

CSSE2010 Course Notes

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August 7, 2021

1 Introduction

Following on from the lecture notes as prescribed by the CSSE2010 course, the course notes will be outputting through looking at the levels of abstraction of a computer.

[insert graph of abstraction in computers]

2 Digital logic level

2.1 Bytes and bits

- Computers represent everything in **binary**
- **Bit** = binary digit (0 or 1)
- **Byte** = 8 bits
- Modern computers deal with words which are usually a power of 2 number of bytes. For example:
 - 1, 2, 4 or 8 bytes = 8, 16, 32 or 64 bits

2.2 Representing whole unsigned numbers in binary

2.2.1 Conversion from binary to decimal and vice versa

Converting binary to decimal

- Add values of each position where bit is 1
- Example: $1010011 = 128 + 32 + 4 + 2 + 1 = 167$

Converting Decimal to Binary

Method 1

- Rewrite ' n ' as the sum of powers of 2 (*by repeating subtracting largest powers of 2 not greater than n*)
- Assemble binary number from 1's in bit positions corresponding to those powers of 2, 0's elsewhere

Method 2

Building up bits from the right (least significant bit (*LSB*) to most (*MSB*))

- Divide n by 2
- Remainder of division (*0 or 1*) is next bit
- Repeat with $n = \text{quotient}$

2.2.2 Least and Most Significant Bits

The most significant bit is denoted as **MSB**, likewise, the least significant bit is denoted as **LSB**. These bits can be found on a binary number typically as the leftmost and rightmost bits respectively.

[arrow pointing to example of MSB and LSB in binary number]

2.3 Basic Digital Logic

2.3.1 Digital circuits and Logic gates

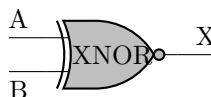
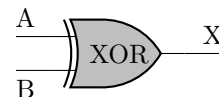
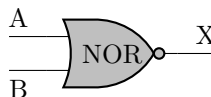
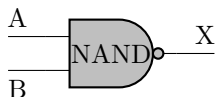
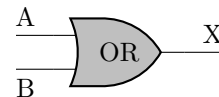
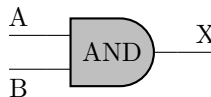
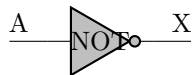
Digital Circuits

- Only two logical levels present (i.e. binary)
 - Logic '0'; usually small voltage (*e.g. around 0 volts*)
 - Logic '1'; usually larger voltage (*e.g. 0.8 to 5 volts, depending on the "logic family", i.e. type/size of transistors*)

Logic gates

- Are the building blocks of computers
- Each gate has
 - One or more inputs
 - Exactly one output
- Perform logic operations *or functions*
 - 7 basic types: **NOT, AND, OR, NAND, NOR, XOR, XNOR**
 - Inputs and outputs can have only two states **1** and **0**; can be called "true" and "false" respectively.
 - Logic symbol, truth table, boolean expression, timing diagram

2.3.2 Basic Logic Gates



2.4 Boolean Logic

2.4.1 Boolean Logic Functions

- Logic functions can be expressed as expressions involving:
 - variables (**literals**), e.g A, B, X
 - functions, e.g +, ., \oplus , \overline{X}
- Rules about how this works are called **Boolean algebra**
- Variables and functions can only take on values 0 or 1

2.4.2 Boolean Algebra conventions

Conventions

- Inversion: [insert overline] (overline)
 - e.g. NOT(A) = \overline{A} (A bar)
- AND: dot(.) or implied (by adjacency)
 - e.g. AND(A,B) = AB = A.B
- OR: plus sign (+)
 - e.g. OR(A,B,C) = A+B+C

Other examples

- XOR(A,B) = $A \oplus B = \overline{A}B + A\overline{B}$
- NAND(A,B,C) = \overline{ABC}
- NOR(A,B) = $\overline{A+B}$

2.4.3 Summary of logic function representations

There are four representations of logic functions (*assume function of n inputs*)

- Truth
 - Lists output for all 2^n combinations of inputs (Best to list inputs in a systematic way)
- Boolean function (or equation)
 - Describes the conditions under which the function output is 1
- Logic Diagram
 - Combination of logic symbols joined by wires
- Timing Diagram

2.4.4 Logic function implementation

- Any logic function can be implemented as the **OR** of **AND** combinations of the inputs
 - Called 'sum of products'
- Example:

– Consider truth table

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- For each '1' in the output column, write down the **AND** combination of inputs that give 1
- OR** these together

Equivalent functions

- Sum of products does not necessarily produce circuit with minimum number of gates
- Can '*manipulator*' boolean functions to give an equivalent function
 - Use rules of boolean algebra
- Example: $Z = AB + AC = A(B+C)$

Boolean identities

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = A$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\bar{A} = 0$	$A + \bar{A} = 1$
Commutative law	$AB = BA$	$B + A = A + B$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A+BC = (A+B).(A+C)$	$A.(B + C) = AB + AC$
Absorption law	$A(A+B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A}\bar{B}$

2.4.5 Number bases

[Flow diagram of Binary(base 2), Decimal(base 10), Hex(base 16), Octal(base 8)] [May need to add further details of the table (in regard to the individual binary representations)]

MSB				LSB	
$2^{(n-1)}$	$2^{(n-2)}$..	2^1	2^0	Excess- $2^{(n-1)}$; $-2^{(n-1)} \leq x \leq (2^{(n-1)} - 1)$
$-2^{(n-1)}$	$2^{(n-2)}$..	2^1	2^0	2's comp ; $-2^{(n-1)} \leq x \leq (2^{(n-1)} - 1)$
$-2^{(n-1)} - 1$	$2^{(n-2)}$..	2^1	2^0	1's comp ; $-(2^{(n-1)} - 1) \leq x \leq (2^{(n-1)} - 1)$
+/-	$2^{(n-2)}$..	2^1	2^0	Sign-Mag ; $-(2^{(n-1)} - 1) \leq x \leq (2^{(n-1)} - 1)$
$2^{(n-1)}$	$2^{(n-2)}$..	2^1	2^0	Unsigned ; $0 \leq x \leq 2^n - 1$

[fix peculiar exponential layout]

2.4.6 Equivalent Circuits

- All circuits can be constructed from **NAND** or **NOR** gates
 - These are called 'complete' gates
- Examples: [insert NOT AND OR logic gates]
- Reason: Easier to build **NAND** and **NOR** gates from transistors

2.5 Binary Arithmetic

2.5.1 Binary addition

- Addition is quite simple in binary

Addend	0	0	1	1
Augend	0	1	0	1
	0	1	1	0
	0	0	0	1

- Above ignores carry in

[make sense of the table above]

Decimal	8-bit unsigned	Decimal	2's complement
10	00001010	10	00001010
+ 243	+ 11110011	+ (-13)	+ 11110011
253	11111101	-3	11111101

- Format matters upon interpreting the number
- Whatever the format is the bit-wise addition (which leads to the hardware circuit) is the same.
- Two's complement; there is no need to do anything with the carry out from the **MSB** to get the correct result

- But in one's complement, the carry out from the MSB will have to be added back to the result to get the correct answer; this is one drawback of one's complement representation

Overflow in binary addition

Decimal	8-bit unsigned	Decimal	2's complement
15	00001111	125	01111101
+ 243	+ 11110011	+ 4	+ 00000100
258	00000010	129	10000001

Overflow: Not enough bits to represent the answer. The result goes out of range thus outputting an incorrect answer.

- Unsigned: carry-out from the **MSB** → overflow
- 2's comp: carry-in to the **MSB** and carry-out from the **MSB** are different → overflow
- Equivalently, overflow occurs if (in 2's comp)
 - * Two negatives added together give a positive, or;
 - * Two positives added together give a negative

2.5.2 Adders and addition of binary words

A device which adds 2 bits (*with no carry-in*) is called a 'half-adder' [insert half adder diagrams]

Addition of binary words

- Have to be able to deal with carry-in

A	B	Cin	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
• 0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

- $S = \overline{A}BC + A\overline{B}C + \dots$

- $S = A \oplus B \oplus Cin$

[insert full adder diagram with relevant sum and carry-out equation]

Binary Adder

- Can cascade full adders to make binary adder
 - Example: for 4 bits ... [insert 4 bit binary adder diagram]
- This is a ripple-carry adder

2.5.3 Binary subtraction

- $\mathbf{A} - \mathbf{B}$; usually implemented as $\mathbf{A} + (-\mathbf{B})$
 - \mathbf{A} and \mathbf{B} are multi-bit quantities
 - '+' in this case means addition (not **OR**)
 - $-\mathbf{B}$ means negative \mathbf{B} - the two's complement of \mathbf{B}
- Two's complement of \mathbf{B} can be calculated by flipping bits and adding 1.

[add relevant equations and interpretations]

3 Microarchitecture level

4 Instruction set architecture level

5 Assembly language level

6 Problem-oriented language level