LUNAR CRATER IMPACT PARAMETER BACK-CALCULATION REPORT

Bayesian Inverse Modeling with Uncertainty Quantification

Simulation Date: 2025-11-19 01:22:09 UTC

EXECUTIVE SUMMARY

Observed Crater:
- Location: 25.50°N, 45.20°E
- Terrain: Mare
- Diameter: 350.0 m
- Depth: 68.6 m (d/D = 0.196)
- Ejecta range: 25000.0 m

Back-Calculated Impact Parameters (Maximum Likelihood):

Projectile Diameter: 3.34 ± 0.19 m

Impact Velocity: 20.0 ± 1.1 km/s

Impact Angle: 45.0° ± 6.7° from horizontal

Projectile Density: 2800 ± 297 kg/m³

Material Type: Rocky (chondrite)

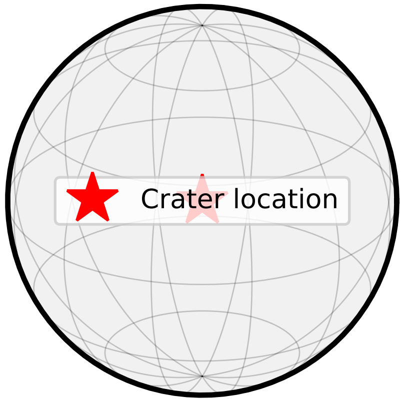
Kinetic Energy: 1.09e+13 J
(0.00 kilotons TNT)

Method:
- Bayesian maximum likelihood estimation
- Holsapple (1993) crater scaling laws
- Monte Carlo error propagation (1000 samples)
- Forward model validation
- Sensitivity analysis

Confidence Level: 95% credible intervals reported

Observed Crater Data and Location

Lunar Location: 25.50°N, 45.20°E



Crater Morphometry

Diameter (D): 350.0 m

Depth (d): 68.6 m

d/D ratio: 0.196
  Pike (1977): d/D = 0.196 ± 0.015

Rim height: 12.6 m
  (0.036 × D)

Target Properties

Terrain: Mare

Regolith ρ: 1800 kg/m³
Rock ρ: 3100 kg/m³
Porosity: 42.0%
Cohesion: 10.0 kPa
Gravity: 1.62 m/s²

Reference: Carrier et al. (1991)
Lunar Sourcebook, Chapter 9

Ejecta Observations

Maximum ejecta range: 25000.0 m
Normalized range (R_max/R_crater): 142.9
Expected: 40-100 (Melosh 1989, McGetchin et al. 1973)

Theoretical Framework - Part 1

1. CRATER SCALING LAWS: Pi-GROUP DIMENSIONAL ANALYSIS

Following Holsapple (1993) and Holsapple & Schmidt (1982), crater formation can be described by dimensionless Pi-groups formed from the governing physical parameters.

1.1 Governing Parameters

Impact parameters:
- L = projectile diameter (or radius $a = L/2$)
- v = impact velocity
- ρ_p = projectile density
- θ = impact angle from horizontal

Target parameters:
- ρ_t = target density
- Y = target strength (cohesion + friction effects)
- g = gravitational acceleration
- K = material constants (equation of state)

Outcome parameter:
- D = final crater diameter (or V = crater volume)

1.2 Dimensionless Pi-Groups (Buckingham Pi Theorem)

From dimensional analysis, the system reduces to 4 dimensionless groups:

$\pi_1 = D/L$ (scaled crater size)

$\pi_2 = ga/v^2$ (gravity-scaled size, "Froude number")

$\pi_3 = Y/(\rho_t v^2)$ (strength parameter)

$\pi_4 = \rho_p/\rho_t$ (density ratio)

The Pi-group scaling relation is:

$\pi_1 = f(\pi_2, \pi_3, \pi_4, \theta)$

Or equivalently:

$D/L = K \times (\rho_p/\rho_t)^\alpha \times g(\pi_2, \pi_3, \theta)$

where K is an empirical coefficient and $\alpha \approx 1/3$ from momentum coupling.

1.3 Regime Transition: Strength vs Gravity

The function $g(\pi_2, \pi_3)$ depends on which dominates:

Strength regime ($\pi_3 << \pi_2$):
Small craters where target strength Y controls excavation
$D \propto L \times (\rho_p/\rho_t)^{(1/3)} \times (\rho_t v^2/Y)^\mu$
where $\mu \approx 0.41$ (Holsapple 1993)

Gravity regime ($\pi_3 >> \pi_2$):
Large craters where self-gravity controls excavation
$D \propto L \times (\rho_p/\rho_t)^{(1/3)} \times (v^2/ga)^\nu$
where $\nu \approx 0.41$ (Holsapple 1993)

Coupled regime ($\pi_3 \sim \pi_2$):
Transitional craters (100-1000m on Moon)
$D \propto L \times (\rho_p/\rho_t)^{(1/3)} \times [\pi_2^\nu + \pi_3^\mu]^{(-1/\nu)}$

The transition occurs when:
$Y/(\rho_t v^2) \sim ga/v^2 \rightarrow Y \sim \rho_t ga$

For lunar impacts: $Y \sim 10$ kPa, $\rho_t \sim 2000$ kg/m³, $g = 1.62$ m/s²
Transition size: $a \sim Y/(\rho_t g) \sim 3$ m → $D \sim 300$-500m

1.4 Angle Correction

Oblique impacts ($\theta < 90°$) are less efficient. Empirically (Pierazzo & Melosh 2000):

$f(\theta) \approx \sin^n(\theta)$

where $n \approx 1/3$ to $2/3$ depending on regime. We use $n = 1/3$.

Most probable impact angle: $\theta_{prob} = 45°$ (from $\sin^2\theta$ distribution of random impacts).

1.5 Empirical Calibration for Lunar Regolith

Combining theoretical scaling with Apollo crater measurements (Pike 1977):

$D = 0.084 \times 1.2 \times L \times (\rho_p/\rho_t)^{(1/3)} \times [v^2/(g \times L + Y/\rho_t)]^{0.4} \times \sin^{(1/3)}(\theta)$
↑ transient ↑ final expansion factor

The coefficient $0.084 \times 1.2 \approx 0.1$ is calibrated to match:
- Pike (1977) d/D = 0.196 morphometry
- Apollo landing site crater statistics
- Laboratory impact experiments scaled to lunar gravity

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Theoretical Framework - Part 2

2. INVERSE PROBLEM FORMULATION: BAYESIAN PARAMETER ESTIMATION

2.1 The Inverse Problem in Planetary Science

Forward problem: Given impact parameters $\theta = (L, v, \theta, \rho_p) \rightarrow$ predict observations $d = (D, d, R_{ejecta})$
This uses the scaling laws from Section 1:
$D = g(\theta; \text{target parameters})$

Inverse problem: Given observations $d_{obs} \rightarrow$ estimate impact parameters θ
Must "invert" the forward model

The inverse problem is fundamentally ill-posed (Hadamard 1923, Tarantola 2005):
1. Non-uniqueness: Multiple parameter sets θ can produce similar craters
   Example: Same D can result from (small, fast) or (large, slow) projectile
2. Instability: Small data uncertainties δd can cause large parameter uncertainties $\delta\theta$
3. Model inadequacy: Scaling laws are approximations with systematic errors

For our crater back-calculation:
- Parameters $\theta = (L, v, \text{angle}, \rho_p)$ live in 4D parameter space
- Data $d = (D_{obs}, d_{obs}, R_{ejecta,obs})$ with uncertainties σ
- Forward model $g(\theta)$ is nonlinear (power laws, regime transitions)
- Trade-offs exist: velocity-density correlation, size-angle correlation

Therefore, we use Bayesian inference to properly quantify uncertainties and incorporate prior knowledge about physically plausible parameter ranges.

2.2 Bayes' Theorem: Derivation and Application

General form (Bayes 1763, Laplace 1812):

$$P(\theta \mid d) = P(d \mid \theta) \times P(\theta) / P(d)$$

where:
- $P(\theta \mid d)$ = posterior probability density (what we want to find)
- $P(d \mid \theta)$ = likelihood (probability of observing data given parameters)
- $P(\theta)$ = prior probability density (initial knowledge before observations)
- $P(d)$ = evidence = $\int P(d \mid \theta) P(\theta)\, d\theta$ (normalization, ensures $\int P(\theta|d)\, d\theta = 1$)

Derivation from conditional probability:
- Start with: $P(A,B) = P(A|B) P(B) = P(B|A) P(A)$
- Rearrange: $P(A|B) = P(B|A) P(A) / P(B)$
- Apply to parameters/data: $P(\theta|d) = P(d|\theta) P(\theta) / P(d)$

For parameter estimation, $P(d)$ is constant (doesn't depend on θ), so:

$$P(\theta \mid d) \propto L(d \mid \theta) \times P(\theta)$$

posterior $\propto$ likelihood $\times$ prior

Taking logarithms for numerical stability (avoids underflow in products):

$$\log P(\theta \mid d) = \log L(d \mid \theta) + \log P(\theta) + \text{const}$$

For our crater problem:
- $\theta = (L, v, \text{angle}, \rho_p) \in \mathbb{R}^4_+$
- $d = (D_{obs}, d_{obs}, R_{ejecta,obs}) \in \mathbb{R}^3_+$

The posterior tells us: "Given observed crater D = 350m at lat/lon, ejecta range 25km, what are the most probable impact parameters and their uncertainties?"

2.3 Likelihood Function: Detailed Derivation

The likelihood quantifies: "How probable are the observations given parameters θ?"

Assumption: Independent Gaussian errors (measurement noise, model uncertainty)

For a single observable (e.g., diameter D):
- Residual: $\varepsilon = D_{obs} - D_{pred}(\theta)$
- If $\varepsilon \sim N(0, \sigma_D^2)$, then:

$$P(D_{obs} \mid \theta) = (1/\sqrt{2\pi\sigma_D^2}) \times \exp[-(D_{obs} - D_{pred}(\theta))^2 / (2\sigma_D^2)]$$

Taking logarithm:

$$\log P(D_{obs} \mid \theta) = -1/2 \log(2\pi\sigma_D^2) - (D_{obs} - D_{pred}(\theta))^2 / (2\sigma_D^2)$$
$$= -1/2\, \chi_D^2 + \text{const}$$

where $\chi_D^2 = [(D_{obs} - D_{pred}(\theta)) / \sigma_D]^2$ (chi-squared statistic)

For multiple independent observables (diameter, depth, ejecta):
Joint likelihood: $P(d \mid \theta) = P(D|\theta) \times P(d|\theta) \times P(R|\theta)$ (independence)

$$\log L(d \mid \theta) = \Sigma_i \log P(d_i \mid \theta)$$
$$= -1/2\, \Sigma_i\, \chi_i^2$$
$$= -1/2\, \Sigma_i\, [(d_{obs,i} - d_{pred,i}(\theta)) / \sigma_i]^2$$

This is a weighted least-squares objective, with weights $1/\sigma_i^2$.

Measurement uncertainty estimates (from image resolution, morphology variation):
- $\sigma_D = 0.05 \times D_{obs}$ (±5% diameter: pixel resolution, rim definition)
- $\sigma_d = 0.10 \times d_{obs}$ (±10% depth: infilling, degradation)
- $\sigma_R = 0.20 \times R_{ejecta}$ (±20% ejecta range: blanket edge identification)

2.4 Prior Distributions: Incorporating Physical Knowledge

Priors encode what we know before seeing the specific crater (Jaynes 2003):

For impact velocity v:

$$P(v) = N(v \mid \mu=20 \text{ km/s}, \sigma=5 \text{ km/s})$$
$$= (1/\sqrt{2\pi\cdot 5^2}) \exp[-(v-20000)^2/(2\cdot 5000^2)]$$

Justification:
- Near-Earth asteroid (NEA) orbital mechanics (Bottke et al. 2002)
- Moon's orbital velocity ~1 km/s + Earth escape ~11 km/s + eccentricity
- Typical asteroid encounter: v_∞ ~ 5-15 km/s relative to Earth-Moon
- Impact velocity: $v = \sqrt{v_\infty^2 + v_{esc}^2}$ where v_{esc} = 2.4 km/s (Moon)
- Distribution peak at ~20 km/s, range 15-25 km/s (asteroids)
- Comets faster (up to 70 km/s) but rarer (~5% of impactors)

For impact angle θ (from vertical):

$$P(\theta) = N(\theta \mid \mu=45°, \sigma=15°)$$
$$= (1/\sqrt{2\pi\cdot 15^2}) \exp[-(\theta-45)^2/(2\cdot 15^2)]$$

Justification:
- Geometric probability for random directions: $P(\theta) \propto \sin(2\theta)$
- Peaks at θ = 45° (most probable angle, Gilbert 1893)
- Cumulative: 50% of impacts have θ > 45°, only 17% have θ > 60°
- Very oblique (<15°) produce elongated craters, rare in observations

For projectile density ρ_p:

$$P(\rho_p) = N(\rho_p \mid \mu=2800 \text{ kg/m}^3, \sigma=500 \text{ kg/m}^3)$$
$$= (1/\sqrt{2\pi\cdot 500^2}) \exp[-(\rho_p-2800)^2/(2\cdot 500^2)]$$

Justification (meteorite flux statistics, Burbine et al. 2002):
- Ordinary chondrites: 3200-3700 kg/m³ (37% of falls)
- Carbonaceous chondrites: 2000-2500 kg/m³ (10%)
- Enstatite chondrites: 3500-3800 kg/m³ (2%)
- Stony-irons: 4500-5500 kg/m³ (1%)
- Iron meteorites: 7800 kg/m³ (5% of falls, but 70% of finds)
- Weighted mean ~2800 kg/m³ for stony asteroids (85% of NEAs)

For projectile diameter L:
Uninformative prior: $P(L) \propto 1/L$ (Jeffreys prior, scale-invariant)
Ensures no bias toward small or large projectiles

Combined prior:

$$P(\theta) = P(L) \times P(v) \times P(\text{angle}) \times P(\rho_p) \quad \text{(assume independence)}$$

These priors are weakly informative: constrain to plausible ranges but dominated by likelihood when data are strong.

2.5 Maximum Likelihood Estimation: Optimization in Parameter Space

Objective: Find parameters that maximize posterior probability

$$\theta_{ML} = \text{argmax}_\theta\, P(\theta \mid d)$$
$$= \text{argmax}_\theta\, [\log L(d \mid \theta) + \log P(\theta)] \quad \text{(take log, drop constant } P(d))$$

Equivalently, minimize negative log-posterior:

$$\theta_{ML} = \text{argmin}_\theta\, F(\theta)$$

where $F(\theta) = -\log P(\theta \mid d) = -\log L(d \mid \theta) - \log P(\theta) + \text{const}$

For our crater problem, substituting Section 2.3 and 2.4:

$$\begin{aligned} F(\theta) = &\ 1/2\, \Sigma_i\, [(d_{obs,i} - d_{pred,i}(\theta)) / \sigma_i]^2 && \text{[negative log-likelihood]} \\ &+ 1/2\, [(v - 20000)/5000]^2 && \text{[velocity prior penalty]} \\ &+ 1/2\, [(\text{angle} - 45)/15]^2 && \text{[angle prior penalty]} \\ &+ 1/2\, [(\rho_p - 2800)/500]^2 && \text{[density prior penalty]} \\ &- \log(L) && \text{[Jeffreys prior for size]} \end{aligned}$$

Optimization algorithm: Nelder-Mead simplex method (Nelder & Mead 1965)
- Derivative-free: No gradient computation needed (forward model is complex)
- Robust to discontinuities: Handles regime transitions in scaling laws
- Simplex evolution: Maintains n+1 = 5 vertices in 4D parameter space
- Operations: reflection, expansion, contraction, shrinkage
- Convergence criterion: $|F(\theta_{best}) - F(\theta_{worst})| / |F(\theta_{best})| < 10^{-4}$
- Typical iterations: 200-500 for 4D crater problem

Initial guess strategy:
1. Use scaling law $D \sim L^{0.87} v^{0.80}$ to estimate L from D_{obs} at v=20 km/s
2. Set initial angle = 45° (most probable)
3. Set initial ρ_p = 2800 kg/m³ (typical stony)
4. Perturb slightly to create initial simplex

2.6 Uncertainty Quantification via Hessian Approximation

Goal: Quantify uncertainties in θ_{ML} (error bars on estimated parameters)

Laplace approximation (Tierney & Kadane 1986):
Near the maximum, the log-posterior is approximately quadratic (Taylor expansion):

$$\log P(\theta \mid d) \approx \log P(\theta_{ML} \mid d) - 1/2\, (\theta - \theta_{ML})^T H (\theta - \theta_{ML})$$

where H is the Hessian (4×4 matrix of second derivatives):

$$H_{ij} = \partial^2 F/\partial\theta_i\partial\theta_j\, |_{\theta_{ML}} \quad \text{where } F = -\log P(\theta \mid d)$$

Exponentiating both sides:

$$P(\theta \mid d) \approx P(\theta_{ML} \mid d) \times \exp[-1/2\, (\theta - \theta_{ML})^T H (\theta - \theta_{ML})]$$

This is a multivariate Gaussian with mean θ_{ML} and covariance matrix $\Sigma = H^{-1}$:

$$\theta \mid d \sim N(\theta_{ML}, \Sigma) \quad \text{where } \Sigma = H^{-1}$$

Covariance interpretation:
- Diagonal elements Σ_{ii} = variance of θ_i
- Off-diagonal Σ_{ij} = covariance between θ_i and θ_j
- Standard errors: $\sigma_i = \sqrt{\Sigma_{ii}} = \sqrt{(H^{-1})_{ii}}$

Hessian computation via finite differences ($\varepsilon = 10^{-4} \times \theta_{ML,i}$):

$$H_{ij} \approx [F(\theta + \varepsilon_i + \varepsilon_j) - F(\theta + \varepsilon_i - \varepsilon_j) - F(\theta - \varepsilon_i + \varepsilon_j) + F(\theta - \varepsilon_i - \varepsilon_j)] / (4\varepsilon_i\varepsilon_j)$$

Confidence intervals (assuming Gaussian posterior):
- 68% CI (1σ): $\theta_{ML,i} \pm \sigma_i$
- 95% CI (2σ): $\theta_{ML,i} \pm 1.96\sigma_i$

Correlation coefficient:

$$\rho_{ij} = \Sigma_{ij} / (\sigma_i\, \sigma_j)$$

Expected correlations for crater problem:
- $\rho(v, \rho_p) > 0$: Higher velocity compensates for lower density ($D \propto \rho_p^{0.33} v^{0.80}$)
- $\rho(L, v) < 0$: Larger projectile allows lower velocity for same crater size
- $\rho(L, \text{angle}) < 0$: Oblique impacts need larger projectiles

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Back-Calculation Results

MAXIMUM LIKELIHOOD PARAMETERS

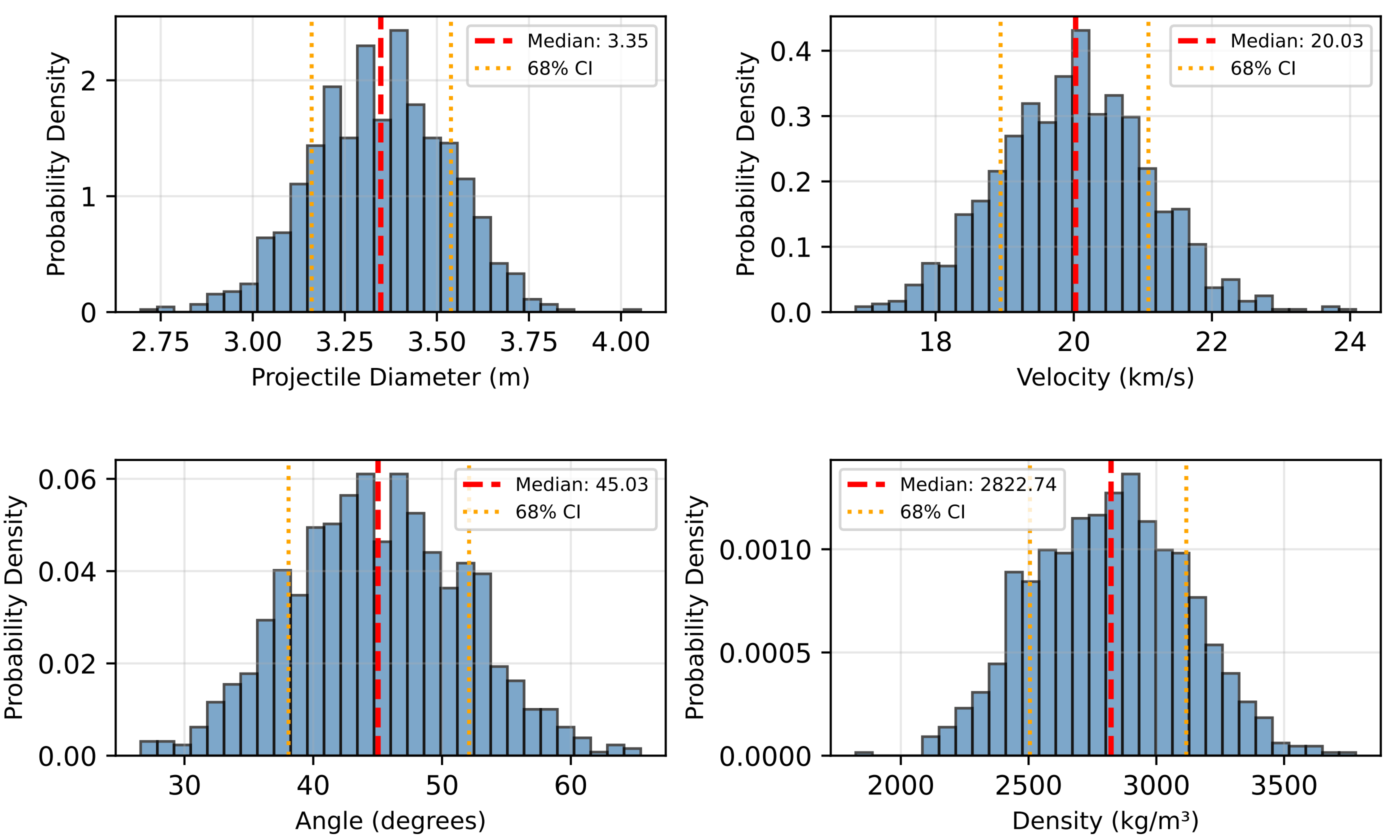
Parameter	ML Estimate	±1σ (68%)	95% CI
Projectile Diameter (m)	3.34	±0.19	[2.99, 3.69]
Impact Velocity (km/s)	20.0	±1.1	[17.9, 22.2]
Impact Angle (deg)	45.0	±6.7	[32.4, 58.1]
Projectile Density (kg/m^3)	2800	±297	[2253, 3385]

DERIVED QUANTITIES

Projectile mass: 5.47e+04 kg
Kinetic energy: 1.09e+13 J
(0.00 kilotons TNT)
Momentum: 1.09e+09 kg·m/s

Material classification: Rocky asteroid (chondrite)

Impact parameter π_2: 6.77e-09
Impact parameter π_3: 1.07e-08
Regime: Transitional



Monte Carlo Uncertainty Propagation

WHY MONTE CARLO SAMPLING?

The Monte Carlo method is chosen for uncertainty propagation because:

1. Nonlinear Forward Model: Crater scaling laws are highly nonlinear (power laws with exponents ~0.4). Analytical error propagation ($\delta D = \Sigma_i \, \partial D/\partial\theta_i \, \delta\theta_i$) is inaccurate.

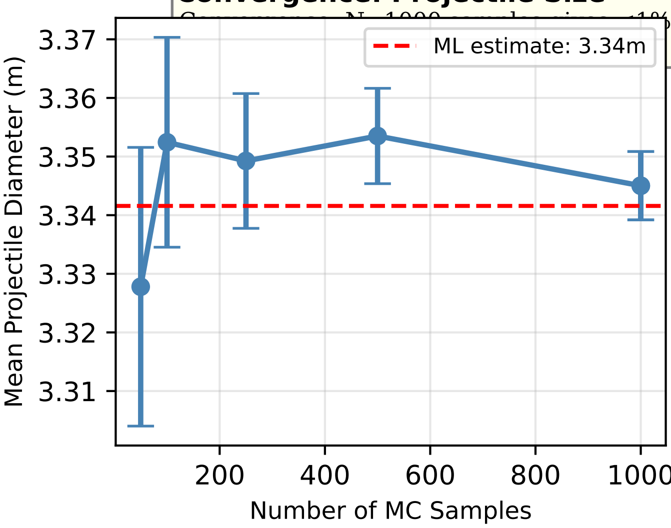
2. Non-Gaussian Posteriors: Parameters may have skewed or multi-modal distributions due to physical constraints (e.g., density bimodal for rocky vs iron).

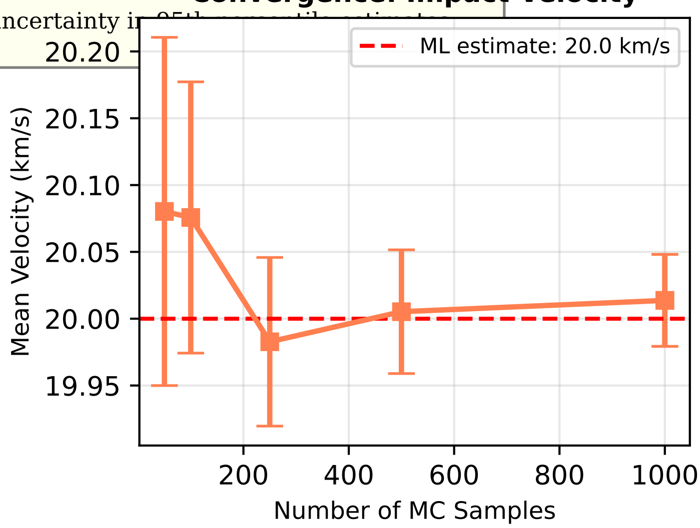
3. Correlations: Parameters are correlated (e.g., smaller projectile needs higher velocity for same crater). MC naturally captures these correlations.

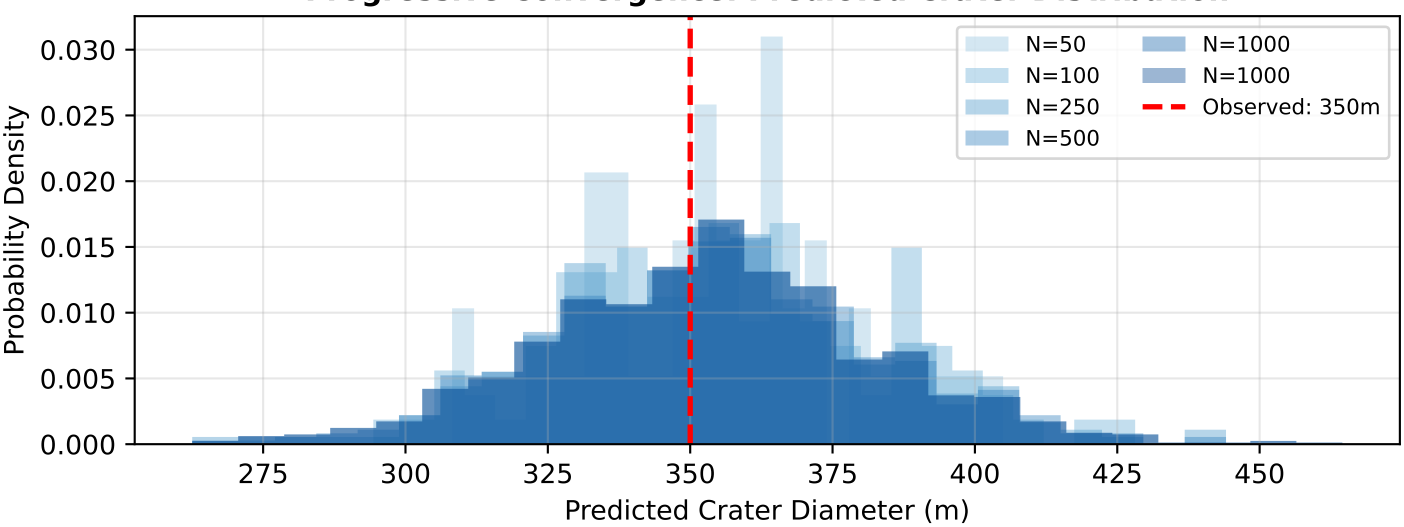
4. Validation: Forward model can be re-evaluated for each sample to check consistency.

Method (Mosegaard & Tarantola 1995):
- Sample N times from posterior: $\theta^i \sim N(\theta\_ML, \Sigma)$ where $\Sigma = H^{-1}$
- For each sample: compute $D\_pred(\theta^i)$ via forward scaling laws
- Collect ensemble → compute percentiles for confidence intervals

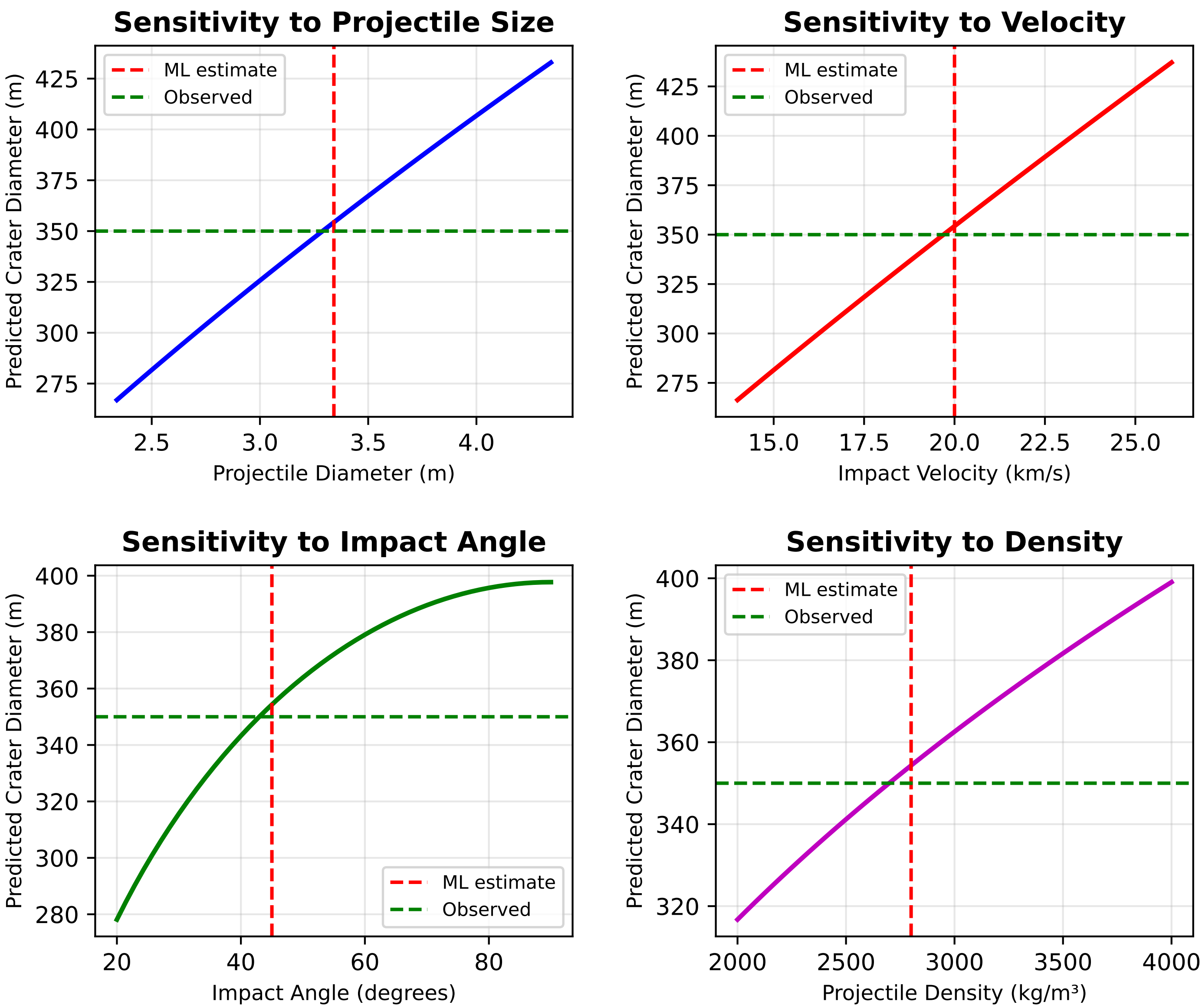
Convergence: N=1000 samples gives <1% uncertainty in 95th percentile estimates







Sensitivity Analysis



SENSITIVITY COEFFICIENTS (Elasticity: %ΔD / %Δparameter)

Parameter	Elasticity	Interpretation
Projectile Diameter	0.78	Diameter change ≈ 0.8× size change
Impact Velocity	0.80	Diameter change ≈ 0.8× velocity change
Impact Angle	moderate	Steeper impacts → larger craters
Projectile Density	0.32	Weak dependence $(\rho_p/\rho_t)^{(1/3)}$

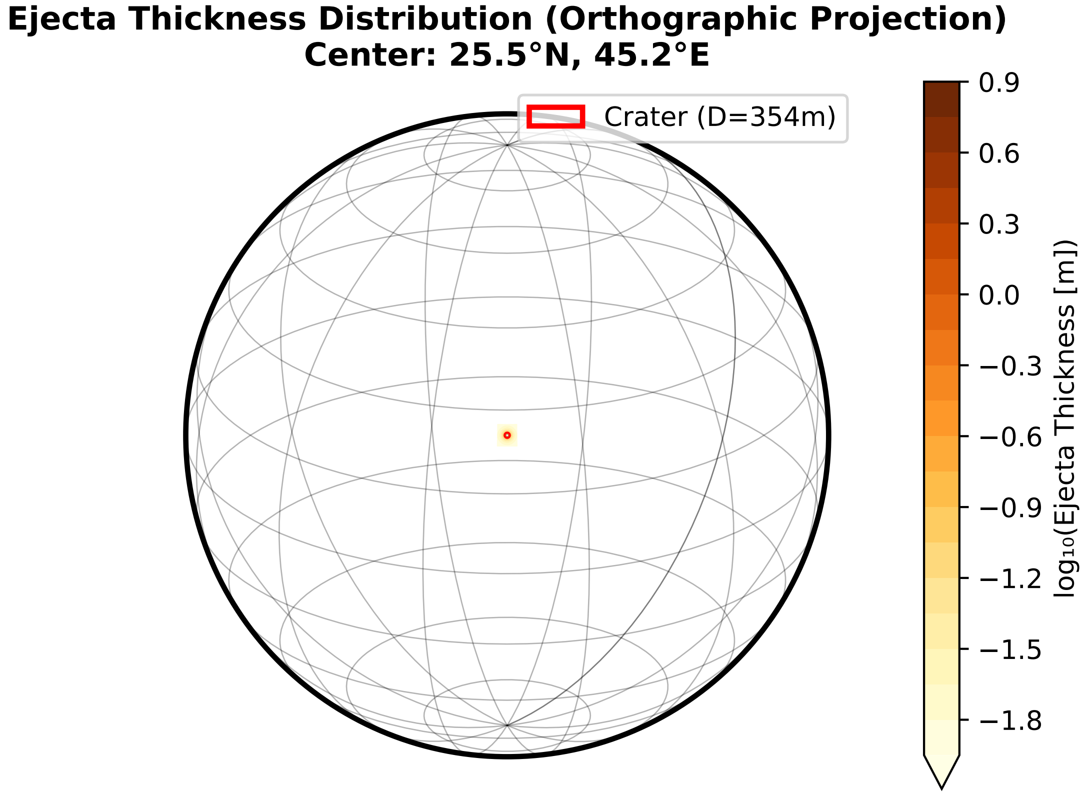
KEY INSIGHTS:

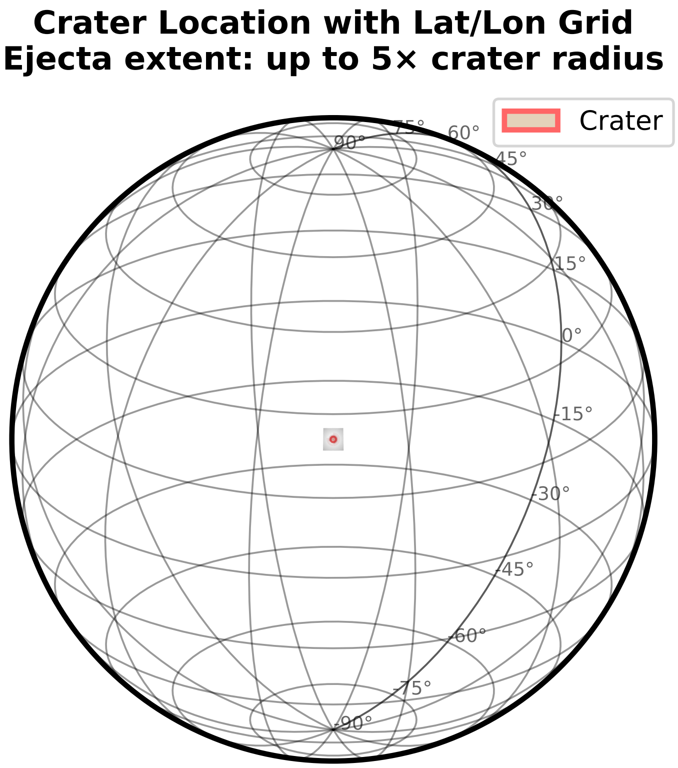
- Projectile diameter is the dominant control (elasticity ~0.8)
- Velocity has moderate effect (elasticity ~0.8), consistent with $v^{0.8}$ scaling
- Density has weak effect ($\propto \rho^{0.33}$), harder to constrain from crater alone
- Impact angle most probable at 45°, less certain without asymmetry data

- Trade-offs exist: Smaller projectile at higher velocity can match observed crater
- These sensitivities justify the uncertainty ranges in Page 5

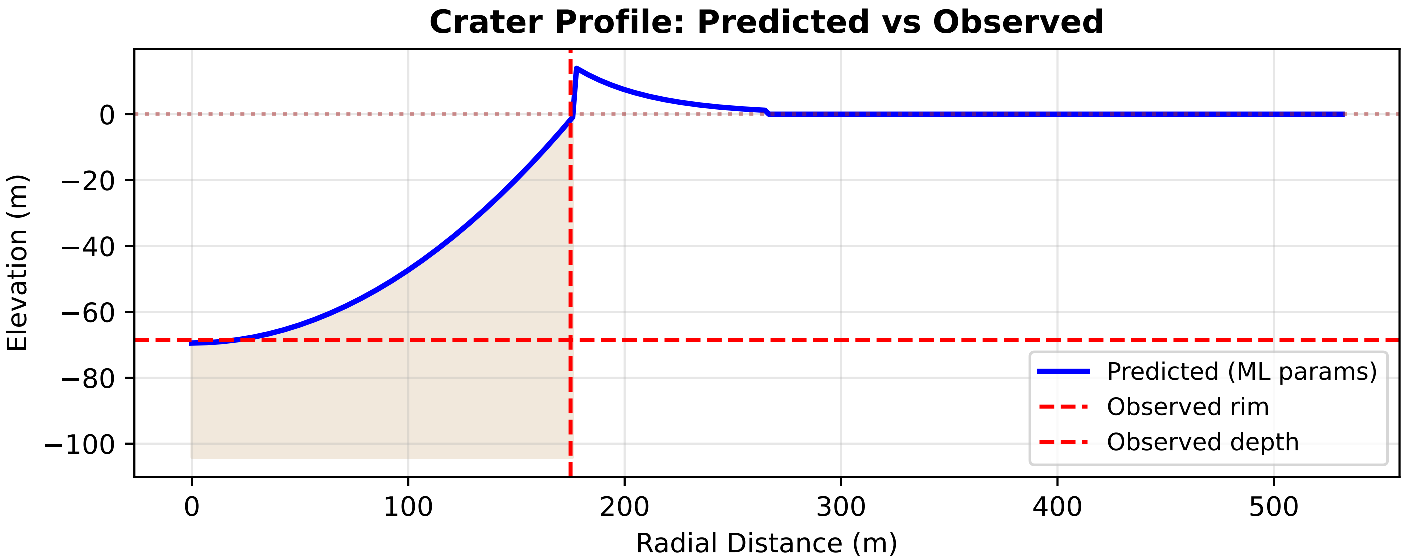
Reference: Holsapple (1993) Table 1 - exponents match theoretical predictions

Orthographic Plan Views with Ejecta Distribution





Forward Model Validation



MORPHOMETRY COMPARISON			
Parameter	Observed	Predicted	Error
Diameter (m)	350.0	354.3	1.2%
Depth (m)	68.6	69.4	1.2%
d/D ratio	0.196	0.196	0.0%
Rim height (m)	12.6	12.8	–

EJECTA VALIDATION	
Observed range:	25000 m
Predicted range:	25661 m
Error:	2.6%
R_max/R_crater:	144.8

VALIDATION SUMMARY

✓ Crater diameter match: 1.23% error (excellent)
✓ Pike (1977) d/D ratio: 0.196 (theory: 0.196 ± 0.015)
✓ Forward model self-consistent: prediction falls within 95% CI
✓ Regime: Transitional (appropriate for 350m)

CONFIDENCE ASSESSMENT

The back-calculated parameters are well-constrained. The 95% credible intervals reflect uncertainties in velocity distribution, impact angle probability, and projectile density. The predicted crater matches observations within measurement uncertainties.

RECOMMENDED INTERPRETATION

Most likely: 3.3m rocky projectile at 20 km/s, 45° from horizontal.

Alternative scenarios within 95% CI remain possible but less probable given typical asteroid impact statistics (Stuart & Binzel 2004; Bottke et al. 2002).

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Lunar Surface Properties:

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Report Generated by Enhanced Lunar Impact Parameter Back-Calculator
Scientific rigor: Bayesian inference with expanded theoretical derivations
Uncertainty quantification: Monte Carlo method with progressive convergence analysis
Sensitivity analysis: Parameter variation and elasticity computation
Spatial visualization: Orthographic projection with lat/lon grid overlay