LIGHT: Enhancing Learning Path Recommendation via Knowledge Topology-Aware Sequence Optimization

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Abstract

Learning path recommendation (LPR) aims to provide individualized and effective learning item routes by modeling learners' learning histories and goals, which has been widely considered a essential task in the field of personalized education. Indeed, considerable research efforts have been dedicated to this direction in recent years, focusing on step-based and sequence-based modeling approaches. However, most of existing studies overlook the complementarity between explicit and implicit relationships among knowledge concepts, while failing to harmonize static knowledge structures with dynamic path generation. To this end, in this paper, we propose **LIGHT**, a know**L**edge topology-aware sequence opt**I**mization model for enhancing learnin**G** pat**H** recommenda**T**ion. Specifically, we first construct a composite concept graph that incorporates explicit prerequisite relationships and implicit collaborative relationships, achieved by mining interaction statistics and collaborative signals from learners' learning processes. Next, we design a complementary contrastive fusion module to fully capture the interplay between the two relational views of concepts through graph structure learning and contrastive constraints, which enhances the effectiveness of the learned representations. Following this, we introduce a knowledge topology-aware modeling module that integrates structural semantics clustering with candidate path sampling. Finally, we develop a bidirectional sensing path optimization network to deeply model and optimize the sampled paths from a sequential perspective, thereby enhancing modeling efficiency while preserving structural semantics. Extensive experiments on three real-world educational datasets clearly demonstrate the effectiveness of the proposed LIGHT model in the LPR task.

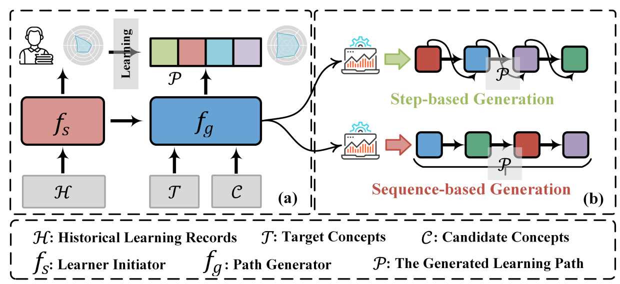

Figure 1: The core elements of learning path recommendation task, including learner initiator f_s and path generator f_g.

CCS Concepts

• **Information systems → Recommender systems**; • **Applied computing → E-learning**.

Keywords

Educational data mining, learner modeling, learning path recommendation, sequence optimization

ACM Reference Format:
Xiaoshan Yu, Shangshang Yang, Ziwen Wang, Siyu Song, Haiping Ma, Zhiguang Cao, and Xingyi Zhang. 2025. LIGHT: Enhancing Learning Path Recommendation via Knowledge Topology-Aware Sequence Optimization. In *Proceedings of the 48th International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR '25), July 13–18, 2025, Padua, Italy.* ACM, New York, NY, USA, 10 pages. https://doi.org/10.1145/3726302.3730022

1 Introduction

With the emergence of various online education platforms (e.g., Coursera [11] and XuetangX [44]), personalized learning [6] has become a prevalent mode of education, offering tailored services

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ACM ISBN 979-8-4007-1592-1/2025/07
https://doi.org/10.1145/3726302.3730022

that efficiently address individual learners' needs and help them achieve their educational goals. Learning path recommendation (LPR) [20], as a fundamental and crucial task in personalized learning, has garnered increasing attention in recent years due to its ability to adaptively generate appropriate learning paths based on learners' proficiency levels, effectively guiding them toward mastery of the target knowledge. In general, LPR primarily consists of two core modules: a learner initiator f_s, which effectively evaluates the learner's initial competence, and a path generator f_g, which constructs personalized learning paths, as illustrated in Figure 1(a).

Existing research on learning path recommendation can be broadly classified into two categories: step-based recommendation and sequence-based recommendation (as shown in Figure 1(b)). The step-based approach involves recommending learning items to students incrementally, with real-time interaction. This method requires students to provide immediate feedback on their performance, enabling the system to adapt planning dynamically. For instance, CSEAL [15] models LPR as a Markov decision process and leverages reinforcement learning to simulate interactions, progressively recommending concepts based on the student's responses. Unlike the former, the second category generates complete personalized resource paths for learners from a global perspective, as seen in SRC [1], which proposes a ranking-based concept-aware method under the set-to-sequence paradigm to generate learning paths.

Despite the impressive results achieved by these approaches, they overlook the complementarity of explicit and implicit relationships between knowledge concepts and fail to harmonize static knowledge structures with dynamic path generation, resulting in a loss of logical coherence in the learning path and suboptimal learning outcomes. To this end, in this paper, we aim to deeply model the learning path recommendation task from the perspective of sequence optimization. However, this goal is relatively challenging due to several factors: (1) *How to learn concept representations effectively?* Learning elements in paths are not self-connecting, which often contain collaborative signals conveyed during previous interactions, leading to increased difficulty in learning representations. (2) *How to efficiently explore the path space while preserving the logical consistency of the path during optimization?* In fact, the complete path search space is vast, with its complexity growing exponentially as the number of learned elements increases, making a well-designed optimization strategy essential.

To address these challenges, we propose a know**L**edge topology-aware sequence opt**I**mization model for achieving a more productive learnin**G** pat**H** recommenda**T**ion (**LIGHT**). Specifically, we first construct a composite concept graph that incorporates explicit prerequisite relationships and implicit collaborative relationships by mining interaction statistics and collaborative signals from learners' learning processes. We then design a complementary contrastive fusion module to effecitvely capture the complementarity between the two relational views of concepts, which is achieved through graph structure learning and contrastive constraints, enabling the learning of more robust and informative representations. Subsequently, we propose a knowledge topology-aware modeling module that consists of the structural semantics clustering and the candidate path sampling. Finally, we develop a bidirectional sensing path optimization network to comprehensively model and optimize the sampled paths from a sequence perspective, enhancing the search efficiency while maintaining the structural semantics. Extensive experiments on three educational datasets clearly demonstrate the effectiveness of the proposed LIGHT model in the LPR task.

2 Related Work

2.1 Learning Path Recommendation

Learning path recommendation (LPR) [20] aims to provide individualized and effective learning item routes by modeling learners' learning histories and goals, which has been widely considered a essential task in the field of personalized education [18, 32, 37, 38, 43]. Existing learning path recommendation approaches are mainly grouped into two categories, respectively, step-based recommendation methods and sequence-based recommendation methods. The first category of methods [13, 15, 16, 28, 41] primarily recommends learning elements to learners in a step-wise manner based on their ongoing interactions and feedback during the learning session. For example, CSEAL [15] frames learning path recommendation as a Markov decision-making process, leveraging the learner's evolving knowledge state and cognitive structures to dynamically recommend appropriate learning items. GEHRL [13], in contrast, proposes a graph enhanced hierarchical reinforcement learning framework to support step-by-step learning path generation that meets multiple learning goals. Another category of methods [1, 19, 39, 42, 45] focuses on constructing complete sequences of learning items directly for learners with fully consideration of learning demands and conceptual associations. For instance, SRC [1] presents a set-to-sequence path recommendation model that ranks and generates learning paths for the learner by mining the relationships among candidate concepts. Despite their potential strengths, these methods fail to sufficiently explore and exploit the interplay between explicit prerequisite relationships and implicit collaborative information among knowledge concepts in the path generation process.

2.2 Sequence Optimization

Sequence optimization [3] refers to the process of refining and improving a sequence of decisions, tasks, or elements to achieve a specified objective, and it plays a crucial role in various domains, including vehicle routing planning [25, 35, 40], industry design [12], and personalized route recommendation [26, 33]. Sequence optimization can broadly be categorized into static-based and dynamic-based approaches from the modeling perspective. Specifically, static-based sequence optimization methods [2, 8, 46] involves constructing and optimizing a sequence by relying on stable, predetermined parameters before execution begins. In contrast, dynamic sequence optimization [31, 34] entails continuously adjusting sequences in response to real-time feedback and changing conditions, optimizing performance dynamically. For example, Ptr-Net [31] utilizes neural attention to adaptively select members of the input sequence as outputs, thereby addressing variable-size output dictionary challenges and providing effective approximations for complex geometric problems. However, modeling learning path recommendation from the perspective of sequence optimization remains unexplored.

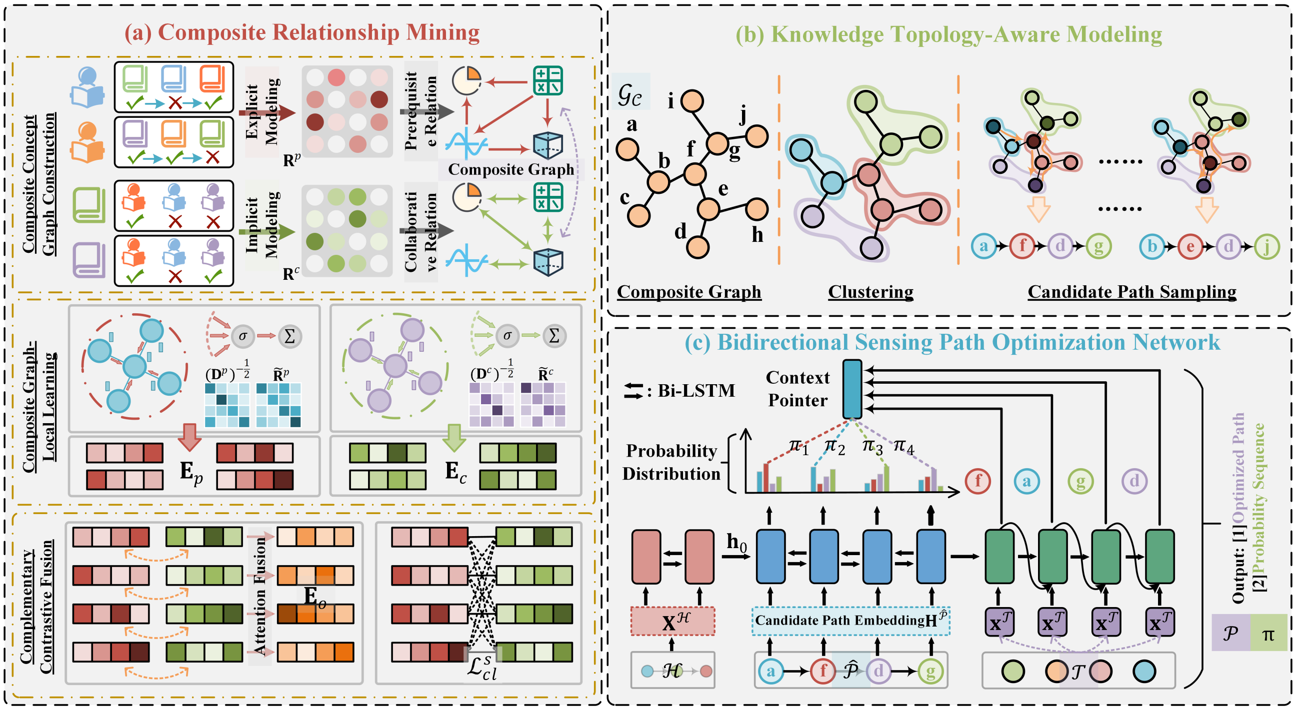

Figure 2: The overview architecture of our proposed LIGHT model. (a) The composite relationship mining module. (b) The knowledge topology-aware modeling module. (c) The bidirectional sensing path optimization network. Best viewed in color.

3 Problem Formulation

In this section, we formally define the learning path recommendation (LPR) task. In each learning session, a learner $s \in \mathcal{S}$ with a historical learning record sequence $\mathcal{H} = \{h_1, h_2, \ldots, h_M\}$ aims to improve the mastery in a specific learning target $\mathcal{T} = \{t_1, t_2, \ldots, t_N\}$, where $h_i = \{c_i, y_i\}$ contains a learned concept c_i and the corresponding performance score $y_i \in \{0, 1\}$, and t_i is one of the target concept intended to be acquired. The goal of the LPR task is to recommend and generate an optimal learning path $\mathcal{P} = \{p_1, p_2, \ldots, p_L\}$ including L learning items for the learner s, based on a given set $C = \{c_1, c_2, \ldots, c_K\}$ containing K candidate knowledge concepts, so that maximize the learner's capability improvement on the target $\mathcal{T}$ after progressively completing the learning process. Notably, the **Learning Gain** metric [15] is widely adopted to quantify the effect of recommended paths, as it depicts the normalized improvement of student proficiency, which is formally defined as follows:

$$LG_{\mathcal{P}} = \frac{E_{end} - E_{start}}{E_{sup} - E_{start}}, \tag{1}$$

where E_{start} and E_{end} denote the student's mastery level of the target concepts before and after the learning path $\mathcal{P}$, respectively, which can be assessed through the corresponding exams, and E_{sup} indicates the theoretical upper bound of the mastery level.

Problem Definition. ***(Learning Path Recommendation).*** *Given a learner's historical learning sequence $\mathcal{H}$, the target concepts set $\mathcal{T}$, and a set of candidate concepts C, the task involves selecting L distinct items from C and constructing them into an optimal learning path $\mathcal{P}$ that aims to enhance the learner's proficiency in $\mathcal{T}$. To achieve this, the LPR requires effectively learning two functions: $f_s(\cdot; \omega_s)$, which models the learner's historical interactions (as the learner initiator), and $f_g(\cdot; \omega_g)$, which generates the desired learning path (as the path generator). The ultimate goal is to maximize the expected learning outcomes, with the optimization objective formalized as:*

$$\begin{cases} \max\limits_{\omega_s, \omega_g} \mathbb{E}_{\mathcal{P} \sim p(\mathcal{P}|\mathcal{H}, \mathcal{T}, C; \omega_g)} \left[\dfrac{E_{end}(\mathcal{P}) - E_{start}(\mathcal{H}; \omega_s)}{E_{sup} - E_{start}(\mathcal{H}; \omega_s)} \right] \\ \text{s.t.} \quad p_i \in C, \forall i \in \{1, 2, \ldots, L\}; \quad p_i \neq p_j, \forall i \neq j. \end{cases} \tag{2}$$

4 Methodology

In this section, we present the LIGHT framework in detail. As depicted in Figure 2, the architecture of LIGHT is primarily composed of three key components: the composite relationship mining module, the knowledge topology-aware modeling module, and the bidirectional sensing path optimization network.

4.1 Composite Relationship Mining

Relational mining of knowledge concepts[5, 22] has been recognized as contributing to cognitive modeling in personalized learning tasks, both in exploring the micro-level logic of knowledge structures and in modeling the macro-level evolution of learners' competencies. Given the explicit and implicit connections between concepts in the learning path recommendation process, we meticulously design two sub-modules–the composite concept graph construction and the complementary contrastive fusion–to integratively mine this composite relationship. The details are as follows.

4.1.1 ***Composite Concept Graph Construction***. To fully exploit the associations between knowledge concepts for more in-depth modeling of learning items, we first construct a composite concept graph $\mathcal{G}_C = (\mathcal{G}_{exp}, \mathcal{G}_{imp}) = (C, \mathcal{R}_{exp} \cup \mathcal{R}_{imp})$, where $C = \{c_1, c_2, \ldots, c_K\}$ represents the set of the concept nodes, and

$\mathcal{R}_{exp}$ and $\mathcal{R}_{imp}$ are the set of explicit and implicit relationships among concept nodes, respectively. First, inspired by previous work [5, 30], which shows that explicit relationship between concepts can often be determined through statistical feature capturing the ordered interaction behaviors of students with different concepts, we construct the transition matrix to mine the prerequisite relation between concepts. Specifically, we start by constructing the probability matrix $\mathbf{P} \in \mathbb{R}^{K\times K}$, where each element $\mathbf{P}_{ij}$ quantifies the frequency with which concept c_j is correctly answered immediately after concept c_i: $\mathbf{P}_{ij} = \frac{Count_{ij}}{Total_i}$, where $Count_{ij}$ is the number of times concept c_j is correctly answered immediately after concept c_i, and $Total_i = \sum_k Count_{ik}$ denotes the total number of times concept c_i is answered correctly. Afterwards, we utilize the min-max scaling to normalize $\mathbf{P}$: $\widetilde{\mathbf{P}}_{ij} = \frac{\mathbf{P}_{ij} - min(\mathbf{P})}{max(\mathbf{P}) - min(\mathbf{P})}$, where $min(\mathbf{P})$ and $max(\mathbf{P})$ denote the minimum and maximum values in $\mathbf{P}$, respectively, and thus we can obtain the binary transition matrix $\mathbf{T} \in \{0,1\}^{K\times K}$, where $\mathbf{T}_{ij} = 1$ if $\widetilde{\mathbf{P}}_{ij}$ exceeds the threshold $\alpha = (\frac{1}{K^2}\sum_{i,j}\widetilde{\mathbf{P}}_{ij})^3$, and $\mathbf{T}_{ij} = 0$ otherwise. Finally, the prerequisite relation matrix $\mathbf{R}^p \in \{0,1\}^{K\times K}$ is formulated to capture the directional dependencies between concepts, as follows:

$$\mathbf{R}^p_{ij} = \begin{cases} 1 & \text{if } \mathbf{T}_{ij} = 1 \text{ and } \mathbf{T}_{ji} = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

where $\mathbf{R}^p_{ij}$ denotes the explicit prerequisite relationship between concepts c_i and c_j, indicating that mastery concept c_i is required before advancing to concept c_j, which helps ensure a logical and effective learning progression through the knowledge structure.

To further explore the implicit relationship between knowledge concepts, which often contain richer underlying information, we leverage collaborative information derived from learner interaction data during exercise-solving—an aspect that has been largely overlooked in prior research. Concretely, for each pair of concepts c_i and c_j, we extract response vectors $\mathbf{r}_i$ and $\mathbf{r}_j$, which capture the response patterns of the same X students who have engaged with exercises related to these concepts. The vector $\mathbf{r}_i = [r_{i1}, r_{i2}, \dots, r_{iX}]$, where $r_{ik} \in \{0,1\}$ aggregates the response from these students, with each entry r_{ik} indicating whether student s_k answered the exercise on concept c_i correctly. Similarly, $\mathbf{r}_j$ is constructed in the same manner for concept c_j. Therefore, we could construct the collaborative relation matrix $\mathbf{R}^c \in \{0,1\}^{K\times K}$, which depicts the collaborative similarity between concepts as follows:

$$\mathbf{R}^c_{ij} = \mathbf{R}^c_{ji} = \begin{cases} 1 & \text{if } sim(\mathbf{r}_i, \mathbf{r}_j) > \beta, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

where $sim(\mathbf{r}_i, \mathbf{r}_j) = \frac{\mathbf{r}_i \cdot \mathbf{r}_j}{\|\mathbf{r}_i\|\|\mathbf{r}_j\|}$ denotes the cosine similarity of $\mathbf{r}_i$ and $\mathbf{r}_j$, implying the strength of the collaborative relationship between concepts c_i and c_j, and β is a pre-determined distance threshold. Indeed, the effectiveness of this implicit relationship construction strategy is supported by the mutual information $\mathcal{I}(\mathbf{r}_i;\mathbf{r}_j)$ between response vectors $\mathbf{r}_i$ and $\mathbf{r}_j$, which quantifies the shared information and justifies the connection between concepts:

$$\mathcal{I}(\mathbf{r}_i;\mathbf{r}_j) = \sum_{\mathbf{r}_i,\mathbf{r}_j} p(\mathbf{r}_i,\mathbf{r}_j) \log \frac{p(\mathbf{r}_i,\mathbf{r}_j)}{p(\mathbf{r}_i)p(\mathbf{r}_j)}. \tag{5}$$

where $p(\mathbf{r}_i, \mathbf{r}_j)$ represents the joint probability distribution of the response vectors $\mathbf{r}_i$ and $\mathbf{r}_j$, and $p(\mathbf{r}_i)$ and $p(\mathbf{r}_j)$ denotes the the marginal probability distributions of $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively. Finally, by constructing both the explicit and implicit relationships, we can obtain the complete composite concept graph $\mathcal{G}_C = (C, \mathcal{R}_{exp} \cup \mathcal{R}_{imp}) = (C, \{(c_i, c_j, exp) \mid \mathbf{R}^r_{ij} = 1\} \cup \{(c_i, c_j, imp) \mid \mathbf{R}^c_{ij} = 1\})$.

4.1.2 *Complementary Contrastive Fusion*.

With the aim of fully exploiting the structural relationships in the constructed composite concept graph $\mathcal{G}_C$ and to account for their complementary aspect, we novelly design a complementary contrastive fusion module to effectively learn concept representations. We first model the graph local information of knowledge concepts including explicit and implicit relationships by introducing the graph convolutional layers [10]. Given the initial node feature matrix $\mathbf{E}_0$, the prerequisite relation matrix $\mathbf{R}^p$, and the collaborative relation matrix $\mathbf{R}^c$, the information propagation process at l-th layer for both the explicit and implicit graphs is meticulously defined as follows:

$$\begin{cases} \mathbf{E}_p^{(l+1)} = \sigma(\bar{\mathbf{R}}^p \mathbf{E}_p^{(l)} \mathbf{W}_p^{(l)} + \mathbf{b}_p^{(l)}) \\ \mathbf{E}_c^{(l+1)} = \sigma(\bar{\mathbf{R}}^c \mathbf{E}_c^{(l)} \mathbf{W}_c^{(l)} + \mathbf{b}_c^{(l)}) \end{cases}, \tag{6}$$

where $\mathbf{E}_p^{(0)} = \mathbf{E}_c^{(0)} = \mathbf{E}_0 \in \mathbb{R}^{K\times d}$ are the initial node representations of the explicit graph and the implicit graph, respectively, d denotes the feature dimension, $\mathbf{W}_p^{(l)}, \mathbf{W}_c^{(l)} \in \mathbb{R}^{d_l \times d_{l+1}}$, $\mathbf{b}_p^{(l)}, \mathbf{b}_c^{(l)} \in \mathbb{R}^{d_l}$ are trainable parameters, d_l represents the output feature dimension of l-th layer and $d_0 = d$, $\sigma(\cdot)$ denotes the non-linear activation function. Here, $\bar{\mathbf{R}}^p, \bar{\mathbf{R}}^c \in \mathbb{R}^{K\times K}$ are the normalized symmetric relation matrices derived from $\mathbf{R}^p$ and $\mathbf{R}^c$ calculated as:

$$\begin{cases} \bar{\mathbf{R}}^p = (\mathbf{D}^p)^{-\frac{1}{2}} \tilde{\mathbf{R}}^p (\mathbf{D}^p)^{-\frac{1}{2}}, \ \tilde{\mathbf{R}}^p = \mathbf{R}^p + \mathbf{I}_K \\ \bar{\mathbf{R}}^c = (\mathbf{D}^c)^{-\frac{1}{2}} \tilde{\mathbf{R}}^c (\mathbf{D}^c)^{-\frac{1}{2}}, \ \tilde{\mathbf{R}}^c = \mathbf{R}^c + \mathbf{I}_K \end{cases}, \tag{7}$$

where $\mathbf{I}_K$ denotes the identity matrix with dimension of K, while $\mathbf{D}^p$ and $\mathbf{D}^c$ represent the explicit and implicit relation degree matrices, respectively. Specifically, $\mathbf{D}^p_{ii} = \sum_j \mathbf{R}^p_{ij}$ and $\mathbf{D}^c_{ii} = \sum_j \mathbf{R}^c_{ij}$. Finally, through iterative information propagation and aggregation across the graph structure, we derive the output neighbor-aware explicit and implicit node representations, denotes as $\mathbf{E}_p$ and $\mathbf{E}_c$.

Immediately after that, the two learned node representations $\mathbf{E}_p$ and $\mathbf{E}_c$, are integrated through a self-attention mechanism, which adaptively balances the contribution of each view. The fusion process is modeled as an attention-weighted sum, where the attention coefficients,λ_i, are computed based on the relevance of each view's transformed features. This strategy ensures that the most informative aspects of both explicit and implicit relationships are emphasized, enhancing the overall quality of the learned representations:

$$\begin{cases} \mathbf{E}_o = \sum_{i\in\{p,c\}} \lambda_i \mathbf{E}_i, \\ \lambda_i = \frac{exp(\mathbf{E}_i \mathbf{W}_i)}{\sum_{j\in\{p,c\}} exp(\mathbf{E}_j \mathbf{W}_j)}, \end{cases} \tag{8}$$

where $\mathbf{W}_i \in \mathbb{R}^{d\times 1}$ denotes the transformation matrix, $\lambda_i \in \mathbb{R}^{K\times 1}$ represents the learned attention coefficient, and $\mathbf{E}_o \in \mathbb{R}^{K\times d}$ is the enriched final concept representation. Meanwhile, to ensure the robustness of the learned features during the fusion process, we employ contrastive learning [21] as a regularization constraint. This is accomplished by maximizing the mutual information between paired nodes from a probabilistic perspective, effectively promoting the capture of shared information in the representations while preserving their unique characteristics:

$$\mathcal{L}_{cl}^{s} = -\mathbb{E}_{(\mathbf{E}_p,\mathbf{E}_c)}\Big[log\frac{exp(sim(\mathbf{E}_p,\mathbf{E}_c)/\tau)}{\sum_{\mathbf{E}_p^{'},\mathbf{E}_c^{'}} exp(sim(\mathbf{E}_p^{'},\mathbf{E}_c^{'})/\tau)}\Big]. \tag{9}$$

where $sim(\cdot)$ denotes the similarity measure, τ is the temperature parameter, and the expectation is taken over all positive pairs $(\mathbf{E}_p, \mathbf{E}_c)$ and negative pairs $(\mathbf{E}_p^{'}, \mathbf{E}_c^{'})$. The contrastive loss $\mathcal{L}_{cl}^{s}$ aligns the two views in the latent space while preserving the distinctiveness of each view, thereby improving the final representation $\mathbf{E}_o$.

4.2 Knowledge Topology-Aware Modeling

In the context of learning path recommendation, it is crucial to capture the intricate topological structures inherent in knowledge concepts. This involves not only representing individual knowledge points but also understanding how these concepts are organized and interrelated within a broader knowledge relation graph. In this section, we introduce knowledge topology-aware modeling to harness structural semantics for generating effective candidate paths.

Considering the logical nature of elements in the path generation process, we first employ a relational semantics-based clustering [14, 36] to capture the global topological associations between concepts. Specifically, we cluster the concept pool into L distinct categories based on the learned knowledge representation $\mathbf{E}_o = [\mathbf{E}_{o,1}, \mathbf{E}_{o,2}, \dots, \mathbf{E}_{o,K}]^T$, which encapsulates the rich relational information from both explicit and implicit graphs, and the clustering process are defined as follows:

$$\mathbf{Z} = \arg\min_{\mathbf{Z}} \sum_{i=1}^{K}\sum_{j=1}^{L} \mathbf{Z}_{ij} \cdot \underbrace{\|\mathbf{E}_{o,i} - \boldsymbol{\mu}_j\|^2}_{f(\mathbf{E}_{o,i};\Theta)} \quad , \tag{10}$$

where $\mathbf{Z} \in \{0,1\}^{K\times L}$ is the cluster assignment matrix (i.e., $\mathbf{Z}_{ij} = 1$ indicates that i-th concept node is assigned to the j-th cluster), $\boldsymbol{\mu}_j = \frac{\sum_{i=1}^{K} \mathbf{Z}_{ij}\cdot \mathbf{E}_{o,i}}{\sum_{i=1}^{K} \mathbf{Z}_{ij}} \in \mathbb{R}^d$ denotes the centroid of the j-th cluster, and $f(\mathbf{E}_{o,i};\Theta) = \|\mathbf{E}_{o,i} - \boldsymbol{\mu}_j\|^2$ represents the clustering function that defined as the squared Euclidean distance. Therefore, based on $\mathbf{Z}$, we can obtain L concept clusters, denoted as $C_{cls} = \{C_1, C_2, \dots, C_L\}$, where $C_j = \{c_i | \mathbf{Z}_{ij} = 1, 1 \le i \le K\}$ and $\sum_{j=1}^{L} |C_j| = K$.

After obtaining structural semantic-aware concept clusters, we further utilize a significance probability-based sampling to initially construct candidate learning paths. Given the i-th cluster C_i, our goal is to sample a representative concept element $\hat{p}_i$ from each cluster C_i, and this sampling process can be expressed as:

$$\hat{p}_i = Sample(C_i) = \arg\max_{c_j \in C_i} \Big(\frac{exp(-\|\mathbf{E}_{o,j} - \boldsymbol{\mu}_i\|^2)}{\sum_{c_k \in C_i} exp(-\|\mathbf{E}_{o,k} - \boldsymbol{\mu}_i\|^2)}\Big). \tag{11}$$

where $Sample(\cdot)$ is the sampling strategy based on a softmax-weighted probability distribution over the mentioned squared Euclidean distance. This form ensures that the probability of selecting a concept varies with its distance from the centroid, thereby somewhat guaranteeing logical consistency of the knowledge path. Repeating this sampling process, eventually we are able to obtain the candidate path $\hat{\mathcal{P}} = \{\hat{p}_i, \hat{p}_2, \dots, \hat{p}_L\}$. The significance of this sampling strategy lies in its ability to generate diverse candidate paths that capture both the structural relationships and global topological associations among concepts. Moreover, by incorporating for the inherent uncertainty, it enhances the likelihood that the generated paths align with the learner's cognitive structure.

4.3 Bidirectional Sensing Path Optimization Network

Considering the sequential and progressive nature of learning items in the knowledge acquisition process [1, 15], it is necessary to conduct in-depth modeling and optimization of candidate paths from the sequence dimension. Remarkably, we model the learning path planning process as a sequence optimization problem. However, it is challenging to incorporate both the local relational features and global topological information of the candidate knowledge concepts into the optimization process. Therefore, we develop a bidirectional sensing path optimization network to optimize learner-adapted paths, which includes the bidirectional path learning module and an attention-based pointer neural network.

In essence, the optimization process of the learning path can be succinctly formalized as follows, by disassembling the core content:

$$\begin{aligned} \mathcal{P}, \pi &= f_g(C, f_s(\mathcal{H};\omega_s), \mathcal{T};\omega_g), \\ &= \underbrace{f_g^{O}\Big(\underbrace{f_g^{\mathcal{G}}(C;\omega_g^{\mathcal{G}})}_{\text{graph modeling}}, \underbrace{f_s(\mathcal{H};\omega_s)}_{\text{initiator}}, \mathcal{T};\omega_g^{O}\Big)}_{\text{path optimization network}}, \end{aligned} \tag{12}$$

where $\mathcal{P}$ and π represent the output path and the corresponding probability sequences, and $f_s(\cdot;\omega_s)$ and $f_g(\cdot;\omega_g)$ denote the learner initiator and path generator, which are used to model the student's historical learning records and generate learning paths, respectively. In particular, in this paper, $f_g(\cdot;\omega_g)$ includes $f_g^{\mathcal{G}}(\cdot;\omega_g^{\mathcal{G}})$ and $f_g^{O}(\cdot;\omega_g^{O})$ for relation graph modeling-based candidate path generation and path optimization, respectively. Based on the former, we can obtain the candidate path representation $\mathbf{H}^{\hat{\mathcal{P}}} = f_g^{\mathcal{G}}(C;\omega_g^{\mathcal{G}}) \in \mathbb{R}^{L\times d}$, while the latter we describe in detail in this section. First, to effectively model and trace the learner's initial ability state, which is crucial for optimizing the path structure, we employ a bidirectional long short-term memory network (Bi-LSTM) [7] for f_s. Considering that each step h_i in $\mathcal{H}$ includes the learning concept c_i and the corresponding answer score y_i, which are typically modeled together in knowledge tracing [24, 30], the learner's proficiency level is modeled as follows:

$$\mathbf{h}_0 = BiLSTM(\underbrace{\big[[\mathbf{x}_{c_1}\|\mathbf{x}_{y_1}], [\mathbf{x}_{c_2}\|\mathbf{x}_{y_2}], \dots, [\mathbf{x}_{c_M}\|\mathbf{x}_{y_M}]\big]^T}_{\mathbf{X}^{\mathcal{H}}} \mathbf{W}^{\mathcal{H}}), \tag{13}$$

where $\mathbf{x}_{c_i}, \mathbf{x}_{y_i} \in \mathbb{R}^d$ are the embedding of concept c_i and response y_i, $\mathbf{X}^{\mathcal{H}} \in \mathbb{R}^{M\times 2d}$ is the input sequence embedding of history interactions, $\mathbf{h}_0 \in \mathbb{R}^{2d}$ is the output initial knowledge state of the learner, $\mathbf{W}^{\mathcal{H}} \in \mathbb{R}^{2d\times d}$ is a trainable matrix, $\|$ represents the operation of concatenating, and $BiLSTM(\cdot)$ denotes the Bi-LSTM network.

To effectively model the probability distribution of expected actions under known sub-paths in path optimization, we employ a pointer network [31] to achieve the optimization goal, using Bi-LSTM as the encoder to ensure consistency and stability in modeling the knowledge state. Given the knowledge state $\mathbf{h}_{i-1}$ at step i-1, obtained by the encoder that captures sequential relationships in both directions, and $\mathcal{P}_{<i} = \{p_1, p_2, \ldots, p_{i-1}\}$ denoting the concept path learned prior to step i, we can model the probability distribution of knowledge concept at step i using attention-based aggregation as:

$$d_i^j = (\mathbf{h}_{i-1}\mathbf{W}_1 + \mathbf{H}_j^{\hat{\mathcal{P}}}\mathbf{W}_2 + \mathbf{x}^{\mathcal{T}}\mathbf{W}_3 + \mathbf{b})\mathbf{w}^T, 1 \leq j \leq L, \tag{14}$$

where $d_i^j \in \mathbb{R}$ denotes the probability of the j-th knowledge concept, $\mathbf{H}_j^{\hat{\mathcal{P}}}$ is the j-th concept representation, $\mathbf{x}^{\mathcal{T}}$ is the target concept embedding, and $\mathbf{W}_1 \in \mathbb{R}^{2d\times d}, \mathbf{W}_2, \mathbf{W}_3 \in \mathbb{R}^{d\times d}, \mathbf{b}, \mathbf{w} \in \mathbb{R}^d$ are trained projection parameters. Immediately afterward, we apply the softmax operation to obtain the normalized probability distribution of the knowledge concepts while avoiding repetitiveness, as follows:

$$\pi(p_i|\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{i-1}, \mathcal{P}_{<i}) = \frac{exp(\mathbf{d}_i)}{\sum_{p_j\in\hat{\mathcal{P}}/\mathcal{P}_{<i}} exp(\mathbf{d}_j)}. \tag{15}$$

where $\mathbf{d}_i = \|_{j=1}^{L} d_i^j$ represents the concept probability distribution at the i-th step. We then sample from the remaining unselected concepts using the derived probability $\pi(p_i)$ to select p_i. Finally, we obtain the constructed path $\mathcal{P} = \{p_1, p_2, \ldots, p_L\}$ and the corresponding probabilities $\pi = \{\pi_{p_1}, \pi_{p_2}, \ldots, \pi_{p_L}\}$ for each step.

4.4 Model Training

Referring to previous work [1, 15], to optimize the full model during training, we use the policy gradient method [29] to calculate the loss of path planning. that there are no real labels such as E_{end}, E_{start}, and E_{sup} in the path optimization process, we construct a cognitive simulator in this paper to compute $LG_{\mathcal{P}}$, which will be described in more detail in the experiment section. Specifically, we treat $LG_{\mathcal{P}}$ as the path reward, and the main loss is formulated as:

$$\begin{aligned}\mathcal{L}_\Omega &= -\mathbb{E}_{\mathcal{P}\sim\pi(\cdot|\Omega)}[J(\mathcal{P})],\\ &= -\frac{1}{|\mathcal{S}|}\sum_{i=1}^{|\mathcal{S}|} LG_{\mathcal{P}_i}\sum_{j=1}^{L} log\ \pi(\mathcal{P}_i^j \mid \Omega),\end{aligned} \tag{16}$$

where $J(\cdot)$ denotes the reward function, $\Omega = (\omega_s, \omega_g)$ represents the trainable model parameters. Meanwhile, we can get the whole contrastive loss according to Equation (9):

$$\mathcal{L}_{cl} = \sum_{s\in\mathcal{S}} \mathcal{L}_{cl}^s. \tag{17}$$

Ultimately, the final optimization objective function is derived combining the two aforementioned losses:

$$\mathcal{L} = \mathcal{L}_\Omega + \gamma \cdot \mathcal{L}_{cl}. \tag{18}$$

where γ is the weight coefficient to control the influence of complementary contrastive fusion. We can train the whole framework and optimize the parameters by gradient descent.

5 Experiments

In this section, we conduct sufficient experiments on three educational datasets to validate the effectiveness of our proposed LIGHT model. Specifically, we aim to address the following research questions (**RQs**) to guide and structure the experimental analysis:

Table 1: The statistics of all datasets.

Statistics	ASSIST09	ASSIST12	SLP-Math
#Learners	4,163	12,025	4,668
#Exercises	17,751	44,984	226
#Knowledge concepts	123	260	40
#Interaction records	338,001	1,019,885	582,532
Avg. concepts per exercise	1.0	1.0	1.0
Avg. exercises per concept	172.73	173.02	5.65
Avg. interaction length	81.19	84.81	124.79

- **RQ1:** How effective and superior is the proposed LIGHT model in addressing the learning path recommendation task?
- **RQ2:** Do the designed key components contribute to enhancing the performance of our proposed LIGHT model?
- **RQ3:** How do the hyper-parameter settings impact the performance of LIGHT, and what configurations yield the best results?
- **RQ4:** How does the LIGHT model facilitate the mining of both explicit and implicit concept relationships, and what insights can be gained from their representational information?

5.1 Experimental Setting

5.1.1 ***Dataset Descriptions.*** In the experiments, we used three educational datasets to valid the effectiveness of our LIGHT on the learning path recommendation task. The dataset statistics are summarized in Table 1, with details provided as follows:

- **ASSIST09** [4]: ASSIST09 is a dataset gathered from the ASSISTments online educational tutoring system, comprising rich sequences of student interactions with various exercise. It captures detailed information on student responses, exercise attempts, and timestamps, and has been widely utilized in educational tasks.
- **ASSIST12** [23]: Similar to ASSIST09, ASSIST12 is also a dataset collected from ASSISTments platform. Differently, ASSIST12 offers a broader scope, incorporating more extensive interaction data and introducing additional affective state information.
- **SLP-Math** [17]: SLP is a publicly available benchmark dataset collected from an online learning platform called Smart Learning Partner (SLP). It specifically captures detailed learning data from secondary school students across multiple subjects, offering a wealth of educational content.

5.1.2 ***Baseline Approaches.*** To demonstrate the superiority of the proposed model, we compare it with three categories of approaches: rule-based, reinforcement learning (RL)-based, and sequence (Seq)-based methods, encompassing a total of seven algorithms. The details of these comparison methods are as follows:

- **RAND**: It is a stochastic path generation strategy in which concepts are randomly selected from the candidate concept pool to construct learning path of formulated length.
- **CLUSTER** [14]: It clusters concepts using representations learned by the simulator and randomly samples a specific number of samples from the clusters to constitute a recommended path.
- **RANK** [1]: It utilizes the simulator to return the learning effect of each concept being learned in turn and uses the magnitude of that effect to select the best concept as the response for the current step, which is similar to the greedy rule.

Table 2: Performance of LIGHT and all baseline methods on three datasets. "$*$" denotes the statistically significant improvement of LIGHT model compared to the best baseline (i.e., two-sided t-test with p<0.05). Bold: the best, Underline: the runner-up.

Methods		Rule-Based			RL-Based		Seq-Based		Ours
Datasets	Steps	RAND	CLUSTER	RANK	CSEAL	GEHRL	GRU4Rec	SRC	**LIGHT**
ASSIST09	n=5	$-0.0835_{\pm 0.0007}$	$-0.0096_{\pm 0.0006}$	$0.0863_{\pm 0.0007}$	$0.0952_{\pm 0.0028}$	$0.1022_{\pm 0.0011}$	$-0.0957_{\pm 0.0075}$	$\underline{0.1745_{\pm 0.0008}}$	$\mathbf{0.2013^{*}_{\pm 0.0010}}$
	n=10	$-0.0960_{\pm 0.0008}$	$-0.2021_{\pm 0.0014}$	$0.0301_{\pm 0.0017}$	$0.1227_{\pm 0.0033}$	$0.1274_{\pm 0.0016}$	$0.1001_{\pm 0.0066}$	$\underline{0.2139_{\pm 0.0005}}$	$\mathbf{0.2527^{*}_{\pm 0.0012}}$
	n=20	$-0.0714_{\pm 0.0012}$	$-0.2196_{\pm 0.0017}$	$0.0964_{\pm 0.0013}$	$0.1764_{\pm 0.0026}$	$0.1953_{\pm 0.0014}$	$0.1706_{\pm 0.0145}$	$\underline{0.2648_{\pm 0.0004}}$	$\mathbf{0.2932^{*}_{\pm 0.0014}}$
	n=30	$0.0022_{\pm 0.0008}$	$0.1484_{\pm 0.0008}$	$0.1540_{\pm 0.0008}$	$0.2549_{\pm 0.0021}$	$0.2406_{\pm 0.0009}$	$0.2165_{\pm 0.0031}$	$\underline{0.3190_{\pm 0.0006}}$	$\mathbf{0.3405^{*}_{\pm 0.0015}}$
ASSIST12	n=5	$-0.1896_{\pm 0.0018}$	$-0.0679_{\pm 0.0014}$	$-0.1822_{\pm 0.0022}$	$-0.1034_{\pm 0.0041}$	$\underline{-0.0523_{\pm 0.0020}}$	$-0.1915_{\pm 0.0050}$	$-0.2110_{\pm 0.0010}$	$\mathbf{0.0168^{*}_{\pm 0.0019}}$
	n=10	$-0.0496_{\pm 0.0010}$	$-0.0370_{\pm 0.0017}$	$-0.0239_{\pm 0.0013}$	$0.0156_{\pm 0.0039}$	$0.0084_{\pm 0.0028}$	$-0.0497_{\pm 0.0030}$	$\underline{0.0225_{\pm 0.0020}}$	$\mathbf{0.0374^{*}_{\pm 0.0002}}$
	n=20	$0.0590_{\pm 0.0011}$	$\underline{0.1082_{\pm 0.0014}}$	$0.1023_{\pm 0.0005}$	$0.0876_{\pm 0.0053}$	$0.0959_{\pm 0.0023}$	$0.0971_{\pm 0.0068}$	$0.0618_{\pm 0.0005}$	$\mathbf{0.1239^{*}_{\pm 0.0017}}$
	n=30	$0.1764_{\pm 0.0014}$	$0.1388_{\pm 0.0013}$	$0.1681_{\pm 0.0017}$	$0.1565_{\pm 0.0036}$	$\underline{0.1771_{\pm 0.0019}}$	$0.1738_{\pm 0.0038}$	$0.1708_{\pm 0.0009}$	$\mathbf{0.1985^{*}_{\pm 0.0022}}$
SLP-Math	n=5	$-0.1383_{\pm 0.0061}$	$-0.2329_{\pm 0.0040}$	$0.3504_{\pm 0.0035}$	$0.2162_{\pm 0.0012}$	$0.1127_{\pm 0.0015}$	$-0.1512_{\pm 0.0006}$	$\underline{0.4559_{\pm 0.0010}}$	$\mathbf{0.4776^{*}_{\pm 0.0021}}$
	n=10	$-0.0412_{\pm 0.0062}$	$0.0298_{\pm 0.0042}$	$0.3624_{\pm 0.0023}$	$0.2459_{\pm 0.0023}$	$0.1638_{\pm 0.0010}$	$0.1387_{\pm 0.0242}$	$\underline{0.4845_{\pm 0.0010}}$	$\mathbf{0.5018^{*}_{\pm 0.0018}}$
	n=20	$0.2213_{\pm 0.0017}$	$\underline{0.4134_{\pm 0.0006}}$	$0.4049_{\pm 0.0014}$	$0.3093_{\pm 0.0016}$	$0.3064_{\pm 0.0019}$	$0.2336_{\pm 0.0232}$	$0.3508_{\pm 0.0003}$	$\mathbf{0.4835^{*}_{\pm 0.0023}}$
	n=30	$0.2427_{\pm 0.0032}$	$0.4365_{\pm 0.0008}$	$0.4184_{\pm 0.0022}$	$0.2774_{\pm 0.0021}$	$0.2845_{\pm 0.0012}$	$0.2609_{\pm 0.0047}$	$\underline{0.4352_{\pm 0.0027}}$	$\mathbf{0.4648^{*}_{\pm 0.0017}}$

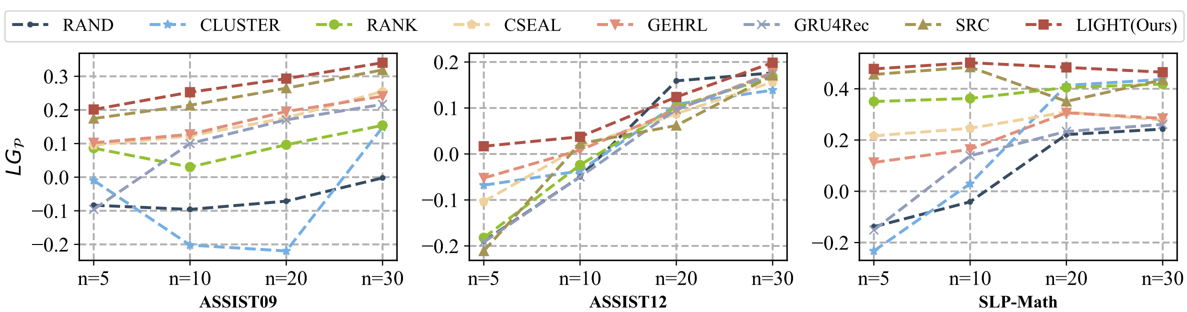

Figure 3: Performance of LIGHT and all baseline methods on three datasets with respect to the learning path length.

- **CSEAL** [15]: It frames LPR as a Markov decision process, leveraging the learner's evolving knowledge state and cognitive structures to dynamically recommend appropriate learning items.
- **GEHRL** [13]: It utilizes a high-level agent as a sub-goal selector to select sub-goals for the low-level agent to achieve thereby realizing the recommendation of learning paths.
- **GRU4Rec** [9]: It is a classical sequence recommendation model that inputs past sequences so as to predict the probability distribution of the next interaction. We adapt it in learning path recommendation to realize the construction of learning elements.
- **SRC** [1]: It represents a classical and effective approach to learning path recommendation by framing the path generation task as a set-to-sequence ranking-based concept-aware process.

5.1.3 ***Evaluation Metrics.*** The purpose of learning path recommendation is to maximize the learning effect of learners on the target concepts, so the evaluation metric used in this paper is $LG_{\mathcal{P}}$ in Eq. (1). However, there is no true label for the evolution of learners' abilities during the learning process, making it difficult to directly assess the learning effect. To address this, we draw on previous practices [1, 15] and accurately trace ability changes by constructing a simulator that pre-trains students using dynamic learning data. In our experiments, we use the DKT [24] model as the simulator model, one of the most well-established knowledge tracing methods, which employs a recurrent neural network (RNN) [27] to effectively model the learner-exercise interaction process. Specifically, this simulator is data-driven, with the knowledge tracing model trained on temporal data. The model takes the learner's past learning sequence as input and outputs the probability of correctly answering the current concept. Once training is complete, the simulator can be used to simulate learners' learning progress on paths recommended by various models, allowing us to obtain the corresponding $LG_{\mathcal{P}}$ for learning effect evaluation.

5.1.4 ***Experimental Settings.*** In our experiments, we selected a fixed number of learners as a test set and support optimization by performing data sampling during training. To ensure the reliability of the results, we conducted multiple rounds of experiments, repeating each experiment and record the corresponding means and standard deviations. Additionally, we performed significance analysis on the results. We implemented all methods with PyTorch

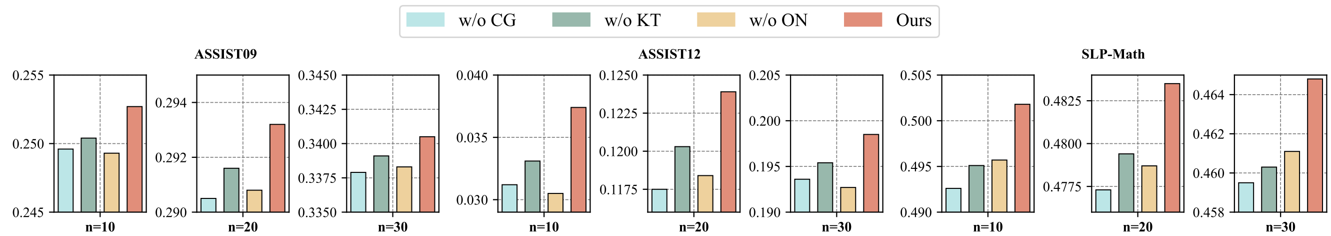

Figure 4: Performance of ablation studies conducted on three datasets, where "w/o" means removing the target module.

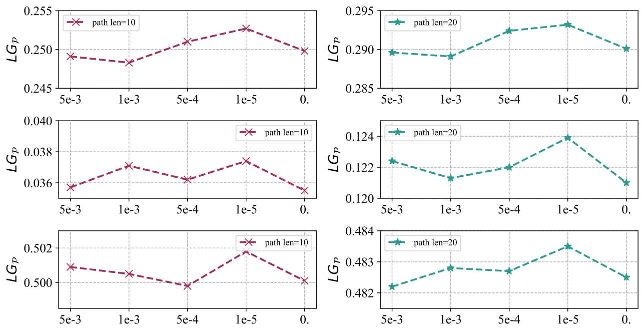

Figure 5: Sensitivity analysis of coefficient γ of the contrastive loss on the ASSIST09/12 and SLP-Math datasets.

by Python. The dimension size of embeddings d is set as 128. The number of graph layers is set to 2. We use the Adam algorithm as the optimizer, where the learning rate was searched in [$1e$-2, $5e$-3, $1e$-3, $5e$-4, $1e$-4]. The distance threshold in implicit relation mining is set to 0.9, and the coefficient γ of contrastive loss is set to $1e$-5.

5.2 Performance Comparison (RQ1)

Table 2 presents the experimental results of the proposed LIGHT model's performance in learning path recommendation, compared to baseline models across the three datasets. The best results for each model are highlighted in bold, while the second-best results are underlined. Particularly, we performed t-tests to assess statistical significance, with asterisks marking notable improvements as indicated in the table. According to the results, several observations can be made: (1) Our LIGHT model consistently outperforms all baseline models across the datasets, with significant improvements that underscore its effectiveness. Specifically, compared to the second-best method, our model delivers an average improvement of 16.59% across all datasets, with a remarkable gain of 33.14% on the ASSIST12 dataset in particular. (2) LIGHT demonstrates even more significant performance on the SLP-Math dataset, achieving a learning effect of nearly 0.5 with a path length of only 5. This result may be attributed to the more structured nature of the SLP-Math data, which likely contributes to the higher performance potential of our model. (3) SRC demonstrates some advantages over other methods on the ASSIST09 dataset, while GEHRL performs well on the ASSIST12 dataset. Although both methods fall short of the proposed LIGHT model, their performance highlights the effectiveness of modeling knowledge relationships and path optimization in the task of learning path recommendation. To further investigate the impact of path length on model performance, we present the experimental results more clearly using line charts, as illustrated in Figure 3. From the results, we observe that there is a linear relationship between the performance of all models and the path length within a certain range, where longer paths lead to improved performance. Notably, this trend exhibits some fluctuations depending on the dataset type and the specific recommendation strategies employed by different methods, such as the CLUSTER method demonstrates a reverse growth trend on the ASSIST09 dataset.

Table 3: Sensitivity analysis of the distance threshold β in implicit relation mining on the ASSIST12 dataset.

Params		$\beta = 0.8$	$\beta = 0.85$	$\beta = 0.9$	$\beta = 0.95$
Rec Steps	n=5	$0.0152_{\pm 0.0009}$	$0.0159_{\pm 0.0016}$	$\mathbf{0.0168_{\pm 0.0019}}$	$0.0155_{\pm 0.0022}$
	n=10	$0.0364_{\pm 0.0007}$	$0.0367_{\pm 0.0025}$	$\mathbf{0.0374_{\pm 0.0002}}$	$0.0361_{\pm 0.0017}$
	n=20	$0.1229_{\pm 0.0012}$	$0.1231_{\pm 0.0020}$	$\mathbf{0.1239_{\pm 0.0017}}$	$0.1224_{\pm 0.0010}$
	n=30	$0.1975_{\pm 0.0023}$	$0.1973_{\pm 0.0013}$	$\mathbf{0.1985_{\pm 0.0022}}$	$0.1978_{\pm 0.0015}$

5.3 Ablation Study (RQ2)

We conducted a comprehensive ablation study to investigate the contribution of each module within the LIGHT framework on the three datasets, defining the following variations: 1) **w/o CG**: removing the composite graph relationship mining; 2) **w/o KT**: removing the knowledge topology-aware modeling module; 3) **w/o ON**: removing the path optimization network. Notably, to ensure the model's runnability, our construction of the variants does not involve directly removing the target module; instead, it involves replacing it with the simple baseline module. As illustrated in Figure 4, the results reveal insightful observations: (1) Compared to LIGHT, all variants exhibit relative performance degradation, highlighting the contribution of the designed sub-modules to our proposed model. (2) The significant performance drop occurs when the path optimization network is removed, which proves that optimizing for candidate paths is beneficial for modeling the learner's state and capturing the conceptual requirements. (3) A more pronounced drop in model performance is observed after removing the composite graph on certain metrics, highlighting the necessity of the proposed explicit and implicit relationship mining module.

5.4 Parameter Sensitivity Analysis (RQ3)

To answer RQ3, we performed a comprehensive parameter sensitivity analysis in this section to assess the impact of hyper-parameters, including the weight coefficient γ of the contrastive loss, the distance threshold β and the number of graph network layers in the composite relation mining. Specifically, we first set the value of γ to {$5e$-3, $1e$-3, $5e$-4, $1e$-5, 0.}, and show the experimental results on the

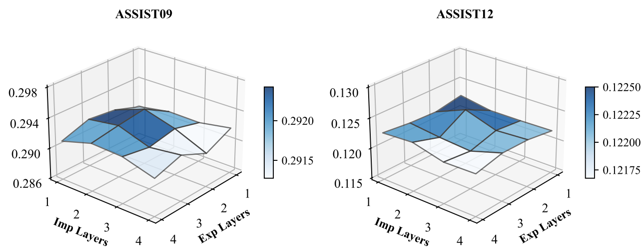

Figure 6: Sensitivity analysis of the number of layers for explicit and implicit graph on ASSIST09 and ASSIST12 datasets.

three datasets. As shown in Figure 5, the model achieves its best performance on the three datasets when the coefficient is set to $1e$-5, with path lengths of 10 and 20. In fact, as the coefficients vary, the model's performance does not follow a fixed pattern, and there is no clear linear trend in its behavior. In particular, when this value is 0—meaning the contrastive loss is disabled—a significant performance degradation is observed. Thus, while this value is small at the point of optimal performance, we believe it is essential as a constraint to ensure effective representational learning. We also conducted a sensitivity analysis for β on the ASSIST12 dataset. Due to the data statistical nature, setting β too low introduces excessive edges and redundant information, so we tested the values of {0.8, 0.85, 0.9, 0.95}. As presented in Table 3, the model achieves optimal performance when it is set to 0.9. Notably, larger beta values lead to a significant performance decline, likely due to inadequate concept modeling from an overly sparse edge structure. Furthermore, we performed parameter experiments on both explicit and implicit graph network layers in the concept relationship mining process. For each layer type, we selected values from the set {1, 2, 3, 4} and tested all possible pairwise combinations. The corresponding results are illustrated in Figure 6. From the results, we observe that the model performance reaches its peak when the layers are set to 2, and the impact of varying the number of layers on performance remains relatively stable across different combinations.

5.5 Case Study (RQ4)

In this section, we conducted an interesting case study on the ASSIST09 dataset to answer RQ4. In this case study, we aim to investigate the association between the explicit and the implicit concept representations learned in LIGHT, as well as their differences compared to the final representations obtained through complementary contrastive fusion, to gain a deeper understanding of the role of the proposed composite relationship mining. Specifically, we visualize the three types of concept representations learned in LIGHT using t-SNE for dimensionality reduction and employ a marginal histogram to intuitively display their feature distributions, as shown in Figure 7. We can observe that the blue explicit representations exhibit a more centralized distribution, indicating a higher level of concentration and proximity, whereas the orange implicit representations are more dispersed. This suggests greater diversity in the concept representations mined from implicit relationships, which is reasonable and aligns with the expectation that collaborative information mining introduces more variation compared to the relatively structured prerequisite features. Additionally, The two

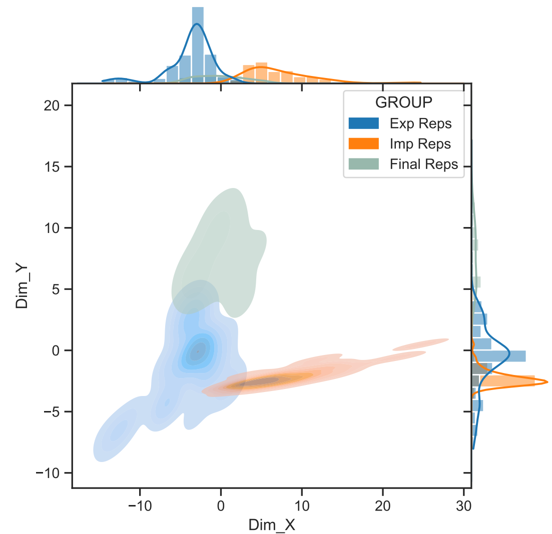

Figure 7: Case study of feature associations among explicit, implicit, and final concept representations in the composite relationship mining of our proposed LIGHT model.

regions exhibit some overlap but remain more distinct, suggesting that the mining of explicit and implicit relationships is indeed complementary. Notably, the final fused representations exhibit a more cohesive and homogeneous distribution, demonstrating their capacity to capture the nuances of both relational features and effectively integrate them for more comprehensive concept modeling.

6 Conclusion

In this paper, we introduced a novel framework termed as **LIGHT** (a know**L**edge topology-aware sequence opt**I**mization model for achieving a more productive learnin**G** pat**H** recommenda**T**ion), which aims to enhance the generation and optimization of learning paths from a sequence optimization perspective. Our approach comprises several key components. Initially, we constructed a composite concept graph that includes explicit prerequisite relationship and implicit collaborative relationship, followed by a complementary contrastive fusion module to fully capture the complementarity of the two relational views between concepts. Subsequently, we proposed a knowledge topology-aware modeling module comprised of the structural semantics clustering and the candidate path sampling to learning conceptual topology. Finally, to deeply optimize the sampled paths from a sequence perspective, we developed a bidirectional sensing path optimization network. Extensive experiments were conducted on three real-world educational datasets to substantiate the efficacy of our LIGHT framework in addressing the LPR task. We hope this work could lead to further studies.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (No. U21A20512, No.62107001, No.62302010), in part by China Postdoctoral Science Foundation (No.2023M740015), in part by the Postdoctoral Fellowship Program (Grade B) of China Postdoctoral Science Foundation (No.GZB20240002), and in part by National Research Foundation, Singapore under its AI Singapore Programme (AISG Award No: AISG3-RP-2022-031).

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