Problem 8_4

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a) Compute the state transition matrix phi(t,t0)

```
syms t t0


PHI1 = [exp(t)*cos(2*(t)) exp(-2*(t))*sin(2*(t));...
       -exp(t)*sin(2*(t)) exp(-2*(t))*cos(2*(t))];

PHI2 = [exp(t0)*cos(2*(t0)) exp(-2*(t0))*sin(2*(t0));...
       -exp(t0)*sin(2*(t0)) exp(-2*(t0))*cos(2*(t0))];

% Property 5.3 states
% PHI(t,s)PHI(s,tau) = PHI(t,tau)

% Property 5.4 states
% PHI(t,tau)^-1 = PHI(tau,t)


PHI = PHI1*inv(PHI2);
PHI = simplify(PHI)
```

b) Compute the matrix A(t) that corresponds to the given state transition matrix

```
% Phi_dot = A(t)*PHI
PHI_dot = diff(PHI,t);

A = PHI_dot*inv(PHI);
A = simplify(A)
```

c) Compute the eigen values of A(t)

```
lambda = eig(A);
lambda = simplify(lambda)
```

d) Classify this system in terms of Lyapunov stability

We can look at the stability of the sytem by analyzing the the state transitioin matrix to see if it become arbitrarily large. The sin and cos functions are bounded between [-1,1], and the exponential functions are bounded between [0,0) depending on the value of t. Thus we can analyze the stability of PHI by looking at is as t -> infinity. By inspection, we can see that `|PHI|` -> inf and t -> inf. Thus the system is not stable.

```
parmas.m
```

```
PHI =

[ sin(2*t)*sin(2*t0)*exp(2*t0 - 2*t) + cos(2*t)*cos(2*t0)*exp(t - t0),
 cos(2*t0)*sin(2*t)*exp(2*t0 - 2*t) - cos(2*t)*sin(2*t0)*exp(t - t0)]
[ cos(2*t)*sin(2*t0)*exp(2*t0 - 2*t) - cos(2*t0)*sin(2*t)*exp(t - t0),
 cos(2*t)*cos(2*t0)*exp(2*t0 - 2*t) + sin(2*t)*sin(2*t0)*exp(t - t0)]


A =

[ (3*cos(4*t))/2 - 1/2,     2 - (3*sin(4*t))/2]
[ - (3*sin(4*t))/2 - 2, - (3*cos(4*t))/2 - 1/2]


lambda =

 - (7^(1/2)*1i)/2 - 1/2
   (7^(1/2)*1i)/2 - 1/2


Undefined variable "parmas" or class "parmas.m".

Error in Prob8_4 (line 46)
parmas.m
```

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