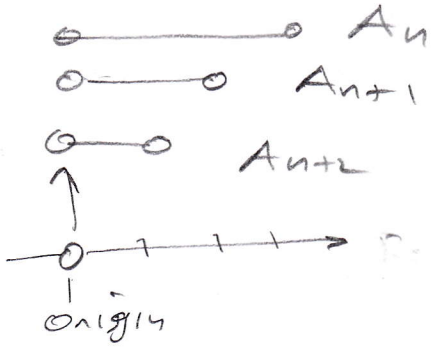
(9) find family of Intervals A_n $n=1,2,\ldots$ such that $A_{n+1} \subset A_n$ $\forall n$ and

$$\bigcap_{n=1}^{\infty} A_n = \emptyset \quad \text{(empty set)}$$

Basically, A_n must be a subset interval of the prior A_{n-1}. So choose the open interval

$$\boxed{A_n = \left(0, \tfrac{1}{n}\right)}$$



note that

$$\lim_{n\to\infty} A_n = \emptyset \text{, in other words } (0,0) = \emptyset$$

since $\lim_{n\to\infty} \frac{1}{n} = 0$

Proof:

Given an arbitrary $n \in \mathbb{N}$,

$A_n = \left(0, \frac{1}{n}\right)$

$A_{n+1} = \left(0, \frac{1}{n+1}\right)$ note that $\frac{1}{n+1} < \frac{1}{n}$

therefore every element of A_{n+1} is an element of A_n, then $A_{n+1} \subset A_n$

Also $\lim_{n\to\infty} A_n = \emptyset$ and the intersection of Any set with the empty set is, by definition $\emptyset$.