1. Using matrix operations, describe the solutions for the following family of equations:

x + 2y - 3z = 5

2x + y - 3z = 13

-x + y = -8

First I rewrite the equations to make sure that variables are in the same order in each and to make any 0, 1, or negative coefficients more clear:

1x + 2y + -3z = 5

2x + 1y + -3z = 13

-1x + 1y + 0z = -8

Then I restate it in matrix form, as Ax = b. I want to solve for x.

= (or Ax = b), so x = A-1b.

Now I need to find the inverse of my 3X3 matrix (A-1). I recall that the inverse of a matrix is the 1/determinant of the matrix \* the matrix adjoint.

Let me find the determinant of my matrix by rewriting the first two columns, then taking the sum of “blue” products and subtracting the “red” products

(1 \* 1 \* 0) + (2 \* -3 \* -1) + (-3 \* 2 \* 1) – (-1 \* 1 \* -3) – (1 \* -3 \* 1) – (0 \* 2 \* 2)

0 + 6 + -6 - 3 - -3 - 0

0

To find the matrix adjoint, I would have to calculate a matrix of minors and cofactor matrix. However, since my determinant is 0, I can’t multiply by 1/0, which is undefined, so I stop here.

Instead, I’ll try to get my matrix into reduced row echelon form.

I’ll add row three to row one and make that sum the new row one:

Now I’ll double row three and add that to row two, to replace row two:

I’ll multiply row three by -1 for a new row three:

Now I’ll subtract row two from row one to replace row one:

And divide row two by 3:

I change around the order to make it fit the convention:

Almost there! I add row two to row one for a new row 1.

x and y are pivot variables, and z is free

I can rewrite a new system of equations that’s much easier:

x – z = 7, or x = z + 7

y – z = -1, or y = z – 1

The number of solutions is infinite: pick your z, subtract one and you have your y. Or add 7, and you have your x. In a three-dimensional world where the z axis comes toward you positively, perpendicular to your body plane with you facing the origin in positive z space, the solution would be a line that would start far in front of you, in negative z space, far to your right and a bit above you, in the XY quadrant I, and as it approached you, it would angle to the left and down.

2. Provide a solution for #1, using R functions of your choice.

First, I enter my A (3 X 3 matrix) and b (constant vector):

A<-rbind(c(1,2,-3),c(2,1,-3),c(-1,1,0))

b<-c(5,13,-8)

I then ask R to solve the set of equations:

solve(A,b)

This produces an error:

Error in solve.default(A, b) :

Lapack routine dgesv: system is exactly singular: U[3,3] = 0

An unsurprising error, given that the determinant is 0. I instead use the pracma package’s rref tool to give me the reduced row echelon form.

install.packages("pracma")

library(pracma)

rref(cbind(A,b))

This gives me my reduced row echelon form, which allows me to come up with my solution set of equations:

b

[1,] 1 0 -1 7

[2,] 0 1 -1 -1

[3,] 0 0 0 0

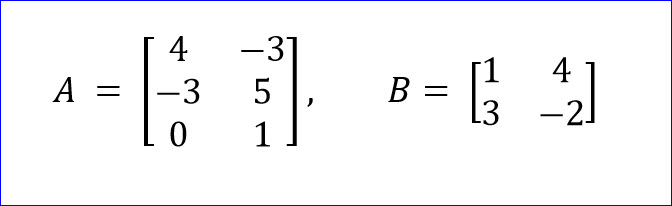
This evaluates to:

x – z = 7 (or x = z + 7)

y – z = -1 (or y = z - 1)

In short, a much faster way to solve the problem!

 3. Solve for AB by hand:



AB =

AB =

4. Solve AB from #3 using R functions of your choice.

A<-rbind(c(4,-3),c(-3,5),c(0,1))

B<-rbind(c(1,4),c(3,-2))

> A %\*% B

[,1] [,2]

[1,] -5 22

[2,] 12 -22

[3,] 3 -2