

AMS 301  
Ma, Pei  
Set 6

Sec 5.1: 16abc, 30, 36  
Sec 5.2: 8, 38, 54  
Sec 5.3: 6, 9, 15, 21, 22  
Sec 5.4: 2, 3ab, 7, 10, 11, 12, 48

Section 5.1  
16.

- (a) How many different outcomes are possible when a pair of dice, one red and one white, are rolled two successive times?

Let  $R = W = 1, 2, 3, 4, 5, 6$  where  $R$  is the set outcomes of rolling the red die and  $W$  is of the white die.

Possible outcomes in one roll can be expressed as  $R \times W$ , and if we take the cardinality,  $O = |R \times W| = |R| \times |W| = 6 * 6 = 36$ , which is the number of possible outcomes in one roll.

The number of outcomes in two successive rolls will be  $O \times O$ , and the number of possible outcomes in two rolls is  $|O \times O| = 36 * 36 = 1296$ .

- (b) What is the probability that each die shows the same value on the second roll as on the first roll?

Since there are 36 possible outcomes for a single roll, there are 36 possible outcomes for two rolls to have the same values. So, the probability that each die shows the same value on the second roll as on the first roll is  $\frac{36}{1296} = 0.46\%$ .

- (c) What is the probability that the sum of the two dice is the same on both rolls?

The possible sums are 2 to 12 (11 different outcomes). The possible outcomes for each sum are as following, 1 to 6 for sums 2 to 7, and 5 to 1 for sums 8 to 12. So, if the sum of the two rolls are the same, the total outcomes for a sum  $x$  will be the number of outcomes for that sum squared. For example, a sum of 7 has 6 outcomes for one roll, so for two rolls to be equal, there are  $6^2$  outcomes. If we sum up the outcomes for each sum,  $1^2 + 2^2 + \dots + 6^2 + 5^2 + \dots + 1^2 = 146$ . So, the probability that the sum of the two dice is the same on both rolls is  $146/1296 = 11.3\%$ .

30. How many times is the digit 5 written when listing all numbers from 1 to 100,000? Comment on the answers **(a)** 4, **(b)**  $5 \times 10^4$ , and **(c)**  $1 + 10 + 100 + 1000$ .

- (a) I don't know how they got 4. A singular number 55555 has five 5s, which is already greater than 4.
- (b) This answer is correct.
- (c) If we consider the number of numbers with 5 in the ones place, we can see by observation, that starting from 5, every 10 numbers will have a 5. Since there are 100,000 numbers, this means there are at least 10,000 numbers with at least one digit being 5.

36. If two different integers between 1 to 100 inclusive are chosen at random, what is the probability that the difference of the two numbers is 15?

The minimum integer is 1 in the pair (1, 16) and the maximum is 100 in the pair (85, 100). Then, we can see that there are 85 pairs. There are  $\binom{100}{2} = \frac{100!}{98!2!} = 100 \cdot 99/2 = 4950$  total ways to pick two different integers. So, the probability that two different integers chosen have a difference of 15 is  $85/4950 = 1.7\%$ .

## Section 5.2

8. There are nine white balls and four red balls in an urn. How many different ways are there to select a subset of six balls, assuming the 13 balls are different? What is the fraction of selections with four whites and two reds?