(8) Prove that if seq $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $m > 0$, the seq $\{ma_n\}_{n=1}^{\infty}$ tends to limit mL

Proof:

(By definition of Limit $a_n \to a$ $\quad \forall \epsilon > 0 \ \exists N \in \mathbb{N}$ such that $\forall n > N \ |a_n - a| < \epsilon$)

so we are given
$\forall \epsilon > 0 \ \exists N \in \mathbb{N}$ such that $\forall n > N \ |a_n - L| < \epsilon$

note:

$$|ma_n - mL| < \epsilon \Rightarrow$$

$$|m||a_n - L| < \epsilon \quad \text{or}$$

$$|a_n - L| < \frac{\epsilon}{m}$$

$\forall \epsilon > 0 \ \exists N \in \mathbb{N}$ such that $\forall n > N \ |a_n - L| < \frac{\epsilon}{m} < \epsilon$

for some number $m > 0$

but $|a_n - L| < \frac{\epsilon}{m} \Rightarrow m|a_n - L| < \epsilon$

since $m > 0$ we can rewrite this as

$|m||a_n - L| < \epsilon$ or $|ma_n - mL| < \epsilon$

therefore $\{ma_n\}_{n=1}^{\infty}$ tends to Limit mL

if $\{a_n\}_{n=1}^{\infty}$ tends to Limit L