Math 565 - Homework 9

Due Monday March 22, 2021

1. Complete problem 4 on page 104 of Do Carmo.

2. Compute the components R_{ijk}^l of the curvature tensor and the sectional curvature for

   (a) the cylinder, and

   (b) the hyperbolic upper half plane.

3. Let $\omega \in \Omega^2(\mathbb{R}^3)$ be given by
$$\omega = e^{xz} dx \wedge dy - \sin(y) z^2 dy \wedge dz.$$
   Compute $d\omega$.

4. Let V be a finite dimensional vector space. Prove that there is a canonical isomorphism (basis-independent) between the space of bilinear maps of $V \times V^*$ ($V \otimes V^*$) and the space of linear maps from V to V denoted $\mathrm{Hom}(V, V)$. **Hint:** To a linear transformation $A : V \to V$ we associate a bilinear map $(v, \varphi) \mapsto \langle Av, \varphi \rangle$ on $V \times V^*$ to $\mathbb{R}$.

5. Show that the restriction of $\sigma = x^1 dx^2 - x^2 dx^1 + x^3 dx^4 - x^4 dx^3$ from $\mathbb{R}^4$ to the sphere S^3 is never zero on S^3.

6. As in problem (1) of Homework 4 prove that the set of all covector fields on M is a $C^\infty(M)$ module.