Launch Pad Noise Reduction using Metamaterial Structures

Master's thesis defense

Université Paris-Saclay - Mathématiques et Applications
Analyse, Modélisation, Simulation

European Space Research and Technology Center
Structures section

P. Marchner

13 September 2018

Outline

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A few words about ESTEC

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- ESA's technical heart
- Located in The Netherlands, $\approx$ 50 km from Amsterdam
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- Development and assessment of ESA's missions
- Test centre for spacecrafts: thermal, acoustic, electromagnetic, vibrations...
- Cooperation with the space industry and research institutes

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- May damage the payload (satellite, space probe...)

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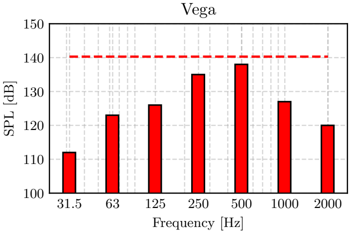
→ Sonic crystal

Objective of the study - Sonic crystal

Idea

- Reduce radiated noise during ignition and lift-off



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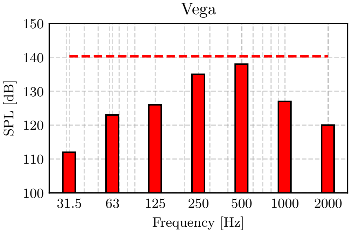

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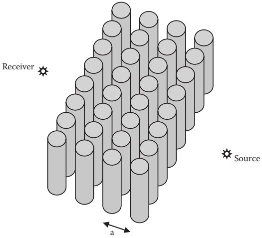
Approach - Helmholtz problems

1. Sound propagation in periodic structures - 2D FEM



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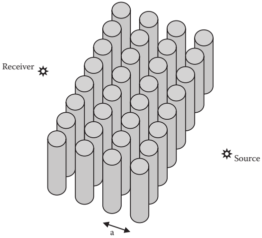

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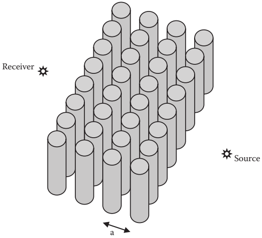

Objective

- Proof of concept of a full scale sonic crystal with simplified physical environment

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- Helmholtz equation

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periodic function $\times$ phase shift: Bloch wavevector $\mathbf{K}$

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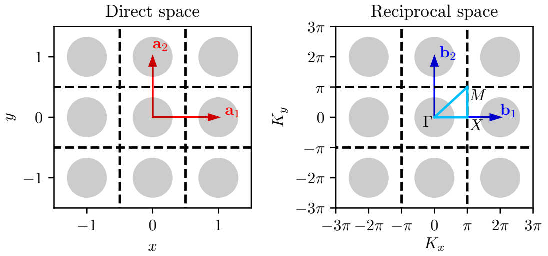
Square periodicity

Irreducible Brillouin Zone (IBZ): Γ-X-M



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Hexagonal periodicity

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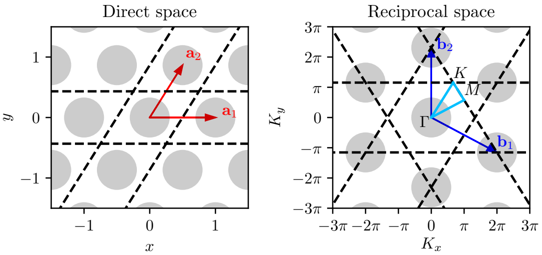

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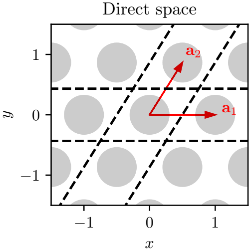
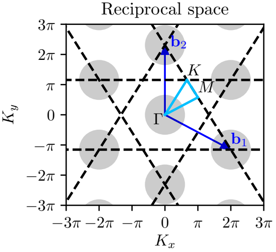




Direct lattice:
$$\mathbf{R} = n\mathbf{a_1} + m\mathbf{a_2}, \ (n, m) \in \mathbb{Z}^2$$

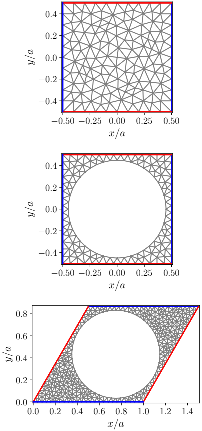
Reciprocal lattice:
$$\mathbf{k} = q\mathbf{b_1} + r\mathbf{b_2}, \ (q, r) \in \mathbb{Z}^2$$

Duality relation:
$$\mathbf{b_i} \cdot \mathbf{a_j} = 2\pi\delta_{ij}$$

Numerical strategy: Variational formulation for $\widetilde{p}$

1. Constrain the domain Ω with periodic boundaries

$$\widetilde{V} = \left\{ u \in H^1(\Omega), u(\mathbf{s}) = u(\mathbf{s} + \mathbf{R}),\ \mathbf{s} \in \partial\Omega \right\}$$



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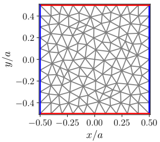
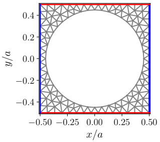
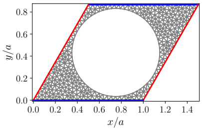
$$\forall \widetilde{q} \in \widetilde{V}, \quad \int_\Omega \nabla\widetilde{p}\,\nabla\widetilde{q}^*\ d\Omega + i\mathbf{K}\int_\Omega \widetilde{p}\,\nabla\widetilde{q}^*\ d\Omega - i\mathbf{K}\int_\Omega \nabla\widetilde{p}\,\widetilde{q}^*\ d\Omega$$
$$+\mathbf{K}^2\int_\Omega \widetilde{p}\,\widetilde{q}^*\ d\Omega = \frac{\omega^2}{c_0^2}\int_\Omega \widetilde{p}\,\widetilde{q}^*\ d\Omega$$







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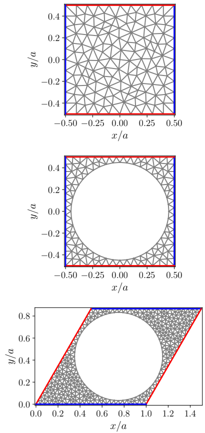
3. Meshing & discretization $\rightarrow$ eigenvalue problem:

$$\mathbb{A}_\mathbf{K}\widetilde{p}_h = \omega^2\mathbb{B}\widetilde{p}_h, \quad \widetilde{p}_h \in \widetilde{V}_h$$



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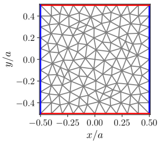

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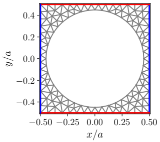

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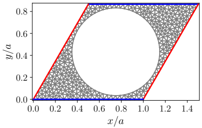
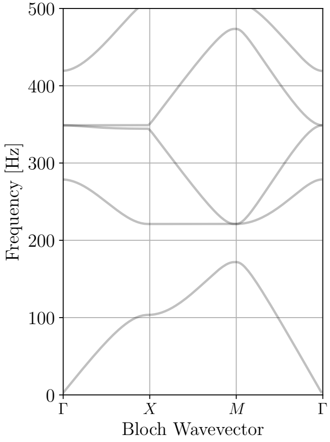


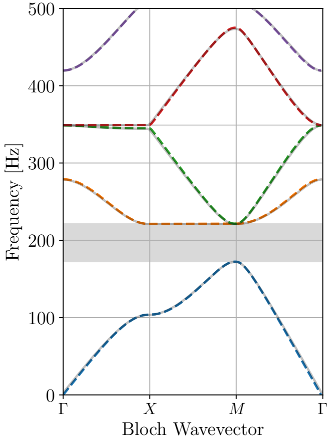
4. Find ω^2 and $\widetilde{p}_h$ for each Bloch wavevector $\mathbf{K}$ inside the Irreducible Brillouin Zone

Band structure, iso-frequency countours - circle, $R = 0.4$



Plane wave expansion

Band structure, iso-frequency countours - circle, $R = 0.4$



Finite element method

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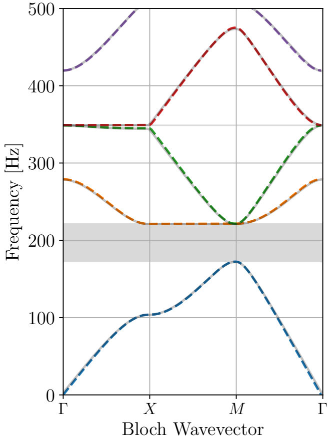

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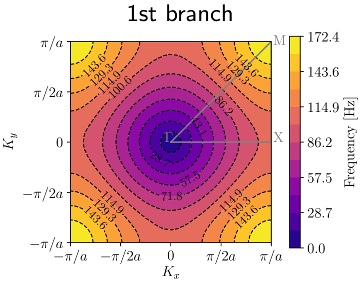

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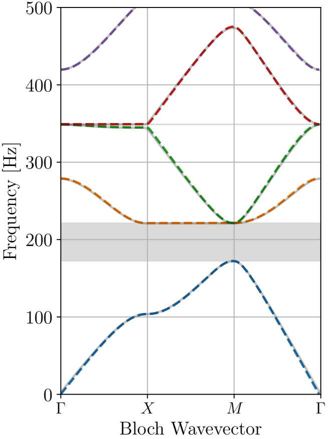
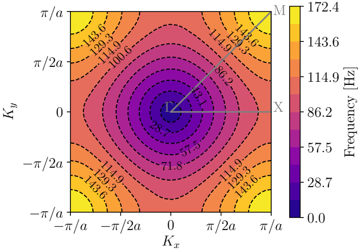
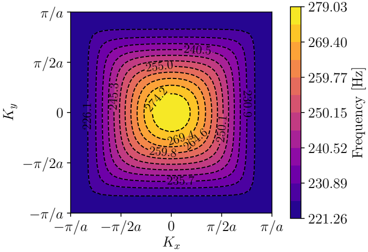


Finite element method

1st branch



2nd branch



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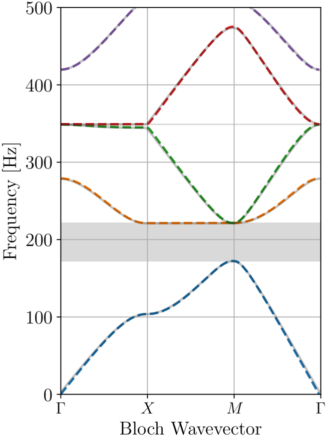
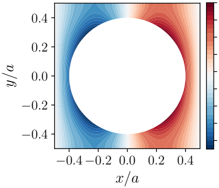
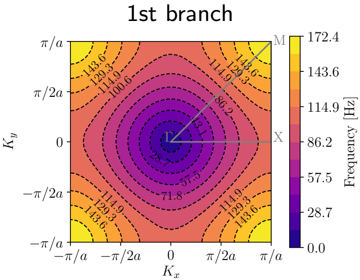
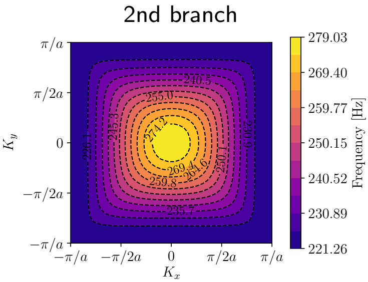


Finite element method

- eigenmode 0: $(K_x, K_y) = (0, 0)$







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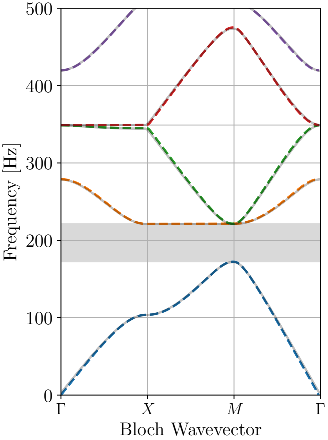

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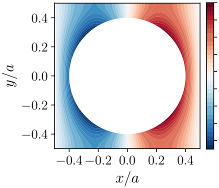
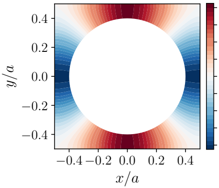


- eigenmode 1: $(K_x, K_y) = (0, 0)$



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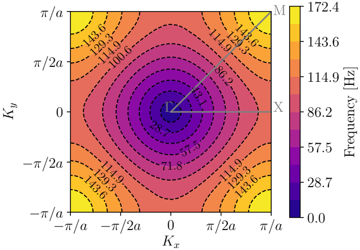

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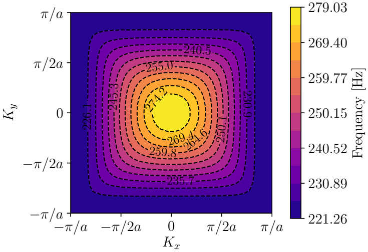
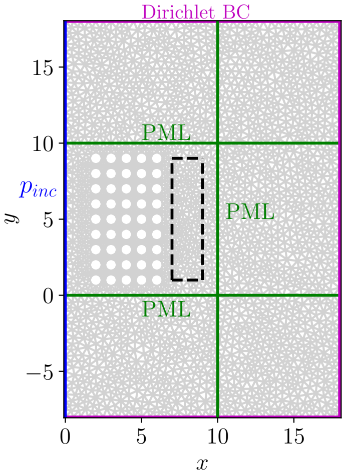


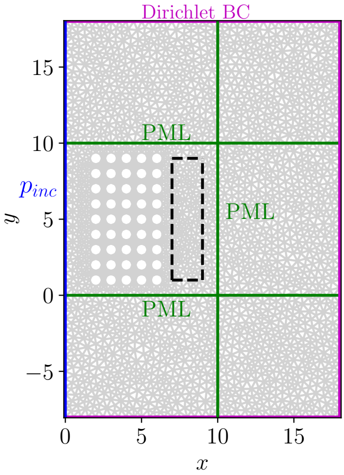
Outline

Finite medium - Multiple scattering



Set up:

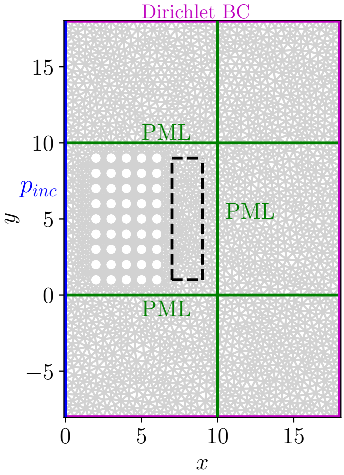
Finite medium - Multiple scattering



Set up:

- Plane wave input

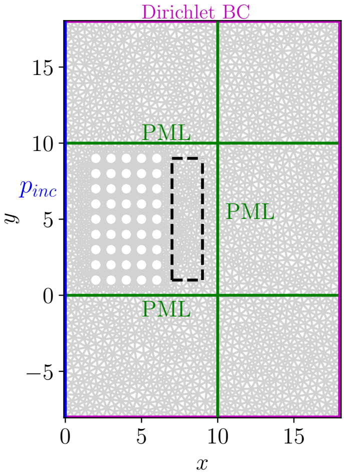
Finite medium - Multiple scattering



Set up:

- Plane wave input
- Radiation condition: Perfectly matched layer (PML)

Finite medium - Multiple scattering



Set up:

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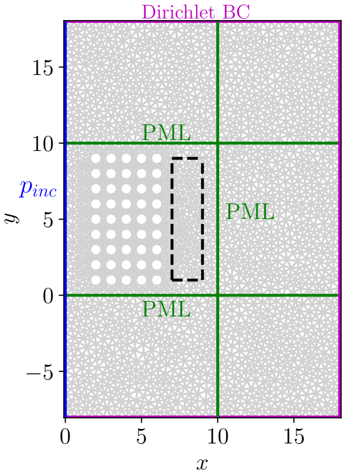
- Radiation condition: Perfectly matched layer (PML)
- Dirichlet condition on outer boundary

Finite medium - Multiple scattering



Set up:

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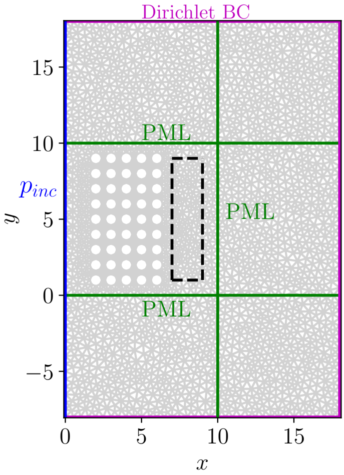
- Neumann condition on inner boundaries

Finite medium - Multiple scattering



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PML - Bermudez function:

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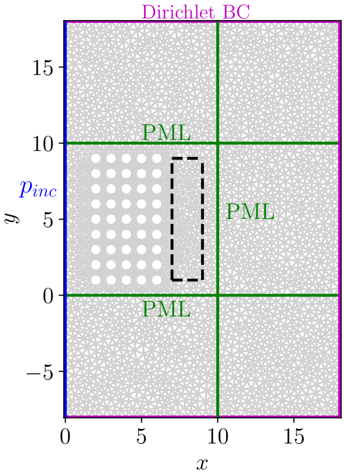
Finite medium - Multiple scattering



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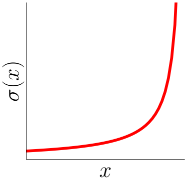

Helmholtz-PML variational problem

Functional space: $V = \left\{ u \in H^1(\Omega), u = 0 \text{ on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \right\}$

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Variational formulation: Find p in V such as:

$$\forall q \in V, \quad \int_{\Omega} \left(\frac{\alpha(x)}{\alpha(y)} \frac{\partial p}{\partial x} \frac{\partial q^*}{\partial x} + \frac{\alpha(y)}{\alpha(x)} \frac{\partial p}{\partial y} \frac{\partial q^*}{\partial y} - k_0^2 \frac{p q^*}{\alpha(x)\alpha(y)} \right) d\Omega = \int_{\Gamma_1} p_{\text{inc}} q^* d\gamma.$$

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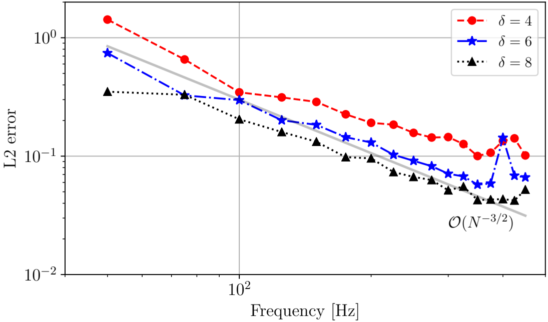
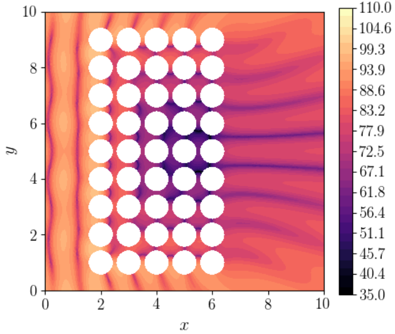
δ: PML width, N: elem per wavelength



Pressure map in dB $\propto \log(p)$

9 × 5 Circles, 172 Hz



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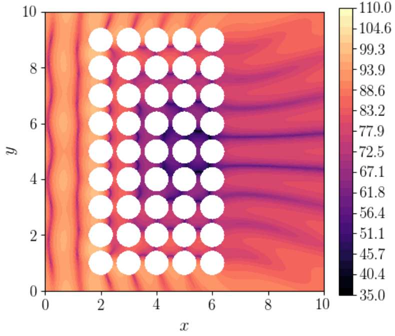

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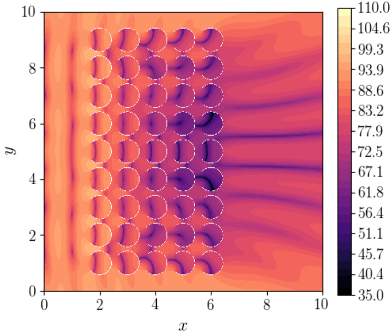

downstream attenuation $\approx +17.0$ dB
upstream attenuation $\approx +4.9$ dB

Outline

Sum up

Work done:

- Development and validation of finite element codes with the Python FEniCS platform
- Characterization of 2D sonic crystals of arbitrary shape
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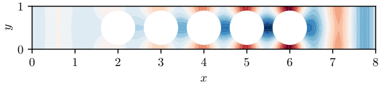
What I did not talk about:

- 1D sonic crystal
- 1-direction FEM transmission problem



- physical behaviour & interpretation
- locally resonant sonic crystal

Outline

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Boundary element method - BEM

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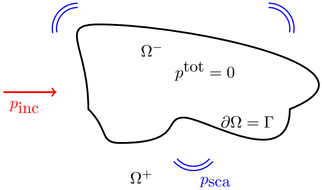

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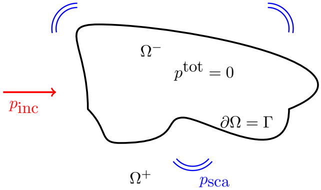

$$p^{\text{tot}} = p^{\text{inc}} + p^{\text{sca}}, \quad r = |\mathbf{x} - \mathbf{y}|$$

$$\Delta p^{\text{tot}} + \frac{\omega^2}{c_0^2} p^{\text{tot}} = 0 \quad \text{in } \Omega^+$$

$$\frac{\partial p^{\text{tot}}}{\partial \mathbf{n}} = 0 \quad \text{on } \Gamma$$

$$\lim_{r \to +\infty} r \left(\frac{\partial p^{\text{sca}}}{\partial r} - i k_0 p^{\text{sca}} \right) = 0$$

Unique solution $p^{\text{tot}} \in H^1_{\text{loc}}(\overline{\Omega^+})$

Mathematical resolution

Variational form $\rightarrow$ Compute p^{tot} in Ω^+ by an integral representation:

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esa

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Jump relations: e.g $K\phi = \gamma_0^-(\mathcal{K}\phi) + \frac{1}{2}\phi = \gamma_0^+(\mathcal{K}\phi) - \frac{1}{2}\phi$

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Apply γ_0^+,γ_1^+ on integral representation $\rightarrow$ Boundary integral equations (BIE)

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Local geometry approximation

Neumann-to-Dirichlet map

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Regularize hypersingular operator D

esa

Available solvers

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Bempp - open source

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Acceleration techniques:

- reduce memory & time complexity to $\mathcal{O}(N \log N)$ or more
- based on fast matrix-vector product

Outline

Sound hard unit sphere scattering: $k_0 a \approx 10$

$\rightarrow$ 1Pa plane wave at 546 Hz, $\approx$ 10 elements per wavelength: 3919 dofs

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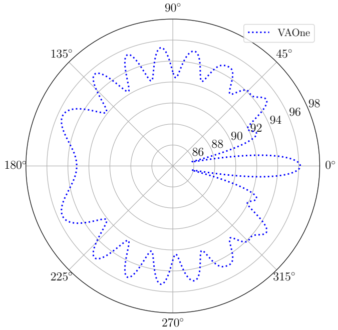
p^{tot} directivity (dB) at $R = 3$



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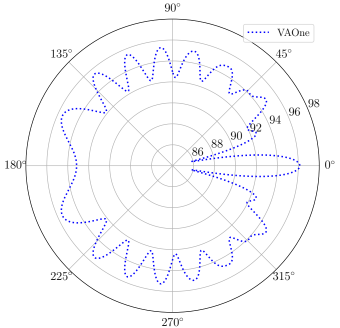
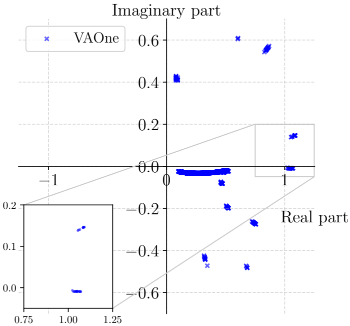


BIE - eigenvalues distribution



VAOne: 14 iterations, increasing with frequency

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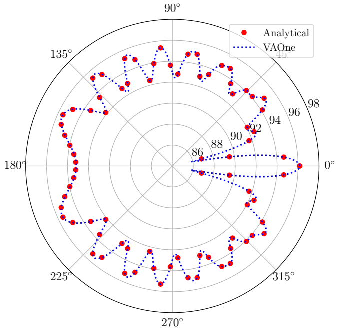

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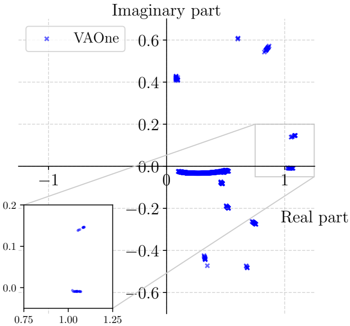

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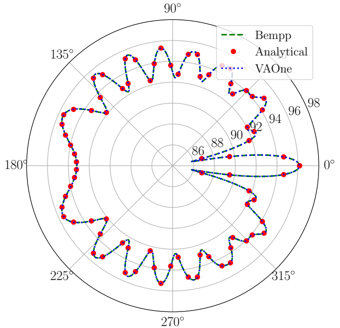

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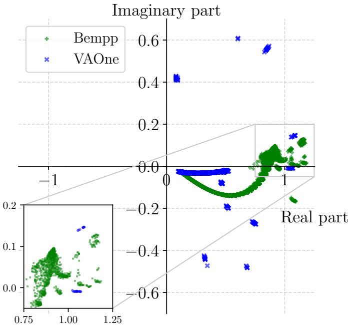


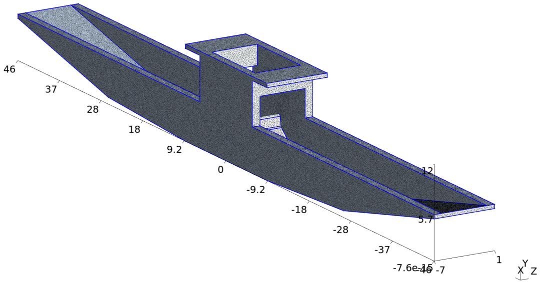
VAOne: 14 iterations, increasing with frequency

Bempp: 5 iterations, quasi constant with frequency

Outline

Scattering problem for the launch pad with sonic crystal

- Vega launch pad geometry: reflective surface



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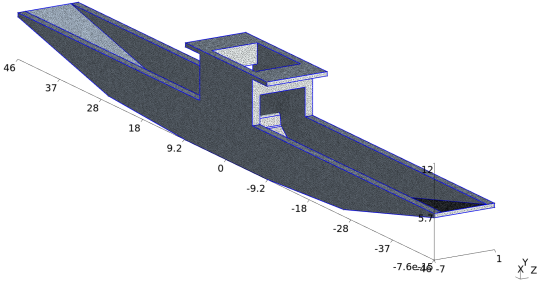

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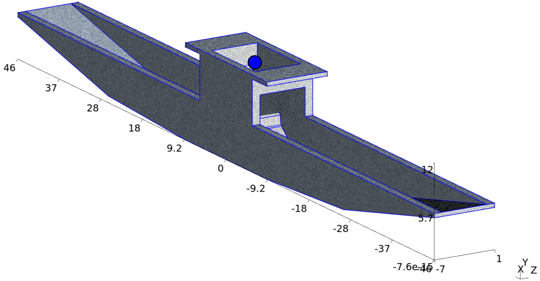
  1. Ignition: $(x, y, z) = (0, 11, -3)$



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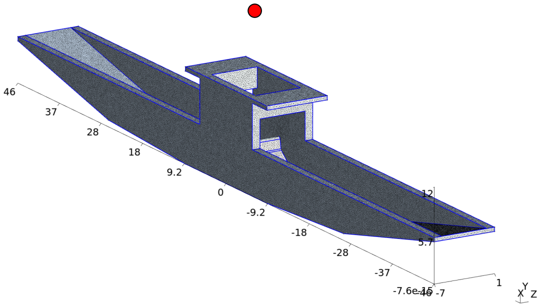
- Source: monopole at 1 Pa
  1. Ignition: $(x, y, z) = (0, 11, -3)$
  2. Liftoff: $(x, y, z) = (0, 30, -3)$



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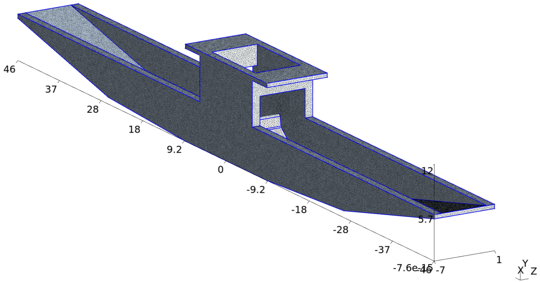

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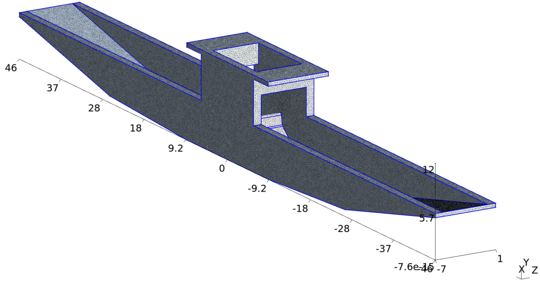

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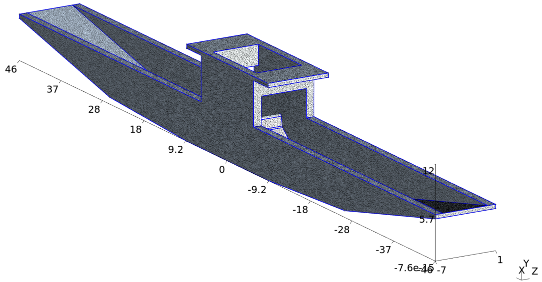

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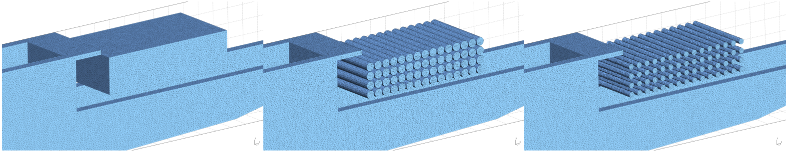
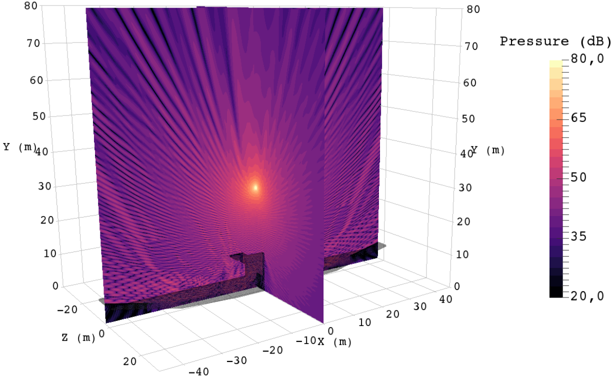
- 3 media: 15×4 sonic crystal



Rigid | SC High | SC Low

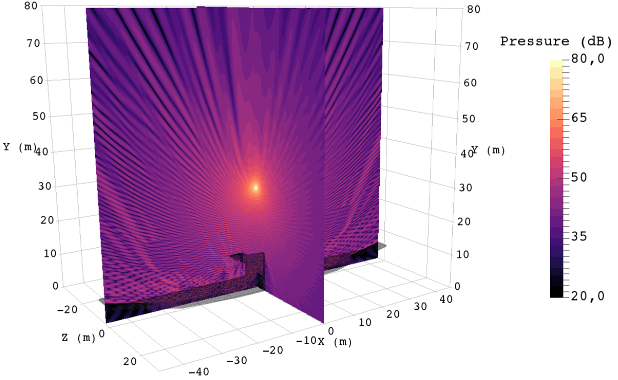
Computation example - Lift-off

- Machine: Intel Xenon 3.3GHz, 8 cores, 256 GB of RAM



Computation example - Lift-off

- Machine: Intel Xenon 3.3GHz, 8 cores, 256 GB of RAM
- 350 Hz, 230k dofs, more than 100 wavelengths across X



Computation example - Lift-off

- Machine: Intel Xenon 3.3GHz, 8 cores, 256 GB of RAM
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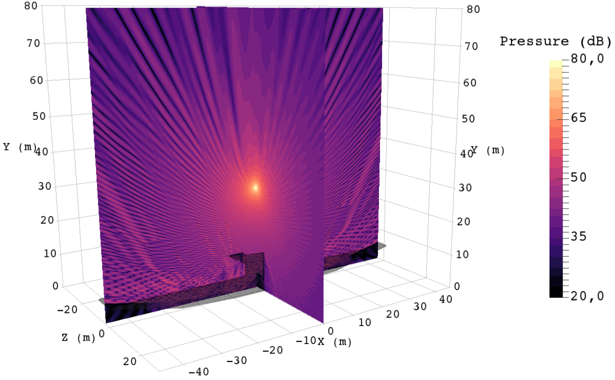

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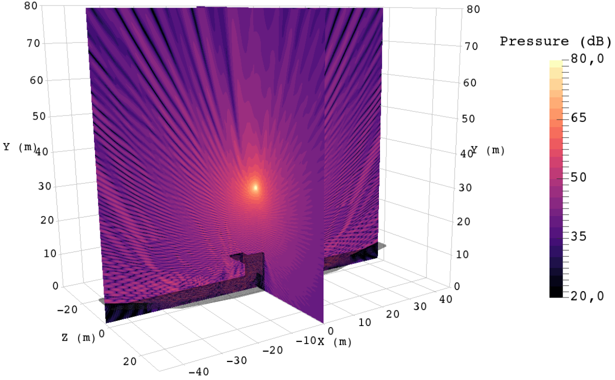
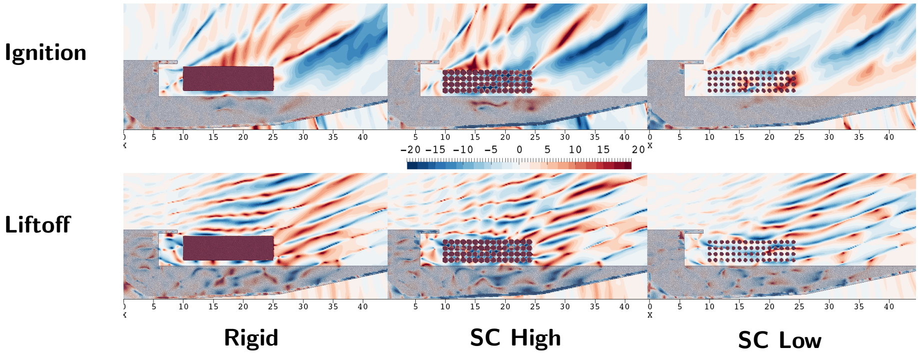
- Lift-off: 6h30min, 82 iterations, full RAM



Engineering the Launch Pad - Insertion loss (dB)

- 100 Hz XY plane - near field



+ positive attenuation / - extra reflection

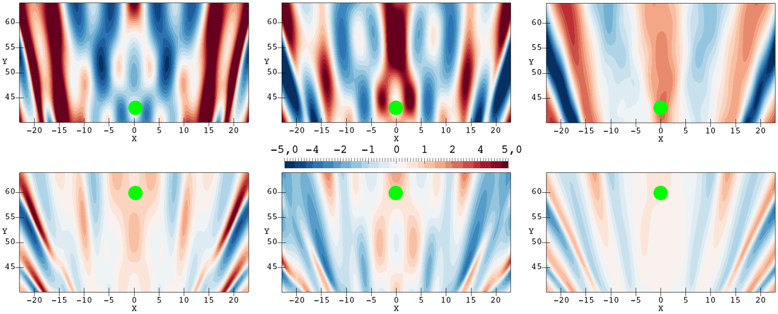
- SC High: Beginning of the Γ-X band gap
- SC Low: 1st diffraction branch

Engineering the Launch Pad - Insertion loss (dB)

- 100 Hz XY plane - far field: fairing tip

Ignition

Liftoff



Rigid **SC High** **SC Low**

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Engineering the Launch Pad - Insertion loss (dB)

- 140 Hz XY plane - near field

Ignition

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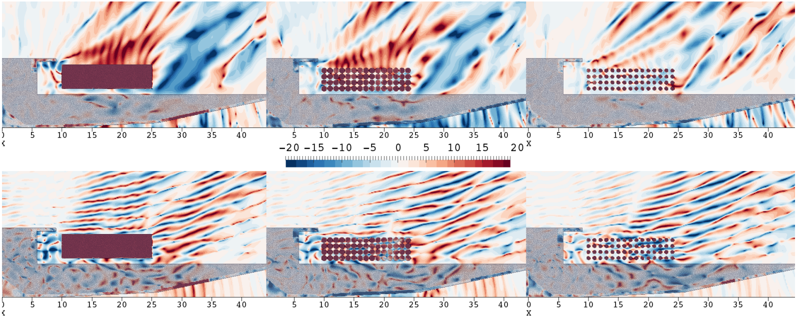

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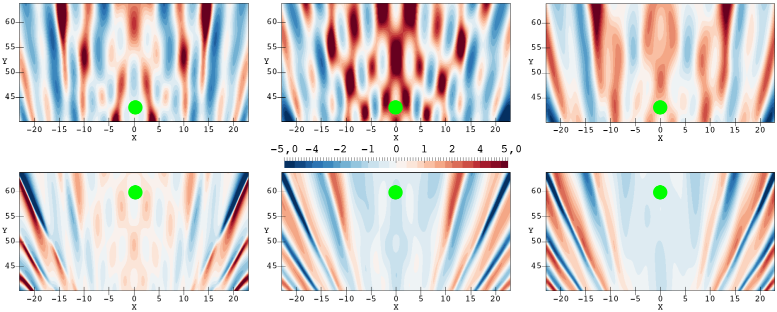

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Outline

Current BEM limitations and future improvements

1. **High frequency limit and mesh size**:
   frequency $\propto$ number of nodes $\propto$ matrix size
   - FMM: algorithm limitation when $ka \geq 500$
   - $\mathcal{H}$-matrix: memory requirement for low rank approximation

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   - Strong influence of the linear system preconditioning
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3. **Acoustic source**:
   - More realistic source: multiple monopoles, multiple right hand sides
   - Source close to the surface: singularity
   - Ground impedance: modify Green's function

Outline

Concluding remarks

Work done:

- Validation and comparison of 2D and 3D Helmholtz problems
- Insights of the sonic crystal behaviour for a real scale application
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Interests for ESA:

- What are the peculiar properties of sonic crystals ? $\checkmark$
- How to design a large scale sonic crystal ? $\checkmark$
- Are sonic crystals interesting for the launch pad noise reduction ? $\checkmark$

Thank You !

Questions ?