

---

## Table of Contents

Problem 2 .....	1
a) .....	1
c) .....	1
d) .....	1
e) .....	2

## Problem 2

```
A = [0      1   0          0;...
      0 -0.1 -0.98      1;...
      0      0   0          1;...
      0  0.1 10.78     -11];
```

```
B = [0      0;...
      0.1 -0.1;...
      0      0;...
      -0.1  1.1];
```

**a)**

Compute the modes of the system

```
[V,J] = eig(A);
```

**c)**

Describe how much each input effects each of the modes

```
w = inv(V);
```

```
% Force affect on modes
```

```
F_M1 = w(1,:)*B(:,1); % First mode
F_M2 = w(2,:)*B(:,1); % Second mode
F_M3 = w(3,:)*B(:,1); % Third mode
F_M4 = w(4,:)*B(:,1); % Fourth mode
```

```
% Torque affect on modes
```

```
T_M1 = w(1,:)*B(:,2); % First mode
T_M2 = w(2,:)*B(:,2); % Second mode
T_M3 = w(3,:)*B(:,2); % Third mode
T_M4 = w(4,:)*B(:,2); % Fourth mode
```

**d)**

Determine an initial condition  $x_0$  such that if  $x(0) = x_0$ , then  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$ ; We need  $x_0$  to not affect the nodes where the eigen values are non zero. this can be done by constructing  $x_0$  to be in the null space of  $w_k$  that corresponds to the unstable nodes.

---

```

wn = [w(1,:);w(4,:)];    % The two vectors corresponding to the
unstable nodes
[~,~,v] = svd(wn);       % Compute the svd to get the null space
vn = v(:,3:4);           % Extract the null space
P = vn*vn'/norm(vn*vn'); % Create a projection matrix that will
project any
                           % vector onto the null space of wn.

% Test
% x_array = P*randn(4,10);
% temp = w*x_array;
% temp = round(temp,6,'significant');
% % temp(4,:) = 0;
% expm(J*80)*temp    %machine precision error

```

## e

If the rank of the controllability matrix is the same as the number of states, then the system is fully controllable.

```

% Controllability matrix with only force
CO_F = ctrb(A,B(:,1));
rank(CO_F);

% Controllability matrix with only torque
CO_T = ctrb(A,B(:,2));
rank(CO_T);

```

*Published with MATLAB® R2018a*