

Exercise 9. (Q2.4): An unbounded sequence (a_n) and a convergent sequence (b_n) with $(a_n - b_n)$ bounded.

This is not possible. Let $\lim b_n = L$, then given an $\epsilon \in \mathbb{R} > 0$, there exists an $N \in \mathbb{N}$, such that whenever $n > N$,

$$\begin{aligned} |b_n - L| &< \epsilon \\ -\epsilon + L &< b_n < \epsilon + L \\ -L - \epsilon &< -b_n < -L + \epsilon \\ a_n - L - \epsilon &< a_n - b_n < a_n - L + \epsilon, \end{aligned}$$

which shows that since (a_n) isn't bounded, neither can $(a_n - b_n)$ be bounded.