Baye's Theorem:

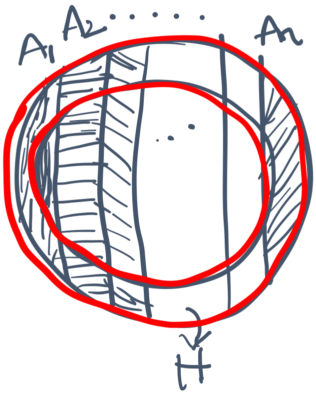
Let an event H be explained by a set of mutually exclusive and exhaustive events $A1, A2, \ldots. An$ defined on a sample space S where $P(H) \neq 0$. Also the unconditional probs. $P(A1)$, $P(A2) \ldots . P(An)$ and conditional probs. $P(H/A1)$, $P(H/A2), \ldots . P(H/An)$ are known. Then the conditional probs. $P(Ai/H)$ of a specified event Ai where H is stated to be actually occurred is given by

$P(Ai/H) = \{P(Ai)\, P(H/Ai)\} / \Sigma\, P(Ai)\, P(H/Ai)$, $\quad (I = 1, 2, 3 \ldots \ldots n)$

This is known as Baye's Theorem.

Proof:

Let A1, A2,….An be n mutually exclusive and exhaustive events and H is the ring shaped region. Then clearly $H = A1H + A2H + \ldots \ldots + AnH$ ………………….(1)



where A1H, A2H,…,AnH are mutually exclusive. Therefore

$P(H) = P(A1H) + P(A2H) + \ldots \ldots + P(AnH)$…………………………..(2)

We know from the theorem of compound prob. that

$P(AB) = P(A)\, P(B/A) = P(B) P(A/B)$.

Therefore using this $\quad P(HAi) = P(H)P(Ai/H) = P(Ai)P(H/Ai)$…….(3)

$\Rightarrow$ $P(Ai/H) = P(Ai)P(H/Ai)/P(H)$………………………………….(4)

Now $P(H) = P(A1)P(H/A1) + P(A2)P(H/A2) + \ldots \ldots + P(An)P(H/An)$

(Using the theorem of compound prob.)

$\Rightarrow$ $P(H) = \Sigma\, P(Ai)P(H/Ai)$

Therefore from equation (4), we have $P(Ai/H) = P(Ai)P(H/Ai) / \Sigma\, P(Ai)P(H/Ai)$

Problem:1 Suppose that 5 men out of 100 and 25 women out of 1000 are colour blind. A colour blind person is chosen at random. What is the probability of his being a male (assuming that males and females are in equal proportion)?

Soln.: Let M = A person is male

F = A person is female

C = A person is colour blind.

Here we want to find $P(M/C)$.

It is given that $P(M) = P(F) = 1/2$, $\quad P(C/M) = 5/100$ and $P(C/F) = 25/1000$.

Now by Baye's theorem $P(M/C) = P(M).P(C/M)/\{P(M).P(C/M) + P(F)P(C/F)\}$

= 2/3 (Ans).

Problem:2 The contents of 3 vessels 1,2 and 3 are as follows: 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black ball; 3 white, 1 red ,2 black ball. A vessel is chosen at random and from it, 2 balls are drawn at random. The two balls are one red and one white. What is the probability that they come from the 2nd vessel ?

Soln.: Let v1 = selected vessel is no.1,

v2 = selected vessel is no.2,

v3 = selected vessel is no.3, and

A = balls drawn are 1 red and 1 white.

Since the vessels are chosen at random, P(v1) = P(v2) = P(v3) = 1/3.

We have to calculate P(v2/A)

Now $P(A/v1) = ({}^{2}C_{1}.\,{}^{1}C_{1}.\,{}^{3}C_{0})/\,{}^{6}C_{2} = 2/15$,

$P(A/v2) = ({}^{2}C_{1}.\,{}^{3}C_{1}.\,{}^{1}C_{0})/\,{}^{6}C_{2} = 2/5$,

$P(A/v3) = ({}^{3}C_{1}.\,{}^{1}C_{1}.\,{}^{2}C_{0})/\,{}^{6}C_{2} = 1/5$.

By Baye's theorem,

$P(v2/A) = P(v2).P(A/v2)/\{P(v1).P(A/v1) + P(v2).P(A/v2) + P(v3).P(A/v3)$

= 6/11 (Ans.)

Problem:3 Given that the sample space 'S' is the population of adults in a small town(who have completed the requirements for a college degree). They are categorized according to their sex and employment as follows: out of 900 population 500 are male of which 460 are employed and the rest are unemployed while among 400 female only 140 are employed. If an individual be chosen at random(for a tour throughout the country to publicize the advantages of establishing new inelestries in the town) what is the probability that an employed individual chosen is a male?

Sol.: Let M be an event that a male is chosen and E be the event that one chosen is employed.

We are to find P(M/E).

Given:	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

According to conditional probability

$P(M/E) = P(M \cap E)/P(E) = 460/600 = 23/30 = 0.7666$ (Ans.)

Discrete Probability Distribution:

Binomial Distribution

Bernoulli Process

If a random expt. be s.t. it has only 2 outcomes referred as 'success' & 'failure',the repeated trials are independent and probability of success remains constant from trial to trial, then such a process is called a "Bernoulli process" & the trialsare called "Bernoulli trials".

Strictly speaking, the Bernoulli possess has the following properties:

1. The experiment consists of n trials.
2. Each trial results in an outcome which can be classified as a success or afailure.
3. The prob. of success, denoted by p remains constant from trial to trial.
4. The repeated trials are independent.

If the above assumptions are not met then the theory we shall develop here doesnot apply.

Def: Thus Binomial distribution is $b(r; n,p) = {}^nC_r\, q^{n-r} p^r$

A r.v, X is said to follow Binomial distribution if it assumes only non negativevalues whose prob. mass function is given by

$$P(X=x) = f(x) = \begin{cases} {}^nC_x\, q^{n-x} p^x, & x=0,1,2,...,n; \quad q = 1-p \\ 0 \quad , & \text{otherwise} \end{cases}$$

It is denoted by $b(x;n,p)$.

Corresponding Cummulative Distribution is

$B(x;n,p) = \sum_{t=x} b(t;n,p)$, $x = 0,1,2,\ldots,n$