Logical Equivalences

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Given any statement variables 'p', 'q' and 'r', a tautology '$\mathbf{t}$' and contradiction '$\mathbf{c}$', the following logical equivalences hold.

1 Commutative laws

- $p \wedge q \equiv q \wedge p$
- $p \vee q \equiv q \vee p$

2 Associative laws

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$

3 Distributive laws

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

4 Identity laws

- $p \wedge \mathbf{t} \equiv p$
- $p \vee \mathbf{c} \equiv p$

5 Negation laws

- $p \vee {\sim}p \equiv \mathbf{t}$
- $p \wedge {\sim}p \equiv \mathbf{c}$

6 Double negative laws

- $\sim(\sim p) \equiv p$

7 Idempotent laws

- $p \vee p \equiv p$
- $p \wedge p \equiv p$

8 Universal bound laws

- $p \vee \mathbf{t} \equiv \mathbf{t}$
- $p \wedge \mathbf{c} \equiv \mathbf{c}$

9 De Morgan's laws

- $\sim(p \wedge q) \equiv \sim p \vee \sim q$
- $\sim(p \vee q) \equiv \sim p \wedge \sim q$

10 Absorption laws

- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

11 Negations of t and c

- $\sim\mathbf{t} \equiv \mathbf{c}$
- $\sim\mathbf{c} \equiv \mathbf{t}$