

⑥ prove that the only prime Triple
(3 primes, 2 apart) is 3, 5, 7

Proof by contradiction.

Assume n is prime such that

$n, n+2, n+4$ are all prime.

but by problem ⑤, we know that at least
one of $n, n+2, n+4$ is divisible by 3

this is true for $n=3$, but for

Any other set of prime numbers,

$n, n+2, n+4$ cannot have one of these
divisible by 3 (since, if it were then
it would not be prime)

So we have a contradiction. Therefore

there cannot be a set of prime $n, n+2, n+4$
unless $n=3$