AMS 301
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Set 7

Sec 5.4: 16, 19, 21, 22, 36, 38, 42c
Sec 6.1: 2b, 4bc, 6, 8, 10, 16
Sec 6.2: 2, 20, 22

Section 5.4

16. Consider the problem of distributing 10 distinct books among three different people with each person getting at least one book. Explain why the following solution strategy is wrong: first select a book to give to the first person in 10 ways; then select a book to give to the second person in nine ways; then select a book to give to the third person in eight ways; and finally distribute the remaining seven books in 7^3 ways.

> There are two issues with this method.
> First, to distribute the remaining 7 books, there would be 3^7 ways, as this is a cases of distributing 7 distinct objects to 3 different people.
> Second, it does not account for repeating cases where the first book each person gets is repeated when distributing the remaining 7 books. For example, lets say person 1 gets book 1. Then, person 2 and 3 gets books 2 and 3 respectively. The final distribution is that person 1 gets 1, 4, 5, and 6. It does not matter what persons 2 or 3 gets. This distribution gets repeated if person 1 gets book 4 as their first book, then gets books 1, 5, and 6 when distribution the rest of the 7 books, and we assume persons 2 and 3 gets the same books to illustrate the case of repeating cases.

19. How many ways are there to distribute three different teddy bears and nine identical lollipops to four children

(a) Without restriction?

> Distribute 3 distinct objects to 4 children, so there are $4^3 = 64$ distributions. Then, to distribute 9 identical objects to 4 different children, there are $C(9+4-1, 9) = \frac{12!}{3!9!} = 220$ distributions. So, there are a total of $64 \times 220 = 14080$ distributions.

(b) With no child getting two or more teddy bears?

> We can only give a child at most 1 teddy bear, which implies that there will always be one child, so there will be $4 * (3 * 2 * 1) = 24$ distributions. Then, there are a total of $24 \times 220 = 5280$ distributions.

(c) With each child getting three “goodies”?

We only need to distribute the distinct objects. So, there will be $4^3 = 64$ distributions. The way the lollipops are distributed is trivial since they're identical.

21. Suppose a coin is tossed 12 times and there are three heads and nine tails. How many such sequences are there in which there are at least five tails in a row?

Let can describe the scenarios with x being tails and | being heads. So, we can write xx|xxxxx|x|x. $x_1 + x_2 + x_3 + x_4 = 9$ where one $x_i \geq 5$ and that $x_i = z_i + 5$. $C(9-(5)+4-1, 4-1) = \frac{(9-5+4-1)!}{(7-4+1)!(4-1)!} = 35$. There are 4 sections that can have the sequence of 5 ore more tails, so the total distributions is $35 \times 4 = 140$.

22. How many binary sequences of length 20 are there that

(a) Start with a run of 0s—that is, a consecutive sequence of (at least) one 0— then a run of 1s, then a run of 0s, then a run of 1s, and finally finish with a run of 0s?

Each sequence will look like 0_1_0_1_0_. We have to distribute the remaining 15 digits into 5 boxes. $C(15 + 5 - 1, 5 - 1) = 3876$ distributions.

(b) Repeat part (a) with the constraint that each run is of length at least 2.

Each sequence will look like 00_11_00_11_00_. We have to distribute the remaining 10 digits into 5 boxes. $C(10 + 5 - 1, 5 - 1) = 1001$ distributions.

36. How many election outcomes in the race for class president are there if there are five candidates and 40 students in the class and

(a) Every candidate receives at least two votes?

(b) One candidate receives at most one vote and all the others receive at least two votes?

(c) No candidate receives a majority of the votes?

(d) Exactly three candidates tie for the most votes?

38. How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 \leq 15$ with $x_i \geq -10$?

42. How many nonnegative integer solutions are there to $x_1 + x_2 + \cdots + x_5 = 20$
(c) With $x_1 = 2x_2$

Section 6.1
2. Build a generating function for ar, the number of integer solutions to the following equations: (b) $e_1 + e_2 + e_3 = r, 0 < e_i < 6$

4. Build a generating function for ar, the number of distributions of r identical objects into (a) Five different boxes with at most three objects in each box (b) Three different boxes with between three and six objects in each box (c) Six different boxes with at least one object in each box (d) Three different boxes with at most five objects in the first box Section 6.2