(4) <u>Claim</u>: every <u>odd</u> natural number is of one of the forms $4n+1$ or $4n+3$ where $n \in \mathbb{Z}$

<u>Proof</u>:

<u>facts</u>: a) $2p$ is even $\forall\, p \in \mathbb{Z}$

b) $2p+1$ is odd $\forall\, p \in \mathbb{Z}$

Case 1

Using (a), $2(2p)$ must be even

then by (b) $2(2p)+1$ is odd $\Rightarrow$ $4p+1$ is odd

Case 2

using (b) and (a)

$2(2p+1)$ is even therefore

$2(2p+1)+1$ is odd, but

$2(2p+1)+1 = 4p+2+1 = 4p+3$ is odd

---

note $m \in \mathbb{N} \Rightarrow m = \{1, 2, 3, \ldots\}$

And for $n \in \mathbb{Z}$, $1 = 4n+1$ if $n=0$

$3 = 4n+3$ if $n=0$

$5 = 4n+1$ if $n=1$

$7 = 4n+3$ if $n=1$

So, for $n=0$,

$4n+1$, $4n+3$ produces an odd natural number

④ cont

Assume $A(n) = 4n+1$ $B(n) = 4n+3$ are true (produce odd natural number)

now show

$A(n+1)$, $B(n+1)$

$A(n+1) = 4\underbrace{\underbrace{(n+1)}_{\text{odd}}}_{\text{even}} + 1 = 4n+4+1 = 4n+5$

produces odd

$B(n+1) = \underbrace{4\underbrace{(n+1)}_{\text{odd}}}_{\text{even}} + \underbrace{3}_{\text{odd}} \Rightarrow$ odd