TÉCNICO LISBOA

Social Choice / Voting

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Voting

- How to make a decision that respects everyone’s wishes?
  - -> consensus?
  - -> majority?
  - -> democratic?

Problems

- Wishes might be incompatible
- When making a choice
- When electing representatives problems with discretizations

Social Choice

Our setting now:

- a set of outcomes
- agents have preferences across them
- for the moment, we won't consider incentive issues:
  - center knows agents' preferences, or they declare truthfully
- the goal: a social choice function: a mapping from everyone's
- preferences to a particular outcome, which is enforced
  - how to pick such functions with desirable properties?

MULTIAGENT SYSTEMS
Algorithmic, Game-Theoretic, and Logical Foundations
Yoav Shoham, Kevin Leyton-Brown

Non-Ranking Voting Schemes

- Plurality
  - pick the outcome which is preferred by the most people
- Cumulative voting
  - distribute e.g., 5 votes each
  - possible to vote for the same outcome multiple times
- Approval voting
  - accept as many outcomes as you “like"

Ranking Voting Schemes

- Plurality with elimination (instant runoff)
  - everyone selects their favorite outcome
  - the outcome with the fewest votes is eliminated
  - repeat until one outcome remains
- Borda
  - assign each outcome a number.
  - The most preferred outcome gets a score of n-1, the next most preferred gets n-2, down to the nth outcome which gets 0.
  - Then sum the numbers for each outcome, and choose the one that has the highest score
- Pairwise elimination
  - in advance, decide a schedule for the order in which pairs will be compared.
  - given two outcomes, have everyone determine the one that they prefer
  - eliminate the outcome that was not preferred, and continue with the schedule

Condorcet Condition

- If there is a candidate who is preferred to every other candidate in pairwise runoff, that candidate should be the winner
- While the Condorcet condition is considered an important property for a voting system to satisfy, there is not always a Condorcet winner
- sometimes, there's a cycle where A defeats B, B defeats C, and C defeats A in their pairwise runoffs

Condorcet Example

499 agents: A > B > C

3 agents: B > C > A

498 agents: C > B > A

- What is the Condorcet winner?
- What would win under plurality voting?
- What would win under plurality with elimination?

Condorcet Example

499 agents: A > B > C

3 agents: B > C > A

498 agents: C > B > A

- What is the Condorcet winner? B
- What would win under plurality voting? A
- What would win under plurality with elimination? C

Sensitivity to Losing Candidate

35 agents: A > C > B

33 agents: B > A > C

32 agents: C > B > A

- What candidate wins under plurality voting?
- What candidate wins under Borda voting?
- Now consider dropping C. Now what happens under both Borda and plurality?

Sensitivity to Losing Candidate

35 agents: A > C > B

33 agents: B > A > C

32 agents: C > B > A

- What candidate wins under plurality voting? A
- What candidate wins under Borda voting? A
  A = 35*2+33 B =33*2+32 C = 32*2+35
- Now consider dropping C. Now what happens under both Borda and plurality? B wins.
  A = 35 B =33+32

Sensitivity to Agenda Setter

35 agents: A > C > B

33 agents: B > A > C

32 agents: C > B > A

- Who wins pairwise elimination, with the ordering A;B;C?
- Who wins with the ordering A;C;B?
- Who wins with the ordering B;C;A?

Sensitivity to Agenda Setter

35 agents: A > C > B

33 agents: B > A > C

32 agents: C > B > A

- Who wins pairwise elimination, with the ordering A;B;C? C
- Who wins with the ordering A;C;B? B
- Who wins with the ordering B;C;A? A

Another Pairwise Elimination Problem

- 1 agent: B D C A
- 1 agent: A B D C
- 1 agent: C A B D
- Who wins under pairwise elimination with the ordering A;B;C;D? D.
- What is the problem with this?
- all of the agents prefer B to D
- the selected candidate is Pareto-dominated!

THE MATHEMATICS OF DEMOCRACY

WHAT IS *Arrow's Impossibility Theorem?*

In order to find the "best" ranked voting system, we reviewed several criteria we want it to satisfy. **Arrow's Impossibility Theorem** says that this perfect voting system cannot exist: in an election with at least three candidates, if we want a voting system to satisfy both

- **Pareto Condition**:
  - if everyone prefers A to B, then B is not a winner; and
- **Independence of Irrelevant Alternatives (IIA)**:
  - a group's preference for one candidate over another should not depend on how the remaining candidates are arranged

...then that voting system must be a **dictatorship**, namely there will be a voter whose vote decides the election regardless of how everyone else voted.

Why can't any other voting system satisfy both Pareto criterion and IIA?

FIND THE ANSWER AT HTTPS://MATHEMATICS-DEMOCRACY-INSTITUTE.ORG/MATH-AND-POLITICS-TRIVIA/

- Social Choice
- Representative choice
- Gerrymandering

Representative choice

Party	Votes
A	1000
B	800
C	400

If we have 5 seats. How many seats per party?

https://en.wikipedia.org/wiki/Proportional_representation

Representative choice

Party	Votes
A	1000
B	800
C	400

If we have 5 seats. How many seats per party?

A	1000	45%	2	40%	3	60%
B	800	36%	2	40%	1	20%
C	400	18%	1	20%	1	20%
	2200		5		5	

https://en.wikipedia.org/wiki/Proportional_representation

D'Hondt method

- After all the, successive quotients are calculated for each party. The party with the largest quotient wins one seat, and its quotient is recalculated.

- Quot = V/(s+1)

- V is the total number of votes that party received, and
- s is the number of seats that party has been allocated so far, initially 0 for all parties.

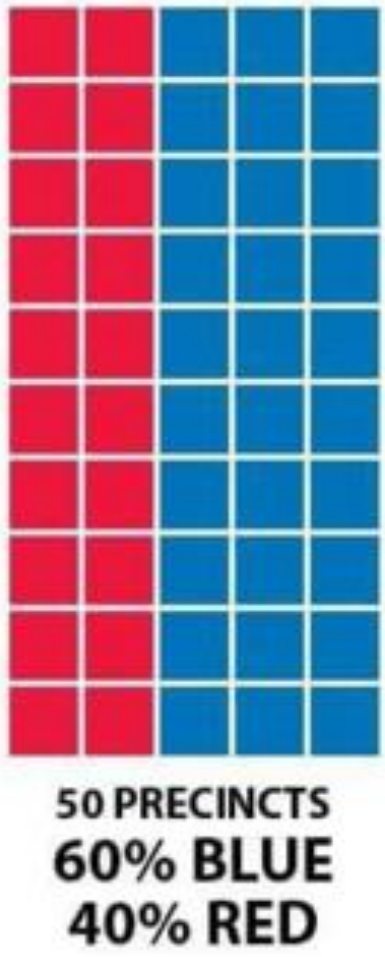
The total votes cast for each party in the electoral district is divided, first by 1, then by 2, then 3, up to the total number of seats to be allocated for the district/constituency. Say there are p parties and s seats. Then a grid of numbers can be created, with p rows and s columns, where the entry in the ith row and jth column is the number of votes won by the ith party, divided by j. The s winning entries are the s highest numbers in the whole grid; each party is given as many seats as there are winning entries in its row.

Representative choice

	Votes	% Vote	FPTP Seats		D'Hondt Seats		Sainte-Laguë Seats		Hare-LR Seats	
Con	258,794	47.4%	8	72.7%	6	54.5%	6	54.5%	5	45.5%
Lab	204,011	37.4%	3	27.3%	5	45.5%	4	36.4%	4	36.4%
LD	33,604	6.2%					1	9.1%	1	9.1%
Brexit Party	15,728	2.9%							1	9.1%
Ashfield Ind	13,498	2.5%								
Green	10,375	1.9%								
Others	9,743	1.8%								
Deviation				25.3%		15.2%		10.1%		9.1%

https://electoral-reform.org.uk/what-is-the-difference-between-dhondt-sainte-lague-and-hare/

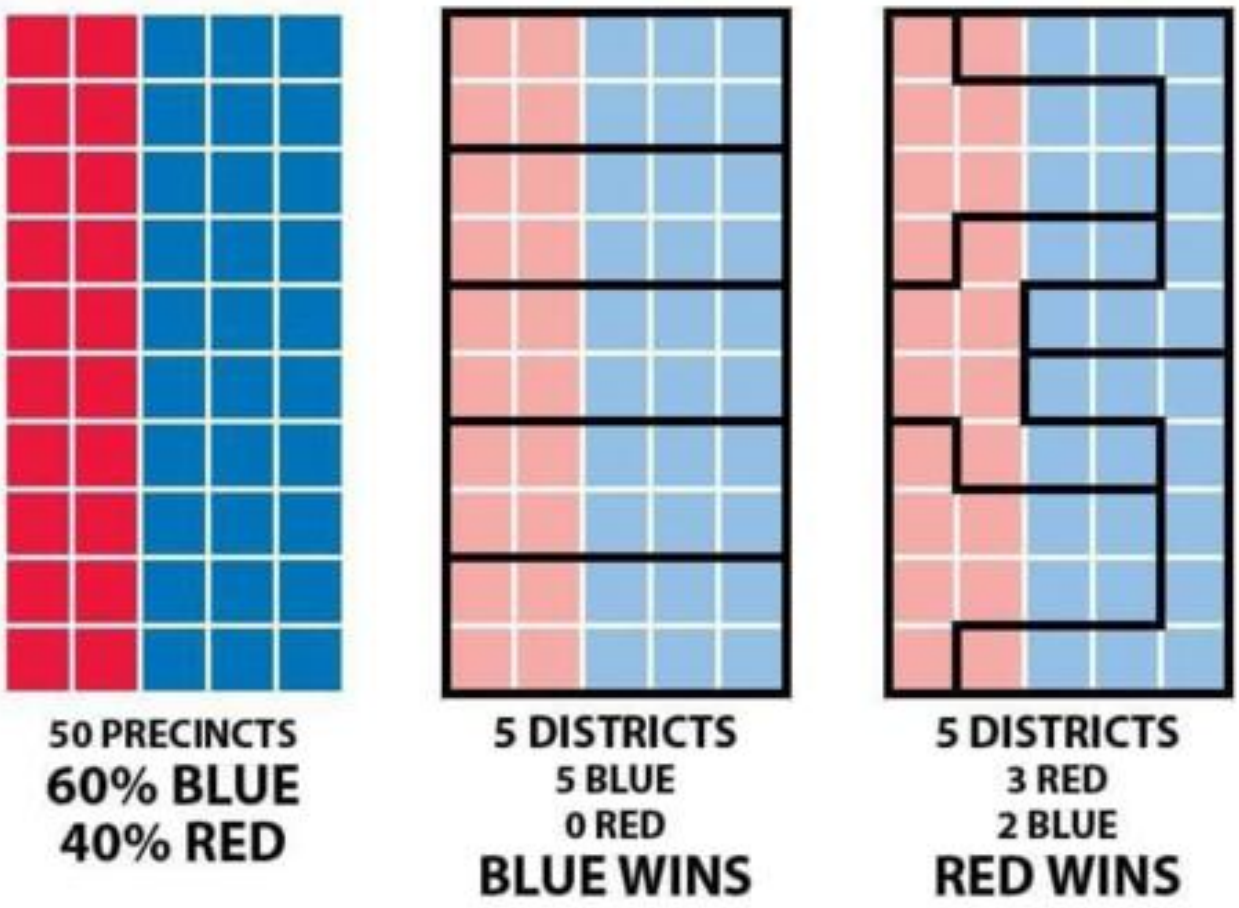
Gerrymandering



Who should be the winner?

If you can choose the districts…

Gerrymandering



https://magazine.caltech.edu/post/rethinking-redistricting