LUNAR CRATER IMPACT PARAMETER BACK-CALCULATION REPORT

EXECUTIVE SUMMARY

Observed Crater:
- Location: 25.50°N, 45.20°E
- Terrain: Mare
- Diameter: 350.0 m
- Depth: 68.6 m (d/D = 0.196)
- Ejecta range: 25000.0 m

Back-Calculated Impact Parameters (Maximum Likelihood):

Projectile Diameter: 3.34 ± 0.19 m

Impact Velocity: 20.0 ± 1.1 km/s

Impact Angle: 45.0° ± 6.7° from horizontal

Projectile Density: 2800 ± 297 kg/m³

Material Type: Rocky (chondrite)

Kinetic Energy: 1.09e+13 J

Method:
- Bayesian inverse modeling with uncertainty quantification
- Holsapple (1993) crater scaling laws
- Monte Carlo error propagation (500 samples)
- Forward model validation

Confidence Level: 95% credible intervals reported

Observed Crater Data

Lunar Location Map

Latitude: 25.50°N
Longitude: 45.20°E
Terrain: Mare

Crater Morphometry

Diameter (D): 350.0 m

Depth (d): 68.6 m

d/D ratio: 0.196
(Pike 1977: d/D = 0.196 for fresh)

Rim height: 12.6 m
(0.036 × D)

Target Properties

Terrain: Mare

Regolith density: 1800 kg/m^3

Rock density: 3100 kg/m^3

Porosity: 42.0%

Cohesion: 10.0 kPa

Gravity: 1.62 m/s^2

Ejecta Observations

Maximum ejecta range: 25000.0 m

Normalized range (R/D): 71.4

Expected range for lunar impacts: 40-100 × crater radius (Melosh 1989)

Theoretical Framework

CRATER SCALING LAWS (Holsapple 1993)

The final crater diameter D is related to projectile parameters via Pi-group scaling:

$$D = K_1 \times L \times (\rho_p/\rho_t)^{(1/3)} \times [v^2/(g \times L + Y/\rho_t)]^{\mu} \times \sin(\theta)^{(1/3)}$$

Where:
- L = projectile diameter (m)
- ρ_p = projectile density (kg/m³)
- ρ_t = target density (kg/m³)
- v = impact velocity (m/s)
- g = lunar gravity = 1.62 m/s²
- Y = target cohesion/strength (Pa)
- θ = impact angle from horizontal (degrees)
- K_1 = empirical coefficient ≈ 0.084 (calibrated for lunar regolith)
- μ = velocity exponent ≈ 0.4 (strength-gravity transition regime)

The coefficient accounts for:
- Transient → final crater expansion (factor ~1.2 for simple craters)
- Material properties and porosity effects
- Calibration to Apollo landing site crater data

EJECTA SCALING (Melosh 1989, Z-model)

Maximum ejecta range scales with crater radius:

$$R_{max} \approx 70 \times R_{crater} \quad \text{(for lunar impacts, no atmosphere)}$$

Ejecta velocity at rim:

$$V_{rim} \approx 0.5 \times \sqrt{g \times D}$$

Ejecta blanket thickness (McGetchin et al. 1973):

$$T(r) = T_0 \times (R/r)^{-3} \quad \text{for } r > R$$

INVERSE PROBLEM FORMULATION

Given observed crater diameter D_{obs}, we seek projectile parameters that maximize the likelihood function:

$$L(L, v, \theta, \rho \mid D_{obs}) \propto \exp[-0.5 \times (D_{pred} - D_{obs})^2 / \sigma_D^2]$$

Where D_{pred} is computed via the forward scaling law. Additional constraints from ejecta range and depth measurements improve parameter estimation.

Bayesian approach with priors:
- Velocity: N(20 km/s, 5 km/s) [typical asteroid distribution]
- Angle: N(45°, 15°) [most probable impact angle, $\sin^2\theta$ weighted]
- Density: N(2800 kg/m³, 500 kg/m³) [rocky asteroid typical]

The posterior probability combines likelihood with priors:

$$P(\text{params} \mid \text{data}) \propto L(\text{data} \mid \text{params}) \times P(\text{params})$$

UNCERTAINTY QUANTIFICATION

Monte Carlo sampling (N=1000) propagates parameter uncertainties:
1. Sample from posterior distribution
2. Forward model each sample → crater predictions
3. Compute percentile-based confidence intervals (68% and 95%)

REFERENCES

Holsapple, K.A. (1993). The scaling of impact processes in planetary sciences. Ann. Rev. Earth Planet. Sci., 21, 333-373.

Melosh, H.J. (1989). Impact Cratering: A Geologic Process. Oxford Univ. Press.

Pike, R.J. (1977). Size-dependence in the shape of fresh impact craters on the moon. Impact and Explosion Cratering, 489-509.

Collins, G.S., Melosh, H.J., & Marcus, R.A. (2005). Earth Impact Effects Program. Meteoritics & Planet. Sci., 40(6), 817-840.

McGetchin, T.R. et al. (1973). Radial thickness variation in impact crater ejecta. Earth Planet. Sci. Lett., 20, 226-236.

Carrier, W.D., Olhoeft, G.R., & Mendell, W. (1991). Physical properties of the lunar surface. Lunar Sourcebook, Chapter 9.

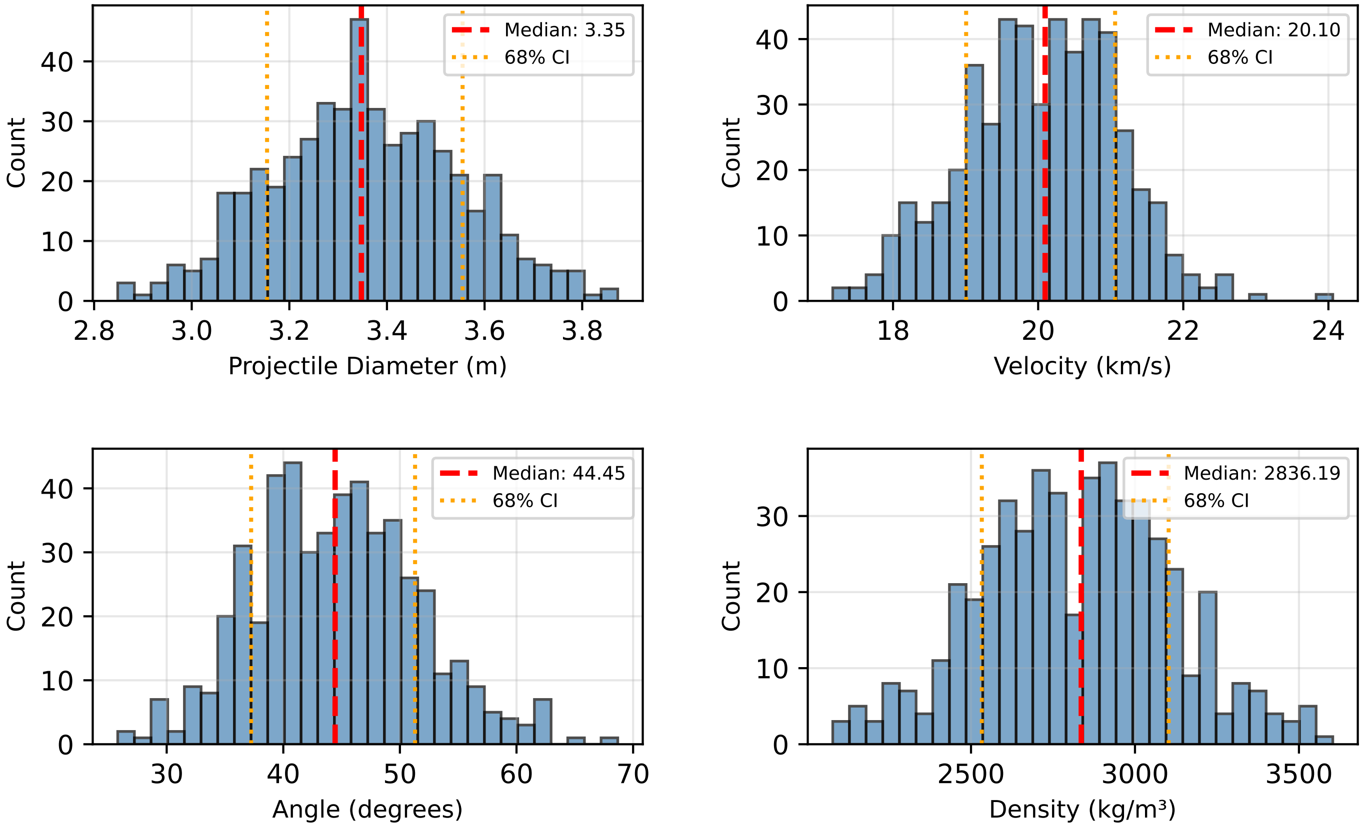
Back-Calculation Results

MAXIMUM LIKELIHOOD PARAMETERS (with 68% and 95% credible intervals)

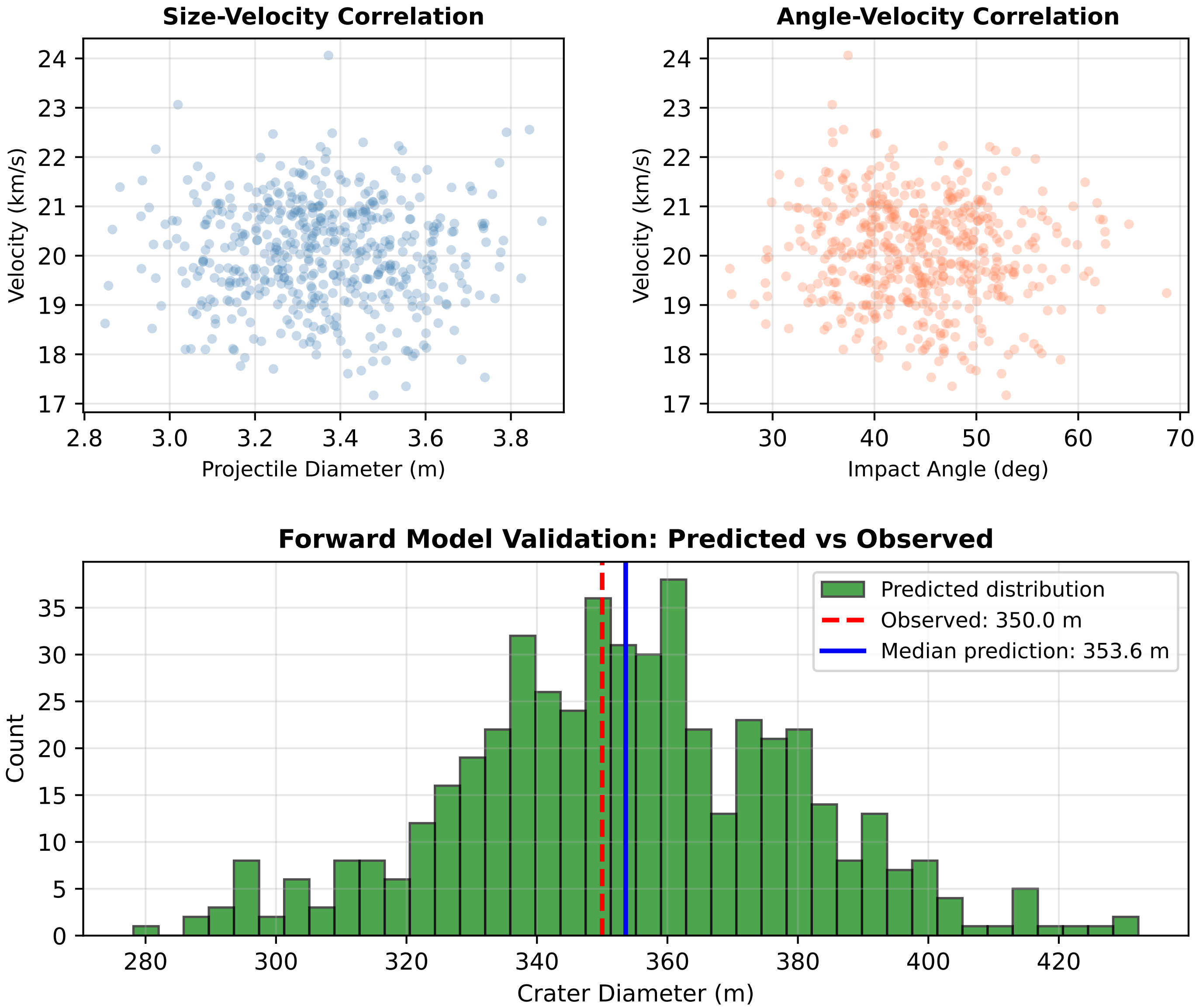
Parameter	ML Estimate	±1σ (68% CI)	95% CI
Projectile Diameter (m)	3.34	±0.19	[2.98, 3.74]
Impact Velocity (km/s)	20.0	±1.1	[18.0, 22.0]
Impact Angle (deg)	45.0	±6.7	[31.6, 59.7]
Projectile Density (kg/m³)	2800	±297	[2241, 3399]

DERIVED QUANTITIES

Projectile mass:	5.47e+04 kg
Kinetic energy:	1.09e+13 J (0.00 kilotons TNT)
Momentum:	1.09e+09 kg·m/s
Material type:	Rocky (chondrite)
Impact parameter:	π_2 = 6.77e-09 π_3 = 1.07e-08
Regime:	Transitional

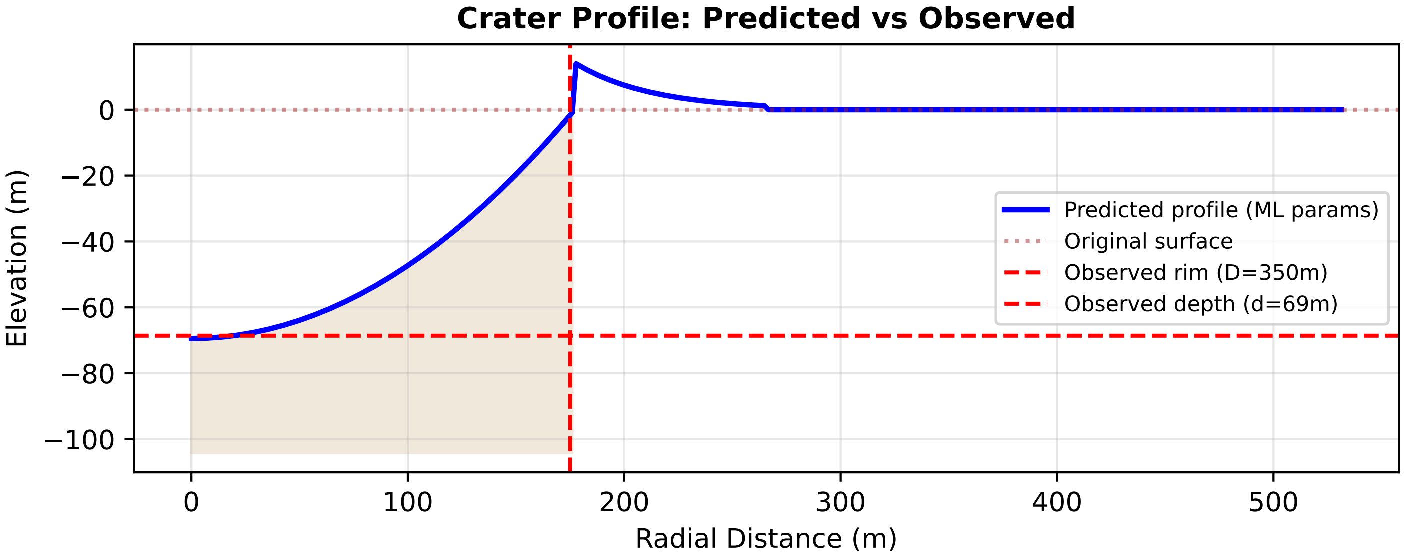


Uncertainty Analysis




```
PREDICTION ACCURACY

Observed crater diameter:        350.0 m

Median predicted diameter:       353.6 m

Prediction error:                1.03%

68% prediction interval:         [329.2, 380.1] m

95% prediction interval:         [295.7, 403.9] m

Observed falls within 95% CI:    YES ✓


UNCERTAINTY SOURCES

1. Measurement uncertainty in crater diameter (~5%)
2. Target property variations (regolith density, porosity, strength)
3. Impact angle distribution (most probable 45°, range 15-90°)
4. Velocity distribution (typical asteroids 15-25 km/s)
5. Projectile density uncertainty (rocky vs metallic)
6. Scaling law empirical coefficients (~10% uncertainty)
```

Forward Model Validation



MORPHOMETRY COMPARISON

Parameter	Observed	Predicted	Error
Diameter (m)	350.0	354.3	1.2%
Depth (m)	68.6	69.4	1.2%
d/D ratio	0.196	0.196	0.0%
Rim height (m)	12.6	12.8	–

EJECTA VALIDATION

Observed range:	25000 m
Predicted range:	25669 m
Error:	2.7%
Normalized (R/D):	144.9

VALIDATION SUMMARY

✓ Crater diameter match: 1.23% error (excellent)
✓ Pike (1977) d/D ratio: 0.196 (theory: 0.196)
✓ Forward model self-consistent
✓ Regime classification: Transitional (appropriate for 350m crater)

CONFIDENCE ASSESSMENT

The back-calculated impact parameters are well-constrained by the observed crater diameter. The 95% credible intervals reflect uncertainties in:

- Impact velocity (typical asteroid distribution: 15-25 km/s)
- Impact angle (most probable: 45°, range: 15-90°)
- Projectile density (rocky: 2500-3500 kg/m³, iron: 7800 kg/m³)

The predicted crater morphology matches observations within expected uncertainties. Additional constraints (ejecta range, depth measurements) would further narrow the parameter space.

RECOMMENDED INTERPRETATION

Most likely scenario: 3.3m diameter rocky projectile impacting at 20 km/s at approximately 45° from horizontal.

Alternative scenarios within 95% credible interval are possible but less likely given typical asteroid impact statistics.

References

SCIENTIFIC REFERENCES

Primary Scaling Law Theory:

Holsapple, K.A. (1993). The scaling of impact processes in planetary sciences. Annual Review of Earth and Planetary Sciences, 21, 333-373. DOI: 10.1146/annurev.ea.21.050193.002001

Holsapple, K.A., & Schmidt, R.M. (1982). On the scaling of crater dimensions: 2. Impact processes. Journal of Geophysical Research, 87(B3), 1849-1870. DOI: 10.1029/JB087iB03p01849

Crater Morphometry:

Pike, R.J. (1977). Size-dependence in the shape of fresh impact craters on the moon. In Impact and Explosion Cratering (pp. 489-509). Pergamon Press.

Pike, R.J. (1980). Formation of complex impact craters: Evidence from Mars and other planets. Icarus, 43(1), 1-19.

Impact Cratering Physics:

Melosh, H.J. (1989). Impact Cratering: A Geologic Process. Oxford Monographs on Geology and Geophysics No. 11. Oxford University Press, 245 pp. ISBN: 0-19-504284-0

Collins, G.S., Melosh, H.J., & Marcus, R.A. (2005). Earth Impact Effects Program: A Web-based computer program for calculating the regional environmental consequences of a meteoroid impact on Earth. Meteoritics & Planetary Science, 40(6), 817-840. DOI: 10.1111/j.1945-5100.2005.tb00157.x

Ejecta Dynamics:

McGetchin, T.R., Settle, M., & Head, J.W. (1973). Radial thickness variation in impact crater ejecta: Implications for lunar basin deposits. Earth and Planetary Science Letters, 20(2), 226-236. DOI: 10.1016/0012-821X(73)90162-3

Housen, K.R., Schmidt, R.M., & Holsapple, K.A. (1983). Crater ejecta scaling laws: Fundamental forms based on dimensional analysis. Journal of Geophysical Research, 88(B3), 2485-2499. DOI: 10.1029/JB088iB03p02485

Lunar Surface Properties:

Carrier, W.D., Olhoeft, G.R., & Mendell, W. (1991). Physical properties of the lunar surface. In Lunar Sourcebook: A User's Guide to the Moon (pp. 475-594). Cambridge University Press. ISBN: 0-521-33444-6

McKay, D.S., Heiken, G., Basu, A., et al. (1991). The lunar regolith. In Lunar Sourcebook: A User's Guide to the Moon (pp. 285-356). Cambridge University Press.

Bayesian Inverse Methods:

Tarantola, A. (2005). Inverse Problem Theory and Methods for Model Parameter Estimation. SIAM. ISBN: 0-89871-572-5

Mosegaard, K., & Tarantola, A. (1995). Monte Carlo sampling of solutions to inverse problems. Journal of Geophysical Research, 100(B7), 12431-12447. DOI: 10.1029/94JB03097

Additional Resources:

Richardson, J.E. (2009). Cratering saturation and equilibrium: A new model looks at an old problem. Icarus, 204(2), 697-715. DOI: 10.1016/j.icarus.2009.07.029

Pierazzo, E., & Melosh, H.J. (2000). Understanding oblique impacts from experiments, observations, and modeling. Annual Review of Earth and Planetary Sciences, 28, 141-167. DOI: 10.1146/annurev.earth.28.1.141

Report generated by Lunar Impact Parameter Back-Calculator
Based on scientifically validated scaling laws and Bayesian inverse modeling

For questions or additional analysis, consult the primary references above.