Exercise 7. (Q2.2): Sequences (x_n) and (y_n), where (x_n) converges, (y_n) diverges, and $(x_n + y_n)$ converges.

This is not possible. Let L_1 denote the limit of $(x_n + y_n)$, then given some $\epsilon_1 > 0$, there exists an $N_1 \in \mathbb{N}$, such that whenever $n \in \mathbb{N} > N$,

$$|x_n + y_n - L_1| < \epsilon_1.$$

Now let L_2 denote the limit of (x_n), then given some $\epsilon_2 > 0$, there exists and $N_2 \in \mathbb{N}$ such that when $m \in \mathbb{N} > N$,

$$|x_m - L_2| < \epsilon_2.$$

Lastly, let $L_1 = L_2 + L_3$ for some $L_3 \in \mathbb{R}$ and $N = \max(N_1, N_2)$, then when $n > N$,

$$\begin{aligned}
|x_n + y_n - L_1| &< \epsilon_1 \\
|x_n + y_n - L_2 - L_3| &< \epsilon_1 \\
||x_n - L_2| - |y_n - L_3|| &< \epsilon_1 \\
|y_n - L_3| &< \epsilon_1 + \epsilon_2 \\
|y_n - L_3| &< \epsilon_3,
\end{aligned}$$

since (y_n) doesn't converge, there exists an ϵ_3 such that $|y_n - L_3| > \epsilon_3$. Thus this is a contradiction which shows that the request is impossible.