



# Math for Data Science & Regression



# Pre-requisites

Hope you have gone through the self-learning content for this session on the PRISM portal.



# By the End of this Session, You Will:

## Core Concepts

- Understanding foundational mathematical concepts such as calculus, linear algebra, probability, and statistics.
- Familiarity with mathematical notations, symbols, and terminology commonly used in data science.

## Analytical Skills Development

- Ability to analyze and interpret data using mathematical techniques.
- Proficiency in applying mathematical models and algorithms to solve data-driven problems.
- Ability to perform hypothesis testing and statistical inference for decision-making.
- Aptitude for visualizing and presenting data using mathematical techniques and tools.



# Recap

# Poll Time

Q. A vector quantity has which of the following properties?

- a. Magnitude and direction
- b. Magnitude only
- c. Direction only
- d. Neither magnitude nor direction



# Poll Time

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- a. Magnitude and direction**
- b. Magnitude only
- c. Direction only
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# Linear Algebra

# Introduction to Linear Algebra

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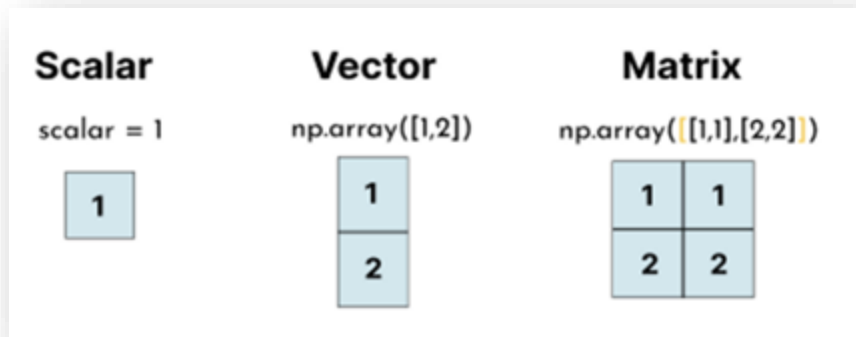
- Linear algebra is a foundational tool for machine learning. For example, linear regression is a machine learning algorithm that uses linear algebra to find the best-fit line for a set of data points.
- Linear algebra concepts can be used to represent data in data science. Data in machine learning is often represented as vectors or matrices.
- Linear algebra concepts can be used to manipulate and transform this data in ways that are useful for machine learning algorithms.
- Linear algebra is used to solve linear equation systems, linear regression. Linear algebra is a powerful tool for solving linear equations.

The building blocks of Linear algebra are:

1. **Scalar:** A single number
2. **Vector:** A One-dimensional array of numbers
3. **Matrix:** A two-dimensional array of numbers



# Building Blocks of Linear Algebra



## Using Numpy:

```
scalar = 1  
vector = np.array([1,2])  
matrix = np.array([[1,1],[2,2]])
```



```
print('vector shape:', vector.shape)  
print('matrix shape:', matrix.shape)
```



```
vector shape: (2,)  
matrix shape: (2, 2)
```

# Pop Quiz

Q. What is the result of adding two scalars?

- a. A Scalar
- b. A Vector
- c. A Matrix
- d. None of the above



# Pop Quiz

Q. What is the result of adding two scalars?

- a. **A Scalar**
- b. A Vector
- c. A Matrix
- d. None of the above





# Vectors and Vector Spaces, Matrix Algebra

# Matrix Multiplication

- Multiplication of two matrices is only defined if the number of columns in the first matrix matches the number of rows in the second matrix.
- Matrix Multiplication can be computed using the dot() function in Python.

## Mathematics:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 & 4 \\ 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 9 & 9 \\ 18 & 18 & 18 \\ 27 & 27 & 27 \end{bmatrix}$$

$1 \times 4 + 1 \times 5 = 9$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 & 4 \\ 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 9 & 9 \\ 18 & 18 & 18 \\ 27 & 27 & 27 \end{bmatrix}$$

$1 \times 4 + 1 \times 5 = 9$

...

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 & 4 \\ 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 9 & 9 \\ 18 & 18 & 18 \\ 27 & 27 & 27 \end{bmatrix}$$

$3 \times 4 + 3 \times 5 = 27$

## Python:

```
# dot product
matrix1 = np.array([[1,1],
                    [2,2],
                    [3,3]])
matrix2 = np.array([[4,4,4],
                    [5,5,5]])

print('matrix 1, matrix 2 dot product\n', matrix1.dot(matrix2), '\n')
print('matrix 2, matrix 1 dot product\n', matrix2.dot(matrix1), '\n')
```

matrix 1, matrix 2 dot product  
[[ 9 9 9]  
[18 18 18]  
[27 27 27]]

matrix 2, matrix 1 dot product  
[[24 24]  
[30 30]]

# Case Study: Product Recommendation

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An online retail company wants to provide personalized product recommendations to its customers based on their preferences. The company has collected data on customer ratings for different product categories. The customer ratings range from 1 (low preference) to 5 (high preference) for each product category.

**Let's consider the following scenario:**

Customer A has rated the product categories as follows:

Shoes: 4  
T-shirts: 3  
Trousers: 5

Customer B has rated the product categories as follows:

Shoes: 5  
T-shirts: 2  
Trousers :4

The company wants to calculate the similarity score between the preferences of Customer A and Customer B to determine their level of similarity in product preferences.

# Example

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The similarity score will be calculated using the dot product of the vectors representing the customer ratings.

Customer A's Ratings Vector: [4, 3, 5]

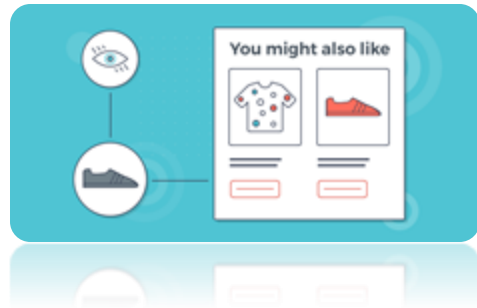
Customer B's Ratings Vector: [5, 2, 4]

To calculate the similarity score between the preferences of Customer A and Customer B, we can use the cosine similarity measure.

Next, we calculate the dot product of the two vectors, which is the sum of the element-wise multiplication of the corresponding components.

Dot product =  $(4 * 5) + (3 * 2) + (5 * 4) = 20 + 6 + 20 = 46$

Then, we calculate the magnitude (Euclidean norm) of each vector, which is the square root of the sum of the squares of the components.



## Example

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Magnitude of Customer A's preferences vector =  $\sqrt{(4^2) + (3^2) + (5^2)} = \sqrt{16 + 9 + 25} = \sqrt{50} \approx 7.071$

Magnitude of Customer B's preferences vector =  $\sqrt{(5^2) + (2^2) + (4^2)} = \sqrt{25 + 4 + 16} = \sqrt{45} \approx 6.708$

Finally, we calculate the cosine similarity score by dividing the dot product by the product of the magnitudes.

Cosine Similarity = Dot product / (Magnitude of Customer A's preferences vector \* Magnitude of Customer B's preferences vector) =  $46 / (7.071 * 6.708) \approx 0.927$

The similarity score between the preferences of Customer A and Customer B, based on the cosine similarity measure, is approximately 0.927. This indicates a relatively high level of similarity in their product preferences.

Note: The cosine similarity ranges from -1 to 1, where 1 indicates identical preferences, 0 indicates no similarity, and -1 indicates opposite preferences. In this case, the value of 0.927 indicates a strong positive similarity.

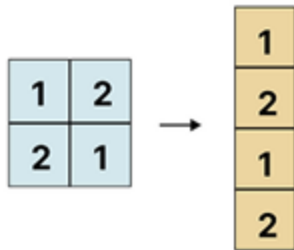




# Reshape

- A vector is often seen as a matrix with one column and it can be reshaped into a matrix by specifying the number of columns and rows using *reshape()*.

## Mathematics:



## Python:

```
# reshape
vector = np.array([1,2,2,1])
print('reshaped vector\n', vector.reshape(2,2), '\n')

matrix = np.array([[1,2],[2,1]])
print('reshaped matrix\n', matrix.reshape(4,1))
```

reshaped vector

```
[[1 2]
 [2 1]]
```

reshaped matrix

```
[[1]
 [2]
 [2]
 [1]]
```

# Pop Quiz

Q. A car rental agency charges a flat rate of \$40 per day and an additional \$0.25 per mile driven. Write the equation in slope-intercept form ( $y = mx + b$ ) to represent the total cost ( $y$ ) as a function of the number of miles driven ( $x$ ).

- a.  $y = 0.25x$
- b.  $y = 60x + 0.25$
- c.  $y = 40x + 0.25$
- d.  $y = 0.25x + 40$



# Pop Quiz

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- b.  $y = 60x + 0.25$
- c.  $y = 40x + 0.25$
- d.  $y = 0.25x + 40$**



# Transpose

- Transpose swaps the rows and columns of the matrix, so that an  $j \times k$  matrix becomes  $k \times j$ . To transpose a matrix, you use *matrix.T*.

## Mathematics:

1	1
2	2
3	3

 → 

1	2	3
1	2	3

## Python:

```
# transpose
matrix = np.array([[1,1],
                  [2,2],
                  [3,3]])
print('transposed matrix\n', matrix.T, '\n')
```

transposed matrix  
[[1 2 3]  
[1 2 3]]

# Pop Quiz

Q. A company wants to analyze the sales data of its products. The sales data is stored in a matrix with 10 rows and 5 columns. The company wants to use the transpose of the matrix to analyze the data. Which of the following is the correct use of the transpose in this case?

- a. The transpose will change the shape of the matrix from 10 rows and 5 columns to 5 rows and 10 columns. This will make it easier for the company to analyze the data by grouping the products together.
- b. The transpose will swap the rows and columns of the matrix. This will make it easier for the company to analyze the data by grouping the sales data for each product together.
- c. The transpose will not change the shape of the matrix. This is because the transpose only swaps the rows and columns of the matrix, it does not change the number of rows or columns.



# Pop Quiz

Q. A company wants to analyze the sales data of its products. The sales data is stored in a matrix with 10 rows and 5 columns. The company wants to use the transpose of the matrix to analyze the data. Which of the following is the correct use of the transpose in this case?

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- c. The transpose will not change the shape of the matrix. This is because the transpose only swaps the rows and columns of the matrix, it does not change the number of rows or columns.



# Identity and Inverse Matrix

- To create a 3 x 3 identity matrix in Python, you can use `numpy.identity(3)`.

```
1 # identity matrix
  I3 = np.identity(3)
  print('3x3 identity matrix\n', I3, '\n')
```

3x3 identity matrix  
[[1. 0. 0.]  
 [0. 1. 0.]  
 [0. 0. 1.]]

- The dot product of the matrix itself (stated as  $M$  below) and the inverse of the matrix is the identity matrix which follows the equation

$$MM^{-1} = M^{-1}M = I_n$$

- There are two things to take into consideration with matrix inverse:
  - 1) The order of the matrix and matrix inverse does not matter even though most matrix dot products are different when the order changes;
  - 2) Not all matrices have an inverse.



# Identity and Inverse Matrix

---

To compute inverse of the matrix, you can use `np.linalg.inv()`.

```
# inverse of the matrix
matrix = np.array([[1,1,1],
                   [0,0,2],
                   [2,0,3]])
print('inverse of the matrix\n', np.linalg.inv(matrix), '\n')
```

```
inverse of the matrix
[[ 0.   -0.75  0.5 ]
 [ 1.    0.25 -0.5 ]
 [ 0.    0.5   0.  ]]
```

# Linear Equation Used Case

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Suppose you need to solve the given equation:

$$3a + 2b = 7$$

$$a - b = 1$$

Let's use dot product to solve for a and b:

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

Let's us represent the coefficient matrix as  $M$ , variable vector as  $x$  and output vector  $y$

$$M \cdot x = y$$

$$M^{-1} \cdot M \cdot x = M^{-1} \cdot y$$

$$I_n \cdot x = M^{-1} \cdot y$$

$$x = M^{-1} \cdot y$$

# Pop Quiz

Q. Which of the following is the correct use of an identity matrix?

- a. To represent the origin in vector space
- b. To simplify matrix multiplication
- c. To solve systems of linear equations
- d. All of the listed



# Pop Quiz

Q. Which of the following is the correct use of an identity matrix?

- a. To represent the origin in vector space
- b. To simplify matrix multiplication
- c. To solve systems of linear equations

**d. All of the listed**





# Activity 1

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Try calculating the value of a and b using the given formula:

```
np.linalg.inv(M).dot(y)
```

# Summary

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- ✓ Introduced participants to the fundamental concepts and principles of linear algebra.
- ✓ Covered various operations on vectors, including addition, subtraction, scalar multiplication, and dot product.
- ✓ Discussed matrix representation, properties, and operations such as addition, subtraction, and scalar multiplication.
- ✓ Explored matrix algebra, including matrix multiplication, inverse, and transpose operations.
- ✓ Covered concepts such as determinants, rank, and solving systems of linear equations using matrices.

# THANK YOU!

Please complete your assessments and review the self-learning content for this session on the **PRISM** portal.







# Linear Transformations, Eigen Values and Eigen Vectors



# Pre-requisites

Hope you have gone through the self-learning content for this session on the PRISM portal.



# By the End of this Session, You Will:

- Understand the concept of linear transformations and their applications in various fields.
- Apply the concepts of eigenvalues and eigenvectors in practical problems and real-world scenarios.
- Develop a foundational understanding of principal component analysis (PCA) and its role in data analysis and dimensionality reduction.
- Apply PCA techniques to analyze and interpret data, identifying key patterns and reducing data complexity.

# What have we learned so far?

- **Vectors are used to represent data points in space. This is useful for tasks such as:**

1. Finding the distance between two points
2. Calculating the slope of a line
3. Determining the area of a triangle

- **Matrices are used to represent data sets. This is useful for tasks such as:**

1. Storing data in a compact form
2. Performing calculations on data sets
3. Representing relationships between data points

- **Linear transformations are used to transform data sets. This is useful for tasks such as:**

1. Scaling

- **Vector spaces are the foundation of linear algebra, and they are used in many different areas of data science. This is because linear algebra provides a powerful set of tools for representing, manipulating, and analyzing data.**

# Pop Quiz

Q. What are vectors used for in data science?

- a. To represent data points in space
- b. To represent data sets
- c. To perform calculations on data sets
- d. All of the above



# Pop Quiz

Q. What are vectors used for in data science?

- a. To represent data points in space
- b. To represent data sets
- c. To perform calculations on data sets
- d. All of the above**



# Linear Transformation

---

A linear transformation is a mathematical function that maps vectors from one space to another.

It preserves two essential properties:

- **Additively:**

A linear transformation preserves vector addition. If two vectors,  $u$  and  $v$ , are mapped to vectors  $Tu$  and  $Tv$  respectively, then the transformation of their sum is equal to the sum of the individual transformations:

$$T(u + v) = Tu + Tv$$

- **Scalar Multiplication:**

A linear transformation also preserves scalar multiplication. If a scalar  $c$  is multiplied by a vector  $u$  and mapped to the vector  $Tu$ , then the transformation of the scaled vector is equal to the scaled transformation:

$$T(cu) = c(Tu)$$

# Properties of Linear Transformation

## 1. Linearity:

A linear transformation preserves addition and scalar multiplication.

Consider a linear transformation that scales the input vector by a constant factor and adds a constant value.

For example, let's say we have a linear transformation  $T$  that represents a conversion from Celsius to Fahrenheit temperature.

Given two Celsius temperatures  $u = 20$  and  $v = 10$ , applying the transformation would yield  $T(u) = 68$  and  $T(v) = 50$ . The linearity property ensures that the sum of the transformed temperatures  $T(u + v)$  would be equal to the sum of their individual transformations, i.e.,  $T(20 + 10) = T(30) = 86$ , which corresponds to the Fahrenheit equivalent of the sum of the original temperatures.



# Properties of Linear Transformation

## 2. Homogeneity:

Scaling the input vector scales the output vector accordingly.

Let's consider a linear transformation  $T$  that doubles the values of a given dataset representing the number of hours spent studying for an exam.

If a student initially studied for 5 hours ( $u = 5$ ), the transformed value  $T(u)$  would be 10.

Homogeneity ensures that scaling the input vector by a scalar, such as multiplying the number of hours by 2, would proportionally scale the transformed output as well.

Therefore, if the student now studies for 10 hours ( $cu = 2 * 5 = 10$ ), the transformed value  $T(cu)$  would also be 20.

# Properties of Linear Transformation

## 3. Additivity:

The transformation behaves additively when two vectors are added.

Suppose we have a linear transformation  $T$  that represents a data preprocessing step of summing two variables, such as total revenue from online sales and total revenue from in-store sales.

If the original values are  $u = \$1000$  and  $v = \$500$ , applying the transformation would yield  $T(u) = \$1000$  and  $T(v) = \$500$ .

The additivity property ensures that the transformed sum  $T(u + v)$  would be equal to the sum of their individual transformations, i.e.,  $T(\$1000 + \$500) = T(\$1500) = \$1500$ .

# Pop Quiz

Q. Linearity in linear transformations implies:

- a. The transformation always produces a straight line graph.
- b. The transformation preserves addition and scalar multiplication.
- c. The transformation only applies to linear data sets.
- d. The transformation is irreversible.



# Pop Quiz

Q. Linearity in linear transformations implies:

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# Matrix Representation of Linear Transformation

- **Unique Matrix Representation:**

Each linear transformation has a unique matrix representation that can be used to apply the transformation to vectors.

- **Matrix-Vector Multiplication:**

Matrix-vector multiplication is a way of applying a linear transformation to a vector by multiplying the vector by the transformation's matrix representation.

- **Transformation Matrix and Input Vector:**

The transformation matrix contains the coefficients that define how the linear transformation operates on the input vector. The input vector contains the values of the input vector.

- **Output Vector:**

The product of the transformation matrix and the input vector produces the output vector. Each element of the output vector represents the result of applying the linear transformation to the corresponding element of the input vector.

# Visualizing Linear Transformation

- Suppose that we want to find the  $2 \times 2$  matrix that describes rotation of the diver by 90 degrees in the counterclockwise direction. Consider first the line connecting  $(0,1)$  to  $(0,-1)$  (Figure 1).

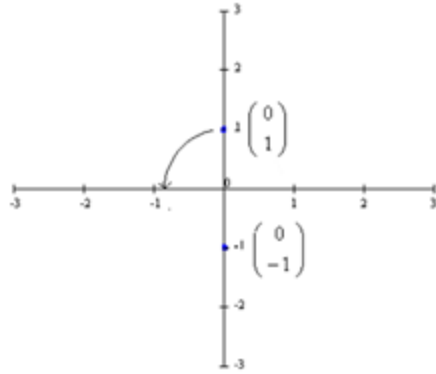


Figure 1

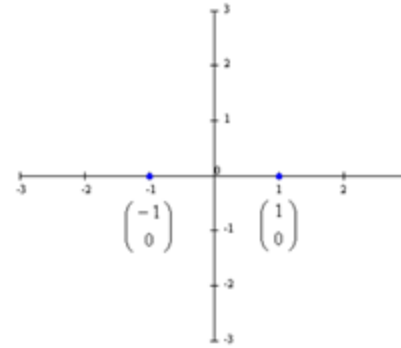


Figure 2

- After rotating this line by 90 degrees in the counterclockwise direction (about the point ) we should get the new line connecting  $(-1,0)$  to  $(1,0)$  (Figure 2).

# Pop Quiz

Q. The matrix representation of a linear transformation:

- a. Captures all possible transformations
- b. Is unique for each linear transformation
- c. Is only applicable to square matrices
- d. Requires non-linear operations



# Pop Quiz

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# Eigen Values

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## Eigenvalues:

- Eigenvalues are scalars that correspond to eigenvectors.
- Eigenvalues represent the scaling factors by which certain vectors are stretched or compressed during a linear transformation.
- The number of eigenvalues of a matrix is equal to the dimension of the matrix.
- The eigenvalues of a matrix can be found by solving the characteristic equation of the matrix.

# Eigen Vectors

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## **Eigenvectors:**

- Eigenvectors are vectors that are only scaled by their corresponding eigenvalues when they undergo a linear transformation.
- Eigenvectors are not rotated or transformed in any other way.
- The direction of an eigenvector is preserved by the linear transformation.
- The magnitude of an eigenvector may be scaled by the linear transformation.


# Relationship

Matrix we are finding the  
eigenvector/eigenvalue of

eigenvalue


$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$



identity matrix

# Eigen Decomposition of a Matrix

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det(A - \lambda I) = 0$$

$$\det \left( \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = 0$$

$$\det \left( \begin{bmatrix} 1 - \lambda & 4 \\ 3 & 2 - \lambda \end{bmatrix} \right) = 0$$

$$(1 - \lambda)(2 - \lambda) - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 2) = 0$$

$$\lambda = 5, -2$$

# Pop Quiz

Q. Eigen decomposition of a matrix  $A$  involves:

- a. Finding the eigenvalues and eigenvectors of  $A$
- b. Breaking down  $A$  into a product of eigenvalues and eigenvectors
- c. Rearranging the elements of  $A$  in a specific order
- d. None of the above



# Pop Quiz

Q. Eigen decomposition of a matrix  $A$  involves:

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- d. None of the above



# Orthogonality of Eigen Vectors

## Theorem:

Let  $A$  be a real symmetric matrix, and let  $\mathbf{v}_1$  and  $\mathbf{v}_2$  be eigenvectors of  $A$  corresponding to distinct eigenvalues  $\lambda_1$  and  $\lambda_2$ , respectively. Then,  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are orthogonal, i.e.,  $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$

## Proof:

Let's consider  $\mathbf{v}_1$  and  $\mathbf{v}_2$  as eigenvectors of  $A$ , corresponding to eigenvalues  $\lambda_1$  and  $\lambda_2$ , respectively.

This can be written as:

$$A\mathbf{v}_1 = \lambda_1 \mathbf{v}_1 \dots (1)$$

$$A\mathbf{v}_2 = \lambda_2 \mathbf{v}_2 \dots (2)$$

We want to show that  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are orthogonal, meaning their dot product is zero:  $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$ .



# Orthogonality of Eigen Vectors

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Taking the dot product of equation (1) with  $\mathbf{v}_2$ , we get:

$$(\mathbf{A}\mathbf{v}_1) \cdot \mathbf{v}_2 = (\lambda_1 \mathbf{v}_1) \cdot \mathbf{v}_2$$

Using the properties of dot product and scalar multiplication, we can rewrite the above equation as:

$$\lambda_1(\mathbf{v}_1 \cdot \mathbf{v}_2) = \mathbf{v}_1 \cdot (\lambda_2 \mathbf{v}_2)$$

Since  $\lambda_1$  and  $\lambda_2$  are distinct eigenvalues,  $\lambda_1 \neq \lambda_2$ . Therefore, we can divide both sides of the equation by  $(\lambda_1 - \lambda_2)$ :  $(\mathbf{v}_1 \cdot \mathbf{v}_2) = 0$

This shows that the dot product of  $\mathbf{v}_1$  and  $\mathbf{v}_2$  is zero, which implies that they are orthogonal.

# Pop Quiz

Q. Which of the following statements is true about the eigenvectors of a symmetric matrix?

- a. The eigenvectors are always orthogonal to each other.
- b. The eigenvectors are never orthogonal to each other.
- c. The orthogonality of eigenvectors depends on the matrix size.
- d. The orthogonality of eigenvectors is unrelated to symmetric matrices.



# Pop Quiz

Q. Which of the following statements is true about the eigenvectors of a symmetric matrix?

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- b. The eigenvectors are never orthogonal to each other.
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- d. The orthogonality of eigenvectors is unrelated to symmetric matrices.



# PCA (Principal Component Analysis)

- PCA is a statistical procedure that is used to reduce the dimensionality of data while preserving as much of the variation as possible.

**Targeting High Variance Data using PCA**



Identifying People with different heights (high variance) is easier even without looking at their faces

**PCA Ignoring Less Variance Data**



Identifying People with same heights (less variance) is most difficult without looking at their faces

# Steps to Calculate PCA

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1. Sort the eigenvalues in descending order. This ensures that the most important principal components are selected first.
2. Calculate the total sum of eigenvalues (total variance). This is the amount of variance that is explained by all of the principal components.
3. Initialize variables. Set a threshold for the desired amount of variance to retain (e.g., 95%). Initialize a cumulative explained variance variable to 0.
4. Iterate over each eigenvalue. For each eigenvalue, calculate the explained variance by dividing it by the total sum of eigenvalues.
5. Add the explained variance to the cumulative explained variance variable. This keeps track of the amount of variance that has been explained by the selected principal components.
6. Check if the cumulative explained variance exceeds or equals the desired threshold. If it does, move to Step 7. If not, proceed to the next eigenvalue.
7. Select the principal components. Add the current eigenvalue to the list of selected principal components.
8. Repeat Step 4 for the remaining eigenvalues until the cumulative explained variance meets or exceeds the desired threshold.
9. Return the list of selected principal components.

# Pop Quiz

Q. The matrix representation of a linear transformation allows for:

- a. Simplifying complex mathematical calculations
- b. Efficient manipulation and analysis of large datasets
- c. Visualizing the transformation using graphical representations
- d. Allowing non-linear transformations to be performed



# Pop Quiz

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- b. Efficient manipulation and analysis of large datasets**
- c. Visualizing the transformation using graphical representations
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# Activity 1

---

## Pre-requisites:

Basic understanding of matrices and vectors.

Understanding of eigenvalues and eigenvectors.

## Scenario:

Imagine that you are a data scientist working for a company that sells clothing. You have a 5 customer reviews, and you want to use eigenvalues and eigenvectors to find the most important features of the reviews.

The data set includes the following features:

[Customer rating, Customer satisfaction, Customer price sensitivity, Customer purchase history]

## Expected outcome:

The result of the activity should be a list of the most important features of the reviews.

## Steps:

1. Create a matrix, A, that represents the data set of customer reviews. (**Dataset:  $A = \text{np.random.randint}(-10, 10, (4, 4))$** )
2. Calculate the eigenvalues and eigenvectors of A.
3. Sort the eigenvalues in descending order.
4. The eigenvectors corresponding to the largest eigenvalues are the most important features of the reviews.

# Activity 2

---

## Pre-requisites:

Basic understanding of matrices and vectors.

Understanding of eigenvalues and eigenvectors.

## Scenario:

Imagine that you are a data scientist working for a company that sells clothing. You have a data set of customer ratings of songs, and you want to use eigenvalues and eigenvectors to find the most important features of the songs. The data set includes the following features:

The data set includes the following features:

[Song popularity, Song genre, Song tempo, Song lyrics]

## Expected outcome:

The result of the activity should be a list of the most important features of the songs.

## Steps:

1. Create a matrix,  $A$ , that represents the data set of customer songs. (**Dataset:  $A = \text{np.random.randint}(-5, 5, (4, 4))$** )
2. Calculate the eigenvalues and eigenvectors of  $A$ .
3. Sort the eigenvalues in descending order.
4. The eigenvectors corresponding to the largest eigenvalues are the most important features of the reviews.

# Summary

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Explored the concept of linear transformations and discover their applications across different fields.



Gained an understanding of eigenvalues and eigenvectors and their importance in the realm of linear algebra.



Applied the concepts of eigenvalues and eigenvectors to solve practical problems and real-world scenarios.



Acquired foundational knowledge of principal component analysis (PCA) and its role in data analysis and dimensionality reduction.



Learned how to use PCA techniques to analyze and interpret data, identifying crucial patterns and simplifying complex datasets.

# Session Feedback

