Autonomous Agents and Multiagent Systems

MSc in Computer Science and Engineering

First Exam (2023-2024)

Instructions

- You have 120 minutes to complete the exam.
- Make sure that your exam has a total of 12 pages. Also, check if there are no missing sheets, then write your full name and student number on this page (and your student number on all pages).
- The exam has 15 questions, with a maximum score of 20 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.
- *If you get stuck in a question, move on.* You should start with the more straightforward questions to secure those points before moving on to the more complex questions.
- *No interaction with the faculty is allowed during the exam.* If you are unclear about a question, clearly indicate the unclear part and answer the question to the best of your ability.
- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.
- This exam is a closed-book assessment, whereby students are NOT allowed to bring books or other reference material into the examination room. You may bring only ONE A4 page of handwritten notes, in your OWN handwriting. Typed notes or a copy of someone else's notes are not allowed.
- You may use a calculator, but any other type of electronic or communication equipment is not allowed.
- **Good luck!**

1 Agent architectures

Question 1. (1 pts.)
Classify greedy agents, such as those implemented in Lab 1, according to their architecture. And discuss their characteristics and disadvantages.

Write your answer here:

Solution 1.
Greedy agents are inherently purely reactive agents, such as those implemented by Brooks's subsumption architecture.
As purely reactive agents

- They are short-sighted, taking the best action according to the current state only.
- They must have sufficient knowledge of the local environment to act, compensating the lack of planning.
- Emergent behavior and complex behavior is hard to engineer. There are no principled methodology to engineer complex behavior from simple actions.
- There is a relationship between the number of behaviors necessary for the agent to be considered effective, and the complexity of the architecture. Hence, when the number of behaviors increase so does the architecture's complexity.
- They are unable to learn.

Question 2. (1 pts.)
Three core social abilities in multi-agent systems are: cooperation, coordination and negotiation. Define each ability and give a concrete example where the social ability is particularly important.

Write your answer here:

Solution 2.
Cooperation is the ability whereby agents pursue the same goal. In multi-robot soccer, agents may assume different roles but play as a team to score a goal.
Coordination is the ability that enables agents to account for inter relations between other agents' actions. Coverage path planning (CPP) requires that every point be visited by at least a robot, coordinating robots can divide sections of the environment among themselves.
Negotiation is the ability to reach agreements between different agents. Autonomous driving may require that agents decide whether to advance or hold in a intersection. Negotiating agents can agree on a configuration where most agents move.

2 Normal-form games

Question 3. (**1 pts.**)
Model the following problem as a normal-form game. Sam and Rosie are dating. They are planning to go together to a restaurant. They have two options of restaurants: the Green Dragon and the Ivy Bush. However, Sam forgot his cell phone at home so he cannot call Rosie to decide on the restaurant. Sam prefers the food at Green Dragon and Rosie prefers Ivy Bush. Both prefer to go together to any of these restaurants than going alone.

Write your answer here:

Solution 3.
N = {Sam, Rosie}
A1 = A2 = {Green Dragon, Ivy Bush}
The payoff matrix of the game:

	Green Dragon	Ivy Bush
Green Dragon	2, 1	0, 0
Ivy Bush	0, 0	1, 2

Sam is represented as player 1 (row) and Rosie as player 2 (column).

Question 4. (**2 pts.**)
Given the following payoff matrix for a 2-agent normal-form game.

	c	d
a	3, 4	5, 1
b	2, 2	8, 3

Find the pure Nash Equilibria and, if possible, the mixed strategy Nash Equilibrium.

Write your answer here:

Solution 4.

In order to find the pure strategy Nash equilibria, we first find the best responses for agent 1 and agent 2 and underline them in the payoff matrix:

	c	d
a	$\underline{3}$, $\underline{4}$	5, 1
b	2, 2	$\underline{8}$, $\underline{3}$

A joint action satisfies the definition of NE if each agent's action is the best response to the other's. We thus have a Nash equilibrium if both payoffs are underlined in a cell of the payoff matrix above. In conclusion, the Nash equilibria is (a, c) and (b, d).

In order to find the mixed strategy NE, let us suppose that Agent 1 believes that Agent 2 will choose *c* with probability p and *d* with probability 1 ,àí p. If Agent 1 best responds with a mixed strategy, then Agent 2 must make him indifferent between *a* and *b*:

$$EU_1(a) = EU_1(b) \to 3p + 5(1 - p) = 2p + 8(1 - p) \to p = \frac{3}{4}$$

Suppose that Agent 2 believes that Agent 1 will choose *a* with probability q and *b* with probability 1 ,àí q. If Agent 2 best responds with a mixed strategy, then Agent 1 must make her indifferent between c and d:

$$EU_2(c) = EU_2(d) \to 4q + 2(1 - q) = q + 3(1 - q) \to q = \frac{1}{4}$$

Hence, the mixed strategy Nash equilibrium is $(\frac{1}{4}, \frac{3}{4}), (\frac{3}{4}, \frac{1}{4})$.

Question 5. (1 pts.)

Discuss the concept of Pareto optimality. Are the pure Nash Equilibria of the game above Pareto optimal?

Write your answer here:

Solution 5.

An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player. The pure Nash Equilibria of the game above are Pareto optimal.

3 Extensive-form games

Consider the following game about two players splitting a pile of 4 coins. Player one can split the pile however he wants. Then, player 2 has the choice to accept or reject the partition. On acceptance, both players receive the partition agreed. Otherwise, both players receive none.

Question 6. (**1 pts.**)

Formally describe the extensive-form version of this game.

Write your answer here:

Solution 6.

$$N = \{1, 2\}$$

$$A = \{0, 1, 2, 3, 4\} \cup \{\mathsf{A}, \mathsf{R}\}$$

$$H = \{\mathsf{D}\} \cup \{\mathsf{D.0}, \mathsf{D.1}, \mathsf{D.2}, \mathsf{D.3}, \mathsf{D.4}\}$$

$$Z = \{\mathsf{A}.0, \mathsf{A}.1, \mathsf{A}.2, \mathsf{A}.3, \mathsf{A}.4\} \cup \{\mathsf{R}.0\, \mathsf{R}.1, \mathsf{R}.2, \mathsf{R}.3, \mathsf{R}.4\}$$

$$\chi(h) = \begin{cases} \{0, 1, 2, 3, 4\} & \text{if} \quad h = \mathsf{D} \\ \{\mathsf{A}, \mathsf{R}\} & \text{if} \quad \textit{otherwise} \end{cases}$$

$$\rho(h) = \begin{cases} 1 & \text{if} \quad h = \mathsf{D} \\ 2 & \text{otherwise} \end{cases}$$

$$\sigma(h, a) = \begin{cases} \mathsf{D}.i & \text{if} \quad h = \mathsf{D} \wedge a = i \ \ \forall i \in \mathbb{N} : (0 \leq i \leq 4) \\ \mathsf{A}.i & \text{if} \quad h = \mathsf{D}.i \wedge a = \mathsf{A} \\ \mathsf{R}.i & \text{if} \quad h = \mathsf{D}.i \wedge a = \mathsf{R} \end{cases}$$

$$u_1(z) = \begin{cases} i & \text{if} \quad z = \mathsf{A}.i \\ 0 & \text{otherwise} \end{cases}$$

$$u_2(z) = \begin{cases} 4 - i & \text{if} \quad z = \mathsf{A}.i \\ 0 & \text{otherwise} \end{cases}$$

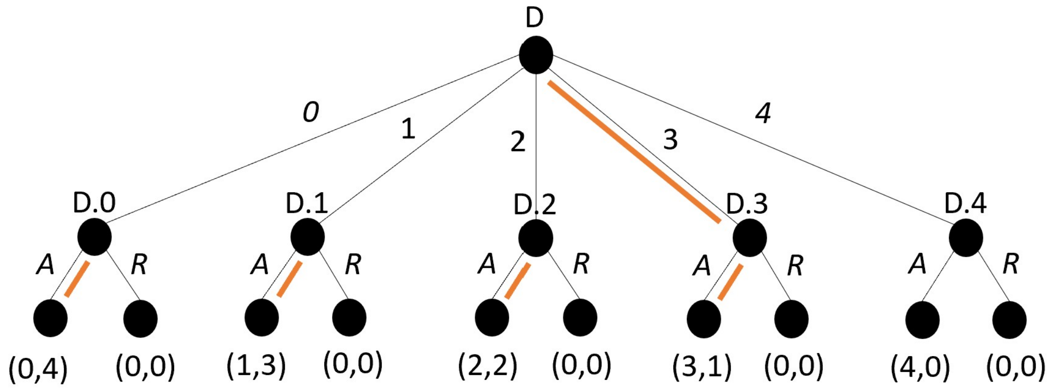
D represents the first node, D.i the nodes after each decision of player 1 (denoting player 1 keeps i to itself) and A.i and R.i the nodes after the decision of player 2.

Question 7. (**2 pts.**)

What rational decision should player 1 take? Justify your answer using the subgame perfect equilibria.

Write your answer here:

Solution 7.



Player 2 is indifferent at D.4, hence it does not have a pure strategy in this subgame. At all other nodes D.i : i < 4 the rational choice for player 2 is to accept. Therefore, the rational choice for player 1 is to take 3 coins.

4 Bayesian games

An agent faces an opponent player who may have two or three actions available. Let $\{X, Y\}$ be the set of actions available to the agent, $\{K, L\}$ be the set of available actions for the opponent player when only two actions are available, and $\{K, L, M\}$ be the set of available actions for the opponent player when all actions are available. With probability p, the opponent has two actions available and the payoff matrix of the game is

	K	L
X	1,1	0,0
Y	0,0	2,2

and, with probability $1 - p$, the opponent has three actions available and the payoff matrix is

	K	L	M
X	1,1	0,0	3,3
Y	0,0	2,2	0,0

Only the opponent player knows if it has two or three actions available.

Question 8. (1 pts.)
Formalize the game above as a Bayesian game. Note that, for the case when only two actions are available for the opponent player, the following game can be equivalently considered (since M is a strictly dominated action in the game below).

	K	L	M
X	1,1	0,0	3,-1
Y	0,0	2,2	0,-1

Write your answer here:

Solution 8.
The Bayesian game is defined by the tuple (N, A, θ, P, u) where:

- $N = \{\text{Agent, Opponent}\}$ is the set of agents. Below, we refer to the agent player with an index of 1 and the opponent player with an index of 2.
- $A_1 = \{X, Y\}$, $A_2 = \{K, L, M\}$ and $A = A_1 \times A_2$.
- $\theta_1 = \{t_1\}$ and $\theta_2 = \{t_2, t_3\}$ are the set of types for each agent and $\theta = \theta_1 \times \theta_2$, where t_2 and t_3 denote, respectively, the cases where the opponent agent has two or three actions available.
- $P(\theta_1 = t_1, \theta_2 = t_2) = p$ and $P(\theta_1 = t_1, \theta_2 = t_3) = 1 - p$ is the prior over types.
- $u = (u_1, u_2)$ is the utility function for each agent such that: (i) $u_1(a_1, a_2, \theta_1 = t_1, \theta_2 = t_2)$ and $u_2(a_1, a_2, \theta_1 = t_1, \theta_2 = t_2)$ are given by the third table above; and (ii) $u_1(a_1, a_2, \theta_1 = t_1, \theta_2 = t_3)$ and $u_2(a_1, a_2, \theta_1 = t_1, \theta_2 = t_3)$ are given by the second table above.

Question 9. (2 pts.)
Assume the agent player (row player) uses a fixed mixed strategy $s_1(X) = s_1(Y) = 0.5$. Calculate the best response for the opponent player with respect to s_1, $\mathrm{BR}_2(s_1)$, as a function of p.

Write your answer here:

Solution 9.
To calculate $\mathsf{BR}_2(s_1)$, we first note that the opponent player (column) player knows his type before deciding on which strategy to pick. This is because strategies for the column player correspond to mappings from the space of player types to the player's action set (or, more generally, to distributions over actions). Hence, $\mathsf{BR}_2(s_1)$ does not depend on p.

We are left with computing the best response to s_1 for each column player type.

If $\theta_2 = t_2$, action M is strictly dominated. Hence, we are left with computing the expected utility for actions K and L.

$$\begin{aligned}\mathsf{EU}_2(K|s_1, \theta_2 = t_2) &= s_1(X) \cdot u_2(a_1 = X, a_2 = K, \theta_1 = t_1, \theta_2 = t_2) \\ &\quad + s_1(Y) \cdot u_2(a_1 = Y, a_2 = K, \theta_1 = t_1, \theta_2 = t_2)\Big] \\ &= \Big[0.5 \cdot 1 + 0.5 \cdot 0\Big] \\ &= 0.5.\end{aligned}$$

$$\begin{aligned}\mathsf{EU}_2(L|s_1, \theta_2 = t_2) &= s_1(X) \cdot u_2(a_1 = X, a_2 = L, \theta_1 = t_1, \theta_2 = t_2) \\ &\quad + s_1(Y) \cdot u_2(a_1 = Y, a_2 = L, \theta_1 = t_1, \theta_2 = t_2)\Big] \\ &= \Big[0.5 \cdot 0 + 0.5 \cdot 2\Big] \\ &= 1.\end{aligned}$$

If $\theta_2 = t_3$, $\mathsf{EU}_2(K|s_1, \theta_2 = t_3) = 0.5$ and $\mathsf{EU}_2(K|s_1, \theta_2 = t_3) = 1$, as in the case above when $\theta_2 = t_2$. For action M,

$$\begin{aligned}\mathsf{EU}_2(M|s_1, \theta_2 = t_3) &= s_1(X) \cdot u_2(a_1 = X, a_2 = M, \theta_1 = t_1, \theta_2 = t_3) \\ &\quad + s_1(Y) \cdot u_2(a_1 = Y, a_2 = M, \theta_1 = t_1, \theta_2 = t_3)\Big] \\ &= \Big[0.5 \cdot 3 + 0.5 \cdot 0\Big] \\ &= 1.5.\end{aligned}$$

Concluding, if $\theta_2 = t_2$, $\mathsf{BR}_2(s_1) = L$, and if $\theta_2 = t_3$, $\mathsf{BR}_2(s_1) = M$.
It was also possible to find a solution in relation to p, which we accepted as a correct answer as well.

5 Repeated games

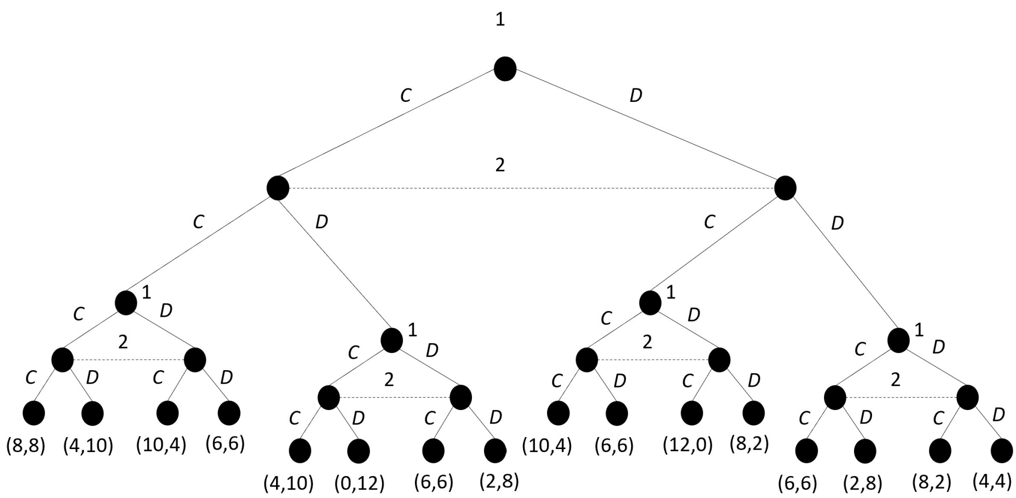
The following stage game of the Prisoner's Dilemma is played repeatedly:

	C	D
C	4, 4	0, 6
D	6, 0	2, 2

Question 10. (**1 pts.**)
If the stage game above is played twice, use an extensive-form game to represent this repeated game.

Write your answer here:

Solution 10.



Question 11. (**1 pts.**)
What is the Nash Equilibrium of this repeated game?

Write your answer here:

Solution 11.
If a stage game G has a unique Nash equilibrium then, for any finite repetitions the game G has a unique outcome: the Nash equilibrium of G is played in every subgame perfect stage. Hence the Nash Equilibrium of the repeated game is the one expected for a Prisoner's Dilemma, {(D,D),(D,D)}.

6 Auctions

Question 12. (1 pts.)
What is a First-Price Auction? Describe the process that it follows.

Write your answer here:

Solution 12.
A first-price auction is a sealed-bid auction that proceeds as follows:
All bidders submit sealed bids simultaneously
No bidder knows the bids of the other bidders
The highest bidder wins and pays the submitted price

Question 13. (2 pts.)
What is a Bayesian-Nash equilibrium of a First-price auction with 3 risk-neutral bidders where the valuations v_1, v_2, v_3 are independent and identically distributed and drawn from $U(0,1)$?

Write your answer here:

Solution 13.
Given that $(\frac{N-1}{N}v_1, \frac{N-1}{N}v_2, ..., \frac{N-1}{N}v_n)$ is a Bayesian-Nash equilibrium for N risk-neutral bidders, taking $N = 3$ into account, we get the following equilibrium: $(\frac{2}{3}v_1, \frac{2}{3}v_2, \frac{2}{3}v_3)$

7 Learning in Games

Consider the following matrix game.

	X	Y
X	-1, -3	2, 4
Y	3, 2	1, 3

Question 14. (**1 pts.**)
For independent learners, centralized learner and joint-action learners define the size of the Q-function each agent needs.

Write your answer here:

Solution 14.
This problem is just a matrix game so no state is needed.

For independent learners $N_{actions}$, for centralized learner and joint-action learners $N_{actions} \times N_{actions}$

Question 15. (**2 pts.**)
For the case of joint action learners using Nash-Q, with no exploration and initialization to 0, show the first 2 steps of learning. In case of a tie use the first actions. (use $\alpha = 0.1$ if needed).

Write your answer here:

Solution 15.

$$
\begin{aligned}
Q_0 = [[0,0],[0,0]] \quad &, \quad Q_1 = [[0,0],[0,0]] && (1)\\
a_0 = X, a_1 = X \quad &, \quad r_0 = -1, r_1 = -3 && (2)\\
Q_0[X,X] \quad &= \quad Q_0[X,X] + \alpha(-1 - Q_0[X,X]) = -\alpha && (3)\\
Q_1[X,X] \quad &= \quad Q_1[X,X] + \alpha(-3 - Q_1[X,X]) = -3\alpha && (4)\\
Q_0 = [[-\alpha,0],[0,0]] \quad &, \quad Q_1 = [[-3\alpha,0],[0,0]] && (5)\\
a_0 = Y, a_1 = Y \quad &, \quad r_0 = 1, r_1 = 3 && (6)\\
Q_0[Y,Y] \quad &= \quad Q_0[Y,Y] + \alpha(1 - Q_0[Y,Y]) = \alpha && (7)\\
Q_1[Y,Y] \quad &= \quad Q_1[Y,Y] + \alpha(3 - Q_1[Y,Y]) = 3\alpha && (8)\\
Q_0 = [[-\alpha,0],[0,\alpha]] \quad &, \quad Q_1 = [[-3\alpha,0],[0,3\alpha]] && (9)
\end{aligned}
$$