🌏 🚀 📊 🧱 50 Startups 🧱 📊 🚀 🌏

✏️ ✏️ Purpose of the Project ✏️ ✏️

1. Exploratory data analysis (EDA) is one of the most significant methods to examine the data we have. In this project, we are going to explore the hidden patterns in the dataset and extract information from them.

2. Implementing Multiple Linear Regression on the dataset for future predicion.

3. Regularization techniques used to address over-fitting

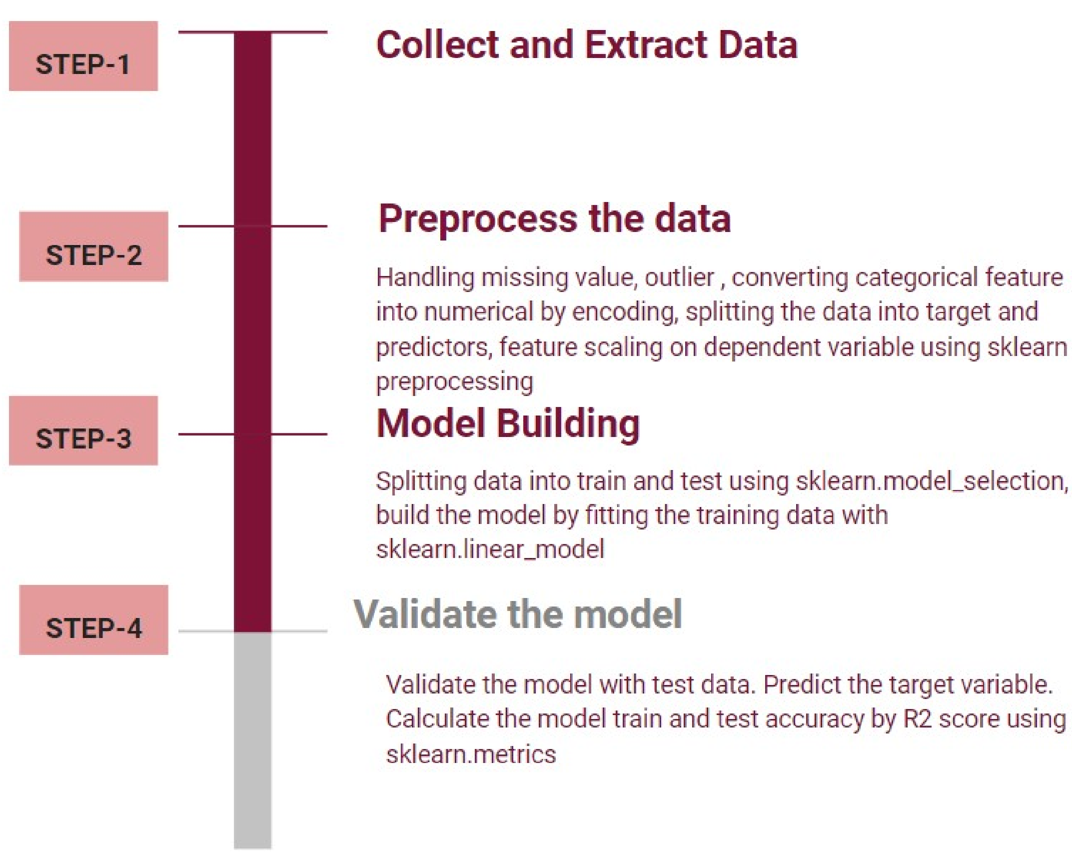
4. Gradient Descent is an optimization algorithm is used that minimize a cost function as far as possible.

5. In addition to above, Polynomial regression is used to describes the fitting of a nonlinear relationship if any.

> - In linear regression, the linearity assumption states that there is a linear relationship between the independent variables (also known as predictors or features) and the dependent variable (also known as the target or response variable). This assumption implies that the relationship between the independent variables and the dependent variable can be described by a straight line.

> - Multiple regression suffers from multicollinearity, autocorrelation, heteroskedasticity.
> - Linear Regression is very sensitive to Outliers. It can terribly affect the regression line and eventually the forecasted values.

work flow of the model :



===

About Dataset

About

Dataset about 50 Startups' expenditures & profits

Column Description

50 startup dataset with columns

- R&D Spend
- Administration
- Marketing Spend
- State
- Profit

Importing Neccessary Libraries

In [1]:
```
import os
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
%matplotlib inline

import warnings
warnings.filterwarnings('ignore')
```

Sklearn package used for Machine Learning

> Model : Linear Regression

In [2]:
```
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler, LabelEncoder, PolynomialFeatures
from sklearn.linear_model import LinearRegression,Lasso, Ridge, ElasticNet,SGDRegressor
from sklearn.metrics import r2_score,mean_absolute_percentage_error, mean_squared_error
from sklearn import metrics
```

Loading and Reading Data

In [3]:
```
dataset = '50_Startups.csv'
df = pd.read_csv(dataset)
df.head(10)
```

Out[3]:

	R&D Spend	Administration	Marketing Spend	State	Profit
0	165349.20	136897.80	471784.10	New York	192261.83
1	162597.70	151377.59	443898.53	California	191792.06
2	153441.51	101145.55	407934.54	Florida	191050.39
3	144372.41	118671.85	383199.62	New York	182901.99
4	142107.34	91391.77	366168.42	Florida	166187.94
5	131876.90	99814.71	362861.36	New York	156991.12
6	134615.46	147198.87	127716.82	California	156122.51
7	130298.13	145530.06	323876.68	Florida	155752.60
8	120542.52	148718.95	311613.29	New York	152211.77
9	123334.88	108679.17	304981.62	California	149759.96

In [4]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 50 entries, 0 to 49
Data columns (total 5 columns):
 #   Column           Non-Null Count  Dtype  
---  ------           --------------  -----  
 0   R&D Spend        50 non-null     float64
 1   Administration   50 non-null     float64
 2   Marketing Spend  50 non-null     float64
 3   State            50 non-null     object 
 4   Profit           50 non-null     float64
dtypes: float64(4), object(1)
memory usage: 2.1+ KB
```

In [5]:
```
len(df.columns)
```

Out[5]:
```
5
```

In [6]:
```
df.shape
```

Out[6]:
```
(50, 5)
```

Descriptive Statistics (Univariate Analysis)

In [7]:
```
df.describe().T
```

Out[7]:

	count	mean	std	min	25%	50%	75%	max
R&D Spend	50.0	73721.6156	45902.256482	0.00	39936.3700	73051.080	101602.8000	165349.20
Administration	50.0	121344.6396	28017.802755	51283.14	103730.8750	122699.795	144842.1800	182645.56
Marketing Spend	50.0	211025.0978	122290.310726	0.00	129300.1325	212716.240	299469.0850	471784.10
Profit	50.0	112012.6392	40306.180338	14681.40	90138.9025	107978.190	139765.9775	192261.83

In [8]:
```

def descriptive_stats(df):
    desc = df.describe().round(2).T
    desc_df = pd.DataFrame(index = df. columns,
                          columns = desc.columns,
                          data = desc)

    plt.figure(figsize=(14,6))
    sns.heatmap(desc, annot=True, cmap='Set2', fmt=".2f")
    plt.xticks(size = 12)
    plt.yticks(size = 12, rotation = 0)
    plt.title('Statisttical Description')
    plt.show()

descriptive_stats(df)
```

Removing Duplicate Rows

In [9]:
```
# Remove duplicates

def drop_dup(df):
    if df.duplicated().any() == True:
        print('The total duplicate row before removing duplicate:', df.duplicated().sum())
        df.drop_duplicates(inplace=True , keep = 'last')  # Remove duplicates
        df = df.reset_index(drop=True) #Reset the index
        print('The total duplicate row after removing duplicate:', df.duplicated().sum(), ' \nshape of datase
    else:
        return 'No duplicate entries'
drop_dup(df)
```

Out[9]: 'No duplicate entries'

Checking Null Value

In [10]:
```
null_details = {'col_name' : df.columns.tolist(),
                'col_type' : [df[col].dtype for col in df.columns],
                'null_col(%)' :  [df[col].isnull().sum()*100/len(df) for col in df.columns]}

df_null_details = pd.DataFrame(null_details)
df_null_details
```

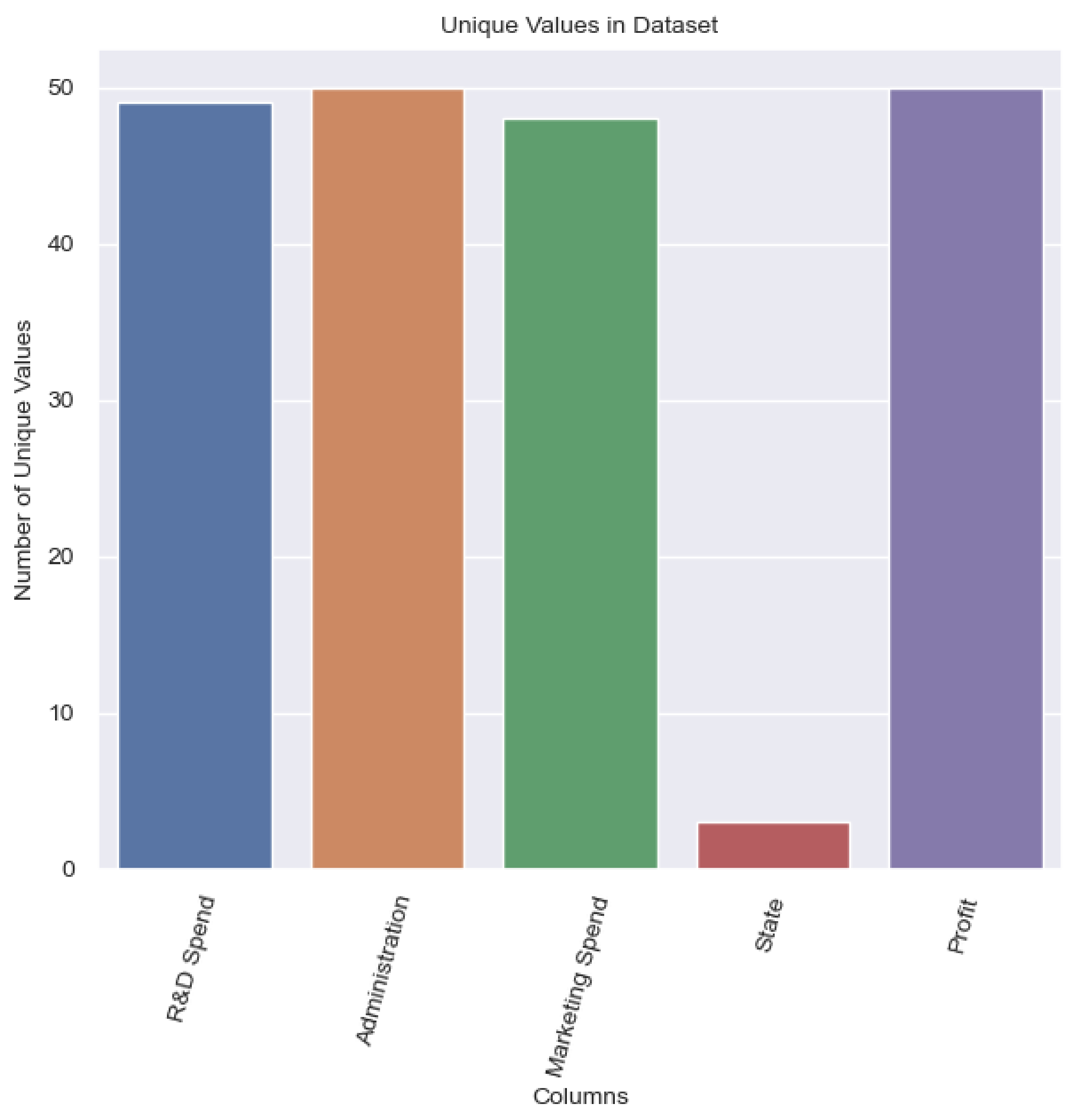
Out[10]:

	col_name	col_type	null_col(%)
0	R&D Spend	float64	0.0
1	Administration	float64	0.0
2	Marketing Spend	float64	0.0
3	State	object	0.0
4	Profit	float64	0.0

Exploratory Data Analysis (EDA) 📈 📊 📉

In [11]:
```
# Count the number of unique values in each column
def check_unquie_count(df):
    unique_counts = df.nunique()
    print(unique_counts)
# Create a bar plot or count plot of unique values
    plt.figure(figsize=(7, 6))
    sns.barplot(x=unique_counts.index, y=unique_counts.values,)

    plt.xticks(rotation=75, fontsize= 10)
    plt.yticks( fontsize= 10 )
    plt.xlabel('Columns',fontsize=10)
    plt.ylabel('Number of Unique Values', fontsize=10)
    plt.title('Unique Values in Dataset', fontsize=10)

# Display the plot
    plt.show()

check_unquie_count(df)
```

```
R&D Spend          49
Administration     50
Marketing Spend    48
State               3
Profit             50
dtype: int64
```



In [12]:
```
num_not_cat = df.select_dtypes(exclude='object')
num_not_cat.dtypes
```

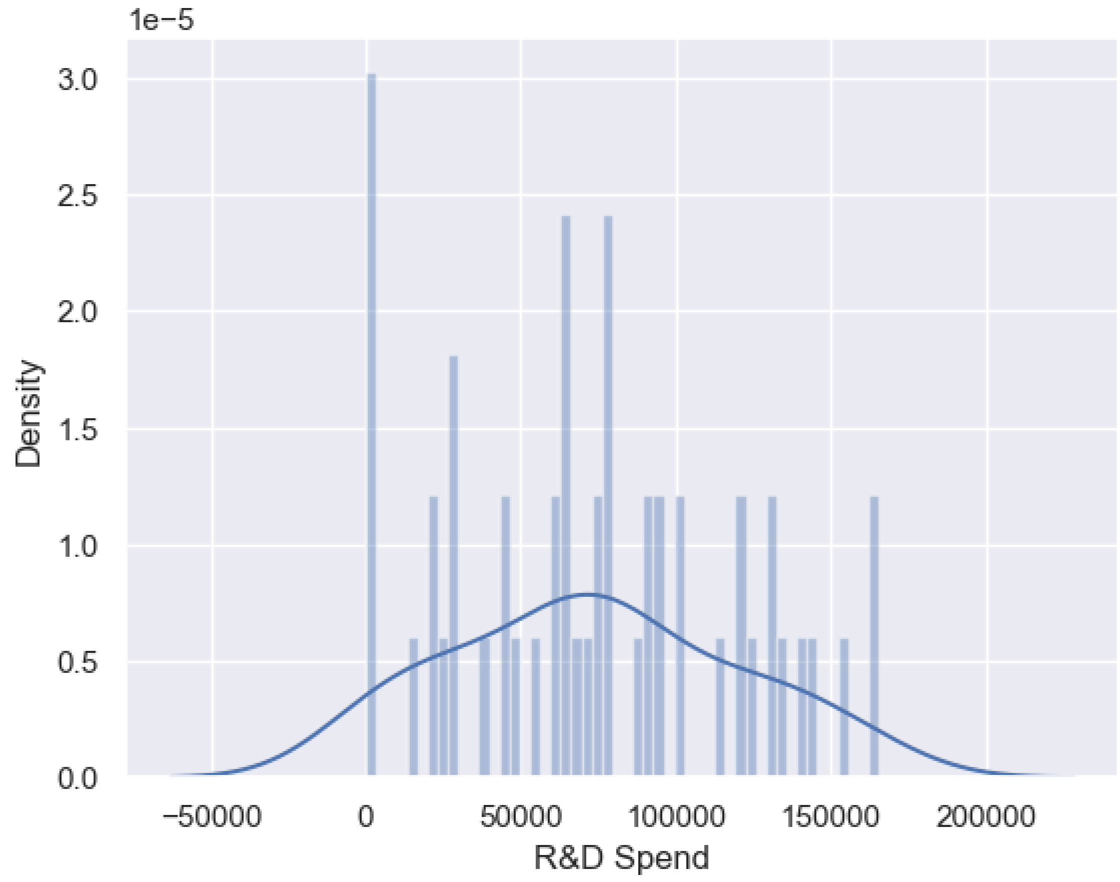
Out[12]:
```
R&D Spend          float64
Administration     float64
Marketing Spend    float64
Profit             float64
dtype: object
```

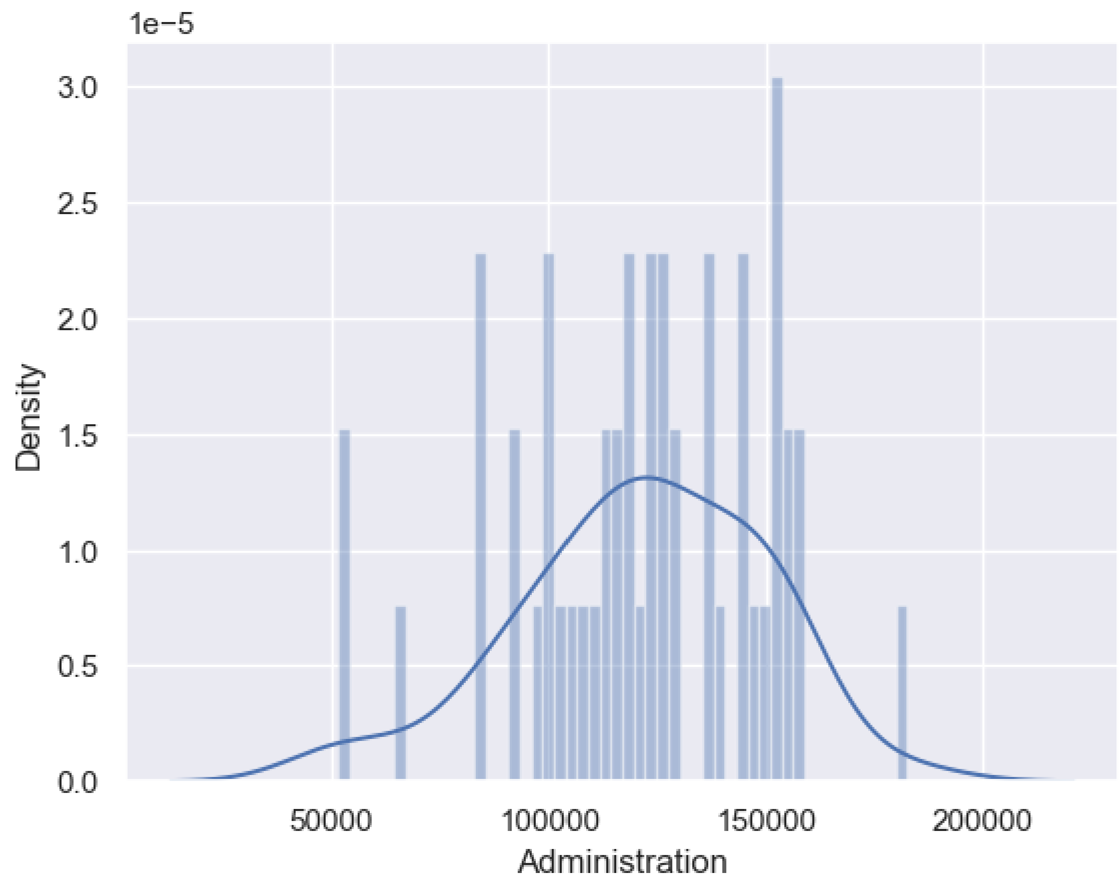
In [13]:
```
# outlier check on all  independent feature except dependent variable
outlier_list = list(num_not_cat.columns)

list_remove=['Profit']
for i in list_remove:
    outlier_list.remove(i)

outlier_list
```

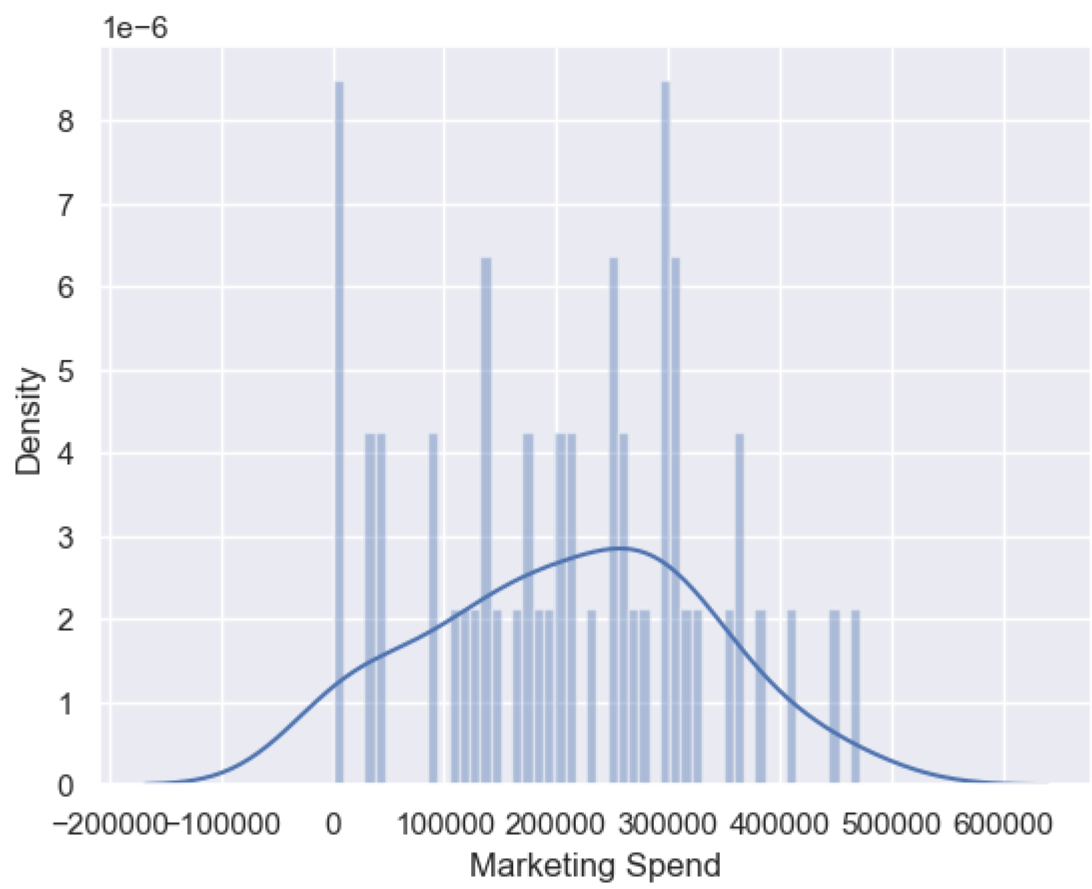
Out[13]:
```
['R&D Spend', 'Administration', 'Marketing Spend']
```

Outlier Check

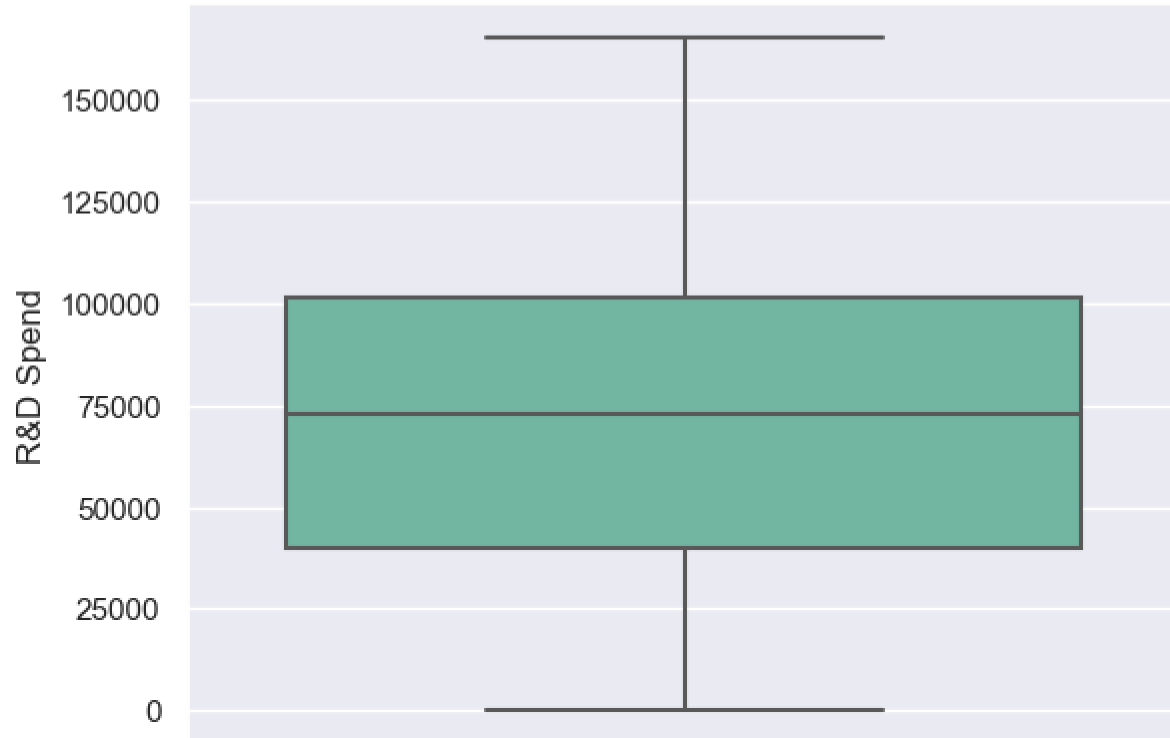
In [14]:
```
def distplots(df,col):
    sns.distplot(df[col], bins=50,kde=True)
    plt.show()

for i in outlier_list:
    distplots(num_not_cat,i)
```

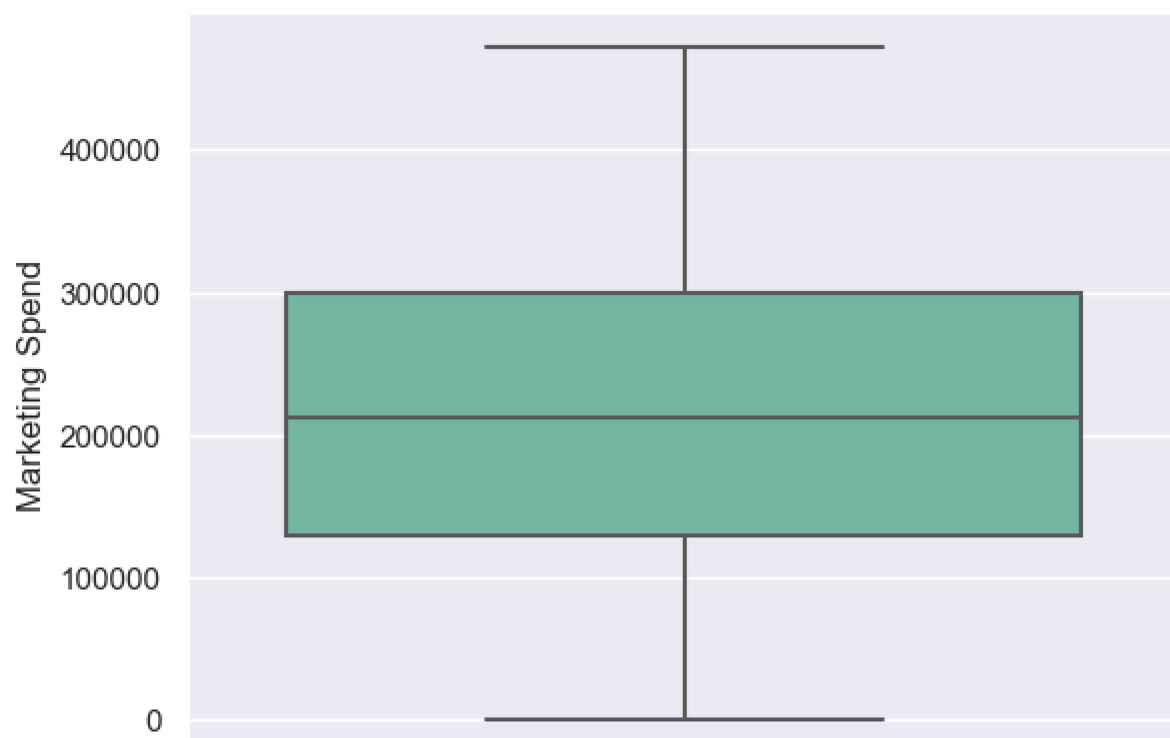






In [15]:
```
def boxplots(df,col):
    sns.boxplot(y = col, data = df, palette ='Set2' )
    plt.show()

for col in outlier_list:
        boxplots(df,col)
```

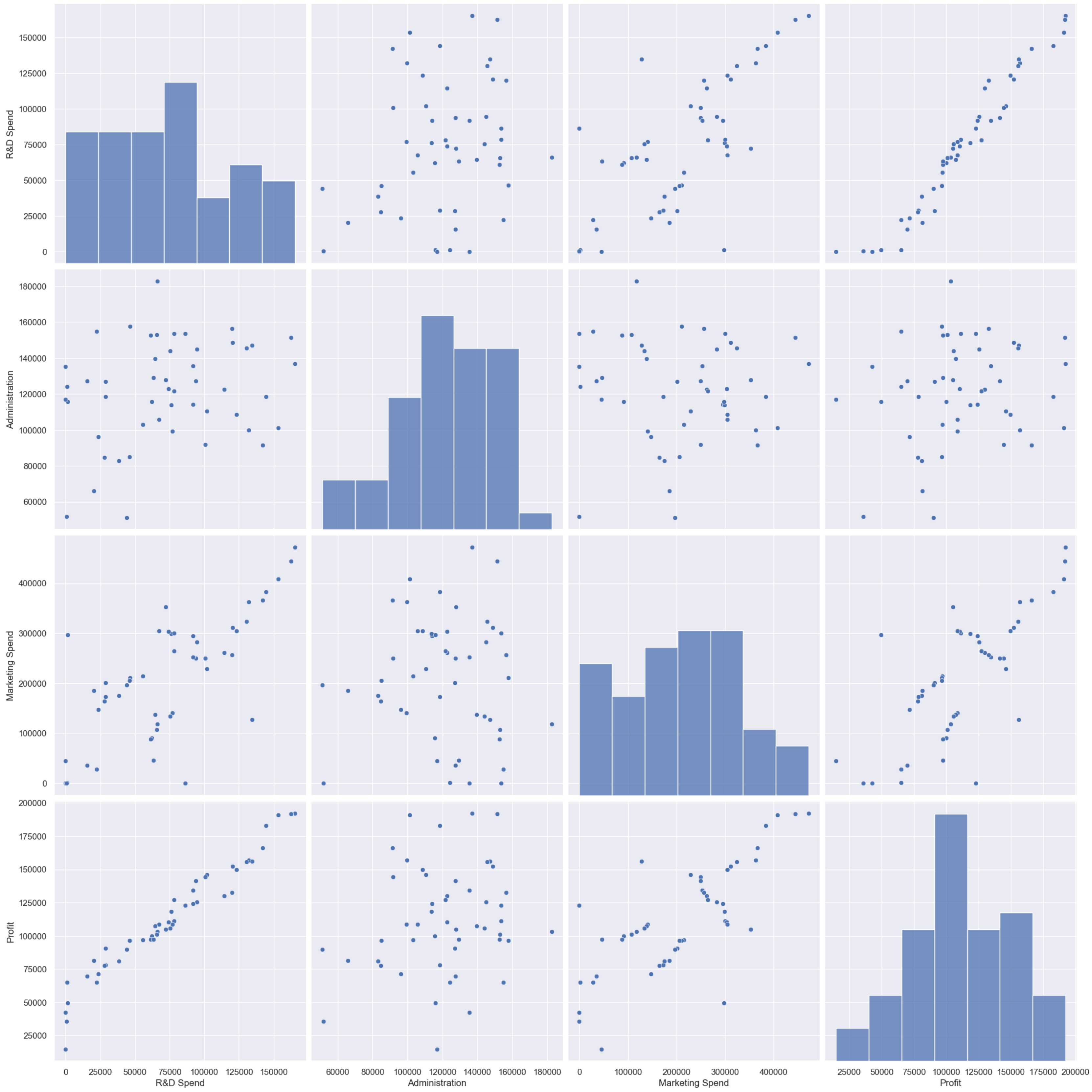




Bivariate Analysis

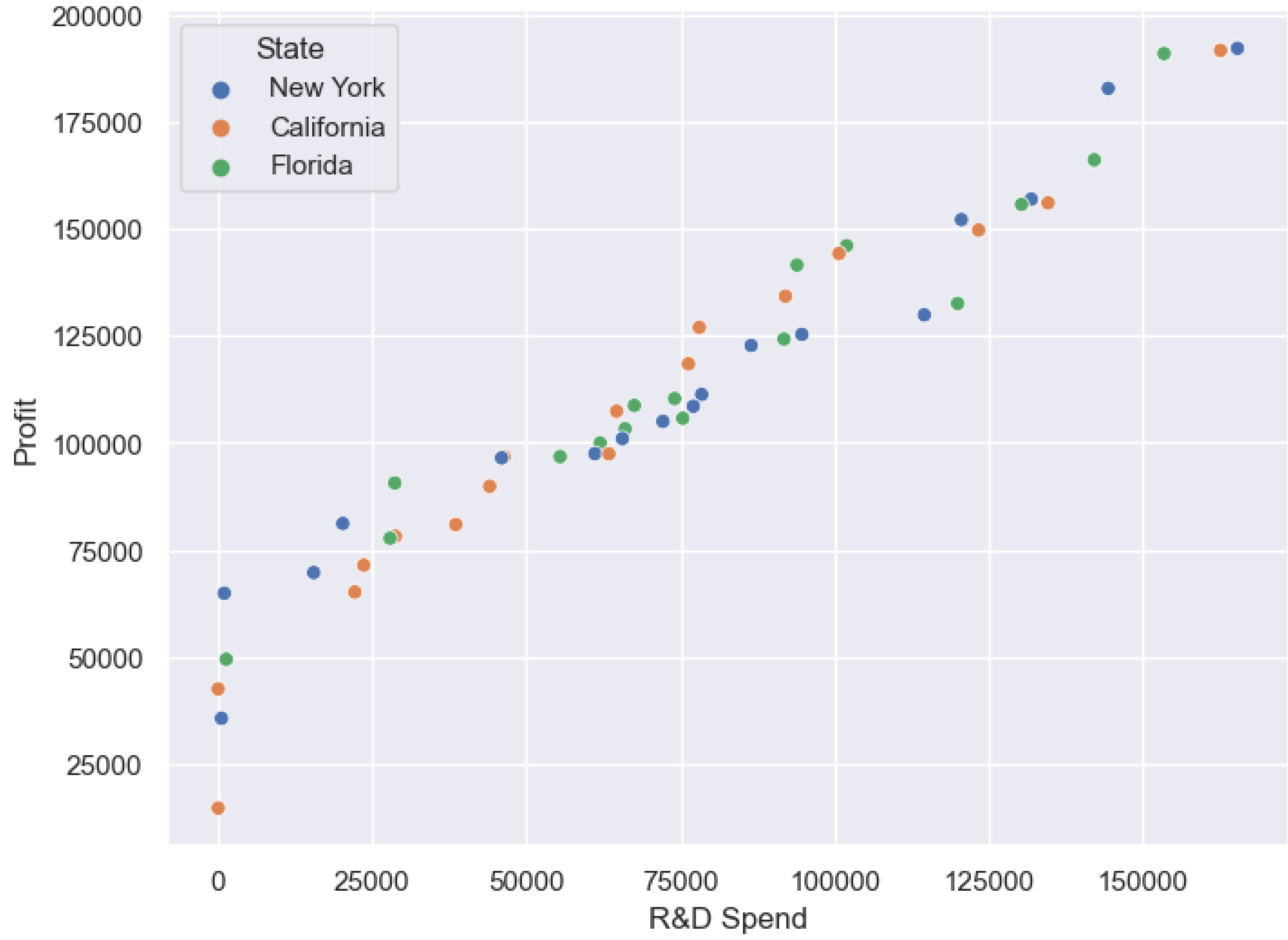
In [16]:
```
sns.pairplot(df, size = 5, kind = 'scatter')
```

Out[16]: <seaborn.axisgrid.PairGrid at 0x1b2b1fa89d0>



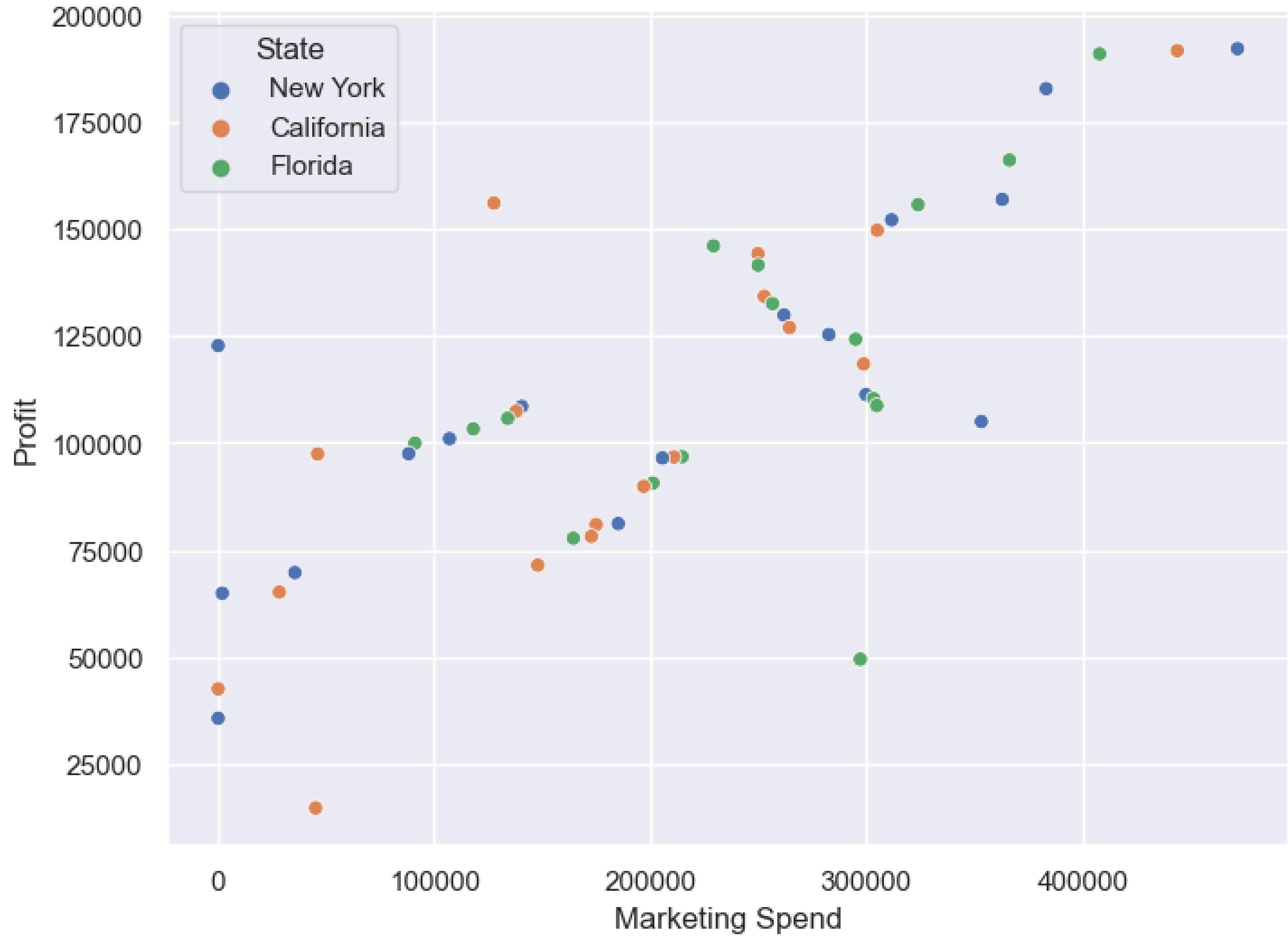
In [17]:
```
plt.figure(figsize = (8,6))
sns.scatterplot(x = df['R&D Spend'],y =df['Profit'],hue=df['State'] )
```

Out[17]: <Axes: xlabel='R&D Spend', ylabel='Profit'>



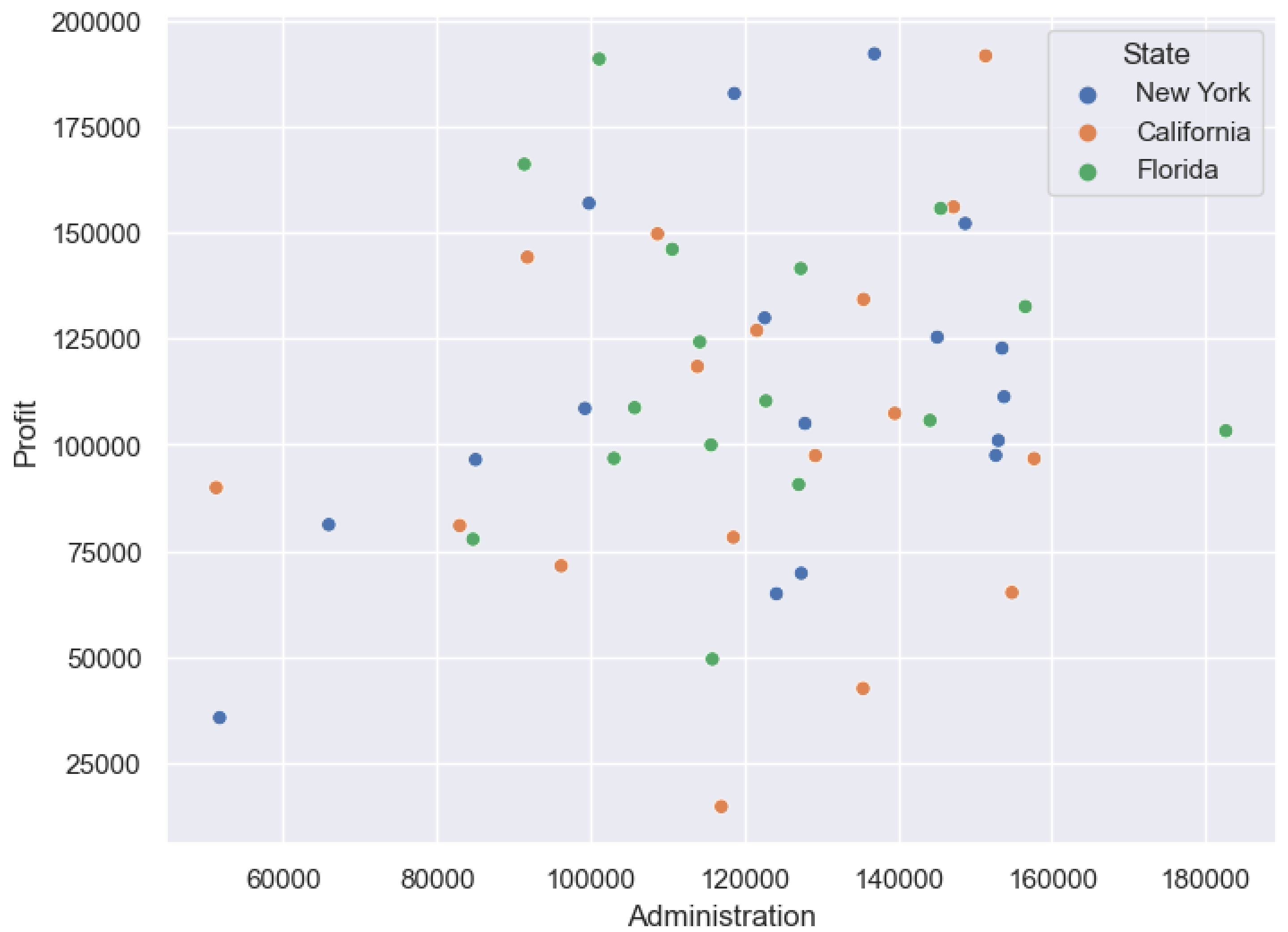
In [18]:
```
plt.figure(figsize = (8,6))
sns.scatterplot(x = df['Marketing Spend'],y =df['Profit'],hue=df['State'] )
```

Out[18]: <Axes: xlabel='Marketing Spend', ylabel='Profit'>



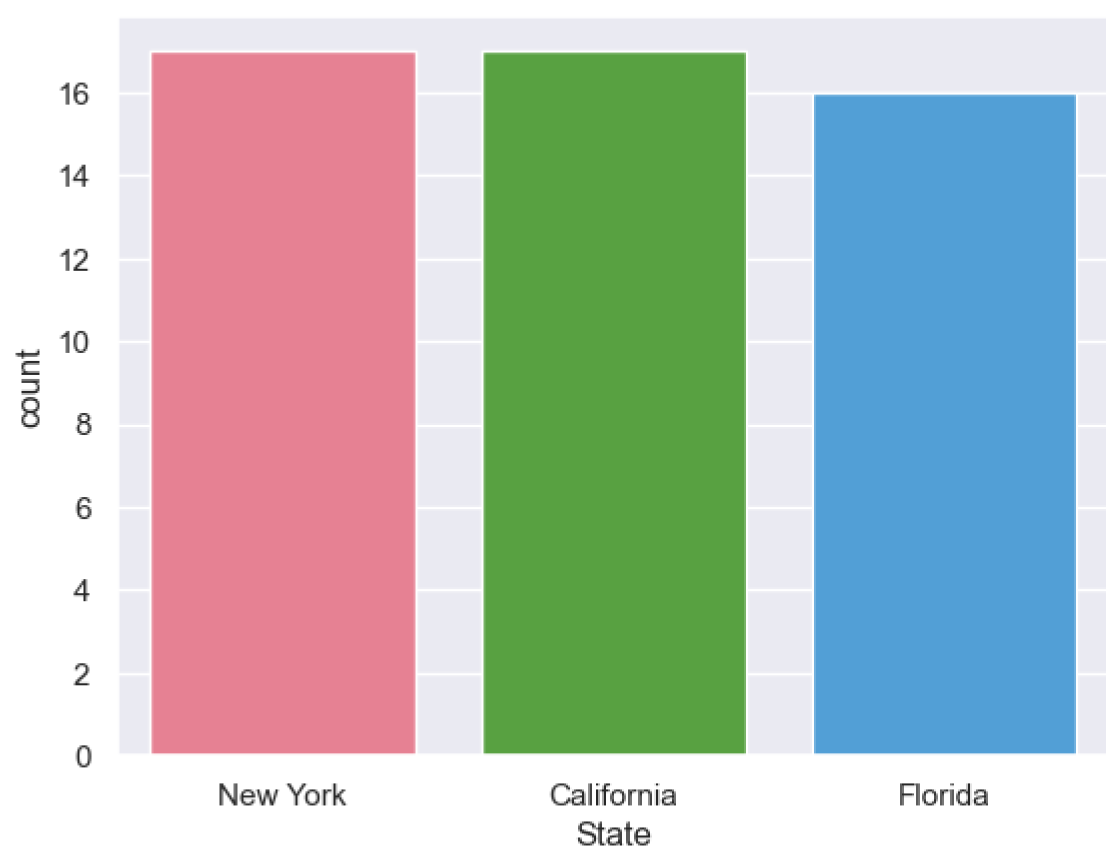
In [19]:
```
plt.figure(figsize = (8,6))
sns.scatterplot(x = df['Administration'],y =df['Profit'],hue=df['State'] )
```

Out[19]: <Axes: xlabel='Administration', ylabel='Profit'>



In [20]:
```
sns.countplot(x = "State", data = df, palette = 'husl', )
```

Out[20]: <Axes: xlabel='State', ylabel='count'>



📈 Preaparation of Data before Training the algorithm

> To train our regression mode, the first step is to split up our data into an target(dependent variables) and features (predictors). And we need to do feature selection only on the predictors or independent variable.

Step-1: Spliting into dependent and independent variable

In [21]:
```
list(df)
```

Out[21]: ['R&D Spend', 'Administration', 'Marketing Spend', 'State', 'Profit']

In [22]:
```
# Assuming 'y' is the column name of the target variable
target = 'Profit'
y = df[[target]]

# Assuming 'X' is the DataFrame containing the feature columns
features = df.drop(target, axis=1)
x = features
```

In [23]:
```
x.head()
```

Out[23]:

	R&D Spend	Administration	Marketing Spend	State
0	165349.20	136897.80	471784.10	New York
1	162597.70	151377.59	443898.53	California
2	153441.51	101145.55	407934.54	Florida
3	144372.41	118671.85	383199.62	New York
4	142107.34	91391.77	366168.42	Florida

In [24]:
```
x.shape
```

Out[24]:
```
(50, 4)
```

In [25]:
```
y.head()
```

Out[25]:

	Profit
0	192261.83
1	191792.06
2	191050.39
3	182901.99
4	166187.94

Step-2: Encoding of categorical feature

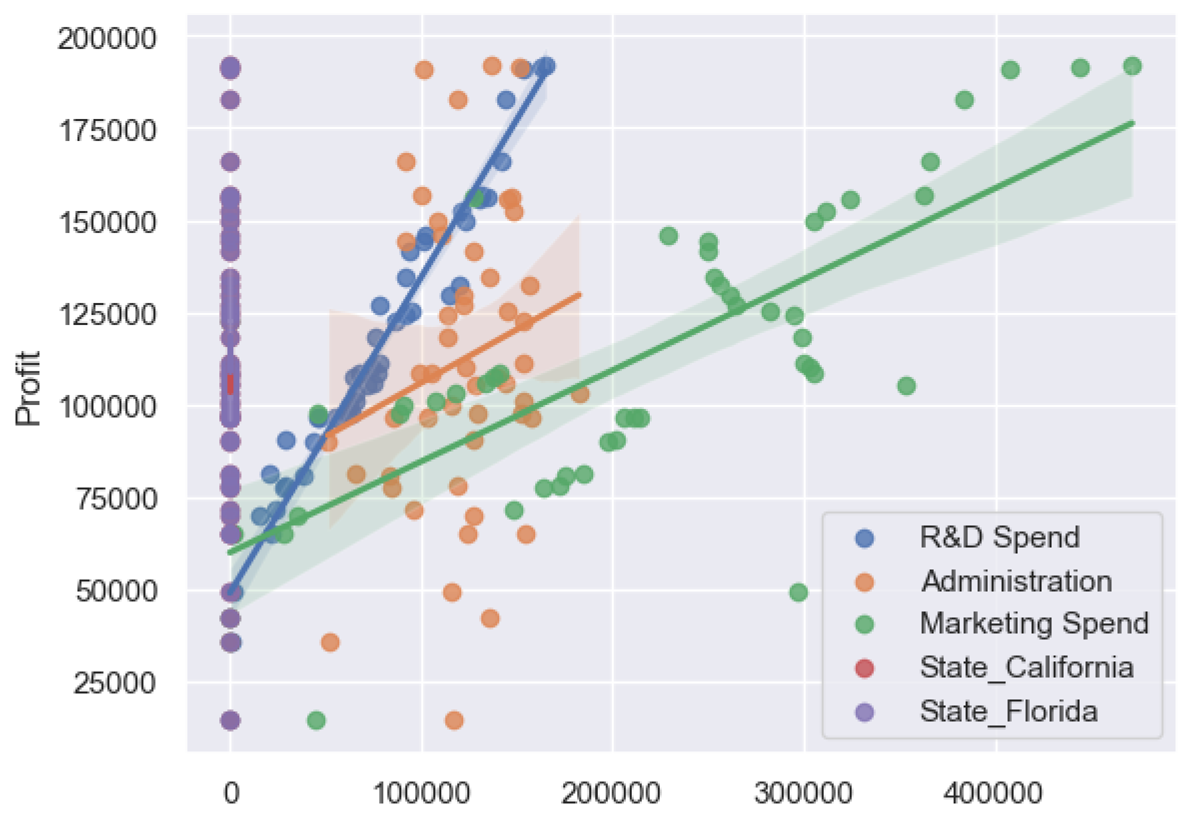
> 'State' feature is object type. In principle, the object type data are treated under encoding after converting the, into category.

In [26]:
```
for col in x.columns:
    if x[col].dtypes == 'object' and x[col].nunique() > 2:
        x[col].astype('category')
        x = pd.get_dummies(x, columns = [col], drop_first = True)

#x = x.drop(['State_New York' ], axis = 1)
x.head()
```

Out[26]:

	R&D Spend	Administration	Marketing Spend	State_California	State_Florida
0	165349.20	136897.80	471784.10	0	0
1	162597.70	151377.59	443898.53	1	0
2	153441.51	101145.55	407934.54	0	1
3	144372.41	118671.85	383199.62	0	0
4	142107.34	91391.77	366168.42	0	1

In [27]:
```
for col in x.columns:
    sns.regplot(x=x[col], y=y, label = col)
    plt.xlabel('')
    plt.legend()
```



step -3: Feature Scaling

- Feature scaling is done only on dependent variables. So we need to split the data into target and dependent variable.

In [28]:
```
sc = StandardScaler()
sc_x = sc.fit_transform(x)
sc_x = pd.DataFrame(sc_x)
```

In [29]:
```
sc_x.head()
```

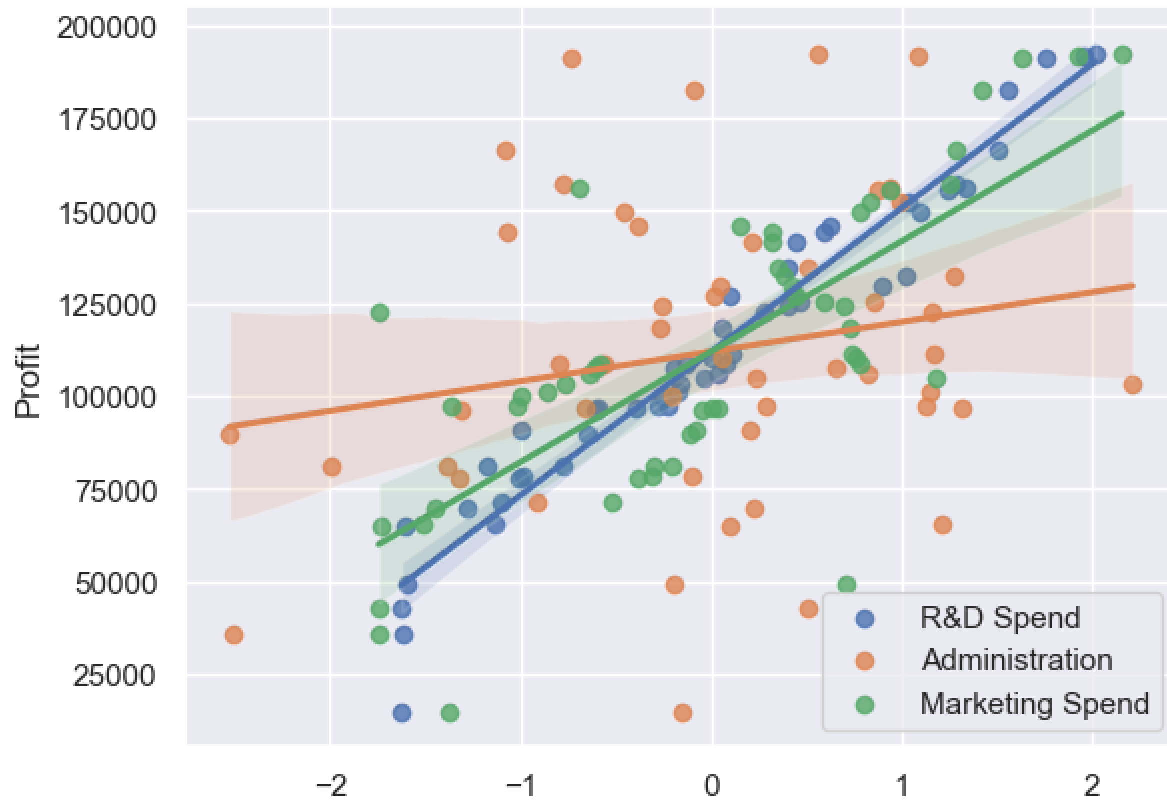
Out[29]:

	0	1	2	3	4
0	2.016411	0.560753	2.153943	-0.717741	-0.685994
1	1.955860	1.082807	1.923600	1.393261	-0.685994
2	1.754364	-0.728257	1.626528	-0.717741	1.457738
3	1.554784	-0.096365	1.422210	-0.717741	-0.685994
4	1.504937	-1.079919	1.281528	-0.717741	1.457738

In [30]:
```
list(x)
```

Out[30]:
```
['R&D Spend',
 'Administration',
 'Marketing Spend',
 'State_California',
 'State_Florida']
```

In [31]:
```
sns.regplot(x=sc_x[0], y=y, label='R&D Spend',)
sns.regplot(x=sc_x[1], y=y, label='Administration',)
sns.regplot(x=sc_x[2], y=y, label='Marketing Spend',)
plt.xlabel('')
plt.legend()
plt.show()
```



step -4: Multi-collinearity check

> There should be no or little mulyicollinearity present for the model building

In [32]:
```
# Finding correlation
plt.figure(figsize=(6,4))
corr = df.corr()
sns.heatmap(corr, annot=True, cmap='Set2')
plt.show()
```

VIF (Multi-collinearity Check)

Multicollinearity refers to a situation in regression analysis where there is a high correlation between two or more predictor variables (also known as independent variables or features). It occurs when the predictor variables in a regression model are highly linearly related to each other, making it difficult to distinguish the individual effects of each variable on the dependent variable.

In the presence of multicollinearity, it becomes challenging to determine the true relationship between the predictor variables and the target variable. This is because multicollinearity can lead to unstable and unreliable estimates of the regression coefficients.

Identifying multicollinearity: Common methods to detect multicollinearity include calculating correlation matrices, variance inflation factors (VIF> 5).

Dealing with multicollinearity: If multicollinearity is detected , several strategies can be employed,

- Dropping one or more correlated variables from the model. (incase more than 1 features have VIF > 5, drop highest and need to perform VIF check again.

In [33]:
```python
from statsmodels.stats.outliers_influence import variance_inflation_factor

variable = sc_x
data = {'VIF' : [variance_inflation_factor(variable, i ) for i in range(len(sc_x.columns))],
        'Features' : x.columns}

vif = pd.DataFrame(data)
```

In [34]:
```python
vif
```

Out[34]:

	VIF	Features
0	2.495511	R&D Spend
1	1.177766	Administration
2	2.416797	Marketing Spend
3	1.335061	State_California
4	1.361299	State_Florida

- for VIF > 5 refers to multicollinearity. For linear regression model building we need no or little multicollinearitywe need to drop the feature. As we don't need multi-collinearity

step-5: Split the data for building the model and prediction

**CASE- I : Test size = 0.3 **

Split the data (70% Train and 30% Test)

In [35]:
```python
# Split the data into training and test for building the model and for prediction
x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=0.3, random_state=121)

print(x_train.shape, x_test.shape, y_train.shape, y_test.shape)
```

```
(35, 5) (15, 5) (35, 1) (15, 1)
```

Approach no - 1 : Multiple LINEAR REGRESSION 📈 📉

Multiple linear equation ;

$$\hat{y} = y + \varepsilon$$

$$\hat{y} = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \ldots + \beta_n \cdot x_n + \varepsilon$$

$$\hat{y} = \beta_0 + \Sigma_1^i \beta_i \cdot x_i + \varepsilon$$

> where β_0 is intercept and β_i's are slopes and ε is error.

- $Response = (Constant + Slope * Predictors) + Error$
- y is the Response, x_i's are the Predictors
- The difference between the actual value and the model's estimate a residual (error).
- These residuals will play a significant role in judging the usefulness of a model.
- If the residuals are small, it implies that the model is a good estimator.

```html
<img src="hyperplane.jpg" alt="" width="400" height="300">
```

Performance Matrix

Here are three common evaluation metrics for regression problems:

> - **Mean Absolute Error** (MAE) is the mean of the absolute value of the errors:
> $$\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

> - **Mean Squared Error** (MSE) is the mean of the squared errors:
> $$\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- **Root Mean Squared Error** (RMSE) is the square root of the mean of the squared errors:

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

📌 Comparing these metrics:

- **MAE** is the easiest to understand, because it's the average error.
- **MSE** is more popular than MAE, because MSE "punishes" larger errors, which tends to be useful in the real world.
- **RMSE** is even more popular than MSE, because RMSE is interpretable in the "y" units.

All of these are **loss functions**, because we want to minimize them.

R^2 Score

$$R^2 = 1 - \frac{SSR}{TSS} = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}$$

- TSS : Total Sum of Square = $\Sigma_i (Actual)_i - (MeanActual)$ = $\sum_i (y_i - \bar{y})^2$
- SSR : Sum of Square residual =$(Residual)^2$ =$(Actual - Predicted)^2$ =$\sum_i (y_i - \hat{y}_i)^2$

Adjusted R^2

$$R^2_{adj} = 1 - \frac{(1-R^2)(N-1)}{(N-p-1)}$$

- N = Total sample size
- p = Number of predictors (No. of Independent variable)

Training

In [36]:
```
# Tain the model with LR model
LR = LinearRegression()
LR.fit(x_train, y_train)
```

Out[36]:
```
LinearRegression()
```

In a Jupyter environment, please rerun this cell to show the HTML representation or trust the notebook.
On GitHub, the HTML representation is unable to render, please try loading this page with nbviewer.org.

In [37]:
```
# find the slopes and intercept from the trained model
intercept =LR.intercept_
slope =LR.coef_

print('The intercept for our linear model is :',intercept.round(3))
print()
print('The coefficients are :',slope.round(3))
```

```
The intercept for our linear model is : [43887.251]

The coefficients are : [[ 8.480000e-01 -1.900000e-02  1.800000e-02  7.453231e+03  3.063963e+03]]
```

In [38]:
```
slope = slope.reshape(5,1)
print(slope.shape, x.columns.shape)
```

```
(5, 1) (5,)
```

In [39]:
```
Lr_data = {'Slope': slope,'Features' : x.columns}
coeff_df = pd.DataFrame(slope, columns=['Slope'], index =x.columns)
coeff_df
```

Out[39]:

	Slope
R&D Spend	0.848180
Administration	-0.019409
Marketing Spend	0.018301
State_California	7453.231161
State_Florida	3063.963122

Prediction of target variable using Linear Regression model

In [40]:
```
# Predict house price by using linear Regression model with test dataset
y_pred_train = LR.predict(x_train)
err_train = y_train - y_pred_train
Train_accuracy = r2_score(y_train, y_pred_train)

y_pred_test = LR.predict(x_test)
err_test = y_test - y_pred_test
Test_accuracy = r2_score(y_test, y_pred_test)
print ('Train accuracy :', Train_accuracy,'\n' 'Test accracy   :', Test_accuracy)
```

```
Train accuracy : 0.9659473642911475
Test accracy   : 0.8990798108840938
```

In [41]:
```
err_train.skew()
```

Out[41]:
```
Profit   -0.306667
dtype: float64
```

In [42]:
```
err_train.kurtosis()
```

Out[42]:
```
Profit    0.42166
dtype: float64
```

According to the histogram the error terms are normally distributed and that is then further confirmed by skewness and kurtosis which are close to zero.

In [43]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 6))

# Plot regression for training data
for i in range(len(x_train.columns)):
    sns.regplot(x=x_train.iloc[:,i ], y=y_pred_train, ax=ax1, )
    ax1.set_title('Training data')
    ax1.set_xlabel('X values')


# Plot regression for test data
for i in range(len(x_test.columns)):
    sns.regplot(x=x_test.iloc[:, i], y=y_pred_test, ax=ax2)
    ax2.set_title('Test data')
    ax2.set_xlabel('X values')

# Adjust spacing between subplots
plt.tight_layout()
# Show the plot
plt.show()
```

In [44]:
```
# mse, rmse, mape
MSE = mean_squared_error(y_test,y_pred_test)
print(f'1. mean squared error (MSE) = ',MSE)
RMSE = np.sqrt(MSE)
print(f'2. root mean squared error (RMSE) = {RMSE}')
MAPE = mean_absolute_percentage_error(y_test,y_pred_test)
print(f'3. mean absolute percentage error (MAPE) = {MAPE}')
ACC = 100-MAPE*100
print(f'4. accuracy of the model = {ACC}')
```

```
1. mean squared error (MSE) =  194052164.52818343
2. root mean squared error (RMSE) = 13930.260748750665
3. mean absolute percentage error (MAPE) = 0.25548939871234455
4. accuracy of the model = 74.45106012876555
```

Approach no 2 : Ordinary Least Square (OLS) Method

In [46]:
```
from statsmodels.regression.linear_model import OLS
import statsmodels.regression.linear_model as smf
```

In [47]:
```
reg_model = smf.OLS(endog = y_train, exog=x_train).fit()   # with unscaled data
reg_model.summary()
```

Out[47]:

OLS Regression Results

Dep. Variable:	Profit	**R-squared (uncentered):**	0.993
Model:	OLS	**Adj. R-squared (uncentered):**	0.992
Method:	Least Squares	**F-statistic:**	854.0
Date:	Mon, 17 Jul 2023	**Prob (F-statistic):**	2.20e-31
Time:	22:16:10	**Log-Likelihood:**	-373.19
No. Observations:	35	**AIC:**	756.4
Df Residuals:	30	**BIC:**	764.2
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
R&D Spend	0.8442	0.065	12.900	0.000	0.711	0.978
Administration	0.2484	0.037	6.718	0.000	0.173	0.324
Marketing Spend	0.0429	0.022	1.932	0.063	-0.002	0.088
State_California	1.253e+04	4782.658	2.620	0.014	2761.040	2.23e+04
State_Florida	8205.2066	4459.135	1.840	0.076	-901.563	1.73e+04

Omnibus:	0.833	**Durbin-Watson:**	2.004
Prob(Omnibus):	0.659	**Jarque-Bera (JB):**	0.362
Skew:	-0.245	**Prob(JB):**	0.834
Kurtosis:	3.093	**Cond. No.**	8.71e+05

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 8.71e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

In [48]:
```
reg_model = smf.OLS(endog = y_test, exog=x_test).fit()   # with unscaled data
reg_model.summary()
```

Out[48]:

OLS Regression Results

Dep. Variable:	Profit	**R-squared (uncentered):**	0.983
Model:	OLS	**Adj. R-squared (uncentered):**	0.975
Method:	Least Squares	**F-statistic:**	115.8
Date:	Mon, 17 Jul 2023	**Prob (F-statistic):**	1.62e-08
Time:	22:16:10	**Log-Likelihood:**	-164.38
No. Observations:	15	**AIC:**	338.8
Df Residuals:	10	**BIC:**	342.3
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
R&D Spend	0.5366	0.155	3.464	0.006	0.191	0.882
Administration	0.4017	0.081	4.948	0.001	0.221	0.583
Marketing Spend	0.1404	0.054	2.602	0.026	0.020	0.261
State_California	-5149.8625	9453.054	-0.545	0.598	-2.62e+04	1.59e+04
State_Florida	-4237.5460	1.47e+04	-0.288	0.779	-3.7e+04	2.86e+04

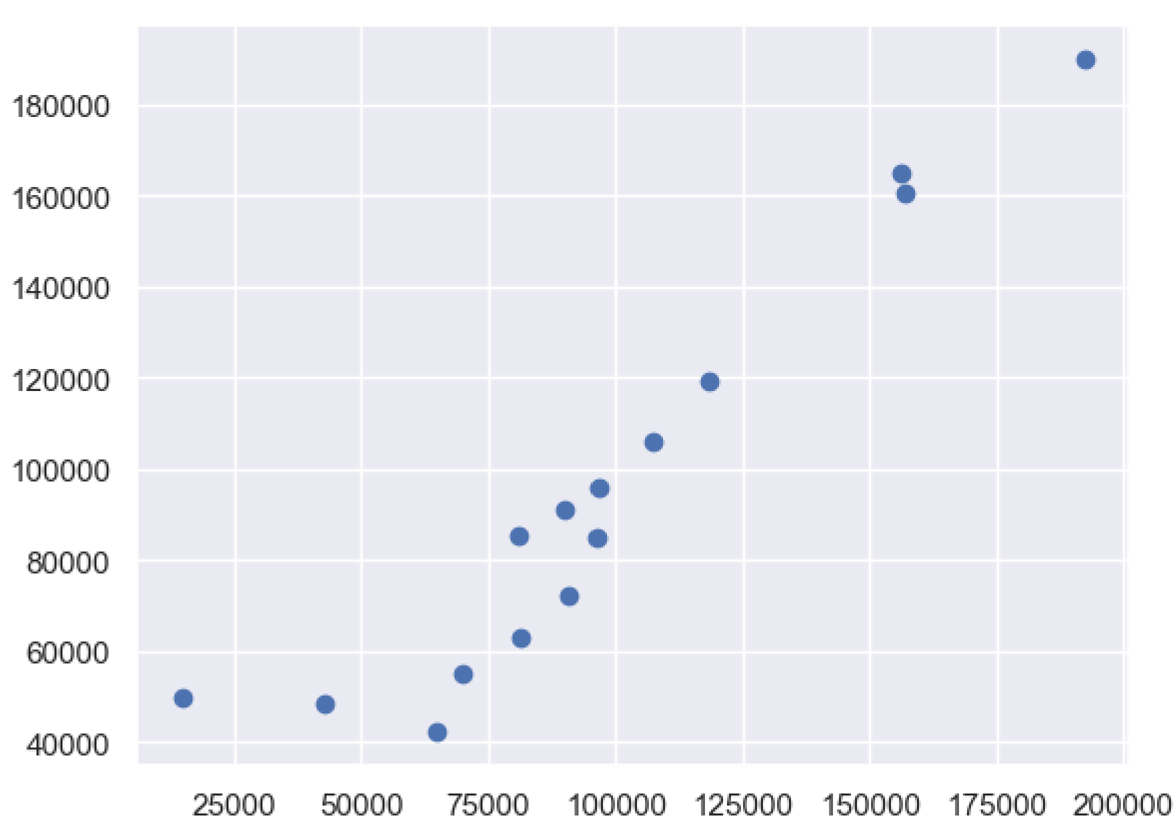
Omnibus:	4.375	**Durbin-Watson:**	2.350
Prob(Omnibus):	0.112	**Jarque-Bera (JB):**	2.026
Skew:	-0.840	**Prob(JB):**	0.363
Kurtosis:	3.645	**Cond. No.**	8.55e+05

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 8.55e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

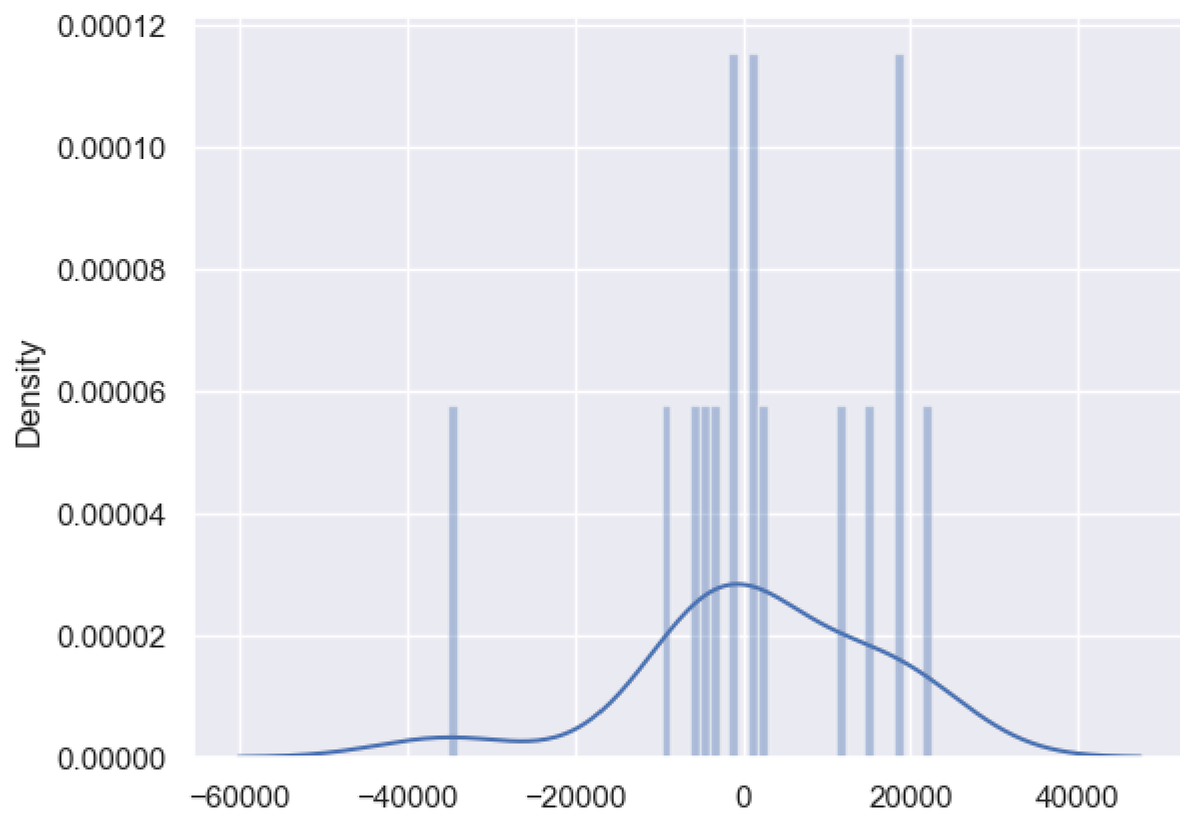
- Endoginity Problem : if dependent variable is set wrongly that is endoginity problem

In [49]:
```
# Check linearity

plt.scatter(y_test, y_pred_test)
```

Out[49]: <matplotlib.collections.PathCollection at 0x1b2b7bb4d30>



In [50]:
```
# Normality of Residual

sns.distplot((y_test - y_pred_test), bins=50)
plt.show()
```



Assumption of LR Model is satisfied

1) Linearity - Satisfied

2) Normality of Residuals- Satisfied

3) Homoscedasticity - Satisfied (there is no outlier and residual is normaly distributed)

4) No autocorrelation - Satisfied

5) No or little Multicollinearity - satisfied

6) No endogenity problem - satisfied

In order to create less complex model when you have a large number of features in your dataset, some of the Regularization techniques used to address over-fitting and feature selection are:

Lasso (L1 reguralization):

> Mathematically, it consists of a linear model trained with ℓ_1 prior as regularizer. The objective function to minimize is:

$$\arg\min_{\beta_o,\beta_j} \sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij})^2 + \lambda \sum_{j=1}^{p} |\beta_j|$$

> The lasso estimate thus solves the minimization of the least-squares penalty. λ controls the strength of reguralization

In [51]:
```
lasso = Lasso(alpha=0.1)
lasso.fit(x_train, y_train)
print("Lasso Model :", (lasso.coef_))
y_pred_train_lasso = lasso.predict(x_train)
y_pred_test_lasso = lasso.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_lasso))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_lasso))
```

```
Lasso Model : [ 8.48177890e-01 -1.94094101e-02  1.83023655e-02  7.45221340e+03
  3.06302820e+03]
Training Accuracy : 0.9659473641429505

Test Accuracy : 0.8990841735485041
```

Ridge :

> Ridge regression addresses some of the problems of **Ordinary Least Squares** by imposing a penalty on the size of coefficients. The ridge coefficients minimize a penalized residual sum of squares,

$$\arg\min_{\beta_o,\beta_j} \sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda \sum_{j=1}^{p}\beta_j^2$$

> $\lambda >= 0$ is a complexity parameter that controls the amount of shrinkage: the larger the value of λ, the greater the amount of shrinkage and thus the coefficients become more robust to collinearity.

> Ridge regression is an L2 penalized model. Add the squared sum of the weights to the least-squares cost function.

In [52]:
```
# Part 2 : Ridge Regression (L2- Regularization)
# closure to zero but not exact zero
# penalty - 0.3

ridge = Ridge(alpha=0.3)
ridge.fit(x_train, y_train)
print("Ridge Model :", (ridge.coef_))
y_pred_train_ridge = ridge.predict(x_train)
y_pred_test_ridge = ridge.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_ridge))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_ridge))
```

```
Ridge Model : [[ 8.47223070e-01 -1.96370335e-02  1.86669800e-02  6.96695621e+03
   2.72415779e+03]]
Training Accuracy : 0.9659193037668692

Test Accuracy : 0.9011734894553887
```

ElasticNet

The first term represents the residual sum of squares, the second term is the L1 regularization term (Lasso), and the third term is the L2 regularization term (Ridge). The Elastic Net regularization combines both L1 and L2 regularization, allowing for a balance between feature selection and coefficient shrinkage. The parameters λ_1 and λ_2 control the strengths of the respective regularization terms.

$$\arg\min_{\beta_o,\beta_j} \sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda_1 \sum_{j=1}^{p}|\beta_j| + \lambda_2 \sum_{j=1}^{p}\beta_j^2$$

In [53]:
```
elastic = ElasticNet(alpha=0.3, l1_ratio=0.1)
elastic.fit(x_train, y_train)
y_pred_train_elastic = elastic.predict(x_train)
y_pred_test_elastic = elastic.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_elastic))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_elastic))
```

```
Training Accuracy : 0.9632279233912138

Test Accuracy : 0.9179513210700663
```

Gradient Descent

> Gradient Descent is a very generic optimization algorithm capable of finding optimal solutions to a wide range of problems. The general idea of Gradient Sescent is to tweak parameters iteratively in order to minimize a cost function. Gradient Descent measures the local gradient of the error function with regards to the parameters vector, and it goes in the direction of descending gradient. Once the gradient is zero, you have reached a minimum.

In [74]:
```
from sklearn.model_selection import train_test_split

x_train, x_test, y_train, y_test = train_test_split(sc_x, y, test_size=0.25, random_state=101)
print('x_train shape :',x_train.shape,'\n''x_test shape :', x_test.shape,'\n' 'y_train shape :',y_train.shape
print()
gd = SGDRegressor()
gd.fit(x_train, y_train)
y_pred_gd_train = gd.predict(x_train)
y_pred_gd_test = gd.predict(x_test)
Train_accuracy_gd =r2_score(y_train, y_pred_gd_train)
Test_accuracy_gd = r2_score(y_test, y_pred_gd_test)

print("GD Trainging Accuracy :", Train_accuracy_gd  )
print("GD Test Accuracy      :",Test_accuracy_gd  )
```

```
x_train shape : (37, 5)
x_test shape : (13, 5)
y_train shape : (37, 1)
y_test shape : (13, 1)

GD Trainging Accuracy : 0.9472665189054107
GD Test Accuracy      : 0.9409293485798228
```

In [55]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 6))

# Plot regression for training data
for i in range(len(x_train.columns)):
    sns.regplot(x=x_train.iloc[:,i ], y=y_pred_gd_train, ax=ax1, )
    ax1.set_title('Training data Gradient Descent')
    ax1.set_xlabel('X values')


# Plot regression for test data
for i in range(len(x_test.columns)):
    sns.regplot(x=x_test.iloc[:, i], y=y_pred_gd_test, ax=ax2)
    ax2.set_title('Test data for Gradient Descent')
    ax2.set_xlabel('X values')

# Adjust spacing between subplots
plt.tight_layout()
# Show the plot
plt.show()
```

Polynomial regression

In [56]:
```
poly = PolynomialFeatures()
x_train_trans = poly.fit_transform(x_train)
x_test_trans = poly.fit_transform(x_test)
lr = LinearRegression()
lr.fit(x_train_trans, y_train)
poly.fit(x_train, y_train)
y_pred_poly_train = lr.predict(x_train_trans)
y_pred_poly_test = lr.predict(x_test_trans)
Train_accuracy_poly =r2_score(y_train, y_pred_poly_train)
Test_accuracy_poly = r2_score(y_test, y_pred_poly_test)

print("Polynomial Regression Trainging Accuracy :", Train_accuracy_poly  )
print("Polynomial Regression Test Accuracy      :",Test_accuracy_poly )
```

```
Polynomial Regression Trainging Accuracy : 0.9643335626660127
Polynomial Regression Test Accuracy      : 0.8670425426922718
```

In [57]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 6))

# Plot regression for training data
for i in range(len(x_train.columns)):
    sns.regplot(x=x_train.iloc[:,i ], y=y_pred_poly_train, ax=ax1, )
    ax1.set_title('Training data')
    ax1.set_xlabel('X values')


# Plot regression for test data
for i in range(len(x_test.columns)):
    sns.regplot(x=x_test.iloc[:, i], y=y_pred_poly_test, ax=ax2)
    ax2.set_title('Test data with Polynomial regression')
    ax2.set_xlabel('X values')

# Adjust spacing between subplots
plt.tight_layout()
# Show the plot
plt.show()
```

**CASE- I : Test size = 0.2 **

Split the data (80% Train and 20% Test)

In [58]:
```
#Split the data into training and test for building the model and for prediction
x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=0.2, random_state=121)

print('x_train shape :',x_train.shape,'\n''x_test shape :', x_test.shape,'\n' 'y_train shape :',y_train.shape

y_pred_train = LR.predict(x_train)
y_pred_test = LR.predict(x_test)


# Validate the actual price of the test data and predicted price
Test_accuracy = r2_score(y_test, y_pred_test)
Train_accuracy = r2_score(y_train, y_pred_train)

print ('Train accuracy :', Train_accuracy,'\n' 'Test accracy   :', Test_accuracy)


# MSE, RMSE, MAPE
MSE = mean_squared_error(y_test,y_pred_test)
print(f'1. mean squared error (MSE) = ',MSE)
RMSE = np.sqrt(MSE)
print(f'2. root mean squared error (RMSE) = {RMSE}')
MAPE = mean_absolute_percentage_error(y_test,y_pred_test)
print(f'3. mean absolute percentage error (MAPE) = {MAPE}')
ACC = 100-MAPE*100
print(f'4. accuracy of the model = {ACC}')
```

```
x_train shape : (40, 5)
x_test shape : (10, 5)
y_train shape : (40, 1)
y_test shape : (10, 1)
Train accuracy : 0.9328286436777007
Test accracy   : 0.9830641471002174
1. mean squared error (MSE) =  29104086.61012637
2. root mean squared error (RMSE) = 5394.820350125328
3. mean absolute percentage error (MAPE) = 0.04542249274560596
4. accuracy of the model = 95.45775072543941
```

In [59]:
```
reg_model = smf.OLS(endog = y_train, exog=x_train).fit()   # with unscaled data
reg_model.summary()
```

Out[59]:

OLS Regression Results

Dep. Variable:	Profit	**R-squared (uncentered):**	0.987
Model:	OLS	**Adj. R-squared (uncentered):**	0.985
Method:	Least Squares	**F-statistic:**	524.2
Date:	Mon, 17 Jul 2023	**Prob (F-statistic):**	7.50e-32
Time:	22:16:15	**Log-Likelihood:**	-437.43
No. Observations:	40	**AIC:**	884.9
Df Residuals:	35	**BIC:**	893.3
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
R&D Spend	0.7605	0.078	9.792	0.000	0.603	0.918
Administration	0.3080	0.041	7.539	0.000	0.225	0.391
Marketing Spend	0.0633	0.028	2.287	0.028	0.007	0.119
State_California	4037.2964	5818.962	0.694	0.492	-7775.823	1.59e+04
State_Florida	3888.8627	5545.837	0.701	0.488	-7369.785	1.51e+04

Omnibus:	0.002	**Durbin-Watson:**	1.949
Prob(Omnibus):	0.999	**Jarque-Bera (JB):**	0.108
Skew:	-0.007	**Prob(JB):**	0.948
Kurtosis:	2.746	**Cond. No.**	8.33e+05

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 8.33e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

In [ ]:
```
# Check linearity

plt.scatter(y_test, y_pred_test)
```

In [ ]:
```
# Normality of Residual

sns.distplot((y_test - y_pred_test), bins=50)
plt.show()
```

1) Linearity - Satisfied

2) Normality of Residuals- Satisfied

3) Homoscedasticity - Satisfied (there is no outlier and residual is normaly distributed, no variance in residual)

4) No autocorrelation - Satisfied

5) No or little Multicollinearity - satisfied

6) No endogenity problem - satisfied

P-values and coefficients in regression analysis work together to tell which relationships in the model are statistically significant and the nature of those relationships.

- The coefficients describe the mathematical relationship between each independent variable and the dependent variable.
- The p-values for the coefficients indicate whether these relationships are statistically significant.

In [60]:
```
reg_model = smf.OLS(endog = y_test, exog=x_test).fit()   # with unscaled data
reg_model.summary()
```

Out[60]:

OLS Regression Results

Dep. Variable:	Profit	**R-squared (uncentered):**	0.995
Model:	OLS	**Adj. R-squared (uncentered):**	0.991
Method:	Least Squares	**F-statistic:**	210.7
Date:	Mon, 17 Jul 2023	**Prob (F-statistic):**	8.29e-06
Time:	22:16:15	**Log-Likelihood:**	-104.46
No. Observations:	10	**AIC:**	218.9
Df Residuals:	5	**BIC:**	220.4
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
R&D Spend	0.6219	0.137	4.543	0.006	0.270	0.974
Administration	0.2576	0.112	2.293	0.070	-0.031	0.547
Marketing Spend	0.1323	0.041	3.203	0.024	0.026	0.238
State_California	1.27e+04	9393.275	1.352	0.234	-1.14e+04	3.68e+04
State_Florida	7324.7987	1.41e+04	0.519	0.626	-2.9e+04	4.36e+04

Omnibus:	1.822	**Durbin-Watson:**	2.054
Prob(Omnibus):	0.402	**Jarque-Bera (JB):**	0.717
Skew:	0.652	**Prob(JB):**	0.699
Kurtosis:	2.855	**Cond. No.**	1.14e+06

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 1.14e+06. This might indicate that there are
strong multicollinearity or other numerical problems.

Reguralization

LASSO

In [61]:
```
lasso = Lasso(alpha=0.1)
lasso.fit(x_train, y_train)
print("Lasso Model :", (lasso.coef_))
y_pred_train_lasso = lasso.predict(x_train)
y_pred_test_lasso = lasso.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_lasso))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_lasso))
```

```
Lasso Model : [ 8.14155288e-01 -3.00339115e-02  2.51026746e-02  1.49832609e+02
  3.03746274e+02]
Training Accuracy : 0.9394005680177581

Test Accuracy : 0.9916556048386072
```

In [ ]:

In [ ]:

Ridge

In [62]:
```
# Part 2 : Ridge Regression (L2- Regularization)
# closure to zero but not exact zero
# penalty - 0.3

ridge = Ridge(alpha=0.3)
ridge.fit(x_train, y_train)
print("Ridge Model :", (ridge.coef_))
y_pred_train_ridge = ridge.predict(x_train)
y_pred_test_ridge = ridge.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_ridge))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_ridge))
```

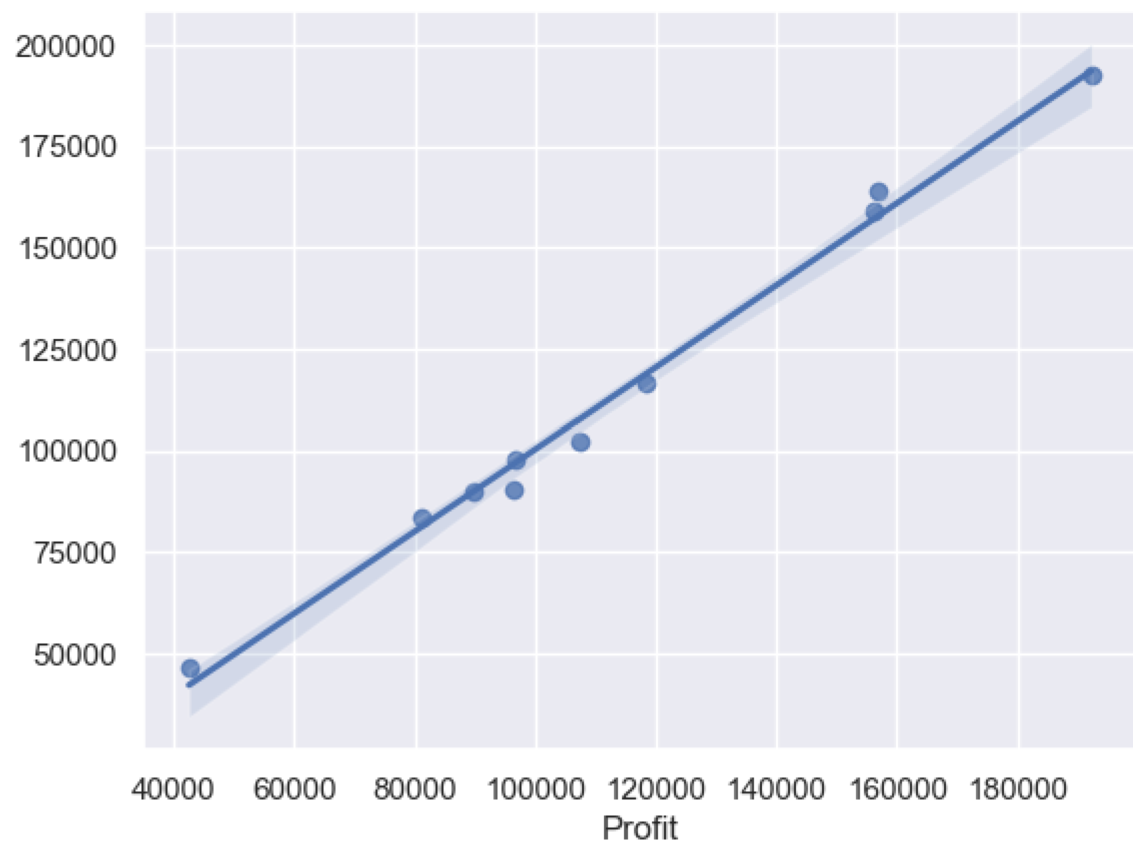
```
Ridge Model : [[ 8.14135029e-01 -3.00295914e-02  2.51212276e-02  1.37464222e+02
   2.88599737e+02]]
Training Accuracy : 0.9394005370541024

Test Accuracy : 0.9916566167602463
```

In [ ]:

In [ ]:

In [63]:
```
sns.regplot(x=y_test, y=y_pred_test_ridge)
```

Out[63]: <Axes: xlabel='Profit'>



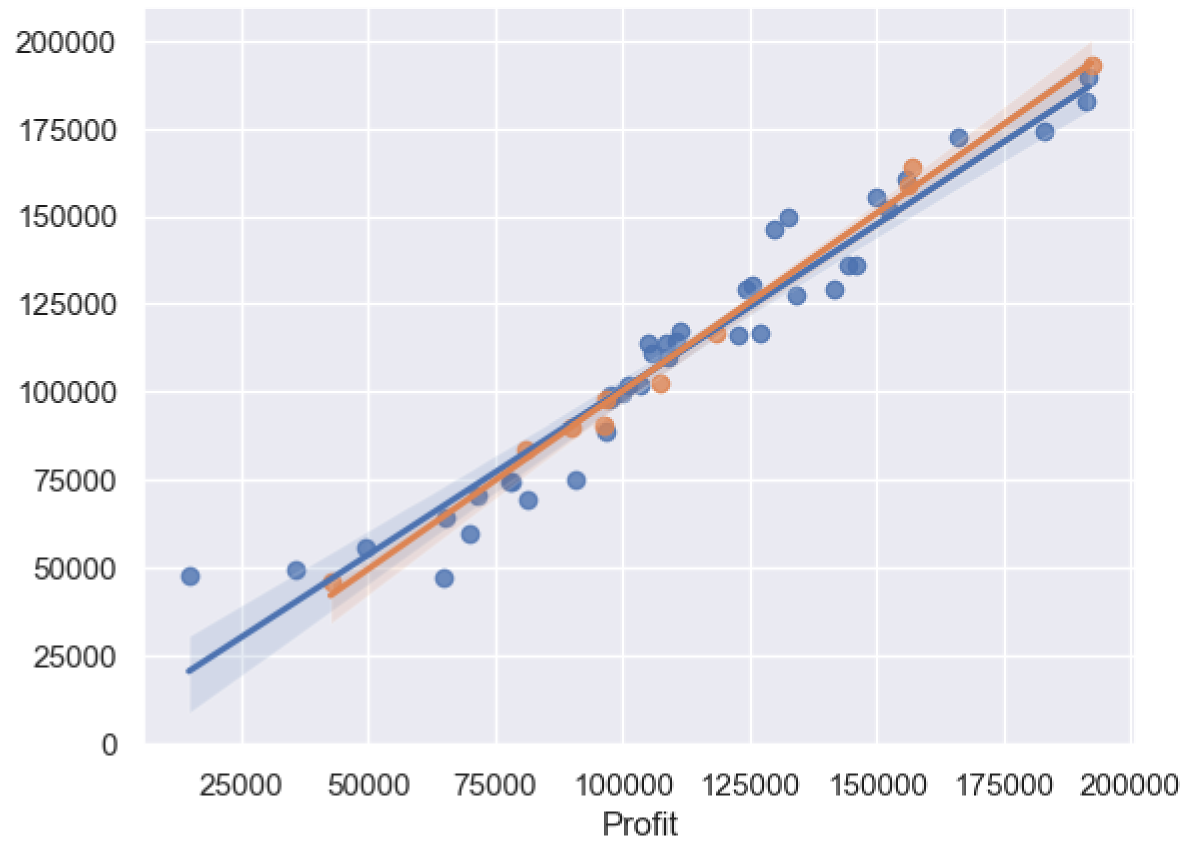
Elastic Net

In [64]:
```
elastic = ElasticNet(alpha=0.3, l1_ratio=0.1)
elastic.fit(x_train, y_train)
y_pred_train_elastic = elastic.predict(x_train)
y_pred_test_elastic = elastic.predict(x_test)
print("Training Accuracy :", r2_score(y_train, y_pred_train_elastic))
print()
print("Test Accuracy :", r2_score(y_test, y_pred_test_elastic))
```

```
Training Accuracy : 0.9393963701399531

Test Accuracy : 0.9916606769091509
```

In [65]:
```
sns.regplot(x=y_train, y=y_pred_train_elastic)
sns.regplot(x=y_test, y=y_pred_test_elastic)
```

Out[65]: <Axes: xlabel='Profit'>



In [ ]:

In [ ]:

Gradient Descent

- Always scaled data is used.

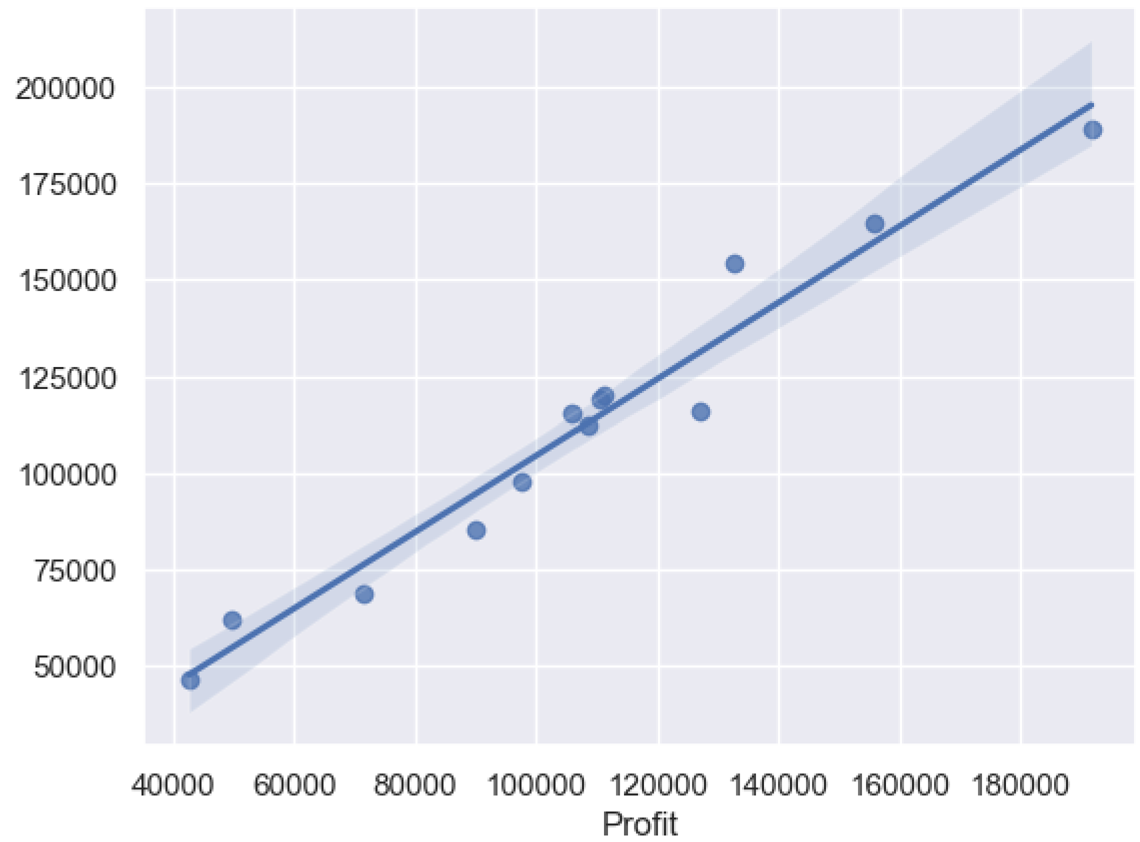
In [66]:
```
from sklearn.model_selection import train_test_split

x_train, x_test, y_train, y_test = train_test_split(sc_x, y, test_size=0.25, random_state=101)
print('x_train shape :',x_train.shape,'\n''x_test shape :', x_test.shape,'\n' 'y_train shape :',y_train.shape
print()
gd = SGDRegressor()
gd.fit(x_train, y_train)
y_pred_gd_train = gd.predict(x_train)
y_pred_gd_test = gd.predict(x_test)
Train_accuracy_gd =r2_score(y_train, y_pred_gd_train)
Test_accuracy_gd = r2_score(y_test, y_pred_gd_test)

print("GD Trainging Accuracy :", Train_accuracy_gd  )
print("GD Test Accuracy      :",Test_accuracy_gd  )
```

```
x_train shape : (37, 5)
x_test shape : (13, 5)
y_train shape : (37, 1)
y_test shape : (13, 1)

GD Trainging Accuracy : 0.9473415978433063
GD Test Accuracy      : 0.9418237075644532
```

The Gradient Descent is a best fit

In [67]:
```
sns.regplot(x=y_test, y=y_pred_gd_test)
```

Out[67]: <Axes: xlabel='Profit'>



Ploynomial Regression

In [68]:
```
poly = PolynomialFeatures()
x_train_trans = poly.fit_transform(x_train)
x_test_trans = poly.fit_transform(x_test)
lr = LinearRegression()
lr.fit(x_train_trans, y_train)
poly.fit(x_train, y_train)
y_pred_poly_train = lr.predict(x_train_trans)
y_pred_poly_test = lr.predict(x_test_trans)
Train_accuracy_poly =r2_score(y_train, y_pred_poly_train)
Test_accuracy_poly = r2_score(y_test, y_pred_poly_test)

print("Polynomial Regression Trainging Accuracy :", Train_accuracy_poly  )
print("Polynomial Regression Test Accuracy      :",Test_accuracy_poly )
```

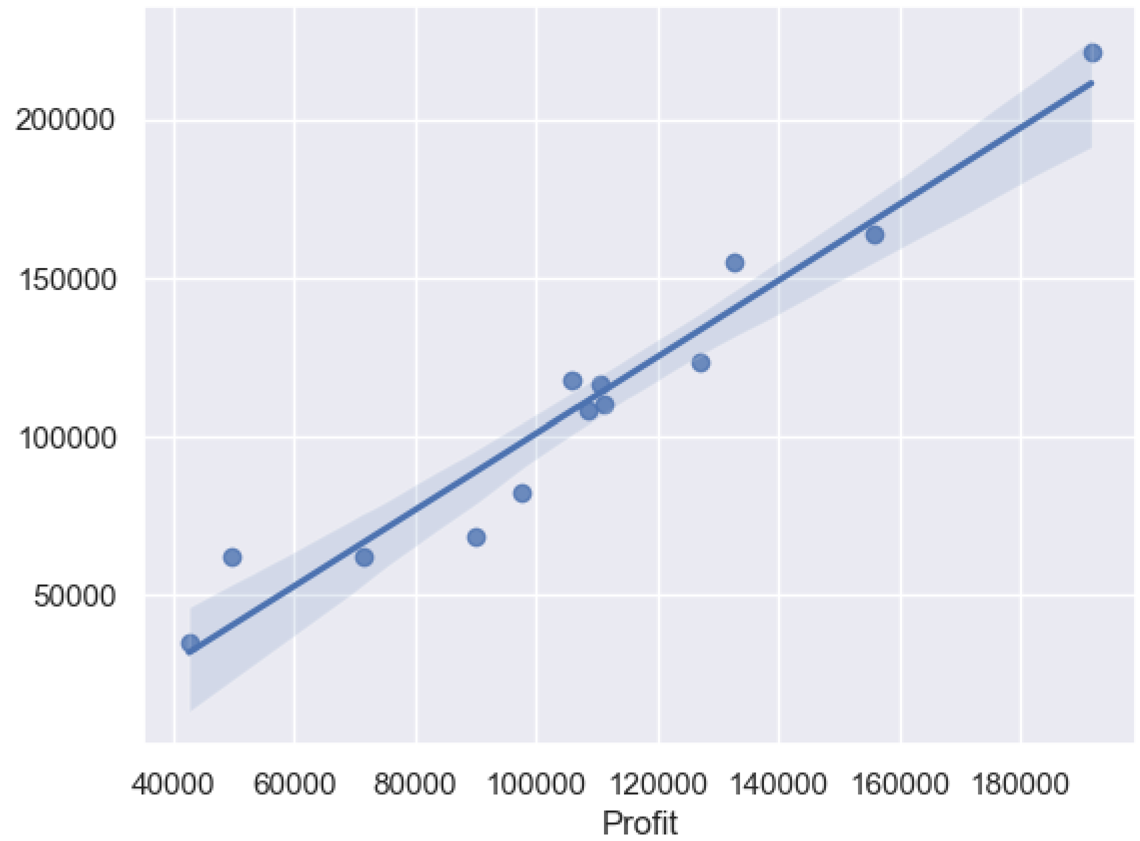
```
Polynomial Regression Trainging Accuracy : 0.9643335626660127
Polynomial Regression Test Accuracy      : 0.8670425426922718
```

In [ ]:

In [ ]:

In [69]:
```
sns.regplot(x=y_test, y=y_pred_poly_test)
```

Out[69]: <Axes: xlabel='Profit'>



Summary of the Model Building

- Test accuracy_MLR (test sample 20%) > Test accuracy_MLR (test sample 30%)
- Train accuracy_MLR (test sample 20%) < Train accuracy_MLR (test sample 30%)
- Reguralization method enhanced the accuracy of both Train and Test

In [93]:
```
row_label=['MLR', 'OLS','Lasso', 'Ridge','ElasticNet', 'Gradient Descent','PolynomialRegression','MLR','OLS'
accuracy_data ={'Test_size' : [0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.2,0.2,0.2,0.2,0.2,0.2,0.2],
               'R2_Train_Accuracy' : [0.965,0.993,0.965,0.96,0.963,0.947,0.964,0.932,0.987,0.939,0.939,0.939
               'R2_Test_Accuracy' :   [0.899,0.983,0.899,0.90,0.917,0.940,0.867,0.983,0.995,0.991,0.991,0.991
accuracy_df =pd.DataFrame(accuracy_data, index =row_label)
accuracy_df
```

Out[93]:

	Test_size	R2_Train_Accuracy	R2_Test_Accuracy
MLR	0.3	0.965	0.899
OLS	0.3	0.993	0.983
Lasso	0.3	0.965	0.899
Ridge	0.3	0.960	0.900
ElasticNet	0.3	0.963	0.917
Gradient Descent	0.3	0.947	0.940
PolynomialRegression	0.3	0.964	0.867
MLR	0.2	0.932	0.983
OLS	0.2	0.987	0.995
Lasso	0.2	0.939	0.991
Ridge	0.2	0.939	0.991
ElasticNet	0.2	0.939	0.991
GradientDescent	0.2	0.947	0.941
Polynomial regression	0.2	0.964	0.867

In [98]:
```
accuracy_df['variance']= abs(accuracy_df['R2_Train_Accuracy'] - accuracy_df['R2_Test_Accuracy'])*100
accuracy_df
```

Out[98]:

	Test_size	R2_Train_Accuracy	R2_Test_Accuracy	variance
MLR	0.3	0.965	0.899	6.6
OLS	0.3	0.993	0.983	1.0
Lasso	0.3	0.965	0.899	6.6
Ridge	0.3	0.960	0.900	6.0
ElasticNet	0.3	0.963	0.917	4.6
Gradient Descent	0.3	0.947	0.940	0.7
PolynomialRegression	0.3	0.964	0.867	9.7
MLR	0.2	0.932	0.983	5.1
OLS	0.2	0.987	0.995	0.8
Lasso	0.2	0.939	0.991	5.2
Ridge	0.2	0.939	0.991	5.2
ElasticNet	0.2	0.939	0.991	5.2
GradientDescent	0.2	0.947	0.941	0.6
Polynomial regression	0.2	0.964	0.867	9.7

Observation of the Model Building based on test sample size (0.2, 0.3)

1. Polynomial Regression is not a suitable model as the variance is quite high, it overfits the data. Which infers there is no non linearity between target and predictors.

2. The variance is minimum in case of Gradient descent.

3. The accuracy is getting better by reducing sample size.

4. In case of smaller sample size, the reguralization is not essential. MLR model is a best fit model for test sample size= 0.2

🙏 Thank You 🙏

In [ ]: