Simulation Models

The Lagrangian is defined as
$L(q, \dot{q}) \triangleq K(q, \dot{q}) - P(q)$
where q represents the generalized coordinates such as position and angles.

The Euler-Lagrange equations are formed by

$$\frac{d}{dt}\left(\frac{\partial L(q, \dot{q})}{\partial \dot{q}}\right) - \frac{\partial L(q)}{q} = \tau_a - B\dot{q} \tag{1}$$

where B is a matrix with positive elements and $\tau = (\tau_1, ..., \tau_n)^\top$ are the generalized forces.

Kinetic Energy of Mechanical Systems

Point mass

$$K = \frac{1}{2} m \left\| \mathbf{v} \right\|^2 \tag{2}$$

where $\mathbf{v}$ is a vector containing the translational velocities.

Multiple point masses

$$K = \frac{1}{2} \sum_{i}^{n} m_i \left\| \mathbf{v_i} \right\|^2 \tag{3}$$

where n is the number of point masses and i is the ith point mass.

Point masses spinning about a point.

$$v_i = w \times p_i$$

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i \mathbf{v}_i \top \mathbf{v}_i \tag{4}$$

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i (\mathbf{w} \times \mathbf{p}_i)^\top (\mathbf{w} \times \mathbf{p}_i) \tag{5}$$

$$K \triangleq \mathbf{w}^\top J \mathbf{w} \tag{6}$$

where J is the inertia matrix

$$J = \sum_{i=1}^{N} m_i ((\mathbf{p}_i \top \mathbf{p}_i) I_3 - \mathbf{p}_i \mathbf{p}_i \top)$$

Potential Energy

Gravity

$$P = mgy + P_0 \tag{7}$$

where P_0 is the potential energy when $y = 0$.
Spring

$$P = \frac{1}{2}ky^2 \tag{8}$$

where k is the spring constant.