Math 290 HW #1 Sections 1 and 2 Mark Petersen

$\frac{19}{20}$

1.1 a) $S_1 = \{6, 7, 8, 9, 10\}$, $|S_1| = 5$

b) $S_2 = \{-10, -9, -8, -7, -6, 6, 7, 8, 9, 10\}$, $|S_2| = 10$

c) $S_3 = \emptyset$, $|S_3| = 0$

d) $S_4 = \{2i\}$, $|S_4| = 1$

e) $S_5 = \{3, 2, 1, 0, -1, \dots\}$, $|S_5| = \infty$

1.2 a) $A_1 = \{x \in S : x \text{ is prime}\}$
b) $A_2 = \{x^3 : x \in S\}$
c) $A_3 = \{x \in S : x \notin \mathbb{N}\}$

1.3 a) $A = \{5x : x \in \mathbb{Z}\}$ ✓
b) $B = \{3 + 5x : x \in \mathbb{Z}\}$ ✓
c) $C = \{x^4 : x \in \mathbb{N}\}$ ✓
d) $D = \{2^x : x \in \mathbb{Z}\}$ ✓

A should be a set so it can't be a number

1.4 a) $\boxed{A = 1}$, $B = \{1\}$, $C = \{\{1\}\}$ — $A \in B$, $B \in C$, $A \in C$
b) $\boxed{A = 1}$, $B = \{1\}$, $C = \{\{1\}\}$ — $A \in B$, $\boxed{B \subseteq C}$, $A \notin C$
c) $A = \{1\}$, $B = \{1, 2\}$, $C = \{\{1, 2\}, \{1\}\}$ ✓ — $A \subsetneq B$, $B \in C$, $A \in C$
d) $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1\}$ ✓ — $A \cap B \subseteq C$, $A \not\subseteq C$, $B \not\subseteq C$
e) $A = \emptyset$, $B = \{1, 2, 3\}$, $C = \{1, 2, 3\}$ ✓ — $A \cap C = \emptyset$, $A \subseteq B$, $|B \cap C| = 3$

You have $B \in C$ not $B \subseteq C$

1.5 let $A = \{1, 2\}$
$P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$, $|P(A)| = 2^{|A|} = 4$

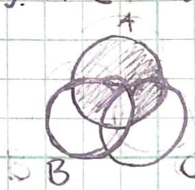
$P(P(A)) = \{$

$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$	
0	0	0	0	$\emptyset$,
0	0	0	1	$\{\{1,2\}\}$,
0	0	1	0	$\{\{2\}\}$,
0	0	1	1	$\{\{2\}, \{1,2\}\}$,
0	1	0	0	$\{\{1\}\}$,
0	1	0	1	$\{\{1\}, \{1,2\}\}$,
0	1	1	0	$\{\{1\}, \{2\}\}$,
0	1	1	1	$\{\{1\}, \{2\}, \{1,2\}\}$,
1	0	0	0	$\{\emptyset\}$,
1	0	0	1	$\{\emptyset, \{1,2\}\}$,
1	0	1	0	$\{\emptyset, \{2\}\}$,
1	1	0	0	$\{\emptyset, \{1\}\}$,
1	1	0	1	$\{\emptyset, \{1\}, \{1,2\}\}$,
1	1	1	0	$\{\emptyset, \{2\}, \{2\}\}$,
1	1	1	1	$\{\emptyset, \{2\}, \{2\}, \{1,2\}\}\}$

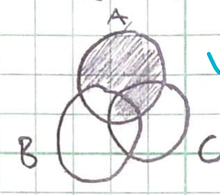
$|P(P(A))| = 2^{|P(A)|} = 2^4 = 16$

1.6 a) $P \cap Q = \{7\}$
b) $P \cup Q = [3,9]$
c) $P - Q = [3,7)$
d) $Q - P = (7,9]$
e) $(R \cap P) - Q = P - Q = [3,7)$
f) $(P \cup Q) \cap R = [3,8]$
g) $P \cup (Q \cap R)$ $[3,8]$

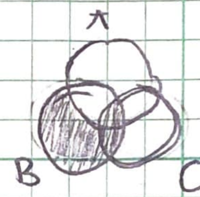
1.7 a) $A-(B \cap C)$



b) $A-(B-C)$

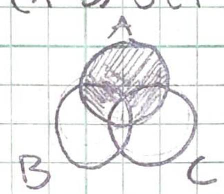


c) $B-(A-C)$



d) $(B \cap C) \cap (B \cup A)$

e) $(A-B) \cup (A-C)$



1.8 a) let $S = \{x : \text{odd integer}\}$ and $T = \{x : \text{even integer}\}$, then $S \cap T = \emptyset$ because a number is either odd or even but never both.

b) $\mathbb{N} \subseteq \mathbb{C}$ thus not disjoint

c) disjoint b/c a number cannot be prime and composite by definition

d) disjoint b/c irrational numbers are real numbers that are not rational.

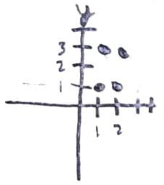
1.9 $U = \{1,2,3,4,5,6,7\}$

$S = \{1,2,3,4,5\}$

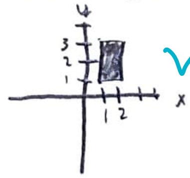
$T = \{4,5,6\}$

Note. $U = S \cup T \cup \{7\}$,
$|S-T| = |\{1,2,3\}| = 3$,
$|T-S| = |\{6\}| = 1$
$|\bar{S}| = |U-S| = |\{6,7\}| = 2$

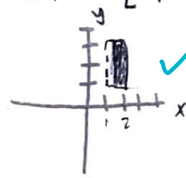
2.1. a) $\{1,2\} \times \{1,3\}$



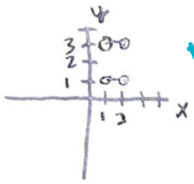
b) $[1,2] \times [1,3]$



c) $(1,2] \times [1,3]$



d) $(1,2] \times \{1,3\}$



(Glad you mentioned it in your email.)

Perfect!

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2.2 Let $A = \{s,t\}$ and $B = \{0,9,7\}$

a) $A \times B = \{(s,0), (s,9), (s,7), (t,0), (t,9), (t,7)\}$

b) $B \times A = \{(0,s), (0,t), (9,s), (9,t), (7,s), (7,t)\}$

c) $A^2 = \{(s,s), (s,t), (t,s), (t,t)\}$

d) $B^2 = \{(0,0), (0,9), (0,7), (9,0), (9,9), (9,7), (7,0), (7,9), (7,7)\}$

e) $\emptyset \times A = \{\ \}$

2.3

a) False. $|A \times B| = |A| \cdot |B| = 3 \cdot 4 = 12$.

b) False. Order matters. ex: $(s,t) \neq (t,s)$.

c) True. Since $\emptyset \subseteq \bigcap_{i \in I} S_i$ then $\bigcap_{i \in I} S_i \subseteq \bigcup_{i \in S} S_i$

d) True. Let $S_i = \mathbb{R}_1 \times \mathbb{R}_2 \times \cdots \times \mathbb{R}_{i-1} \times \{0\} \times \mathbb{R}_{i+1} \times \mathbb{R}_{i+2} \times \cdots \times \mathbb{R}_\infty$

Then $\bigcap_{i=1}^{\infty} S_i = (0,0,\ldots)$ which has only one element.

e) False. A^4 consists of orderd quadruples from A.

2.4 Let A denote the set containing all the letters in the alphabet, W denote the set containing every english word, $W_\alpha = \{x \in W : \alpha \text{ is a letter in the word } x\}$, and $V \subseteq A$ be an index set.

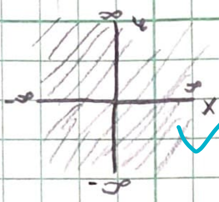
a) $\bigcap_{\alpha \in V} W_\alpha$ where $V = \{a, w, x, y\}$

b) $\bigcup_{\alpha \in V} W_\alpha$ where $V = \{s, t, u\}$

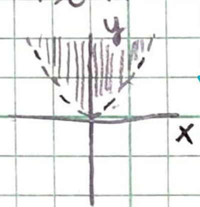
c) $\bigcap_{\alpha \in \{p,r\}} W_\alpha - \bigcup_{\alpha \in V} W_\alpha$ where $V = \{a, e, i, o, u\}$ not empty ex: pry

2.5. Consider the set $P_r = \{(x,y) \in \mathbb{R}^2 : y = x^2 + r, r \in \mathbb{R}\}$

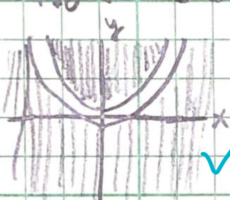
a) $\bigcup_{r\in\mathbb{R}} P_r = \{(x,y) \in \mathbb{R}^2\}$



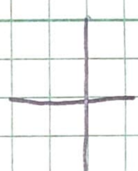
b) $\bigcup_{r>0} P_r = \{(x,y) \in \mathbb{R}^2 : y - x^2 > 0\}$



c) $\bigcup_{r\neq 0} P_r = \{(x,y) \in \mathbb{R}^2 : y - x^2 \neq 0\}$



d) $\bigcap_{r\in\mathbb{R}} P_r = \emptyset$



e) $\bigcap_{r>0} P_r = \emptyset$