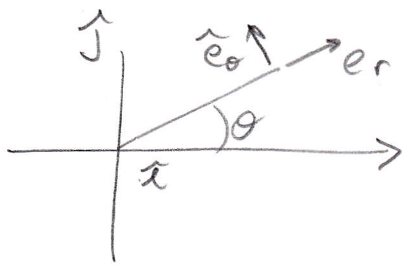

$$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\frac{d\hat{e}_r}{d\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath} = \hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{d\theta} = -\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath} = -\hat{e}_r$$

$$\frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt} = \frac{d\hat{e}_r}{dt} = \dot{\theta}\,\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{d\theta}\frac{d\theta}{dt} = \frac{d\hat{e}_\theta}{dt} = -\dot{\theta}\,\hat{e}_r$$

note

$$\frac{d\hat{e}_r}{dr} = 0$$

$$\frac{d\hat{e}_\theta}{dr} = 0$$

$$\vec{r} = r\,\hat{e}_r \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

$$= \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

let $v_r = \dot{r}$

$v_\theta = r\dot{\theta}$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$\vec{v} = v_r\,\hat{e}_r + v_\theta\,\hat{e}_\theta$$

$$\vec{a} = \dot{\vec{v}} = \ddot{r}\,\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + \dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{dt}$$

$$= \ddot{r}\,\hat{e}_r + \dot{r}\dot{\theta}\,\hat{e}_\theta + \dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$$

$$= \ddot{r}\,\hat{e}_r + 2\dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$$

$$= (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$