6.2) Compute A^t and e^{At} for the following matrices

$$A_1=\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix},\quad A_2=\begin{bmatrix}1&1&0\\0&0&1\\0&0&1\end{bmatrix},\quad A_3=\begin{bmatrix}2&0&0&0\\2&2&0&0\\0&0&3&3\\0&0&0&3\end{bmatrix}$$

A_1) $$A_1A_1=\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&1+1&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1&2&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&1+2&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$A^t=\begin{bmatrix}1&t&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\lambda^t&t\lambda^{t-1}&0\\0&\lambda^t&0\\0&0&\lambda^t\end{bmatrix}\quad ✓$$

$$e^{At}=\mathcal{L}^{-1}(sI-A)^{-1}$$

$$\Rightarrow\begin{bmatrix}s-1&-1&0\\0&s-1&0\\0&0&s-1\end{bmatrix}\quad \text{cofactor}\begin{bmatrix}(s-1)^2&0&0\\(s-1)&(s-1)^2&0\\0&0&(s-1)^2\end{bmatrix}\quad \text{adjugate}=\begin{bmatrix}(s-1)^2&(s-1)&0\\0&(s-1)^2&0\\0&0&(s-1)^2\end{bmatrix}$$

$$\det(sI-A)=(s-1)^3$$

$$(sI-A)^{-1}=\begin{bmatrix}\frac{1}{s-1}&\frac{1}{(s-1)^2}&0\\0&\frac{1}{s-1}&0\\0&0&\frac{1}{s-1}\end{bmatrix}\qquad \mathcal{L}^{-1}(sI-A)^{-1}=e^{t}\begin{bmatrix}1&t&0\\0&1&0\\0&0&1\end{bmatrix}u(t)\quad ✓$$

A_2) $$A=\begin{bmatrix}1&1&0\\0&0&1\\0&0&1\end{bmatrix}\qquad A^2=\begin{bmatrix}1&1&a_{13}+1\\0&0&1\\0&0&1\end{bmatrix}\qquad \text{thus } A^t=\begin{bmatrix}1&1&t-1\\0&0&1\\0&0&1\end{bmatrix}\quad ✓$$

$$e^{At}=\mathcal{L}^{-1}(sI-A)^{-1}$$

$$\Rightarrow\begin{bmatrix}s-1&-1&0\\0&s&-1\\0&0&s-1\end{bmatrix}\quad \text{cofactor}\begin{bmatrix}s(s-1)&0&0\\s-1&(s-1)^2&0\\1&s-1&s(s-1)\end{bmatrix}\quad \text{adjugate}=\begin{bmatrix}s(s-1)&s-1&1\\0&(s-1)^2&s-1\\0&0&s(s-1)\end{bmatrix}$$

$$\det(sI-A)=s(s-1)^2$$

$$(sI-A)^{-1}=\begin{bmatrix}\frac{1}{s-1}&\frac{1}{s(s-1)}&\frac{1}{s(s-1)^2}\\0&\frac{1}{s}&\frac{1}{s(s-1)}\\0&0&\frac{1}{(s-1)}\end{bmatrix}\qquad \frac{1}{s(s-1)}=\frac{1}{s}+\frac{-1}{s-1}$$

$$\frac{1}{s(s-1)^2}=\frac{1}{s}+\frac{-1}{(s-1)}+\frac{1}{(s-1)^2}$$

s^2-2s+1

$$=(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & \frac{1}{s}-\frac{1}{s-1} & \frac{1}{s}-\frac{1}{s-1}+\frac{1}{(s-1)^2} \\ 0 & c & \frac{1}{s}-\frac{1}{s-1} \\ 0 & 0 & \frac{1}{s-1} \end{bmatrix}$$

$$\mathcal{L}^{-1}(sI-A)^{-1} = \begin{bmatrix} e^{t} & 1-e^{t} & 1-e^{t}+te^{t} \\ 0 & 1 & 1-e^{t} \\ 0 & 0 & e^{t} \end{bmatrix} u(t)$$

A3) $A_3 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$ where $B_1 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ & $B_2 = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}$

$$A_3^k = \begin{bmatrix} B_1^k & 0 \\ 0 & B_2^k \end{bmatrix} \quad \& \quad e^{At} = \begin{bmatrix} e^{B_1 t} & 0 \\ 0 & e^{B_2 t} \end{bmatrix}$$

$$B_1^2 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 \\ 2\cdot 2 & 2^2 \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda^k & 0 \\ k\lambda^{(k-1)} & \lambda^k \end{bmatrix} = \begin{bmatrix} 2^k & 0 \\ k\,2^{(k-1)} & 2^k \end{bmatrix}$$

$$B_2^k = \begin{bmatrix} \lambda^k & k\lambda^{(k-1)} \\ 0 & \lambda^k \end{bmatrix} \text{ From first part } = \begin{bmatrix} 3^k & k3^{(k-1)} \\ 0 & 3^k \end{bmatrix}$$

$$A^k = \begin{bmatrix} 2^k & 0 & 0 & 0 \\ k2^{(k-1)} & 2^k & 0 & 0 \\ 0 & 0 & 3^k & k3^{(k-1)} \\ 0 & 0 & 0 & 3^k \end{bmatrix}$$

✓

$$e^{At} = \begin{bmatrix} e^{B_1 t} & 0 \\ 0 & e^{B_2 t} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \qquad \mathcal{L}^{-1}(sI-B_1) = \mathcal{L}^{-1}\begin{bmatrix} s-2 & 0 \\ 2 & s-2 \end{bmatrix}\frac{1}{(s-2)^2} = \begin{bmatrix} e^{2t} & 0 \\ 2te^{2t} & e^{2t} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix} \qquad \mathcal{L}^{-1}(sI-B_2) = \begin{bmatrix} e^{3t} & 3e^{3t}t \\ 0 & e^{3t} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{2t} & 0 & 0 & 0 \\ 2te^{2t} & 0 & 0 & 0 \\ 0 & 0 & e^{3t} & 3te^{3t} \\ 0 & 0 & 0 & e^{3t} \end{bmatrix}$$

7.2 2^{nd} Edition)

Consider the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

a) compute the characteristic & minimum polynomial of A.

$$\det(\lambda I - A) = \begin{vmatrix} s-1 & 1 & 0 \\ 0 & s-2 & 0 \\ 0 & 0 & s-2 \end{vmatrix} = (s-1)(s-2)^2 \Rightarrow (s-1)(s^2-4s+4)$$
$$\Rightarrow s^3 - 4s^2 + 4s - s^2 + 4s - 4 = s^3 - 5s^2 + 8s - 4 \checkmark$$

$$A^3 - 5A^2 + 8A - 4I = 0I \checkmark$$

b) Is this matrix diagonalizable? If so diagonalize it, otherwise compute its Jordan normal form.

$$\lambda = 1 \quad (\lambda I - A) = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \quad \begin{matrix} x_2 = 0 \\ x_2 = 0 \\ -x_2 - x_3 = 0 \end{matrix} \quad v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 2 \quad (A - \lambda I) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} -x_1 + x_2 = 0 \\ x_3 = \text{anything} \\ x_2 = 0 \end{matrix} \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(A - \lambda I)v_3 = v_2 \quad \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \\ v_3 \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{matrix} -x_1 + x_2 = 0 \quad x_1 = -1 \\ x_3 = \text{anything} \\ x_2 = -1 \end{matrix} \quad v_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \checkmark$$

c) Compute e^{At}

$$e^{At} = V e^{Jt} V^{-1} = V \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \end{bmatrix} V^{-1}$$

7.

8.2) For a given matrix A, construct vectors for which (8.2) holds for each of the three norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$.

(8.2) For every matrix A $\exists$ a vector $x^* \in \mathbb{R}^n$ for which $\|A\|_p = \frac{\|Ax^*\|_p}{\|x^*\|_p}$

$\|A\|_2 = \sigma_{max}[A]$ where $\sigma_{max}[A]$ denotes the larges singular value of A

let v_1 correspond to the left eigen of σ_{max}

✓ $\|A\|_2 = \|Ax^*\|_2 / \|x^*\|_2 = \|U\Sigma V^T v_1\| / \|v_1\| = \|U\Sigma e_1\|_2 = \|U e_1 \sigma_{max}\| = \|U e_1\|_2 \|\sigma_{max}\| = \|\sigma_{max}\|$

$\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$

let x^* be $e_i \in \mathbb{R}^n$ where i indicates the non-zero element = to one.

✓ $\|A\|_1 = \|Ax^*\|_1 / \|x^*\|_1 = \|Ae_i\| / \|e_i\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$

$\|A\|_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |a_{ij}|$ or max row sum

let x^* be $v \in \mathbb{R}^n$ where v is filled elements (α) of the same value whose sign is the same as the elements in the max abs row.

$\|A\|_\infty = \|Ax^*\|_\infty / \|x^*\|_\infty = \|Av\|_\infty / \|v\|_\infty$

$$= \left\| \begin{matrix} \alpha a_{11} + \alpha a_{12} + \dots + \alpha a_{1n} \\ \alpha a_{21} + \alpha a_{22} + \dots + \alpha a_{2n} \\ \alpha a_{m1} + \alpha a_{m2} + \dots + \alpha a_{mn} \end{matrix} \right\| / \alpha = \|\alpha\| \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}| / \|\alpha\| = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|$$

8.4) Consider a linear system with a state-transition matrix $\Phi(t,\tau)$ for which

$$\Phi(t,0) = \begin{bmatrix} e^{t} \cos 2t & e^{-2t} \sin 2t \\ -e^{t} \sin 2t & e^{-2t} \cos 2t \end{bmatrix}$$

See Matlab

c) Compute the eigenvalues of $A(t)$

$$\det(A-sI) = \det\begin{bmatrix} 3\cos(\phi)/2 - 1/2 - s & 2-3\sin(\phi)/2 \\ -3\sin(\phi)/2 - 2 & -1/2 - 3\cos(\phi)/2 - s \end{bmatrix} \quad \text{where } \phi = 4t-4t_0$$

$$= \left(3\cos(\phi)/2 - 1/2 - s\right)\left(-1/2 - 3\cos\phi/2 - s\right) - \left(2-3\sin(\phi)/2\right)\left(-3\sin(\phi)/2 - 2\right)$$

$$= -9\cos^2\phi/4 - \cancel{3\cos\phi/4} - s\cos\phi/2 + 1/4 + \cancel{3\cos\phi/4} + 1/2 s + 1/2 s + s\cos\phi/2 + s^2$$
$$\left[+3\cos(\phi) + 4 - 9\sin^2\phi/4 - 3\sin(\phi)\right]$$

$$= -9/4\left(c_\phi^2 + s_\phi^2\right) + 1/4 + s + s^2 + 4 - 3s_\phi + 3c_\phi$$

$$= s^2 + s + 2 - 3s_\phi + 3c_\phi$$

See Matlab

$$\lambda = \frac{-1 \pm \sqrt{1-4(2-3s_\phi+3c_\phi)^2}}{2} = \frac{-1}{2} \pm \frac{\sqrt{1-4(4-6s_\phi+6c_\phi - 6s_\phi + 9s_\phi^2 - 9s_\phi c_\phi + 6c_\phi - 9c_\phi s_\phi + 9c_\phi^2}}{2}$$

$$\lambda = \frac{-1}{2} \pm \sqrt{1-4(13-12s_\phi+12c_\phi - 18c_\phi s_\phi)}$$

using Matlab for the rest

$$\lambda = -1/2 - \tfrac{1}{2}\sqrt{7}\,j \quad \text{and} \quad -1/2 + 1/2\sqrt{7}\,j$$

d) Classify this system in terms of Lyapunove stability

Looking at the state transition matrix it looks unstable.

3.5) verify that $(e^{At})' = e^{A't}$

$$e^{At} = \sum_{k=0}^{\infty}\frac{t^k}{k!}A^k \Rightarrow (e^{At})' = \left(\sum_{k=0}^{\infty}\frac{t^k}{k!}A^k\right)' = \sum_{k=0}^{\infty}\frac{t^k}{k!}(A^k)' = \sum_{k=0}^{\infty}\frac{t^k}{k!}(A')^k = e^{A't}$$

8.6) Consider the Continuous-time LTI system

$$\dot{x} = Ax \qquad \lambda \in \mathbb{R}^n$$

and suppose that there exists a positive constant μ and positive-definite matrices $P, Q \in \mathbb{R}^n$ for the Lyapunov equation

$$A'P + PA + 2\mu P = -Q$$

Show that all eigenvalues of A have real parts less than $-\mu$.

For clearification Let $B = A + \upsilon I$ then

$$B'P_b + P_bB = -Q_b \Rightarrow (A' + \upsilon I)P_b + P_b(A + \upsilon I) = -Q_b \Rightarrow A'P_b + P_bA + 2\upsilon P_b = -Q_b$$

Thus according to Theorem 8.2 all of the eigen values of B have strictly negative real parts. $\text{eig}(B) < 0$

$B = A + \upsilon I = V^{-1}\Lambda V + \upsilon I$ wher V & Λ are the eigen vecters and eigen values of A, then

$B = V^{-1}\Lambda V + \upsilon V^{-1}IV = V^{-1}(\Lambda + \upsilon I)V$ where $\Lambda + \upsilon I$ are the eigen values of B thus $\Lambda + \upsilon I < 0 I$ and $\Lambda < -\upsilon I$

Thus all the eigen values of A have real parts less than $-\upsilon$

8.7) Investigate whether or not the solutions to the following nonlinear systems converge to the given equilibrium point when they start close enough to it

a) The State Space System

$$\dot{x}_1 = -x_1 + x_1(x_1^2 + x_2^2) = f_1(x_1, x_2)$$
$$\dot{x}_2 = -x_2 + x_2(x_1^2 + x_2^2) = f_2(x_1, x_2)$$

whith equilibrium points $x_1 = x_2 = 0$

$$\partial f_1/\partial x_1 |_{eq} = -1 + x_1^2 + x_2^2 |_{eq} = -1$$
$$\partial f_1/\partial x_2 |_{eq} = 0 + 2x_1x_2 |_{eq} = 0$$
$$\partial f_2/\partial x_2 |_{eq} = 0 + 2x_1x_2 |_{eq} = 0$$
$$\partial f_2/\partial x_2 |_{eq} = -1 + x_1^2 + x_2^2 |_{eq} = -1$$

$\dot{x} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{A} x$ thus $x(t) = e^{A}x(0)$ The eigen values of A are negative thus the system will converge to the equilibrium point when near it.

b) The second order system

$$\ddot{w} + g(w)\dot{w} + w = 0 \quad \Rightarrow \quad \ddot{w} = -g(w)\dot{w} - w = f(w,\dot{w})$$

with the equilibrium point $w = \dot{w} = 0$. Determine for which values of $g(0)$ we can guarantee convergence to the origin based on the local linearization

$$\frac{\partial f}{\partial w}\Big|_{eq} = -1$$

$$\frac{\partial f}{\partial \dot{w}}\Big|_{eq} = -g(w)\Big|_{eq} = -g(0)$$

$$\partial \ddot{w} = -g(0)\,\delta\dot{w} - \delta w \qquad \text{where } \delta = x(t) - x_{eq}$$

Let $r(w,\dot{w}) = f(w,\dot{w}) - \left(f(w_{eq},\dot{w}_{eq}) - g(0)\delta\dot{w} - \delta w\right) = O(\|\delta w\|^2, \|\delta\dot{w}\|^2)$

basically r is the error in the taylor series approximation.

?

Which means that there is a constant c and a ball $\bar{B}$ around equilibrium for which

$$\|r\| \le c\left[\|\delta w\|^2 + \|\delta\dot{w}\|^2\right] \qquad \forall w,\dot{w} \in \bar{B}$$

Create the Lyapunov function

$$
\begin{aligned}
V(t) &= \tfrac{1}{2}w(t)^2 + \tfrac{1}{2}\dot{w}(t) \\
\dot{V}(t) &= \delta w(t)\,\delta\dot{w}(t) + \delta\dot{w}(t) f(x) \\
&= \delta w(t)\,\delta\dot{w}(t) + \delta\dot{w}(t)\left[-g(0)\,\delta\dot{w} - \delta w + r\right] \\
&= \delta w(t)\,\delta\dot{w}(t) - \delta\dot{w}(t)\,\delta w - \delta\dot{w}(t)^2 g(0) + \delta\dot{w}(t)\, r \\
&= -\delta\dot{w}^2(t)\, g(0) + \delta\dot{w}(t)\, r \\
&\le -\delta\dot{w}^2(t)\, g(0) + c\,\delta\dot{w}(t)\left[\|\delta w\|^2 + \|\delta\dot{w}\|^2\right] \\
&\le -\|\delta\dot{w}(t)\|^2 g(0) + c\|\delta\dot{w}(t)\|\left[\|\delta w\|^2 + \|\delta\dot{w}\|^2\right] \\
&\le -\|\delta\dot{w}\|^2\left[g(0) - c\left[\|\delta w\|^2 + \|\delta\dot{w}\|\right]/\|\delta\dot{w}\|\right]
\end{aligned}
$$

We need $g(0) > c\left[\|\delta w\|^2 + \|\delta\dot{w}\|\right]/\|\delta\dot{w}\|$

Or in state space form

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -g(0) \end{bmatrix} x \qquad \text{eigen values are } \det(A - \lambda I) = \det\begin{bmatrix} -\lambda & 1 \\ -1 & -g(0)-\lambda \end{bmatrix} = -\lambda(-g(0)-\lambda) + 1$$

$$= \lambda^2 + g(0)\lambda + 1$$

$$\frac{-g(0) \pm \sqrt{g(0)^2 - 4}}{2}$$

$$g(0) \text{ s.t. } \quad -g(0) \pm \sqrt{g(0)^2 - 4} < 0$$

$g(0) > 0$ The $Re\{\lambda's\}$ are strictly negative ✓

$g(0) < 0$ the $Re\{\lambda s\}$ are strictly positive ✓

$g(0) = 0$ The $R(\lambda s)$ are 0 and eigen values are complex ✓

Problem 8_4

Table of Contents

a) Compute the state transition matrix phi(t,t0)

```
syms t t0


PHI1 = [exp(t)*cos(2*(t)) exp(-2*(t))*sin(2*(t));...
       -exp(t)*sin(2*(t)) exp(-2*(t))*cos(2*(t))];

PHI2 = [exp(t0)*cos(2*(t0)) exp(-2*(t0))*sin(2*(t0));...
       -exp(t0)*sin(2*(t0)) exp(-2*(t0))*cos(2*(t0))];

% Property 5.3 states
% PHI(t,s)PHI(s,tau) = PHI(t,tau)

% Property 5.4 states
% PHI(t,tau)^-1 = PHI(tau,t)


PHI = PHI1*inv(PHI2);
PHI = simplify(PHI)
```

b) Compute the matrix A(t) that corresponds to the given state transition matrix

```
% Phi_dot = A(t)*PHI
PHI_dot = diff(PHI,t);

A = PHI_dot*inv(PHI);
A = simplify(A)
```

c) Compute the eigen values of A(t)

```
lambda = eig(A);
lambda = simplify(lambda)
```

d) Classify this system in terms of Lyapunov stability

We can look at the stability of the sytem by analyzing the the state transitioin matrix to see if it become arbitrarily large. The sin and cos functions are bounded between [-1,1], and the exponential functions are bounded between [0,0) depending on the value of t. Thus we can analyze the stability of PHI by looking at is as t -> infinity. By inspection, we can see that |`PHI`| -> inf and t -> inf. Thus the system is not stable.

```
parmas.m
```

```
PHI =

[ sin(2*t)*sin(2*t0)*exp(2*t0 - 2*t) + cos(2*t)*cos(2*t0)*exp(t - t0),
 cos(2*t0)*sin(2*t)*exp(2*t0 - 2*t) - cos(2*t)*sin(2*t0)*exp(t - t0)]
[ cos(2*t)*sin(2*t0)*exp(2*t0 - 2*t) - cos(2*t0)*sin(2*t)*exp(t - t0),
 cos(2*t)*cos(2*t0)*exp(2*t0 - 2*t) + sin(2*t)*sin(2*t0)*exp(t - t0)]


A =

[ (3*cos(4*t))/2 - 1/2,     2 - (3*sin(4*t))/2]
[ - (3*sin(4*t))/2 - 2, - (3*cos(4*t))/2 - 1/2]


lambda =

 - (7^(1/2)*1i)/2 - 1/2
   (7^(1/2)*1i)/2 - 1/2


Undefined variable "parmas" or class "parmas.m".

Error in Prob8_4 (line 46)
parmas.m
```

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Aircraft Dynamics

x_dot = A*x + B*u

```
P.A = [-0.038   18.984   0       -32.174;...
       -0.001  -0.632    1        0;...
        0       -0.759   -0.518   0;...
        0        0        1       0];

P.B =   [10.1       0;...
          0        -0.0086;...
          0.025    -0.011;...
          0         0];
```

a.

Compute the modes of this system The mode is defined as exp(lambda_i*t)Vi

```
[V,lambda] = eig(P.A);
```

b.

Analyze these modes physically by looking at how the physical states play a role in each mode. Are there some modes that deal more with the pitch? Others that have more effect on the angle of attack?

```
V_r = [0.9953    0.9953    1.0000    1.0000;...
       0.0562    0.0562   -0.0005   -0.0005;...
      -0.0191   -0.0191    0.0007    0.0007;...
       0.0500    0.0500   -0.0011   -0.0011];

lambda_r = [-0.5825 ;...
            -0.5825 ;...
            -0.0115 ;...
            -0.0115];...
```

We can analyze these modes physically by looking at their real parts. If the initial state of the aircraft is along the span of any of thses modes, then the states of the aircraft will evolve as X(t) = sigma exp(lambda_i*t)X(0) where X(0) = sigma*V_i and sigma is a scalar. By looking at the eigen vector associated with the mode, we can see how some physical states are affected vs other physical states. In all of the modes, velocity is affected the most. The first two modes seem to have more affect on the pitch and and angle of attack than the other two modes.

c.

Aircraft generally exhibit two longitudinal motions (also called modes), a phugoid and short period mode. Phugoid represents the coupling between the vehicle altitude and the airspeed, while the short period mode (whith much faster dynamics) is the coupling between the angle of attack and the pitch rate. Identify the values from (a) that best represent the phugoid and short period modes respectively.

The first two modes show a strong coupling between the angle of attack and the pitch rate, and they have faster dynamics since the eigen values are larger, these must be the short period modes. By default the other two modes must be the phugoid modes.

d.

Given an input this aircraft wil respond along a linear combination of the modes. Determine how strongly each of the inputs will effect each of the different modes. Will one input effect one physical parameter more strongly?

```
W = inv(V);

% Throttle along mode one
th_v1 = W(1,:)*P.B(:,1)

% Throttle along mode two
th_v2 = W(2,:)*P.B(:,1)

% Throttle along mode three
th_v3 = W(3,:)*P.B(:,1)

% Throttle along mode four
th_v4 = W(4,:)*P.B(:,1)

% Elevator along mode one
el_v1 = W(1,:)*P.B(:,2)

% Elevator along mode two
el_v2 = W(2,:)*P.B(:,2)

% Elevator along mode three
el_v3 = W(3,:)*P.B(:,2)

% Elevator along mode four
el_v4 = W(4,:)*P.B(:,2)
```

```
th_v1 =

  -0.0326 - 0.1605i


th_v2 =

  -0.0326 + 0.1605i
```

```
th_v3 =

   5.0825 + 1.9645i


th_v4 =

   5.0825 - 1.9645i


el_v1 =

  -0.0214 + 0.1176i


el_v2 =

  -0.0214 - 0.1176i


el_v3 =

   0.0213 - 0.0557i


el_v4 =

   0.0213 + 0.0557i
```

Throttle affects modes 3 and 4 the most, and the elevator commands affects all the modes about equally in magnitude. This makes sense since modes 3 and 4 are more for air speed and throttle should affect the air speed most.

TEST

```
% syms c11 c12 c13 c21 c22 c23 c31 c32 c33
% syms v11 v12 v13 v21 v22 v23 v31 v32 v33
% syms e1 e2 e3
% syms x1 x2 x3
% C =[c11 c12 c13; c21 c22 c23; c31 c32 c33];
% V = [v11 v12 v13; v21 v22 v23; v31 v32 v33];
% E = [e1 0 0; 0 e2 0; 0 0 e3];
% X = [x1;x2;x3];
%
% C*E*V*X

% W = inv(P.V)
```

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