MA-107 Summary

1 Supply and Demand

2 Recurrence equations

Equations relating values of y at discrete points in time, expressing value y_t of y at time t as a function of y_{t-1}, one unit of time before time t.

2.1 Solving recurrence equations

Solving recurrence equation refers to finding value of y_t as a function of t.

Problem
Solve the recurrence equation

$$y_t = ay_{t-1} + b$$

for $t \geq 0$ *given that* $y_0 = C$.

Solution
After making sure that the equation you're solving is in the form

$$y_t = ay_{t-1} + b$$

you proceed as follows:

1. *If* $a = 1$ *your solution is* $y_t = y_0 + bt$. *Otherwise:*
2. *Make sure that your recurrence equation is of the form* $y_t = ay_{t-1} + b$ *for* a, b *constants.*
3. *Find* $y^* = \dfrac{b}{1-a}$ *where* a, b *are constants from the equation above.*
4. *Solution is* $y(t) = y^* + (y_0 - y^*)a^t$

2.2 Long term behaviour

Describing behaviour of function y_t as $t \to \infty$.

Problem
Given $y_t = ay_{t-1} + b$ how does y_t behave as t tends to infinity?

Solution
Assume recurrence equation $y_t = ay_{t-1} + b$ and time independent solution y^*, long term behaviour of y_t ***depends on coefficient*** $\mathbf{a}$*:*

- *If $\mathbf{1} < \mathbf{a}$, then the function* ***increases unboundedly****($y_t \to +\infty$) if $y_0 > y^*$ and the function* ***decreases unboundedly*** *$y_t \to -\infty$ $y_0 < y^*$*
- *If $\mathbf{0} < \mathbf{a} < \mathbf{1}$, then the* ***function increases towards*** *y^* ($y_t \to y^*$) if $y_0 > y^*$ and the* ***function decreases towards*** *y^* ($y_t \to y^*$) if $y_0 < y^*$*
- *If $-\mathbf{1} < \mathbf{a} < \mathbf{0}$, then y_t* ***oscillates towards*** *y^*.*
- *If $-\mathbf{1} < \mathbf{a} < \mathbf{0}$, then y_t* ***oscillates unboundedly****.*

2.3 Investment schemes

Comparing return from different investments in terms of their present value.

Problem
Given access to a bank account with interest rate r accrued annually at the end of each year and an asset with value function $V(t)$ calculate present value of selling said asset after t years.

Solution
Present value $P(t)$ is given by the equation

$$P(t) = V(t)(1 + r)^{-t}$$

Problem
Given access to a bank account with continuously compounded interest with rate r and an asset with value function $V(t)$ calculate present value of selling said asset after t years.

Solution
Present value $P(t)$ is given by the equation

$$P(t) = V(t)e^{-tr}$$