① Is the following true?

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

False — Proof by cases

note, there are only two combinations that are ≤ 12:

case 1: $m=1, n=1$ (<12)

case 2: $m=2, n=1$ (<12)

all other combinations are > 12

case 1: $m=1, n=1$ gives $3+5 = 8 \neq 12$

case 2: $m=2, n=1$ gives $6+5 = 11 \neq 12$

for completness, show next two cases:

case 3: $m=1, n=2$ gives $3+10 = 13 > 12$

case 4: $m=3, n=1$ gives $9+5 = 14 > 12$

Another proof (by contradiction)

$3m + 5n = 12$? Assume $3m + 5n = 12$

n must be 1 (otherwise $3m + 5n > 12$)

so $3m + 5 = 12 \Rightarrow 3m = 12 - 5 = 7$

note that there does not exist an $m \in \mathbb{N}$ such that $3m = 7$ therefore a contradiction

and $3m + 5n \neq 12$ for $m, n \in \mathbb{N}$