TÉCNICO LISBOA

Autonomous Agents and Multiagent Systems

MSc in Computer Science and Engineering

Second Exam (2023-2024)

Instructions

- You have 120 minutes to complete the exam.
- Make sure that your exam has a total of 13 pages. Also, check if there are no missing sheets, then write your full name and student number on this page (and your student number on all pages).
- The exam has 16 questions, with a maximum score of 20 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.
- *If you get stuck in a question, move on.* You should start with the more straightforward questions to secure those points before moving on to the more complex questions.
- *No interaction with the faculty is allowed during the exam.* If you are unclear about a question, clearly indicate the unclear part and answer the question to the best of your ability.
- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.
- This exam is a closed-book assessment, whereby students are NOT allowed to bring books or other reference material into the examination room. You may bring only ONE A4 page of handwritten notes, in your OWN handwriting. Typed notes or a copy of someone else's notes are not allowed.
- You may use a calculator, but any other type of electronic or communication equipment is not allowed.
- **Good luck!**

1 Agent architectures

Consider the algorithm in Listing 1 that implements a deliberative agent using the BDI model. Its auxiliary functions are defined as follows:

- brf: Belief revision function that updates the beliefs.
- $option$: Option generation function that produces agent's desires.
- $filter$: Select the best intention(s) that the agent will commit to.
- $plan$: Generates a plan (π) consisting of a list of actions.

Algorithm 1 Deliberative agent

Require: B_0, I_0

$B \leftarrow B_0$
$I \leftarrow I_0$
while true **do**
    get next percept ρ
    $B \leftarrow brf(B, \rho)$
    $D \leftarrow option(B, I)$
    $I \leftarrow filter(B, D, I)$
    $\pi \leftarrow plan(B, I)$
    while not ($empty(\pi)$ or
        $succeeded(I, B)$ or
        $impossible(I, B)$) **do**
        $\alpha \leftarrow head(\pi)$
        $execute(\alpha)$
        $\pi \leftarrow tail(\pi)$
        get next percept ρ
        $B \leftarrow brf(B, \rho)$
        if not $sound(\pi, I, B)$ **then**
            $\pi \leftarrow plan(B, I)$
        end if
    end while
end while

Question 1. (**0.5 pts.**)

The deliberative architecture is based on the BDI model and use the notation of intention I as states that the agent has chosen to pursue. What role do intentions play in the process of means-ends reasoning?

Write your answer here:

Solution 1.
Intentions are the foundation of means-ends reasoning. They play the role of constraining the decisions. They represent commitments to specific goals (desires) and restrict both the beliefs (internal state) and the options that the agent considers to define a plan.

Question 2. (**1 pts.**)
Describe the process of deliberation and identify in Algorithm 1 the lines that implement the deliberation routine.

Write your answer here:

Solution 2.
The deliberation process consists in updating the current intention, it consists of three components: Belief revision (implemented by brf function), option exploration (implemented by $options$ function) and selecting one or more intentions and commit to their achivement (implemented by $filter$). The deliberation processes consists of lines 5-7.

Question 3. (**0.5 pts.**)
There are type of commitment is implemented in Listing 1? Justify your answer.

Write your answer here:

Solution 3.
Single-minded commitment because it maintains a certain intention I through the loop in lines 9-17. The intention is fixed until it either is achieved or is no longer possible.

2 Normal-form games

Question 4. (1 pts.)
Given the following payoff matrix for a 2-agent normal-form game. Find and eliminate strictly dominated actions.

	x	y	z
a	0, 1	1, 3	2, 4
b	4, 5	2, 3	5, 2
c	3, 3	5, 7	3, 1

Write your answer here:

Solution 4.
Action *a* is strictly dominated as $U_1(a,i) < U_1(b,i)$ and $U_1(a,i) < U_1(c,i)$ for $i \in \{x,y,z\}$.
After eliminating action *a* we are left with the following game.

	x	y	z
b	4, 5	2, 3	5, 2
c	3, 3	5, 7	3, 1

Action *z* is strictly dominated as $U_2(i,z) < U_2(i,x)$ and $U_2(i,z) < U_2(i,y)$ for $i \in \{b,c\}$.
After eliminating action *z* we are left with the following game.

	x	y
b	4, 5	2, 3
c	3, 3	5, 7

Question 5. (1 pts.)
Compute the pure Nash Equilibria of the previous game.
Write your answer here:

SolutionSolution 5.5.
To find the Nash Equilibria you need to find the best response for each player given the potential action of the other. These are underlined in the matrix. You can use the simplified matrix for this purpose as it is an equivalent game.

	x	y
b	$\underline{4}$, $\underline{5}$	2, 3
c	3, 3	$\underline{5}$, $\underline{7}$

The NE are (b,x) and (c,y).

Question 6. (**1 pts.**)

Define the concept of mixed strategy Nash equilibrium in normal-form games. Does every normal-form game have a mixed strategy Nash equilibrium?

Write your answer here:

Solution 6.

In a mixed strategy equilibrium, agents choose their actions probabilistically, rather than deterministically. It represents a situation where agents randomize their actions to achieve the best possible expected payoff.

This differs from pure strategy Nash equilibrium, where agents choose a specific action with certainty and the joint action is a Nash equilibrium (as defined in the previous answer).

Every normal-form game with a finite number of actions for each agent, has at least one mixed strategy Nash equilibrium.

Question 7. (**2 pts.**)

If possible, compute the mixed strategy Nash Equilibrium of the previous game. (Hint: use the simplified version obtained from the elimination of the strictly dominated actions).

Write your answer here:

Solution 7.

We start from the simplified game

	x	y
b	4, 5	2, 3
c	3, 3	5, 7

In order to find the mixed strategy NE, let us suppose that Agent 1 believes that Agent 2 will choose *x* with probability p and *y* with probability 1 - p. If Agent 1 best responds with a mixed strategy, then Agent 2 must make him indifferent between *b* and *c*:

$$EU_1(b) = EU_1(c) \rightarrow 4p + 2(1-p) = 3p + 5(1-p) \rightarrow p = \frac{3}{4}$$

Suppose that Agent 2 believes that Agent 1 will choose *b* with probability q and *c* with probability 1 - q. If Agent 2 best responds with a mixed strategy, then Agent 1 must make her indifferent between x and y:

$$EU_2(x) = EU_2(y) \rightarrow 5q + 3(1-q) = 3q + 7(1-q) \rightarrow q = \frac{2}{3}$$

Hence, the mixed strategy Nash equilibrium is $(\frac{2}{3}, \frac{1}{3}), (\frac{3}{4}, \frac{1}{4})$.

Question 8. (**1 pts.**)

Consider the simplified game of the previous questions. Explain how it can be solved by using social conventions.

Write your answer here:

Solution 8.

Using Social Conventions as a coordination mechanism defines an ordering scheme.

Ordering the agents, for example: Agent 1 $\succ$ Agent 2, gives priority to Agent 1, who decides first, hence chooses action b.

For this reason, according to the proposed ordering scheme, the selected joint action is (b, x).

An ordering of actions is not needed in this case.

3 Applications of a Nash equilibrium: Cournot Model

Question 9. (**2 pts.**)
Consider a market with two firms that produce homogeneous products. Firm A has a constant marginal cost of \$20 per unit, while firm B has a constant marginal cost of \$10 per unit. The market demand function is given by $P = 100 - 2Q$, where Q is the total quantity and P is the price. Calculate the Cournot equilibrium output for each firm and the price.

Write your answer here:

Solution 9.
To find the Cournot equilibrium output and price, we can use the following steps:

- Firm A's payoff is $\pi_A = Pq_A - c_A = (100 - 2(q_A + q_B))q_A - 20q_A = -2q_A^2 - 2q_Bq_A + 80q_A$
- Firm B's payoff is $\pi_B = Pq_B - c_B = (100 - 2(q_A + q_B))q_B - 10q_B = -2q_B^2 - 2q_Aq_B + 90q_B$
- Firm A's first order condition: $\frac{d}{dq_A}\pi_A = 0$

  $-4q_A - 2q_B + 80 = 0$

  $q_A = \frac{40-q_B}{2}$
- Firm B's first order condition: $\frac{d}{dq_B}\pi_B = 0$

  $-4q_B - 2q_A + 90 = 0$

  $q_B = \frac{45-q_A}{2}$
- We now need to solve the following pair of equations:

  $q_A = \frac{40-q_B}{2}$

  $q_B = \frac{45-q_A}{2}$
- Hence, the Nash equilibrium is:

  $q_A = \frac{35}{3}$

  $q_B = \frac{50}{3}$
- And the price is $P = \frac{130}{3}$

4 Extensive-Form Games

Consider the following extensive-form game, see Figure 1. Nodes inside the same dashed rectangle belong to the same information set.

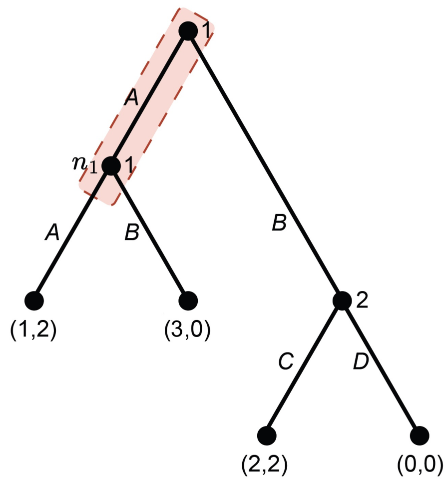

Figure 1: Extensive form game with two players, 1 and 2. Player 1 has actions `A` and `B`. Player 2 has actions `C` and `D`. Nodes inside the same dashed rectangle belong to the same information set.

Question 10. (**1 pts.**)
Does this game satisfy the perfect recall condition? Justify your answer.

Write your answer here:

Solution 10.
No, for example, for the first terminal node, player 1 does not know if he played one or two actions, in this case, `A` or `AA`, respectively.

Question 11. (2 pts.)

Assuming that player 1 plays the behaviour strategy (p, 1-p), compute the behavioural strategy equilibria for both players. Justify your answer.

Write your answer here:

Solution 11.

p is the probability of player 1 choosing `A`.

$$\begin{aligned} \mathsf{EU}(1) &= p^2 + 3p(1-p) + 2(1-p) \\ &= -2p^2 + p + 2 \\ \frac{\partial - 2p^2 + p + 2}{\partial p} &= 0 \\ -4p + 1 &= 0 \\ p &= \frac{1}{4} \end{aligned}$$

The mixed strategy is $(\frac{1}{4}, \frac{3}{4})$,$(1, 0)$ for player 1 and 2, respectively.

5 Bayesian games

Consider the two-agent game below

	Z	W
X	α,3	α,1
Y	1,0	1,β

where $\alpha \in \{0, 2\}$ is known by agent 1 (row agent), $\beta \in \{1, 3\}$ is known by agent 2 (column agent), and $P(\alpha = 0, \beta = 1) = P(\alpha = 0, \beta = 3) = P(\alpha = 2, \beta = 1) = P(\alpha = 2, \beta = 3) = 1/4$.

Question 12. (**3 pts.**)

Formalize the game above as a Bayesian game and find the Bayesian Nash equilibrium.

Write your answer here:

Solution 12.

The Bayesian game is defined by the tuple (N, A, θ, P, u) where:

- $N = \{1, 2\}$ is the set of agents.
- $A_1 = \{X, Y\}$, $A_2 = \{Z, W\}$ and $A = A_1 \times A_2$.
- $\theta_1 = \{0, 2\}$ and $\theta_2 = \{1, 3\}$ are the set of types for each agent and $\theta = \theta_1 \times \theta_2$.
- $P(\theta_1 = 0, \theta_2 = 1) = P(\theta_1 = 0, \theta_2 = 3) = P(\theta_1 = 2, \theta_2 = 1) = P(\theta_1 = 2, \theta_2 = 3) = 1/4$ is the prior over types.
- $u = (u_1, u_2)$ is the utility function for each agent such that $u_1(a_1, a_2, \theta_1, \theta_2)$ and $u_2(a_1, a_2, \theta_1, \theta_2)$ are given by the by payoff matrix above.

For agent 1 it holds that, when $\theta_1 = 0$, Y strictly dominates X; when $\theta_1 = 2$, X strictly dominates Y. Hence, $a_1^*(\theta_1 = 0) = Y$ and $a_1^*(\theta_1 = 2) = X$.
For agent 2, when $\theta_2 = 1$ we have that

$$\begin{aligned}\text{EU}_2(Z) &= P(\theta_1 = 0|\theta_2 = 1) \cdot u_2(a_1^*(\theta_1 = 0), Z, \theta_1 = 0, \theta_2 = 1)\\ &+ P(\theta_1 = 2|\theta_2 = 1) \cdot u_2(a_1^*(\theta_1 = 2), Z, \theta_1 = 2, \theta_2 = 1)\\ &= 1/2 \cdot 0 + 1/2 \cdot 3 = 3/2.\end{aligned}$$

$$\begin{aligned}\text{EU}_2(W) &= P(\theta_1 = 0|\theta_2 = 1) \cdot u_2(a_1^*(\theta_1 = 0), W, \theta_1 = 0, \theta_2 = 1)\\ &+ P(\theta_1 = 2|\theta_2 = 1) \cdot u_2(a_1^*(\theta_1 = 2), W, \theta_1 = 2, \theta_2 = 1)\\ &= 1/2 \cdot 1 + 1/2 \cdot 1 = 1.\end{aligned}$$

Thus, $a_2^*(\theta_2 = 1) = Z$ since $\text{EU}_2(Z) > \text{EU}_2(W)$.
For $\theta_2 = 3$, we have that

$$\begin{aligned}\text{EU}_2(Z) &= P(\theta_1 = 0|\theta_2 = 3) \cdot u_2(a_1^*(\theta_1 = 0), Z, \theta_1 = 0, \theta_2 = 3)\\ &+ P(\theta_1 = 2|\theta_2 = 3) \cdot u_2(a_1^*(\theta_1 = 2), Z, \theta_1 = 2, \theta_2 = 3)\\ &= 1/2 \cdot 0 + 1/2 \cdot 3 = 3/2.\end{aligned}$$

$$
\begin{aligned}
\mathsf{EU}_2(W) &= P(\theta_1 = 0|\theta_2 = 3) \cdot u_2(a_1^*(\theta_1 = 0), W, \theta_1 = 0, \theta_2 = 3) \\
&+ P(\theta_1 = 2|\theta_2 = 3) \cdot u_2(a_1^*(\theta_1 = 2), W, \theta_1 = 2, \theta_2 = 3) \\
&= 1/2 \cdot 3 + 1/2 \cdot 1 = 2.
\end{aligned}
$$

Thus, $a_2^*(\theta_2 = 3) = W$ since $\mathsf{EU}_2(Z) < \mathsf{EU}_2(W)$.
Concluding, the Bayesian Nash equilibrium is given by $a^* = (a_1^*, a_2^*)$, where $a_1^*(\theta_1 = 0) = Y$, $a_1^*(\theta_1 = 2) = X$, $a_2^*(\theta_2 = 1) = Z$, and $a_2^*(\theta_2 = 3) = W$.

Question 13. (**1 pts.**)
Consider the definition of Bayesian games that uses epistemic types. Describe the three possible notions of expected utility, clearly indicating which sets of types each agent has access to.

Write your answer here:

Solution 13.
The three notions of expected utility in the context of Bayesian games are:

- *ex post*, where each agent knows all agents' types.
- *ex interim*, where each agent knows its own type but not the types of other agents.
- *ex ante*, where each agent does not know anybody's type.

6 Learning in Games

Suppose you have an adversary and you are sure that the adversary is either a Tit-for-Tat player (Defects after a defect and Collaborates after a Collaboration) or a Collaborator (always Collaborates) with a 50% probability. You also know they all tend to make mistakes 5% of the time.

Question 14. (1 pts.)
For these agents, write the likelihood function that gives the probability of playing Collaborate/Defect after the other player has Collaborate or Defect. Use A_i^t as the action player i plays at time t.

Write your answer here:

Solution 14.
For the Tit-for-Tat player we have:

$$p(A_i^{t+1} = D | A_j^t = D, Type = T4T) = 0.95 \tag{1}$$
$$p(A_i^{t+1} = C | A_j^t = C, Type = T4T) = 0.95 \tag{2}$$

For the Collaborator player we have:

$$p(A_i^{t+1} = D | A_j^t = D, Type = Col) = p(A_i^{t+1} = D | Type = Col) = 0.05 \tag{3}$$
$$p(A_i^{t+1} = C | A_j^t = C, Type = Col) = p(A_i^{t+1} = C | Type = Col) = 0.95 \tag{4}$$

Question 15. (1 pts.)
Assume that they play the following joint actions $[A_{ij}^2 = (C, C), A_{ij}^1 = (D, D), A_{ij}^0 = (C, D)]$. Player 1 will assume that player 2 is of what type? For the calculation assume that the first joint action was random.

Write your answer here:

Solution 15.

$$\begin{aligned} p(Type_2 = Col | (C,C),(D,D),(C,D)) &\propto p(C|Type_2 = Col)p(D|Type_2 = Col)p(Type_2 = Col) \\ &= 0.95 \times 0.05 \times 0.5 \end{aligned}$$

$$\begin{aligned} p(Type_2 = T4T | (C,C),(D,D),(C,D)) &\propto p(C|D, Type_2 = T4T)p(D|C, Type_2 = T4T)p(Type_2 = T4T) \\ &= 0.05 \times 0.05 \times 0.5 \end{aligned}$$

$$p(Type_2 = Col | (C,C),(D,D),(C,D)) = \frac{0.95 \times 0.05 \times 0.5}{0.95 \times 0.05 \times 0.5 + 0.05 \times 0.05 \times 0.5}$$
$$p(Type_2 = T4T | (C,C),(D,D),(C,D)) = \frac{0.05 \times 0.05 \times 0.5}{0.95 \times 0.05 \times 0.5 + 0.05 \times 0.05 \times 0.5}$$

Question 16. (**1 pts.**)

	L	R
X	3, 4	-1, 0
Y	0, -1	4, 7

Using fictitious play, calculate the beliefs and actions for both agents in the first 3 rounds. Use the following initial beliefs $w_1 = (1.5, 1.)$ and $w_2 = (1.5, 1.)$.

Write your answer here:

Solution 16.

Round	1's action	2's action	1's beliefs	2's beliefs
0			(1.5,1)	(1.5,1)
$\frac{3\times1.5-1\times1}{2.5} = 1.4$ $\frac{0\times1.5+4\times1}{2.5} = 1.6->Y$ $\frac{4\times1.5-1\times1}{2.5} = 2$ $\frac{0\times1.5+7\times1}{2.5} = 2.8->R$				
1	Y	R	(1.5,2)	(1.5,2)
$\frac{3\times1.5-1\times2}{3.5} = 0.7$ $\frac{0\times1.5+4\times2}{3.5} = 2.3->Y$ $\frac{4\times1.5-1\times2}{3.5} = 1.14$ $\frac{0\times1.5+7\times2}{3.5} = 4->R$				
2	Y	R	(1.5,3)	(1.5,3)
3	Y	R	(1.5,4)	(1.5,4)