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Problem 1

```
syms w

A = [ 0 1 0 0;...
    3*w^2 0 0 2*w;...
    0 0 0 1;...
    0 -2*w 0 1];

B = [0 0;...
     1 0;...
     0 0;...
     0 1];

 % a) Show that the system is controllable from u

 % Controllability matrix test. Rank(C) = dim(A)
 Ctr = [ B A*B A^2*B A^3*B];
 rank(Ctr);

 % Eigen vector test. rank([A-lambdaI, B]) = dim(A) for every eigen
 value
 [V,D] = eig(A);
 for i = 1:4
    rank([A-D(i,i)*eye(4), B]);
 end

 % b) Can the system still be controlled if the radial thruster fails?
 What
 % if the tangential thruster fails?

 % radial thruster test
 br = B(:,1);
 Ctr_r = [ br A*br A^2*br A^3*br];
 rank(Ctr_r)

 % tangential thruster test
 bt = B(:,2);
 Ctr_t = [ bt A*bt A^2*bt A^3*bt];
 rank(Ctr_t)
```

Problem 2

```
% a) Linearize the system around the equilibrium point x1 = x2 =x3
=0. Is
% the system controllable.

A = [-1 0 0; 0 -1 0; 0 0 0];
B = [1 0; 0 1; 0 0];
rank(ctrb(A,B))

% b) Linearize the system around the equilbrium point x1=x2=x3 = 1.
Is
% this system controllable.
A = [-1 0 0; 0 -1 0; -1 1 0];
B = [1 0; 0 1; 1 -1];
rank(ctrb(A,B))
```

Problem 3

```
% Consider an LTI system
A = [-1 0; 0 -1];
B = [-1;1];
C = [1 0; 0 1];
D = [2;1];

% a) Is this system realizable
rank(ctrb(A,B));

% b ) Perform a similarity transformation to obtain a controllable
% realization.

Ctr = ctrb(A,B); % Get the first n_bar l.i. columns of the
controllability matrix.
                 % Also, get the null space of Ctr.

T = [Ctr(:,1),[1;1]];  % Create the similarity transformation

A_bar = inv(T)*A*T;
B_bar = inv(T)*B;
C_bar = C*T;

% Verify

syms s
C*inv(s*eye(2) -A)*B;

C_bar(:,1)*inv(s - A_bar(1,1))*B_bar(1)

C_bar*inv(s*eye(2)-A_bar)*B_bar
```

Problem 4

```
A = [ 6  4  1;...
    -5 -4  0;...
    -4 -3 -1];

B = [1 -1 -1]';

alpha = charpoly(A);

C = ctrb(A,B);

temp = [1 alpha(2), alpha(3);...
        0   1        alpha(2);...
        0   0           1];

T = C*temp
```

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