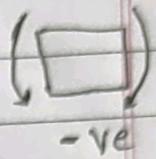


Bending stress/moment (M)

convention -

unit - NM

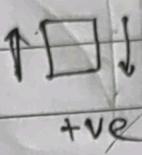
+ve



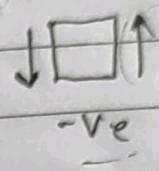
-ve

Transverse shear stress (V)  
(Force)

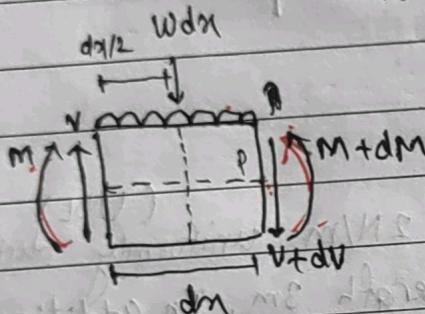
unit - N



+ve



-ve



$$\sum F = 0$$

$$\sum M = 0$$

$$W = \frac{-dV}{dx}$$

$$V = wdx + v + dv$$

$$\therefore W = -dv/dx$$

$\sum M = 0$  ... calculating at point P

$$-M dx - V dx + wdx \cdot \frac{dx}{2} + M + dm = 0$$

$$-V dx + \left( -\frac{dV}{dx} \right) \frac{dx}{2} dm + dm = 0$$

$$-V dx = \frac{dV}{dx} dm + dm$$

$$-V dx + dm = 0$$

IMP

$$V = dM/dx$$

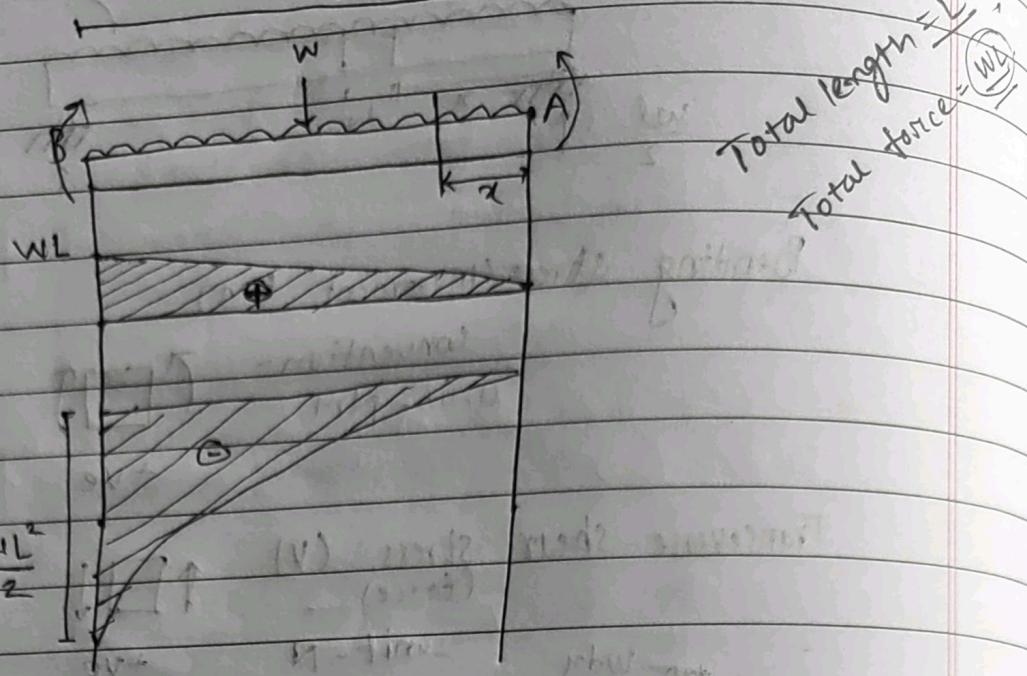
whenever shear force is 0, bending moment has maximum value : or minimum

$$W = -\frac{d}{dx}(V) = -\frac{d^2 M}{dx^2}$$

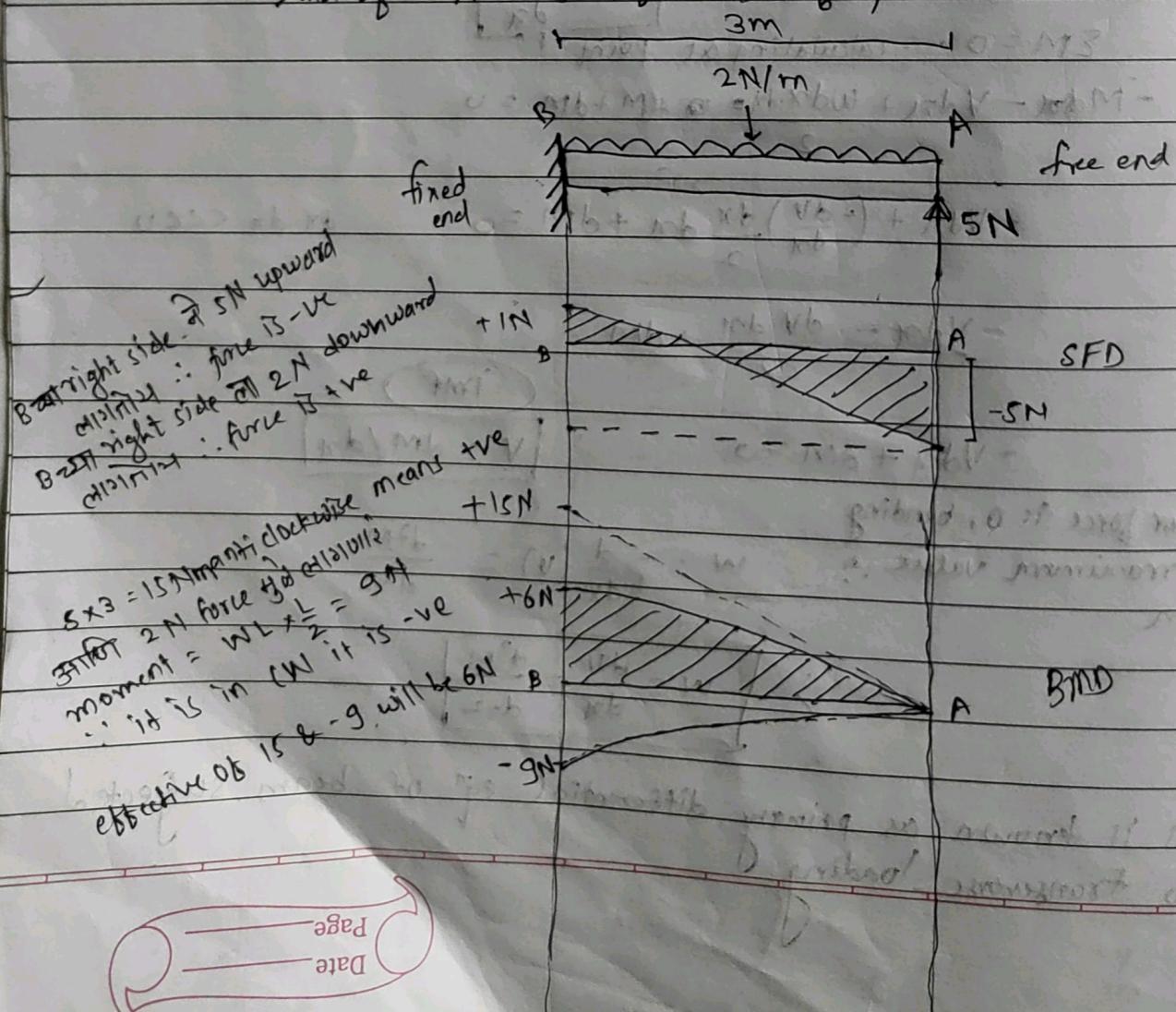
$$\therefore -W = \frac{dV}{dm} = \frac{d^2 M}{dx^2}$$

This is known as primary differential eqn of beam subjected to transverse loading.

# \* Shear force & bending moment diagram

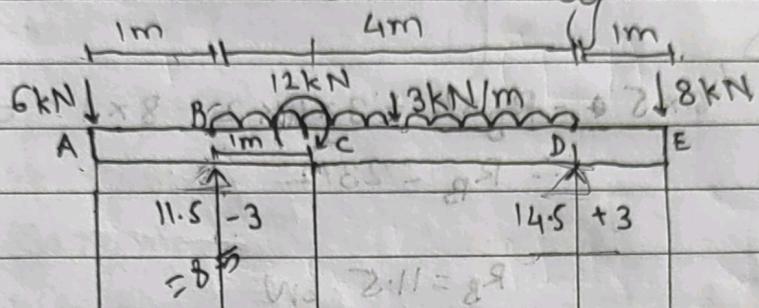


- i) A cantilever beam suppose  $2 \text{ N/m}$  uniformly distributed load throughout the span of length  $3\text{m}$ . In addition, A concentrated load of  $5\text{N}$  acts at free end of upward. Draw S.F. & B.M. diag.



2) An overhung simple supported beam of length 6m is supported by 1m from each end. The middle 4m is loaded with uniformly distributed load of 3 kN/m. At the left hand end, 6 kN & right hand end, 8 kN is also supported by beam. Additionally one positive couple 12 kNm is acting at a point 2m from left hand end. Draw BM & SF diag for beam.

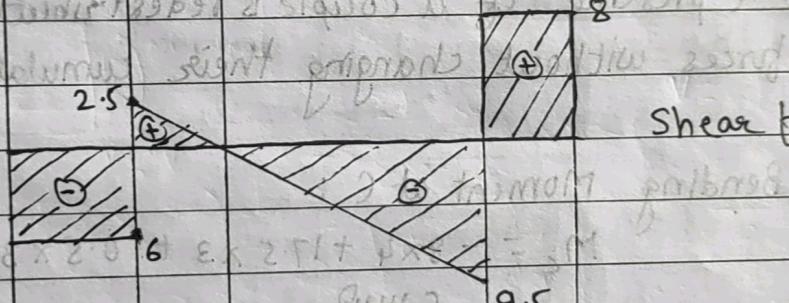
positive couple,  
it's in CW dir.



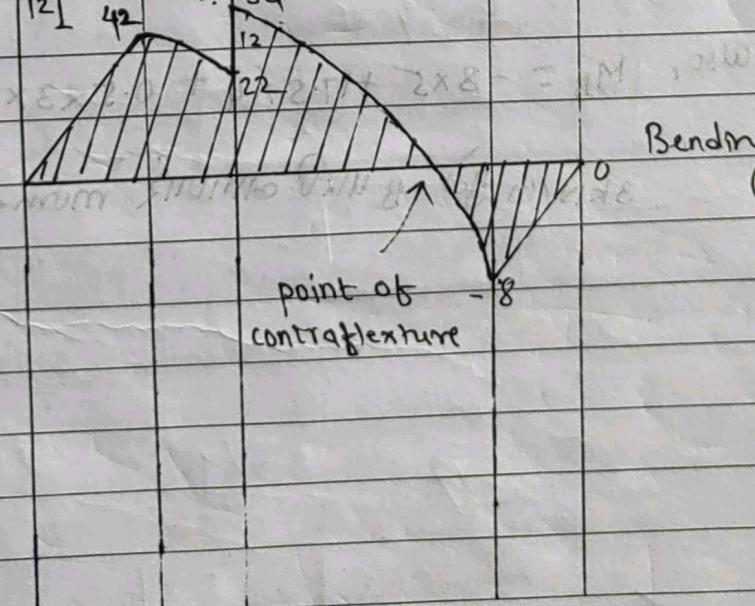
$$11.5 - 3 = 8.5$$

$$14.5 + 3 = 17.5$$

$$W = 2 \cdot 11 = 8.9$$



Shear force Diagram



Bending Moment Diagram

$R_B$  &  $R_D$  are to be calculated (without considering couple) at B,

$$6 \times 1 - 3 \times 4 \times 2 + R_D \times 4 - 8 \times 5 = 0$$

$$\therefore 4R_D = 58$$

$$R_D = 14.5 \text{ kN}$$

at D,

$$6 \times 5 - R_B \times 4 + 3 \times 4 \times 2 - 8 \times 1 = 0$$

$$\therefore R_B = 23/2$$

$$R_B = 11.5 \text{ kN}$$

The presence of a couple is redistributive - the supportive forces without changing their cumulative value.

Bending Moment at C :

$$M_C = -8 \times 4 + 17.5 \times 3 + 0.5 \times 3 \times 3^2 = 34$$

$$3 \text{ kN/m } \overset{\text{C 412}}{\text{Hab}} \text{ moment} = WL \times \frac{L}{2} = 3 \times 3 \times \frac{3}{2} = 0.5 \times 3 \times 3^2$$

$$\text{Also, } M_B = -8 \times 5 + 17.5 \times 4 + 0.5 \times 3 \times 4 \times 4 = 54$$

$$3 \text{ kN/m } \overset{\text{B 412}}{\text{Hab}} \text{ moment} = 3 \times 4 \times \frac{4}{2} = 0.5 \times 3 \times 4^2$$

## Bending Stress in Beams:

- ① Pure bending (No shear force is acting)

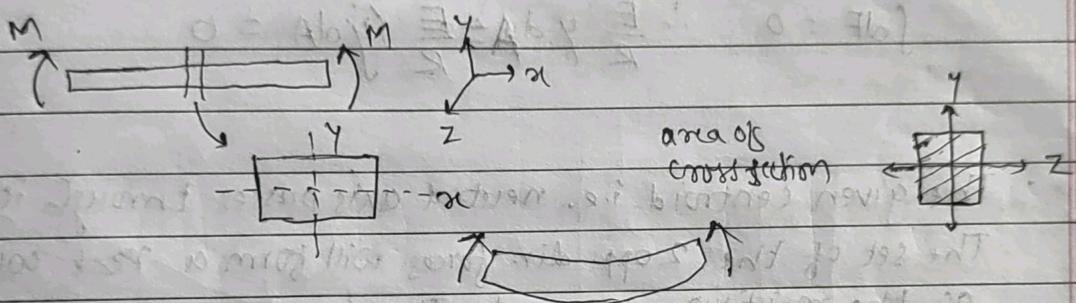


$$M = \text{constant}$$

$$\text{shear force } \frac{dM}{dm} = 0$$

- ② Modulus of elasticity ( $E$ )

Same in tension & compression

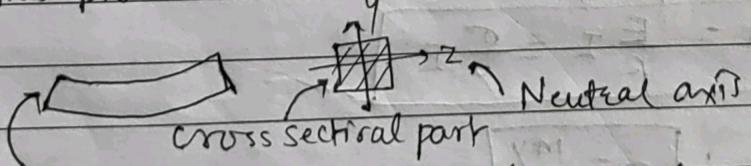


- ③ The plane section remains plane after bending

- ④ The sect<sup>n</sup> under consider<sup>n</sup> is axisymmetric

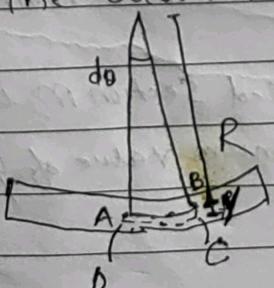
- ⑤ After bending there is a plane which remains unstrained.  
This plane is known as neutral plane

(no change in length)



Cross sectional plane is called neutral plane

- ⑥ The curvature is circular in nature



$$CD \quad AB = (R+y) d\theta$$

$$AB = R d\theta$$

$$\Delta y = y d\theta$$

As there is no axial force acting on the surface, net summation of the force on the cross section is zero.

$$\text{Strain} = \frac{\Delta y}{R d\theta} = \frac{y d\theta}{R d\theta} = \frac{y}{R}$$

$$\text{Stress} = \sigma = \frac{y}{R} \cdot E = \frac{E}{R} \cdot y$$

Force on element

$$dF = \sigma dA = \frac{E}{R} y \cdot dA$$

$$\int dF = 0 \quad \therefore \frac{E}{R} y dA = \frac{E}{R} \int y dA = 0$$

The given centroid i.e. neutral axis passes through it.

The set of the 2 opp. dir. forces will form a rest couple known as the resistivity moment. That is equal to external moment for equilibrium.

$$M = \int y \cdot dF = \int y \sigma dA = \int y \frac{E}{R} y dA$$

$$= \frac{E}{R} \int y^2 dA$$

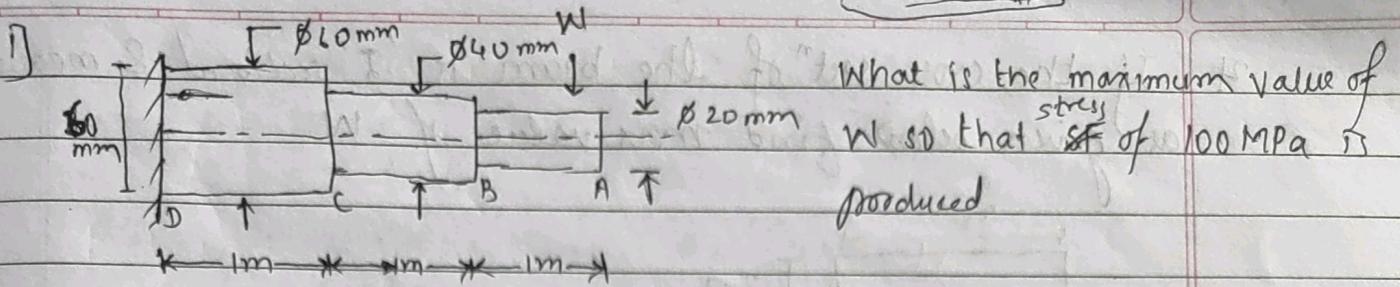
$$M = \frac{E}{R} I = \frac{\sigma}{y} I$$

$$\therefore r = \frac{My}{I}$$

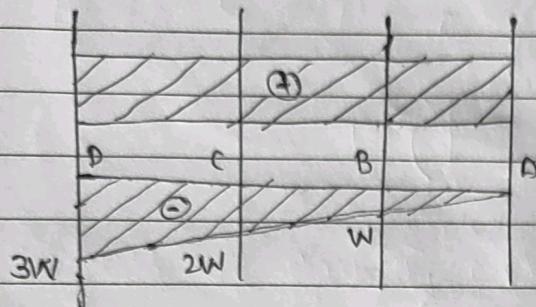
$I$  is second moment of inertia re. moment of area.

$$\sigma = \frac{M}{I/y} \quad I/y \text{ is called Section Modulus at maximum value of } y$$

$$\text{Modulus of rigidity} = G = EI$$



What is the maximum value of  $W$  so that stress of  $100 \text{ MPa}$  is produced



$$I_{AB} = \frac{\pi D^4}{64} = \frac{\pi \times 160000}{64} = \pi \times 2500 = 7857 \text{ mm}^4 = 0.7857 \times 10^4 \text{ mm}^4$$

$$I_{BC} = \frac{\pi D^4}{64} = 125663 \text{ mm}^4 = 12.56 \times 10^4 \text{ mm}^4$$

$$I_{CD} = \frac{\pi D^4}{64} = 63.6172 \times 10^4 \text{ mm}^4$$

$$\text{at } D, \sigma = \frac{My}{I}$$

$$\therefore 100 \times 10^6 = 63.6172 \times 10^4 \times 30$$

$$M = 21.20 \times 10^{11}$$

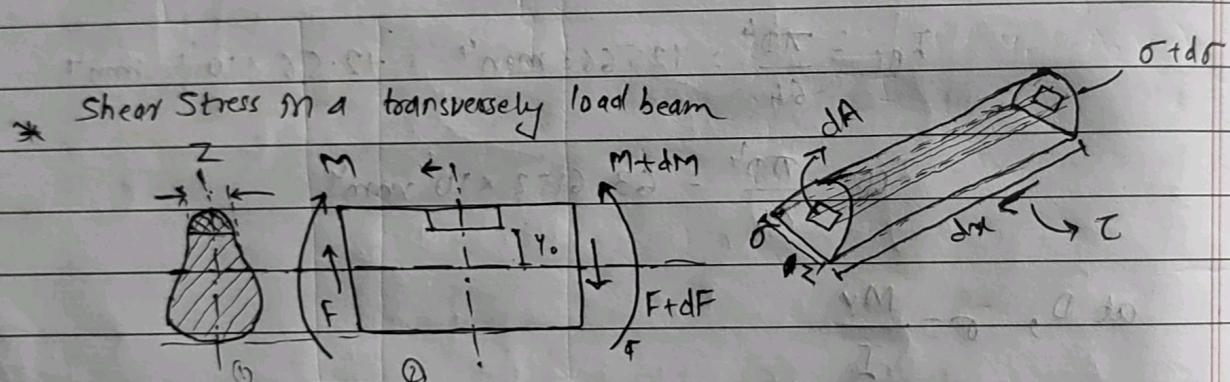
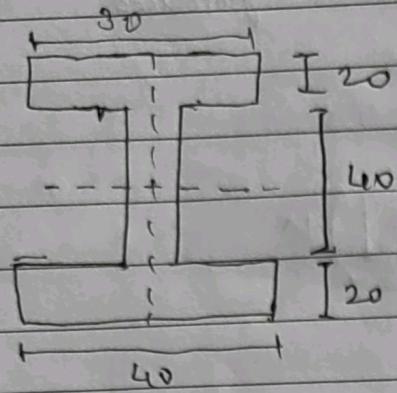
$$\text{at } C, \sigma = \frac{My}{I} \therefore 100 \times 10^6 = \frac{M \times 20}{12.56 \times 10^4}$$

$$M = 6.28 \times 10^{11}$$

$$\text{at } B, \sigma = \frac{My}{I} \quad 100 \times 10^6 = \frac{M \times 10}{0.7857 \times 10^4}$$

$$M = 0.7857 \times 10^{11}$$

Q) If the cross sect<sup>n</sup> of the beam is 'I' sectioned & measurement as given below find max value of  $W$



(1) is cross sect<sup>n</sup> of (2)

The small area  $dA$  on the elemental piece is subjected to normal stress due to bending moment  $M$

Net force acting on element =  $d\sigma \cdot dA$

$$= \frac{dM \cdot y}{I} \cdot dA$$

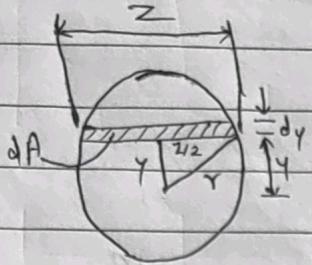
The counter active force is the resistive force restricting the movement & is the shear force.

$$T \cdot z \cdot dx = \frac{dM \cdot y}{I} \cdot dA$$

$$\therefore T = \left( \frac{dM}{dx} \right) \cdot \left( \frac{1}{z I} \right) \cdot (y \cdot dA)$$

$$= \frac{F_y}{z I} dA$$

when cross sect<sup>n</sup> is circular



$$r^2 = y^2 + \frac{z^2}{4}$$

$$z = \frac{F}{ZI} \bar{y}A$$

$$y^2 = r^2 - \frac{z^2}{4} \quad \therefore 2y dy = -\frac{1}{4} \cdot 2z dz$$

$$\therefore y dy = -\frac{z dz}{4}$$

$$dA = z dy$$

$$\bar{y} \cdot A = \int y dA$$

$$= \int y \cdot z dy =$$

$$(F_A + \bar{F}_A) \pi = \int$$

We've to convert it to z limits

as y goes from y to r, z goes from z to 0

$$\bar{y} \cdot A = \int_z^0 -\frac{z^2}{4} dz = \frac{1}{4} \int_0^z z^2 dz = \frac{1}{12} z^3$$

$$\bar{y} A = z^3 / 12$$

$$\therefore z = \frac{F \cdot \bar{y} A}{ZI} = \frac{F \cdot z^3 / 12}{ZI} = \frac{F z^2}{12 I}$$

The moment of inertia of cross sect<sup>n</sup> =  $I = \pi r^4 / 4$

$$I = \frac{F z^2}{12 \pi r^4} = \frac{F z^2}{3 \pi r^4}$$

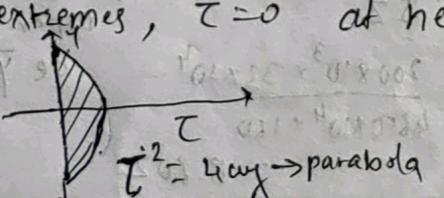
$$I = \frac{F z^2}{3 \pi r^4}$$

This expression is desirable in terms of  $y$   
to get variation of  $I$

$$\therefore I = \frac{F z^2}{3 \pi r^4} = \frac{4 F}{3 \pi} \left( \frac{y^2 - z^2}{r^4} \right)$$

extremes means  $y = \pm r$  neutral axis means  $y = 0$

At extremes,  $I = 0$  at neutral axis  $I$  is maximum



$$I^2 = 4ay \rightarrow \text{parabola}$$

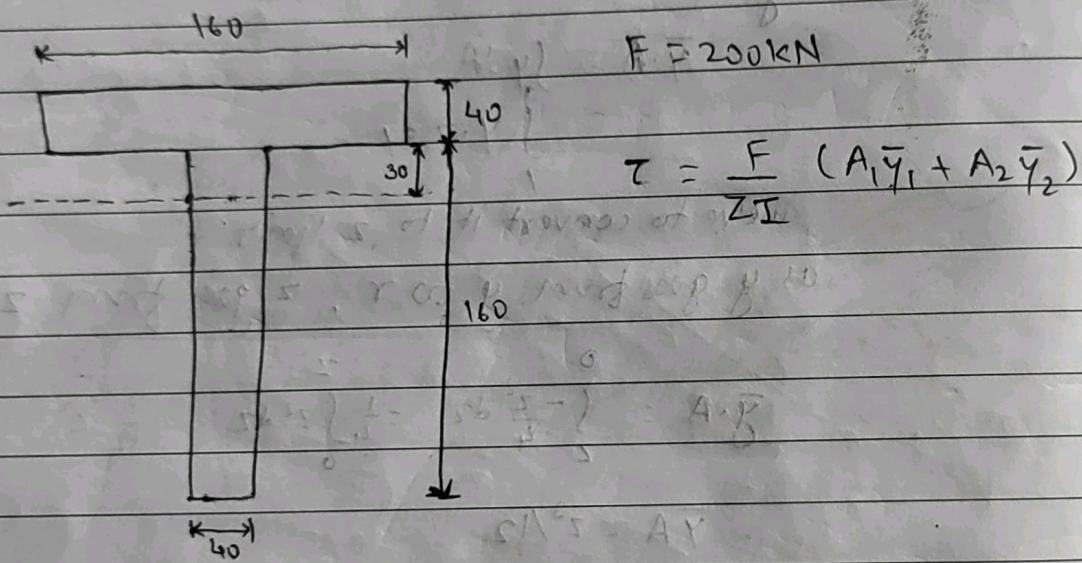
$$I_{max} = \frac{4F}{3\pi r^2}$$

$$\tau_{avg} = \frac{F_{avg}}{\pi r^2}$$

$$\frac{I_{max}}{\tau_{avg}} = \frac{4}{3}$$

The same ratio for rectangular cross sectn,

the ratio  $\frac{I_{max}}{\tau_{avg}} = \frac{3}{2}$



Locat'n of neutral plane is ( $y_0$ ) from top

$$y_0 = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{160 \times 40 \times 20 + 160 \times 40 \times 120}{2 \times 160 \times 40} = 70$$

$\bar{y}_1$  is locat'n of  $y_1$  from  $y_0$  ie, neutral plane

$$\therefore \bar{y}_1 = 50$$

$$\bar{y}_2 = 50$$

$$A_1 \bar{y}_1 = 6400 \times 50 = 320000 \text{ mm}^3$$

$$A_2 \bar{y}_2 = 6400 \times 50 = 320000 \text{ mm}^3$$

$$I = I_1 + I_2 = 1685 \times 10^4 + 2965 \times 10^4 = 4650 \times 10^4 \text{ mm}^4$$

$$\tau = \frac{F (A \bar{y})}{I Z} \quad \because \text{flange is in contact with only upper part } A \text{ will be } A_1, \bar{y} \text{ will be } \bar{y}_1$$

$$= \frac{200 \times 10^3 \times 32 \times 10^3}{4650 \times 10^4 \times 160} = 8.602$$

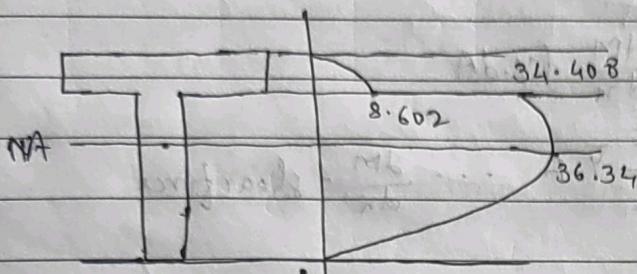
Section just above the web

$$A_b = \frac{160}{40} \times 8.602 = 34.408$$

$$\therefore T \propto \frac{1}{Z}$$

Vertical part  $\approx Z/40$  horizontal part  $\approx 160/34$ .

$\therefore$  Vertical part  $\approx T$  will be 4 times that of hor. part  $Z$

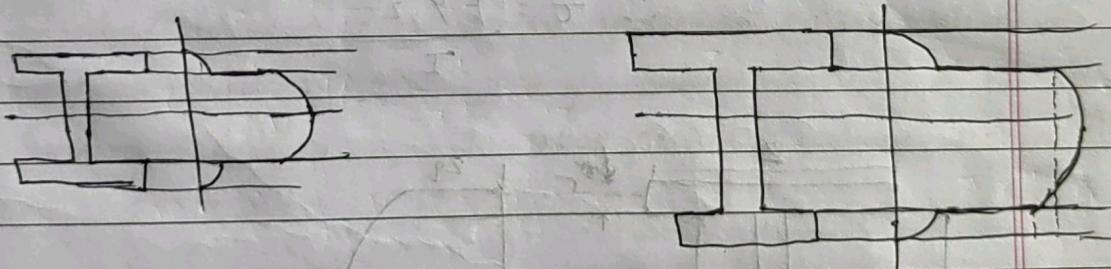


$T$  Through neutral axis, will be calculated as  $T$  above NA -  $T$  below

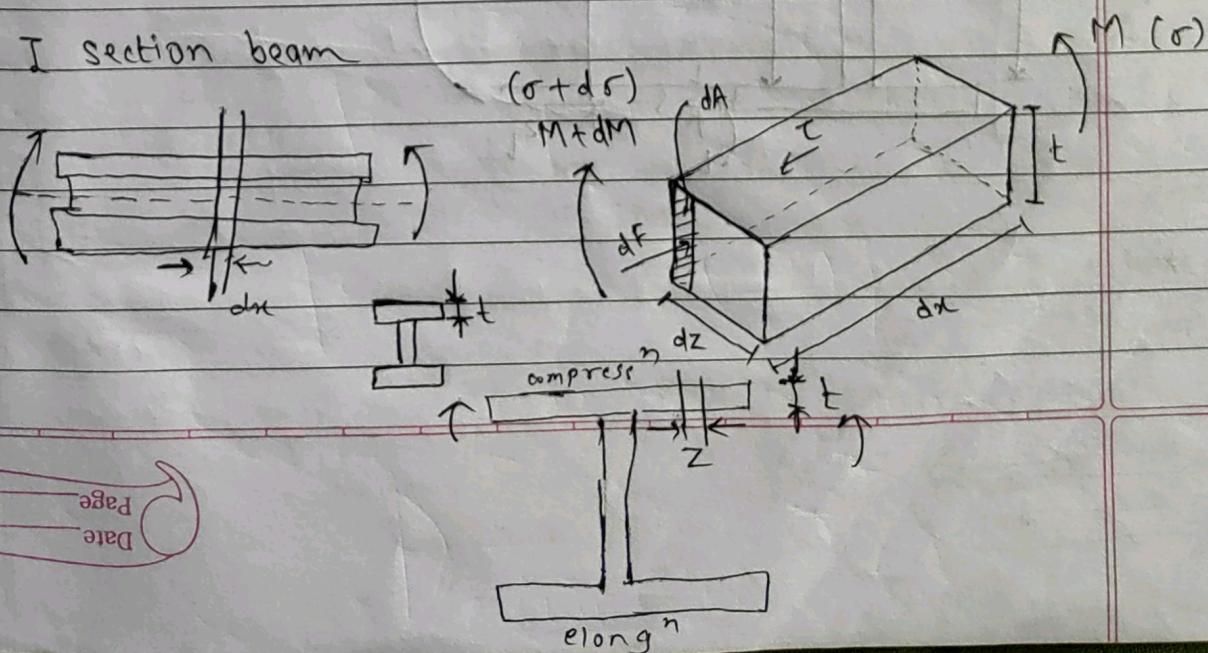
$$NA \text{ or simply } T = \frac{F}{Z} (A_1\bar{y}_1 + A_2\bar{y}_2)$$

however  $A_2\bar{y}_2$  will be in -ve sense here because  
of point being below NA

& thus  $T$  will go on decreasing until it becomes 0



\* I section beam



$$\therefore \text{Net force acting on double shaded area} = d\tau \cdot dA$$

$$dF = \frac{dm \cdot y}{I} \cdot dA$$

$$F = \int \frac{dm y \cdot dA}{I}$$

$\therefore$  syst is in equilibrium, restoring force will be shear force

$$\tau \cdot (tdx) = \int \frac{dm y \cdot dA}{I}$$

$$\therefore q = \tau t = \frac{dm}{I \cdot dy} \int y \cdot dA$$

$$= \frac{F}{I} \bar{y} \cdot A \quad \dots \frac{dm}{dy} = \text{shear force}$$

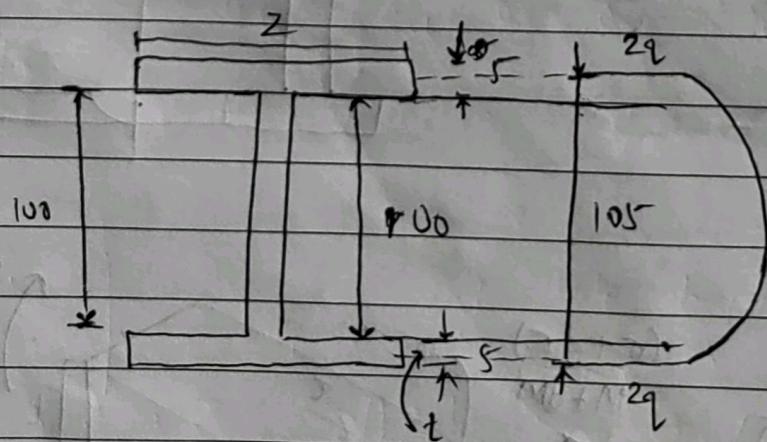
$\tau t$  is known as shear flow which is used for thin section beam analysis.

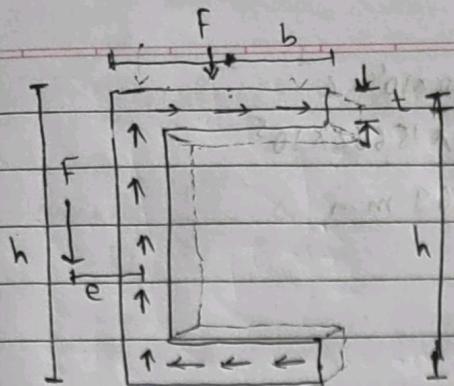
unit of shear flow is  $N/m^2$

$$\text{now, } A = zt$$

$$q = \tau t = \frac{F \bar{y} \cdot zt}{I}$$

$$\therefore \tau = \frac{F \bar{y} \cdot z}{I}$$





back part is girdered

here front part will twist like this



$h \gg t$

$$\text{induced shear force } F = \frac{Ft b^2 h}{4I}$$

$$\text{couple} = \frac{Ft b^2 h}{4I} \cdot \frac{h}{2}$$

let  $F$  be the restoring force applied at a horizontal distance  $e$  as shown

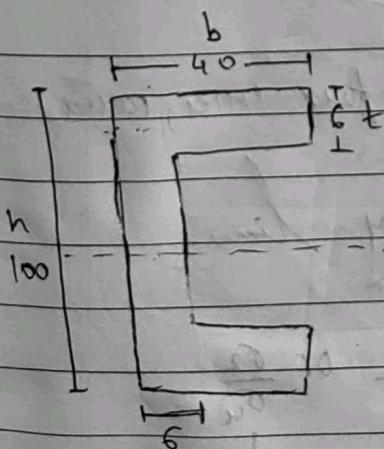
$$\therefore \text{Restoring couple} = Fe$$

for no twisting of the beam,

$$Fe = \frac{Ft b^2 h}{4I} \cdot \frac{h}{2}$$

$$\therefore e = \frac{b^2 h^2 t}{8I}$$

This  $e$  is known as shear centre of cross section. If the (vertical) transverse force is applied through shear centre of the sect', no twisting will occur. The shear centre may be within the body or outside



Moment of Area of top & bottom flange  $= I_1 = I_2$

$$= \frac{bd^3}{12} = \frac{40 \times 6^3}{12} = 720$$

M.O.A through neutral axis  $= MOA + A\bar{y}^2$

$$= 720 + 40 \times 6 \times 47^2 \\ = 5.301 \times 10^5$$

$$MOA \text{ of vertical part} = \frac{bd^3}{12} = \frac{6 \times 100^3}{12} = 5 \times 10^5$$

$$\therefore I_{\text{net}} = I_1 + I_2 + I_3 = (5.301 + 5 + 5.301) \times 10^5 \\ = 15.602 \times 10^5$$

$$e = \frac{b^2 h^2 t}{8 I} = \frac{1600 \times 10^4 \times 6}{8 \times 15.602 \times 10^5}$$

$$e = 7.69 \text{ mm}$$

## \* Theories of failure

failure point : ductile point  $\sigma_y$  (yield stress)

brittle point  $\sigma_u$  (ultimate stress)

They can be estimated for any material by uniaxial tension test by UTM (universal testing machine)

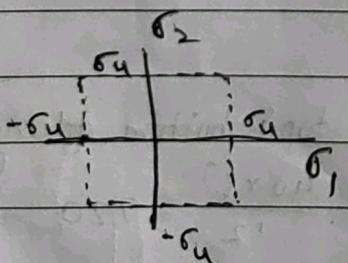
$$\sigma_{12} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + t_{xy}^2}$$

## \* Rankine's theory (maximum normal stress theory)

It states that the brittle material fails when any of the principle stress reaches the ultimate point ( $\sigma_u$ ) (with an assumption, the strain in tension & in compression are same

$$\sigma_1 = \pm \sigma_u \text{ or } \sigma_2 = \pm \sigma_u$$

+ve for tension -ve for compression



any state beyond the four lines causes failure

The lines are called boundary lines.

The normalized form is expressed as  $\frac{\sigma_1}{\sigma_u}$  or  $\frac{\sigma_2}{\sigma_u}$

$\downarrow$  x axis       $\downarrow$  y axis

Reciprocal of above terms is known as factor of safety

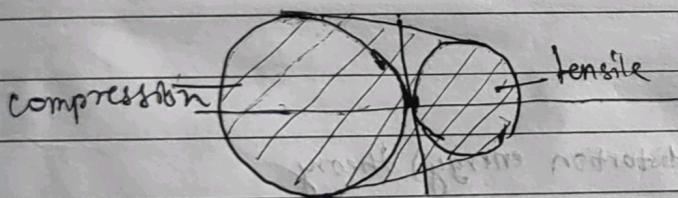
among  $\frac{\sigma_u}{\sigma_1}$  &  $\frac{\sigma_u}{\sigma_2}$  whichever gives minimum value  $\beta$  factor of safety

The brittle material in general, exhibits stronger behaviour in compression

\* Mohr's theory :- c-compressive t-tensile

It states that the material fails under tensile & compressive load according to ( $\sigma_{uc}$  or  $\sigma_{ut}$ ) depending on type of loading when any one of the principle stress crosses the limit.

shaded is safety region



\* Ductile material ( $\sigma_y$ )

Material fails due to shear when induced shear stress reaches the yield point value of the shear stress obtained by simple tension test.

① Maximum shear stress theory (Tresca's criterion)

At principle axis, shear stress is  $\tau_0$  & in its perpendicular the  $T_{yp} = \pm \left( \frac{\sigma_1 - \sigma_2}{2} \right)$

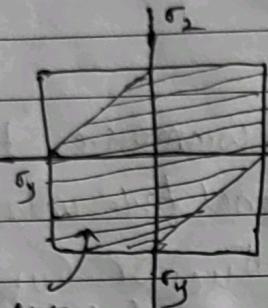
for uniaxial tensile test,  $\tau_0 = \sigma_y$  &  $T_y = \sigma_{yp}/2$

ductile material

The ductile material fails when the maximum shear stress induced exceeds the yield shear stress obtained by simple tension test.

In simple tension test,  $\tau = \frac{\sigma}{2}$

$$\sigma_y = \pm (\sigma_1 - \sigma_2)$$



$$\sigma_y = \sigma_1 - \sigma_2 \quad \text{two lines}$$

$$\sigma_y = \sigma_2 - \sigma_1$$

This is the safe area where failure is not happening

## ② Maximum Strain energy (distortion energy) theory

Distortion means the change of shape & usually it is due to induced shear stress. The element absorb energy for this distortion. The specific distort' energy is defined by the distort' energy absorbed per unit vol. In triaxial stress con', it is given as

$$U = \frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

In uniaxial stress con', as  $\sigma_2 = \sigma_3 = 0$ ,

$$U = \frac{\sigma_1^2}{6G}$$

for biaxial con',  $\sigma_3 = 0$

$$U = \frac{[(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2]}{12G}$$

$$= \frac{1}{12G} (2\sigma_1^2 - 2\sigma_1\sigma_2 + 2\sigma_2^2)$$

Page \_\_\_\_\_ Date \_\_\_\_\_

$$U = \frac{1}{6G} (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

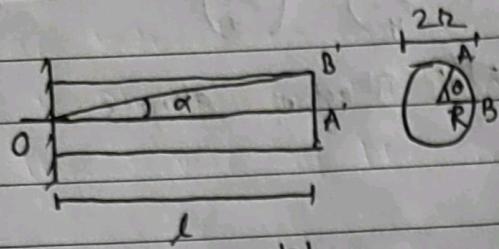
$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2$$

(missed)

$$1 = \left(\frac{\sigma_1}{\sigma_y}\right)^2 - \left(\frac{\sigma_1 \sigma_2}{\sigma_y^2}\right) + \left(\frac{\sigma_2}{\sigma_y}\right)^2$$

This is eqn of ellipse

# Torsion



$$\tan \alpha = \alpha = A'B'/OA$$

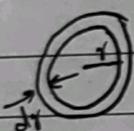
$$AB = R\theta$$

$$\alpha = \frac{R\theta}{l}$$

$$\sigma = E\varepsilon$$

$$\tau_r = \frac{GR\theta}{l}$$

If  $\tau$  is stress developed on elemental area,



$$dF = \tau (2\pi r dr)$$

$$dT = \tau (2\pi r dr) r \quad T \text{ is torque}$$

$$\text{at } r, \tau = \frac{GR\theta}{l}$$

$$\therefore dT = \frac{Gr^2\theta}{l} (2\pi r dr)$$

$$T = \frac{G\theta}{l} \int (2\pi r dr) r^2$$

$$= \frac{G\theta}{l} I_z$$

$J$  or  $I_z$  is moment of area about Z axis

$$I_z = J = I_x + I_y$$

$T = \frac{G\theta J}{l}$
---------------------------

$$T = \frac{G\theta r J}{rl}$$

$$T = \frac{J \tau_r}{r}$$

$$\therefore \boxed{\tau_r = \frac{Tr}{J}}$$

$$\boxed{\tau_{\max} = \frac{TR}{J}}$$

for a circle of  $R$ , priorly we used to take  $J = \pi d^4/64$

$$\text{here, } J = I_z = I_x + I_y = \pi d^4/64 + \pi d^4/64 = \pi d^4/32$$

\* Assumption

homogeneous

Isotropic

Twisting is within elastic limit & small

Compression & Elongation has same effect

\* Solid circular shaft transmitting power

$$\text{Power} = P = T \omega$$

$$\sigma_m = \frac{M_y r}{I}$$

$$r = \frac{d}{2} = d/2$$

$$I = \pi d^4 / 64$$

$$\sigma_m = \frac{M_y d}{2 \times \pi d^4 / 64}$$

$$\sigma_m = \frac{32 M_y}{\pi d^3}$$

$d$  is shaft diameter

$$T_m = \frac{T_{eq} r}{J} = \frac{T_{eq} d}{2 J} = \frac{T_{eq} d}{2 \frac{\pi d^4}{32}} = \frac{16 T_{eq}}{\pi d^3}$$

yield stress  $\approx 0.8 \times$  ultimate stress

$$\sigma_{eq} = \frac{\sigma}{2} + \sqrt{(\sigma/2)^2 + \tau^2}$$

$$T_{eq} = \sqrt{(\sigma/2)^2 + \tau^2}$$

$$M_{eq} = \frac{1}{2} [M + \sqrt{M^2 + T^2}]$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

$$\sigma_y \approx 0.8 \sigma_u$$

$$\tau_y = \frac{1}{2} \sigma_y$$

$\sigma_y$  &  $\sigma_u$  are available by simple tension test (for steel)

## \* Factor of safety (F.S.):

It is a factor by which the magnitude of stress is divided (as the case may be), to limit the stress value induced in an element for the sake of its safety. It is always greater than 1 & chosen judiciously depending on the service of the machine element.

$$[\sigma] = \sigma_g = \frac{\sigma_y}{F.S.}$$

↓ alloy

$$[\tau] = \tau_a = \frac{\tau_y}{F.S.}$$

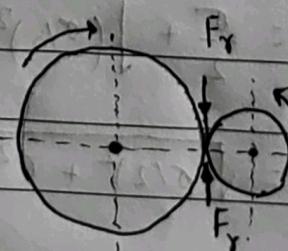
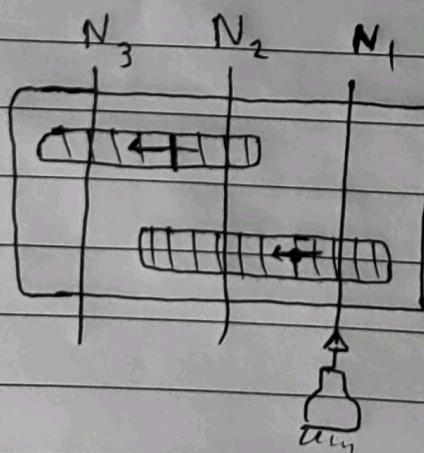
Ex. F.S. = 2       $\sigma_u = 300 \text{ MPa}$

$$\tau_y = 0.8 \sigma_u = 240 \text{ MPa}$$

$$\tau_a = \frac{1}{2} \tau_y = 120 \text{ MPa}$$

$$\tau_a = \frac{240}{2} = 120 \text{ MPa}$$

$$\tau_a = \frac{\tau_y}{F.S.} = 60 \text{ MPa}$$



1] Input power 6 kW, input rpm = 1500, diameter of input gear = 50 mm  
 dia of 1<sup>st</sup> intermediate gear = 100 mm, 2<sup>nd</sup> = 50 mm, dia of output gear = 100 mm. Distance b/w the bearing support for all the shafts is 225 mm. All gears are equally spaced. Find rate diameter of all these shafts with  $\sigma_u = 300$ , F.S. = 2.5

\* ~~according to longitudinal section Hb to account b/w bearing bellow will be diff of gear~~

The rigidity of shaft is given by angle of twist ( $\theta$ )

$$\theta = \frac{TL}{GJ} \quad T - \text{Torque}$$

L - length b/w transmission point

G - Modulus of rigidity

J - polar moment of area

for steel  $E = 210 \text{ GPa}$

1] Compare the shear stress induced & angle of twist b/w shafts of same material & same area of cross section, one being solid & other being hollow with inner to outer diameter ratio 1:2

~~no effect on shear stress~~  
~~area of solid cross sect^n = area of hollow cross sect^n~~

$$\frac{\pi d^2}{4} = \frac{\pi (d_2^2 - d_1^2)}{4}$$

$$d_2 = 2d_1$$

$$d^2 = 3d_1^2$$

$$d = \sqrt{3} d_1$$

$$\frac{T_m(\text{solid})}{T_m(\text{hollow})} = \frac{16 T_{eq} / \pi d^3}{16 T_{eq} / \pi (d_2^3 - d_1^3)} = \frac{8d_1^3 - d_1^3}{3\sqrt{3} d_1^3 - 3\sqrt{3}} = \frac{7}{3\sqrt{3}} = \frac{7\sqrt{3}}{9}$$

$$= 1.3471$$

~~error in assignment~~

$$1 \text{ N/mm}^2 = 9.809 \text{ atm}$$

$\approx 10 \text{ atm}$

# Pressure Vessel

\* The pressure vessels are closed vessels with internal to external pressure difference of diff. shapes (cylindrical or spherical), may be thick or thin walled.

If the internal diameter is ' $d$ ' & wall thickness is ' $t$ ',  
 $\frac{d}{t} > 20$  is classified as thin walled pressure vessel

In industrial practice,  $t$  lies bet<sup>n</sup> 2.8 mm to 5 mm

Thin walled pressure vessel may be of any diameter

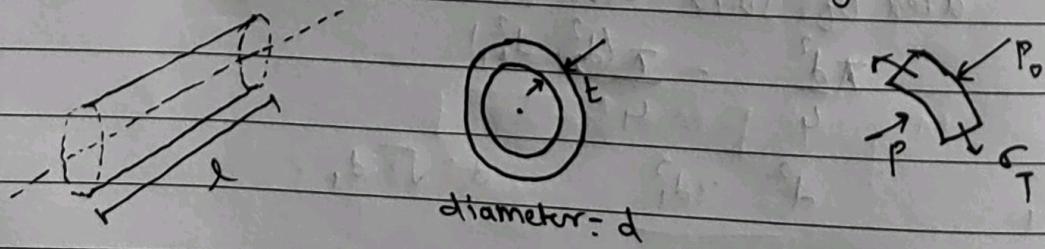
But as the diameter increases, its stability goes weaker & weaker. Diameter upto 1m is of common use

The manufacturing technique of thick walled cylinder, which has more than 12mm thickness is commonly casting.

Material for thick cylinder - cast iron

" " thin " - steel

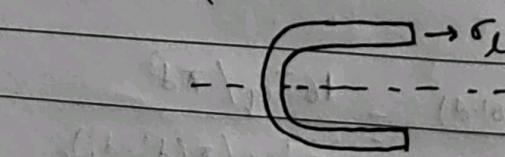
The stress analysis of thin cylinder is a simplified version (i.e. approximate one) of the thick cylinders



$\sigma_L$  - longitudinal

$\sigma_T$  - transverse

r - showpiece in house



$$F = \frac{\pi d^4}{4} \cdot p$$

$$\sigma_e = \frac{F}{A} = \frac{\pi d^2 p}{4\pi dt} = \frac{pd}{4t}$$

The developed shear stress on any element =  $\frac{\sigma_1 - \sigma_2}{2}$

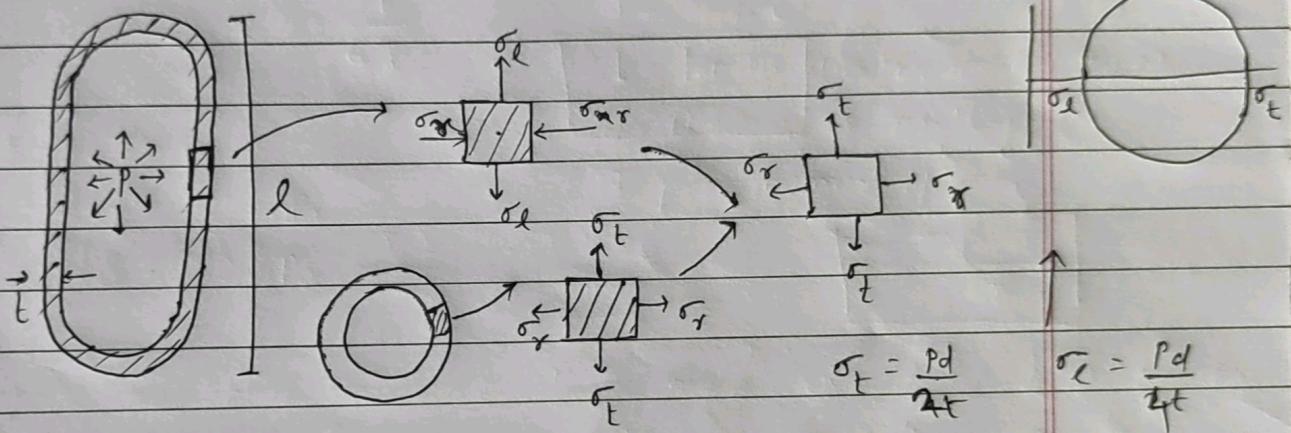
$$\sigma_1 = \sigma_t = \frac{pd}{2t}$$

$$\sigma_2 = \sigma_e = \frac{pd}{4t}$$

$$\sigma_t = \frac{F}{A} = \frac{pd}{2lt} = \frac{pd}{2t}$$

$$= \frac{pd}{8t}$$

$$T_{max} = \frac{\sigma_1 - \sigma_2}{2}$$



A domestic gas cylinder is made of 2.2 mm thick steel sheet whose internal diameter is 300 mm. If the yield point of the material is 260 MPa & factor of safety = 5, estimate the maximum safe pressure it can hold.

$$t = 2.2 \text{ mm}$$

$$d = 300 \text{ mm}$$

$$\sigma_y = 260 \times 10^6 \text{ Pa} \\ = 260 \text{ N/mm}^2$$

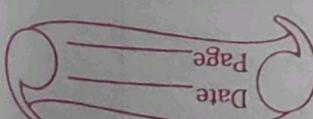
$$\frac{\sigma_y}{f.o.s} = [\sigma_y]$$

$$\frac{260}{5} = [\sigma_y] = 52 \text{ Pa}$$

$$\therefore T_y = \frac{[\sigma_y]}{2} = 26 \text{ Pa}$$

$$T_y = T_a = T_{max} = 26 = \frac{pd}{8t} = \frac{p \times 300}{8 \times 2.2}$$

$$\therefore p = 1.5253 \text{ N/mm}^2$$



$$\sigma_e = \frac{pd}{4t} \quad \therefore 52 = \frac{p \times 300}{4 \times 2.2}$$

$$\sigma_t = \frac{pd}{2t} \Rightarrow 52 = \frac{p \times 300}{2 \times 2.2}$$

$$\therefore p = 1.5253 \text{ N/mm}^2$$

$$\therefore p = 3046$$

$$0.7626 \text{ N/mm}^2$$