# Homework 3. Partially observable Markov decision problems

We will continue with the game we developed in HW2. Consider now that the spider cannot observe in which step of the ladder it is in. However, using its legs, the spider can identify whether it is sitting on the ground, at a web, or at the top level.

## Exercise

(a) Define a POMDP based on the available information. The observations are {g,w,t,e} for ground, web, top level, and empty (no observation).

(b) Compute the belief (the probability that the spider is at a given state) for the following situations:

   (a) After feeling the web with its legs.

   (b) After feeling the web with its legs and playing two turns, assuming that the spider made no observation after each step (i.e., it makes two empty observations).

   (c) After starting and playing 3 times, assuming that the spider made no observation after each step (i.e., it makes three empty observations).

(c) If the belief is:

$$\begin{bmatrix} 0.2 & 0.08 & 0.24 & 0.32 & 0.16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

what is the best action to make? (Use MLS and QMDP)

Consider the following transition matrices and cost function describing the game developed in HW2.

$$\boldsymbol{P}^{play} = \begin{bmatrix} .2 & .4 & .4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & .4 & .4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & .4 & .4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & .4 & .4 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & 0 & .8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .2 & .4 & .4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .2 & 0 & .4 & .4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .2 & .4 & .4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .2 & 0 & .4 & .4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .2 & 0 & 0 & .8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P}^{stop} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The cost function is

$$C^{play} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$
$$C^{stop} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$