LECTURE 1: Elastic Stability of Columns

Introduction: Structural members which carry compressive loads may be divided into two broad categories depending on their relative lengths and cross-sectional dimensions.

Columns: Short, thick members are generally termed columns and these usually fail by crushing when the yield stress of the material in compression is exceeded.

Struts: Long, slender columns are generally termed as struts, they fail by buckling some time before the yield stress in compression is reached. The buckling occurs owing to one the following reasons.

- the strut may not be perfectly straight initially.
- the load may not be applied exactly along the axis of the Strut.
- one part of the material may yield in compression more readily than others owing to some lack of uniformity in the material properties through out the strut.

In all the problems considered so far we have assumed that the deformation to be both progressive with increasing load and simple in form i.e. we assumed that a member in simple tension or compression becomes progressively longer or shorter but remains straight. Under some circumstances however, our assumptions of progressive and simple deformation may no longer hold good and the member become unstable. The term strut and column are widely used, often interchangeably in the context of buckling of slender members.]

At values of load below the buckling load a strut will be in stable equilibrium where the displacement caused by any lateral disturbance will be totally recovered when the disturbance is removed. At the buckling load the strut is said to be in a state of neutral equilibrium, and theoretically it should than be possible to gently deflect the strut into a simple sine wave provided that the amplitude of wave is kept small.

Theoretically, it is possible for struts to achieve a condition of unstable equilibrium with loads exceeding the buckling load, any slight lateral disturbance then causing failure by buckling, this condition is never achieved in practice under static load conditions. Buckling occurs immediately at the point where the buckling load is reached, owing to the reasons stated earlier.

The resistance of any member to bending is determined by its flexural rigidity EI and is The quantity I may be written as $I = Ak^2$,

Where I = area of moment of inertia
A = area of the cross-section
k = radius of gyration.

The load per unit area which the member can withstand is therefore related to k. There will be two principal moments of inertia, if the least of these is taken then the ratio

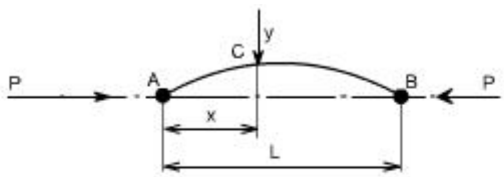
$$\frac{l}{k} \quad \text{i.e.} \quad \frac{\text{length of member}}{\text{least radius of gyration}}$$

Is called the slenderness ratio. It's numerical value indicates whether the member falls into the class of columns or struts.

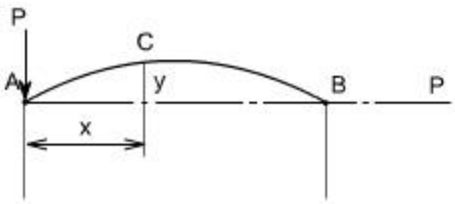
Euler's Theory : The struts which fail by buckling can be analyzed by Euler's theory. In the following sections, different cases of the struts have been analyzed.

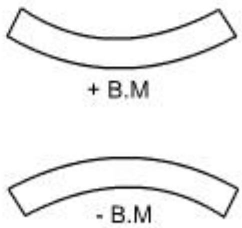
Case A: Strut with pinned ends: Consider an axially loaded strut, shown below, and is subjected to an axial load 'P' this load 'P' produces a deflection 'y' at a distance 'x' from one end.

Assume that the ends are either pin jointed or rounded so that there is no moment at either end.



Assumption: The strut is assumed to be initially straight, the end load being applied axially through centroid.





According to sign convention

$$B.M|_C = -Py$$

Futher,we know that

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = -P.y = M$$

In this equation 'M' is not a function 'x'. Therefore this equation can not be integrated directly as has been done in the case of deflection of beams by integration method.

Thus,

$$EI\frac{d^2y}{dx^2} + Py = 0$$

Though this equation is in 'y' but we can't say at this stage where the deflection would be maximum or minimum.

So the above differential equation can be arranged in the following form $\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$

Let us define a operator

$$D = d/dx$$

$$(D^2 + n^2)\, y = 0 \text{ where } n^2 = P/EI$$

This is a second order differential equation which has a solution of the form consisting of complimentary function and particular integral but for the time being we are interested in the complementary solution only[in this P.I = 0; since the R.H.S of Diff. equation = 0.

Thus $y = A\cos(nx) + B\sin(nx)$

Where A and B are some constants.

Therefore $y = A\cos\sqrt{\frac{P}{EI}}\,x + B\sin\sqrt{\frac{P}{EI}}\,x$

In order to evaluate the constants A and B let us apply the boundary conditions,

(i) at $x = 0$; $y = 0$
(ii) at $x = L$; $y = 0$

Applying the first boundary condition yields A = 0.

Applying the second boundary condition gives

$$B\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$$

Thus either $B = 0$, or $\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$

if B=0, that y0 for all values of x hence the strut has not buckled yet. Therefore, the solution required is

$$\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0 \quad \text{or} \quad \left(L\sqrt{\frac{P}{EI}}\right) = \pi \text{ or } nL = \pi$$

$$\text{or} \quad \sqrt{\frac{P}{EI}} = \frac{\pi}{L} \quad \text{or} \quad P = \frac{\pi^2 EI}{L^2}$$

From the above relationship the least value of P which will cause the strut to buckle, and it is called the **Euler Crippling Load** P_e from which w obtain.

$$\boxed{P_e = \frac{\pi^2 EI}{L^2}}$$

It may be noted that the value of I used in this expression is the least moment of inertia

It should be noted that the other solutions exists for the equation

$$\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0 \qquad \text{i.e. } \sin nL=0$$

The interpretation of the above analysis is that for all the values of the load P, other than those which make sin nL = 0; the strut will remain perfectly straight since

$$y = B \sin nL = 0$$

For the particular value of

$$P_e = \frac{\pi^2 EI}{L^2}$$

$$\sin nL = 0 \quad \text{or } nL = \pi$$

$$\text{Therefore} \quad n = \frac{\pi}{L}$$

$$\text{Hence } y = B \sin nx = B \sin \frac{\pi x}{L}$$

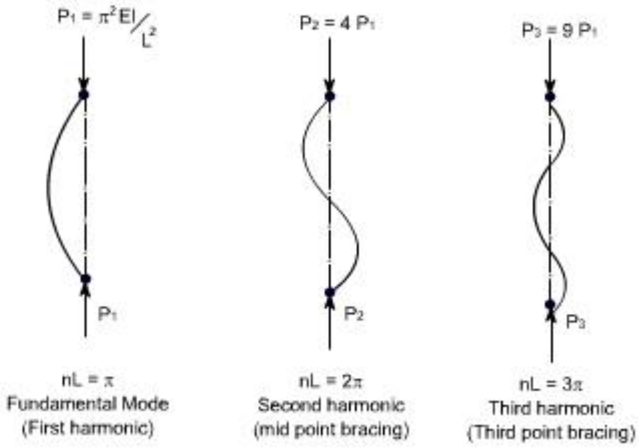
Then we say that the strut is in a state of neutral equilibrium, and theoretically any deflection which it suffers will be maintained. This is subjected to the limitation that 'L' remains sensibly constant and in practice slight increase in load at the critical value will cause the deflection to increase appreciably until the material fails by yielding.

Further it should be noted that the deflection is not proportional to load, and this applies to all strut problems; like wise it will be found that the maximum stress is not proportional to load.

The solution chosen of nL = π is just one particular solution; the solutions nL= 2π, 3π, 5π etc are equally valid mathematically and they do, infact, produce values of 'P_e' which are equally valid for modes of buckling of strut different from that of a simple bow. Theoretically therefore, there are an infinite number of values of P_e , each corresponding with a different mode of buckling.

The value selected above is so called the fundamental mode value and is the lowest critical load producing the single bow buckling condition.

The solution nL = 2π produces buckling in two half waves, 3π in three half-waves etc.



$$L\sqrt{\frac{P}{EI}} = \pi \text{ or } P_1 = \frac{\pi^2 EI}{L^2}$$

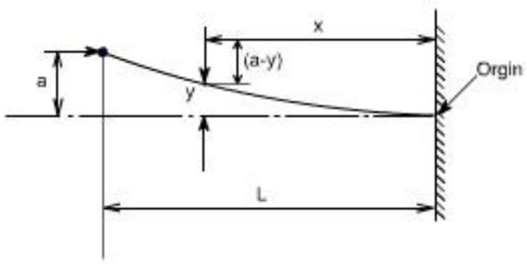
$$\text{If } L\sqrt{\frac{P}{EI}} = 2\pi \text{ or } P_2 = \frac{4\pi^2 EI}{L^2} = 4P_1$$

$$\text{If } L\sqrt{\frac{P}{EI}} = 3\pi \text{ or } P_3 = \frac{9\pi^2 EI}{L^2} = 9P_1$$

If load is applied sufficiently quickly to the strut, then it is possible to pass through the fundamental mode and to achieve at least one of the other modes which are theoretically possible. In practical loading situations, however, this is rarely achieved since the high stress associated with the first critical condition generally ensures immediate collapse.

struts and columns with other end conditions: Let us consider the struts and columns having different end conditions

Case b: One end fixed and the other free:



writing down the value of bending moment at the point C

$$B.M|_C = P(a - y)$$

Hence, the differential equation becomes,

$$EI\frac{d^2y}{dx^2} = P(a - y)$$

On rearranging we get

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}$$

$$\text{Let } \frac{P}{EI} = n^2$$

Hence in operator form, the differential equation reduces to $(D^2 + n^2) y = n^2a$

The solution of the above equation would consist of complementary solution and particular solution, therefore

y_{gen} = A cos(nx) + sin(nx) + P. I

where

P.I is a particular value of y which satisfies the differential equation

Hence $y_{P.I}$ = a

Therefore the complete solution becomes Y = A cos(nx) + B sin(nx) + a

Now imposing the boundary conditions to evaluate the constants A and B

(i) at x = 0; y = 0 ; This yields A = -a

(ii) at x = 0; dy/dx = 0 ; This yields B = 0

Hence, y = –a cos(nx) + a

Further, at x = L; y = a

Therefore a = - a cos(nx) + a or 0 = cos(nL)

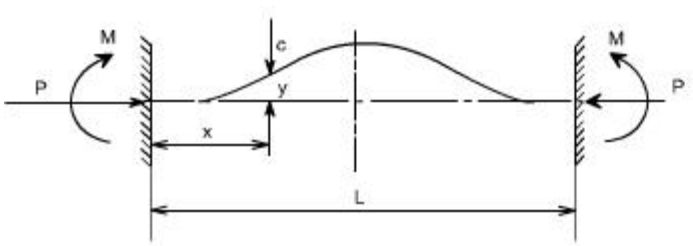
Now the fundamental mode of buckling in this case would be

$$nL = \frac{\pi}{2}$$

$$\sqrt{\frac{P}{EI}}\,L = \frac{\pi}{2}, \text{Therefore, the Euler's crippling load is given as}$$

$$P_e = \frac{\pi^2 EI}{4L^2}$$

Case 3: Strut with fixed ends



Due to the fixed end supports bending moment would also appears at the supports, since this is the property of the support.

Bending Moment at point C = M - P.y

$$EI\frac{d^2y}{dx^2} = M - Py$$

$$\text{or } \frac{d^2y}{dx^2} + \frac{P}{EI} = \frac{M}{EI}$$

$n^2 = \frac{P}{EI}$, Therefore in the operator from, the equation reduces to

$$\left(D^2 + n^2\right)y = \frac{M}{EI}$$

$$y_{general} = y_{complementary} + y_{particular\,integral}$$

$$y\big|_{P.I} = \frac{M}{n^2EI} = \frac{M}{P}$$

Hence the general solution would be

$$y = B\,\text{Cos}nx + A\,\text{Sin}nx + \frac{M}{P}$$

Boundry conditions relevant to this case are at x=0:y=0

$$B = -\frac{M}{P}$$

Also at $x = 0; \frac{dy}{dx} = 0$ hence

A=0

Therefore,

$$y = -\frac{M}{P}\text{Cos}\,nx + \frac{M}{P}$$

$$y = \frac{M}{P}\left(1 - \text{Cos}\,nx\right)$$

Futher, it may be noted that at $x = L; y = 0$

$$\text{Then } 0 = \frac{M}{P}\left(1 - \text{Cos}\,nL\right)$$

Thus, either $\frac{M}{P} = 0$ or $\left(1 - \text{Cos}\,nL\right) = 0$

obviously, $\left(1 - \text{Cos}\,nL\right) = 0$

$\cos nL = 1$

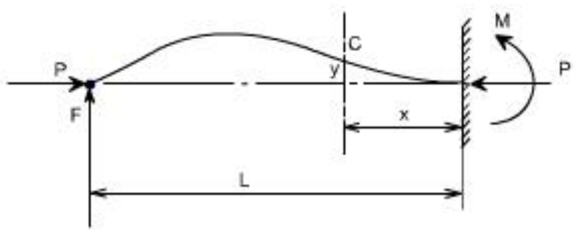
Hence the least solution would be

$nL = 2\pi$

$\sqrt{\frac{P}{EI}}\,L = 2\pi$, Thus, the buckling load or crippling load is

Thus,
$$P_e = \frac{4\pi^2.EI}{L^2}$$

Case 4 : <u>One end fixed, the other pinned</u>



In order to maintain the pin-joint on the horizontal axis of the unloaded strut, it is necessary in this case to introduce a vertical load F at the pin. The moment of F about the built in end then balances the fixing moment.

With the origin at the built in end, the B,M at C is given as

$$EI\frac{d^2y}{dx^2} = -Py + F(L-x)$$

$$EI\frac{d^2y}{dx^2} + Py = F(L-x)$$

Hence

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{F}{EI}(L-x)$$

In the operator form the equation reduces to

$$\left(D^2 + n^2\right)y = \frac{F}{EI}(L-x)$$

$$y_{particular} = \frac{F}{n^2EI}(L-x) \text{ or } y = \frac{F}{P}(L-x)$$

The full solution is therefore

$$y = A\,Cos\,mx + B\,Sin\,nx + \frac{F}{P}(L-x)$$

The boundry conditions relevants to the problem are at x=0;y=0

$$\text{Hence } A = -\frac{FL}{P}$$

$$\text{Also at } x = 0; \frac{dy}{dx} = 0$$

$$\text{Hence } B = \frac{F}{nP}$$

$$\text{or } y = -\frac{FL}{P}Cos\,nx + \frac{F}{nP}Sin\,nx + \frac{F}{P}(L-x)$$

$$y = \frac{F}{nP}\left[Sin\,nx - nL\,Cos\,nx + n(L-x)\right]$$

Also when x = L ; y = 0

Therefore

$$nL\,Cos\,nL = Sin\,nL \quad \text{or } \tan nL = nL$$

The lowest value of nL (neglecting zero) which satisfies this condition and which therefore produces the fundamental buckling condition is nL = 4.49radian

$$\text{or} \sqrt{\frac{P}{EI}}\,L = 4.49$$

$$\frac{P_e}{EI}L^2 = 20.2$$

$$P_e = \frac{2.05\pi^2\,EI}{L^2}$$

Equivalent Strut Length: Having derived the results for the buckling load of a strut with pinned ends the Euler loads for other end conditions may all be written in the same form.

$$\text{i.e. } P_e = \frac{\pi^2 EI}{L^2}$$

Where L is the equivalent length of the strut and can be related to the actual length of the strut depending on the end conditions.

The equivalent length is found to be the length of a simple bow(half sine wave) in each of the strut deflection curves shown. The buckling load for each end condition shown is then readily obtained. The use of equivalent length is not restricted to the Euler's theory and it will be used in other derivations later.

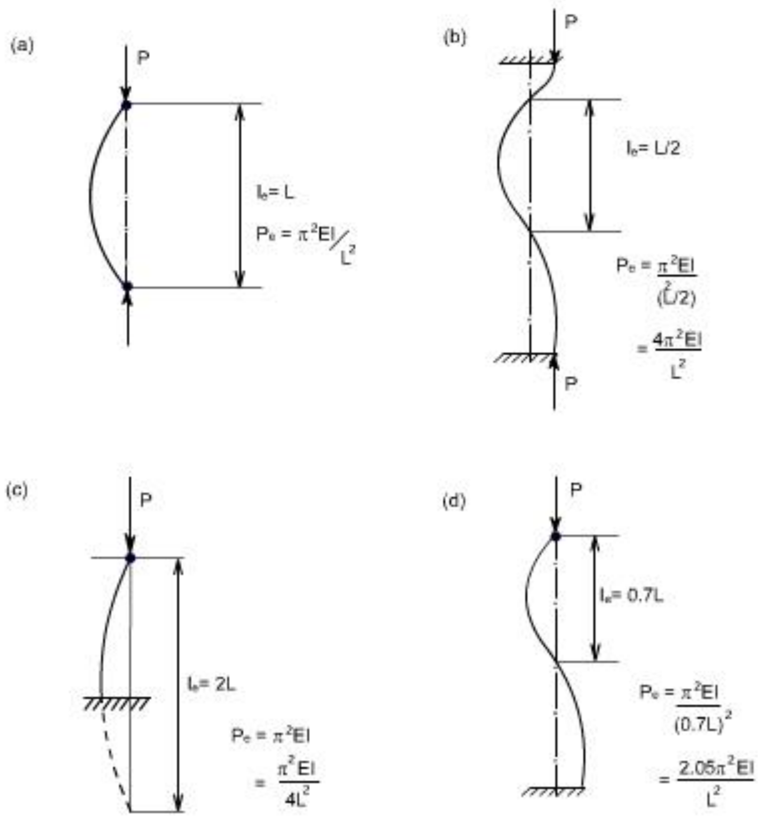
The critical load for columns with other end conditions can be expressed in terms of the critical load for a hinged column, which is taken as a fundamental case.

For case(c) see the figure, the column or strut has inflection points at quarter points of its unsupported length. Since the bending moment is zero at a point of inflection, the free body diagram would indicates that the middle half of the fixed ended is equivalent to a hinged column having an effective length $L_e = L / 2$.

The four different cases which we have considered so far are:

(a) Both ends pinned (c) One end fixed, other free

(b) Both ends fixed (d) One end fixed and other pinned



LECTURE 2: Comparison of Euler Theory with Experiment results

Limitations of Euler's Theory : In practice the ideal conditions are never [i.e. the strut is initially straight and the end load being applied axially through centroid] reached. There is always some eccentricity and initial curvature present. These factors needs to be accommodated in the required formula's.

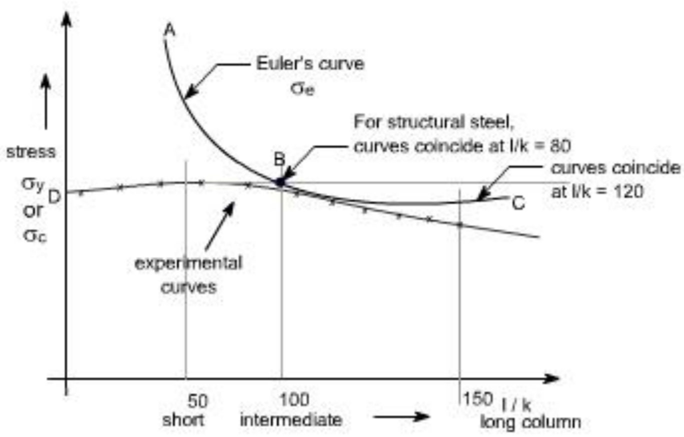
It is realized that, due to the above mentioned imperfections the strut will suffer a deflection which increases with load and consequently a bending moment is introduced which causes failure before the Euler's load is reached. Infact failure is by stress rather than by buckling and the deviation from the Euler value is more marked as the slenderness-ratio l/k is reduced. For values of l/k < 120 approx, the error in applying the Euler theory is too great to allow of its use. The stress to cause buckling from the Euler formula for the pin ended strut is

$$\text{Euler's stress, } \sigma_e = \frac{P_e}{A} = \frac{\pi^2 EI}{Al^2}$$

$$\text{But, } I = Ak^2$$

$$\sigma_e = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

A plot of σ_e versus l / k ratio is shown by the curve ABC.



Allowing for the imperfections of loading and strut, actual values at failure must lie within and below line CBD.

Other formulae have therefore been derived to attempt to obtain closer agreement between the actual failing load and the predicted value in this particular range of slenderness ratio i.e.l/k=40 to l/k=100.

(a) Straight line formulae : The permissible load is given by the formulae

$$P = \sigma_y A\left[1 - n\left(\frac{l}{k}\right)\right]$$ Where the value of index 'n' depends on the material used and the end conditions.

(b) Johnson parabolic formulae : The Johnson parabolic formulae is defined as

$$P = \sigma_y A\left[1 - b\left(\frac{l}{k}\right)^2\right]$$ where the value of index ?b' depends on the end conditions.

(c) Rankine Gordon Formulae :

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

Where P_e = Euler crippling load

P_c = Crushing load or Yield point load in Compression

P_R = Actual load to cause failure or Rankine load

Since the Rankine formulae is a combination of the Euler and crushing load for a strut.

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

For a very short strut P_e is very large hence 1/ P_e would be large so that 1/ P_e can be neglected.

Thus $P_R = P_c$, for very large struts, P_e is very small so 1/ P_e would be large and 1/ P_c can be neglected ,hence $P_R = P_e$

The Rankine formulae is therefore valid for extreme values of 1/k.It is also found to be fairly accurate for the intermediate values in the range under consideration. Thus rewriting the formula in terms of stresses, we have

$$\frac{1}{\sigma A} = \frac{1}{\sigma_e A} + \frac{1}{\sigma_y A}$$

$$\frac{1}{\sigma} = \frac{1}{\sigma_e} + \frac{1}{\sigma_y}$$

$$\frac{1}{\sigma} = \frac{\sigma_e + \sigma_y}{\sigma_e . \sigma_y}$$

$$\sigma = \frac{\sigma_e . \sigma_y}{\sigma_e + \sigma_y} = \frac{\sigma_y}{1 + \frac{\sigma_y}{\sigma_e}}$$

For struts with both ends pinned

$$\sigma_e = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

$$\sigma = \frac{\sigma_y}{1 + \frac{\sigma_y}{\pi^2 E}\left(\frac{l}{k}\right)^2}$$

$$\sigma = \frac{\sigma_y}{1 + a\left(\frac{l}{k}\right)^2}$$

Where $a = \frac{\sigma_y}{\pi^2 EI}$ and the value of 'a' is found by conducting experiments on various materials. Theoretically, but having a value normally found by experiment for various materials. This will take into account other types of end conditions.

Therefore $$\boxed{\text{Rankine load} = \frac{\sigma_y . A}{1 + a\left(\frac{l}{k}\right)^2}}$$

Typical values of 'a' for use in Rankine formulae are given below in table.

Material	σ_y or σ_c MN/m^2	Value of a	
		Pinned ends	Fixed ends
Low carbon steel	315	1/7500	1/30000
Cast Iron	540	1/1600	1/64000
Timber	35	1/3000	1/12000

note a = 4 x (a for fixed ends)

Since the above values of 'a' are not exactly equal to the theoretical values , the Rankine loads for long struts will not be identical to those estimated by the Euler theory as estimated.

Strut with initial Curvature : As we know that the true conditions are never realized , but there are always some imperfections. Let us say that the strut is having some initial curvature. i.e., it is not perfectly straight before loading. The situation will influence the stability. Let us analyze this effect.

by a differential calculus

$$R_0 \approx \frac{1}{d^2y_0/dx^2} \text{ (Approximately)}$$

$$\text{Futher } \frac{E}{R} = \frac{M}{I} \text{ and } \frac{EI}{R} = M$$

$$\text{But for this case } EI\left[\frac{1}{R} - \frac{1}{R_0}\right] = M$$

since strut is having some initial curvature

Now putting

$$\frac{1}{R} = \frac{d^2y}{dx^2} \text{ and } \frac{1}{R_0} = \frac{d^2y_0}{dx^2}$$

Where 'y_0' is the value of deflection before the load is applied to the strut when the load is applied to the strut the deflection increases to a value 'y'. Hence

$$EI\left[\frac{d^2y}{dx^2} - \frac{d^2y_0}{dx^2}\right] = M$$

$$EI\frac{d^2y}{dx^2} - EI\frac{d^2y_0}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = M + EI\frac{d^2y_0}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = -Py + EI\frac{d^2y_0}{dx^2}$$

If the pinended strut is under the action of a load P then obviously the BM would be' – py'

Hence

$$EI\frac{d^2y}{dx^2} + Py = EI\frac{d^2y_0}{dx^2}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{d^2y_0}{dx^2}$$

Again letting

$$\frac{P}{EI} = n^2$$

$$\frac{d^2y}{dx^2} + n^2y = \frac{d^2y_0}{dx^2}$$

The initial shape of the strut y_0 may be assumed circular, parabolic or sinusoidal without making much difference to the final results, but the most convenient form is

$$y_0 = C.\sin\frac{\pi x}{l}$$ where C is some constant or here it is amplitude

Which satisfies the end conditions and corresponds to a maximum deviation 'C'. Any other shape could be analyzed into a Fourier series of sine terms. Then

$$\frac{d^2y}{dx^2} + n^2y = \frac{d^2y_0}{dx^2} = \frac{d^2}{dx^2}\left[C.\sin\frac{\pi x}{l}\right] = \left(-C.\frac{\pi^2}{l^2}\right)\sin\left(\frac{\pi x}{l}\right)$$

The computer solution would be therefore be

$$y_{general} = y_{complementry} + y_{PI}$$

$$y = A\cos nx + B\sin nx + \frac{C.\frac{\pi^2}{l^2}}{\left(\frac{\pi^2}{l^2}\right) - n^2}\sin\left(\frac{\pi x}{l}\right)$$

Boundary conditions which are relevant to the problem are

at x = 0 ; y = 0 thus B = 0

Again when x = l ; y = 0 or x = l / 2 ; dy/dx = 0

the above condition gives B = 0

Therefore the complete solution would be

$$y = \frac{C.\frac{\pi^2}{l^2}}{\left\{\left(\frac{\pi^2}{l^2}\right) - n^2\right\}}\sin\left(\frac{\pi x}{l}\right)$$

Again the above solution can be slightly rearranged.since

$$P_e = \frac{\pi^2 EI}{l^2}$$

hence the term $\dfrac{\frac{\pi^2}{l^2}}{\frac{\pi^2}{l^2} - n^2}$ after multiplying the denominator & numerator by EI is equal to

$$\frac{\frac{\pi^2 EI}{l^2}}{\frac{\pi^2 EI}{l^2} - n^2 EI} = \left[\frac{P_e}{P_e - P}\right]$$

Since $n^2 = \dfrac{P}{EI}$

where P_e = Euler's load P=applied load

Thus

$$y = \frac{C.\frac{\pi^2}{l^2}}{\left\{\left(\frac{\pi^2}{l^2}\right) - n^2\right\}}\sin\left(\frac{\pi x}{l}\right)$$

$$y = \left\{\frac{C.P_e}{P_e - P}\right\}\sin\left(\frac{\pi x}{l}\right)$$

The crippling load is again

$$P = P_e = \frac{\pi^2 EI}{l^2}$$

Since the BM for a pin ended strut at any point is given as

$$M = -Py \text{ and}$$

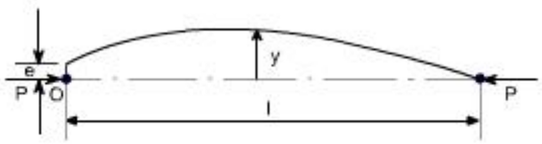
$$\text{Max BM} = P\, y_{max}$$

Now in order to define the absolute value in terms of maximum amplitude let us use the symbol as '^'.

$$\hat{M} = P.\hat{y}$$
$$= C.\frac{P.P_e}{(P_e - p)}$$
$$\text{Therefore } \hat{M} = \frac{C\ P\ P_e}{[P_e - p]} \text{ since } y_{max^m} = \frac{P_e}{[P_e - p]}$$
$$\sin\frac{\pi x}{l} = 1 \text{ when } \frac{\pi x}{l} = \frac{\pi}{2}$$
$$\text{Hence } \hat{M} = \frac{C\ P\ P_e}{[P_e - p]}$$

Strut with eccentric load

Let 'e' be the eccentricity of the applied end load, and measuring y from the line of action of the load.



Then $$EI\frac{d^2y}{dx^2} = -Py$$

or $(D^2 + n^2)\, y = 0$ where $n^2 = P / EI$

Therefore $y_{general} = y_{complementary}$

$= A\sin nx + B\cos nx$

applying the boundary conditions then we can determine the constants i.e.

at $x = 0$; $y = e$ thus $B = e$

at $x = l / 2$; $dy / dx = 0$

Therefore

$$A\cos\frac{nl}{2} - B\sin\frac{nl}{2} = 0$$
$$A\cos\frac{nl}{2} = B\sin\frac{nl}{2}$$
$$A = B\tan\frac{nl}{2}$$
$$A = e\tan\frac{nl}{2}$$

Hence the complete solution becomes

$$y = A\sin(nx) + B\cos(nx)$$

substituting the values of A and B we get

$$y = e\left[\tan\frac{nl}{2}\sin nx + \cos nx\right]$$

Note that with an eccentric load, the strut deflects for all values of P, and not only for the critical value as was the case with an axially applied load. The deflection becomes infinite for tan (nl)/2 = ∞ i.e. nl = π giving the same crippling load $P_e = \frac{\pi^2 EI}{l^2}$. However, due to additional bending moment set up by deflection, the strut will always fail by compressive stress before Euler load is reached.

Since

$$y = e\left[\tan\frac{nl}{2}\sin nx + \cos nx\right]$$

$$y_{max^m}\Big|_{at\ x = \frac{l}{2}} = e\left[\tan\left(\frac{nl}{2}\right)\sin\frac{nl}{2} + \cos\frac{nl}{2}\right]$$

$$= e\left[\frac{\sin^2\frac{nl}{2} + \cos^2\frac{nl}{2}}{\cos\frac{nl}{2}}\right]$$

$$= e\left[\frac{1}{\cos\frac{nl}{2}}\right] = e\ \sec\frac{nl}{2}$$

Hence maximum bending moment would be

$$M_{max^m} = P\ y_{max^m}$$

$$= P\,e\ \sec\frac{nl}{2}$$

Now the maximum stress is obtained by combined and direct strain

$$\sigma = \frac{P}{A} + \frac{M}{Z} \text{ stress due to bending } \frac{\sigma}{y} = \frac{M}{I};$$

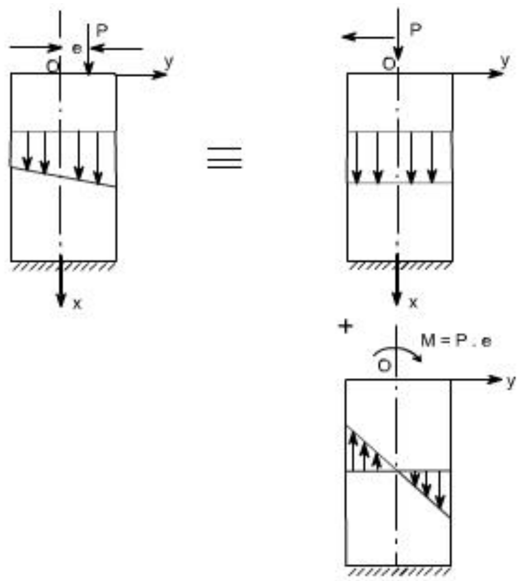
$$M = \sigma\frac{I}{y};\ \sigma_{max} = \frac{M}{Z} \text{ Wher } Z = I/y \text{ is section modulus}$$

The second term is obviously due the bending action.

Then such a loading may be replaced by its statically equivalent of a centrally applied compressive force 'P' and a couple of moment P.e

Consider a short strut subjected to an eccentrically applied compressive force P at its upper end. If such a strut is comparatively short and stiff, the deflection due to bending action of the eccentric load will be negligible compared with eccentricity 'e' and the principal of super-imposition applies.

If the strut is assumed to have a plane of symmetry (the xy - plane) and the load P lies in this plane at the distance 'e' from the centroidal axis ox.



The centrally applied load P produces a uniform compressive $\sigma_1 = P/A$ stress over each cross-section as shown by the stress diagram.

The end moment 'M' produces a linearly varying bending stress $\sigma_2 = My/I$ as shown in the figure.

Then by super-imposition, the total compressive stress in any fiber due to combined bending and compression becomes,

$$\sigma = \frac{P}{A} + \frac{My}{I}$$

$$\sigma = \frac{P}{A} + \frac{M}{I/y}$$

$$\boxed{\sigma = \frac{P}{A} + \frac{M}{Z}}$$