MATH1061 Course Notes

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1 Logic

1.1 Logical Connectives

1.1.1 Basic logical connectives

For a given logical statement come logical connectives. Basic logical connectives include:

- **not** $= \sim$
- **and** $= \wedge$
- **or** $= \vee$
- **exclusive or** $= \oplus$

1.1.2 Logical Equivalence

Given two statement forms, you can show that they are logically equivalent by using a truth table or by using the laws of logical equivalence.
The logical equivalence between two statements is demonstrated by the symbol $\equiv$

[insert truth table]

1.1.3 Conditional logical connectives

Logical connectives and equivalences, for given truth statements 'p' and 'q'

- **if .. then** $= \rightarrow$
- **if and only if** $= \leftrightarrow$
- $p \rightarrow q \equiv \sim p \vee q \equiv \sim q \rightarrow \sim p$

1.1.4 Order of Operations

1. $\sim$
2. $\wedge$ and $\vee$, use parentheses to specify. If no parentheses given, work from left to right.
3. $\rightarrow$ and $\leftrightarrow$, use parentheses to specify. If no parentheses given, work from left to right.

1.2 Necessary and sufficient conditions

For given truth statements 'p' and 'q':

- p is a necessary condition for q means "if $\sim p$ then $\sim q$" or equivalently "if q then p" or "q only if p".
- p is a sufficient condition for q means "if p then q" or equivalently "q if p"

1.3 Definitions

1.3.1 Tautology and contradictions

- A tautology is a statement form which always takes truth values "**true**" for all possible truth values of its variables.
- A contradiction is a statement form which always takes truth values "**false**" for all possible truth values of its variables.

1.3.2 Contrapositive

For given truth statements 'p' and 'q'
The contrapositive of $p \to q$ is $\sim q \to \sim p$

- These are logically equivalent:
  $p \to q \equiv \sim q \to \sim p$

1.3.3 Biconditional

For given truth statements 'p' and 'q'
The biconditional of p and q, denoted $p \leftrightarrow q$, is defined by the following truth table:

[insert truth table]

2 Logical Equivalences

Given any statement variables 'p', 'q' and 'r', a tautology '**t**' and contradiction '**c**', the following logical equivalences hold.

2.1 Commutative laws

- $p \wedge q \equiv q \wedge p$
- $p \vee q \equiv q \vee p$

2.2 Associative laws

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$

2.3 Distributive laws

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

2.4 Identity laws

- $p \wedge \mathbf{t} \equiv p$
- $p \vee \mathbf{c} \equiv p$

2.5 Negation laws

- $p \vee \sim p \equiv \mathbf{t}$
- $p \wedge \sim p \equiv \mathbf{c}$

2.6 Double negative laws

- $\sim(\sim p) \equiv p$

2.7 Idempotent laws

- $p \vee p \equiv p$
- $p \wedge p \equiv p$

2.8 Universal bound laws

- $p \vee \mathbf{t} \equiv \mathbf{t}$
- $p \wedge \mathbf{c} \equiv \mathbf{c}$

2.9 De Morgan's laws

- $\sim(p \wedge q) \equiv \sim p \vee \sim q$
- $\sim(p \vee q) \equiv \sim p \wedge \sim q$

2.10 Absorption laws

- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

2.11 Negations of t and c

- $\sim\mathbf{t} \equiv \mathbf{c}$
- $\sim\mathbf{c} \equiv \mathbf{t}$