(5)

Claim: $\forall n \in \mathbb{Z}$ at least one of the integers $n, n+2, n+4$ is divisiable by 3

note that every 3RD integer is divisiable by 3

ie 1 2 3 4 5 6 7 8 9 . . .

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & | & & & | & & & | \\ & & 1\cdot 3 & & & 2\cdot 3 & & & 3\cdot 3 \end{array}$$

proof by cases

case 1: point selected is divisiable by 3

n is $3|n$

case 2 point selected is 1 digit prior to point divisiable by 3 ($n = p-1$ where $3|p$)

then $n+4 = p-1+4 = p+3$

but $3|p \Rightarrow 3|(p+3)$ ✓

case 3: point selected is 1 digit after point divisable by 3 ($n = p+1$ where $3|p$)

then $n+2 = p+1+2 = p+3$

but since $3|p \Rightarrow 3|(p+3)$

therefore $n, n+2,$ or $n+4$ is divisiable by 3 $\forall n \in \mathbb{Z}$