Rules of Inference

Paddy Maher

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1 Modus Ponens

$$
\begin{array}{l}
p \to q \\
p \\
\therefore q
\end{array}
$$

2 Modus Tollens

$$
\begin{array}{l}
p \to q \\
\sim q \\
\therefore \sim p
\end{array}
$$

3 Generalisation

$$
\begin{array}{l}
p \\
\therefore p \vee q
\end{array}
$$

$$
\begin{array}{l}
q \\
\therefore p \vee q
\end{array}
$$

4 Specialisation

$$p \wedge q$$
$$\therefore p$$

$$p \wedge q$$
$$\therefore q$$

5 Conjunction

$$p$$
$$q$$
$$\therefore p \wedge q$$

6 Elimination

$$p \vee q$$
$$\sim q$$
$$\therefore p$$

$$p \vee q$$
$$\sim p$$
$$\therefore q$$

7 Transitivity

$$p \rightarrow q$$
$$q \rightarrow r$$
$$\therefore p \rightarrow r$$

8 Proof by Division into cases

$$
\begin{array}{l}
p \vee q \\
p \rightarrow q \\
q \rightarrow r \\
\therefore r
\end{array}
$$

9 Contradiction Rule

$$
\begin{array}{l}
\sim p \rightarrow (contradiction) \\
\therefore p
\end{array}
$$