# Problem 1

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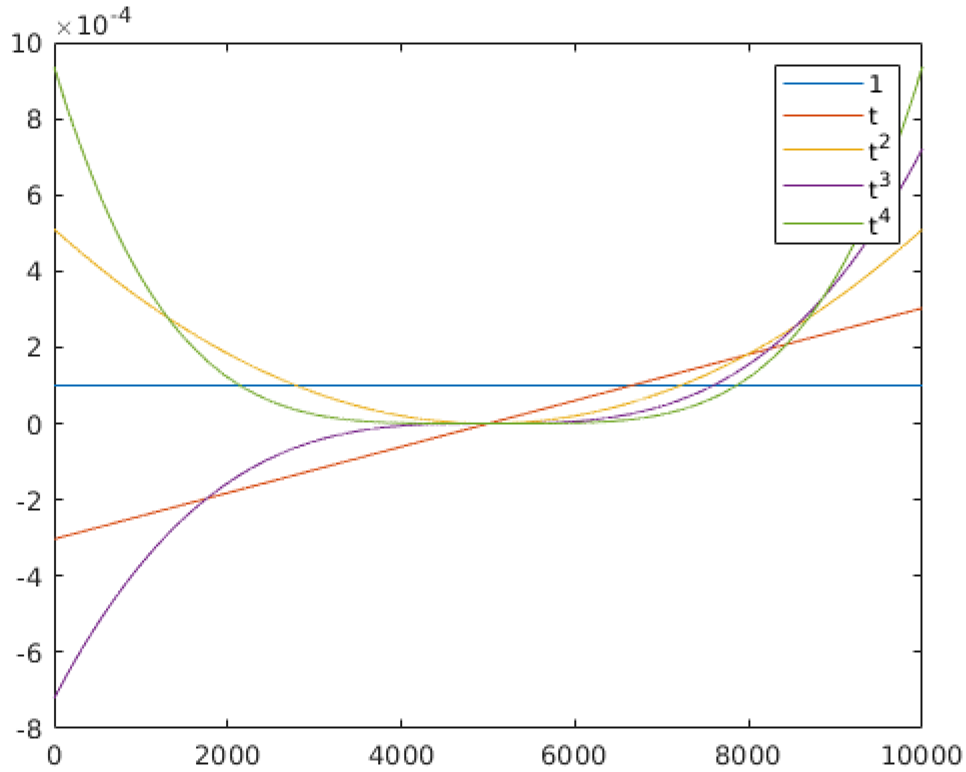
# a:

```
Write a script that numerically generates the first 5 Legendre polynomi
orthogonalization of the set of vectors {1,t,t^2,t^3,t^4} over the
interval [-1,1]. Plot each of these polynomials.
```

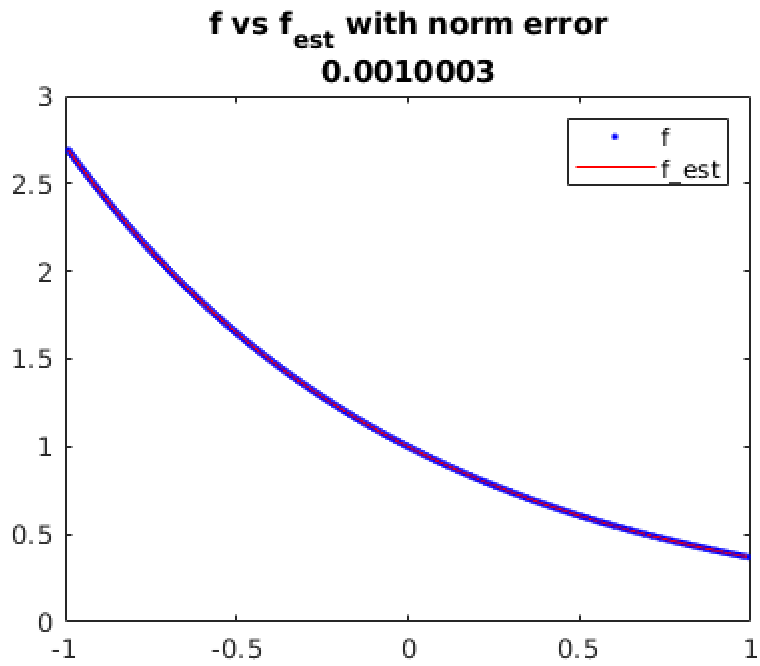
```
num = 10000;
t = linspace(-0.99, .99, num);
p = zeros(num, 5);
for kk = 1:5
 p(:, kk) = t.^(kk-1);
end

e = zeros(num,5);
q = zeros(num,5);


for i = 1:5
   e(:,i) = p(:,i);
   for j = 1:i-1
       e(:,i) = e(:,i) - dot(p(:,i),q(:,j))*q(:,j);
   end
   q(:,i) = e(:,i)/dot(e(:,i),e(:,i));
end

figure(1),clf;
plot(q);
legend('1','t','t^2','t^3','t^4')
```




# b.

```
Perform a linear least-squares approximation of the function f(t)=e^{-t} o
using the Legendre polynomials derived in part 1. Plot f(t) and your
approximation (or expansion) of f(t). Find the norm of the error vector.
That is, find the norm of the difference between f(t) and your
approximation of f(t).
```

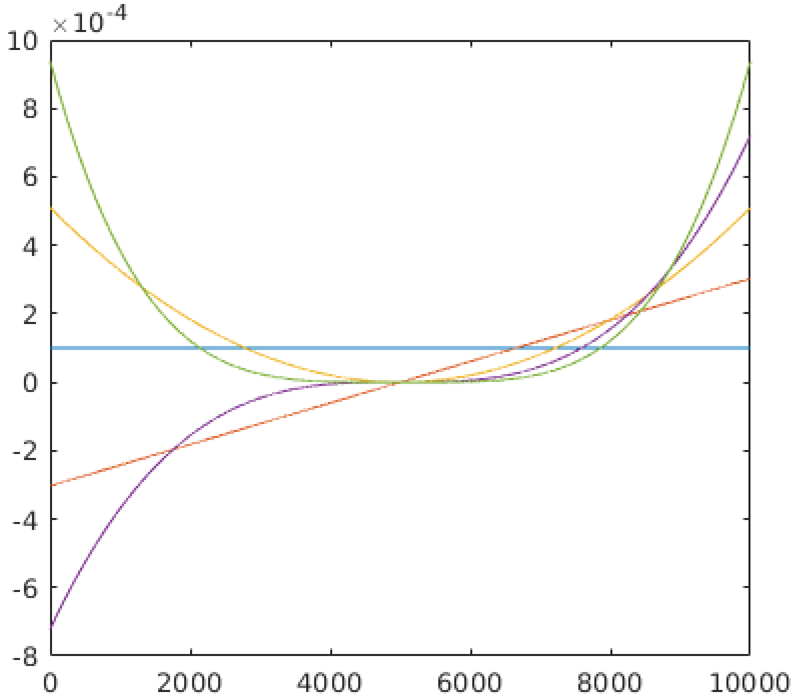
```
f = exp(-t)';

% Perform linear least squares
x = q\f;

f_est = (q*x);

norm_err = sum((f-f_est).^2);

figure(2);
plot(t,f,'b.',t,f_est','r');
legend('f','f\_{est}');
title(["f vs f_{est} with norm error",num2str(norm_err)])
```




# c.

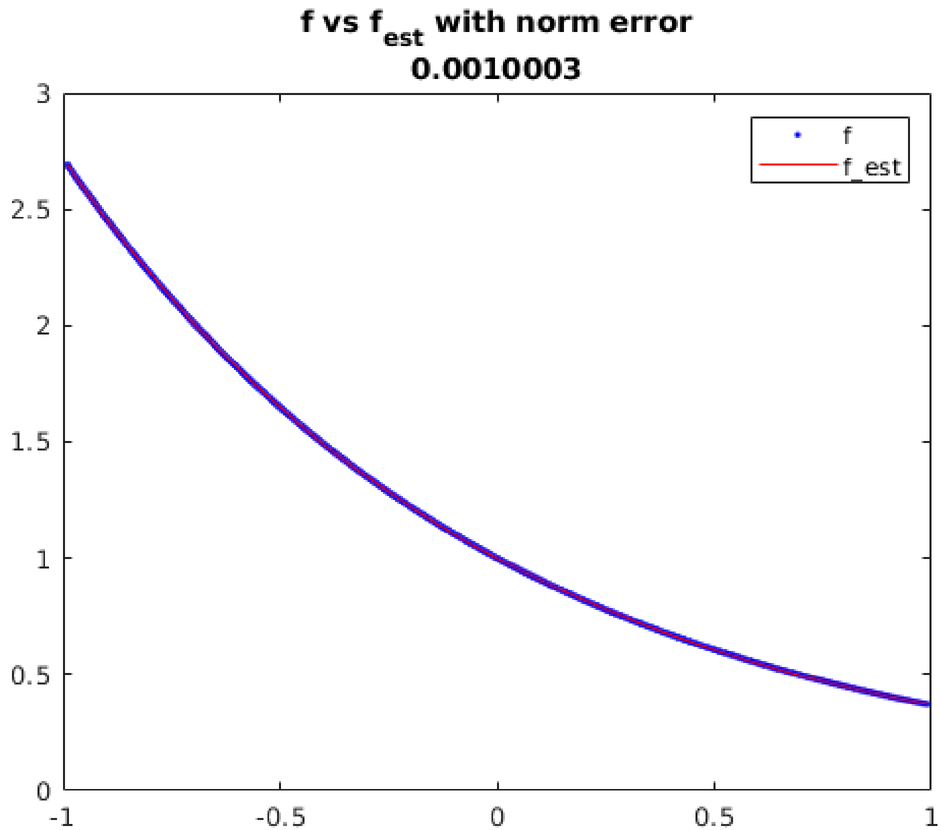
```
Modify your results form (a) to generate the first 5 Chebyshev polynomi
refer to example 2.15.1 on p.120 of the book, and note that
generation of the Chebyschev polynomials will be identical to the
Legendre polynomials but with the use of a weighted inner product.
```

```
w = 1./sqrt(1-t.^2)';

e_c = zeros(num,5);
q_c = zeros(num,5);

for i = 1:5
   e_c(:,i) = p(:,i);
   for j = 1:i-1
       e_c(:,i) = e_c(:,i) - dot(w.*p(:,i),q_c(:,j))*q_c(:,j);
   end
   q_c(:,i) = e_c(:,i)/dot(e_c(:,i),e_c(:,i));
end

figure(3),clf;
plot(q_c);
```




## d.

```
Repeat part (b) using the Chebyshev polynomials instead of the Legendre po

f_c = exp(-t)';

% Perform linear least squares
x_c = q_c\f_c;

f_c_est = (q_c*x_c);

norm_err_c = sum((f_c-f_c_est).^2);

figure(4);
plot(t,f_c,'b.',t,f_c_est','r');
legend('f','f\_{est}');
title(["f vs f_{est} with norm error",num2str(norm_err_c)])
```




# e. Discuss results

Since both span the same space, they can approximate it equally as well.

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