LECTURE 1: INTRODUCTION AND REVIEW

Preamble: Engineering science is usually subdivided into number of topics such as

1. Solid Mechanics
2. Fluid Mechanics
3. Heat Transfer
4. Properties of materials and soon Although there are close links between them in terms of the physical principles involved and methods of analysis employed.

The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviors of solid bodies subjected to various types of loadings. This is usually subdivided into further two streams i.e Mechanics of rigid bodies or simply Mechanics and Mechanics of deformable solids.

The mechanics of deformable solids which is branch of applied mechanics is known by several names i.e. strength of materials, mechanics of materials etc.

Mechanics of rigid bodies: The mechanics of rigid bodies is primarily concerned with the static and dynamic behaviour under external forces of engineering components and systems which are treated as infinitely strong and undeformable Primarily we deal here with the forces and motions associated with particles and rigid bodies.

Mechanics of deformable solids :

Mechanics of solids: The mechanics of deformable solids is more concerned with the internal forces and associated changes in the geometry of the components involved. Of particular importance are the properties of the materials used, the strength of which will determine whether the components fail by breaking in service, and the stiffness of which will determine whether the amount of deformation they suffer is acceptable. Therefore, the subject of mechanics of materials or strength of materials is central to the whole activity of engineering design. Usually the objectives in analysis here will be the determination of the stresses, strains, and deflections produced by loads. Theoretical analyses and experimental results have an equal roles in this field.

Analysis of stress and strain :

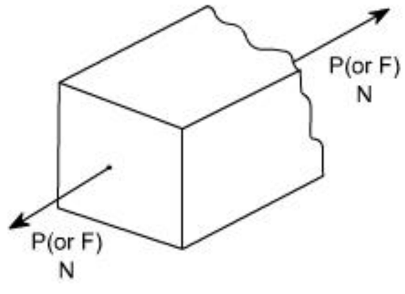
Concept of stress : Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

(i) due to service conditions
(ii) due to environment in which the component works
(iii) through contact with other members
(iv) due to fluid pressures
(v) due to gravity or inertia forces.

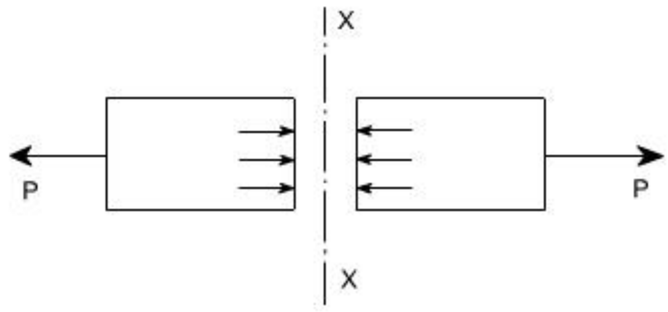
As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

These internal forces give rise to a concept of stress. Therefore, let us define a term stress
Stress:



Let us consider a rectangular bar of some cross sectional area and subjected to some load or force (in Newtons)

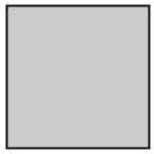
Let us imagine that the same rectangular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown



Now stress is defined as the force intensity or force per unit area. Here we use a symbol σ to represent the stress.

$$\sigma = \frac{P}{A}$$

Where A is the area of the X ? section



Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross ? section.

But the stress distributions may be far from uniform, with local regions of high stress known as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross sectional area, A, we must consider a small area, 'δA' which carries a small load δP, of the total force 'P', Then definition of stress is

$$\boxed{\sigma = \frac{\delta F}{\delta A}}$$

As a particular stress generally holds true only at a point, therefore it is defined mathematically as

$$\boxed{\sigma = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}}$$

Units : The basic units of stress in S.I units i.e. (International system) are N / m^2 (or Pa)

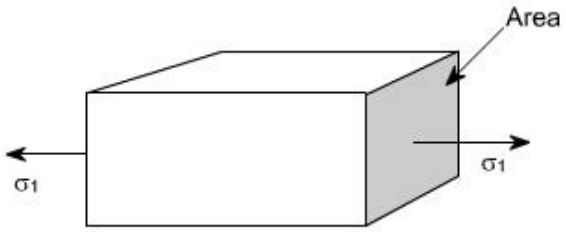
- MPa = 10^6 Pa
- GPa = 10^9 Pa
- KPa = 10^3 Pa

Some times N / mm^2 units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

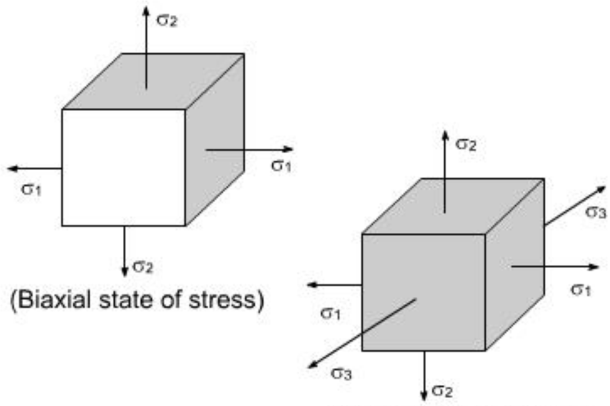
TYPES OF STRESSES : only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

Let us define the normal stresses and shear stresses in the following sections.

Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (σ)



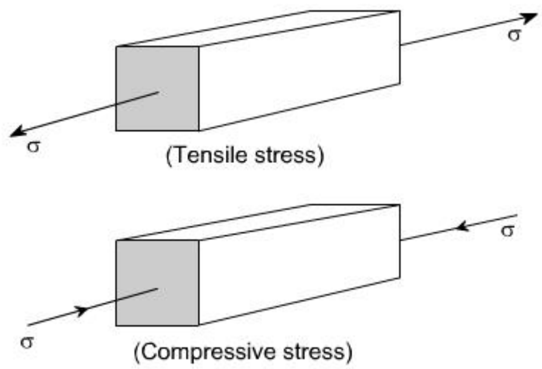
This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :



(Biaxial state of stress)

(Triaxial state of stress)

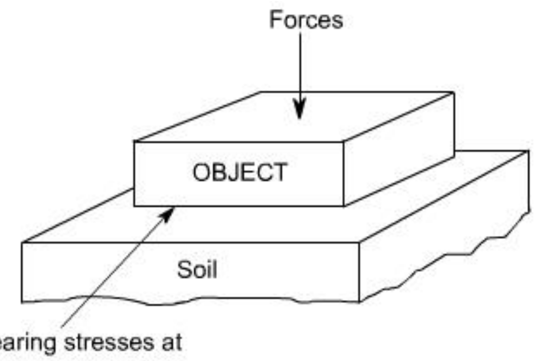
Tensile or compressive stresses : The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area



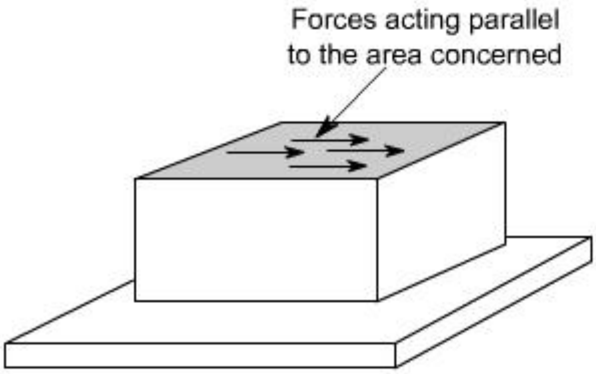
(Tensile stress)

(Compressive stress)

Bearing Stress : When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).



Shear stresses : Let us consider now the situation, where the cross ? sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interistes are known as shear stresses.



The resulting force intensities are known as shear stresses, the mean shear stress being equal to

$$\tau = \frac{P}{A}$$

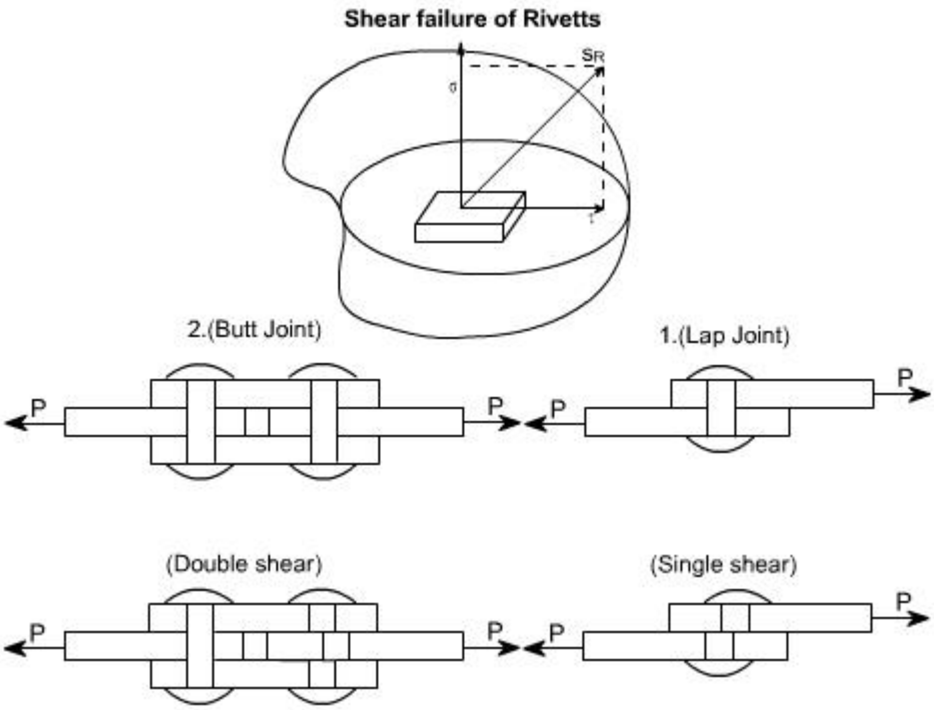
Where P is the total force and A the area over which it acts.

As we know that the particular stress generally holds good only at a point therefore we can define shear stress at a point as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

The greek symbol τ (tau) (suggesting tangential) is used to denote shear stress.

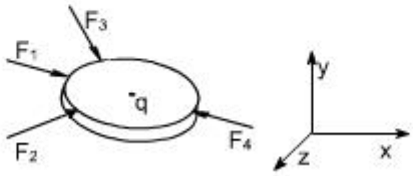
However, it must be borne in mind that the stress (resultant stress) at any point in a body is basically resolved into two components σ and τ one acts perpendicular and other parallel to the area concerned, as it is clearly defined in the following figure.



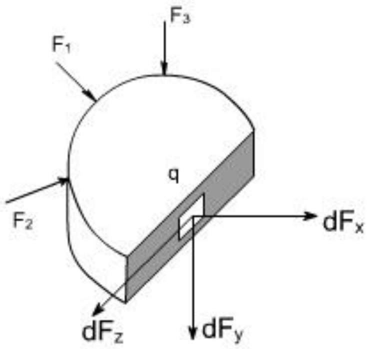
The single shear takes place on the single plane and the shear area is the cross - sectional of the rivett, whereas the double shear takes place in the case of Butt joints of rivetts and the shear area is the twice of the X - sectional area of the rivett.

LECTURE 2: ANALYSIS OF STERSSES

General State of stress at a point : Stress at a point in a material body has been defined as a force per unit area. But this definition is some what ambiguous since it depends upon what area we consider at that point. Let us, consider a point ?q' in the interior of the body



Let us pass a cutting plane through a pont 'q' perpendicular to the x - axis as shown below

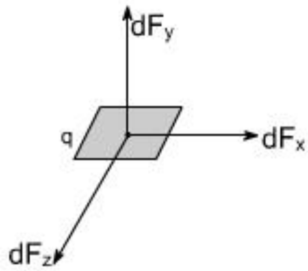


The corresponding force components can be shown like this

- $dF_x = \sigma_{xx}.\ da_x$
- $dF_y = \tau_{xy}.\ da_x$
- $dF_z = \tau_{xz}.\ da_x$

where da_x is the area surrounding the point 'q' when the cutting plane $\perp$ to x - axis.

In a similar way it can be assumed that the cutting plane is passed through the point 'q' perpendicular to the y - axis. The corresponding force components are shown below

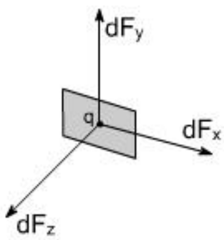


The corresponding force components may be written as

- $dF_x = \tau_{yx}.\ da_y$
- $dF_y = \sigma_{yy}.\ da_y$
- $dF_z = \tau_{yz}.\ da_y$

where da_y is the area surrounding the point 'q' when the cutting plane $\perp^r$ is to y - axis.

In the last it can be considered that the cutting plane is passed through the point 'q' perpendicular to the z - axis.



The corresponding force components may be written as

- $dF_x = \tau_{zx}. da_z$
- $dF_y = \tau_{zy}. da_z$
- $dF_z = \sigma_{zz}. da_z$

where da_z is the area surrounding the point 'q' when the cutting plane $\perp^r$ is to z - axis.

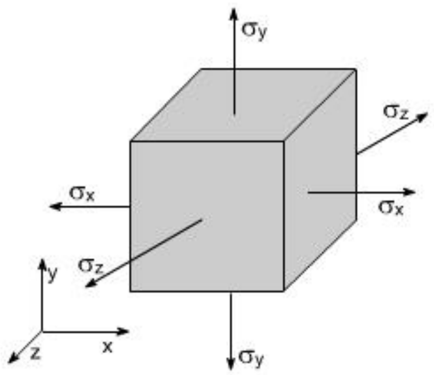
Thus, from the foregoing discussion it is amply clear that there is nothing like stress at a point 'q' rather we have a situation where it is a combination of state of stress at a point q. Thus, it becomes imperative to understand the term state of stress at a point 'q'. Therefore, it becomes easy to express a state of stress by the scheme as discussed earlier, where the stresses on the three mutually perpendicular planes are labeled in the manner as shown earlier. the state of stress as depicted earlier is called the general or a triaxial state of stress that can exist at any interior point of a loaded body.

Before defining the general state of stress at a point. Let us make ourselves conversant with the notations for the stresses.

We have already chosen to distinguish between normal and shear stress with the help of symbols σ and τ .

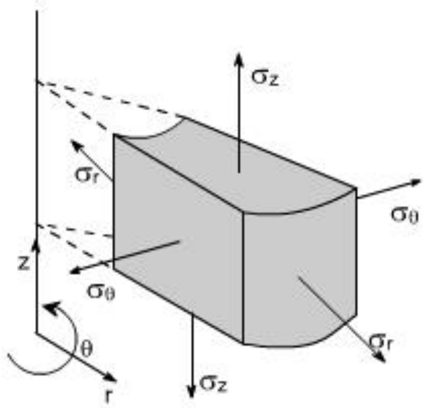
Cartesian - co-ordinate system: In the Cartesian co-ordinates system, we make use of the axes, X, Y and Z

Let us consider the small element of the material and show the various normal stresses acting the faces



Thus, in the Cartesian co-ordinates system the normal stresses have been represented by σ_x, $\sigma_y$and σ_z.

Cylindrical - co-ordinate system: In the Cylindrical - co-ordinate system we make use of co-ordinates r, θ and Z.



Thus, in the Cylindrical co-ordinates system, the normal stresses i.e components acting over a element is being denoted by σ_r, $\sigma_\theta$and σ_z.

Sign convention : The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).

First subscript : it indicates the direction of the normal to the surface.

Second subscript : it indicates the direction of the stress.

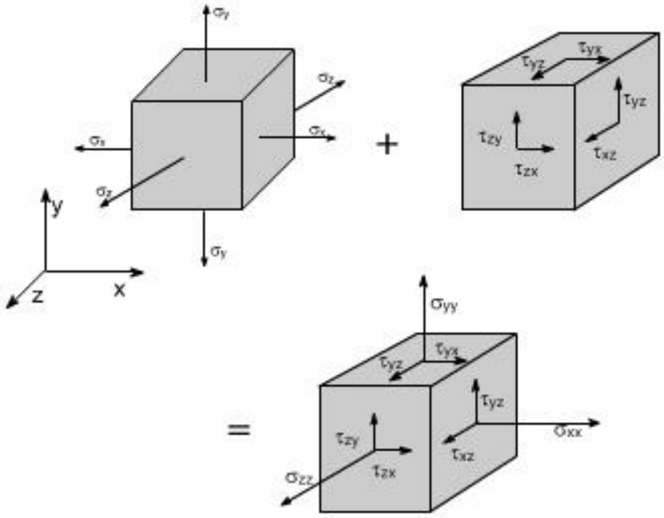
It may be noted that in the case of normal stresses the double script notation may be dispensed with as the direction of the normal stress and the direction of normal to the surface of the element on which it acts is the same. Therefore, a single subscript notation as used is sufficient to define the normal stresses.

Shear Stresses : With shear stress components, the single subscript notation is not practical, because such stresses are in direction parallel to the surfaces on which they act. We therefore have two directions to specify, that of normal to the surface and the stress itself. To do this, we stress itself. To do this, we attach two subscripts to the symbol ' τ' , for shear stresses.

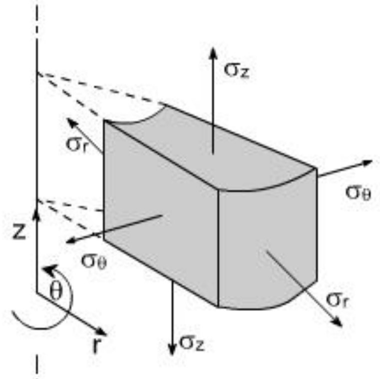
In cartesian and polar co-ordinates, we have the stress components as shown in the figures.

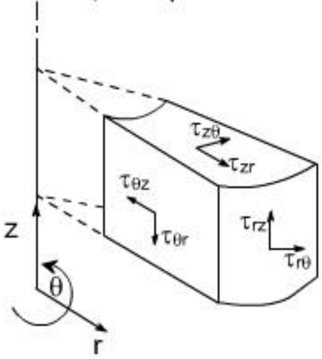
τ_{xy} , τ_{yx} , τ_{yz} , τ_{zy} , τ_{zx} , τ_{xz}

$\tau_{r\theta}$, $\tau_{\theta r}$, $\tau_{\theta z}$, $\tau_{z\theta}$,τ_{zr} , τ_{rz}

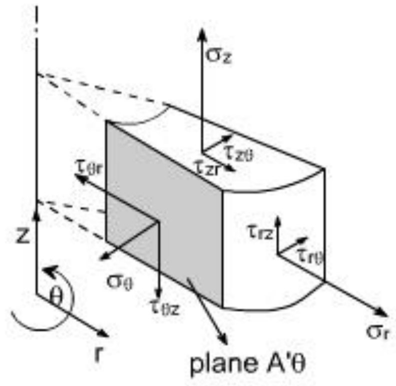


So as shown above, the normal stresses and shear stress components indicated on a small element of material separately has been combined and depicted on a single element. Similarly for a cylindrical co-ordinate system let us shown the normal and shear stresses components separately.





Now let us combine the normal and shear stress components as shown below :



Now let us define the state of stress at a point formally.

State of stress at a point : By state of stress at a point, we mean an information which is required at that point such that it remains under equilibrium. or simply a general state of stress at a point involves all the normal stress components, together with all the shear stress components as shown in earlier figures.

Therefore, we need nine components, to define the state of stress at a point

$$\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \sigma_y & \tau_{yx} & \tau_{yz} \\ \sigma_z & \tau_{zx} & \tau_{zy} \end{matrix}$$

If we apply the conditions of equilibrium which are as follows:

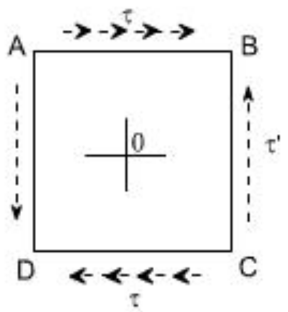
$$\sum F_x = 0 \; ; \; \sum M_x = 0$$
$$\sum F_y = 0 \; ; \; \sum M_y = 0$$
$$\sum F_z = 0 \; ; \; \sum M_z = 0$$

Then we get
$$\tau_{xy} = \tau_{yx}$$
$$\tau_{yz} = \tau_{zy}$$
$$\tau_{zx} = \tau_{xy}$$

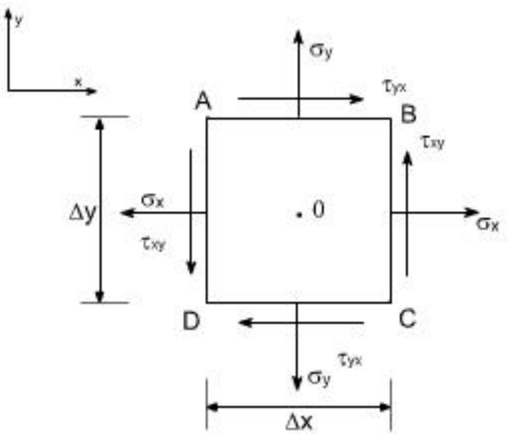
Then we will need only six components to specify the state of stress at a point i.e σ_x , σ_y, σ_z , τ_{xy} , τ_{yz} , τ_{zx}

Now let us define the concept of complementary shear stresses.

Complementary shear stresses: The existence of shear stresses on any two sides of the element induces complementary shear stresses on the other two sides of the element to maintain equilibrium.



on planes AB and CD, the shear stress τ acts. To maintain the static equilibrium of this element, on planes AD and BC, τ' should act, we shall see that τ' which is known as the complementary shear stress would come out to equal and opposite to the τ . Let us prove this thing for a general case as discussed below:



The figure shows a small rectangular element with sides of length Δx, Δy parallel to x and y directions. Its thickness normal to the plane of paper is Δz in z ? direction. All nine normal and shear stress components may act on the element, only those in x and y directions are shown.

Sign convections for shear stresses:

Direct stresses or normal stresses

- tensile +ve

- compressive -ve

Shear stresses:

- tending to turn the element C.W +ve.

- tending to turn the element C.C.W -ve.

The resulting forces applied to the element are in equilibrium in x and y direction. (Although other normal and shear stress components are not shown, their presence does not affect the final conclusion).

Assumption : The weight of the element is neglected.

Since the element is a static piece of solid body, the moments applied to it must also be in equilibrium. Let ?O' be the centre of the element. Let us consider the axis through the point ?O'. the resultant force associated with normal stresses σ_x and σ_y acting on the sides of the element each pass through this axis, and therefore, have no moment.

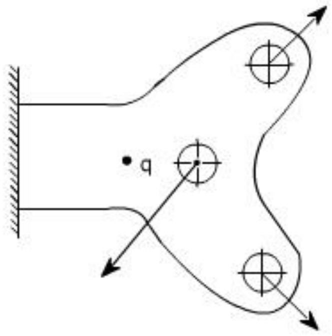
Now forces on top and bottom surfaces produce a couple which must be balanced by the forces on left and right hand faces

Thus, $$\tau_{yx}\ .\ \Delta x\ .\ \Delta z\ .\ \Delta y = \tau_{xy}\ .\ \Delta x\ .\ \Delta z\ .\ \Delta y \Rightarrow \tau_{yx} = \tau_{xy}$$

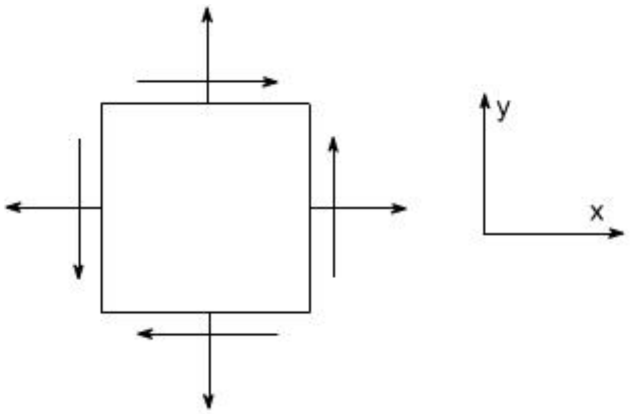
In other word, the complementary shear stresses are equal in magnitude. The same form of relationship can be obtained for the other two pair of shear stress components to arrive at the relations

$$\boxed{\begin{array}{l} \tau_{zy} = \tau_{zy} \\ \tau_{zx} = \tau_{xz} \end{array}}$$

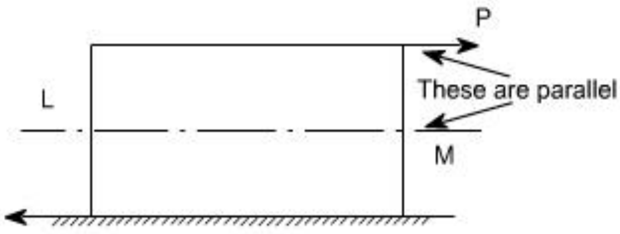
LECTURE 3: ANALYSIS OF STRESSES



Consider a point 'q' in some sort of structural member like as shown in figure below. Assuming that at point 'q' a plane state of stress exist. i.e. the state of state stress is to describe by a parameters σ_x, σ_y and τ_{xy} These stresses could be indicated on the two dimensional diagram as shown below:



This is a common way of representing the stresses. It must be realized that the material is unaware of what we have called the x and y axes. i.e. the material has to resist the loads irrespective less of how we wish to name them or whether they are horizontal, vertical or otherwise further more, the material will fail when the stresses exceed beyond a permissible value. Thus, a fundamental problem in engineering design is to determine the maximum normal stress or maximum shear stress at any particular point in a body. There is no reason to believe a priori that σ_x, σ_y and τ_{xy} are the maximum value. Rather the maximum stresses may associates themselves with some other planes located at ?θ'. Thus, it becomes imperative to determine the values of σ_θ and τ_θ. In order tto achieve this let us consider the following.

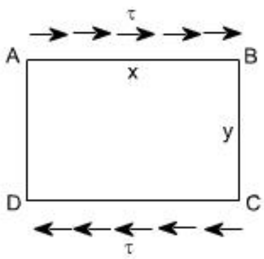


Shear stress: If the applied load P consists of two equal and opposite parallel forces not in the same line, than there is a tendency for one part of the body to slide over or shear from the other part across any section LM. If the cross section at LM measured parallel to the load is A, then the average value of shear stress τ = P/A . The shear stress is tangential to the area over which it acts.

If the shear stress varies then at a point then τ may be defined as

$$\tau = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

Complementary shear stress:



Let ABCD be a small rectangular element of sides x, y and z perpendicular to the plane of paper let there be shear stress acting on planes AB and CD

It is obvious that these stresses will from a couple (τ . xz)y which can only be balanced by tangential forces on planes AD and BC. These are known as complementary shear stresses. i.e. the existence of shear stresses on sides AB and CD of the element implies that there must also be complementary shear stresses on to maintain equilibrium.

Let τ' be the complementary shear stress induced on planes AD and BC. Then for the equilibrium (τ . xz)y = τ' (yz)x

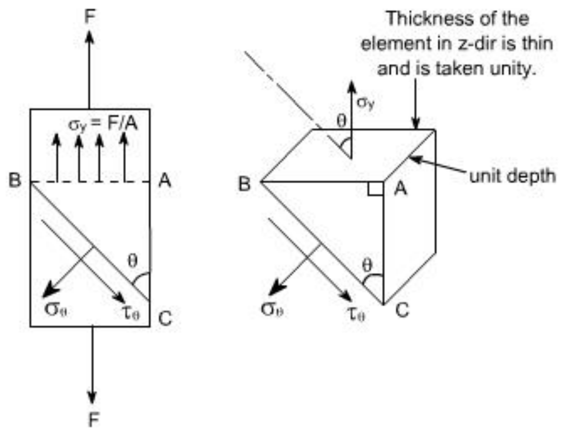
$$\boxed{\tau = \tau'}$$

Thus, every shear stress is accompanied by an equal complementary shear stress.

Stresses on oblique plane: Till now we have dealt with either pure normal direct stress or pure shear stress. In many instances, however both direct and shear stresses acts and the resultant stress across any section will be neither normal nor tangential to the plane.

A plane state of stress is a 2 dimensional state of stress in a sense that the stress components in one direction are all zero i.e $\sigma_z = \tau_{yz} = \tau_{zx} = 0$

Examples of plane state of stress includes plates and shells. Consider the general case of a bar under direct load F giving rise to a stress σ_y vertically



The stress acting at a point is represented by the stresses acting on the faces of the element enclosing the point.

The stresses change with the inclination of the planes passing through that point i.e. the stress on the faces of the element vary as the angular position of the element changes.

Let the block be of unit depth now considering the equilibrium of forces on the triangle portion ABC

Resolving forces perpendicular to BC, gives

$$\sigma_\theta.BC.1 = \sigma_y \sin\theta \;.\; AB \;.\; 1$$

$$\text{but } AB/BC = \sin\theta \text{ or } AB = BC\sin\theta$$

Substituting this value in the above equation, we get

σ_θ.BC.1 = $\sigma_y$sinθ . BCsinθ . 1 or $\boxed{\sigma_\theta = \sigma_y \ .\sin^2 2\theta}$ (1)

Now resolving the forces parallel to BC

$$\tau_\theta.BC.1 = \sigma_y \cos\theta \,.\, AB\sin\theta \,.\, 1$$

$$\text{again } AB = BC\cos\theta$$

$$\tau_\theta.BC.1 = \sigma_y\cos\theta \,.\, BC\sin\theta \,.\, 1 \text{ or } \tau_\theta = \sigma_y\sin\theta\cos\theta$$

$$\boxed{\tau_\theta = \frac{1}{2}.\sigma_y.\sin 2\theta} \qquad (2)$$

If $\theta = 90^0$ the BC will be parallel to AB and $\tau_\theta = 0$, i.e. there will be only direct stress or normal stress.

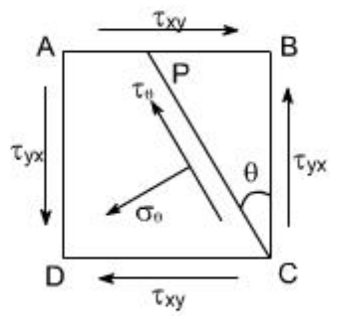
By examining the equations (1) and (2), the following conclusions may be drawn

The value of direct stress σ_θ is maximum and is equal to σ_y when $\theta = 90^0$.

The shear stress τ_θ has a maximum value of 0.5 σ_y when $\theta = 45^0$

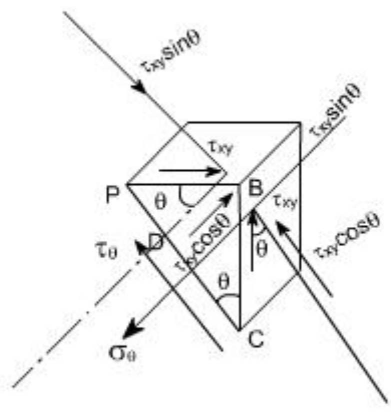
The stresses σ_θ and σ_θ are not simply the resolution of σ_y

Material subjected to pure shear: Consider the element shown to which shear stresses have been applied to the sides AB and DC



Complementary shear stresses of equal value but of opposite effect are then set up on the sides AD and BC in order to prevent the rotation of the element. Since the applied and complementary shear stresses are of equal value on the x and y planes. Therefore, they are both represented by the symbol τ_{xy}.

Now consider the equilibrium of portion of PBC



Assuming unit depth and resolving normal to PC or in the direction of σ_θ

$$\sigma_\theta.PC.1 = \tau_{xy}.PB.\cos\theta.1 + \tau_{xy}.BC.\sin\theta.1$$

$$= \tau_{xy}.PB.\cos\theta + \tau_{xy}.BC.\sin\theta$$

Now writing PB and BC in terms of PC so that it cancels out from the two sides

$$PB/PC = \sin\theta \quad BC/PC = \cos\theta$$
$$\sigma_\theta.PC.1 = \tau_{xy}.\cos\theta\sin\theta PC + \tau_{xy}.\cos\theta.\sin\theta PC$$
$$\sigma_\theta = 2\tau_{xy}\sin\theta\cos\theta$$
$$\sigma_\theta = \tau_{xy}.2.\sin\theta\cos\theta$$

$$\boxed{\sigma_\theta = \tau_{xy}.\sin 2\theta} \qquad (1)$$

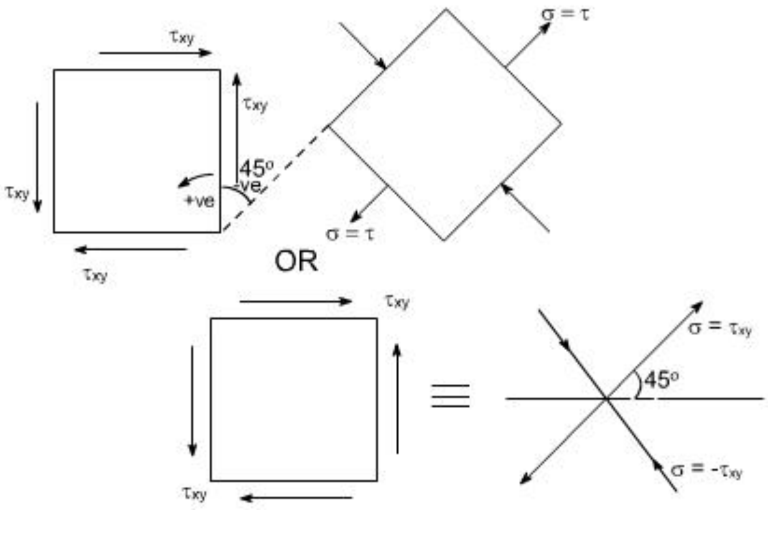
Now resolving forces parallel to PC or in the direction τ_θ.then $\tau_{xy}PC \,.\, 1 = \tau_{xy} \,.\, PB\sin\theta - \tau_{xy} \,.\, BC\cos\theta$

–ve sign has been put because this component is in the same direction as that of τ_θ.

again converting the various quantities in terms of PC we have

$$\tau_{xy}PC \,.\, 1 = \tau_{xy} \,.\, PB.\sin^2\theta - \tau_{xy} \,.\, PC\cos^2\theta$$
$$= -[\, \tau_{xy} (\cos^2\theta - \sin^2\theta) \,]$$
$$= -\tau_{xy}\cos 2\theta \text{ or } \boxed{\tau_\theta = -\tau_{xy} \cos 2\theta} \qquad (2)$$

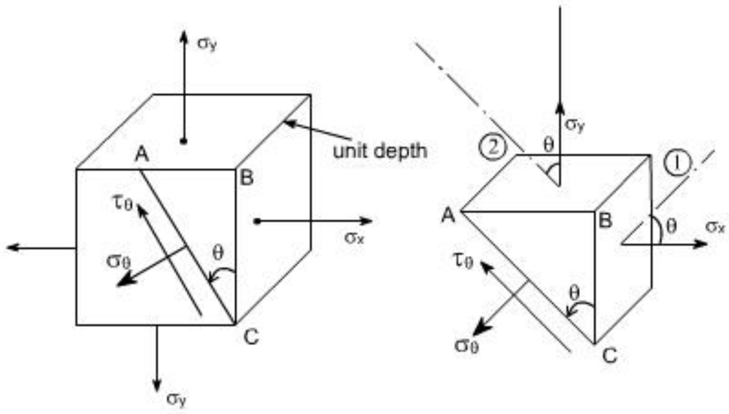
- the negative sign means that the sense of τ_θ is opposite to that of assumed one. Let us examine the equations (1) and (2) respectively
- From equation (1) i.e, $\sigma_\theta = \tau_{xy} \sin 2\theta$
- The equation (1) represents that the maximum value of σ_θ is τ_{xy} when $\theta = 45^0$.
- Let us take into consideration the equation (2) which states that $\tau_\theta = -\tau_{xy} \cos 2\theta$
- It indicates that the maximum value of τ_θ is τ_{xy} when $\theta = 0^0$ or 90^0. it has a value zero when $\theta = 45^0$.
- From equation (1) it may be noticed that the normal component σ_θ has maximum and minimum values of $+\tau_{xy}$ (tension) and $-\tau_{xy}$ (compression) on plane at $\pm 45^0$ to the applied shear and on these planes the tangential component τ_θ is zero.
- Hence the system of pure shear stresses produces and equivalent direct stress system, one set compressive and one tensile each located at 45^0 to the original shear directions as depicted in the figure below:



Material subjected to two mutually perpendicular direct stresses: Now consider a rectangular element of unit depth, subjected to a system of two direct stresses both tensile, σ_x and $\sigma_y$acting right angles to each other.

for equilibrium of the portion ABC, resolving perpendicular to AC

$$\sigma_\theta.\text{ AC.1} = \sigma_y \sin\theta.\text{ AB.1} + \sigma_x \cos\theta.\text{ BC.1}$$



converting AB and BC in terms of AC so that AC cancels out from the sides

$$\sigma_\theta = \sigma_y \sin^2\theta + \sigma_x \cos^2\theta$$

Futher, recalling that $\cos^2\theta - \sin^2\theta = \cos 2\theta$ or $(1 - \cos 2\theta)/2 = \sin^2\theta$

Similarly $(1 + \cos 2\theta)/2 = \cos^2\theta$

Hence by these transformations the expression for σ_θ reduces to

$$= 1/2\sigma_y (1 - \cos 2\theta) + 1/2\sigma_x (1 + \cos 2\theta)$$

On rearranging the various terms we get

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \qquad (3)$$

Now resolving parallel to AC $\quad \tau_\theta.\text{AC.1} = -\sigma_y.\cos\theta.\text{AB.1} + \sigma_x.\text{BC.}\sin\theta\ .1$

The -ve sign appears because this component is in the same direction as that of AC.

Again converting the various quantities in terms of AC so that the AC cancels out from the two sides.

$$\tau_\theta.\text{AC.1} = [\tau_x \cos\theta \sin\theta - \sigma_y \sin\theta \cos\theta]\text{AC}$$

$$\tau_\theta = (\sigma_x - \sigma_y)\sin\theta\cos\theta$$

$$= \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

$$\text{or} \quad \boxed{\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta} \qquad (4)$$

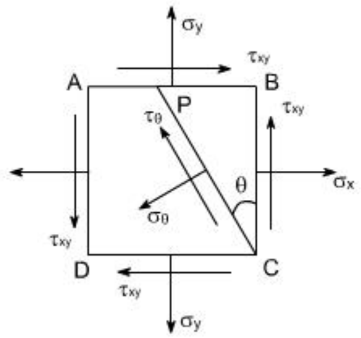
Conclusions : The following conclusions may be drawn from equation (3) and (4)

(i) The maximum direct stress would be equal to σ_x or σ_y which ever is the greater, when $\theta = 0^0$ or 90^0
(ii) The maximum shear stress in the plane of the applied stresses occurs when $\theta = 45^0$

$$\tau_{max} = \frac{(\sigma_x - \sigma_y)}{2}$$

LECTURE 4: Material subjected to combined direct and shear stresses

Now consider a complex stress system shown below, acting on an element of material. The stresses σ_x and σ_y may be compressive or tensile and may be the result of direct forces or as a result of bending. The shear stresses may be as shown or completely reversed and occur as a result of either shear force or torsion as shown in the figure below:



As per the double subscript notation the shear stress on the face BC should be notified as τ_{yx}, however, we have already seen that for a pair of shear stresses there is a set of complementary shear stresses generated such that $\tau_{yx} = \tau_{xy}$

By looking at this state of stress, it may be observed that this state of stress is combination of two different cases:

(i) Material subjected to pure state of stress shear. In this case the various formulas deserved are as follows

$$\sigma_\theta = \tau_{yx}\sin 2\theta$$
$$\tau_\theta = \tau_{yx}\cos 2\theta$$

(ii) Material subjected to two mutually perpendicular direct stresses. In this case the various formula's derived are as follows.

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta$$
$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

To get the required equations for the case under consideration, let us add the respective equations for the above two cases such that

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

- These are the equilibrium equations for stresses at a point. They do not depend on material proportions and are equally valid for elastic and inelastic behaviour
- This eqn gives two values of 2θ that differ by 180^0 .Hence the planes on which maximum and minimum normal stresses occur ate 90^0 apart.

For σ_θ to be a maximum or minimum $\frac{d\sigma_\theta}{d\theta} = 0$

Now

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\frac{d\sigma_\theta}{d\theta} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta.2 + \tau_{xy}\cos 2\theta.2$$
$$= 0$$

i.e. $-(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta.2 = 0$

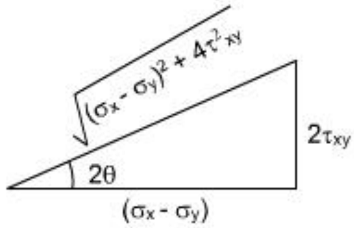
$\tau_{xy}\cos 2\theta.2 = (\sigma_x - \sigma_y)\sin 2\theta$

Thus, $\boxed{\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$

From the triangle it may be determined

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$\sin 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$



Substituting the values of cos2 θ and sin2 θ in equation (5) we get

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cdot\frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$+ \frac{\tau_{xy}\cdot 2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$= \frac{(\sigma_x + \sigma_y)}{2} + \frac{1}{2}\cdot\frac{(\sigma_x - \sigma_y)^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$+ \frac{1}{2}\frac{4\tau^2_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

or

$$= \frac{(\sigma_x + \sigma_y)}{2} + \frac{1}{2}\cdot\frac{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\cdot\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

Hence we get the two values of σ_θ, which are designated σ_1 as σ_2 and respectively,therefore

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

The σ_1 and σ_2 are termed as the principle stresses of the system.

Substituting the values of $\cos 2\theta$ and $\sin 2\theta$ in equation (6) we see that

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{1}{2}(\sigma_x - \sigma_y)\frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}} - \frac{\tau_{xy}\cdot(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$\tau_\theta = 0$$

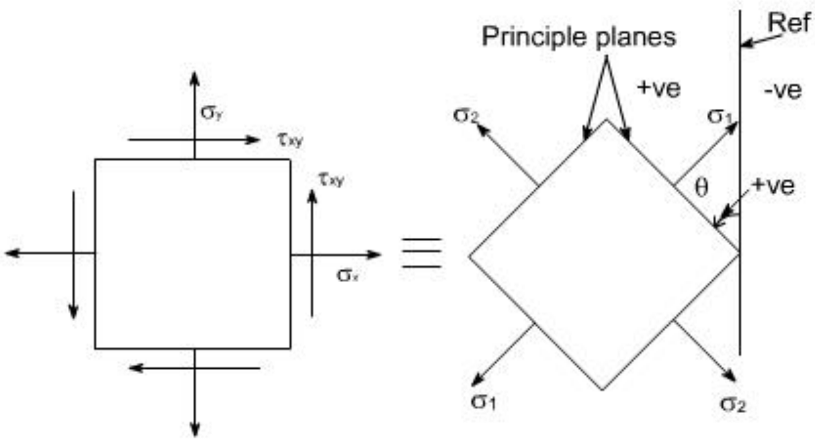
This shows that the values of shear stress is zero on the principal planes.

Hence the maximum and minimum values of normal stresses occur on planes of zero shearing stress. The maximum and minimum normal stresses are called the principal stresses, and the planes on which they act are called principal plane the solution of equation

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

will yield two values of 2θ separated by 180^0 i.e. two values of θ separated by 90^0 .Thus the two principal stresses occur on mutually perpendicular planes termed principal planes.

Therefore the two dimensional complex stress system can now be reduced to the equivalent system of principal stresses.



Let us recall that for the case of a material subjected to direct stresses the value of maximum shear stresses

$\tau_{max^m} = \frac{1}{2}(\sigma_x - \sigma_y)$ at $\theta = 45^0$, Thus, for a 2-dimensional state of stress, subjected to principle stresses

$\tau_{max^m} = \frac{1}{2}(\sigma_1 - \sigma_2)$, on substituting the values if σ_1 and σ_2, we get

$$\tau_{max^m} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

Alternatively this expression can also be obtained by differentiating the expression for τ_θ with respect to θ i.e.

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\frac{d\tau_\theta}{d\theta} = -\frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta.2 + \tau_{xy}\sin 2\theta.2$$

$$= 0$$

$$\text{or } (\sigma_x - \sigma_y)\cos 2\theta + 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = \frac{(\sigma_y - \sigma_x)}{2\tau_{xy}} = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

Recalling that

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

Thus,

$$\boxed{\tan 2\theta_p . \tan 2\theta_s = 1}$$

Therefore, it can be concluded that the equation (2) is a negative reciprocal of equation (1) hence the roots for the double angle of equation (2) are 90^0 away from the corresponding angle of equation (1).

This means that the angles that angles that locate the plane of maximum or minimum shearing stresses form angles of 45^0 with the planes of principal stresses.

Further, by making the triangle we get

$$\cos 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}}$$

$$\sin 2\theta = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}}$$

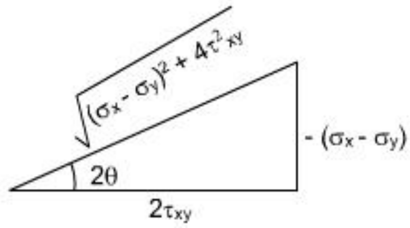
Therefore by substituting the values of $\cos 2\theta$ and $\sin 2\theta$ we have

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{1}{2}.-\frac{(\sigma_x - \sigma_y).(\sigma_x - \sigma_y)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}} - \frac{\tau_{xy}.2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}}$$

$$= -\frac{1}{2}.\frac{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2_{xy}}}$$

$$\tau_\theta = \pm\frac{1}{2}.\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$



Because of root the difference in sign convention arises from the point of view of locating the planes on which shear stress act. From physical point of view these sign have no meaning.

The largest stress regard less of sign is always know as maximum shear stress.

Principal plane inclination in terms of associated principal stress:

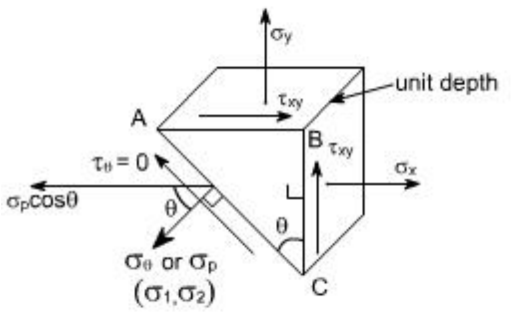
We know that the equation
$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

yields two values of q i.e. the inclination of the two principal planes on which the principal stresses s1 and s2 act. It is uncertain, however, which stress acts on which plane unless equation.

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

is used and observing which one of the two principal stresses is obtained.

Alternatively we can also find the answer to this problem in the following manner



Consider once again the equilibrium of a triangular block of material of unit depth, Assuming AC to be a principal plane on which principal stresses σ_p acts, and the shear stress is zero.

Resolving the forces horizontally we get: σ_x .BC . 1 + τ_{xy} .AB . 1 = σ_p . $\cos\theta$. AC dividing the above equation through by BC we get

$$\sigma_x. + \tau_{xy}\frac{AB}{BC} = \sigma_p.\cos\theta.\frac{AC}{BC}$$

or

$$\sigma_x + \tau_{xy}\tan\theta = \sigma_p$$

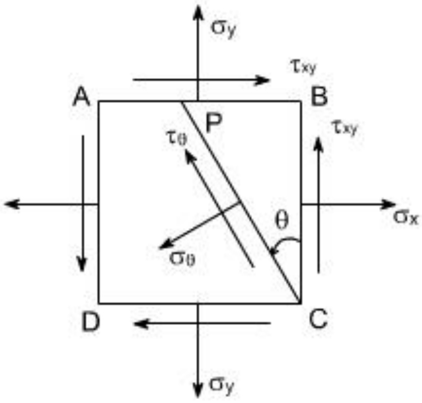
Thus

$$\boxed{\tan\theta = \frac{\sigma_p - \sigma_x}{\tau_{xy}}}$$

LECTURE 5: GRAPHICAL SOLUTION / MOHR'S STRESS CIRCLE

The transformation equations for plane stress can be represented in a graphical form known as Mohr's circle. This graphical representation is very useful in depending the relationships between normal and shear stresses acting on any inclined plane at a point in a stresses body.

To draw a Mohr's stress circle consider a complex stress system as shown in the figure



The above system represents a complete stress system for any condition of applied load in two dimensions

The Mohr's stress circle is used to find out graphically the direct stress σ and sheer stress τ on any plane inclined at θ to the plane on which σ_x acts. The direction of θ here is taken in anticlockwise direction from the BC.

STEPS: In order to do achieve the desired objective we proceed in the following manner

(i) Label the Block ABCD.

(ii) Set up axes for the direct stress (as abscissa) and shear stress (as ordinate)

(iii) Plot the stresses on two adjacent faces e.g. AB and BC, using the following sign convention.

Direct stresses – tensile positive; compressive, negative

Shear stresses ---? tending to turn block clockwise, positive

? tending to turn block counter clockwise, negative

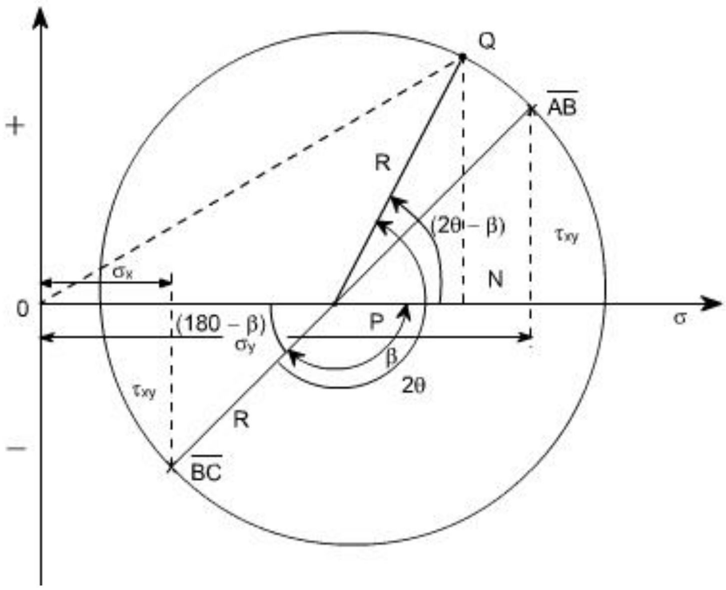
[i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]

This gives two points on the graph which may than be labeled as $\overline{AB}$ and $\overline{BC}$ respectively to denote stresses on these planes.

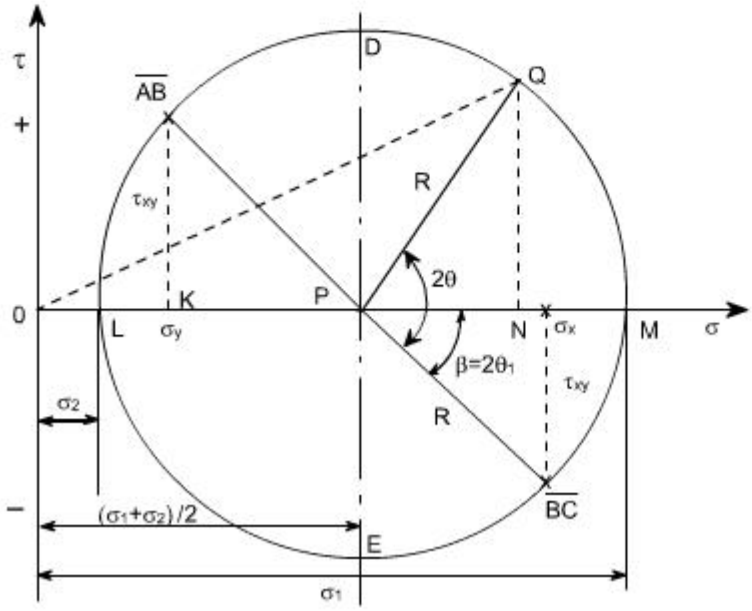
(iv) Join $\overline{AB}$ and $\overline{BC}$.

(v) The point P where this line cuts the s axis is than the centre of Mohr's stress circle and the line joining $\overline{AB}$ and $\overline{BC}$ is diameter. Therefore the circle can now be drawn.

Now every point on the circle then represents a state of stress on some plane through C.



Proof:



Consider any point Q on the circumference of the circle, such that PQ makes an angle 2θ with BC, and drop a perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an angle θ to BC. Here we have assumed that $\sigma_x > \sigma_y$

Now let us find out the coordinates of point Q. These are ON and QN.

From the figure drawn earlier

ON = OP + PN

OP = OK + KP

$$OP = \sigma_y + 1/2\,(\sigma_x - \sigma_y) = \sigma_y/2 + \sigma_y/2 + \sigma_x/2 - \sigma_y/2 = (\sigma_x + \sigma_y)/2$$

$$PN = R\cos(2\theta - \beta)$$

hence

$$ON = OP + PN = (\sigma_x+\sigma_y)/2 + R\cos(2\theta-\beta) = (\sigma_x+\sigma_y)/2 + R\cos2\theta\cos\beta + R\sin2\theta\sin\beta$$

now make the substitutions for Rcosβ and Rsinβ. $R\cos\beta = \dfrac{(\sigma_x - \sigma_y)}{2};\ R\sin\beta = \tau_{xy}$

Thus, $ON = (\sigma_x+\sigma_y)/2 + 1/2\,(\sigma_x+\sigma_y)\cos2\theta + \tau_{xy}\sin2\theta$ (1)

Similarly $QM = R\sin(2\theta-\beta) = R\sin2\theta\cos\beta - R\cos2\theta\sin\beta$

Thus, substituting the values of R cosβ and Rsinβ, we get

$$QM = 1/2\,(\sigma_x - \sigma_y)\sin2\theta - \tau_{xy}\cos2\theta \quad (2)$$

If we examine the equation (1) and (2), we see that this is the same equation which we have already derived analytically

Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at θ to BC in the original stress system.

N.B: Since angle $\overline{BC}$ PQ is 2θ on Mohr's circle and not θ it becomes obvious that angles are doubled on Mohr's circle. This is the only difference, however, as They are measured in the same direction and from the same plane in both figures.

Further points to be noted are :

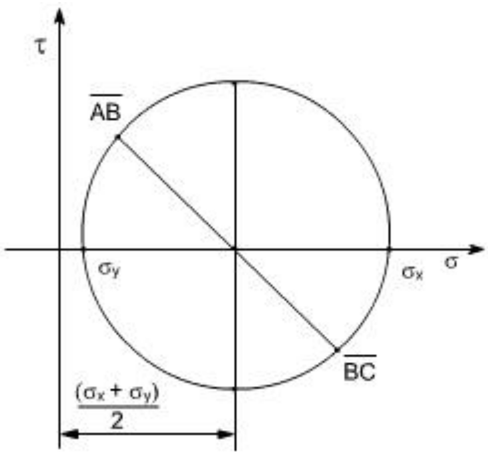
(1) The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses σ_1 and $2\theta_1$ gives the angle of the plane θ_1 from BC. Similar OL is the other principal stress and is represented by σ_2

(2) The maximum shear stress is given by the highest point on the circle and is represented by the radius of the circle.

This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie on the stress axis midway between σ_x and σ_y . [since $+\tau_{xy}$ & $-\tau_{xy}$ are shear stress & complimentary shear stress so they are same in magnitude but different in sign.]

(3) From the above point the maximum sheer stress i.e. the Radius of the Mohr's stress circle would be $\frac{(\sigma_x - \sigma_y)}{2}$

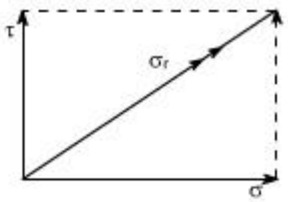
While the direct stress on the plane of maximum shear must be midway between σ_x and σ_y i.e $\frac{(\sigma_x + \sigma_y)}{2}$



(4) As already defined the principal planes are the planes on which the shear components are zero.

Therefore are conclude that on principal plane the sheer stress is zero.

(5) Since the resultant of two stress at 90^0 can be found from the parallogram of vectors as shown in the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.



(6) The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highly recommended.

LECTURE 6: ILLUSRATIVE PROBLEMS:

Let us discuss few representative problems dealing with complex state of stress to be solved either analytically or graphically.

PROB 1: A circular bar 40 mm diameter carries an axial tensile load of 105 kN. What is the Value of shear stress on the planes on which the normal stress has a value of 50 MN/m^2 tensile.

Solution: Tensile stress σ_y= F / A = 105 x 10^3 / π x (0.02)^2 = 83.55 MN/m^2

Now the normal stress on an obliqe plane is given by the relation

$\sigma_\theta = \sigma_y \sin^2\theta \Rightarrow$ 50 x 10^6 = 83.55 MN/m^2 x 10^{6}sin$^2\theta$ $\Rightarrow$ $\theta = 50^0 68'$

The shear stress on the oblique plane is then given by

τ_θ = 1/2 $\sigma_y$sin2θ = 1/2 x 83.55 x 10^6 x sin 101.36 = 40.96 MN/m^2

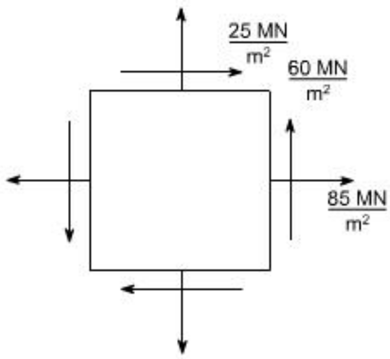
Therefore the required shear stress is 40.96 MN/m^2

PROB 2: For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows:

(a) 85 MN/m^2 tensile
(b) 25 MN/m^2 tensile at right angles to (a)
(c) Shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and (b) act; the sheer couple acting on planes carrying the 25 MN/m^2 stress is clockwise in effect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

Solution: The problem may be attempted both analytically as well as graphically. Let us first obtain the analytical solution



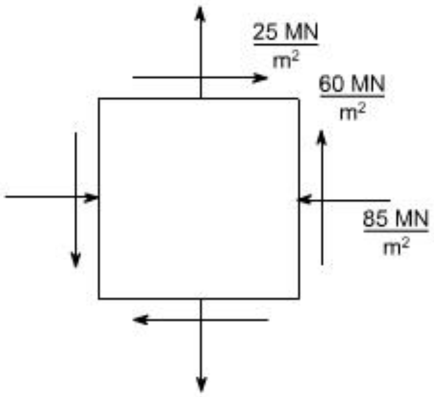
The principle stresses are given by the formula

$$\sigma_1 \text{ and } \sigma_2$$
$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$
$$= \frac{1}{2}(85 + 25) \pm \frac{1}{2}\sqrt{(85 + 25)^2 + (4 \text{x} 60^2)}$$
$$= 55 \pm \frac{1}{2}.60\sqrt{5} = 55 \pm 67$$
$$\Rightarrow \sigma_1 = 122 \text{ MN/m}^2$$
$$\sigma_2 = -12 \text{ MN/m}^2 (\text{compressive})$$

For finding out the planes on which the principle stresses act us the equation $\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$

The solution of this equation will yeild two values θ i.e they θ_1 and θ_2 giving θ_1= 31^{0}71' & θ_2= 121^{0}71'

(b) In this case only the loading (a) is changed i.e. its direction had been changed. While the other stresses remains unchanged hence now the block diagram becomes.



Again the principal stresses would be given by the equation.

$$\sigma_1 \;\&\; \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$= \frac{1}{2}(-85 + 25) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= \frac{1}{2}(-60) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= -30 \pm \frac{1}{2}\sqrt{12100 + 14400}$$

$$= -30 \pm 81.4$$

$$\sigma_1 = 51.4 \text{ MN/m}^2;\ \sigma_2 = -111.4 \text{ MN/m}^2$$

Again for finding out the angles use the following equation.

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

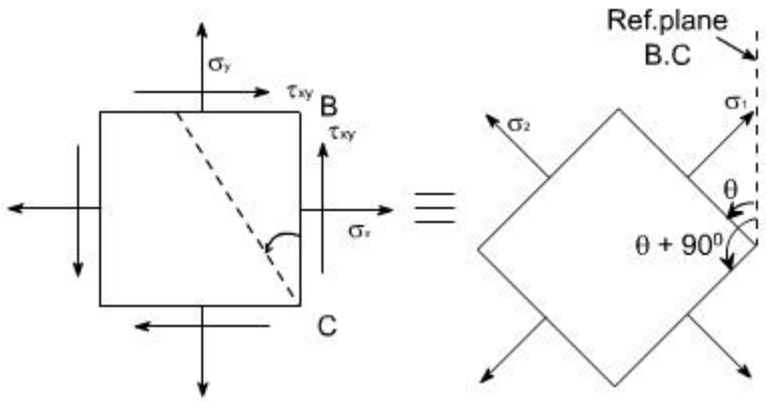
$$= \frac{2 \times 60}{-85 - 25} = +\frac{120}{-110}$$

$$= -\frac{12}{11}$$

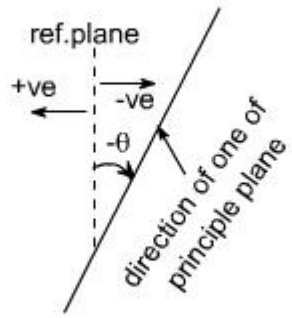
$$2\theta = \tan\left(-\frac{12}{11}\right)$$

$$\Rightarrow \quad \theta = -23.74^0$$

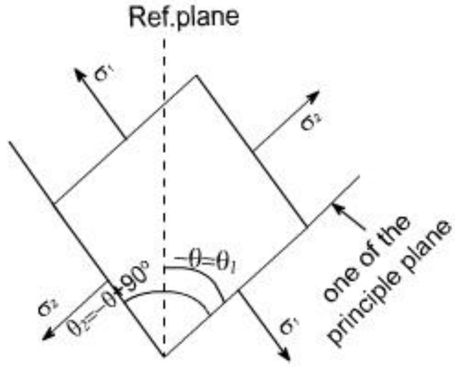
Thus, the two principle stresses acting on the two mutually perpendicular planes i.e principle planes may be depicted on the element as shown below:



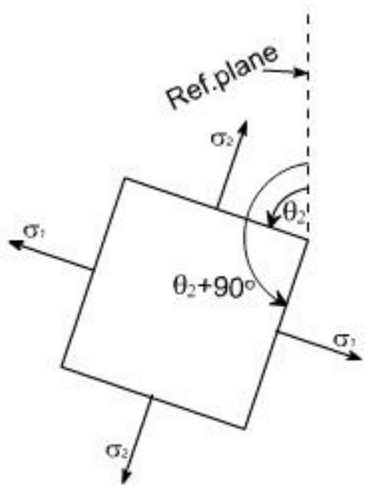
So this is the direction of one principle plane & the principle stresses acting on this would be σ_1 when is acting normal to this plane, now the direction of other principal plane would be $90^0 + \theta$ because the principal planes are the two mutually perpendicular plane, hence rotate the another plane $\theta + 90^0$ in the same direction to get the another plane, now complete the material element if θ is negative that means we are measuring the angles in the opposite direction to the reference plane BC .



Therefore the direction of other principal planes would be $\{-\theta + 90\}$ since the angle $-\theta$ is always less in magnitude then 90 hence the quantity ($-\theta + 90$) would be positive therefore the Inclination of other plane with reference plane would be positive therefore if just complete the Block. It would appear as



If we just want to measure the angles from the reference plane, than rotate this block through 180^0 so as to have the following appearance.



So whenever one of the angles comes negative to get the positive value,

first Add 90^0 to the value and again add 90^0 as in this case $\theta = -23^0 74'$

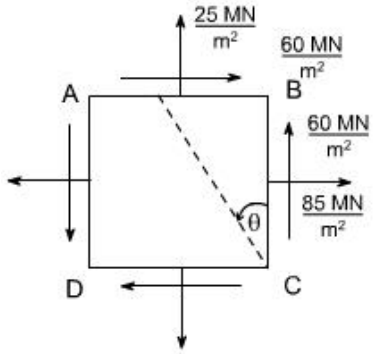
so $\theta_1 = -23^0 74' + 90^0 = 66^0 26'$ .Again adding 90^0 also gives the direction of other principle planes

i.e $\theta_2 = 66^0 26' + 90^0 = 156^0 26'$

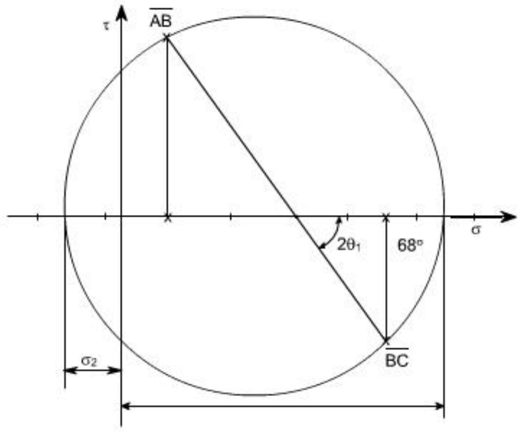
This is how we can show the angular position of these planes clearly.

GRAPHICAL SOLUTION:

Mohr's Circle solution: The same solution can be obtained using the graphical solution i.e the Mohr's stress circle, for the first part, the block diagram becomes



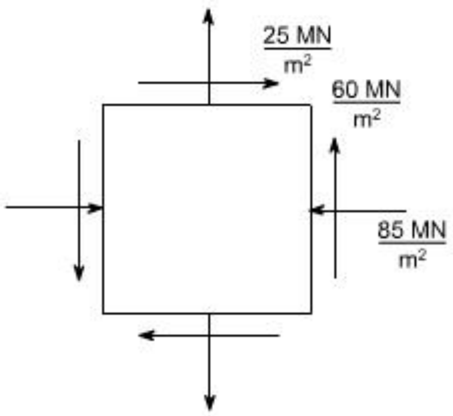
Construct the graphical construction as per the steps given earlier.



Taking the measurements from the Mohr's stress circle, the various quantities computed are

- σ_1 = 120 MN/m^2 tensile
- σ_2 = 10 MN/m^2 compressive
- $\theta_1 = 34^0$ counter clockwise from BC
- $\theta_2 = 34^0 + 90 = 124^0$ counter clockwise from BC

Part Second : The required configuration i.e the block diagram for this case is shown along with the stress circle.

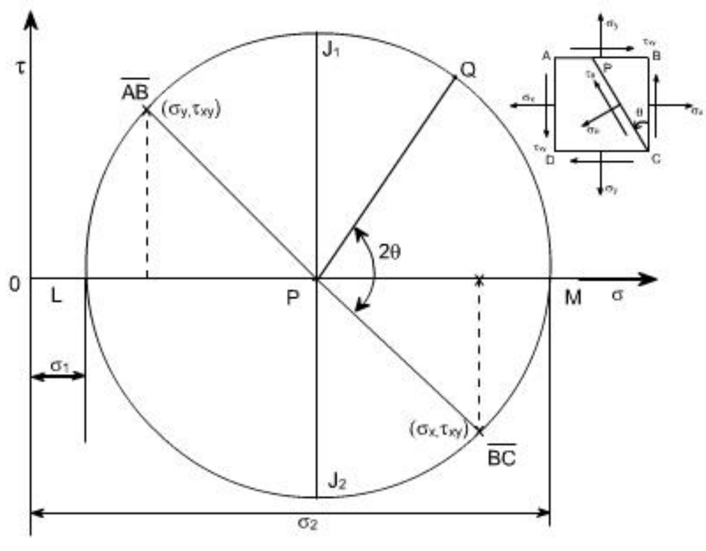


By taking the measurements, the various quantites computed are given as

- σ_1 = 56.5 MN/m^2 tensile
- σ_2 = 106 MN/m^2 compressive
- $\theta_1 = 66^0 15'$ counter clockwise from BC
- $\theta_2 = 156^0 15'$ counter clockwise from BC

Salient points of Mohr's stress circle:

1. complementary shear stresses (on planes 90^0 apart on the circle) are equal in magnitude
2. The principal planes are orthogonal: points L and M are 180^0 apart on the circle (90^0 apart in material)
3. There are no shear stresses on principal planes: point L and M lie on normal stress axis.
4. The planes of maximum shear are 45^0 from the principal points D and E are 90^0 , measured round the circle from points L and M.
5. The maximum shear stresses are equal in magnitude and given by points D and E
6. The normal stresses on the planes of maximum shear stress are equal i.e. points D and E both have normal stress co-ordinate which is equal to the two principal stresses.



As we know that the circle represents all possible states of normal and shear stress on any plane through a stresses point in a material. Further we have seen that the co-ordinates of the point ?Q' are seen to be the same as those derived from equilibrium of the element. i.e. the normal and shear stress components on any plane passing through the point can be found using Mohr's circle. Worthy of note:

1. The sides AB and BC of the element ABCD, which are 90^0 apart, are represented on the circle by $\overline{AB}$ P and $\overline{BC}$ P and they are 180^0 apart.
2. It has been shown that Mohr's circle represents all possible states at a point. Thus, it can be seen at a point. Thus, it, can be seen that two planes LP and PM, 180^0 apart on the diagram and therefore 90^0 apart in the material, on which shear stress τ_θ is zero. These planes are termed as principal planes and normal stresses acting on them are known as principal stresses. Thus , σ_1 = OL and σ_2 = OM
3. The maximum shear stress in an element is given by the top and bottom points of the circle i.e by points J_1 and J_2 ,Thus the maximum shear stress would be equal to the radius of i.e. $\tau_{max}= 1/2(\sigma_1 - \sigma_2)$,the corresponding normal stress is obviously the distance OP = $1/2(\sigma_x + \sigma_y)$, Further it can also be seen that the planes on which the shear stress is maximum are situated 90^0 from the principal planes (on circle), and 45^0 in the material.
4. The minimum normal stress is just as important as the maximum. The algebraic minimum stress could have a magnitude greater than that of the maximum principal stress if the state of stress were such that the centre of the circle is to the left of orgin.

i.e. if $\sigma_1 = 20$ MN/m^2 (say)

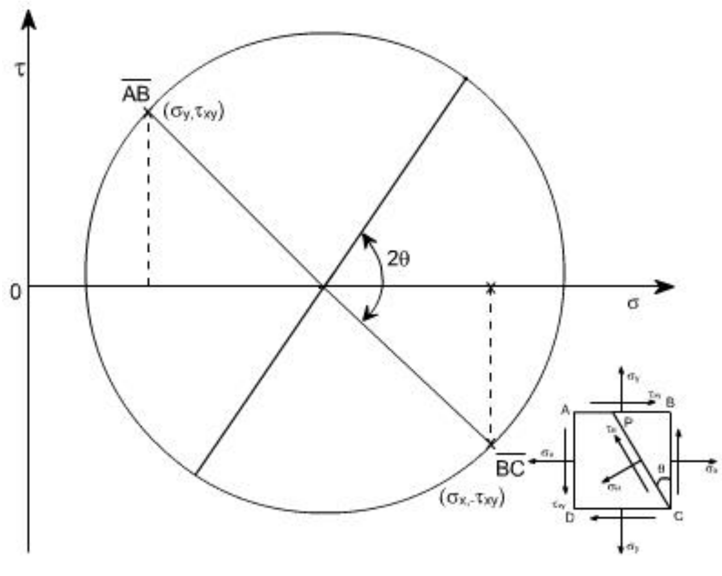
$\sigma_2 = -80$ MN/m^2 (say)

Then $\tau_{max}{}^{m} = (\sigma_1 - \sigma_2 / 2) = 50$ MN/m^2

If should be noted that the principal stresses are considered a maximum or minimum mathematically e.g. a compressive or negative stress is less than a positive stress, irrespective or numerical value.

5. Since the stresses on perpendicular faces of any element are given by the co-ordinates of two diametrically opposite points on the circle, thus, the sum of the two normal stresses for any and all orientations of the element is constant, i.e. Thus sum is an invariant for any particular state of stress.

Sum of the two normal stress components acting on mutually perpendicular planes at a point in a state of plane stress is not affected by the orientation of these planes.



This can be also understand from the circle Since AB and BC are diametrically opposite thus, what ever may be their orientation, they will always lie on the diametre or we can say that their sum won't change, it can also be seen from analytical relations

We know

$$\sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

on plane BC; $\theta = 0$

$\sigma_{n1} = \sigma_x$

on plane AB; $\theta = 270^0$

$\sigma_{n2} = \sigma_y$

Thus $\sigma_{n1} + \sigma_{n2} = \sigma_x + \sigma_y$

6. If $\sigma_1 = \sigma_2$, the Mohr's stress circle degenerates into a point and no shearing stresses are developed on xy plane.
7. If $\sigma_x + \sigma_y = 0$, then the center of Mohr's circle coincides with the origin of $\sigma - \tau$ co-ordinates.

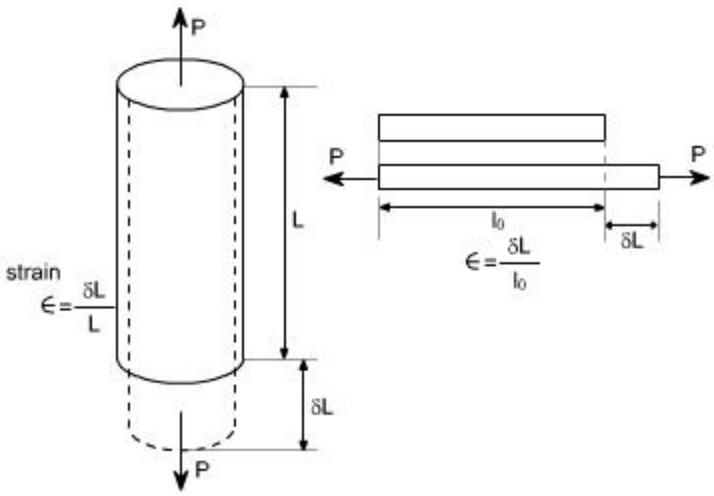
LECTURE 7: ANALYSIS OF STRAINS

CONCEPT OF STRAIN

Concept of strain : if a bar is subjected to a direct load, and hence a stress the bar will change in length. If the bar has an original length L and changes by an amount δL, the strain produce is defined as follows:

$$\text{strain}(\epsilon) = \frac{\text{change in length}}{\text{orginal length}} = \frac{\delta L}{L}$$

Strain is thus, a measure of the deformation of the material and is a non-dimensional Quantity i.e. it has no units. It is simply a ratio of two quantities with the same unit.

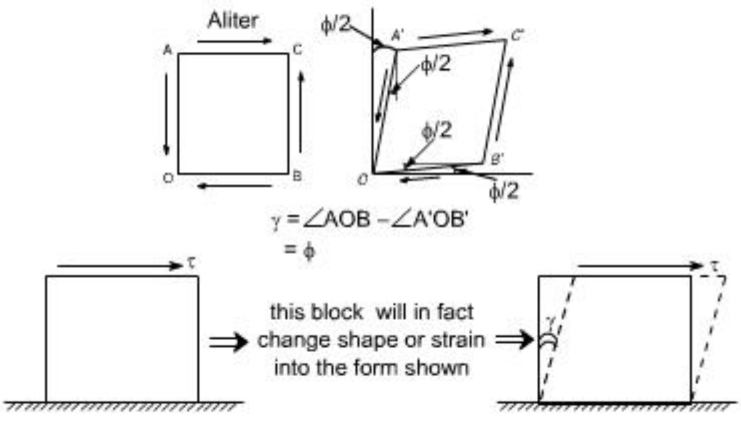


Since in practice, the extensions of materials under load are very small, it is often convenient to measure the strain in the form of strain x 10^{-6} i.e. micro strain, when the symbol used becomes $\mu \in$.

Sign convention for strain: Tensile strains are positive whereas compressive strains are negative. The strain defined earlier was known as linear strain or normal strain or the longitudinal strain now let us define the shear strain.

Definition: An element which is subjected to a shear stress experiences a deformation as shown in the figure below. The tangent of the angle through which two adjacent sides rotate relative to their initial position is termed shear strain. In many cases the angle is very small and the angle it self is used, (in radians), instead of tangent, so that $\gamma = \angle$ AOB - $\angle$ A'OB' = ϕ

Shear strain: As we know that the shear stresses acts along the surface. The action of the stresses is to produce or being about the deformation in the body consider the distortion produced by shear stress on an element or rectangular block



This shear strain or slide is ϕ and can be defined as the change in right angle. or The angle of deformation γ is then termed as the shear strain. Shear strain is measured in radians & hence is non-dimensional i.e. it has no unit. So we have two types of strain i.e. normal stress & shear stresses.

Hook's Law : A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed.

Hook's law therefore states that Stress (σ) α strain($\in$)

$$\boxed{\frac{\text{stress}}{\text{strain}} = \text{constant}}$$

Modulus of elasticity : Within the elastic limits of materials i.e. within the limits in which Hook's law applies, it has been shown that

Stress / strain = constant

Poisson's ratio: If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to σ / E . There will also be a strain in all directions at right angles to σ . The final shape being shown by the dotted lines.

It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain. The ratio of the lateral strain to longitudinal strain is known as the poison's ratio .

Poison's ratio (μ) = – lateral strain / longitudinal strain

For most engineering materials the value of μ his between 0.25 and 0.33.

Three dimensional state of strain : Consider an element subjected to three mutually perpendicular tensile stresses σ_x , σ_y and σ_z as shown in the figure below.

The negative sign indicating that if σ_y and σ_z are positive i.e. tensile, these they tend to reduce the strain in x direction thus the total linear strain is x direction is given by

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Principal strains in terms of stress: In the absence of shear stresses on the faces of the elements let us say that σ_x , σ_y , σ_z are in fact the principal stress. The resulting strain in the three directions would be the principal strains.

i.e. We will have the following relation.

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right]$$

$$\epsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right]$$

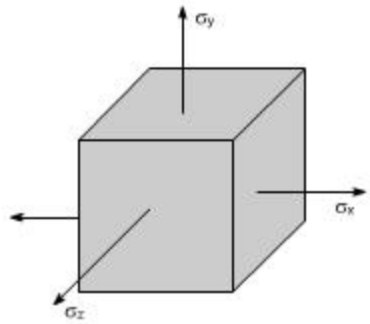
This constant is given by the symbol E and is termed as the modulus of elasticity or Young's modulus of elasticity

$$E = \frac{\text{strain}}{\text{stress}} = \frac{\sigma}{\epsilon}$$

$$= \frac{P/A}{\delta L/L}$$

Thus $\boxed{E = \frac{PL}{A\delta L}}$

The value of Young's modulus E is generally assumed to be the same in tension or compression and for most engineering material has high, numerical value of the order of 200 GPa



If σ_y and σ_z were not present the strain in the x direction from the basic definition of Young's modulus of Elasticity E would be equal to

$$\epsilon_x = \sigma_x / E$$

The effects of σ_y and σ_z in x direction are given by the definition of Poisson's ratio 'μ' to be equal as $-\mu\,\sigma_y/E$ and $-\mu\,\sigma_z/E$

For Two dimensional strain: system, the stress in the third direction becomes zero i.e $\sigma_z = 0$ or $\sigma_3 = 0$

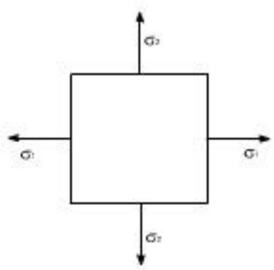
Although we will have a strain in this direction owing to stresses σ_1 & σ_2 .

Hence the set of equation as described earlier reduces to

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\,\sigma_2\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\,\sigma_1\right]$$

$$\epsilon_3 = \frac{1}{E}\left[-\mu\,\sigma_1 - \mu\,\sigma_2\right]$$



Hence a strain can exist without a stress in that direction

$$\text{i.e if } \sigma_3 = 0;\ \epsilon_3 = \frac{1}{E}\left[-\mu\sigma_1 - \mu\sigma_2\right]$$

Also

$$\epsilon_1 . E = \sigma_1 - \mu\sigma_2$$

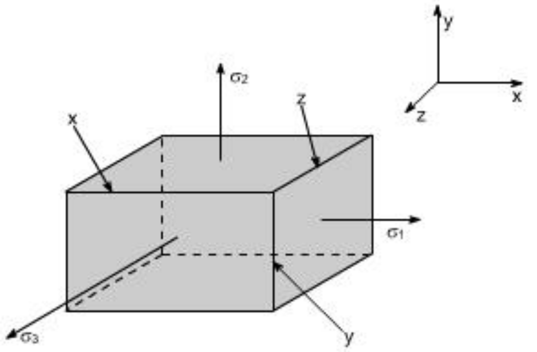
$$\epsilon_2 . E = \sigma_2 - \mu\sigma_1$$

so the solution of above two equations yields

$$\boxed{\begin{aligned}\sigma_1 &= \frac{E}{(1-\mu^2)}\left[\epsilon_1 + \mu\epsilon_2\right]\\ \sigma_2 &= \frac{E}{(1-\mu^2)}\left[\epsilon_2 + \mu\epsilon_1\right]\end{aligned}}$$

Hydrostatic stress : The term Hydrostatic stress is used to describe a state of tensile or compressive stress equal in all directions within or external to a body. Hydrostatic stress causes a change in volume of a material, which if expressed per unit of original volume gives a volumetric strain denoted by ϵ_v. So let us determine the expression for the volumetric strain.

Volumetric Strain:



Consider a rectangle solid of sides x, y and z under the action of principal stresses σ_1 , σ_2 , σ_3 respectively.

Then ϵ_1 , ϵ_2 , and ϵ_3 are the corresponding linear strains, than the dimensions of the rectangle becomes

$$(x + \epsilon_1 . x); (y + \epsilon_2 . y); (z + \epsilon_3 . z)$$

hence the

$$\begin{aligned}\text{Volumetric strain} &= \frac{\text{Increase in volume}}{\text{Original volume}}\\ &= \frac{x(1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3)z - xyz}{xyz}\\ &= (1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3) - 1 \cong \epsilon_1 + \epsilon_2 + \epsilon_3 \ \left[\text{Neglecting the products of } \epsilon\text{'s}\right]\end{aligned}$$

ALITER : Let a cuboid of material having initial sides of Length x, y and z. If under some load system, the sides changes in length by dx, dy, and dz then the new volume (x + dx) (y + dy) (z +dz)

- New volume = xyz + yzdx + xzdy + xydz
- Original volume = xyz
- Change in volume = yzdx +xzdy + xydz
- Volumetric strain = (yzdx +xzdy + xydz) / xyz = $\epsilon_x + \epsilon_y + \epsilon_z$
- Neglecting the products of epsilon's since the strains are sufficiently small.

Volumetric strains in terms of principal stresses: As we know that

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Futher Volumetric strain $= \epsilon_1 + \epsilon_2 + \epsilon_3$

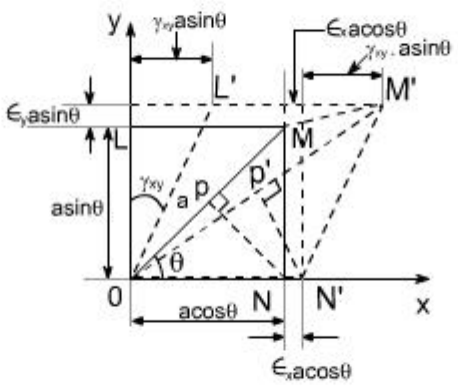
$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$

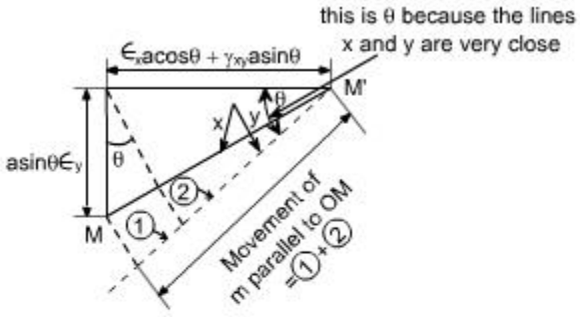
$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1-2\mu)}{E}$$

hence the

$$\boxed{\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1-2\mu)}{E}}$$

Strains on an oblique plane ---- (a) Linear strain





Consider a rectangular block of material OLMN as shown in the xy plane. The strains along ox and oy are $\in_x$ and $\in_y$, and γ_{xy} is the shearing strain.

Then it is required to find an expression for $\in_\theta$, i.e the linear strain in a direction inclined at θ to OX, in terms of $\in_x$,$\in_y$, γ_{xy} and θ.

Let the diagonal OM be of length 'a' then ON = a cos θ and OL = a sin θ , and the increase in length of those under strains are $\in_x a\cos\theta$ and $\in_y a \sin\theta$ (i.e. strain x original length) respectively.

If M moves to M', then the movement of M parallel to x axis is $\in_x a\cos\theta + \gamma_{xy}\sin\theta$ and the movement parallel to the y axis is $\in_y a\sin\theta$

Thus the movement of M parallel to OM , which since the strains are small is practically coincident with MM'. and this would be the summation of portions (1) and (2) respectively and is equal to

$$= (\epsilon_y\ a\sin\theta)\sin\theta + (\epsilon_x\ a\cos\theta + \gamma_{xy} a\sin\theta)\cos\theta$$

$$= a\left[\epsilon_y\ \sin\theta.\sin\theta + \epsilon_x\ \cos\theta.\cos\theta + \gamma_{xy}\ \sin\theta.\cos\theta\right]$$

hence the strain along OM

$$= \frac{\text{extension}}{\text{original length}}$$

$$\epsilon_\theta = \epsilon_x\ \cos^2\theta + \gamma_{xy}\sin\theta.\cos\theta + \epsilon_y\ \sin^2\theta$$

$$\epsilon_\theta = \epsilon_x\ \cos^2\theta + \epsilon_y\ \sin^2\theta + \gamma_{xy}\ \sin\theta.\cos\theta$$

$$\text{Recalling } \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\text{or } 2\cos^2\theta - 1 = \cos 2\theta$$

$$\cos^2\theta = \left[\frac{1+\cos 2\theta}{2}\right]$$

$$\sin^2\theta = \left[\frac{1-\sin 2\theta}{2}\right]$$

hence

$$\epsilon_\theta = \epsilon_x\ \left[\frac{1+\cos 2\theta}{2}\right] + \epsilon_y\ \left[\frac{1-\sin 2\theta}{2}\right] + \gamma_{xy} a\sin\theta.\cos\theta$$

$$= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

This expression is identical in form with the equation defining the direct stress on any inclined plane θ with $\in_x$ and $\in_y$ replacing σ_x and σ_y and ½ γ_{xy} replacing τ_{xy} i.e. the shear stress is replaced by half the shear strain

Shear strain: To determine the shear stain in the direction OM consider the displacement of point P at the foot of the perpendicular from N to OM and the following expression can be derived as

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]$$

In the above expression ½ is there so as to keep the consistency with the stress relations. Futher -ve sign in the expression occurs so as to keep the consistency of sign convention, because OM' moves clockwise with respect to OM it is considered to be negative strain.

The other relevant expressions are the following :

Principal planes :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

Principal strains :

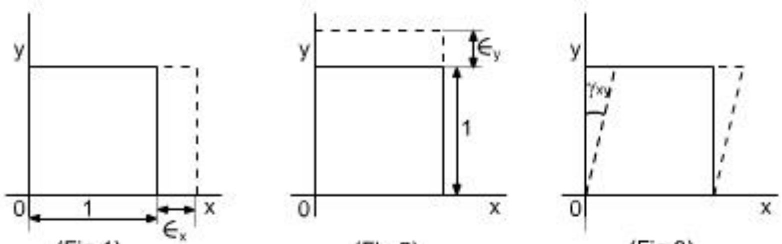
$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Maximum shear strains :

$$\frac{\gamma_{max}}{2} = \pm\sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Let us now define the plane strain condition

Plane Strain : In xy plane three strain components may exist as can be seen from the following figures:



(Fig.1) (Fig.2) (Fig.3)

Therefore, a strain at any point in body can be characterized by two axial strains i.e $\in_x$ in x direction, $\in_y$ in y - direction and γ_{xy} the shear strain.

In the case of normal strains subscripts have been used to indicate the direction of the strain, and $\in_x$, $\in_y$ are defined as the relative changes in length in the co-ordinate directions.

With shear strains, the single subscript notation is not practical, because such strains involves displacements and length which are not in same direction. The symbol and subscript γ_{xy} used for the shear strain referred to the x and y planes. The order of the subscript is unimportant. γ_{xy} and γ_{yx} refer to the same physical quantity. However, the sign convention is important. The shear strain γ_{xy} is considered to be positive if it represents a decrease the angle between the sides of an element of material lying parallel the positive x and y axes. Alternatively we can think of positive shear strains produced by the positive shear stresses and vice-versa.

Plane strain : An element of material subjected only to the strains as shown in Fig. 1, 2, and 3 respectively is termed as the plane strain state.

Thus, the plane strain condition is defined only by the components , $\in_y$, γ_{xy} : $\in_z = 0$; $\gamma_{xz} = 0$; $\gamma_{yz} = 0$

It should be noted that the plane stress is not the stress system associated with plane strain. The plane strain condition is associated with three dimensional stress system and plane stress is associated with three dimensional strain system.

LECTURE 8: PRINCIPAL STRAIN

For the strains on an oblique plane we have an oblique we have two equations which are identical in form with the equation defining the direct stress on any inclined plane θ .

$$\in_\theta = \left\{\frac{\in_x + \in_y}{2}\right\} + \left\{\frac{\in_x - \in_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\in_x - \in_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]$$

Since the equations for stress and strains on oblique planes are identical in form, so it is evident that Mohr's stress circle construction can be used equally well to represent strain conditions using the horizontal axis for linear strains and the vertical axis for half the shear strain.

It should be noted, however that the angles given by Mohr's stress circle refer to the directions of the planes on which the stress act and not the direction of the stresses themselves.

The direction of the stresses and therefore associated strains are therefore normal (i.e. at 90^0) to the directions of the planes. Since angles are doubled in Mohr's stress circle construction it follows therefore that for a true similarity of working a relative rotation of axes of $2 \times 90^0 = 180^0$ must be introduced. This is achieved by plotting positive sheer strains vertically downwards on the strain circle construction.

The sign convention adopted for the strains is as follows:

- **Linear Strains :** extension - positive and compression - negative
- { Shear of strains are taken positive, when they increase the original right angle of an unstrained element. }

Shear strains : for Mohr's strains circle sheer strain γ_{xy} - is +ve referred to x - direction the convention for the shear strains are bit difficult. The first subscript in the symbol γ_{xy} usually denotes the shear strains associated with direction. e.g. in γ_{xy}? represents the shear strain in x - direction and for γ_{yx}? represents the shear strain in y - direction. If under strain the line associated with first subscript moves counter clockwise with respect to the other line, the shearing strain is said to be positive, and if it moves clockwise it is said to be negative.

N.B: The positive shear strain is always to be drown on the top of $\in_x$ .If the shear stain γ_{xy} is given]

Mohr's strain circle: For the plane strain conditions can we derivate the following relations

$$\in_\theta = \left\{\frac{\in_x + \in_y}{2}\right\} + \left\{\frac{\in_x - \in_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta \qquad (1)$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\in_x - \in_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right] \qquad (2)$$

Re writing the equation (1) as below :

$$\left[\in_\theta - \left(\frac{\in_x + \in_y}{2}\right)\right] = \left\{\frac{\in_x - \in_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta \qquad (3)$$

squaring and adding equations (2) and (3)

$$\left[\in_\theta - \left(\frac{\in_x + \in_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left[\left\{\frac{\in_x - \in_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta\right]^2 + \left[\frac{1}{2}(\in_x - \in_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]^2$$

$$\left[\in_\theta - \left(\frac{\in_x + \in_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left(\frac{\in_x + \in_y}{2}\right)^2 + \frac{\gamma^2_{xy}}{4}$$

Now as we know that

$$\in_{1,2} = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\in_1 + \in_2 = \in_x + \in_y$$

$$\left(\frac{\in_1 - \in_2}{2}\right)^2 = \left(\frac{\in_x - \in_y}{2}\right)^2 + \frac{\gamma^2_{xy}}{4}$$

Therefore the equation gets transformed to

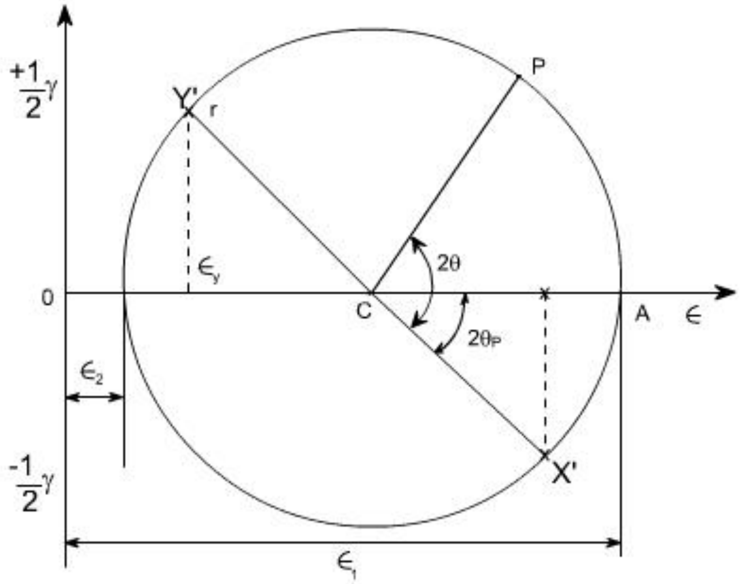
$$\left[\in_\theta - \left(\frac{\in_1 + \in_2}{2}\right)\right]^2 + \left[\frac{\gamma_\theta}{2}\right]^2 = \left(\frac{\in_1 - \in_2}{2}\right)^2 \qquad (4)$$

If we plot equation (4) we obtain a circle of radius $\left(\frac{\in_1 - \in_2}{2}\right)$ with center at $\left(\frac{\in_1 + \in_2}{2}, 0\right)$

A typical point P on the circle given the normal strain and half the sheer strain $1/2\gamma_{xy}$ associated with a particular plane. We note again that an angle subtended at the centre of Mohr's circle by an arc connecting two points on the circle is twice the physical angle in the material.

Mohr's strain circle : Since the transformation equations for plane strain are similar to those for plane stress, we can employ a similar form of pictorial representation. This is known as Mohr's strain circle.

The main difference between Mohr's stress circle and stress circle is that a factor of half is attached to the shear strains.



Points X' and Y' represents the strains associated with x and y directions with $\in$ and $\gamma_{xy}/2$ as co-ordiantes

Co-ordinates of X' and Y' points are located as follows :

$$X' = \left(\in_x, -\frac{\gamma_{xy}}{2}\right)$$

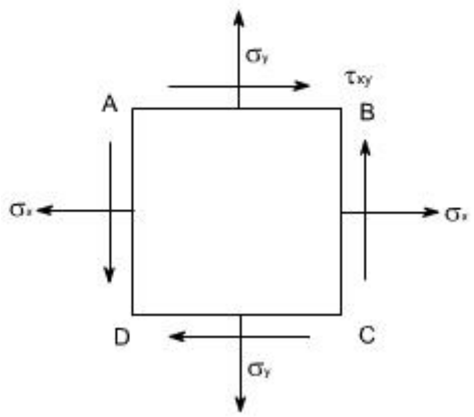
$$Y' = \left(\in_y, +\frac{\gamma_{xy}}{2}\right)$$

In x ? direction, the strains produced, the strains produced by σ_x,and $-\tau_{xy}$ are $\in_x$ and $-\gamma_{xy}/2$

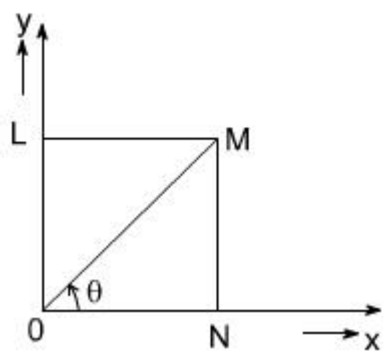
where as in the Y - direction, the strains are produced by $\in_y$ and $+\gamma_{xy}$ are produced by σ_y and $+\tau_{xy}$

These co-ordinated are consistent with our sign notation (i.e. + ve shear stresses produces produce +ve shear strain & vice versa)

on the face AB is τ_{xy}+ve i.e strains are ($\in_y$, $+\gamma_{xy}/2$) where as on the face BC, τ_{xy} is negative hence the strains are ($\in_x$, $-\gamma_{xy}/2$)



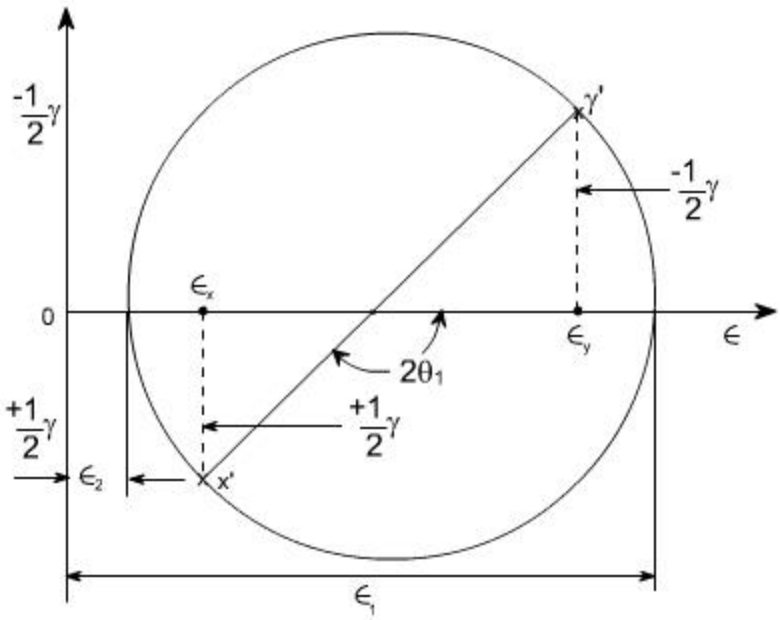
A typical point P on the circle gives the normal strains and half the shear strain, associated with a particular plane we must measure the angle from x ? axis (taken as reference) as the required formulas for $\in_\theta$, $-1/2\ \gamma_\theta$ have been derived with reference to x-axis with angle measuring in the c.c.W direction



CONSTRUCTION : In this we would like to locate the points x' & y' instead of AB and BC as we have done in the case of Mohr's stress circle.

steps

1. Take normal or linear strains on x-axis, whereas half of shear strains are plotted on y-axis.
2. Locate the points x' and y'
3. Join x' and y' and draw the Mohr's strain circle
4. Measure the required parameter from this construction.



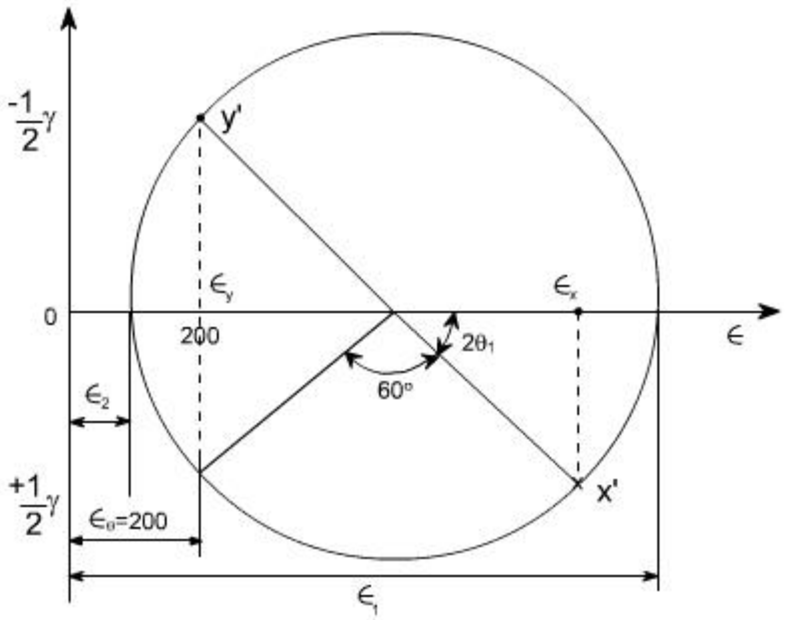
Note: positive shear strains are associated with planes carrying positive shear stresses and negative strains with planes carrying negative shear stresses.

ILLUSTRATIVE EXAMPLES :

1. At a certain point, a material is subjected to the following state of strains: $\epsilon_x = 400 \times 10^{-6}$ units, $\epsilon_y = 200 \times 10^{-6}$ units, $\gamma_{xy} = 350 \times 10^{-6}$ radians

Determine the magnitudes of the principal strains, the direction of the principal strains axes and the strain on an axis inclined at 30^0 clockwise to the x ? axis.

Solution: Draw the Mohr's strain circle by locating the points x' and y'



By Measurement the following values may be computed

$\epsilon_1 = 500 \text{ X } 10^{-6}$ units

$\epsilon_2 = 100 \times 10^{-6}$ units

$\theta_1 = 60^0 /2 = 30^0$

$\theta_2 = 90 + 30 = 12^0$

$\epsilon_{30} = 200 \times 10^{-6}$ units

The angles being measured c.c.w. from the direction of ϵ_x.

PROB 2.: A material is subjected to two mutually perpendicular strains $\in_x = 350 \times 10^{-6}$ units and $\in_y = 50 \times 10^{-6}$ units together with an unknown sheer strain γ_{xy} if the principal strain in the material is 420×10^{-6} units Determine the following.

(a) Magnitude of the shear strain
(b) The other principal strain
(c) The direction of principal strains axes
(d) The magnitude of the principal stresses
(e) If E = 200 GN / m^2; $\gamma = 0.3$

Solution : The Mohr's strain circle can be drawn as per the procedure described earlier. from the graphical construction, the following results may bre obtained :

(i) Shear strain $\gamma_{xy} = 324 \times 10^{-6}$ radians
(ii) other principal strain = -20×10^{-6}
(iii) direction of principal strain = $47^0 / 2 = 23^0\ 30'$
(iv) direction of other principal strain = $90^0 + 23^0\ 30' = 113^0\ 30'$

In order to determine the magnitude of principle stresses, the computed values of $\in_1$and $\in_2$ from the graphical construction may be substituted in the following expressions

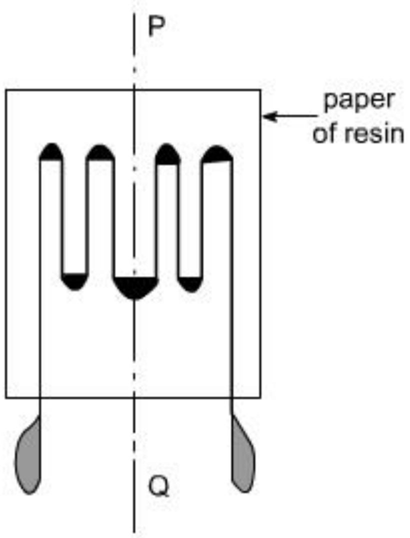
$$\sigma_1 = \frac{(\in_1 + \gamma\in_2)}{(1-\gamma^2)}.E = 91\ \frac{MN}{m^2}$$

$$\sigma_2 = \frac{(\in_2 + \gamma\in_1)}{(1-\gamma^2)}.E = 23\ \frac{MN}{m^2}$$

Use of strain Gauges : Although we can not measure stresses within a structural member, we can measure strains, and from them the stresses can be computed, Even so, we can only measure strains on the surface. For example, we can mark points and lines on the surface and measure changes in their spacing angles. In doing this we are of course only measuring average strains over the region concerned. Also in view of the very small changes in dimensions, it is difficult to archive accuracy in the measurements

In practice, electrical strain gage provide a more accurate and convenient method of measuring strains.

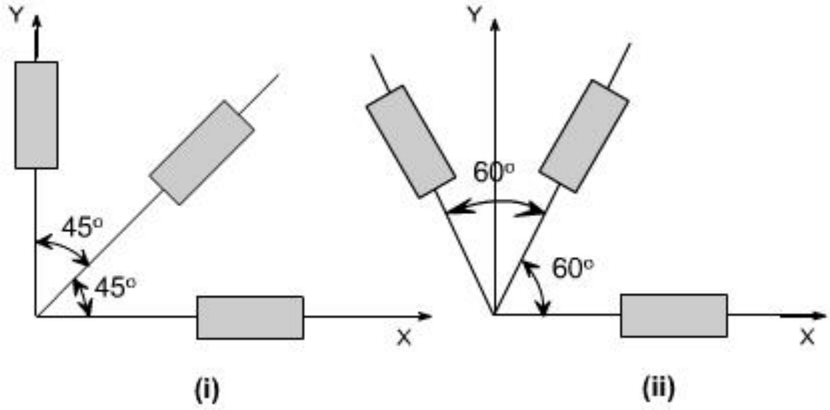
A typical strain gage is shown below.

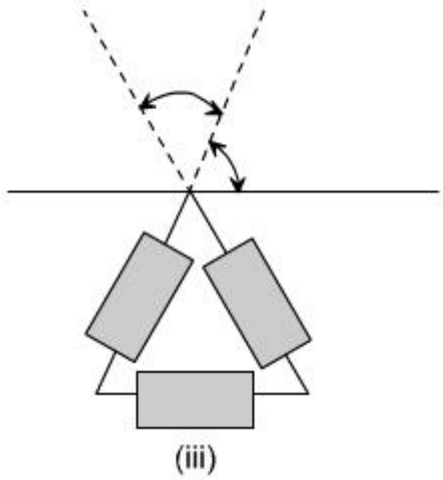


The gage shown above can measure normal strain in the local plane of the surface in the direction of line PQ, which is parallel to the folds of paper. This strain is an average value of for the region covered by the gage, rather than a value at any particular point.

The strain gage is not sensitive to normal strain in the direction perpendicular to PQ, nor does it respond to shear strain. therefore, in order to determine the state of strain at a particular small region of the surface, we usually need more than one strain gage.

To define a general two dimensional state of strain, we need to have three pieces of information, such as $\in_x$, $\in_y$ and γ_{xy} referred to any convenient orthogonal co-ordinates x and y in the plane of the surface. We therefore need to obtain measurements from three strain gages. These three gages must be arranged at different orientations on the surface to from a strain rosette. Typical examples have been shown, where the gages are arranged at either 45^0 or 60^0 to each other as shown below :

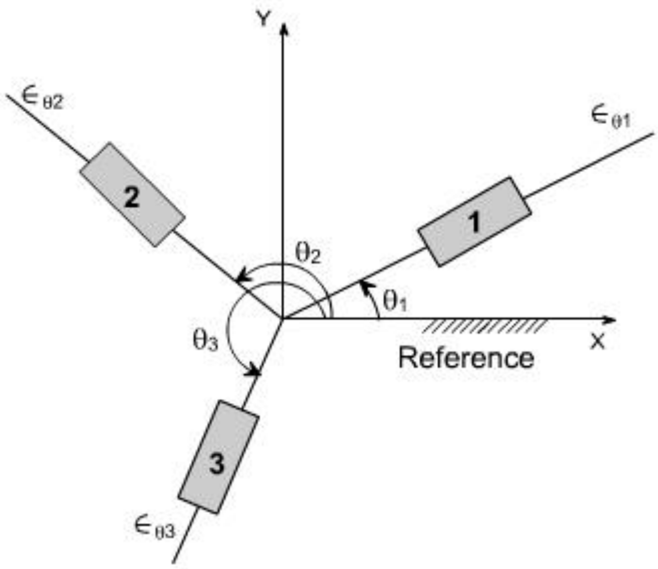




A group of three gages arranged in a particular fashion is called a strain rosette. Because the rosette is mounted on the surface of the body, where the material is in plane stress, therefore, the transformation equations for plane strain to calculate the strains in various directions.

Knowing the orientation of the three gages forming a rosette, together with the in ? plane normal strains they record, the state of strain at the region of the surface concerned can be found. Let us consider the general case shown in the figure below, where three strain gages numbered 1, 2, 3, where three strain gages numbered 1, 2, 3 are arranged at an angles of θ_1 , θ_2 , θ_3 measured c.c.w from reference direction, which we take as x ? axis.

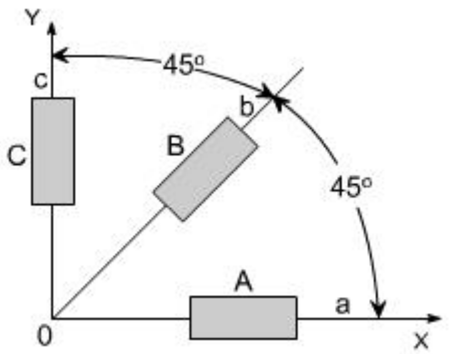
Now, although the conditions at a surface, on which there are no shear or normal stress components. Are these of plane stress rather than the plane strain, we can still use strain transformation equations to express the three measured normal strains in terms of strain components $\in_x$, $\in_y$, $\in_z$ and γ_{xy} referred to x and y co-ordinates as



$$\in_{\theta 1} = \in_x \cos^2\theta_1 + \in_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1.\cos\theta_1$$
$$\in_{\theta 2} = \in_x \cos^2\theta_2 + \in_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2.\cos\theta_2$$
$$\in_{\theta 3} = \in_x \cos^2\theta_3 + \in_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3.\cos\theta_3$$

This is a set of three simultaneous linear algebraic equations for the three unknowns $\in_x$, $\in_y$, γ_{xy} to solve these equation is a laborious one as far as manually is concerned, but with computer it can be readily done. Using these later on, the state of strain can be determined at any point.

Let us consider a 45^0 degree stain rosette consisting of three electrical ? resistance strain gages arranged as shown in the figure below :



The gages A, B,C measure the normal strains $\in_a$, $\in_b$, $\in_c$ in the direction of lines OA, OB and OC.

Thus

$$\in_{\theta 1} = \in_x \cos^2\theta_1 + \in_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1.\cos\theta_1$$

for $\theta_1 = 0; \in_{\theta 1} = \in_a$

$$\boxed{\in_{\theta 1} = \in_a} \qquad (1)$$

again

$$\in_{\theta 2} = \in_x \cos^2\theta_2 + \in_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2.\cos\theta_2$$

for gage B; $\theta_2 = 45^0$

$$\in_{\theta 2} = \in_b$$

$$\in_b = \in_x \cos^2 45^0 + \in_y \sin^2 45^0 + \gamma_{xy}\sin 45^0.\cos 45^0$$

$$\in_b = \in_x \frac{1}{2} + \in_y \frac{1}{2} + \frac{1}{2}\gamma_{xy} = \frac{\in_x + \in_y + \gamma_{xy}}{2}$$

$$\gamma_{xy} = 2\in_b - (\in_x + \in_y)$$

since $\in_x = \in_a$

$$\boxed{\gamma_{xy} = 2\in_b - (\in_a + \in_y)} \qquad (2)$$

for the gage C

$$\in_{\theta 3} = \in_x \cos^2\theta_3 + \in_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3.\cos\theta_3$$

for $\theta_3 = 90^0; \in_{\theta 3} = \in_c$

or $\boxed{\in_c = \in_y}$ (3)

Thus, substituting the relation (3) in the equation (2) we get

$\gamma_{xy} = 2\in_b - (\in_a + \in_c)$ and other equation becomes

$\in_x = \in_a$; $\in_y = \in_c$

Since the gages A and C are aligned with the x and y axes, they give the strains $\in_x$ and $\in_y$ directly

Thus, $\in_x$, $\in_y$ and γ_{xy} can easily be determined from the strain gage readings. Knowing these strains, we can calculate the strains in any other directions by means of Mohr's circle or from the transformation equations.

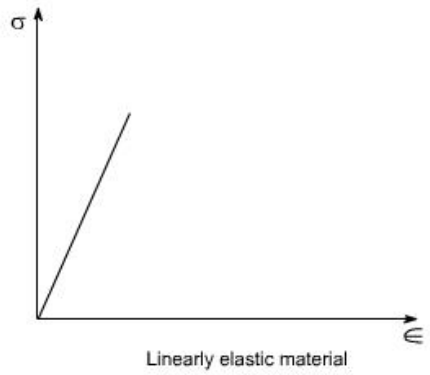
The 60^0 Rosette: For the 60^0 strain rosette, using the same procedure we can obtain following relation.

$$\in_x = \in_a$$
$$\in_y = \frac{1}{3}(2.\in_b + 2.\in_c - \in_a)$$
$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\in_c - \in_b)$$

LECTURE 9: STRESS - STRAIN RELATIONS

Stress / Strain Relations: The Hook's law, states that within the elastic limits the stress is proportional to the strain since for most materials it is impossible to describe the entire stress strain curve with simple mathematical expression, in any given problem the behavior of the materials is represented by an idealized stress strain curve, which emphasizes those aspects of the behaviors which are most important is that particular problem.

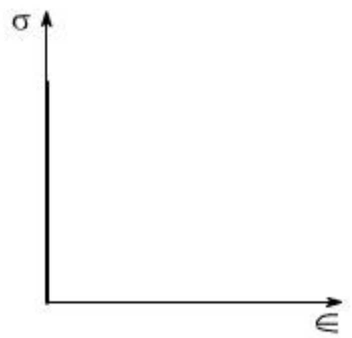
(i) Linear elastic material: A linear elastic material is one in which the strain is proportional to stress as shown below:



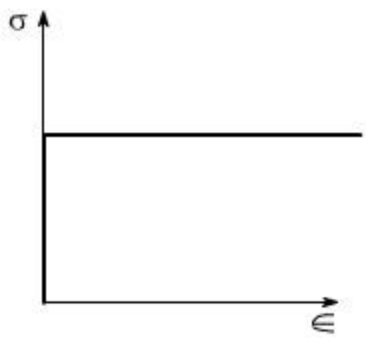
Linearly elastic material

There are also other types of idealized models of material behavior.

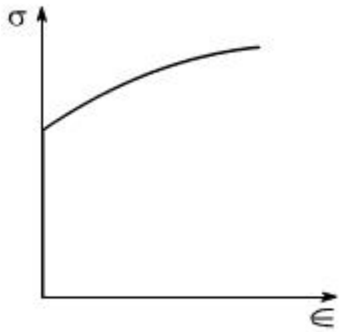
(ii) Rigid Materials: It is the one which do not experience any strain regardless of the applied stress.



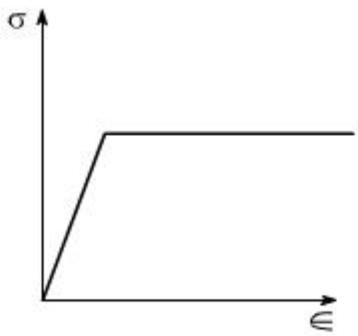
(iii) Perfectly plastic(non-strain hardening): A perfectly plastic i.e non-strain hardening material is shown below:



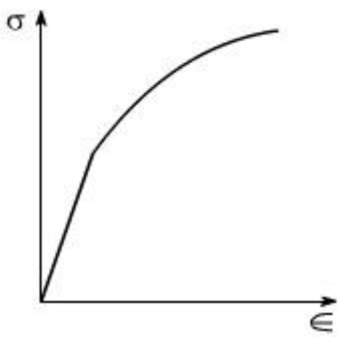
(iv) Rigid Plastic material(strain hardening): A rigid plastic material i.e strain hardening is depicted in the figure below:



(v) Elastic Perfectly Plastic material: The elastic perfectly plastic material is having the characteristics as shown below:



(vi) Elastic Plastic material: The elastic plastic material exhibits a stress Vs strain diagram as depicted in the figure below:



Elastic Stress and strain Relations :

Previously stress strain relations were considered for the special case of a uni-axial loading i.e. only one component of stress i.e. the axial or normal component of stress was coming into picture. In this section we shall generalize the elastic behavior, so as to arrive at the relations which connect all the six components of stress with the six components of elastic stress. Further, we would restrict ourselves to linearly elastic material.

Before writing down the relations let us introduce a term ISOTROPY

ISOTROPIC: If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is isotropic or in other words we can say that isotropy of a material in a characteristics, which gives us the information that the properties are the same in the three orthogonal directions x y z, on the other hand if the response is dependent on orientation it is known as an-isotropic.

Examples of an-isotropic materials, whose properties are different in different directions are

(i) Wood
(ii) Fibre reinforced plastic
(iii) Reinforced concrete

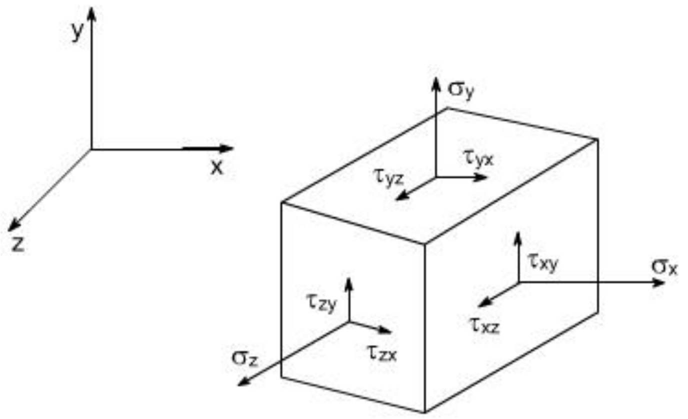
HOMOGENIUS: A material is homogenous if it has the same composition through out body. Hence the elastic properties are the same at every point in the body. However, the properties need not to be the same in all the direction for the material to be homogenous. Isotropic materials have the same elastic properties in all the directions. Therefore, the material must be both homogenous and isotropic in order to have the lateral strains to be same at every point in a particular component.

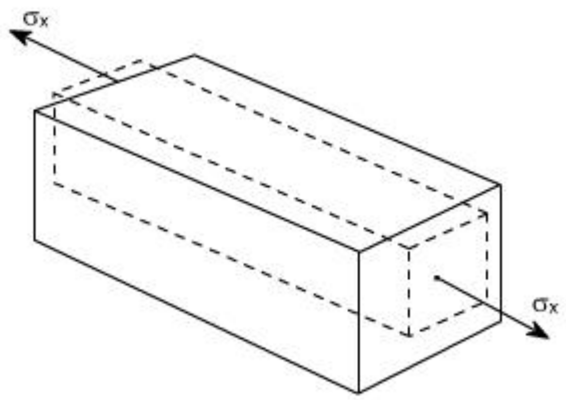
Generalized Hook's Law: We know that for stresses not greater than the proportional limit.

$$\epsilon = \frac{\sigma}{E} \text{ or } \mu = -\frac{|\epsilon_{lateral}|}{|\epsilon_{axial}|}$$

These equation expresses the relationship between stress and strain (Hook's law) for uniaxial state of stress only when the stress is not greater than the proportional limit. In order to analyze the deformational effects produced by all the stresses, we shall consider the effects of one axial stress at a time. Since we presumably are dealing with strains of the order of one percent or less. These effects can be superimposed arbitrarily. The figure below shows the general tri-axial state of stress.

Let us consider a case when σ_x alone is acting. It will cause an increase in dimension in X-direction whereas the dimensions in y and z direction will be decreased.

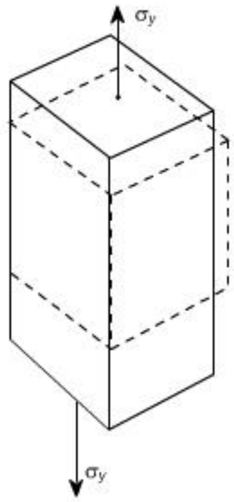




$$\epsilon_x = \frac{\sigma_x}{E}, \epsilon_y = -\mu\epsilon_x ; \epsilon_z = -\mu\epsilon_x$$

Therefore the resulting strains in three directions are $\epsilon_x = \frac{\sigma_x}{E} ; \epsilon_y = -\mu\frac{\sigma_x}{E} ; \epsilon_z = -\mu\frac{\sigma_x}{E}$

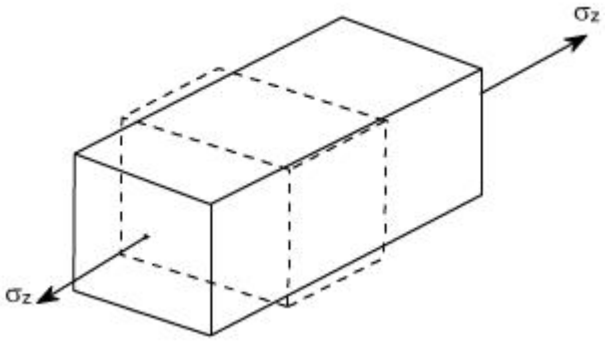
Similarly let us consider that normal stress σ_y alone is acting and the resulting strains are



$$\epsilon_y = \frac{\sigma_y}{E}, \epsilon_x = -\mu\epsilon_y ; \epsilon_z = -\mu\epsilon_y$$

$$\epsilon_y = \frac{\sigma_y}{E} ; \epsilon_x = -\mu\frac{\sigma_y}{E} ; \epsilon_z = -\mu\frac{\sigma_y}{E}$$

Now let us consider the stress σ_z acting alone, thus the strains produced are



$$\epsilon_z = \frac{\sigma_z}{E}, \epsilon_y = -\mu\epsilon_z ; \epsilon_x = -\mu\epsilon_z$$

$$\epsilon_z = \frac{\sigma_z}{E} ; \epsilon_y = -\mu\frac{\sigma_z}{E} ; \epsilon_x = -\mu\frac{\sigma_z}{E}$$

Thus the total strain in any one direction is

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z) \qquad (1)$$

In a similar manner, the total strain in the y and z directions become

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z) \qquad (2)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y) \qquad (3)$$

In the following analysis shear stresses were not considered. It can be shown that for an isotropic material's a shear stress will produce only its corresponding shear strain and will not influence the axial strain. Thus, we can write Hook's law for the individual shear strains and shear stresses in the following manner.

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad (4)$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \quad (5)$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \quad (6)$$

The Equations (1) through (6) are known as Generalized Hook's law and are the constitutive equations for the linear elastic isotropic materials. When these equations isotropic materials. When these equations are used as written, the strains can be completely determined from known values of the stresses. To engineers the plane stress situation is of much relevance (i.e. $\sigma_z = \tau_{xz} = \tau_{yz} = 0$), Thus then the above set of equations reduces to

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\epsilon_z = -\mu\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} \text{ and } \tau_{xy} = \frac{\gamma_{xy}}{G}$$

Their inverse relations can be also determined and are given as

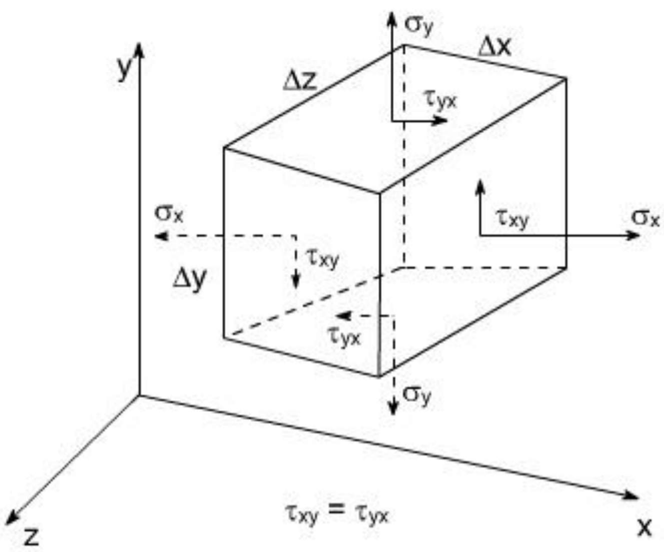
$$\sigma_x = \frac{E}{(1-\mu^2)}(\epsilon_x + \mu\,\epsilon_y)$$

$$\sigma_y = \frac{E}{(1-\mu^2)}(\epsilon_y + \mu\,\epsilon_x)$$

$$\tau_{xy} = G.\gamma_{xy}$$

Hook's law is probably the most well known and widely used constitutive equations for an engineering materials.? However, we can not say that all the engineering materials are linear elastic isotropic ones. Because now in the present times, the new materials are being developed every day. Many useful materials exhibit nonlinear response and are not elastic too.

Plane Stress: In many instances the stress situation is less complicated for example if we pull one long thin wire of uniform section and examine ? small parallepiped where x ? axis coincides with the axis of the wire



So if we take the xy plane then σ_x , σ_y , τ_{xy} will be the only stress components acting on the parrallepiped. This combination of stress components is called the plane stress situation

A plane stress may be defined as a stress condition in which all components associated with a given direction (i.e the z direction in this example) are zero

$$\boxed{\sigma_z = \tau_{zx} = \tau_{zy} = 0}$$

Plane strain: If we focus our attention on a body whose particles all lie in the same plane and which deforms only in this plane. This deforms only in this plane. This type of deformation is called as the plane strain, so for such a situation.

$\in_z = \gamma_{zx} = \gamma_{zy} = 0$ and the non zero terms would be $\in_x$, $\in_y$ & γ_{xy}

i.e. if strain components $\in_x$, $\in_y$ and γ_{xy} and angle θ are specified, the strain components $\in_x'$, $\in_y'$ and γ_{xy}' with respect to some other axes can be determined.

ELASTIC CONSTANTS

In considering the elastic behavior of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and γ .

It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be found out . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity = Stress / strain
(ii) G = Shear Modulus or Modulus of rigidity= Shear stress / Shear strain
(iii) μ = Possion's ratio = – lateral strain / longitudinal strain
(iv) K = Bulk Modulus of elasticity = Volumetric stress / Volumetric strain

Where

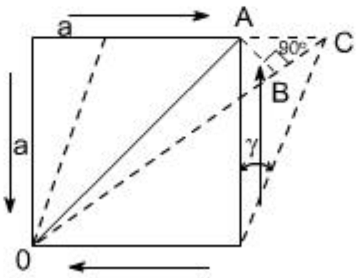
- Volumetric strain = sum of linear stress in x, y and z direction.
- Volumetric stress = stress which cause the change in volume.

Let us find the relations between them

LECTURE 10: RELATION AMONG ELASTIC CONSTANTS

Relation between E, G and υ : Let us establish a relation among the elastic constants E, G and υ. Consider a cube of material of side 'a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 45^0.



Therefore strain on the diagonal OA = Change in length / original length

Since angle between OA and OB is very small hence OA ≅ OB therefore BC, is the change in the length of the diagonal OA

$$\text{Thus, strain on diagonal OA} = \frac{BC}{OA} = \frac{AC\cos 45^0}{OA}$$

$$OA = \frac{a}{\sin 45^0} = a.\sqrt{2}$$

$$\text{hence} \qquad \text{strain} = \frac{AC}{a\sqrt{2}}.\frac{1}{\sqrt{2}} = \frac{AC}{2a}$$

but AC = $a\gamma$

where γ = shear strain

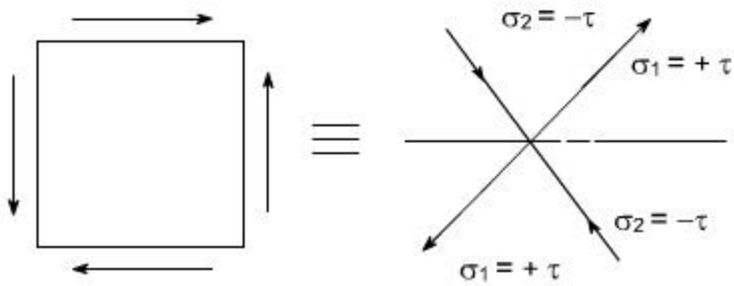
$$\text{Thus, the strain on diagonal} = \frac{a\gamma}{2a} = \frac{\gamma}{2}$$

From the definition

$$G = \frac{\tau}{\gamma} \text{ or } \gamma = \frac{\tau}{G}$$

$$\text{thus, the strain on diagonal} = \frac{\gamma}{2} = \frac{\tau}{2G}$$

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45^0 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.



Thus, for the direct state of stress system which applies along the diagonals:

$$\text{strain on diagonal} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{\tau}{E} - \mu\frac{(-\tau)}{E} = \frac{\tau}{E}(1+\mu)$$

equating the two strains one may get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\text{or} \qquad \boxed{E = 2G(1+\mu)}$$

We have introduced a total of four elastic constants, i.e E, G, K and γ. It turns out that not all of these are independent of the others. Infact given any two of then, the other two can be found.

Again $\quad E = 3K(1-2\gamma)$

$$\Rightarrow \frac{E}{3(1-2\gamma)} = K$$

if $\gamma = 0.5$ $K = \infty$

$$\epsilon_v = \frac{(1-2\gamma)}{E}(\epsilon_x + \epsilon_y + \epsilon_z) = 3\frac{\sigma}{E}(1-2\gamma)$$

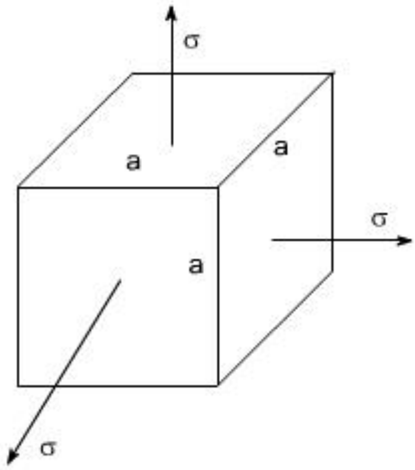
(for $\epsilon_x = \epsilon_y = \epsilon_z$ hydrostatic state of stress)

$\epsilon_v = 0$ if $\gamma = 0.5$

irrespective of the stresses i.e, the material is incompressible.

When γ = 0.5 Value of k is infinite, rather than a zero value of E and volumetric strain is zero, or in other words, the material is incompressible.

Relation between E, K and υ : Consider a cube subjected to three equal stresses σ as shown in the figure below



The total strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress σ is given as

$$= \frac{\sigma}{E} - \gamma\frac{\sigma}{E} - \gamma\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1 - 2\gamma)$$

volumetre strain = 3.linear strain

volumetre strain = $\epsilon_x + \epsilon_y + \epsilon_z$

or thus, $\epsilon_x = \epsilon_y = \epsilon_z$

volumetric strain = $3\frac{\sigma}{E}(1 - 2\gamma)$

By definition

Bulk Modulus of Elasticity (K) = $\frac{\text{Volumetric stress}(\sigma)}{\text{Volumetric strain}}$

or

Volumetric strain = $\frac{\sigma}{k}$

Equating the two strains we get

$$\frac{\sigma}{k} = 3.\frac{\sigma}{E}(1 - 2\gamma)$$

$$\boxed{E = 3K(1 - 2\gamma)}$$

Relation between E, G and K : The relationship between E, G and K can be easily determained by eliminating υ from the already derived relations

E = 2 G (1 + μ) and E = 3 K (1 – 2μ)

Thus, the following relationship may be obtained

$$\boxed{E = \frac{9\,GK}{(3K + G)}}$$

Relation between E, K and γ : From the already derived relations, E can be eliminated

$$E = 2G(1+\gamma)$$
$$E = 3K(1-2\gamma)$$
Thus, we get
$$3k(1-2\gamma) = 2G(1+\gamma)$$
therefore
$$\gamma = \frac{(3K-2G)}{2(G+3K)}$$
or
$$\boxed{\gamma = 0.5(3K-2G)(G+3K)}$$

Engineering Brief about the elastic constants : We have introduced a total of four elastic constants i.e E, G, K and υ. It may be seen that not all of these are independent of the others. Infact given any two of them, the other two can be determined. Futher, it may be noted that

$$E = 3K(1-2\gamma)$$
or
$$K = \frac{E}{(1-2\gamma)}$$
$$\text{if } \gamma = 0.5;\ K = \infty$$
$$\text{Also } \epsilon_v = \frac{(1-2\gamma)}{E}(\sigma_x + \sigma_y + \sigma_z)$$
$$= \frac{(1-2\gamma)}{E}.3\sigma \text{ (for hydrostatic state of stress i.e } \sigma_x = \sigma_y = \sigma_z = \sigma \text{)}$$

hence if υ = 0.5, the value of K becomes infinite, rather than a zero value of E and the volumetric strain is zero or in other words, the material becomes incompressible

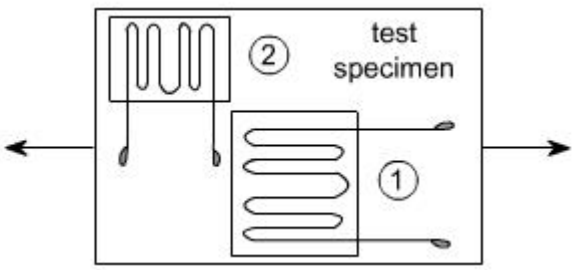
Further, it may be noted that under condition of simple tension and simple shear, all real materials tend to experience displacements in the directions of the applied forces and Under hydrostatic loading they tend to increase in volume. In other words the value of the elastic constants E, G and K cannot be negative

Therefore, the relations

- E = 2 G (1 + υ)
- E = 3 K (1 – 2υ)
- Yields $\boxed{-1 \le \nu \le 0.5}$

In actual practice no real material has value of Poisson's ratio negative . Thus, the value of υ cannot be greater than 0.5, if however υ > 0.5 than ϵ_v = –ve, which is physically unlikely because when the material is stretched its volume would always increase.

Determination of Poisson's ratio: Poisson's ratio can be determined easily by simultaneous use of two strain gauges on a test specimen subjected to uniaxial tensile or compressive load. One gage is mounted parallel to the longitudnal axis of the specimen and other is mounted perpendicular to the longitudnal axis as shown below:



LECTURE 11: MECHANICAL PROPERTIES

Mechanical Properties: In the course of operation or use, all the articles and structures are subjected to the action of external forces, which create stresses that inevitably cause deformation. To keep these stresses, and, consequently deformation within permissible limits it is necessary to select suitable materials for the Components of various designs and to apply the most effective heat treatment. i.e. a Comprehensive knowledge of the chief character tics of the semi-finished metal products & finished metal articles (such as strength, ductility, toughness etc) are essential for the purpose.

For this reason the specification of metals, used in the manufacture of various products and structure, are based on the results of mechanical tests or we say that the mechanical tests conducted on the specially prepared specimens (test pieces) of standard form and size on special machines to obtained the strength, ductility and toughness characteristics of the metal.

The conditions under which the mechanical test are conducted are of three types

(1) **Static:** When the load is increased slowly and gradually and the metal is loaded by tension, compression, torsion or bending.
(2) **Dynamic:** when the load increases rapidly as in impact
(3) **Repeated or Fatigue:** (both static and impact type) . i.e. when the load repeatedly varies in the course of test either in value or both in value and direction Now let us consider the uni-axial tension test.

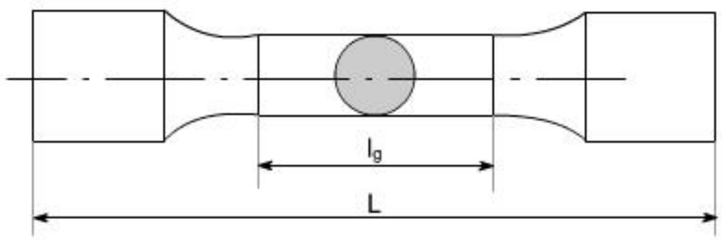
[For application where a force comes on and off the structure a number of times, the material cannot withstand the ultimate stress of a static tool. In such cases the ultimate strength depends on no. of times the force is applied as the material works at a particular stress level. Experiments one conducted to compute the number of cycles requires to break to specimen at a particular stress when fatigue or fluctuating load is acting. Such tests are known as fatigue tests]

Uniaxial Tension Test: This test is of static type i.e. the load is increased comparatively slowly from zero to a certain value.

Standard specimen's are used for the tension test.

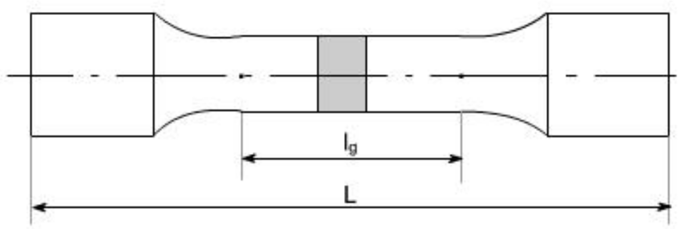
There are two types of standard specimen's which are generally used for this purpose, which have been shown below:

Specimen I: This specimen utilizes a circular X-section.



[specimen with circular X-section]

Specimen II: This specimen utilizes a rectangular X-section.

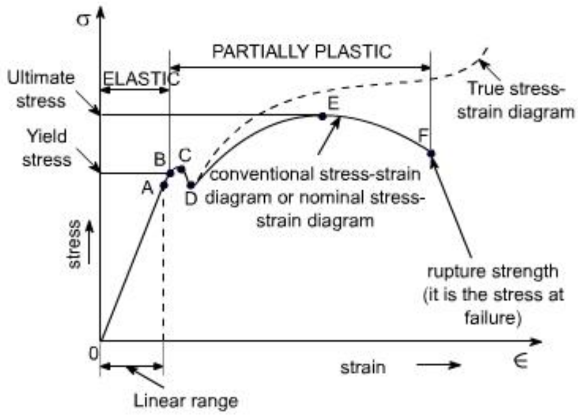


[specimen with rectangular X-section]

l_g = gauge length i.e. length of the specimen on which we want to determine the mechanical properties. The uniaxial tension test is carried out on tensile testing machine and the following steps are performed to conduct this test.

(i) The ends of the specimen's are secured in the grips of the testing machine.
(ii) There is a unit for applying a load to the specimen with a hydraulic or mechanical drive.
(iii) There must be a some recording device by which you should be able to measure the final output in the form of Load or stress. So the testing machines are often equipped with the pendulum type lever, pressure gauge and hydraulic capsule and the stress Vs strain diagram is plotted which has the following shape.

A typical tensile test curve for the mild steel has been shown below



Nominal stress / Strain OR Conventional Stress ? Strain diagrams: Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

True stress / Strain Diagram: Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.

SALIENT POINTS OF THE GRAPH:

(A) So it is evident form the graph that the strain is proportional to strain or elongation is proportional to the load giving a st.line relationship. This law of proportionality is valid upto a point A.

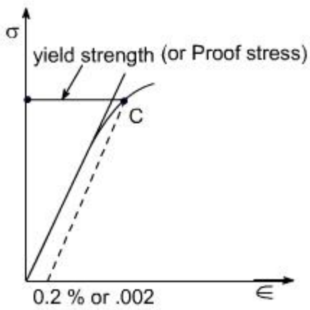
or we can say that point A is some ultimate point when the linear nature of the graph ceases or there is a deviation from the linear nature. This point is known as **the limit of proportionality or the proportionality limit**.

(B) For a short period beyond the point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as **Elastic Limit** .

(C) **and (D)** - Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus permanent deformation or permanent set when load is removed. These two points are termed as upper and lower yield points respectively. The stress at the yield point is called the yield strength.

A study a stress strain diagrams shows that the yield point is so near the proportional limit that for most purpose the two may be taken as one. However, it is much easier to locate the former. For material which do not posses a well define yield points, In order to find the yield point or yield strength, an offset method is applied.

In this method a line is drawn parallel to the straight line portion of initial stress diagram by off setting this by an amount equal to 0.2% of the strain as shown as below and this happens especially for the low carbon steel.



(E) A further increase in the load will cause marked deformation in the whole volume of the metal. The maximum load which the specimen can with stand without failure is called the load at the ultimate strength.

The highest point 'E' of the diagram corresponds to the ultimate strength of a material.

- σ_u = Stress which the specimen can with stand without failure & is known as Ultimate Strength or Tensile Strength.
- σ_u is equal to load at E divided by the original cross-sectional area of the bar.

(F) Beyond point E, the bar begins to forms neck. The load falling from the maximum until fracture occurs at F.

[Beyond point E, the cross-sectional area of the specimen begins to reduce rapidly over a relatively small length of bar and the bar is said to form a neck. This necking takes place whilst the load reduces, and fracture of the bar finally occurs at point F]

Note: Owing to large reduction in area produced by the necking process the actual stress at fracture is often greater than the above value. Since the designers are interested in maximum loads which can be carried by the complete cross section, hence the stress at fracture is seldom of any practical value.

Percentage Elongation: ' δ ':

The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs.

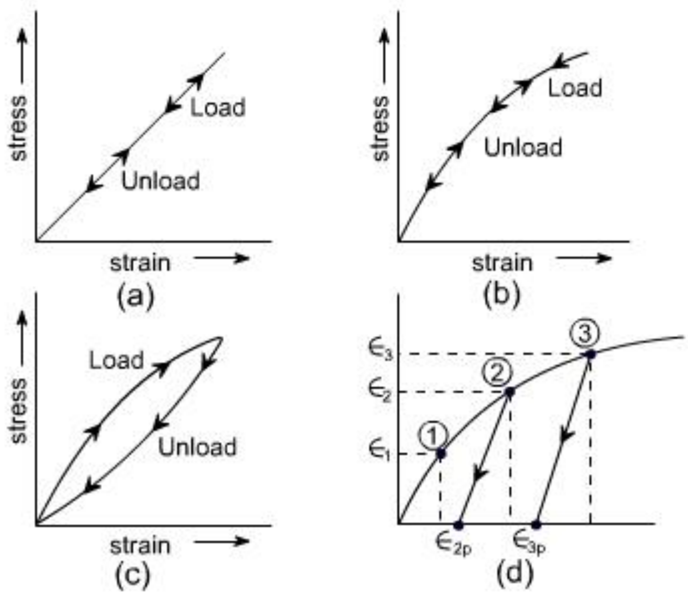
It is the ratio of the extension in length of the specimen after fracture to its initial gauge length, expressed in percent.

$$\delta = \frac{(l_1 - l_g)}{l_1} \times 100$$

where l_l = gauge length of specimen after fracture(or the distance between the gage marks at fracture) and l_g= gauge length before fracture(i.e. initial gauge length)

For 50 mm gage length, steel may here a % elongation δ of the order of 10% to 40%.

Elastic Action: The elastic is an adjective meaning capable of recovering size and shape after deformation. Elastic range is the range of stress below the elastic limit.



Many engineering materials behave as indicated in Fig(a) however, some behaves as shown in figures in (b) and (c) while in elastic range. When a material behaves as in (c), the σ vs $\in$ is not single valued since the strain corresponding to any particular 'σ ' will depend upon loading history.

Fig (d): It illustrates the idea of elastic and plastic strain. If a material is stressed to level (1) and then relased the strain will return to zero beyond this plastic deformation remains.

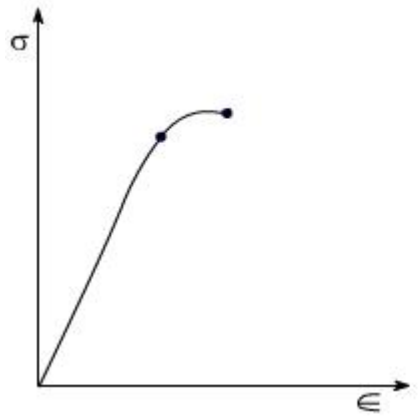
If a material is stressed to level (2) and then released, the material will recover the amount ($\in_2 - \in_{2p}$), where $\in_{2p}$ is the plastic strain remaining after the load is removed. Similarly for level (3) the plastic strain will be $\in_{3p}$.

Ductile and Brittle Materials: Based on this behaviour, the materials may be classified as ductile or brittle materials

Ductile Materials: It we just examine the earlier tension curve one can notice that the extension of the materials over the plastic range is considerably in excess of that associated with elastic loading. The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials.

Brittle Materials: A brittle material is one which exhibits a relatively small extensions or deformations to fracture, so that the partially plastic region of the tensile test graph is much reduced.

This type of graph is shown by the cast iron or steels with high carbon contents or concrete.



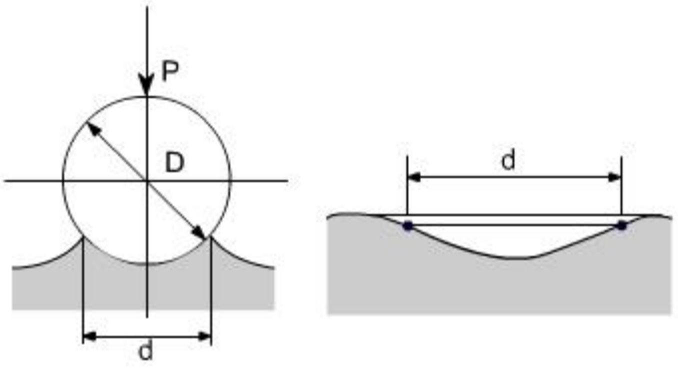
Conditions Affecting Mechanical Properties: The Mechanical properties depend on the test conditions

(1) It has been established that lowering the temperature or increasing the rate of deformation considerably increases the resistance to plastic deformation. Thus, at low temperature (or higher rates of deformation), metals and alloys, which are ductile at normal room temperature may fail with brittle fracture.
(2) Notches i.e. sharp changes in cross sections have a great effect on the mechanical properties of the metals. A Notch will cause a non uniform distribution of stresses. They will always contribute lowering the ductility of the materials. A notch reduces the ultimate strength of the high strength materials. Because of the non uniform distribution of the stress or due to stress concentration.
(3) Grain Size: The grain size also affects the mechanical properties.

Hardness: Hardness is the resistance of a metal to the penetration of another harder body which does not receive a permanent set.

Hardness Tests consists in measuring the resistance to plastic deformation of layers of metals near the surface of the specimen i.e. there are Ball indentation Tests.

Ball indentation Tests: This method consists in pressing a hardened steel ball under a constant load P into a specially prepared flat surface on the test specimen as indicated in the figures below :



After removing the load an indentation remains on the surface of the test specimen. If area of the spherical surface in the indentation is denoted as F sq. mm. Brinell Hardness number is defined as :

$$Bhn = P / F$$

F is expressed in terms of D and d

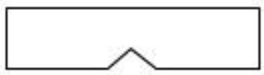
D = ball diameter

d = diametric of indentation and Brinell Hardness number is given by $Bhn = \frac{2P}{\pi D(D - \sqrt{D^2 - d^2})}$

Then, there is also **Vicker's Hardness Number** in which the ball is of conical shape.

IMPACT STRENGTH

Static tension tests of the unnotched specimen's do not always reveal the susceptibility of metal to brittle fracture. This important factor is determined in impact tests. In impact tests we use the notched specimen's



this specimen is placed on its supports on anvil so that blow of the striker is opposite to the notch the impact strength is defined as the energy A, required to rupture the specimen,

$$\text{Impact Strength} = A / f$$

Where f = It is the cross section area of the specimen in cm^2 at fracture & obviously at notch.

The impact strength is a complex characteristic which takes into account both toughness and strength of a material. The main purpose of notched bar tests is to study the simultaneous effect of stress concentration and high velocity load application

Impact test are of the severest type and facilitate brittle friction. Impact strength values can not be as yet be used for design calculations but these tests as rule provided for in specifications for carbon & alloy steels.Futher, it may be noted that in impact tests fracture may be either brittle or ductile. In the case of brittle fracture, fracture occurs by separation and is not accompanied by noticeable plastic deformation as occurs in the case of ductile fracture.

LECTURE 12

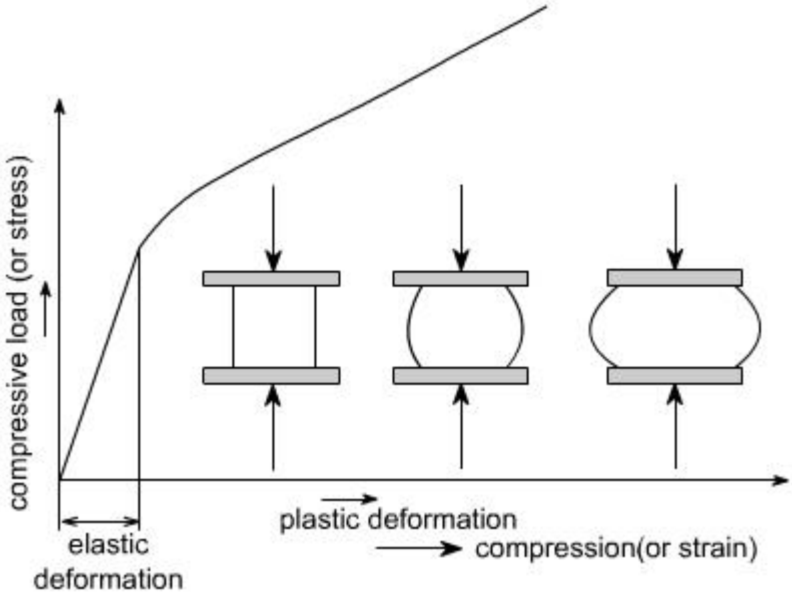
Compression Test: Machines used for compression testing are basically similar to those used for tensile testing often the same machine can be used to perform both tests.

Shape of the specimen: The shape of the machine to be used for the different materials are as follows:

- **(i)** **For metals and certain plastics:** The specimen may be in the from of a cylinder
- **(ii)** **For building materials:** Such as concrete or stone the shape of the specimen may be in the from of a cube.

Shape of stress stain diagram

(a) Ductile materials: For ductile material such as mild steel, the load Vs compression diagram would be as follows



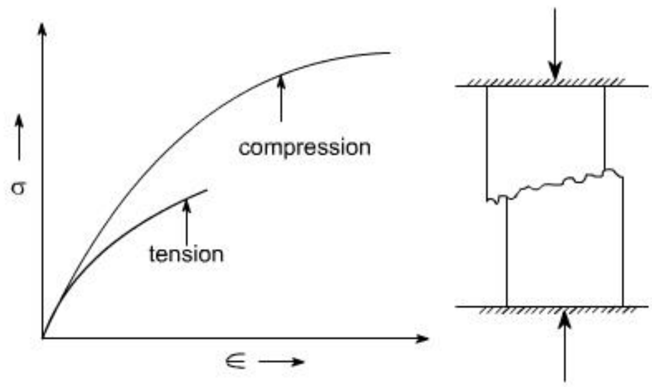
(1) The ductile materials such as steel, Aluminum, and copper have stress ? strain diagrams similar to ones which we have for tensile test, there would be an elastic range which is then followed by a plastic region.
(2) The ductile materials (steel, Aluminum, copper) proportional limits in compression test are very much close to those in tension.
(3) In tension test, a specimen is being stretched, necking may occur, and ultimately fracture fakes place. On the other hand when a small specimen of the ductile material is compressed, it begins to bulge on sides and becomes barrel shaped as shown in the figure above. With increasing load, the specimen is flattened out, thus offering increased resistance to further shortening (which means that the stress ? strains curve goes upward) this effect is indicated in the diagram.

Brittle materials (in compression test)

Brittle materials in compression typically have an initial linear region followed by a region in which the shortening increases at a higher rate than does the load. Thus, the compression stress ? strain diagram has a shape that is similar to the shape of the tensile diagram.

However, brittle materials usually reach much higher ultimate stresses in compression than in tension.

For cast iron, the shape may be like this



Brittle materials in compression behave elastically up to certain load, and then fail suddenly by splitting or by craking in the way as shown in figure. The brittle fracture is performed by separation and is not accompanied by noticeable plastic deformation.

Hardness Testing: The tem ?hardness' is one having a variety of meanings; a hard material is thought of as one whose surface resists indentation or scratching, and which has the ability to indent or cut other materials.

Hardness test: The hardness test is a comparative test and has been evolved mainly from the need to have some convenient method of measuring the resistance of materials to scratching, wear or in dentation this is also used to give a guide to overall strength of a materials, after as an inspection procedure, and has the advantage of being a non ? destructive test, in that only small indentations are lift permanently on the surface of the specimen.

Four hardness tests are customarily used in industry namely

- **(i)** Brinell
- **(ii)** Vickers
- **(iii)** Rockwell
- **(iv)** Shore Scleroscopy

The most widely used are the first two.

In the Brinell test the indenter is a hardened steel ball which is pressed into the surface using a known standard load. The diameter of resulting indentation is than measured using a microscope & scale.

Units: The units of Brinell Hardness number in S.I Unit would have been N/mm^2 or Mpa

To avoid the confusion which would have been caused of her wise Hardness numbers are quotes as kgf / mm^2

Brinell Hardness test: In the Brinell hardness test, a hardened steel ball is pressed into the flat surface of a test piece using a specified force. The ball is then removed and the diameter of the resulting indentation is measured using a microscope.

The Brinell Hardness no. (BHN) is defined as **BHN = P / A**

where P = Force applied to the ball.

A = curved area of the indentation

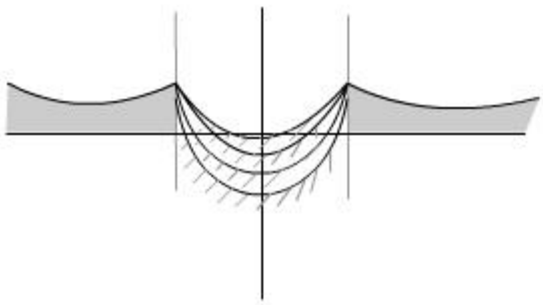
It may be shown that $$A = \frac{1}{2}\pi D\left[D - \sqrt{D^2 - d^2}\right]$$

D = diameter of the ball,

d = the diameter of the indentation.

In the Brinell Test, the ball diameter and applied load are constant and are selected to suit the composition of the metal, its hardness, and selected to suit the composition of the metal, its hardness, the thickness etc. Further, the hardness of the ball should be at least 1.7 times than the test specimen to prevent permanent set in the ball.

Disadvantage of Brinell Hardness Test: The main disadvantage of the Brinell Hardness test is that the Brinell hardness number is not independent of the applied load. This can be realized from. Considering the geometry of indentations for increasing loads. As the ball is pressed into the surface under increasing load the geometry of the indentation charges.



Here what we mean is that the geometry of the impression should not change w.r.t. load, however the size it impression may change.

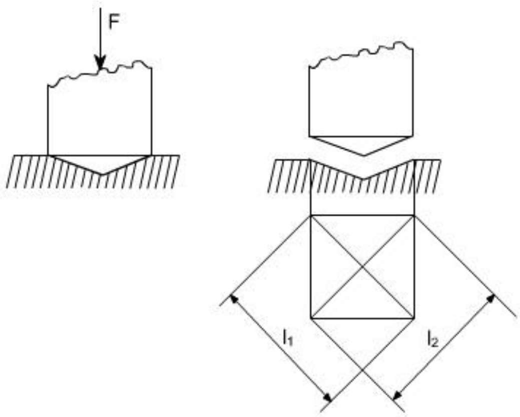
Vickers Hardness test: The Vicker's Hardness test follows a procedure exactly a identical with that of Brinell test, but uses a different indenter. The steel ball is replaced by a diamond, having the from of a square ? based pyramid with an angle of 136^0 between opposite faces. This is pressed into the flat surface of the test piece using a specified force, and the diagonals of the resulting indentation measured is using a microscope. The Hardness, expressed as a Vicker's pyramid number is defined as the ratio F/A, where F is the force applied to the diamond and A is the surface area of the indentation.

$$A = \frac{\frac{1}{2}l^2}{\sin\frac{1}{2}(136^0)}$$

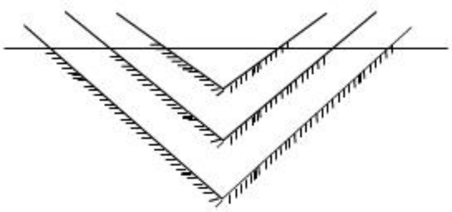
$$= \frac{l^2}{.854v_x} \Rightarrow H_V = \frac{F}{\frac{l^2}{.854}}$$

$$H_V = \frac{.854F}{l^2}$$

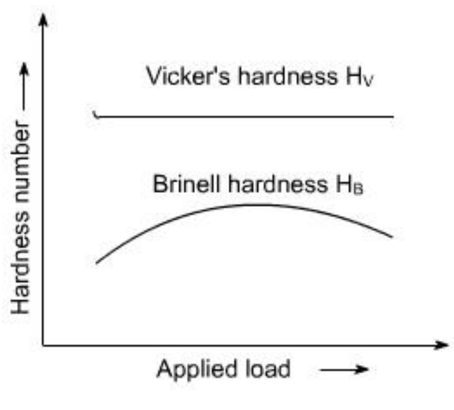
It may be shown that where l is the average lengthof the diagonal is $l = \frac{1}{2}(l_1 + l_2)$



In the Vicker Test the indenters of pyramidal or conical shape are used & this overcomes the disadvantage which is faced in Brinell test i.e. as the load increases, the geometry of the indentation's does not change



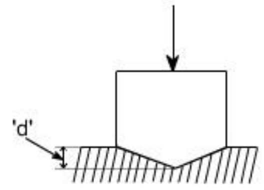
The Variation of Hardness number with load is given below.



Advantage: Apart from the convenience the vicker's test has certain advantages over the Brinell test.

(i) Harder material can be tested and indentation can be smaller & therefore less obtrusive or damaging. Upto a 300 kgf /mm^2 both tests give the same hardness number but above too the Brinell test is unreliable.

Rockwell Hardness Test : The Rockwell Hardness test also uses an indenter when is pressed into the flat surface of the test piece, but differs from the Brinell and Vicker's test in that the measurement of hardness is based on the depth of penetration, not on the surface area of indentation. The indenter may be a conical diamond of 120^0 included angle, with a rounded apex. It is brought into contact with the test piece, and a force F is applied.



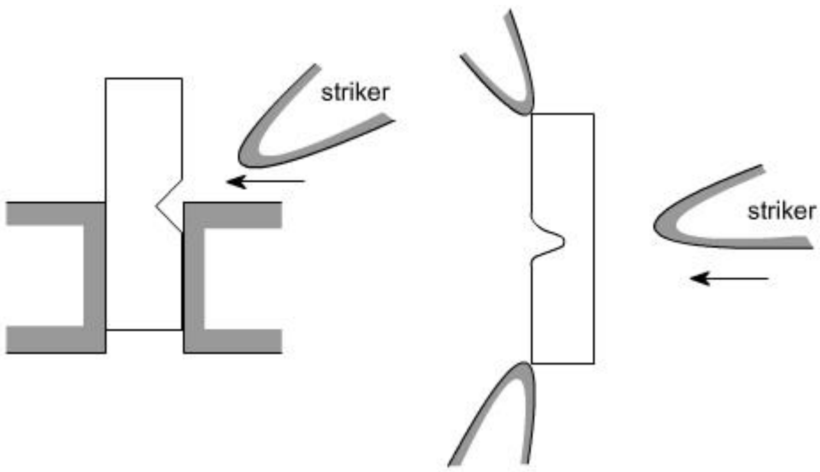
Advantages : Rockwell tests are widely applied in industry due to rapidity and simplicity with which they may be performed, high accuracy, and due to the small size of the impressions produced on the surface.

Impact testing: In an impact test' a notched bar of material, arranged either as a cantilever or as a simply supported beam, is broken by a single blow in such a way that the total energy required to fracture it may be determined.

The energy required to fracture a material is of importance in cases of shock loading' when a component or structure may be required to absorb the K.E of a moving object.

Often a structure must be capable of receiving an accidental shock load' without failing completely, and whether it can do this will be determined not by its strength but by its ability to absorb energy. A combination of strength and ductility will be required, since large amounts of energy can only be absorbed by large amounts of plastic deformation. The ability of a material to absorb a large amount of energy before breaking is often referred as toughness, and the energy absorbed in an impact test is an obvious indication of this property.

Impact tests are carried out on notched specimens, and the notches must not be regarded simply as a local reduction in the cross sectional area of the specimen, Notches and , in fact, surface irregularities of many kind give rise to high local stresses, and are in practice, a potential source of cracks.



[cantilever type specimen] [simply supporter specimen]

The specimen may be of circular or square cross ? section arranged either as a cantilever or a simply supported beam.

<u>Toughness:</u> It is defined as the ability of the material to withstand crack i.e to prevent the transfer or propagation of cracks across its section hence causing failures. Cracks are propagated due to stress concentration.

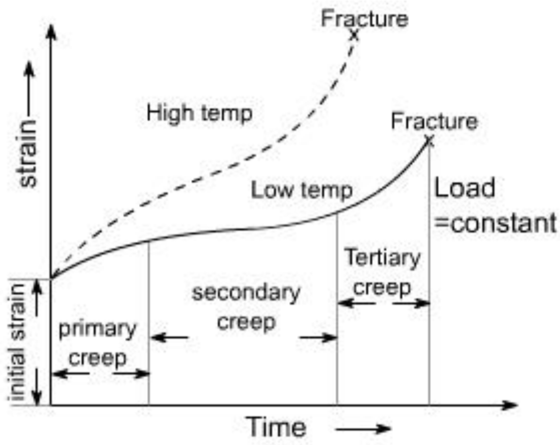
Creep: Creep is the gradual increase of plastic strain in a material with time at constant load. Particularly at elevated temperatures some materials are susceptible to this phenomena and even under the constant load, mentioned strains can increase continually until fractures. This form of facture is particularly relevant to the turbines blades, nuclear rectors, furnaces rocket motors etc.

The general from of strain versus time graph or creep curve is shown below.

The general form of $\in$ Vs t graph or creep curve is shown below for two typical operation conditions, In each case the curve can be considered to exhibit four principal features

(a) An initial strain, due to the initial application of load. In most cases this would be an elastic strain.
(b) A primary creep region, during which the creep rate (slope of the graph) dimensions.

(c) A secondary creep region, when the creep rate is sensibly constant.
(d) A tertiary creep region, during which the creep rate accelerate to final fracture.



[Typical creep curve]

It is obvious that a material which is susceptible to creep effects should only be subjected to stresses which keep it in secondary (st.line) region throughout its service life. This enables the amount of creep extension to be estimated and allowed for in design.

Practice Problems: PROB 1: A standard mild steel tensile test specimen has a diameter of 16 mm and a gauge length of 80 mm such a specimen was tested to destruction, and the following results obtained.

- Load at yield point = 87 kN
- Extension at yield point = 173 x 16^{-6} m
- Ultimate load = 124 kN
- Total extension at fracture = 24 mm
- Diameter of specimen at fracture = 9.8 mm
- Cross - sectional area at fracture = 75.4 mm^2
- Cross - sectional Area ?A' = 200 mm^2

Compute the followings:

(i) Modulus of elasticity of steel
(ii) The ultimate tensile stream
(iii) The yield stress
(iv) The percentage elongation
(v) The Percentage reduction in Area.

PROB 2: A light alloy specimen has a diameter of 16mm and a gauge Length of 80 mm. When tested in tension, the load extension graph proved linear up to a load of 6kN, at which point the extension was 0.034 mm. Determine the limits of proportionality stress and the modulus of elasticity of material.

Note: For a 16mm diameter specimen, the Cross ? sectional area A = 200 mm^2

This is according to tables Determine the limit of proportion try stream & the modulus of elasticity for the material.

Ans: 30 MN /m^2 , 70.5 GN /m^2

solution:

$$\text{Limit of proportionaly stress} = \frac{6\text{ kN}}{200\times10^{-6}}$$

$$= 30\ \text{MN/m}^2$$

$$\text{Young Modulus} \qquad E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{strain} = \frac{.034}{80}$$

$$E = 30\times10^{6} \Big/ \frac{.034}{80}$$

$$= 70.5\ \text{GN/m}^2$$