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Aircraft Dynamics

x_dot = A*x + B*u

```
P.A = [-0.038   18.984   0       -32.174;...
       -0.001  -0.632    1        0;...
        0       -0.759   -0.518   0;...
        0        0        1       0];

P.B =   [10.1       0;...
          0        -0.0086;...
          0.025    -0.011;...
          0         0];
```

a.

Compute the modes of this system The mode is defined as exp(lambda_i*t)Vi

```
[V,lambda] = eig(P.A);
```

b.

Analyze these modes physically by looking at how the physical states play a role in each mode. Are there some modes that deal more with the pitch? Others that have more effect on the angle of attack?

```
V_r = [0.9953    0.9953    1.0000    1.0000;...
       0.0562    0.0562   -0.0005   -0.0005;...
      -0.0191   -0.0191    0.0007    0.0007;...
       0.0500    0.0500   -0.0011   -0.0011];

lambda_r = [-0.5825 ;...
            -0.5825 ;...
            -0.0115 ;...
            -0.0115];...
```

We can analyze these modes physically by looking at their real parts. If the initial state of the aircraft is along the span of any of thses modes, then the states of the aircraft will evolve as X(t) = sigma exp(lambda_i*t)X(0) where X(0) = sigma*V_i and sigma is a scalar. By looking at the eigen vector associated with the mode, we can see how some physical states are affected vs other physical states. In all of the modes, velocity is affected the most. The first two modes seem to have more affect on the pitch and and angle of attack than the other two modes.

c.

Aircraft generally exhibit two longitudinal motions (also called modes), a phugoid and short period mode. Phugoid represents the coupling between the vehicle altitude and the airspeed, while the short period mode (whith much faster dynamics) is the coupling between the angle of attack and the pitch rate. Identify the values from (a) that best represent the phugoid and short period modes respectively.

The first two modes show a strong coupling between the angle of attack and the pitch rate, and they have faster dynamics since the eigen values are larger, these must be the short period modes. By default the other two modes must be the phugoid modes.

d.

Given an input this aircraft wil respond along a linear combination of the modes. Determine how strongly each of the inputs will effect each of the different modes. Will one input effect one physical parameter more strongly?

```
W = inv(V);

% Throttle along mode one
th_v1 = W(1,:)*P.B(:,1)

% Throttle along mode two
th_v2 = W(2,:)*P.B(:,1)

% Throttle along mode three
th_v3 = W(3,:)*P.B(:,1)

% Throttle along mode four
th_v4 = W(4,:)*P.B(:,1)

% Elevator along mode one
el_v1 = W(1,:)*P.B(:,2)

% Elevator along mode two
el_v2 = W(2,:)*P.B(:,2)

% Elevator along mode three
el_v3 = W(3,:)*P.B(:,2)

% Elevator along mode four
el_v4 = W(4,:)*P.B(:,2)
```

```
th_v1 =

  -0.0326 - 0.1605i


th_v2 =

  -0.0326 + 0.1605i
```

```
th_v3 =

   5.0825 + 1.9645i


th_v4 =

   5.0825 - 1.9645i


el_v1 =

  -0.0214 + 0.1176i


el_v2 =

  -0.0214 - 0.1176i


el_v3 =

   0.0213 - 0.0557i


el_v4 =

   0.0213 + 0.0557i
```

Throttle affects modes 3 and 4 the most, and the elevator commands affects all the modes about equally in magnitude. This makes sense since modes 3 and 4 are more for air speed and throttle should affect the air speed most.

TEST

```
% syms c11 c12 c13 c21 c22 c23 c31 c32 c33
% syms v11 v12 v13 v21 v22 v23 v31 v32 v33
% syms e1 e2 e3
% syms x1 x2 x3
% C =[c11 c12 c13; c21 c22 c23; c31 c32 c33];
% V = [v11 v12 v13; v21 v22 v23; v31 v32 v33];
% E = [e1 0 0; 0 e2 0; 0 0 e3];
% X = [x1;x2;x3];
%
% C*E*V*X

% W = inv(P.V)
```

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