

③ $\forall n \in \mathbb{Z} \quad (n^2 + n + 1 \text{ is odd})$

True

Proof: $\forall n \in \mathbb{Z} \quad n^2 + n + 1 = n(n+1) + 1$

but $n(n+1)$ must be even

since an odd \cdot even is even and

$n(n+1)$ is either even \cdot odd or odd \cdot even

Therefore $n(n+1) + 1$ is an even plus one

$\Rightarrow n^2 + n + 1 \text{ is odd } \forall n \in \mathbb{Z}$

_____ addendum

JUST in case you need proof of odd \cdot even = even

$\forall p \in \mathbb{Z}$

$2p$ is even

$2p+1$ is odd

$(2p)(2p+1) = 4p^2 + 2p = 2(2p^2 + p) = 2m \text{ is even}$
(where $m = 2p^2 + p$)