FDM schemes, target: 2^{nd} order, stable

$$u' = \frac{u_{i+1} - u_{i-1}}{2\Delta x} \qquad 2^{nd} \text{ order}$$

stability: $u = e^{ikx}$

$$u' = i\tilde{k} e^{ikx} = \frac{e^{ik(x+\Delta x)} - e^{ik(x-\Delta x)}}{2\Delta x} = \frac{1}{2\Delta x} e^{ikx}\left(e^{ik\Delta x} - e^{ik-\Delta x}\right) = \frac{1}{\Delta x}\sin(k\Delta x)\, e^{ikx}$$

$$\tilde{k} = \frac{1}{\Delta x}\sin(k\Delta x)$$

small Δx: $\sin(k\Delta x) \sim k\Delta x \qquad \tilde{k} = k$

$$u + \varepsilon \to u = \varepsilon u^{ikx}$$

$$u' = \varepsilon' u^{i\tilde{k}x} + \varepsilon i k u^{i\tilde{k}x} = \frac{\varepsilon}{\Delta x}\sin(k\Delta x)$$

$$\frac{-3u_i + 4u_{i+1} - u_{i+2}}{2\Delta x} \qquad 2^{nd} \text{ order}$$

$$u = e^{ikx}$$

$$u' = i\tilde{k}\cancel{e^{ikx}} = \frac{1}{2\Delta x}\cancel{e^{ikx}}\left(-3 + 4e^{ik\Delta x} - e^{2ik\Delta x}\right)$$

$$i\tilde{k} = \frac{1}{\Delta x}\left(2e^{ik\Delta x} - \frac{1}{2}e^{2ik\Delta x} - \frac{3}{2}\right)$$

$$\tilde{k}\Delta x = -i\left(2e^{ik\Delta x} - \frac{1}{2}e^{2ik\Delta x} - \frac{3}{2}\right)$$

$$u' = \frac{-u_{i+2} + 8u_{i+1} - 8u_{i-1} + u_{i-2}}{12\Delta x} \qquad 4^{th} \text{ order}$$

$$u' = i\tilde{k}e^{ikx} = \frac{1}{12\Delta x}\left(+e^{-2ik\Delta x} - 8e^{-ik\Delta x} + 8e^{ik\Delta x} - e^{2ik\Delta x}\right)e^{ikx}$$

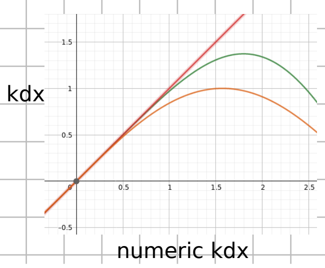
$$= \frac{1}{2i}\frac{i}{6\Delta x}\left(-e^{2ik\Delta x} - e^{-2ik\Delta x} + 8e^{-ik\Delta x} + 8e^{ik\Delta x}\right)e^{ikx}$$

$$= \frac{i}{6\Delta x}\left(-\sin(2k\Delta x) + 8\sin(k\Delta x)\right)e^{ikx}$$

$$\tilde{k}\Delta x = -\frac{1}{6}\sin(2k\Delta x) + \frac{4}{3}\sin(k\Delta x)$$

$k\Delta x \to 0$: $\qquad \tilde{k}\Delta x = -\frac{1}{6}2k\Delta x + \frac{4}{3}k\Delta x$

$$\tilde{k} = k$$



red exact
green forth order
yellow second order

$$u'' = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \qquad 2^{nd} \text{ order}$$

$$u = e^{ikx}$$

$$u'' = -\tilde{k}^2 \cancel{e^{ikx}} = \frac{1}{\Delta x^2}\left(e^{ik\Delta x} - 2 + e^{-ik\Delta x}\right) \cancel{e^{ikx}}$$

$$-\tilde{k}^2 \Delta x^2 = 2\cos(k\Delta x) - 2$$

$$u'' = \frac{2u_i - 5u_{i+1} + 4u_{i+2} - u_{i+3}}{\Delta x^2} \qquad 2^{nd} \text{ order}$$

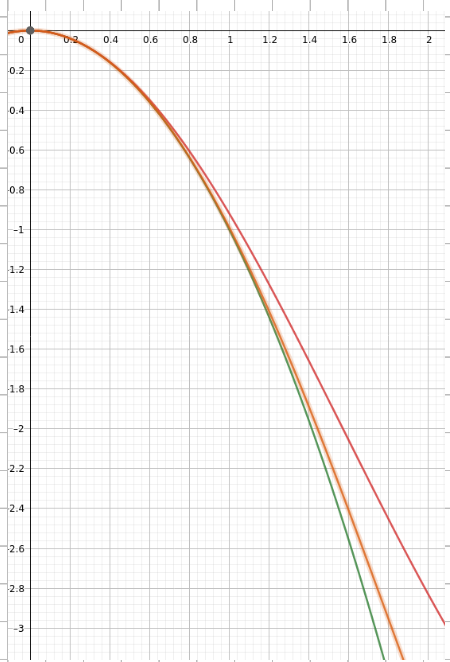
$$u'' = -\tilde{k}^2 = \frac{1}{\Delta x^2}\left(2 - 5e^{ik\Delta x} + 4e^{2ik\Delta x} - e^{3ik\Delta x}\right)$$

$$u'' = \frac{1}{12\Delta x^2}\left(-u_{i+2} + 16u_{i+1} - 30u_i + 16u_{i-1} - u_{i-2}\right) \qquad 4^{th} \text{ order}$$

$$-\tilde{k}^2 \Delta x^2 = \frac{8}{3}\cos(k\Delta x) - \frac{1}{3}\cos(2k\Delta x) + \frac{5}{2}$$

numeric kdx

(kdx)^2



green exact
yellow forth order
red second order

G populations

$$O_\alpha E = RT\frac{\partial E}{\partial u_\alpha} + u_\alpha E$$

$$O E = RT\frac{\partial E}{\partial u} + uE$$

$$= RTu + RT\frac{\partial U}{u} + uE$$

$$O^2 E = RT\frac{\partial}{\partial u}\left(RT\frac{\partial E}{\partial u} + uE\right) + u\left(RT\frac{\partial E}{\partial u} + uE\right)$$

$$= RT\left(R\frac{\partial T}{\partial u}\frac{\partial E}{\partial u} + RT\frac{\partial^2 E}{\partial u^2} + E + u\frac{\partial E}{\partial u}\right) + uRT\frac{\partial E}{\partial u} + u^2 E$$

with $\frac{\partial E}{\partial u} = u$ $\quad \frac{\partial^2 E}{\partial u} = 1$, $\frac{\partial T}{\partial u} = \frac{\partial U}{\partial u} = 0$

$$RT\left(RT + U + \frac{u^2}{2} + u^2\right) + u^2RT + u^2U + u^2\frac{u^2}{2}$$

$$RT\left(RT + U + \frac{u^2}{2} + 2u^2\right) + u^2U + u^2\frac{u^2}{2}$$

$$\frac{p}{\rho}\frac{p}{\rho} + \frac{p}{\rho}U + \frac{p}{\rho}\frac{u^2}{2} + 2\frac{p}{\rho}u^2 + u^2U + u^2\frac{u^2}{2}$$

$$\cdot\rho =$$

$$\frac{\partial E}{\partial u} = \frac{\partial U}{\partial u} + u = c_v\frac{\partial T}{\partial u} + u$$

$$\frac{\partial T}{\partial u} = \frac{\partial T}{\partial u}$$

$$\frac{\partial^2 E}{\partial u^2} = \frac{\partial}{\partial u}\left(c_v\frac{\partial T}{\partial u} + u\right) = c_v\frac{\partial^2 T}{\partial u^2} + 1$$

$$H = U + \frac{p}{\rho}$$

$$\left(H + \frac{u^2}{2}\right)u$$

$$= Uu + \frac{u^2}{2}u + pu$$

$$= Eu + \frac{p}{\rho}u$$

$$= Eu + RTu$$

same if $\frac{\partial U}{\partial u} = 0$

$$\left(H + \frac{u^2}{2}\right)p^{eq} + u^2 p$$

$$\left(H + \frac{u^2}{2}\right)\left(RT + u^2\right) + u^2 p$$

$$\left(U + \frac{p}{\rho} + \frac{u^2}{2}\right)\left(\frac{p}{\rho} + u^2\right) + u^2 p$$

$$U\frac{p}{\rho} + Uu^2 + \frac{p}{\rho}\frac{p}{\rho} + \frac{p}{\rho}u^2 + \frac{u^2}{2}\frac{p}{\rho} + \frac{u^2}{2}u^2 + u^2 p$$

$$RT\left(R\frac{\partial T}{\partial u}\frac{\partial E}{\partial u} + RT\frac{\partial^2 E}{\partial u^2} + E + u\frac{\partial E}{\partial u}\right) + uRT\frac{\partial E}{\partial u} + u^2E \overset{!}{=} U\frac{p}{\rho^2} + \frac{Uu^2}{\rho} + \frac{p}{\rho}\frac{p}{\rho}\frac{1}{\rho} + \frac{p}{\rho}\frac{1}{\rho}u^2 + \frac{u^2}{2}\frac{p}{\rho}\frac{1}{\rho} + u^2\frac{p}{\rho}$$

$$\frac{p}{\rho}R\frac{\partial T}{\partial u}\frac{\partial E}{\partial u} + \frac{p}{\rho}\frac{\partial^2 E}{\partial u^2} + \frac{p}{\rho}u\frac{\partial E}{\partial u} + u\frac{p}{\rho}\frac{\partial E}{\partial u} = -\frac{p}{\rho}U - \frac{p}{\rho}\frac{u^2}{2} - u^2U - u^2\frac{u^2}{2} + U\frac{p}{\rho^2} + \frac{Uu^2}{\rho} + \frac{p}{\rho}\frac{p}{\rho}\frac{1}{\rho} + \frac{p}{\rho}\frac{1}{\rho}u^2 + \frac{u^2}{2}\frac{p}{\rho}\frac{1}{\rho} + u^2\frac{p}{\rho}$$

$$\mu = \left(\frac{1}{\omega} - \frac{1}{2}\right) P\delta t$$

$$\nu = \frac{\mu}{\rho} = \left(\frac{1}{\omega} - \frac{1}{2}\right) RT\delta t \qquad \frac{1}{\omega} - \frac{1}{2} = \frac{\nu}{RT\delta t}$$

$$\omega = \frac{1}{\frac{\nu}{RT\delta t} + \frac{1}{2}} = \frac{2RT\delta t}{2\nu + RT\delta t}$$

$$\mathrm{Pr} = \frac{C_p \mu}{\kappa} = \frac{\omega_1(2-\omega)}{\omega(2-\omega_1)} \qquad \omega_1 = \frac{2\omega \mathrm{Pr}}{\omega(\mathrm{Pr}-1)+2}$$

$$\rho u A = cst$$

$$\rho_{in} u_{in} \cancel{A} = \rho_{out} u_{out} \cancel{A}$$

$$\frac{\rho_{in}}{RT_{in}} = p_{in} = p_{out} = \frac{\rho_{out}}{RT_{out}}$$

$$(f_+ + f_0 + f_-)(f_+ + f_-) = f_+^2 + 2f_+ f_- + f_0 f_+ + f_0 f_- + f_-^2 = \rho u$$