Design of Machine Elements

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Basic Procedure of Design of Machine Elements

- Specify functions of element
- ↓
- Determine Load acting on element
- ↓
- Select suitable Material for element
- ↓
- Determine failure mode (s) of element
- ↓
- Determine geometric dimensions of element based on above failure mode
- ↓
- Check & modify dimensions for assembly & manufacture
- ↓
- Check design at critical cross-sections
- ↓
- Prepare detail engineering drawing of element

Load Determination

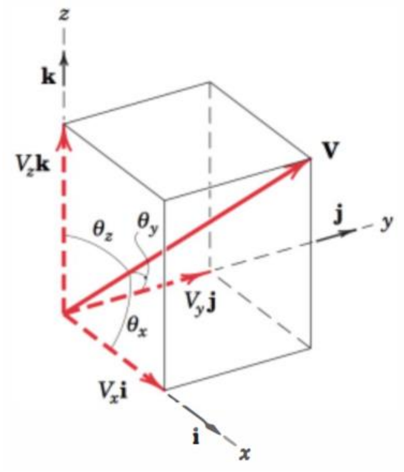
Load: external forces, Moments etc.

External Forces → Applied forces, Reactive forces

Vector *representing physical quantities*

- Magnitude
- Direction
- Line of action
- Point of application

Vectors → Free vector, Sliding vector, Fixed vector



External and Internal Effects

Principle of Transmissibility

Force Classification

Action and Reaction

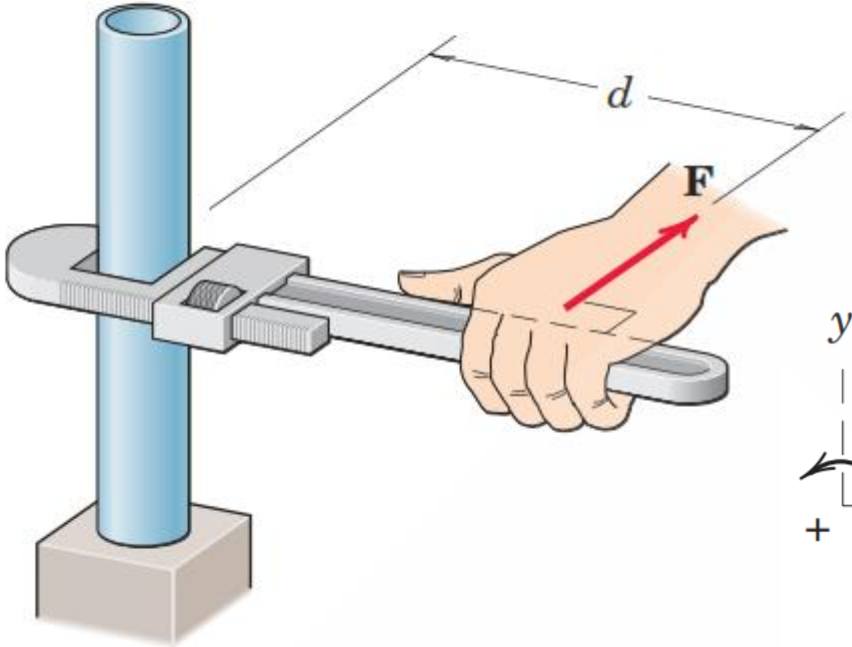
Concurrent Forces

Equilibrium and Free-Body Diagrams

$$\sum \mathbf{F} = 0$$

$$\sum \mathbf{M} = 0$$

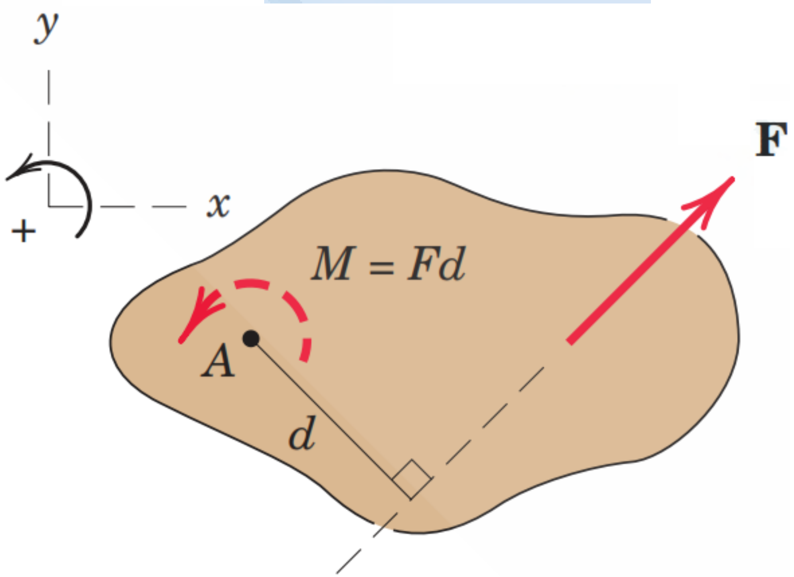
Moment about a Point

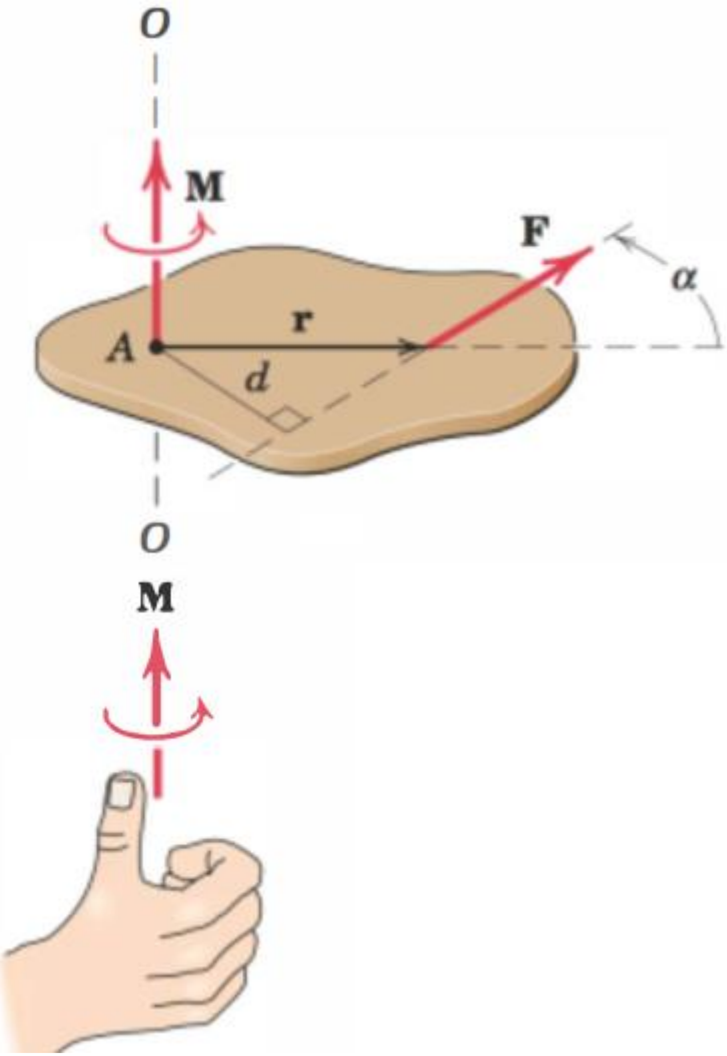


$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

$$M = Fr \sin \alpha = Fd$$

$$\boxed{M = Fd}$$



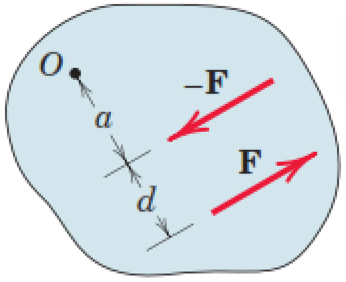


Varignon's Theorem

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

Couple

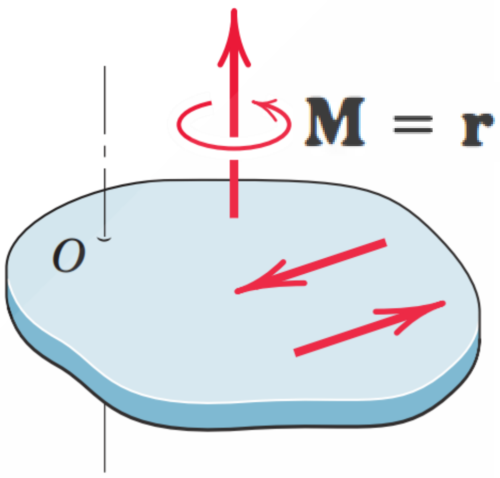
The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.



(a)

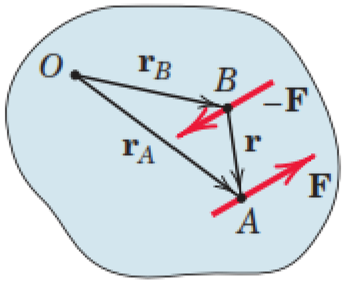
$$M = Fd$$

couple vector **M**



$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

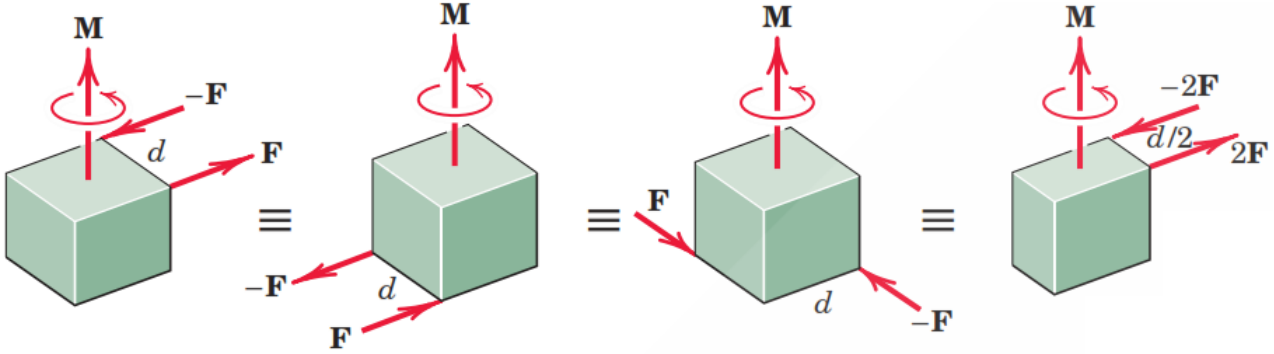
The moment of a couple is a *free vector*, whereas the moment of a force about a point (which is also the moment about a defined axis through the point) is a *sliding vector* whose direction is along the axis through the point. As in the case of two dimensions, a couple tends to produce a pure rotation of the body about an axis normal to the plane of the forces which constitute the couple.



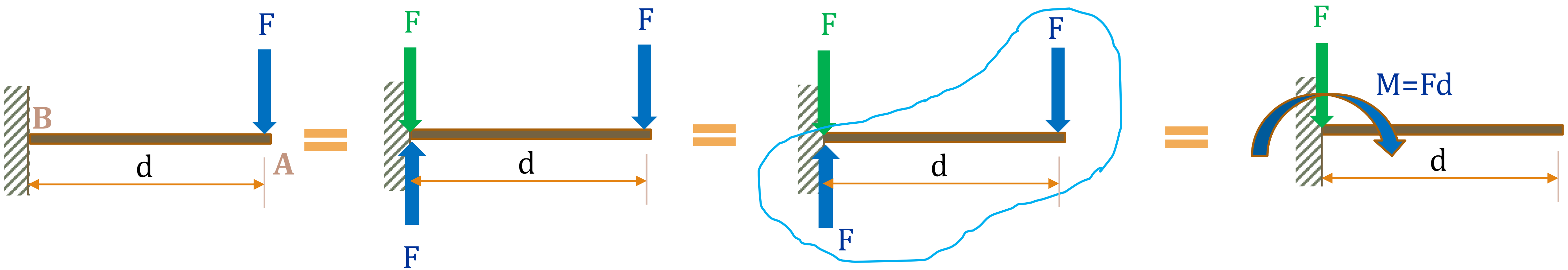
(b)

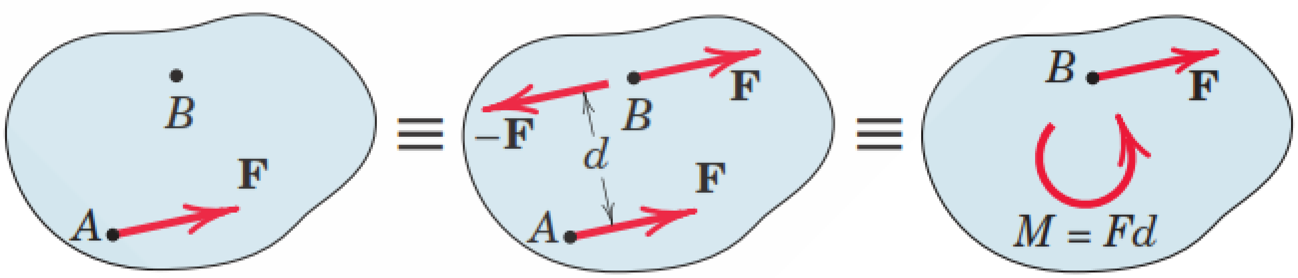
$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

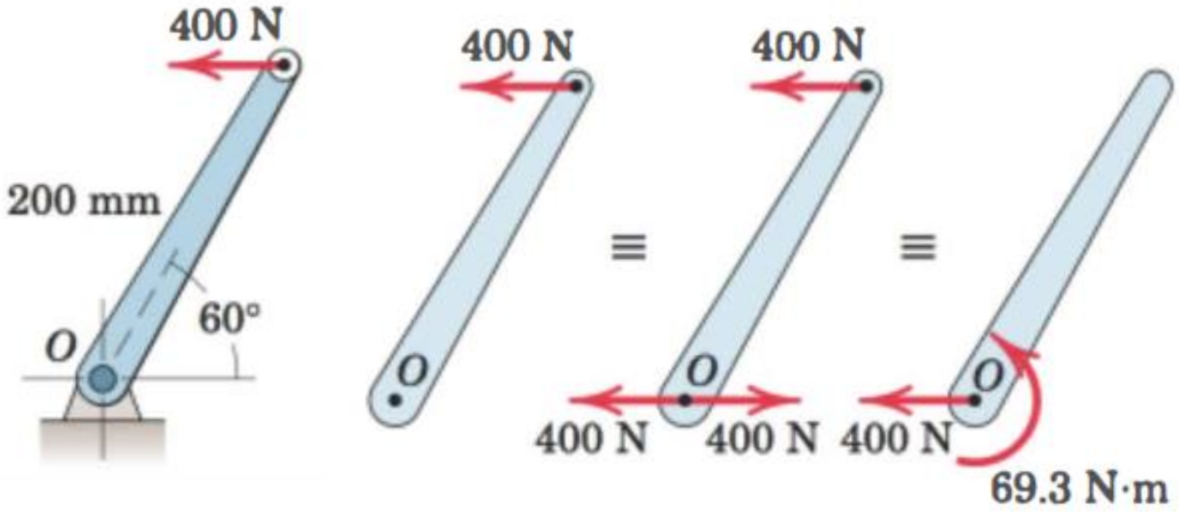
Equivalent Couples

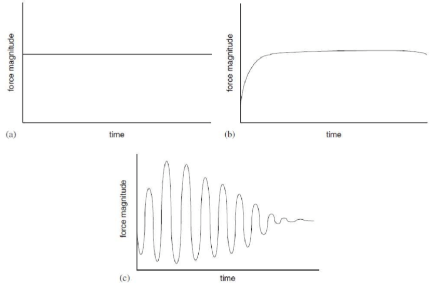


Force-Couple Systems









Load Classification

- *w.r.t time*
  - Static load
  - Cyclic load
  - Impact load
- *w.r.t the area over which it is applied*
  - Concentrated load
  - Distributed load
- *w.r.t location & method of application*
  - Normal load
  - Shear load
  - Torsion load
  - Bending load
  - Combined load

- Magnitude
- Direction
- Line of action
- Point of application

Design for Static Loading

Design for Cyclic/Fluctuating/ Fatigue/Variable Loading

Design for Impact Loading

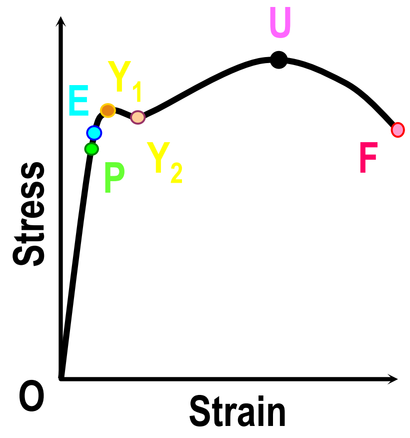
The behavior of machine parts is entirely different when they are subjected to time varying loading

Design Criteria

- One of the major decisions confronting the designer is the selection of appropriate “Design Criteria” or “Failure-prevention”. This is largely influenced by the **Modes of Failure** of the machine elements or structural elements.
- Designer should find the nature of action in the member that may cause it to fail.
- Some quantity such as stress, deflection etc. which characterizes the action that may cause its failure
- The action that initiates failure frequently is referred to as the Mode of Failure

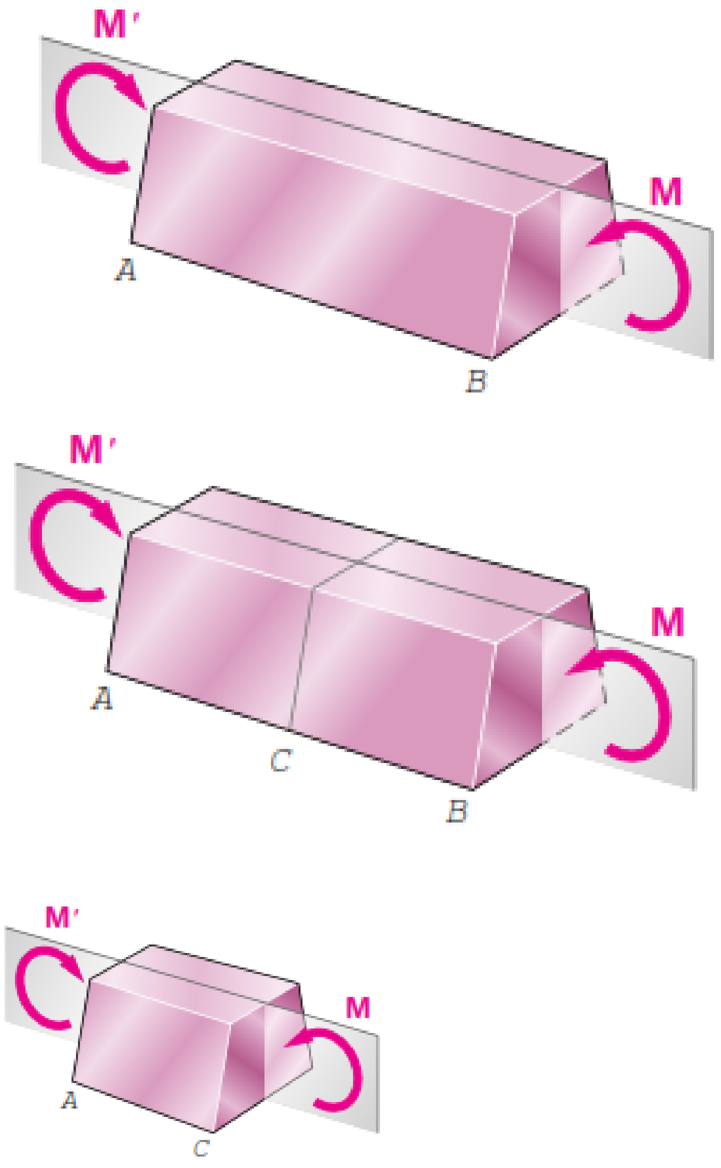
Common Modes of Failures

- Yielding → Shearing, Crushing, Bending
- Fracture
- Excessive elastic deflection
- Buckling
- Wear
- Corrosion etc…



Design Analysis of Curved Beams

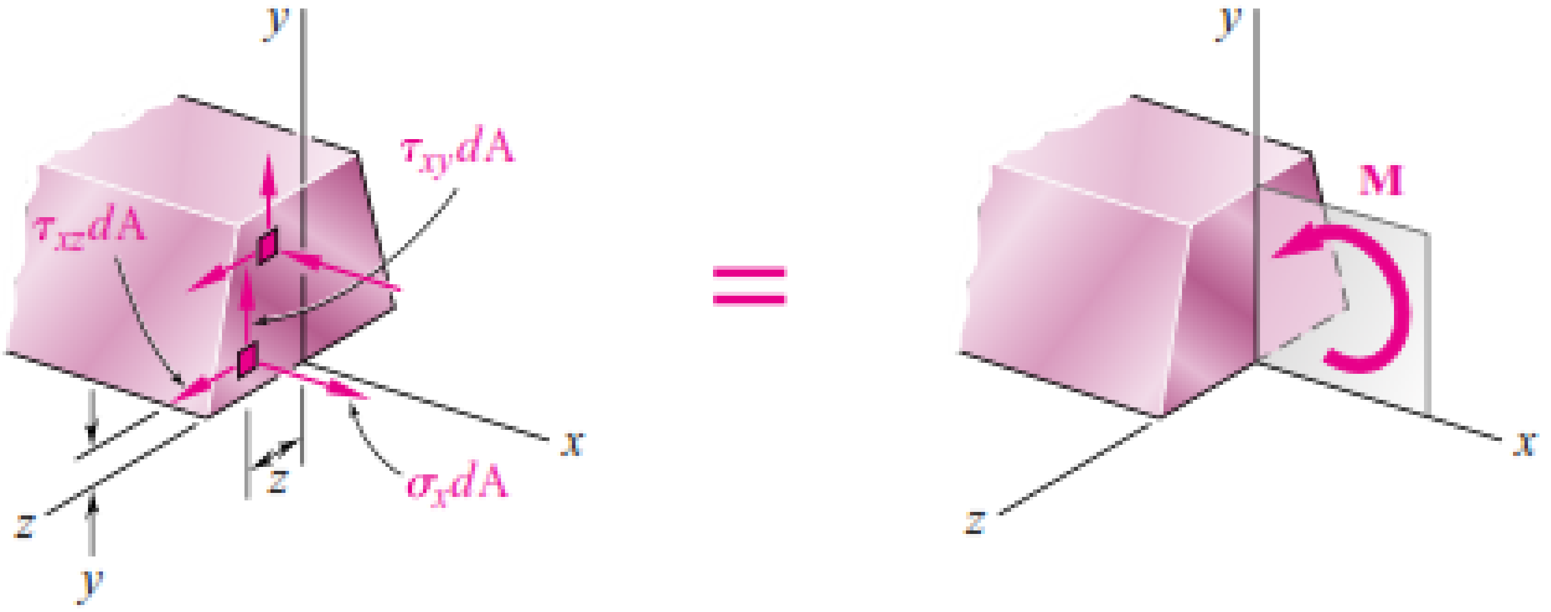
Curved Beams



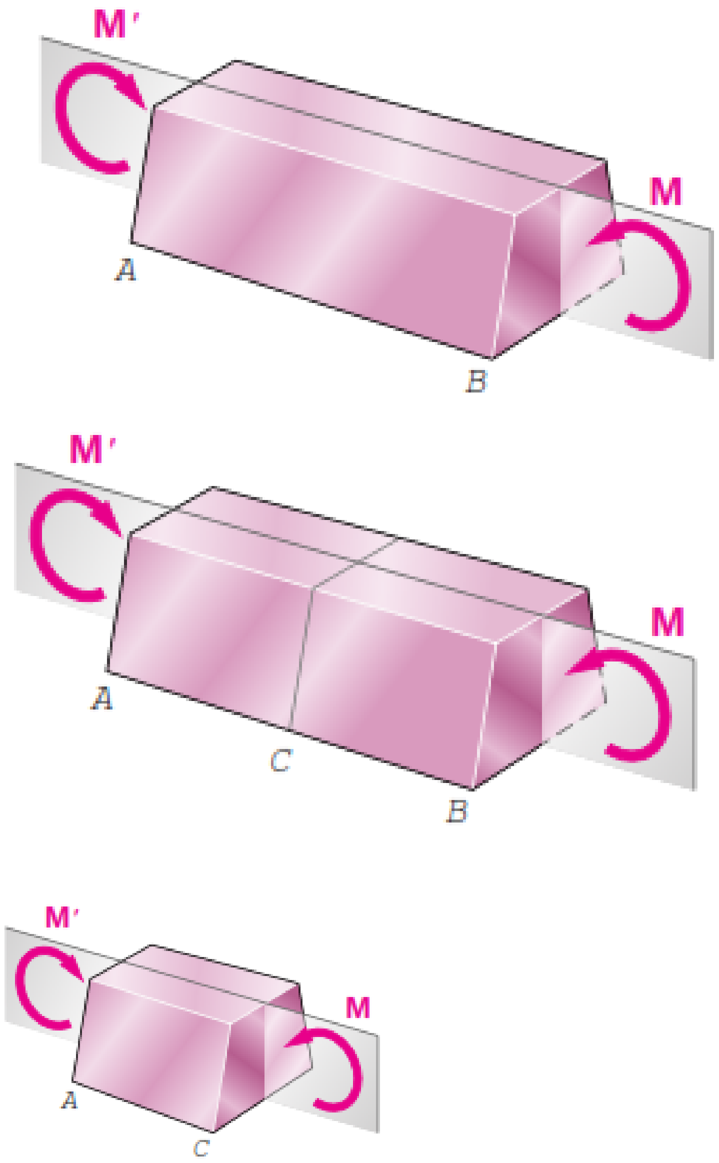
Design Analysis of Straight Beams

Consider a prismatic member AB possessing a plane of symmetry & subjected to equal & opposite couples M & M’ acting in that plane.

We observed that if a section is passed through the member AB at some arbitrary point C, the conditions of equilibrium of the portion AC of the member require that the internal forces in the section at C be equivalent to the couple M.

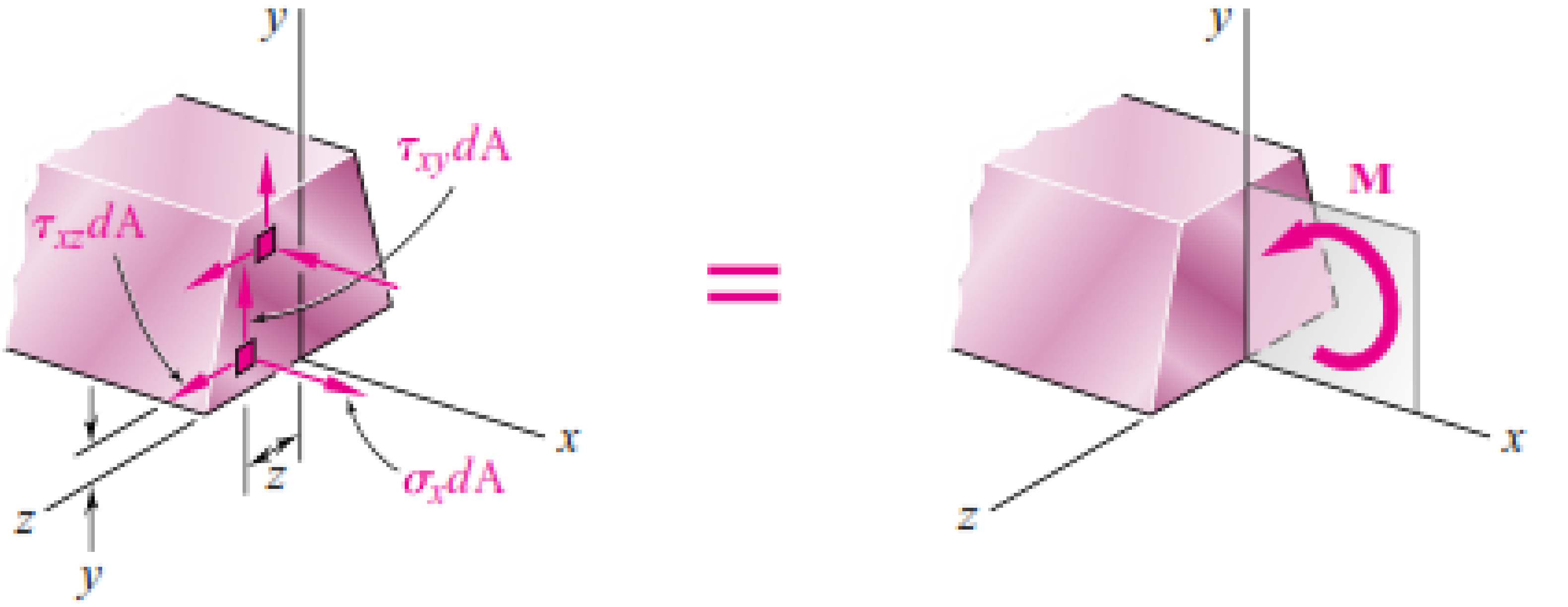


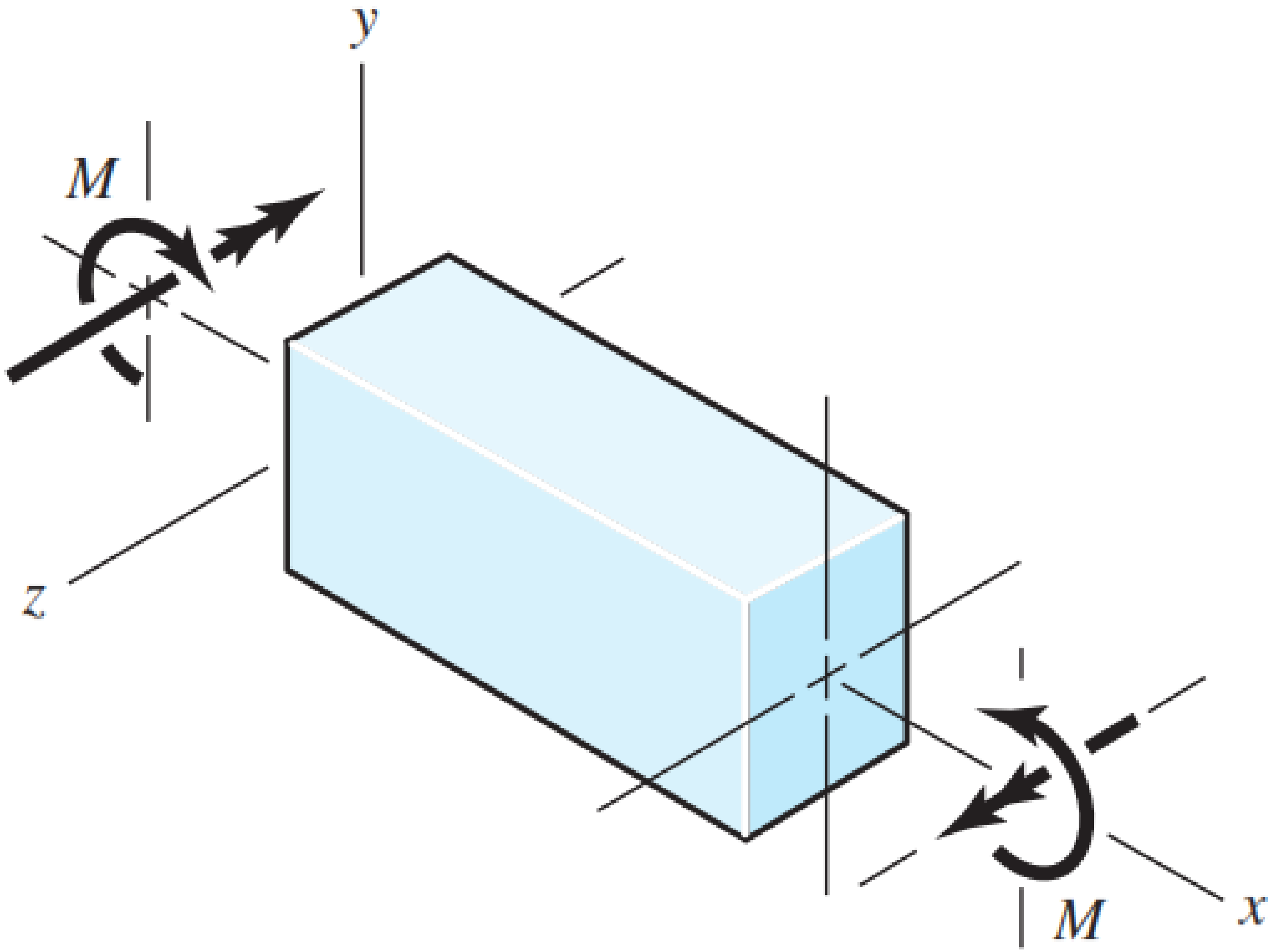
Design Analysis of Straight Beams



Thus, the internal forces in any cross-section of a symmetric member in pure bending are equivalent to a couple.

The Moment of the couple formed by internal forces is referred as Bending Moment





y
M
z
M
x

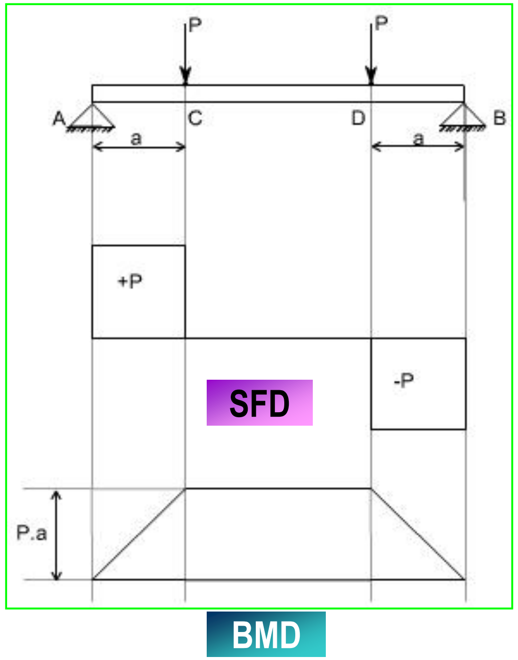
Concept of pure bending

Elastic Flexure formula : Bernoulli-Euler Flexure Formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{M}{Z}$$

Section Modulus $Z = I/y_{max}$



The following assumptions must be satisfied

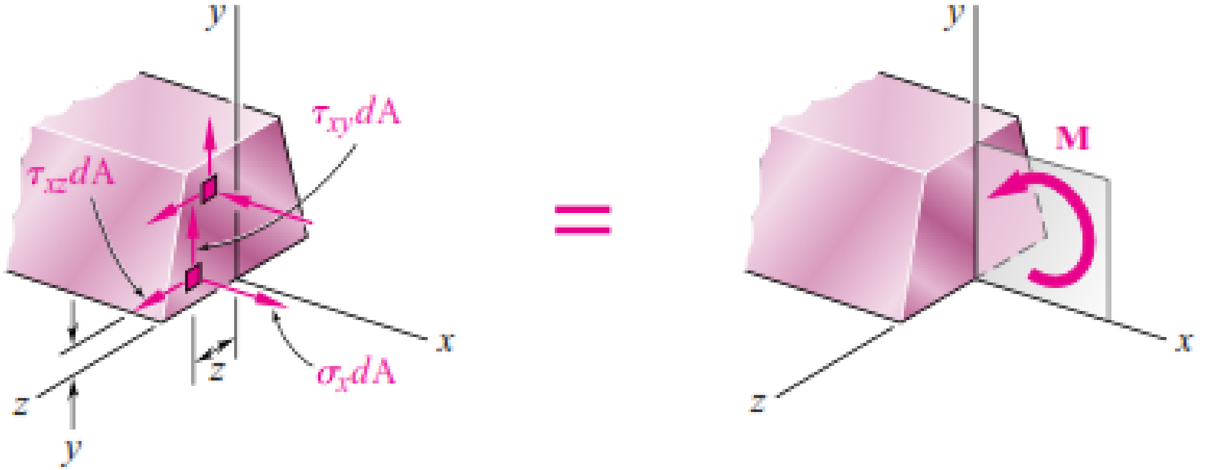
- The beam is subjected to pure bending (shear force=0, no torsion, no axial loads).
- The beam is initially straight with a c/s that is constant throughout the length.
- The beam has an axis of symmetry in the plane of bending.
- Plane c/s of the beam remain plane during bending.
- Couples are assumed to be loaded in the plane of symmetry.
- The beam material is homogeneous, isotropic & obeys Hooke’s law.

Design Analysis of Straight Beams

From statics

Any couple actually consists of two equal & opposite forces.

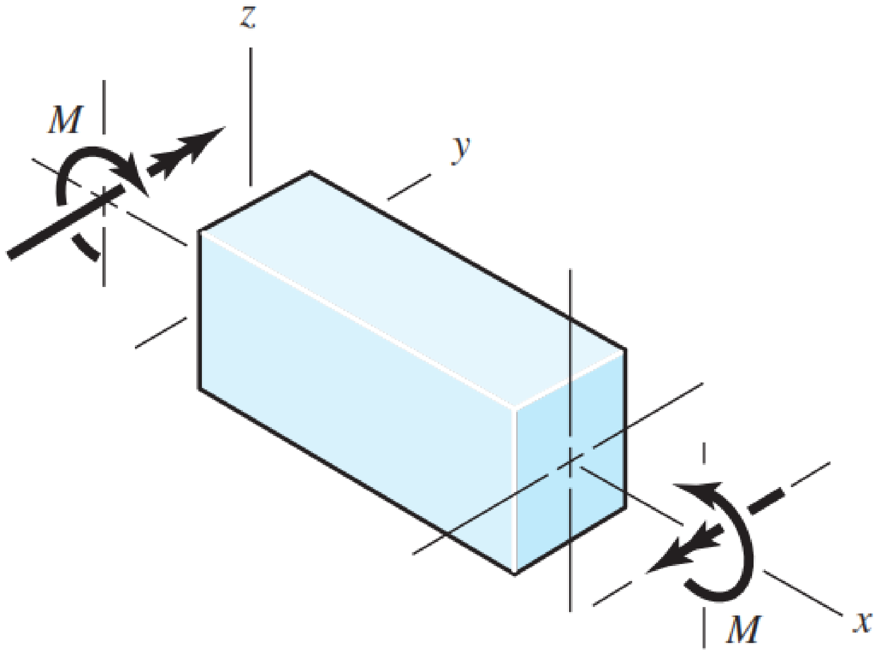
The sum of the components of these forces is equal to zero.



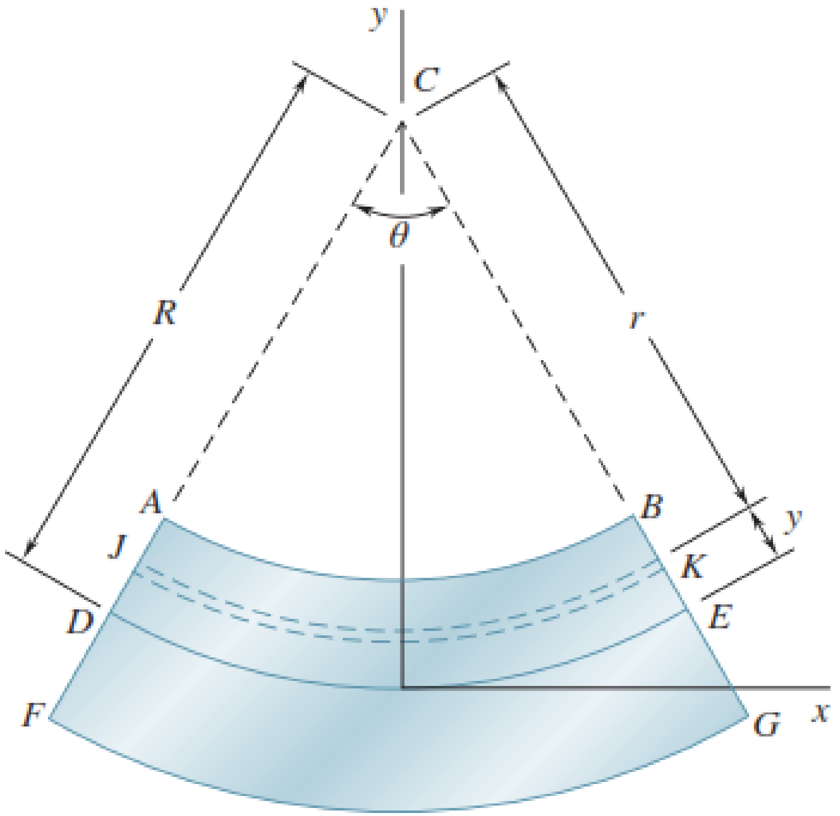
x components: $\int \sigma_x dA = 0$

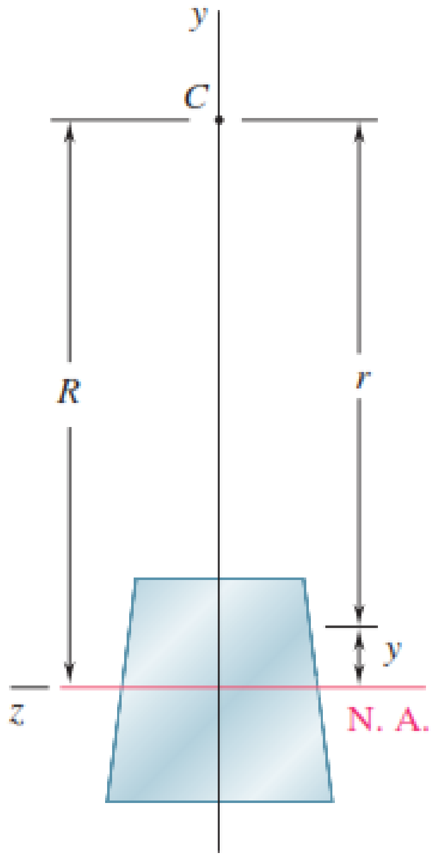
moments about y axis: $\int z\, \sigma_x dA = 0$

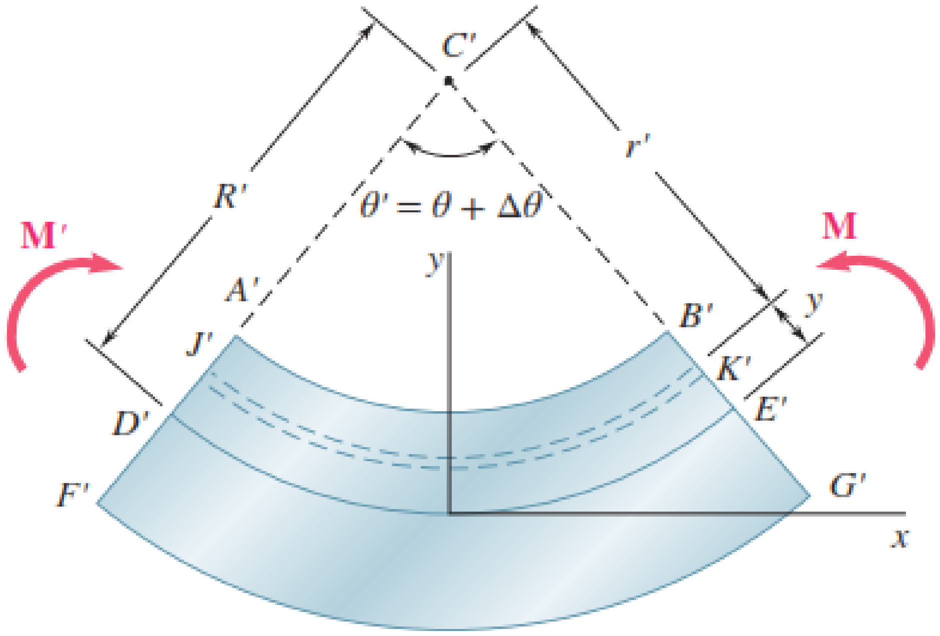
moments about z axis: $\int(-y\, \sigma_x dA) = M$



Design Analysis of Curved Beams







Assumption

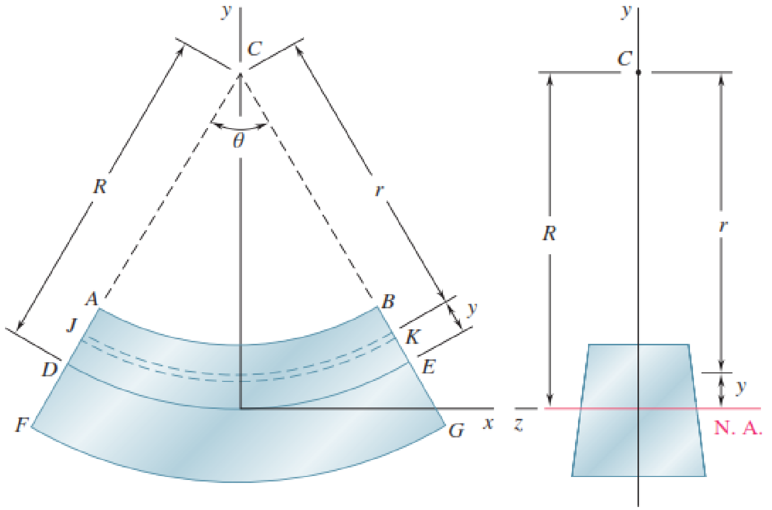
Our discussion will be limited to curved members of uniform cross-section possessing a plane of symmetry in which the bending couples are applied.

It will be assumed that all stresses remain below the proportional limit.

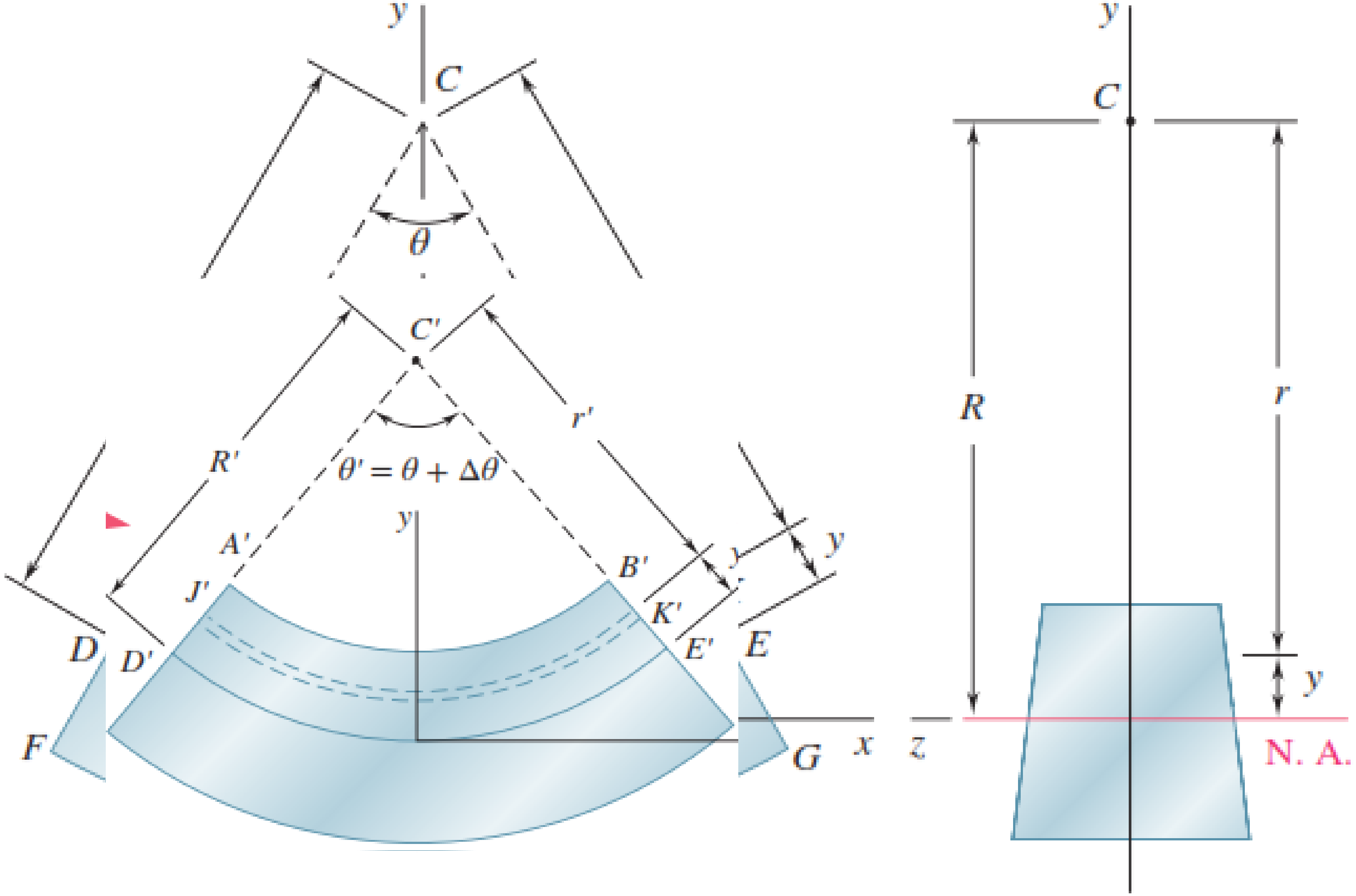
When the initial curvature of the member is large i.e., if its radius of curvature & the dimensions of the cross-section of the member are of the same order of magnitude, we must use a different method of analysis, which was first introduced by the German Engineer **E. Winkler** (1835-1888)

In its unstressed state, its upper & lower surfaces intersect the vertical XY plane along arcs of circle AB & FG centered at C.

Now apply two equal & opposite couples M & M' in the plane of symmetry of the member.



Design Analysis of Curved Beams



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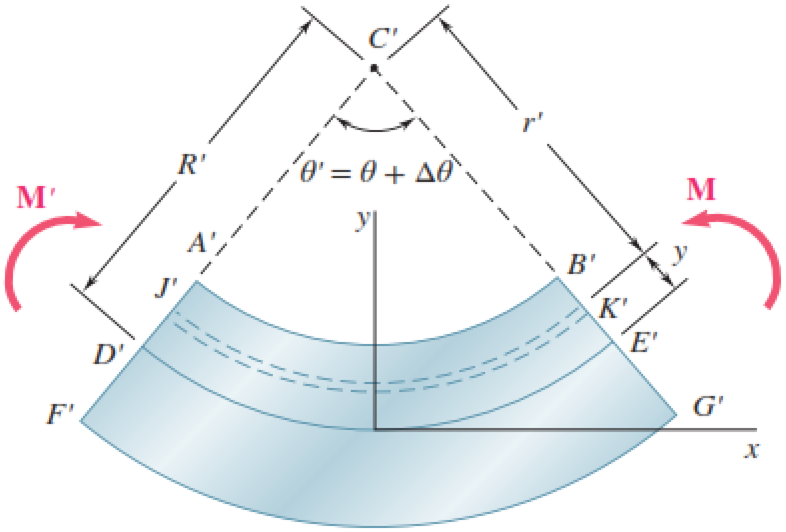


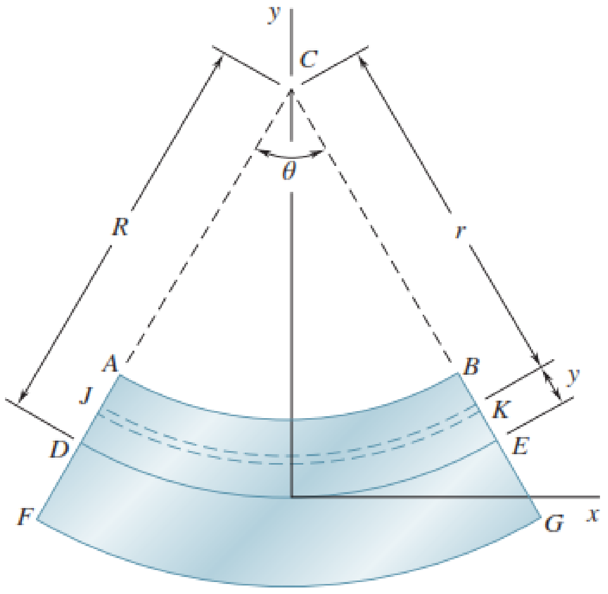
(Containing 'c')

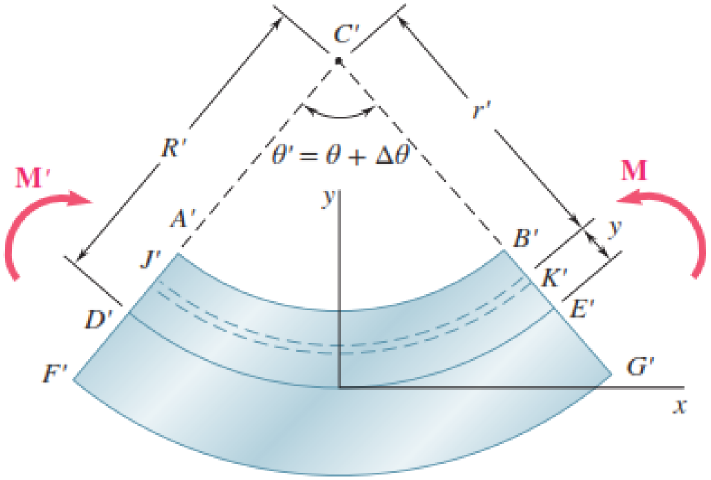
Because of the requirement that transverse Section remain plane and that the various arcs of circle will be transformed into circular & Concentric arcs with a center C' (different from 'C' (before bending))

after bending

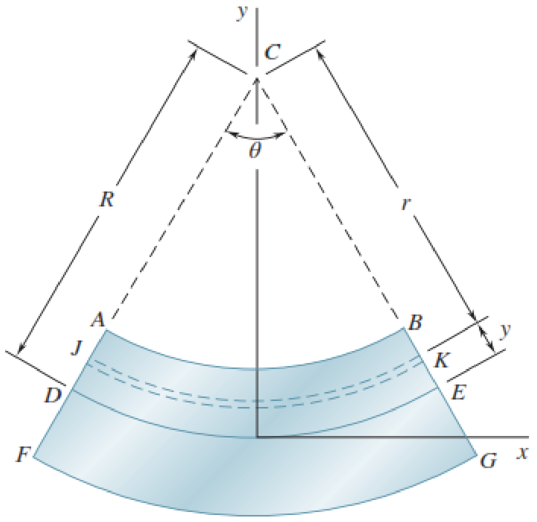
If the couples M and M' are directed as shown, the curvature of various arcs of circle will increase.

$\therefore \quad A'C' < AC$





The Couples M & M' will cause the length of the upper/inner surface of the curved member to decrease & the length of the lower/outer surface to increase.



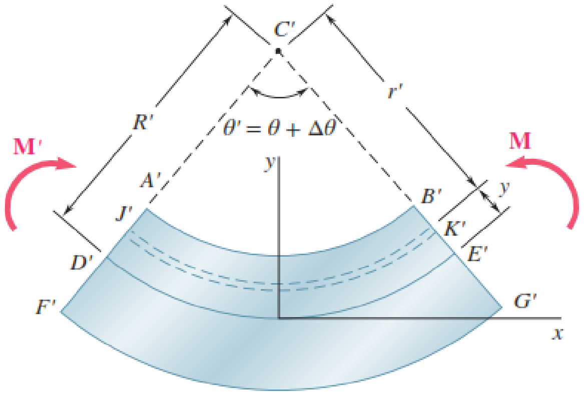
$\therefore$ $A'B' < AB$

$F'G' > FG$

& $\theta' = \theta + \Delta\theta$ - - - - - - - (1)

But neutral surface must exist in the member, the length of which remains constant

The intersection of the neutral surface with the plane of symmetry (i.e. xy plane) has been represented by the arc DE of radius 'R' (before bending) & by the arc D'E' of radius R' (after bending)



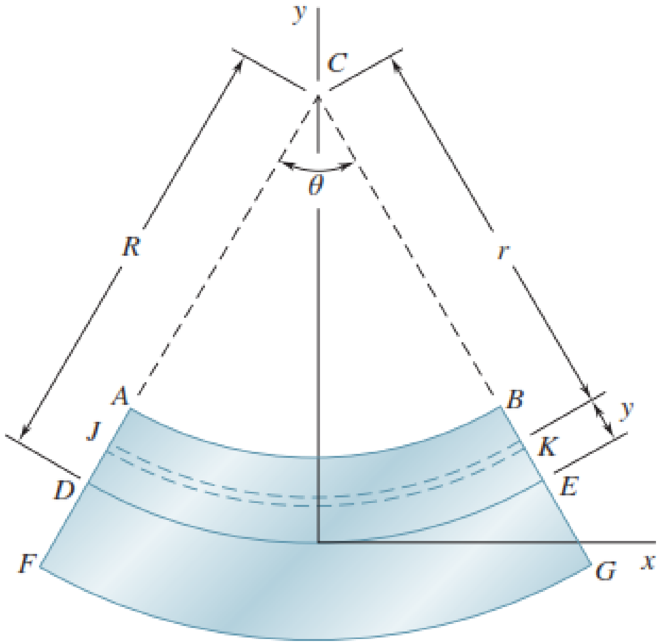
As length of the DE (neutral surface) remains constant

$\therefore$ $R.\theta = R'\theta'$ - - - - - - (2)

Let us consider an arc of circle 'JK' located at a distance 'y' above the neutral surface.

r = radius of the arc JK before bending

r' = " " " J'K' after bending.



Length of the arc JK before bending $= r.\theta$

" " " " J'K' after bending $= r'\theta'$

Deformation of JK : $\delta = r'\theta' - r.\theta$ where $r = R - y$ ---- (3)

$r' = R' - y$

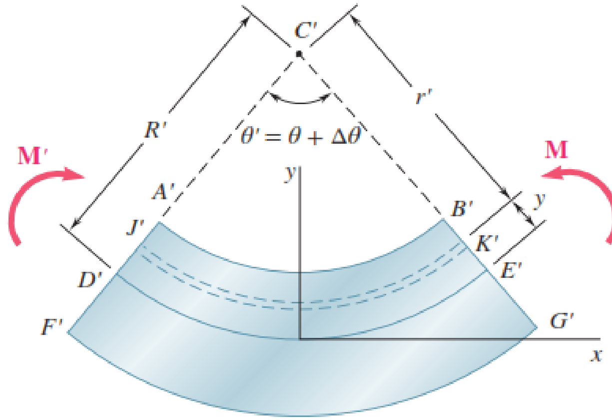
$r = R' - y$

$\delta = (R'-y)\theta' - (R-y)\theta$

$\delta = (R'\theta' - R\theta) - y(\theta' - \theta)$

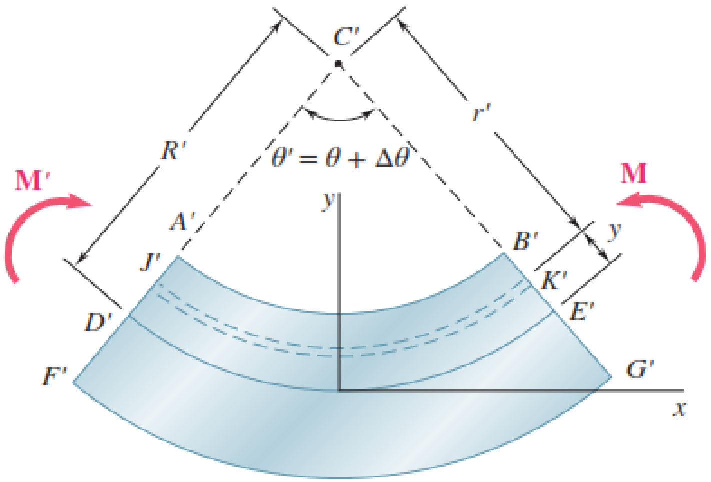
$\delta = 0 - y.\Delta\theta$ as $R'\theta' = R\theta$

$\delta = -y.\Delta\theta$ ---- (4) $\theta' = \theta + \Delta\theta$ $\therefore \theta' - \theta = \Delta\theta$



Normal strain in the element JK: $\epsilon_x = \dfrac{\text{Deformation } (\delta)}{\text{original length } (r\theta)} = \dfrac{\delta}{r\theta}$

$$= -y \cdot \frac{\Delta\theta}{r\theta} = -\frac{\Delta\theta}{\theta} \cdot \frac{y}{r}$$



$$\epsilon_x = -\frac{\Delta\theta}{\theta} \cdot \frac{y}{r} \quad \text{.........(5)}$$

$$\boxed{\epsilon_x = -\frac{\Delta\theta}{\theta} \cdot \frac{y}{R-y}}$$

As $r = R-y$ .....(6)

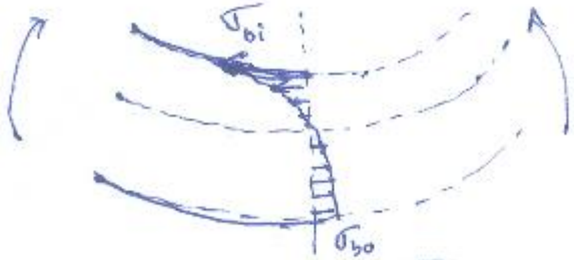
The relation obtained shows that, while each transverse section remains plane, the normal strain (ϵ_x) does not vary linearly with the distance from the neutral surface

Apply Hooke's Law,

Normal stress $\sigma_x = E \cdot \epsilon_x$

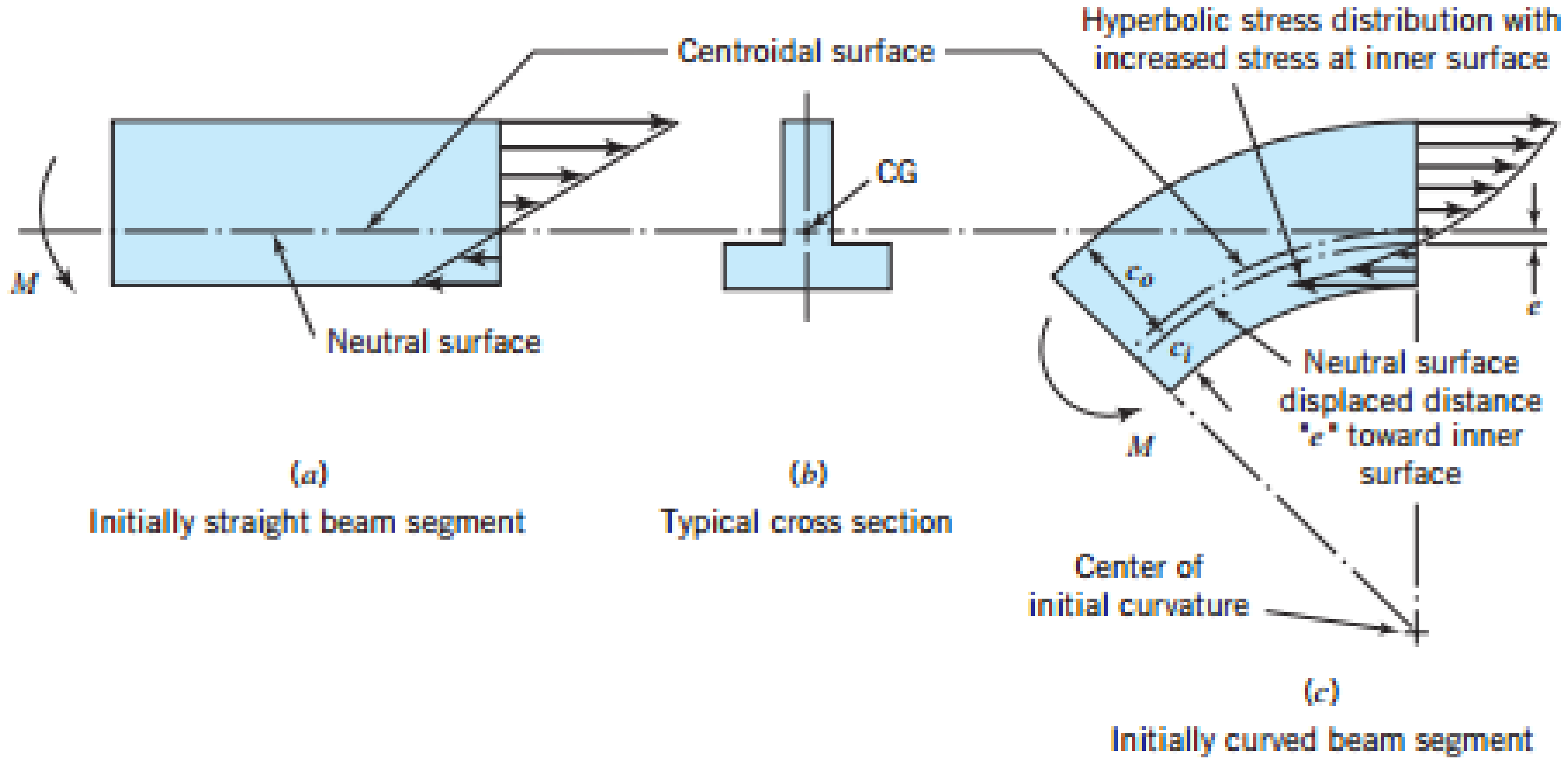
$$\boxed{\sigma_x = -\frac{E\Delta\theta}{\theta} \cdot \frac{y}{R-y}} \quad \text{-----} \quad (7)$$

- Normal stress σ_x does not vary linearly with the distance 'y' from the neutral surface. Plotting σ_x vs. y, we obtain an arc of Hyperbola.



$$\boxed{\sigma_x = -\frac{E\Delta\theta}{\theta} \cdot \frac{R-r}{r}}$$

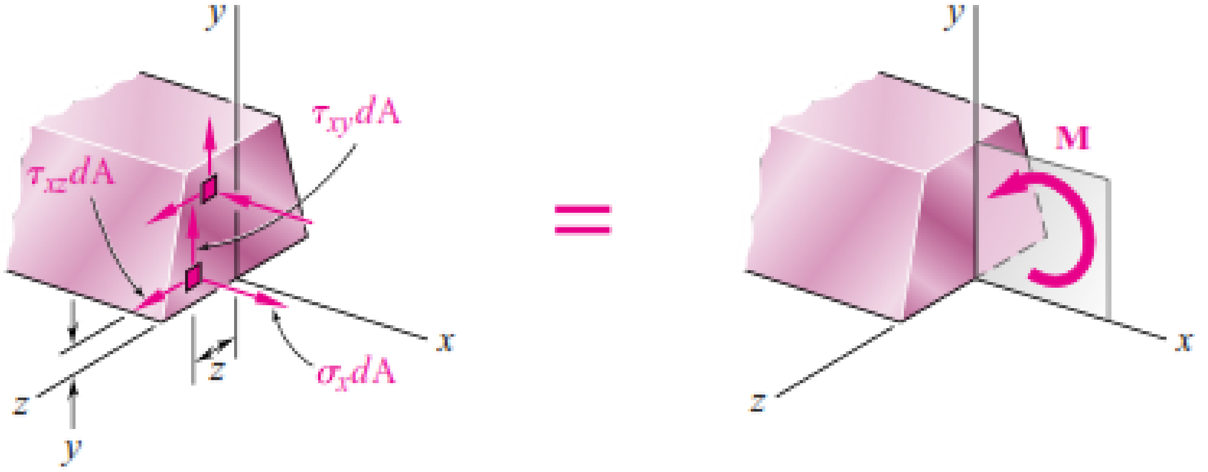
Obj: (a) Det. R
(b) Det. $\frac{E \cdot \Delta\theta}{\theta}$


Centroidal surface
Hyperbolic stress distribution with
increased stress at inner surface
CG
M
Neutral surface
c_o
c_i
e
Neutral surface
displaced distance
"e" toward inner
surface
M
(a)
Initially straight beam segment
(b)
Typical cross section
Center of
initial curvature
(c)
Initially curved beam segment

Design Analysis of Straight Beams

From statics

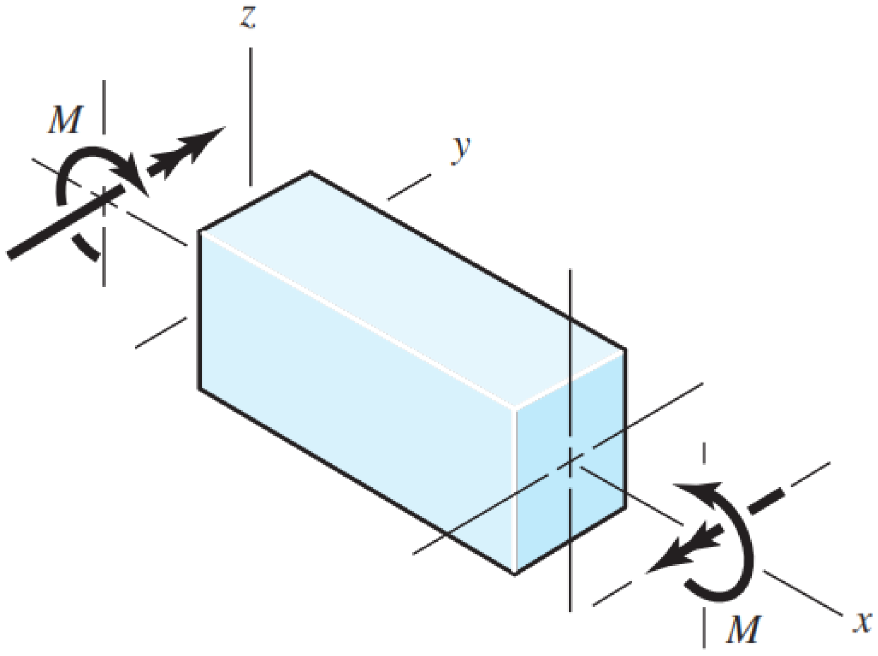
Any couple actually consists of two equal & opposite forces.
The sum of the components of these forces is equal to zero.



x components: $\int \sigma_x dA = 0$

moments about y axis: $\int z \, \sigma_x dA = 0$

moments about z axis: $\int (-y \, \sigma_x dA) = M$



Obj: (a) Det. R
(b) Det. $\frac{E \cdot \Delta\theta}{\theta}$

The elementary forces acting on any transverse Section must be Statically equivalent to the bending couple M.

From Statics, any couple 'M' actually consists of two equal & opposite forces (ie. $\pm\sigma_x dA$)

The sum of the elementary internal resisting forces (ie. above equal & opposite forces $\pm\sigma_x dA$) acting on the transverse section (along x-dir^n) must be zero

$$\therefore \int \sigma_x dA = 0 \qquad \text{- - - - - (9)}$$

and that the sum of their moments about the Z-axis must be equal to B.M
(Sum of the moment of these resisting forces about z-axis)

$$\therefore \int -y\sigma_x dA = M \qquad \text{- - - - - (10)}$$

From eq^n (8) & (9)

$$\int \sigma_x . dA = 0$$

$$-\frac{E\Delta\theta}{\theta}\int \frac{R-r}{r} dA = 0$$

from eqⁿ (8) & (9)

$$\int \sigma_x \cdot dA = 0$$

$$-\frac{E\Delta\theta}{\theta}\int \frac{R-r}{r}\, dA = 0$$

$$\therefore \int \frac{R-r}{r}\, dA = 0 \quad \text{where } -\frac{E\Delta\theta}{\theta} \neq 0$$

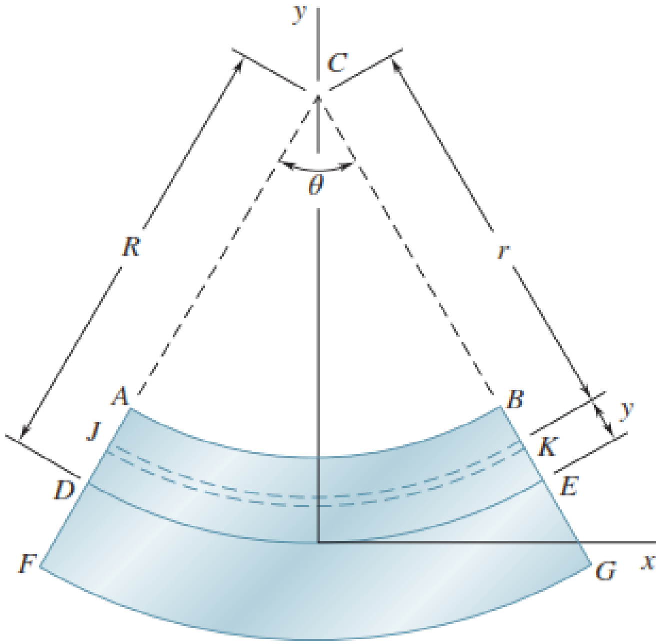
$$R\int \frac{dA}{r} - \int dA = 0$$

$$\therefore R\int \frac{dA}{r} = A$$

$$\boxed{R = \frac{A}{\int \frac{dA}{r}}}$$

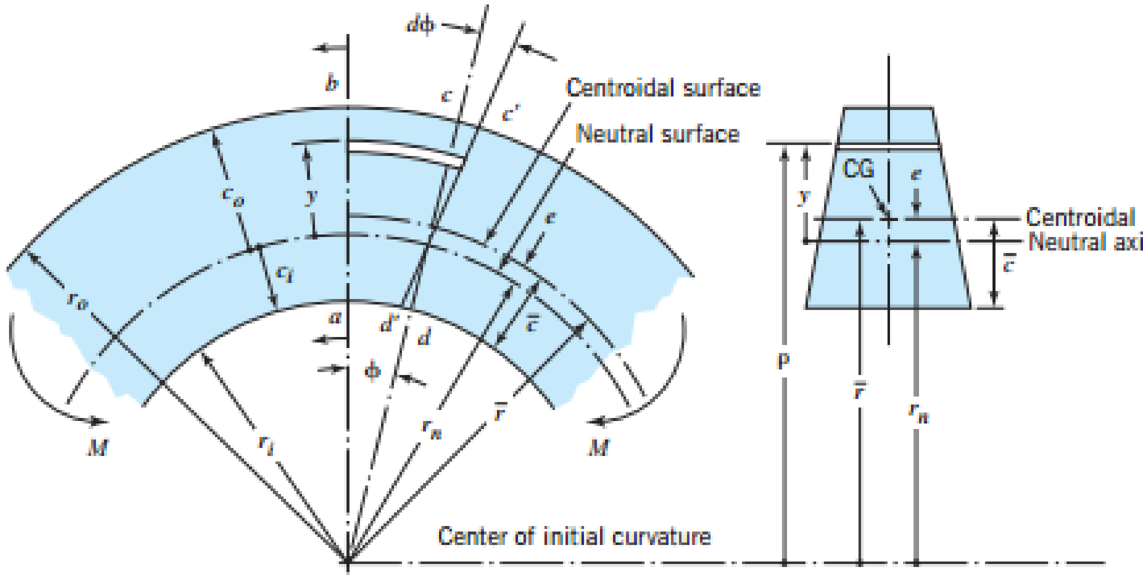
⇒ Distⁿ 'R' from the centre of curvature 'C' to the neutral surface.

- - - - - - - - - - (11)



Distance of the centroid of the cross-section from centre of curvature 'c'

$$\bar{r} = \frac{\int r\,dA}{A} \quad \text{- - - - - - (12)}$$



From eqⁿ (11) & (12), we can write

$$R \neq \bar{r}$$

$$\boxed{R = \frac{A}{\int \frac{dA}{r}}}$$

⇒ Distⁿ 'R' from the centre of curvature 'C' to the neutral surface. - - - - - - - - (11)

Remarks:

The neutral axis of a transverse section does not pass through the centroid of that section

If a beam has significant curvature, then the NA will no longer be coincident with the centroidal axis

From Eq. (10)

$$\int -y\,\sigma_x\,dA = M$$

$$\int \frac{E\Delta\theta}{\theta}\cdot\frac{R-r}{r}\cdot y\,dA = M$$

$$\frac{E\Delta\theta}{\theta}\int\frac{(R-r)}{r}\cdot(R-r)\,dA = M$$

as y $r = R-y \;\therefore\; y = R-r$

$$\frac{E\Delta\theta}{\theta}\int\frac{(R-r)^2}{r}\,dA = M$$

$$\frac{E\Delta\theta}{\theta}\left[R^2\int\frac{dA}{r} - 2R\int dA + \int r\,dA\right] = M$$

$$\frac{E\Delta\theta}{\theta}\left[R.A - 2RA + \bar{r}A\right] = M$$

as $R = \dfrac{A}{\int\frac{dA}{r}} \;\therefore\; A = R\int\frac{dA}{r}$

$\bar{r} = \dfrac{\int r\,dA}{A} \;\therefore\; \int r\,dA = A\bar{r}$

$\therefore \quad \frac{E\Delta\theta}{\theta}\left[A(\bar{r}-R)\right] = M$

$$\therefore \quad \boxed{\frac{E\Delta\theta}{\theta} = \frac{M}{A(\bar{r}-R)}}$$

Note: $\Delta\theta > 0$; $M > 0$

from eqn (13) $\bar{r}-R > 0$

$\therefore R < \bar{r}$

Thus, the NA is always located betn the Centroid of the section & the Centre of Curvature of the member. [NA shifts toward the Centre of Curvature by a distance 'e'.]

$$\sigma_x = -\frac{E\Delta\theta}{\theta}\,\frac{y}{R-y}$$

$$\sigma_x = -\frac{M.y}{A(\bar{r}-R)(R-y)} = -\frac{M.y}{Ae(R-y)} \quad \text{---} (14)$$

where $e = \bar{r}-R$

= eccentricity betn Centroidal & Neutral axis

$$\sigma_x = -\frac{M.y}{Ae(R-y)} \quad \text{-------} (15)$$

$$\sigma_b = \sigma_x = \frac{M(r-R)}{Aer} \quad \text{as} \quad r = R-y \quad \text{--} (16)$$

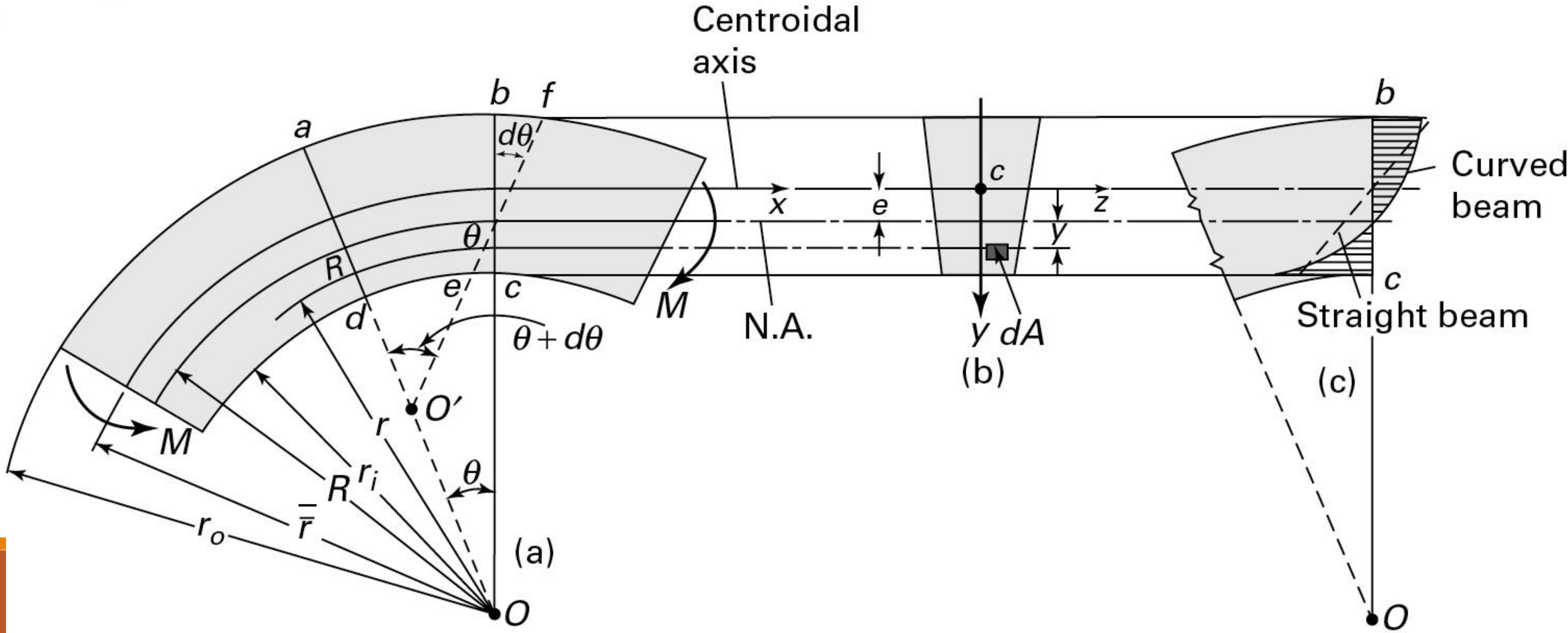
$$\sigma_x = -\frac{M \cdot y}{Ae(R-y)} \quad \text{------} (15)$$

$$\sigma_b = \sigma_x = \frac{M(r-R)}{Aer} \quad \text{as} \quad r = R-y \quad \text{---} (16)$$

$$\sigma_{bi} = \frac{M(r_i - R)}{A e r_i} \quad \text{--- Inner fibre}$$

$$\sigma_{bo} = \frac{M(r_o - R)}{A e r_o} \quad \text{--- outer fibre}$$

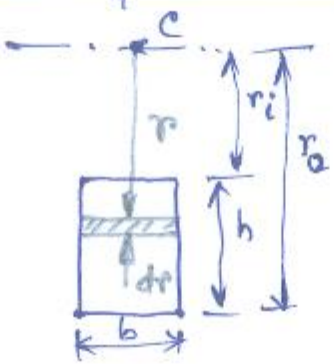
$$\sigma_{bi} > \sigma_{bo}$$



Location of Neutral Axis in Curved Beams

Obj: ⓐ Det. R

Rectangle or Rectangular C/s

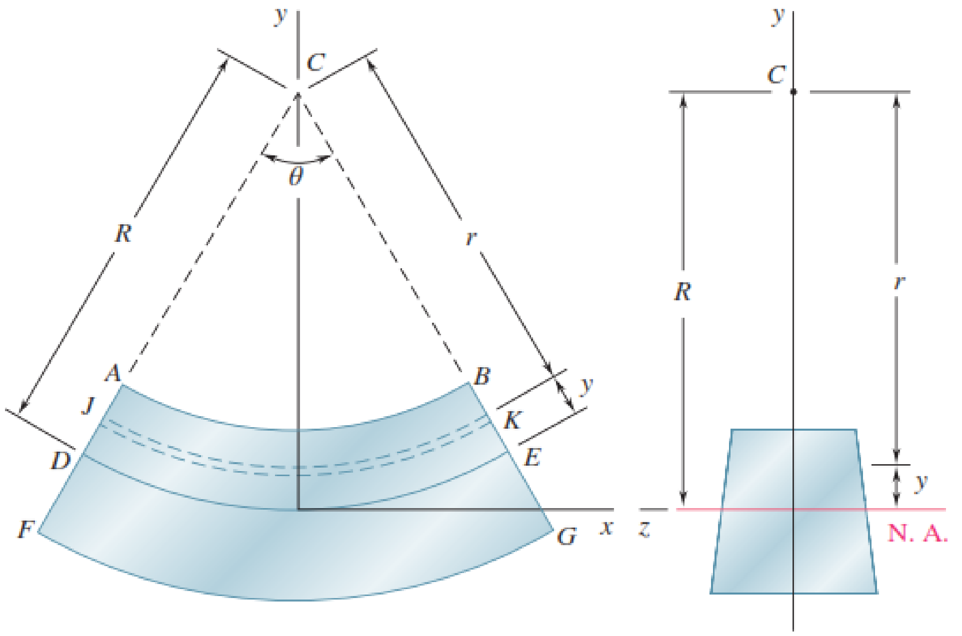


$A = b.h \quad ; \quad r_i < r_o$

$dA = b.dr$

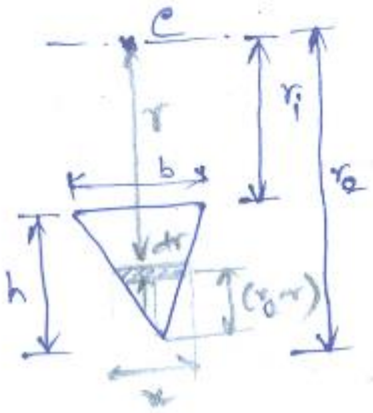
$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} = \frac{b.h}{\int_{r_i}^{r_o} b\frac{dr}{r}} = \frac{h}{\int_{r_i}^{r_o} \frac{dr}{r}} = \frac{h}{\ln(r_o/r_i)}$$

$$\bar{r} = \frac{1}{A}\int_{r_i}^{r_o} r\,dA = \frac{1}{bh}\int_{r_i}^{r_o} b.r\,dr = \frac{1}{bh}\cdot\frac{b}{2}(r_o+r_i)(r_o-r_i) = \frac{r_o+r_i}{2} = r_i + h/2$$



Location of Neutral Axis in Curved Beams

Triangle or Triangular c/s



$$A = \frac{1}{2} b h$$

Properties of Δ : $\dfrac{x}{(r_o - r)} = \dfrac{b}{h}$ $\quad \therefore \quad x = \dfrac{b}{h}(r_o - r)$

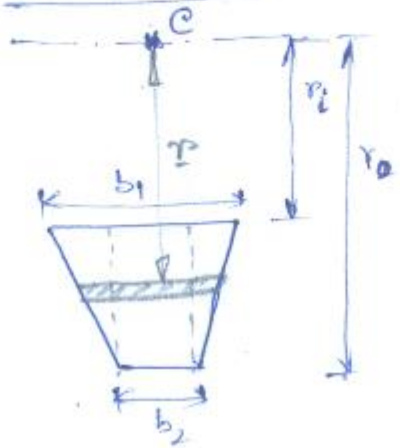
$$dA = x \cdot dr = \frac{b}{h}(r_o - r)\,dr$$

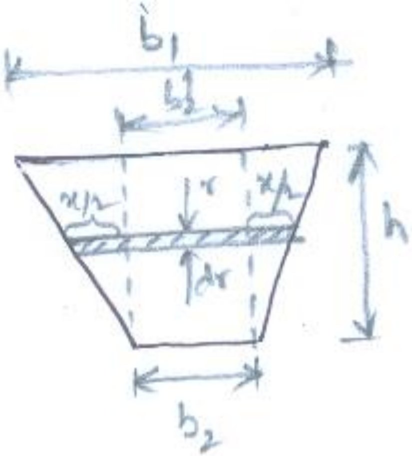
$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} = \frac{\frac{1}{2} b \cdot h}{\int_{r_i}^{r_o} \frac{b}{h}\left(\frac{r_o - r}{r}\right) dr} = \frac{\frac{1}{2} h}{\frac{1}{h}\left[r_o \ln(r_o/r_i) - (r_o - r_i)\right]}$$

$$R = \frac{\frac{1}{2} h}{\frac{1}{h}\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]} = \frac{h}{2\left[\left(\frac{r_o}{h}\right)\ln\left(\frac{r_o}{r_i}\right) - 1\right]}$$

Location of Neutral Axis in Curved Beams

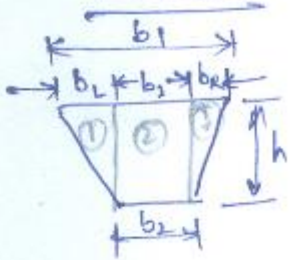
Trapezoid





Area:

Method 1



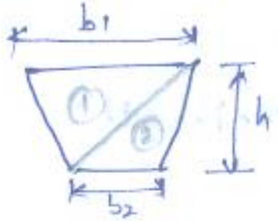
$$A = A_1 + A_2 + A_3 \quad \& \quad b_1 = b_L + b_2 + b_R$$

$$= \frac{1}{2} h b_L + h b_2 + \frac{1}{2} h . b_R$$

$$= \frac{1}{2} h \left(b_L + 2b_2 + b_R \right)$$

$$= \frac{1}{2} h \left(b_1 + b_2 \right)$$

Method 2



$$A = A_1 + A_2$$

$$= \frac{1}{2} b_1 h + \frac{1}{2} b_2 h = \frac{h}{2}(b_1 + b_2)$$

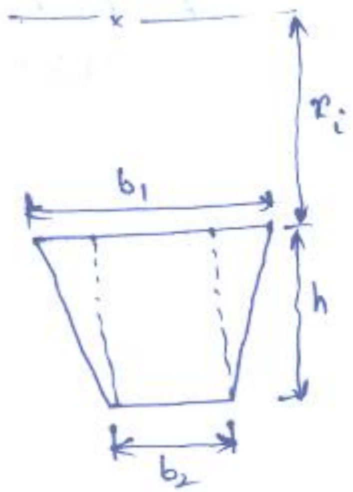
$$dA = x . dr + b_2 . dr$$

$$dA = \frac{b_1 - b_2}{h} (r_o - r) dr + b_2 dr$$

$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} = \frac{\frac{1}{2} h (b_1 + b_2)}{\int_{r_i}^{r_o} \left(\frac{b_1 - b_2}{h} \right) \left(\frac{r_o - r}{r} \right) dr + \int_{r_i}^{r_o} b_2 . \frac{dr}{r}}$$

$$R = \frac{\frac{1}{2} h (b_1 + b_2)}{\left(\frac{b_1 - b_2}{h} \right) \left[r_o \ln (r_o / r_i) - (r_o - r_i) \right] + b_2 \ln (r_o / r_i)} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_o - b_2 r_i) \ln (r_o / r_i) - h (b_1 - b_2)}$$

Location of Neutral Axis in Curved Beams



$$\bar{r} = r_i + \frac{\sum A_i y_i}{\sum A_i}$$

$$= r_i + \frac{\left\{\frac{1}{2}(b_1 - b_2)h\right\} \times \frac{h}{3} + \left\{b_2 \cdot h\right\} \times \frac{h}{2}}{\frac{1}{2}(b_1 - b_2)h + b_2 h}$$

$$= r_i + \frac{\frac{1}{6}h\left[(b_1 - b_2)h + 3b_2 h\right]}{\frac{1}{2}h(b_1 + b_2)}$$

$$= r_i + \frac{h(b_1 + 2b_2)}{3(b_1 + b_2)}$$

$\bar{r}$ = radius of Centroidal Axis

Method#1

$$\bar{r} = \frac{\int r\,dA}{A} = \frac{1}{A}\int r\,dA$$

Now $\quad dA = \left(\frac{b_1-b_2}{h}\right)(r_o-r)\,dr + b_2\,dr$

$$A = \frac{1}{2}h(b_1+b_2)$$

$$\int_{r_i}^{r_o} r\,dA = \int_{r_i}^{r_o} r\left[\left(\frac{b_1-b_2}{h}\right)(r_o-r)\right]dr + \int_{r_i}^{r_o} b_2 \cdot r\,dr$$

$$= \left(\frac{b_1-b_2}{h}\right)\left[r_o\int_{r_i}^{r_o} r\,dr - \int_{r_i}^{r_o} r^2\,dr\right] + b_2\int_{r_i}^{r_o} r\,dr$$

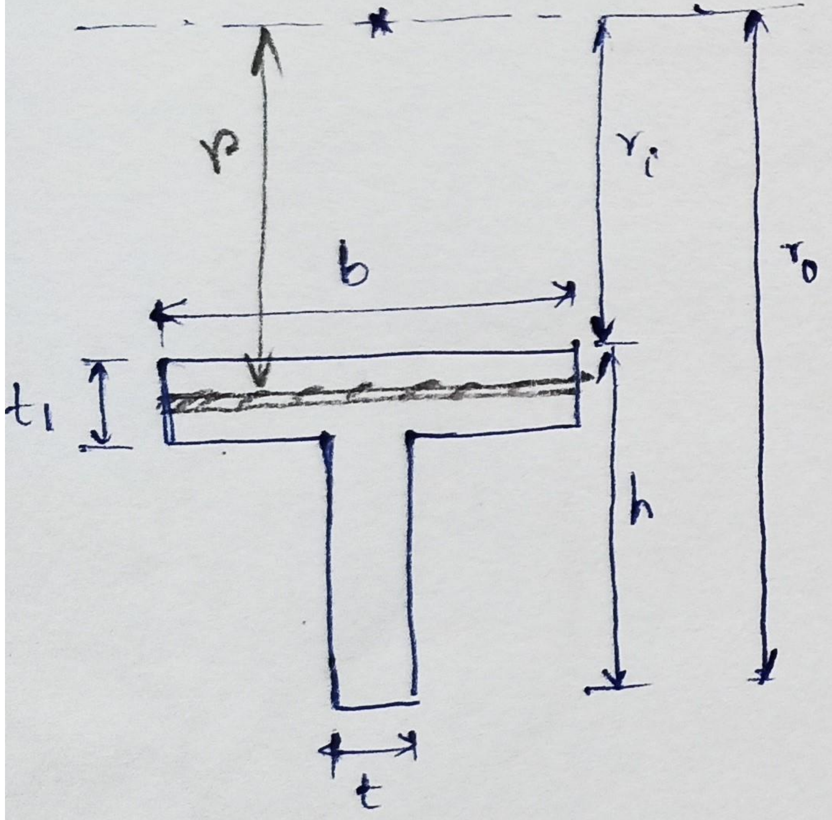
$$= \left(\frac{b_1-b_2}{h}\right)\left[\frac{r_o}{2}(r_o^2-r_i^2) - \frac{1}{3}(r_o^3-r_i^3)\right] + \frac{b_2}{2}(r_o^2-r_i^2)$$

$$\bar{r} = \frac{1}{A}\int_{r_i}^{r_o} r\,dA$$

$$= \frac{1}{\frac{1}{2}h(b_1+b_2)} \cdot \left[\left(\frac{b_1-b_2}{h}\right)\left\{\frac{r_o}{2}(r_o^2-r_i^2) - \frac{1}{3}(r_o^3-r_i^3)\right\} + \frac{b_2}{2}(r_o^2-r_i^2)\right]$$

Location of Neutral Axis in Curved Beams

T- Section



$$\text{Area} \quad A = b.t_1 + (h-t_1).t$$

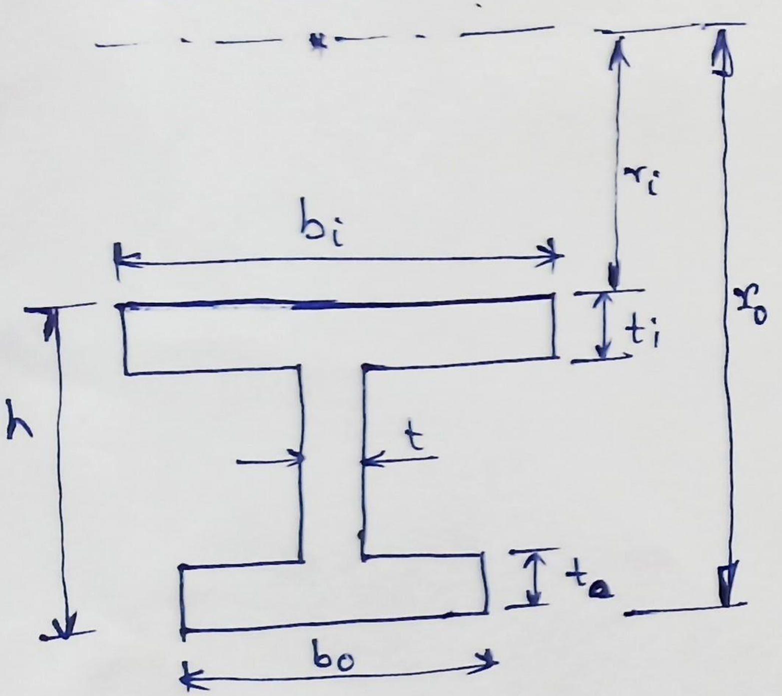
$$\int \frac{dA}{r} = \int_{r_i}^{r_i+t_1} b\frac{dr}{r} + \int_{r_i+t_1}^{r_o} t.\frac{dr}{r}$$

$$= b\ln\left(\frac{r_i+t_1}{r_i}\right) + t\ln\left(\frac{r_o}{r_i+t_1}\right)$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{bt_1 + (h-t_1)t}{b.\ln\left(\frac{r_i+t_1}{r_i}\right) + t.\ln\left(\frac{r_o}{r_i+t_1}\right)}$$

Location of Neutral Axis in Curved Beams

I- Section



Area $A = b_i t_i + (h - t_i - t_o).t + b_o t_o$

$$\int \frac{dA}{r} = \int_{r_i}^{r_i+t_i} b_i \frac{dr}{r} + \int_{r_i+t_i}^{r_o-t_o} t \frac{dr}{r} + \int_{r_o-t_o}^{r_o} b_o \frac{dr}{r}$$

$$= b_i \ln\left(\frac{r_i+t_i}{r_i}\right) + t \ln\left(\frac{r_o-t_o}{r_i+t_i}\right) + b_o . \ln\left(\frac{r_o}{r_o-t_o}\right)$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{b_i t_i + (h - t_i - t_o) t + b_o t_o}{b_i \ln\left(\frac{r_i+t_i}{r_i}\right) + t . \ln\left(\frac{r_o-t_o}{r_i+t_i}\right) + b_o \ln\left(\frac{r_o}{r_o-t_o}\right)}$$

SUMMARY

$$R = \frac{A}{\int \frac{dA}{r}}$$

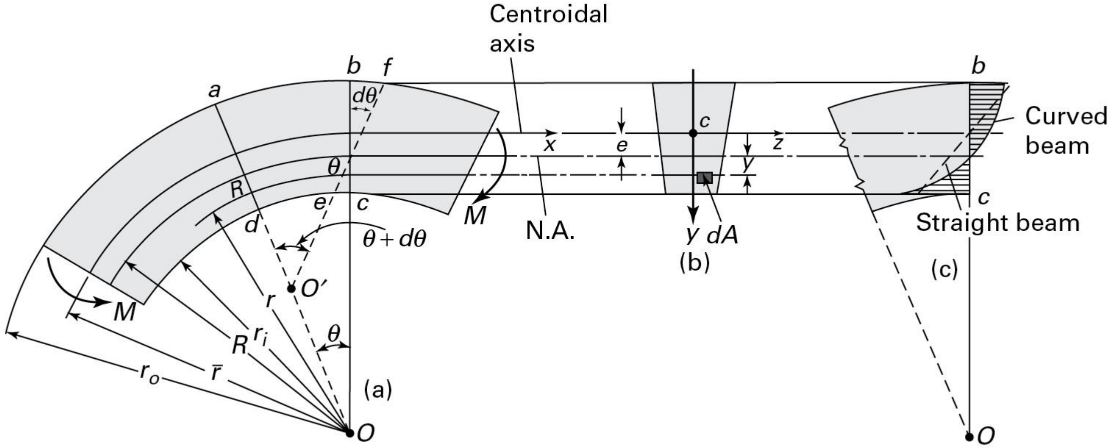
⇒ Dist° 'R' from the centre of curvature 'C' to the neutral surface.

Distance of the centroid of the cross-section from centre of curvature 'c'

$$\bar{r} = \frac{\int r\,dA}{A}$$

$$\sigma_x = -\frac{M \cdot y}{Ae(R-y)} \quad \text{------ (15)}$$

$$\sigma_b = \sigma_x = \frac{M(r-R)}{Aer} \quad \text{as} \quad r = R-y \quad \text{--- (16)}$$



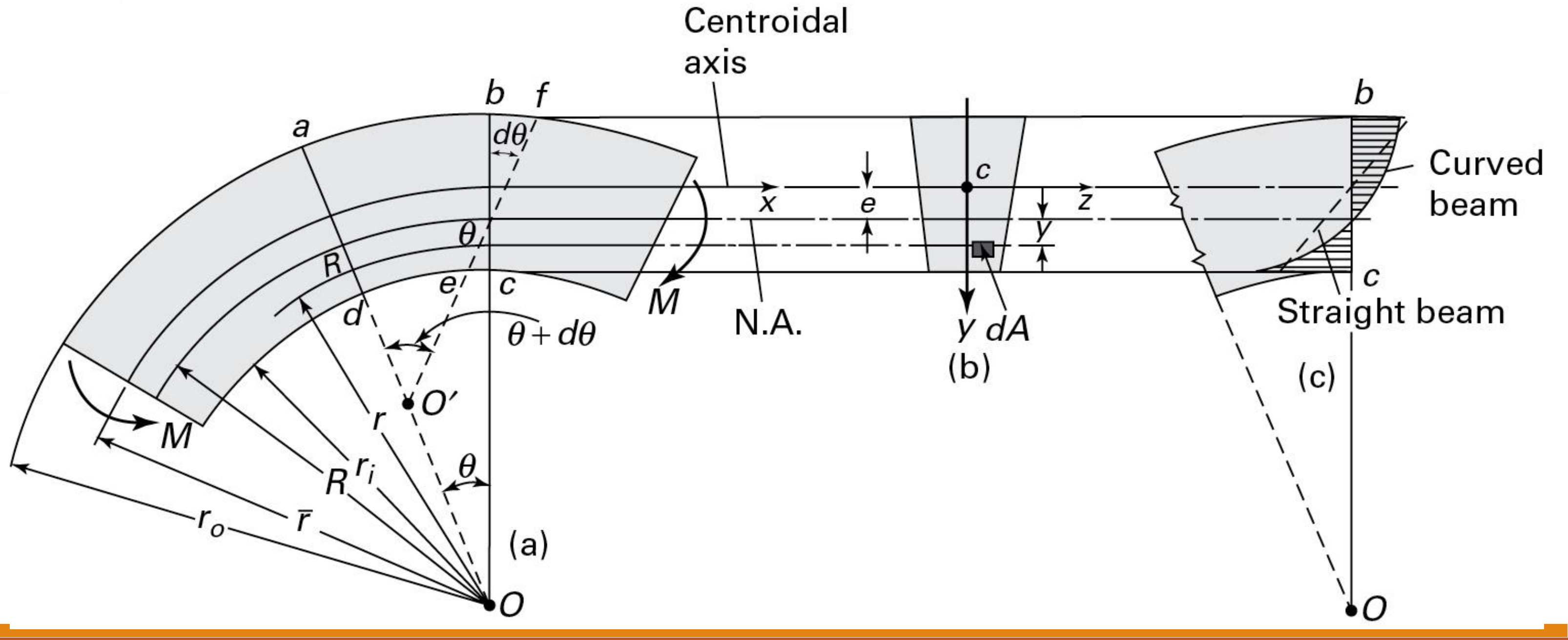
SUMMARY

$$\sigma_b = \sigma_x = \frac{M(r-R)}{Aer} \quad \text{so} \quad r = R-y \;\cdots\; (16)$$

$$\sigma_{bi} = \frac{M(r_i - R)}{Aer_i} \;\cdots\; \text{Inner fibre}$$

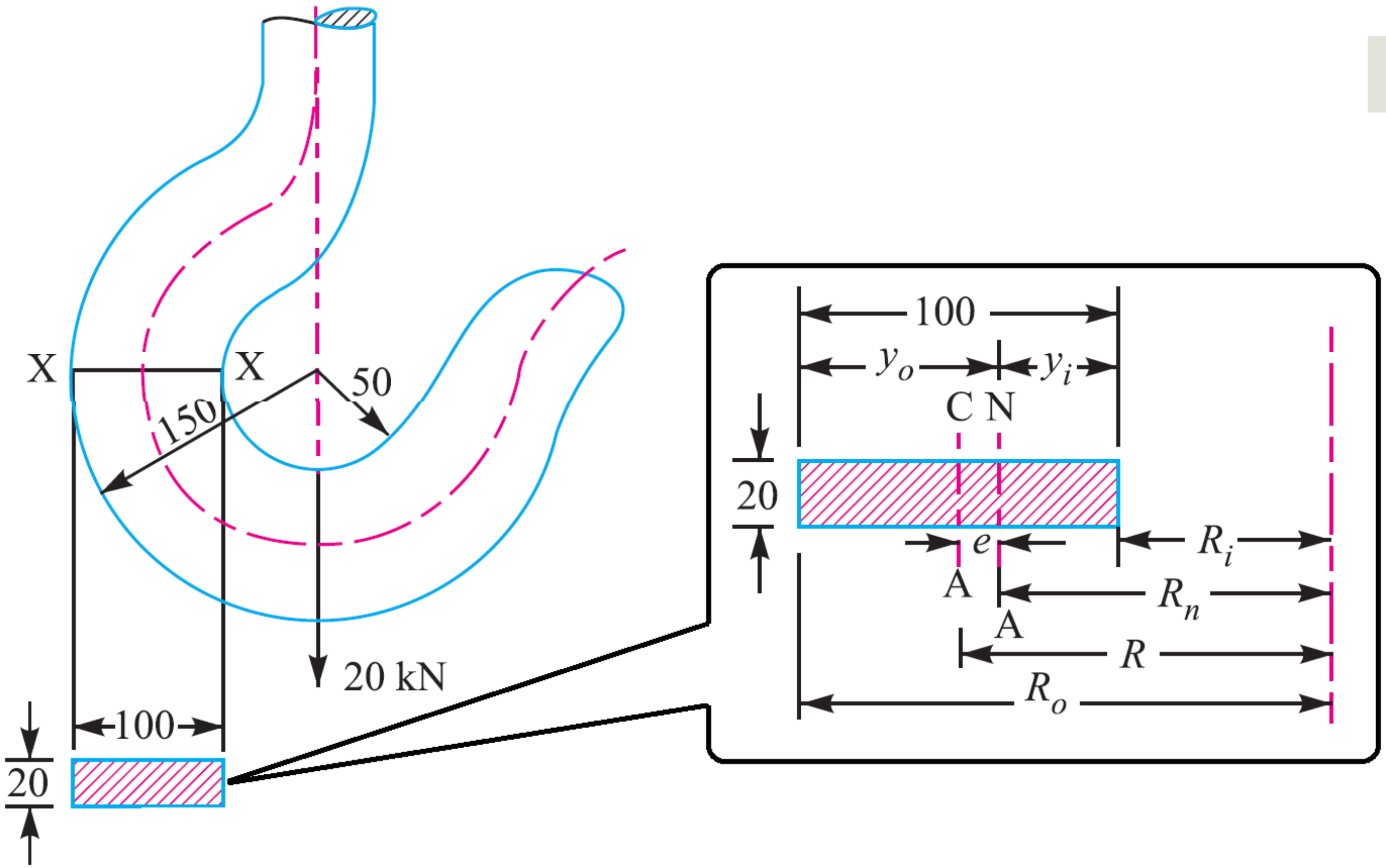
$$\sigma_{bo} = \frac{M(r_o - R)}{Aer_o} \;\cdots\; \text{outer fibre}$$

$$\sigma_{bi} > \sigma_{bo}$$



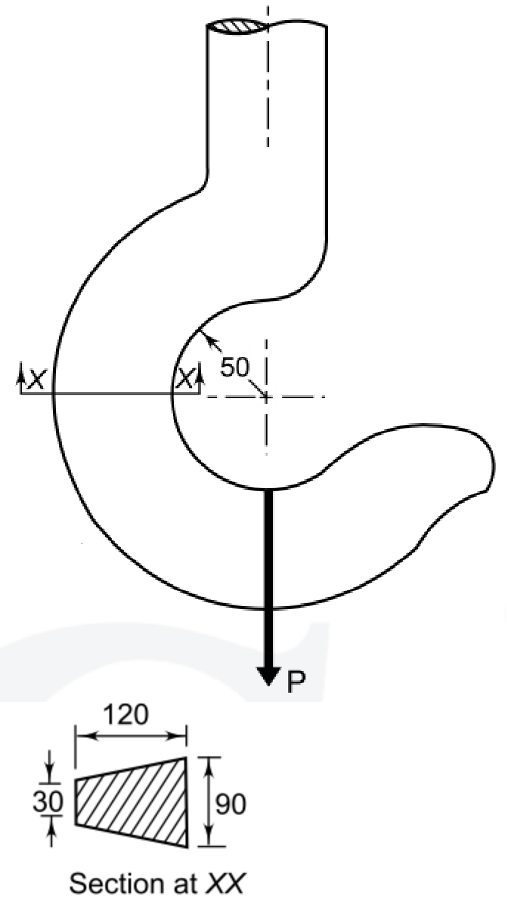
Design for Static Loading

Design of Crane hooks and C-frames



Problem Statement:

A crane hook having an approximate trapezoidal cross-section carrying load of 95 KN as shown in Fig. It is made of plain carbon steel 45C8 & assume FoS is 3.5. Find out the stress distribution and maximum stress in the cross-section at XX.



Main Steps in Design Analysis of Crane Hook

Step 1 - Check Given data & materials

Step 2 - Calculation of A, R & $\bar{r}$

Step 3 - Check the loading pattern (Direction, line of action & magnitude)

Step 4 - Identify the stresses induced due to above loading pattern

Step 5 - Estimation of load induced stresses

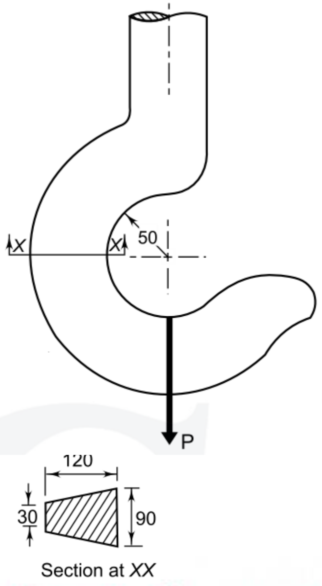
- **Step 5.1** - Estimation of stress due to bending (i.e., bending stress)
- **Step 5.2** - Estimation of direct tensile stress
- **Step 5.3** - Determine the maximum stress due to combined effect.

Step 6 - Calculation of allowable stresses, Apply failure criteria & estimate dimensions of cross-section

Step 1 Given data & Materials

$$b_i = 90 \text{ mm} \quad b_o = 30 \text{ mm} \quad h = 120 \text{ mm}$$

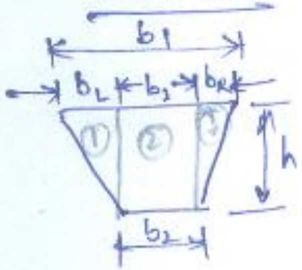
$$r_o = 170 \text{ mm} \quad r_i = 50 \text{ mm}$$



Step 2 - Calculation of A, R & r

Calculation of A

Area: Method I



$A = A_1 + A_2 + A_3$ & $b_1 = b_L + b_2 + b_R$

$= \frac{1}{2} h b_L + h b_2 + \frac{1}{2} h . b_R$

$= \frac{1}{2} h (b_L + 2b_2 + b_R)$

$= \frac{1}{2} h (b_1 + b_2) = \frac{1}{2}[h(b_i + b_o)] = \frac{1}{2}[(120)(90+30)]$

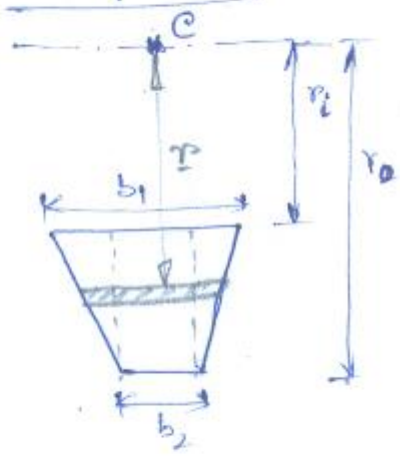
$= 7200 \text{ mm}^2$

Calculation of R

$$\boxed{R = \frac{A}{\int \frac{dA}{r}}}$$

⇒ Dist^o 'R' from the Centre of Curvature 'C' to the neutral Surface.

Trapezoid

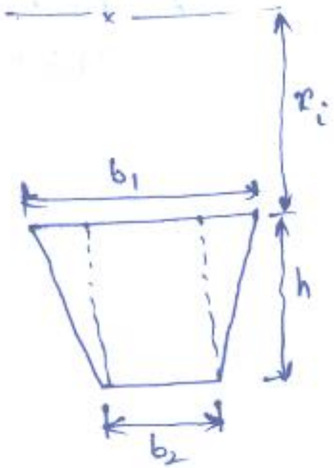


Calculation of R

$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} = \frac{\frac{1}{2}h(b_1+b_2)}{\int_{r_i}^{r_o}\left(\frac{b_1-b_2}{h}\right)\left(\frac{r_o-r}{r}\right)dr + \int_{r_i}^{r_o} b_2 \cdot \frac{dr}{r}}$$

$$R = \frac{\frac{1}{2}h(b_1+b_2)}{\left(\frac{b_1-b_2}{h}\right)\left[r_o \ln(r_o/r_i) - (r_o-r_i)\right] + b_2 \ln(r_o/r_i)} = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1 r_o - b_2 r_i)\ln(r_o/r_i) - h(b_1-b_2)} = 89.1816\text{ mm}$$

Calculation of $\bar{r}$



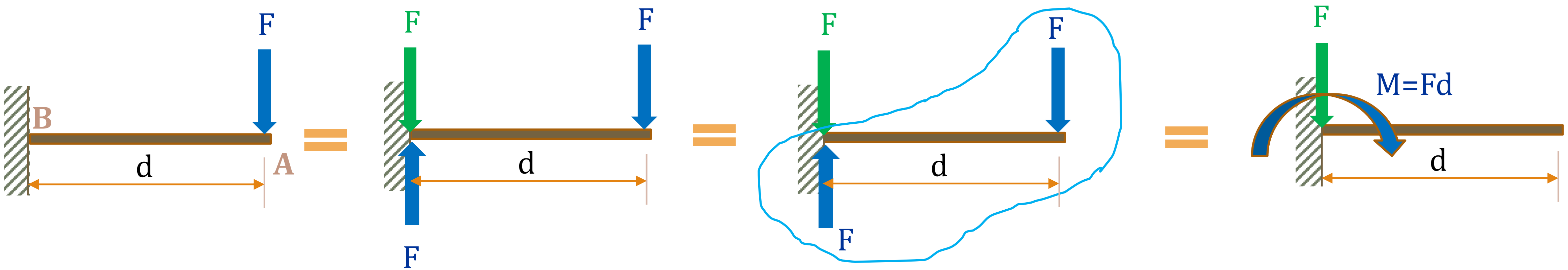
$$\bar{r} = r_i + \frac{\sum A_i y_i}{\sum A_i}$$

$$= r_i + \frac{\left\{\frac{1}{2}(b_1-b_2)h\right\} \times \frac{h}{3} + \left\{b_2 \cdot h\right\} \times \frac{h}{2}}{\frac{1}{2}(b_1-b_2)h + b_2 h}$$

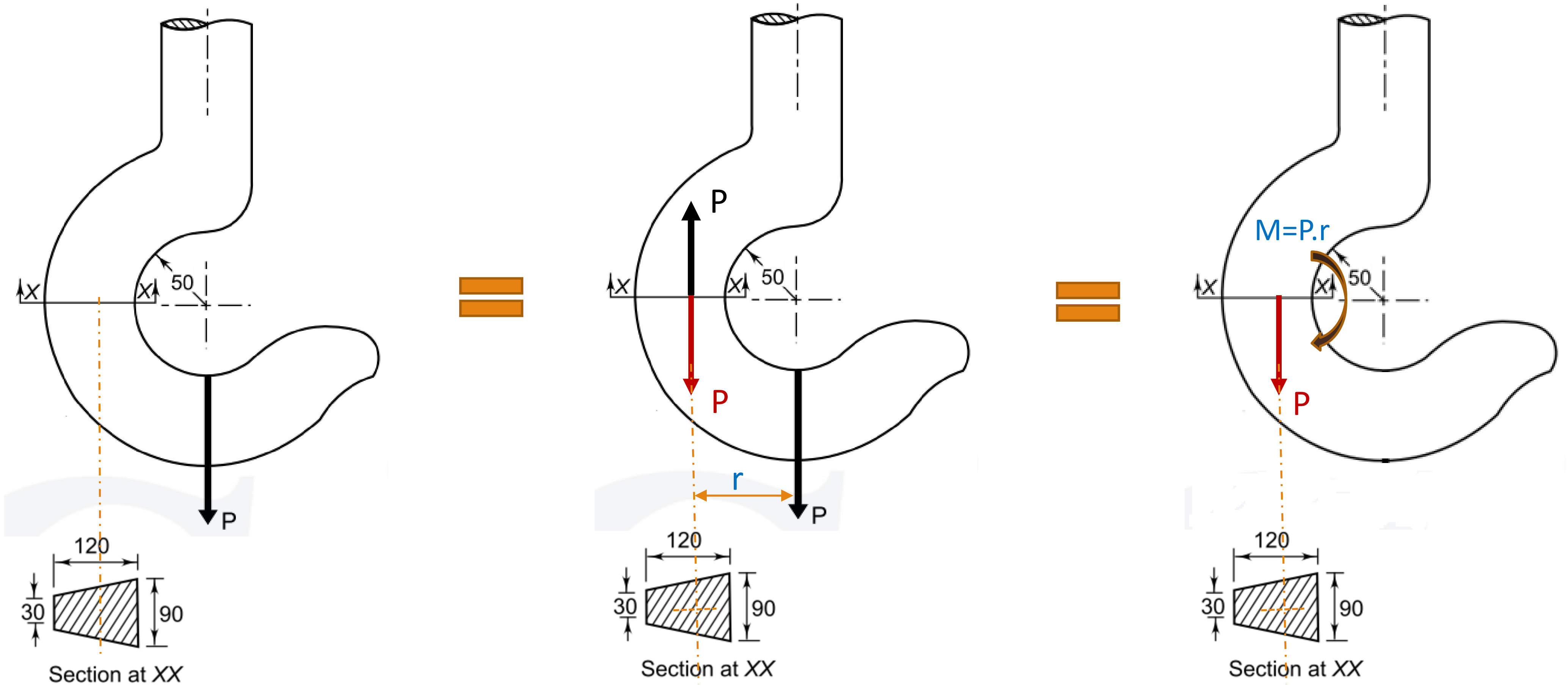
$$= r_i + \frac{\frac{1}{6}h\left[(b_1-b_2)h + 3b_2 h\right]}{\frac{1}{2}h(b_1+b_2)}$$

$$= r_i + \frac{h(b_1+2b_2)}{3(b_1+b_2)} = 50 + \frac{120(90+2\times 30)}{3(90+30)} = 100\text{ mm}$$

Force-Couple Systems

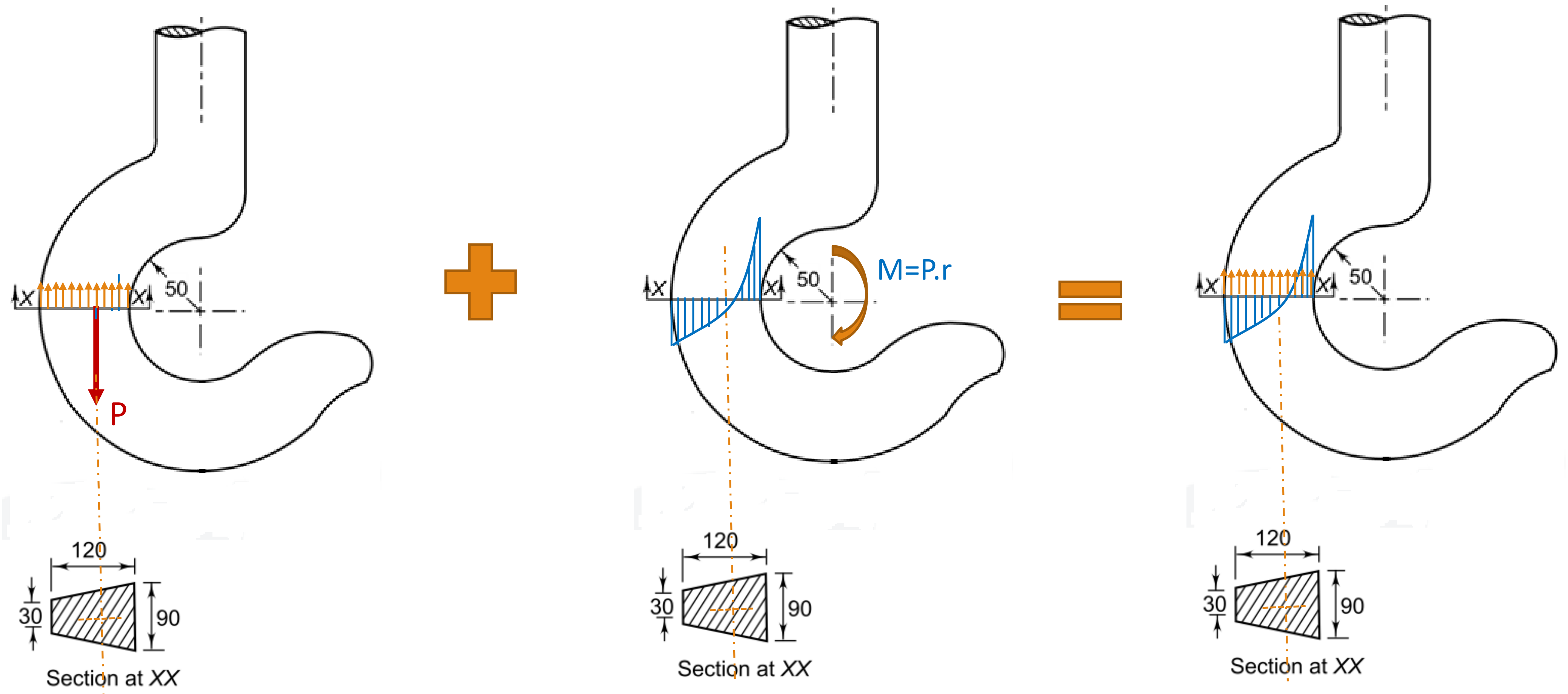


Step 3 - Check the loading pattern (Direction, line of action & magnitude)



Step 4 - Identify the stresses induced due to above loading pattern

Step 4 - Identify the stresses induced due to above loading pattern



Eccentricity betn Centroidal & Neutral Axis

$$e = \bar{r} - R = (100 - 89.2)\ \text{mm} = 10.8\ \text{mm}$$

$$\sigma_{bi} = \frac{M(r_i - R)}{A e r_i} \quad \text{--- Inner fibre}$$

$$\sigma_{bo} = \frac{M(r_o - R)}{A e r_o} \quad \text{--- outer fibre}$$

$$\sigma_{bi} > \sigma_{bo}$$

$$M_b = PR = (100\ P)\ \text{N-mm}$$

Estimation of stress due to bending (i.e., bending stress)

$$\sigma_{bi} = \frac{M(r_i - R)}{A e r_i} = \frac{(100P)(39.1816)}{(7200)(10.8184)(50)}$$

Moment of the force 'P' about Centroidal axis

$$M_b = P.\bar{r} = 95\times10^3 \times 100 = 95\times10^5\ \text{N-mm}$$

Tensile Stress due to bending moment (M):

$$\sigma_t = \frac{M.y}{A.e.(R-y)}$$

$S_{yt} = 380 \text{ N/mm}^2 \quad (fs) = 3.5$

$$\sigma_{max.} = \frac{S_{yt}}{(fs)} = \frac{380}{3.5} = 108.57 \text{ N/mm}^2$$

Stress due to bending at the inner fibre

$$\sigma_{bi} = \frac{M(R - r_i)}{A(\bar{r} - R)\, r_i}$$

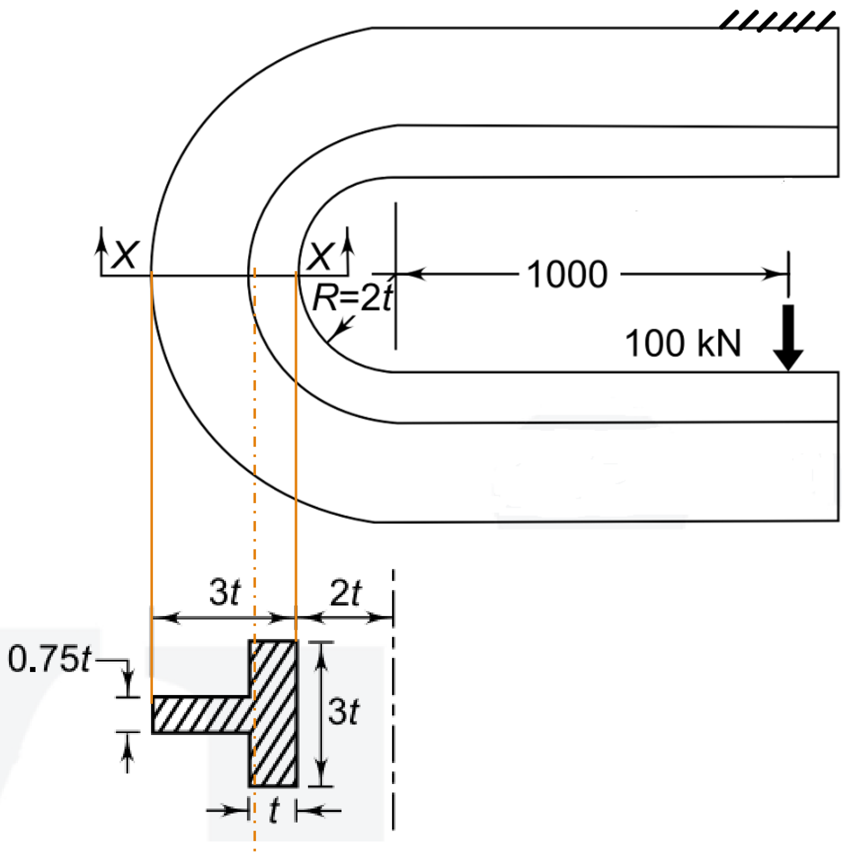
$$= \frac{95 \times 10^{3} (89.2 - 50)}{7200 \times 10.8 \times 50} \text{ N/mm}^2 = 95.578 \text{ N/mm}^2$$

Tensile Stress due to P :

$$\sigma_t = \frac{P}{A} = \frac{95 \times 10^3}{7200} = 13.2 \text{ N/mm}^2$$

Problem Statement:

The C-frame having T-section of a 100 kN capacity press is shown in Fig.. The material of the frame is grey Cast Iron FG 200 & the FoS is 3. Determine the dimensions of the frame.



Main Steps in Design Analysis of C-frame

Step 1 - Check Given data & materials

Step 2 - Calculation of A, R & $\bar{r}$

Step 3 - Check the loading pattern (Direction, line of action & magnitude)

Step 4 - Identify the stresses induced due to above loading pattern

Step 5 - Estimation of stresses

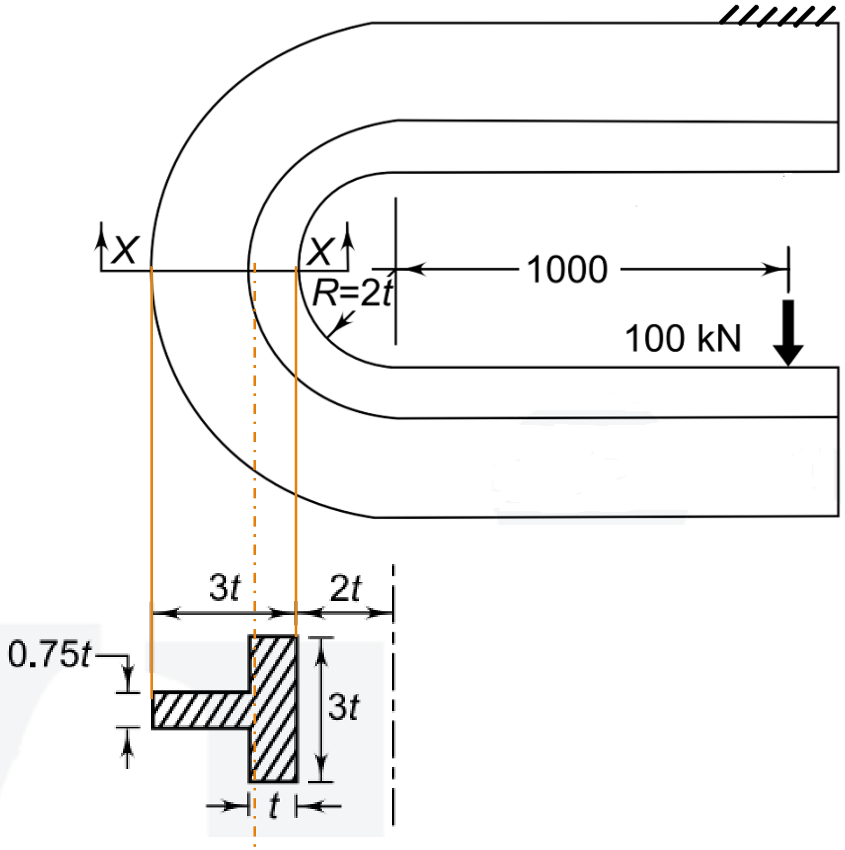
- **Step 5.1** - Estimation of stress due to bending (i.e., bending stress)
- **Step 5.2** - Estimation of direct tensile stress
- **Step 5.3** - Determine the combined effect.

Step 6 - Calculation of allowable stresses, Apply failure criteria & estimate dimensions of cross-section

Step 1 Given data & Materials

Given Data: $P = 100\,kN = 100 \times 10^3\,N$

$r_i = 2t$, $r_o = 3t + 2t = 5t$



Step 2 - Calculation of A, R & r

Calculation of A

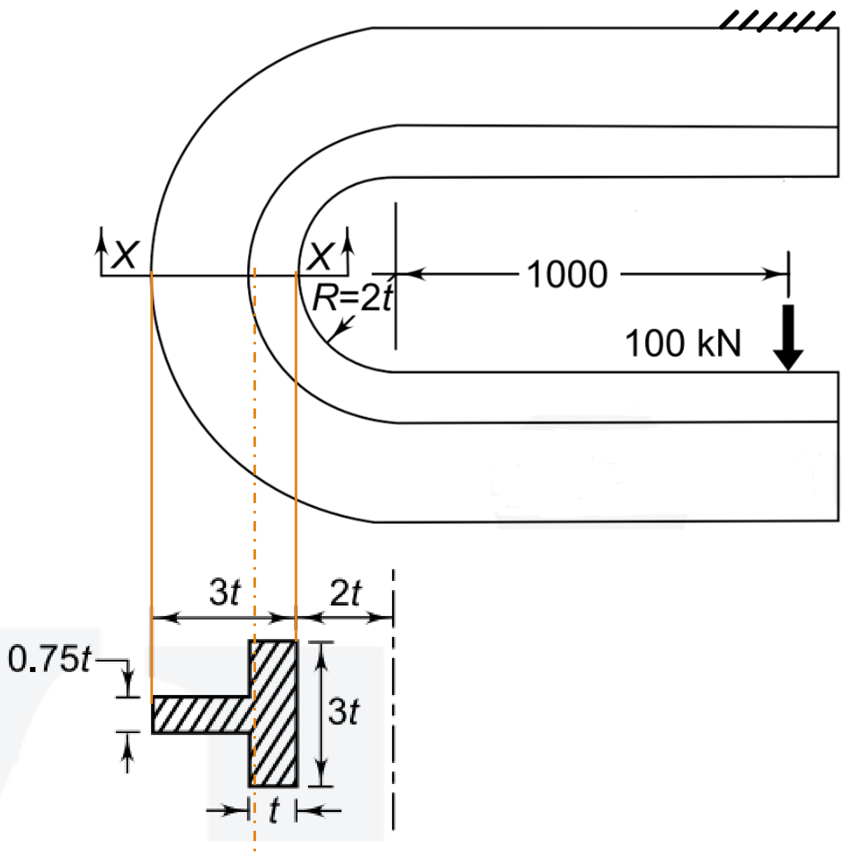
$$\text{Area } A = 3t \cdot t + (3t - t) \times 0.75t = 4.5t^2$$

Calculation of $\bar{r}$

$$\bar{r} = r_i + \frac{3t \cdot t \cdot t/2 + 2t \times 0.75t\,(t + 2t/2)}{3t \cdot t + 2t \times 0.75t}$$

$$= 2t + \frac{1.5t^3 + 3t^3}{4.5t^2} = 3t$$

Step 2 - Calculation of A, R & r



Calculation of R

$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}}$$

$$\int_{r_i}^{r_o} \frac{dA}{r} = \int_{r_i}^{r_i+t} 3t \cdot \frac{dr}{r} + \int_{r_i+t}^{r_o} 0.75t \frac{dr}{r} = 3t\int_{2t}^{3t} \frac{dr}{r} + 0.75t \int_{3t}^{5t} \frac{dr}{r}$$

$$= 3t \ln\left(\frac{3t}{2t}\right) + 0.75t \ln\left(\frac{5t}{3t}\right) = 3t \cdot \ln(3/2) + 0.75t \ln(5$$

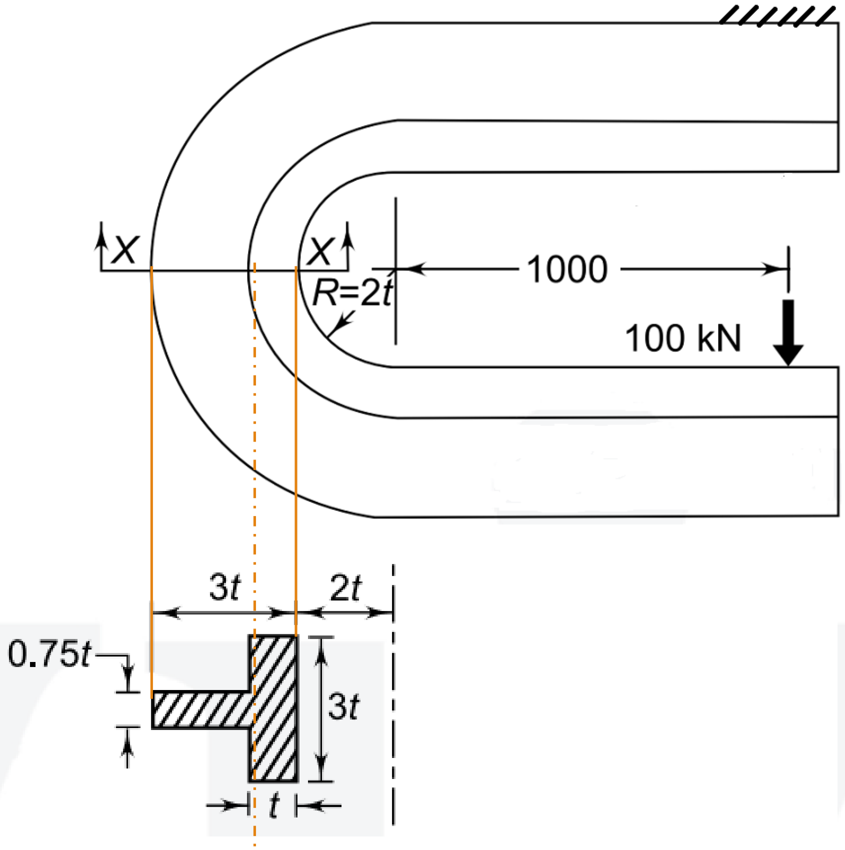
$$= 3 \times 0.405t + 0.75 \times .5108$$

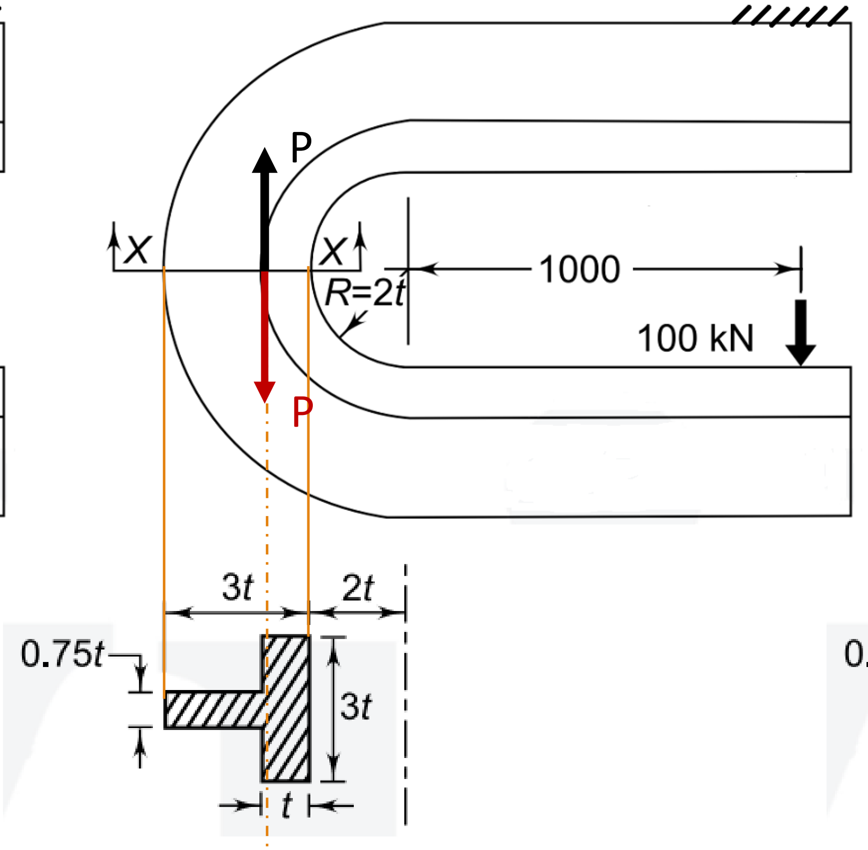
$$= 1.6t$$

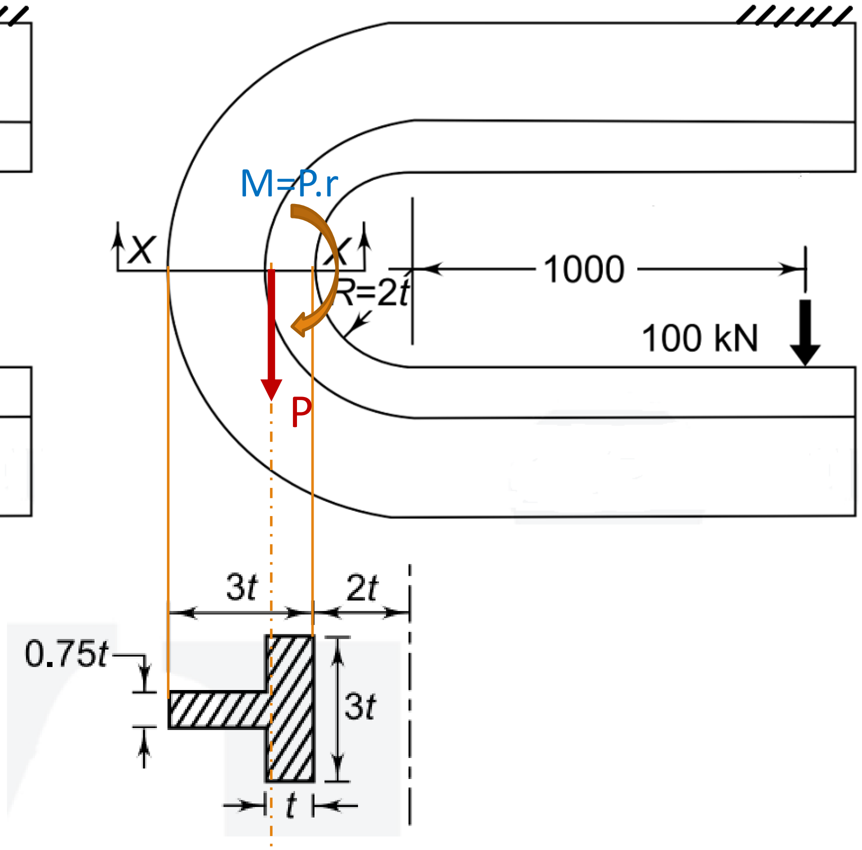
$$R = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} = \frac{4.5t^2}{1.6t} = 2.8125t$$

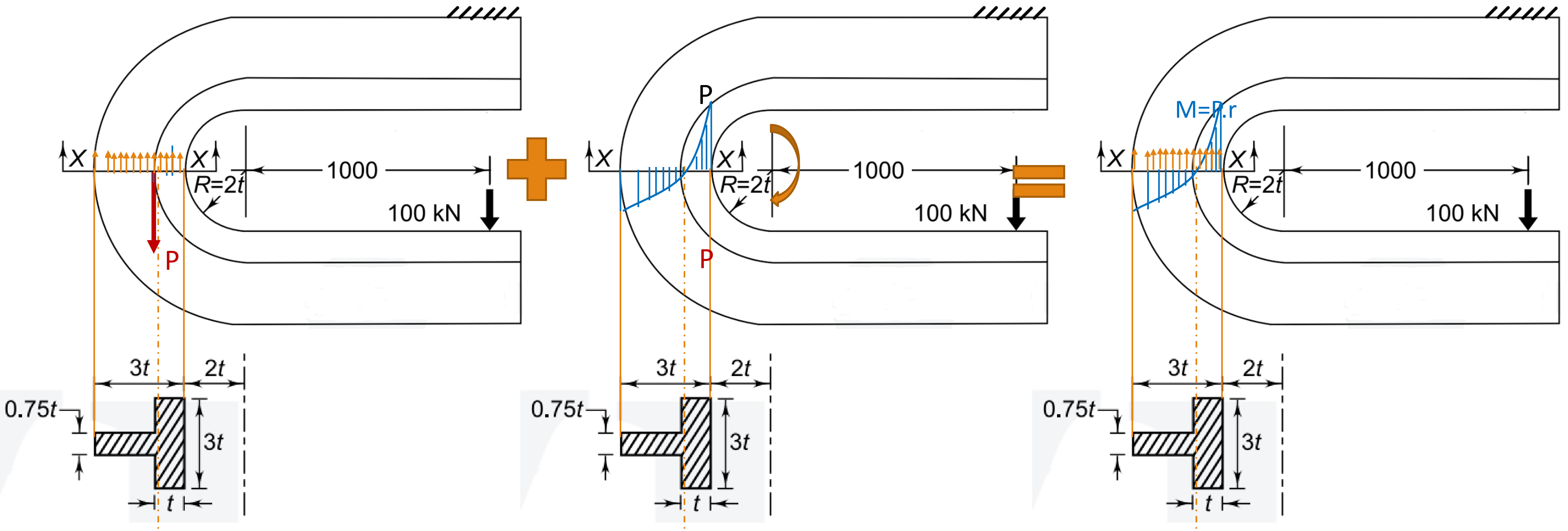
$$\therefore e = \bar{r} - R = 3t - 2.8125t = 0.1875t$$

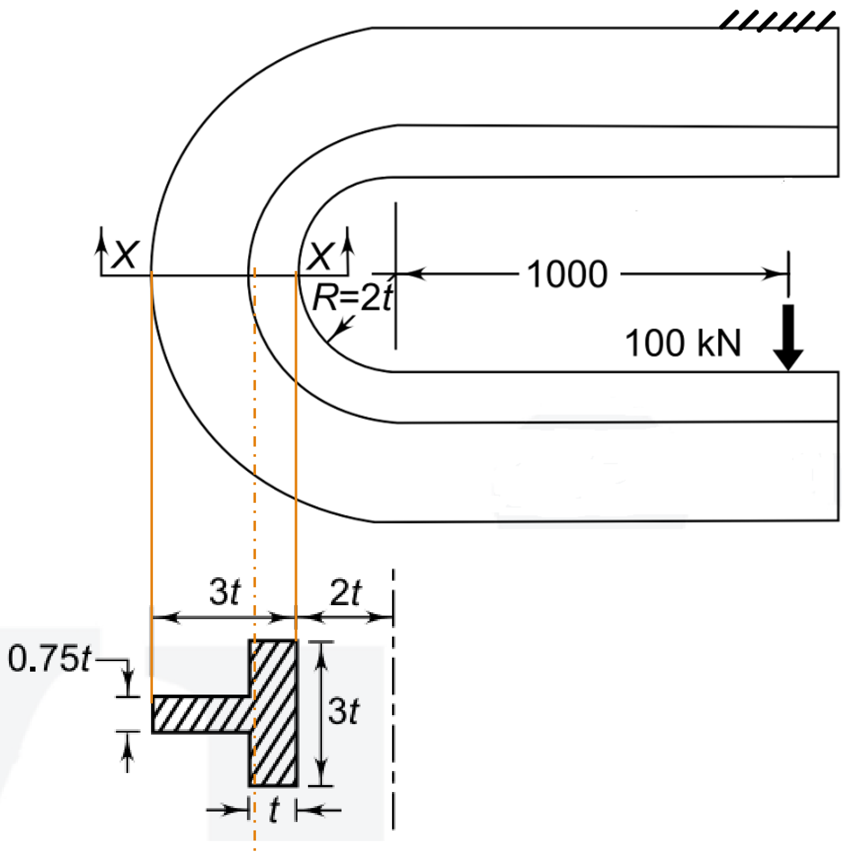
Force–Couple Systems








X
X
R=2t
1000
100 kN
P
3t
2t
0.75t
3t
t
P
M=P.r



Moment of the force 'P' about centroidal axis

$$M = P \times (\bar{r} + 1000)$$

$$= 100 \times 10^3 (3t + 1000) \quad \text{(clockwise)} \quad (-)\text{ve}$$

Bending stress

$$\sigma_{bi} = \frac{M(r_i - R)}{A e r_i}$$

$$= -\frac{100 \times 10^3 (3t + 1000)(2t - 2.8125t)}{4.5t^2 \times 0.1875t \times 2t} \ \text{N/mm}^2$$

Direct Tensile stress due to P

$$\sigma_t = \frac{P}{A} = \frac{100 \times 10^3}{A} = \frac{100 \times 10^3}{4.5t^2} \ N/mm^2$$

Max^m stress at inner fibre

$$\sigma_{max} = \sigma_{bi} + \sigma_t$$

$$\sigma_{max} \leq [\sigma]$$

$$[\sigma] = \frac{S_{ut}}{FOS} = \frac{200}{3} = 66.67 \ N/mm^2$$

$$t^3 - 2512.8\,t - 72650 = 0$$

$t = 99.2$ mm

$t = 100$ mm

Thank You