definitions: $E=U+\frac{u^2}{2}$ $\quad U=\int_0^T c_v\,dT$ $\quad H=U+\frac{p}{\rho}$ $\quad \frac{p}{\rho}=RT$ $\quad \mathcal{O}E=RT\frac{\partial E}{\partial u}+uE$

$$\mathcal{O}^2E = RT\frac{\partial}{\partial u}\left(RT\frac{\partial E}{\partial u}+uE\right)+u\left(RT\frac{\partial E}{\partial u}+uE\right)$$

$$= RT\left(R\frac{\partial T}{\partial u}\frac{\partial E}{\partial u}+RT\frac{\partial^2 E}{\partial u^2}+E+u\frac{\partial E}{\partial u}\right)+uRT\frac{\partial E}{\partial u}+u^2E$$

with $\frac{\partial E}{\partial u}=u$ $\quad \frac{\partial^2 E}{\partial u}=1$, $\frac{\partial T}{\partial u}=\frac{\partial U}{\partial u}=0$

$$\mathcal{O}^2E = RT\left(RT+U+\frac{u^2}{2}+u^2\right)+u^2RT+u^2U+u^2\frac{u^2}{2}$$

$$= RT\left(RT+U+\frac{u^2}{2}+2u^2\right)+u^2U+u^2\frac{u^2}{2}$$

$$= \frac{p}{\rho}\frac{p}{\rho}+\frac{p}{\rho}U+\frac{p}{\rho}\frac{u^2}{2}+2\frac{p}{\rho}u^2+u^2U+u^2\frac{u^2}{2}$$

$\neq$

solution for $\rho\mathcal{O}\mathcal{O}E$ (eq. 38 with $\alpha=\beta=x$ for a 1D system)

$$\rho\mathcal{O}\mathcal{O}E = \left(H+\frac{u^2}{2}\right)\overset{eq}{p}+u^2p$$

$$= \left(H+\frac{u^2}{2}\right)\left(RT+u^2\right)+u^2p$$

$$= \left(U+\frac{p}{\rho}+\frac{u^2}{2}\right)\left(\frac{p}{\rho}+u^2\right)+u^2p$$

$$= U\frac{p}{\rho}+Uu^2+\frac{p}{\rho}\frac{p}{\rho}+\frac{p}{\rho}u^2+\frac{u^2}{2}\frac{p}{\rho}+\frac{u^2}{2}u^2+u^2p$$

$$\mathcal{O}\mathcal{O}E = U\frac{p}{\rho^2}-U\frac{u^2}{\rho}+\frac{p}{\rho}\frac{p}{\rho^2}+\frac{p}{\rho^2}u^2+\frac{u^2}{2}\frac{p}{\rho^2}+\frac{u^2}{2}u^2\frac{1}{\rho}+u^2\frac{p}{\rho}$$