$

Name : Adarsh Arya
Roll No. : 20ME8022
Reg. No. : 20U10095
Q : 3(d)
Session : 2022-23

Question 3 (e):



$x_1 = 20$ mm
$x_2 = 5$ mm
$(y_1) = 5$ mm
$y_2 = 35$ mm
$A_1 = 400$ mm^2;
$A_2 = 500$ mm^2

$$\therefore \bar{X} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{8000 + 2500}{900} = \frac{35}{3} = 11.67 \text{ mm}$$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{2000 + 17500}{900} = \frac{65}{3} = 21.67 \text{ mm}$$

Moment of Inertia:

$$I_x = I_{x_1} + I_{x_2}$$
$$= I_{G_{1x}} + A_1(y_1 - \bar{Y})^2 + I_{G_{2x}} + A_2(y_2 - \bar{Y})^2$$
$$= \frac{40 \times 10^3}{12} + 400\left(5 - \frac{65}{3}\right)^2 + \frac{10 \times 50^3}{12} + 500\left(35 - \frac{65}{3}\right)^2$$
$$= 3.10 \times 10^5 \text{ mm}^4 = 3.1 \times 10^1 \text{ cm}^4$$

$$I_y = I_{y_1} + I_{y_2} = I_{G_{1y}} + A_1(x_1 - \bar{X})^2 + I_{G_{2y}} + A_2(x_2 - \bar{X})^2$$
$$= \frac{10 \times 40^3}{12} + 400\left(20 - \frac{35}{3}\right)^2 + \frac{50 \times 10^3}{12} + 500\left(5 - \frac{35}{3}\right)^2$$
$$= 1.1 \times 10^5 \text{ mm}^4 = 1.1 \times 10^1 \text{ cm}^4 \ (107500 \text{ mm}^4)$$

Product of Inertia:

$$P_{xy} = (I_{xy})_1 + (I_{xy})_2 = 400(8.3)(16.67) + 500(-6.67)(-13.33)$$

$$\simeq 1 \times 10^5 \text{ mm}^4 \simeq 1 \times 10^1 \text{ cm}^4$$

$$\therefore A(3.1, 1) \quad ; \quad B(1.1, -1)$$

$$\tan 2\theta_p = \frac{2P_{xy}}{I_x - I_y} = \frac{2}{3.1 - 1.1} = 1 \Rightarrow \theta_p = 45/2$$

Name: Adarsh Arya
Roll : 20ME8022
Reg No.: 20U10095
Q: 3(e)
Session: 2022-23

3e) Mohr's Circle:

Result Table:

	cm^4 (0.5×10^1)	Direction from x-axis (Anticlockwise)
OC	2.1	–
R	1.4	–
I_1	3.5	157.5°
I_2	0.7	67.5°
P_1	1.4	22.5°
P_2	-1.4	112.5°

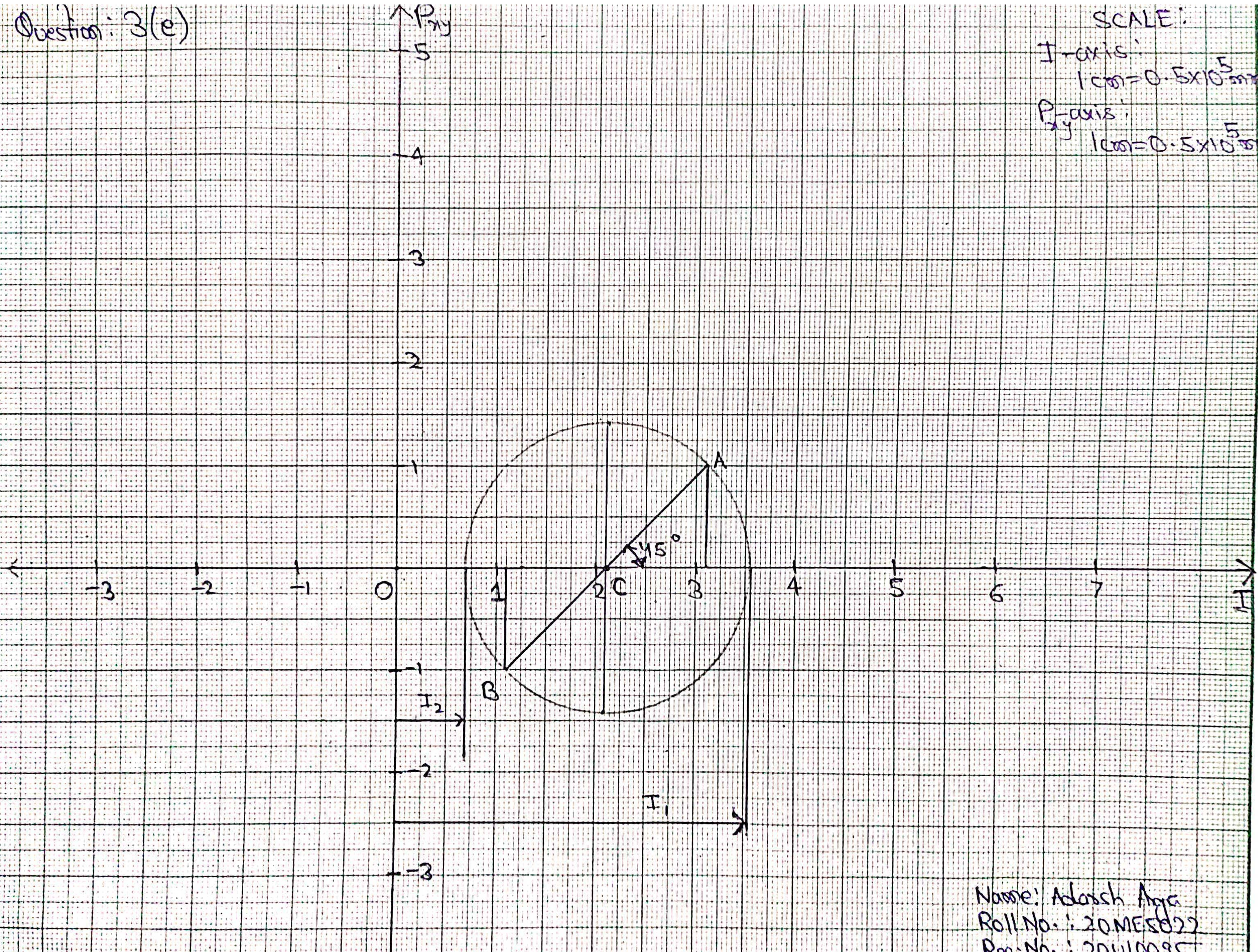
Name: Adarsh Arya
Roll no.: 20ME8022
Reg. no.: 20U10095
Q: 3(e)
Session: 2022-23

Question: 3(e)

SCALE:

I-axis:
1 cm = 0.5×10^5 mm^4

P_{xy}-axis:
1 cm = 0.5×10^5 mm^4



Name: Adarsh Arya
Roll No. : 20ME8022
Reg. No. : 20U10095
Q: 3(e)
Session: 2022-23