Adaptive Sig. Proc. & Machine Intel.

Supplement: Exploiting non-circularity in communications

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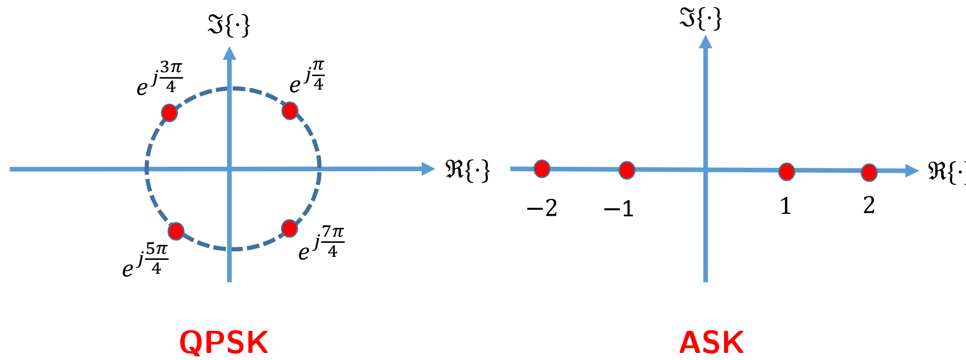
Circularity
Constellations in communications, 4 symbols

Consider a communication system with 4 complex-valued symbols.

The most widely used modulation schemes are quadrature phase shift keying (**QPSK**) and amplitude shift keying (**ASK**).



Although these constellations are arranged so that the distances of each point to its nearest neighbour is equal in both cases, the **QPSK is more compact**.

QPSK second-order statistics:

$$\text{covariance}: \quad c = E[zz^*] = 1$$
$$\text{pseudocov.}: \quad p = E[zz] = 0$$

ASK second-order statistics:

$$\text{covariance}: \quad c = E[zz^*] = 2.5$$
$$\text{pseudocov.}: \quad p = E[zz] = 2.5$$

In the case of the **QPSK** there is no power difference or correlation between the real and imaginary components, resulting in the impropriety measure of $\rho = 0$.
In the case of the **ASK** all the information is on the real axis, resulting in the impropriety measure of $\rho = 1$ (real-valued signals are maximally non-circular).

Noncircularity in communications: More on constellations

Constellations used in digital communications are mostly second-order circular. For instance, for QPSK the estimation of improperness gives:

$$\text{covariance}: \quad c = E[|z|^2] = [P\{z \,\big|\, z = \tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\} + P\{z \,\big|\, z = \tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}i\}$$
$$+ P\{z \,\big|\, z = -\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}i\} + P\{z \,\big|\, z = -\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\}] \times 1 = 1$$

$$\text{pseudocov.}: \quad p = E[z^2] = [P\{z \,\big|\, z = \tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\} + P\{z \,\big|\, z = -\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}i\}] \times \tfrac{1}{2}i$$
$$+ [P\{z \,\big|\, z = \tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}i\} + P\{z \,\big|\, z = -\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\}] \times (-\tfrac{1}{2}i) = 0$$

The only exception is BPSK, for which

$$E[|z|^2] = E[z^2] = P\{z \,|\, z = 1\} \times 1 + P\{z \,\big|\, z = -1\} \times (-1)^2 = 1$$

Constellations used in communications

Bandwidth	Constellations	Modulations	Coding Domain
Narrow-band(NB) (WLAN 802.11b, cellular 3G)	BPSK, OQPSK*	DSSS, CCK	Time-domain
Wide-band(WB) (WLAN 802.11 a/g/n/ac, cellular 4G)	BPSK,QPSK,M-QAM	OFDM	Frequency-domain

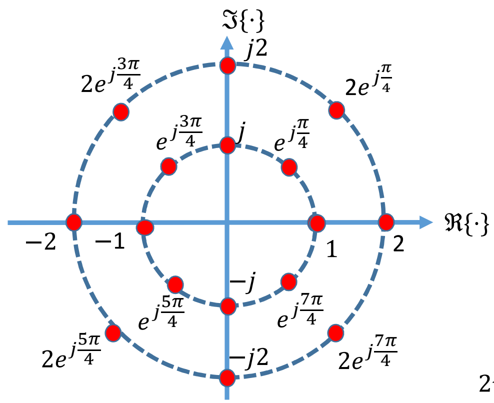
It is worth noting that the OQPSK (Offset-QPSK) waveform used in WLAN 802.11b and cellular CDMA standards is second-order circular, however, it introduces improper complex noise into the system. For more detail, we refer to

A. Mirbagheri, K. N. Plataniotis, and S. Pasupathy, "An enhanced widely linear CDMA receiver with OQPSK modulation", IEEE Transactions on Communications, vol. 54, no. 2, pp. 261-272, 2006.

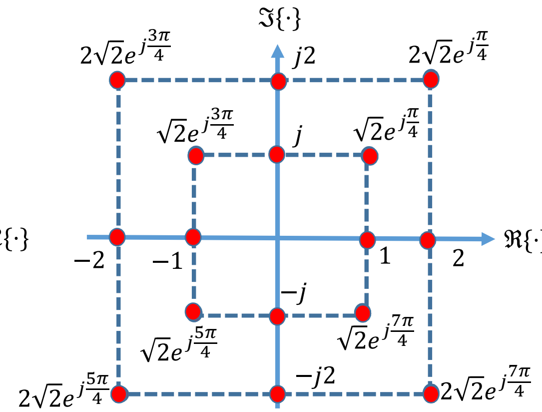
Circularity in communications
Constellations in communications, 16 symbols

Now, consider a communication system with 16 complex-valued symbols.

The most widely used modulation schemes are the amplitude and quadrature phase shift keying **APSK**) and quadrature amplitude modulation **QAM**).
Note that the constellation for **16-APSK** is more **compact** than that of the **16-QAM**.



16-APSK



16-QAM

16-APSK second-order statistics:

$c = E[zz^*] = 2.5$

$p = E[zz] = 0$

16-QAM second-order statistics:

$c = E[zz^*] = 3.75$

$p = E[zz] = 0$

Although **both methods are proper**, **only the 16-APSK is circular** (losely speaking).
Note that **circular** constellations offer better **energy efficiency**, whereas **non-circular constellations are more resilient to noise**, especially when using widely-linear processing.

Summary: Circularity in symbol constellations

The effects of circularity and impropriety on modulation schemes are summarized below:

1. **Circular** constellations, such as QPSK and 16-APSK, are more **energy efficient** as compared to their non-circular counterparts. This is due to compactness of circular constellations, and their **higher entropy**.
2. Communication systems with **non-circular constellations** are **more resilient to noise**. This stems from the larger average distance between the constellation points in non-circular constellations, compared to their circular counterparts.
3. In communication systems with **non-circular constellations** the step from strictly-linear to **widely-linear** processing generally results in **improved performance**.

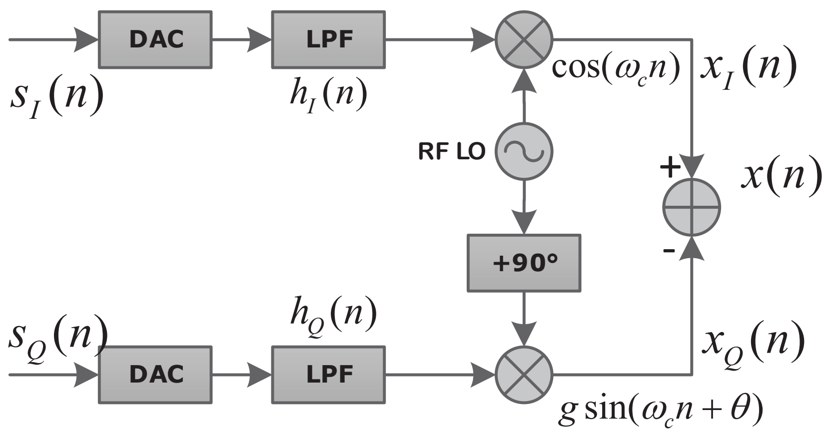
Some examples:

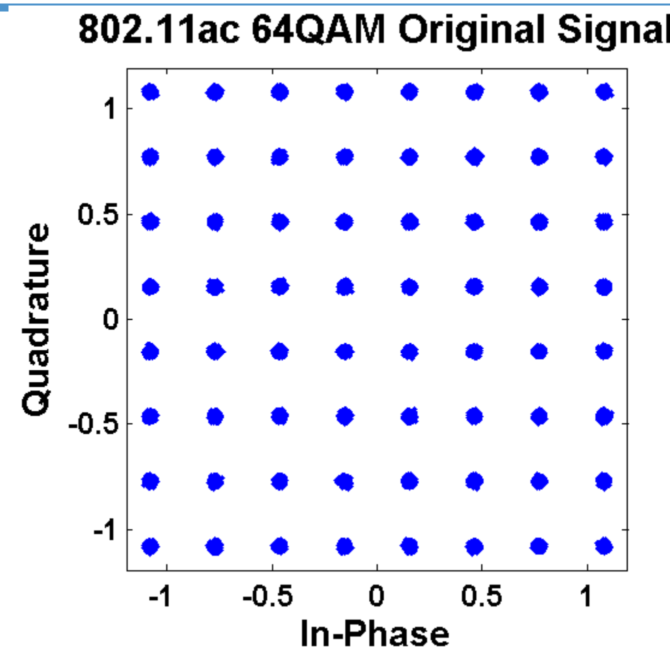
○ The long-term evolution (LTE) standard for high speed mobile phone communications switches between QPSK (**circular**) and 16-QAM (**non-circular but proper**), based on the conditions of communication channels.

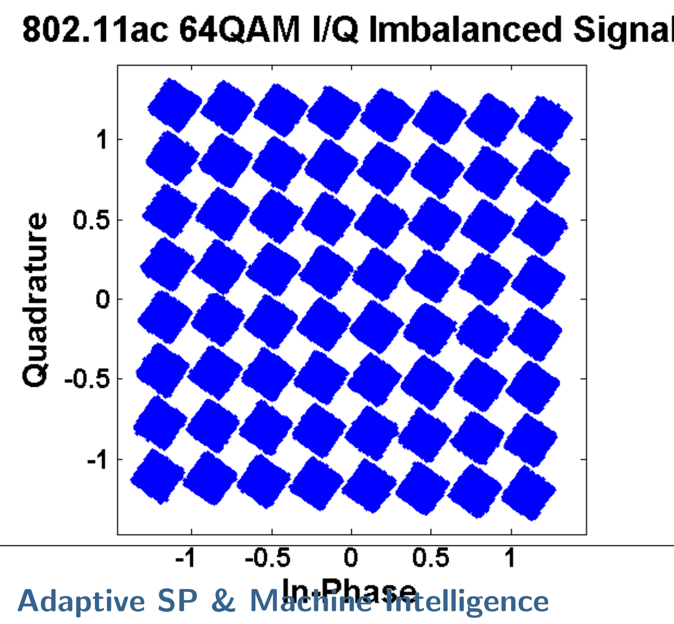
○ Satellite television broadcast standards use **M-APSK** (**circular and proper**) and **M-QAM** (**non-circular but proper**) schemes, with $M \in \{8, 16, 32\}$.

○ In optical communications, different types of **ASK** (**non-circular and improper**) are used.

Noncircularity arising from I/Q imbalance







Consider the baseband discrete-time input signal, $s(n)$, which is complex circular, e.g., 64-QAM. After passing through an I/Q imbalanced modulator, the output $x(n)$ becomes noncircular, that is

$$x(n) = \mu(n) * s(n) + \nu(n) * s^*(n)$$

where

$$\mu(n) = 1/2[h_I(n) + g h_Q(n) e^{-j\theta}]$$

$$\nu(n) = 1/2[h_I(n) - g h_Q(n) e^{-j\theta}]$$

Proof: Noncircularity and I/Q imbalance

Derivation:

The modulated passband signal $x_p(n)$ is given by

$$\begin{aligned} x_p(n) &= [s_I(n) * h_I(n)] \cos \omega_c n - [s_Q(n) * h_Q(n)] g \sin(\omega_c n + \varphi) \\ &= \underbrace{[s_I(n) * h_I(n) + g \sin \varphi s_Q(n) * h_Q(n)]}_{x_I(n)} \cos \omega_c n - \underbrace{g \cos \varphi}_{x_Q(n)} \sin \omega_c n \end{aligned}$$

Upon extracting the baseband signal from $x_p(n)$, and taking the in-phase and quadrature branches as the real and imaginary parts of $x(n)$, we have

$$\begin{aligned} x(n) &= x_I(n) + j x_Q(n) \\ &= \underbrace{\frac{1}{2}[h_I(n) + g e^{-j\varphi} h_Q(n)]}_{\mu(n)} * s(n) + \underbrace{\frac{1}{2}[h_I(n) - g e^{-j\varphi} h_Q(n)]}_{\nu(n)} * s^*(n) \end{aligned}$$

where $s(n) = s_I(n) + j s_Q(n)$

In a narrow-band scenario, the I/Q imbalance becomes frequency-independent, that is, $h_I(n) = h_Q(n) \approx \delta(n)$, and so

$$x(n) = \underbrace{\frac{1}{2}[1 + g e^{-j\varphi}]}_{\mu} s(n) + \underbrace{\frac{1}{2}[1 - g e^{-j\varphi}]}_{\nu} s^*(n)$$

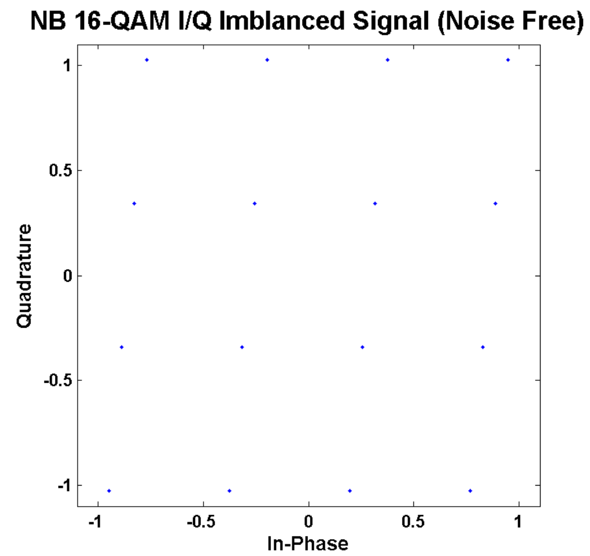
Impact of I/Q Imbalance in NB and WB COMMS

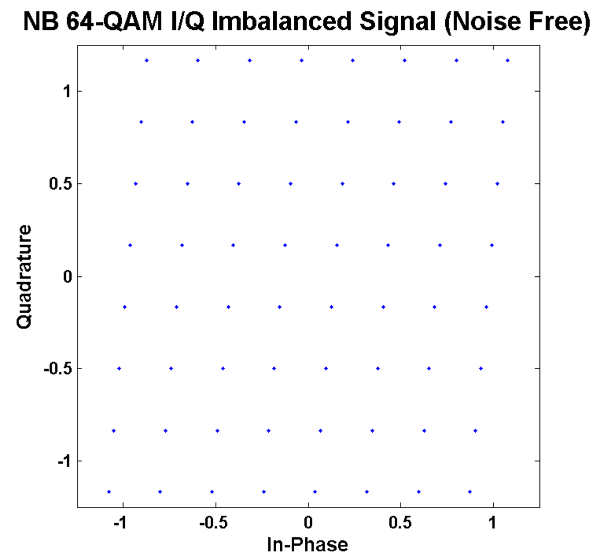
Consider frequency-independent I/Q imbalance.

Then, the time-domain narrowband (NB) I/Q imbalance can be modelled as

$$x(n) = \mu s(n) + \nu s^*(n)$$

Observe that the input signal $s(n)$ is interfered with its self-image $s^*(n)$, leading to the rotation and scaling of the constellation graph.





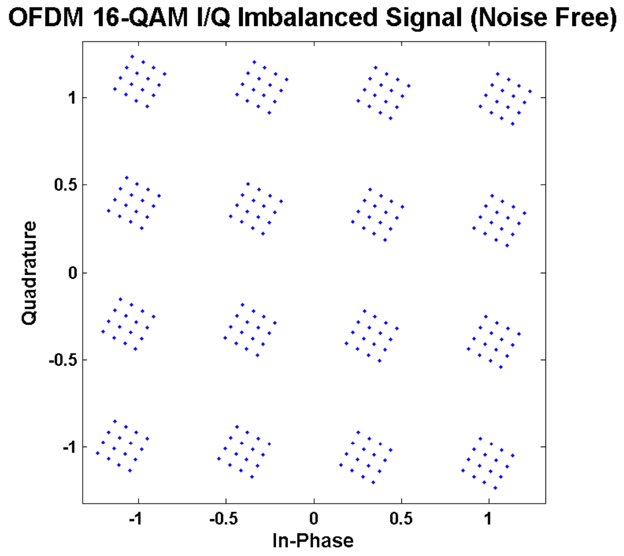
Impact of I/Q Imbalance in NB and WB COMMS, contd.

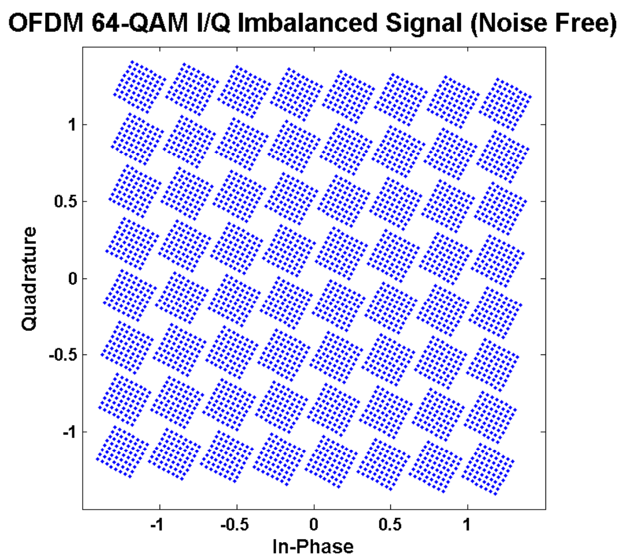
In wideband (WB) scenarios, OFDM is commonly used to combat frequency-selective fading. The mathematical interpretation of OFDM is iFFT. Upon taking iFFT on both sides, we obtain the frequency-domain WB I/Q imbalance model,

$$X_k(m) = \mu S_k(m) + \nu S^*_{-k}(m)$$

where k and m refer, respectively, to the subcarrier index and baseband OFDM symbol index.

Here, the input signal on a subcarrier k is interfered by the signal on the image subcarrier $-k$, causing a rotation and scaling of the whole graph. More importantly, for M-QAM constellation, each dot will ergodically scatter to M different positions, yielding a total of M^2 constellation dots.





Some literature

The following references give more insight:

1. F. Sterle, "Widely linear MMSE transceivers for MIMO channels," *IEEE Transactions on Signal Processing*, vol. 55, no. 8, pp. 4258-4270, August 2007.
2. D. Mattera, L. Paura , and F. Sterle, "MMSE WL equalizer in presence of receiver IQ imbalance," *IEEE Transactions on Signal Processing*, vol. 56, no. 4, pp. 1735-1740, April 2008.
3. D. P. Mandic and V. S. L. Goh, "Complex valued nonlinear adaptive filters: Noncircularity, widely linear, and neural models", Wiley, 2009.
4. P. J. Schreier and L. L. Scharf, *Statistical signal processing of complex-valued data: The theory of improper and noncircular signals*, Cambridge University Press, 2010.
5. Mustafa Ergen, *Mobile Broadband including WiMAX and LTE*, Springer, 2009.
6. H. J. R. Dutton, *Understanding optical communications*, IBM, 1998.
7. S. Kanna, M. Xiang, D. Mandic, *Real-time detection of rectilinear sources for wireless communications*, Proceedings of the IEEE ISWCS Conference, pp. 542–545, 2015.

Finally, for the latest standards on digital video broadcasting (**DVB**) we refer to dvb.org/standards.

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