Differential and Complex Algebraic Geometry

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Introduction

First and foremost I want to clarify that by no means am I an expert on the presented topics or any topics in mathematics for that matter. I have wanted to study complex geometry for quite some time but was unable take a course or do research in the area. So this is me exploring complex geometry and some of the topics that are parallel. As it happens, this requires a full course on differential geometry and Lie groups. I'm taking these notes from 3-4 books as well as supplemental notes found online. Of course these will all be cited at the end. The end goal is to prepare myself for spectral/noncommutative geometry. However, the functional analysis side of things would be a novel on it's own. Some functional analysis will pop up here and there in these notes but only in the context of geometry. I am not going to go too deep into operator algebras or anything like that. Most of this information is new to me at the time of writing. Also worth noting, I am writing this with a VERY casual approach so my language and verbiage may not be as formal or precise, I'm writing what my brain is thinking. I am documenting my journey of learning geometry and whatever may branch off of it that interests me.

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1 The Matrix Exponential

1.1 The Exponential Map and Matrix Groups

Given an $n \times n$ matrix A, we want to find a way to find e^A. We can actually just do this with the usual power series:

$$e^A = I_n + \sum_{p \geq 1} \frac{A^p}{p!} = \sum_{p \geq 0} \frac{A^p}{p!} \tag{1}$$

Using an inductive proof, we can show that this is well defined. But we won't write it out here.

contents

how to cite thm in title

Theorem 1.1. *uhhh*

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nothing here yet

Example 1.1. *temp*

contents

this means something

Definition 1.1. *temp*