Exercise 18. (Q10): Prove that if the series $\sum_{k=1}^{\infty} |a_n|$ converges, then the series $\sum_{k=1}^{\infty} a_k$ converges.

Proof: We assume directly that $\sum_{k=1}^{\infty} |a_n|$ converges, then given an $\epsilon > 0$, there exists an $N \in \mathbb{N}$, such that whenever $n > m \in \mathbb{N} > N$,

$$\sum_{k=m}^{n} |a_k| < \epsilon.$$

Well,

$$\sum_{k=m}^{n} |a_k| \geq \left| \sum_{k=m}^{n} a_k \right|,$$

thus

$$\left| \sum_{k=m}^{n} a_k \right| < \epsilon,$$

which is the Cauchy convergent series condition for the series $\sum_{k=1}^{\infty} a_k$, therefore, $\sum_{k=1}^{\infty} a_k$ converges if $\sum_{k=1}^{\infty} |a_n|$ converges. ∎