ECEN 773 Project

EKF With Parameter Estimation

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Outline

Problem Statement

Consider the LTI Discrete system

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + Bu_{k+1} + \mathbf{w}$$
$$\mathbf{y}_{k+1} = C\mathbf{x}_{k+1} + \mathbf{v}$$

where $\mathbf{w}$ is process noise and $\mathbf{v}$ is observation noise with zero mean multivariate normal distribution with covariances Q and R, $\mathcal{N}(0, Q)$, $\mathcal{N}(0, R)$.

and the observer

$$\dot{\hat{\mathbf{x}}}_{k+1} = A\hat{\mathbf{x}}_k + Bu_{k+1}$$
$$\hat{\mathbf{y}}_{k+1} = C\hat{\mathbf{x}}_{k+1}$$

Bayes' Inference

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{\text{likelihood} \times \text{prior}}{\text{normalizing factor}}$$

Prior

Let $\mathbf{x}_k$ be a multivariate normal random process at time step k with mean $\hat{x}_k$ and covariance P_k, $\mathcal{N}(\hat{x}_k, P_k)$ Then

$$\mathbf{x}_{k+1|k} = A\mathbf{x}_k + Bu_{k+1} + \mathbf{w}$$

$$\mathbf{E}\{x_{k+1|k}\} = \hat{x}_{k+1|k} = A\hat{x}_k + Bu_{k+1}$$

$$\mathbf{E}\{x_{k+1|k} - \hat{x}_k)(x_{k+1|k} - \hat{x}_k)^\top\} = P_{k+1|k} = AP_kA^\top + Q$$

and the prior is

$$p(x) = k_x \exp\left(-\frac{1}{2}(\mathbf{x} - \hat{x})^\top P^{-1}(\mathbf{x} - \hat{x})\right)$$

where k_x is a constant coefficient of a Gaussian PDF.

Likelihood

Let $\mathbf{y}_k$ be a multivariate normal random process at time step k. Then

$$\mathbf{y}_{k+1|k+1} = C\mathbf{x}_{k+1} + \mathbf{v_k}$$

and the likelihood is

$$p(y|k) = k_y \exp\left(-\frac{1}{2}(\mathbf{y} - Cx)^\top R^{-1}(\mathbf{y} - Cx)\right)$$

where k_y is a constant coefficient of a Gaussian PDF.

Posterior

The posterior,

$$p(x|y) = k_{xy} \exp\left(-\frac{1}{2}(\mathbf{y} - Cx)^\top R^{-1}(\mathbf{y} - Cx)\right) \exp\left(-\frac{1}{2}(\mathbf{x} - \hat{x})^\top P^{-1}(\mathbf{x} - \hat{x})\right)$$

,
is maximized when

$$\frac{\partial}{\partial x}\left[-\frac{1}{2}(\mathbf{y} - Cx)^\top R^{-1}(\mathbf{y} - Cx) - \frac{1}{2}(\mathbf{x} - \hat{x})^\top P^{-1}(\mathbf{x} - \hat{x}\right] = 0$$

i.e. when

$$\hat{x} = \left(P^{-1} + C^\top R^{-1} C\right)^{-1} \left(P^{-1}\hat{x} + H^\top R^{-1} y\right)$$

Kalman Filter

Predict	
Predicted (*a priori*) state estimate	$\hat{x}_{k\|k-1} = A\hat{x}_{k-1\|k-1} + Bu_k$
Predicted (*a priori*) error covariance	$P_{k\|k-1} = AP_{k-1\|k-1}A^\top + Q$
Update	
Innovation	$z = y_k - C\hat{x}_{k\|k-1}$
Innovation Covariance	$S_k = R + CP_{k\|k-1}C^\top$
Kalman gain	$K_k = P_{k\|k-1}CS_k^{-1}$
Updated (*a posteriori*) state estimate	$\hat{x}_{k\|k} = \hat{x}_{k\|k-1} + K_k z$
Updated (*a posteriori*) estimate covariance	$P_{k\|k} = (I - K_k C)P_{k\|k-1}$

Problem Statement

$$
\begin{aligned}
\mathbf{x} &= f(\mathbf{x}, u) + \mathbf{w} \\
\mathbf{y} &= C\mathbf{x} + \mathbf{v}
\end{aligned}
$$

where **w** is process noise and **v** is observation noise with zero mean multivariate normal distribution with covariances Q and R, $\mathcal{N}(0, Q)$, $\mathcal{N}(0, R)$.

EXF Discrete

Predict	
Predicted (*a priori*) state estimate	$\hat{x}_{k\|k-1} = f(\hat{x}_{k-1\|k-1}, u_k)$
Predicted (*a priori*) error covariance	$P_{k\|k-1} = FP_{k-1\|k-1}F^\top + Q$
Update	
Innovation	$z = y_k - C\hat{x}_{k\|k-1}$
Innovation Covariance	$S_k = R + CP_{k\|k-1}C^\top$
Kalman gain	$K_k = P_{k\|k-1}CS_k^{-1}$
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Updated (*a posteriori*) estimate covariance	$P_{k\|k} = (I - K_k C)P_{k\|k-1}$

where F is the jacobian of $f(x, u)$ with respect to x.

EKF Parameter Estimation

Augment the states with the parameters s.t.

$$x_a = \begin{bmatrix} x \\ p \end{bmatrix}$$

and the system becomes

$$\mathbf{x}_a = f(\mathbf{x}_a, u) + \mathbf{w}$$
$$\mathbf{y} = C\mathbf{x}_{\mathbf{new}} + \mathbf{v}$$

System

$$
\begin{aligned}
x &= f(x, u) \\
y &= C\mathbf{x}
\end{aligned}
$$

where

$$
\begin{aligned}
x &= [\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, Jx, Jy, Jz]^\top \\
y &= [\phi, \theta, \psi]^\top \\
f &: \mathbf{R}^9 \to \mathbf{R}^9 \\
C &: \mathbf{R}^9 \to \mathbf{R}^3
\end{aligned}
$$

where Jx, Jy, Jz are the moments of inertia and the parameters to be estimated.

Observability Theorem

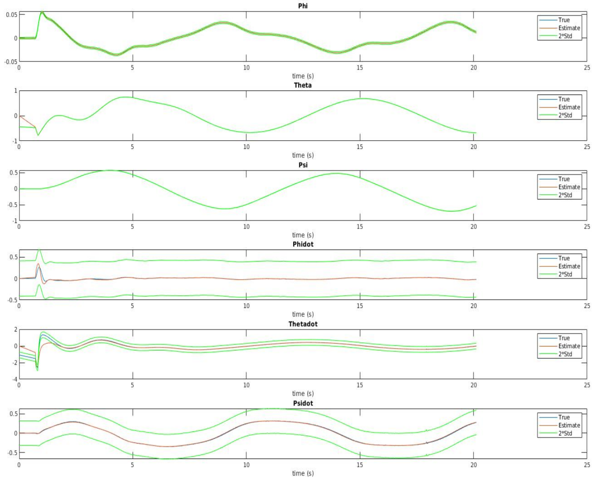
Using the non lienar system

$$\begin{aligned} \dot{x} &= f(x) \\ y &= Cx \end{aligned}$$

Observability: Let $\mathcal{O}_s$ be the observation space containing all repeated lie derivatives of y, then the system is locally observable in x_o iff $\dim(\frac{\partial \mathcal{O}_s(x_o)}{\partial x}) = n$. Where n is the number of states.

$$\mathcal{O}_s = \begin{bmatrix} y = Cx \\ \dot{y} = Cf(x) \\ \ddot{y} = C\frac{\partial f}{\partial x} f(x) \\ \vdots \end{bmatrix}$$

The rank of $\frac{\partial \mathcal{O}_s}{\partial x}$ for the WB is 9. Thus observable at certain configurations.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Estimate
2*Std
time (s)
Theta
time (s)
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Psidot
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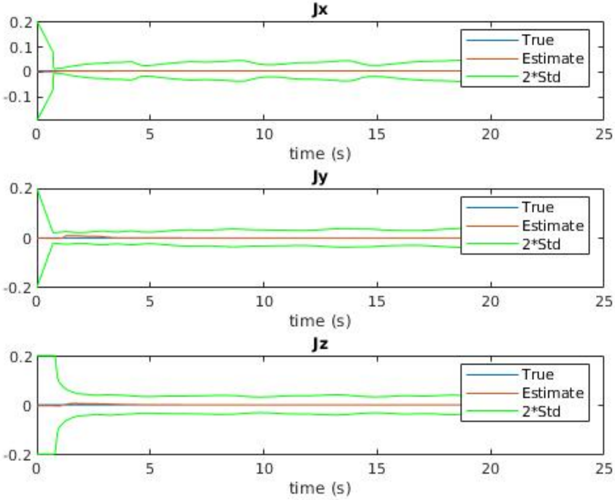
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Jz
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time (s)
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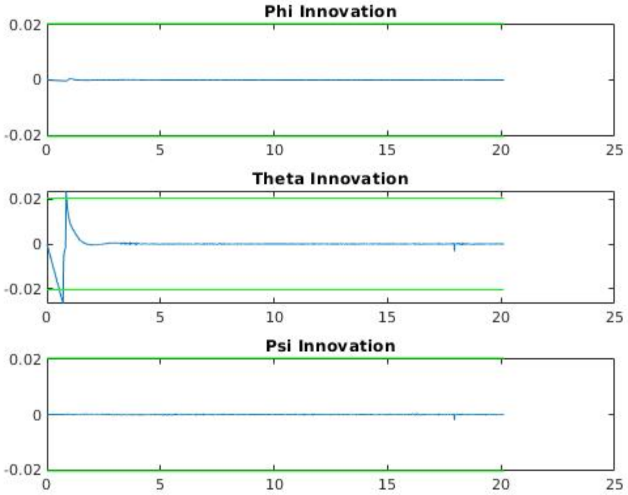
Estimate
2*Std


Phi Innovation
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Psi Innovation
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