Conditional Probability

Defn.: The probability of occurrence of an event B when it is known that some other event A has already occurred is called conditional probability of B given A.

Theorem of Compound Probability (Multiplication law of Probability): The probability of the simultaneous occurrence of two events A and B is equal to the probability of A multiplied by the conditional probability of B, given that A has occurred (or it is equal to the probability of B multiplied by the conditional probability of A given that B has occurred).

In symbols P(AB) = P(A).P(B/A)
= P(B).P(A/B)
(This theorem is also known as Muliplication theorem)

In particular if A and B are independent then P(B/A) is same as P(B), Then for two independent event P(AB)=P(A).P(B)
These can be generalized as
P(A1A2……..An)=P(A1).P(A2)…….P(An) for independent events A1,A2……An.

Problem: 1
A salesman has a 80% chance of making a sale to each customer. The behavior of successive customers is assumed to be independent. If two customers X and Y enter the shop, what is the prob. that the salesman will make a sale?

Sol.: Let A and B denote the events 'sale to customer X' and 'sale to the customer Y' respectively. We have to find prob. that a sale is made to at least one of the two customers X and Y i.e. to find P(A+B). Since complementary event is that no sale is made i,e no sale to X as well as no sale to Y.

Hence P(A+B)= 1-P($\bar{A}\bar{B}$)
=1-P($\bar{A}$).P($\bar{B}$)
As A and B are independent.
P($\bar{A}$)=1-P(A)=1-(80/100)=20/100=.2

<u>Alt. method:</u>
P(A+B)=P(A)+P(B)-P(AB)
=P(A)+P(B)-P(A).P(B)
=4/5+4/5-4/5.4/5

$P(\bar{B})$=1-P(B)=1-(80/100)=20/100=.2 =8/5-16/25

P(A+B)= 1-$P(\bar{A})$.$P(\bar{B})$ =1-.04=.96 =24/25=96/100=.96

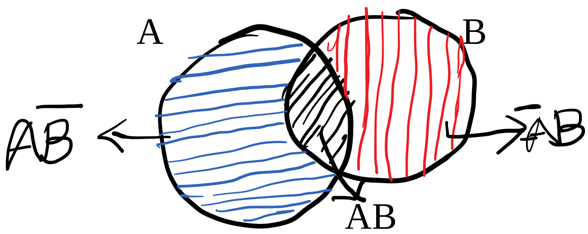
Problem:2

There is a 50-50 chance that a contractor's farm A will bid for the construction of a multi storeyed building. Another farm B submits a bid and the prob. is 3/4 that it will get the job, provided A does not bid. If the farm A submits a bid, the prob. that the farm B will get the job is only 1/3. What is the prob. that farm B will get the job?

Sol.:

Let A= event that farm A submits the bid

B=event that farm B gets the job.



Then since the chance of submitting the bid is 50-50, therefore

P(A)=1/2

$P(B/\bar{A})$=3/4

P(B/A)=1/3

Then P(B)=$P(AB\cup\bar{A}B)$=P(AB)+$P(\bar{A}B)$=P(A)P(B/A)+$P(\bar{A})P(B/\bar{A})$

=13/24.

Problem:3

A problem of statistics is given to three students A,B,C whose chances of solving it are 1/2,1/3,1/4 respectively. What is the probability that the problem will be solved?

<u>Drawing with/ without replacement</u>

Problem: 4

Two drawings each of 4 balls are made from a bag containing 6 white and 7 green balls. What is the chance that the first drawing will give 4 white and the second 4 greens balls,

(a) when the balls are replaced before the 2^{nd} drawing

(b) when the balls are not replaced before the 2^{nd} drawing

Sol.: (a) 4 white balls can be drawn out of 6 white balls in $6c4$ ways in the 1^{st} drawing.

Therefore P(drawing 4 white balls) = $6c4/13c4$……..(A)

Since these ball are replaced before 2nd drawing, therefore in the 2^{nd} drawing 4 green balls can be drawn out of 7 green balls is $7c4$

P(drawing 4 green balls) = $7c4/13c4$………………..(B)

Since (A) and (B) are independent, therefore

required probability = $(6c4/13c4).(7c4/13c4)$ = 21/20449

(b) In the first draw

P(drawing 4 white balls) = $6c4/13c4$

If these ball are not replaced , then in the 2^{nd} drawing total no. of balls = 2+7 =9.

Now 4 green balls out of 7 can be drawn in = $7c4$ ways.

Therefore P(drawing 4 green balls) = $7c4/9c4$

Required probability = $(6c4/13c4).(7c4/9c4)$ = 5/858