Chapter 4
Deliberation with Temporal Domain Models

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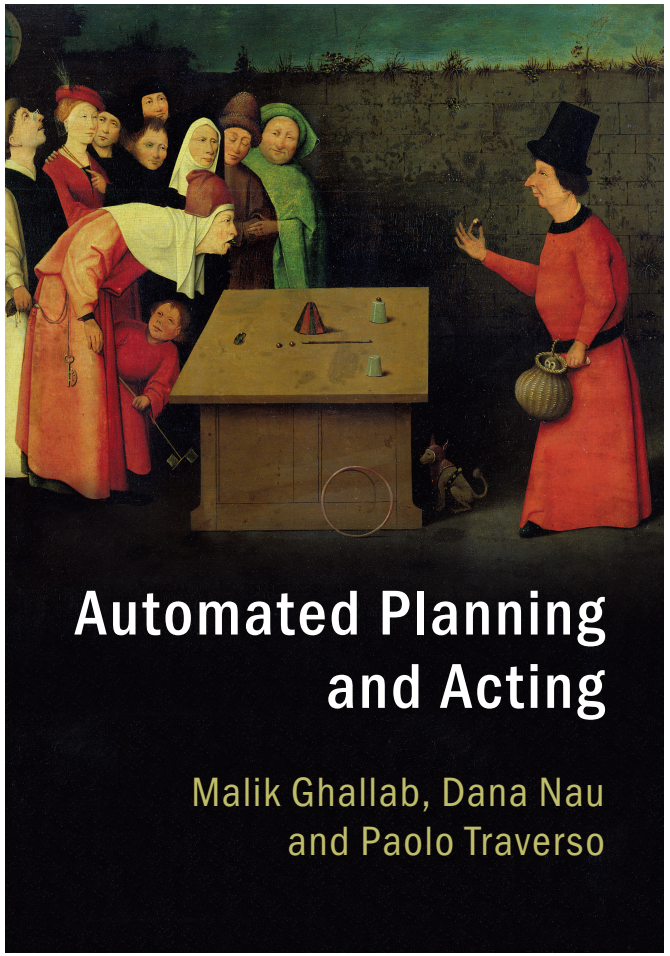
University of Maryland



http://www.laas.fr/planning

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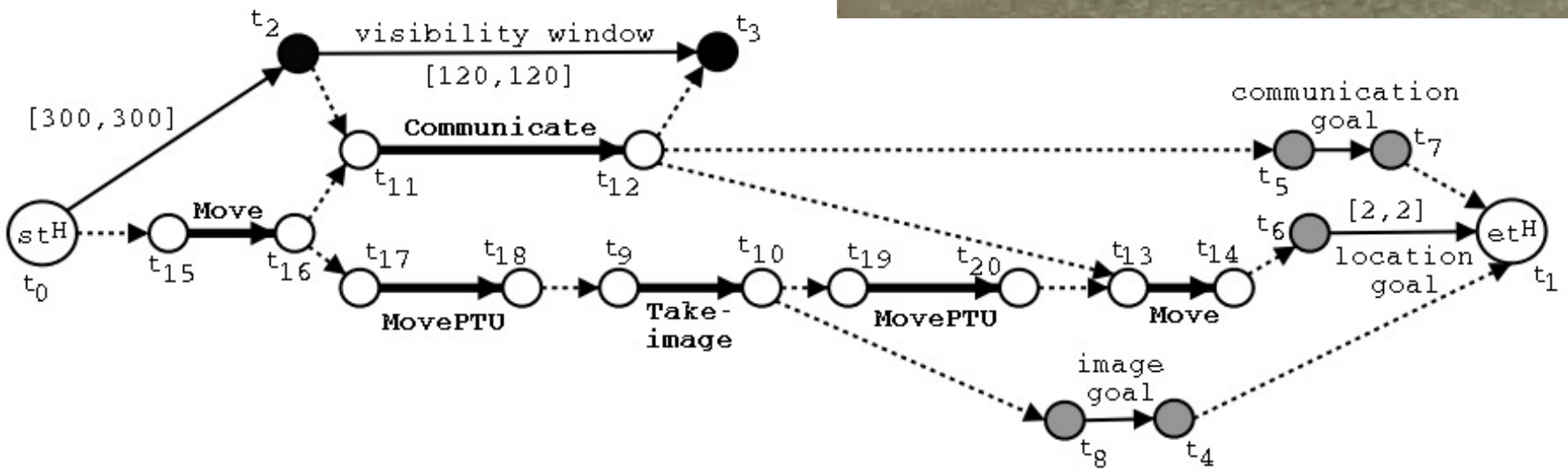
IxTeT

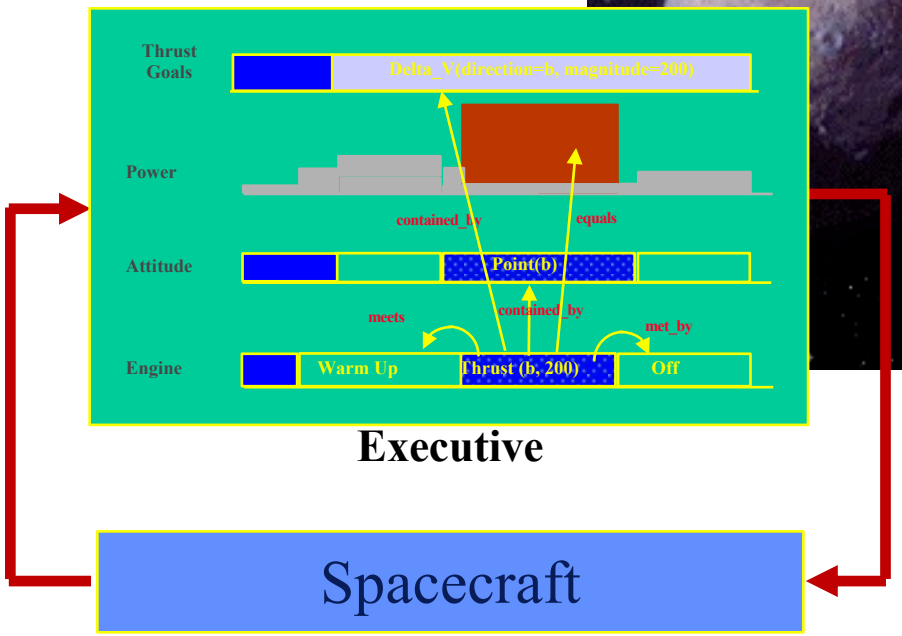
- LAAS/CNRS, Toulouse, France
- mid-1990s
- Video:

  https://www.cs.umd.edu/~nau/apa/ixtet.mov



RAX/PS

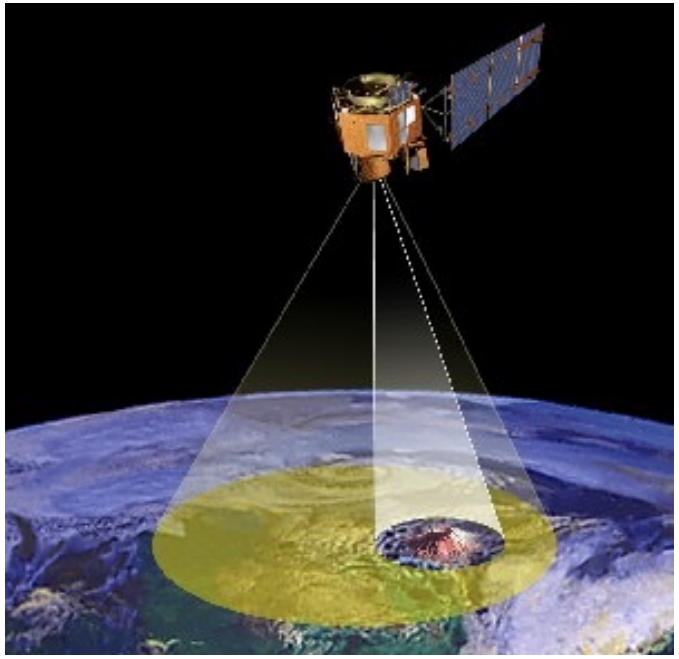
- Planning/control of DS1 spacecraft
- NASA Ames and JPL, 1999



T-ReX

- Planning/control of AUVs
- Monterey Bay Aquarium Research Institute, ≈ 2005-2010

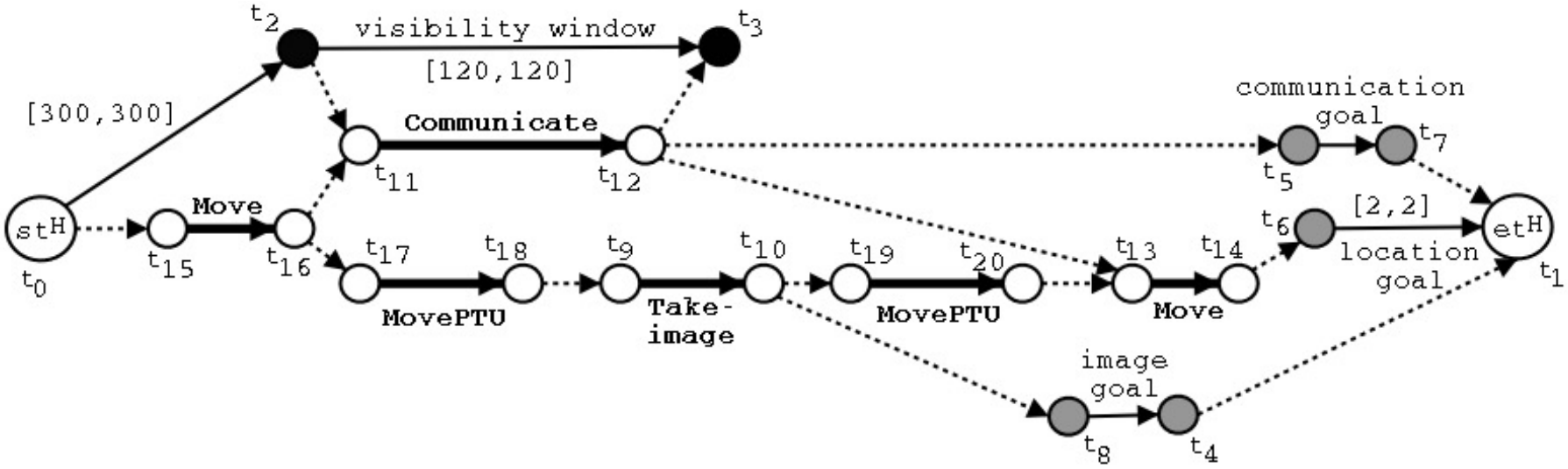
Casper (NASA JPL)



- Planning/control of spacecraft
- NASA JPL, ongoing

Temporal Models

- Constraints on state variables and events
  - Reflect predicted actions and events
- Actions have duration
  - preconditions and effects may occur at times other than start and end
- Time constraints on goals
  - relative or absolute
- Exogenous events expected to occur in the future
- Maintenance actions: maintain a property
  - e.g., track a moving target, keep a door closed
- Concurrent actions
  - interacting effects, joint effects
- Delayed commitment
  - instantiation at acting time



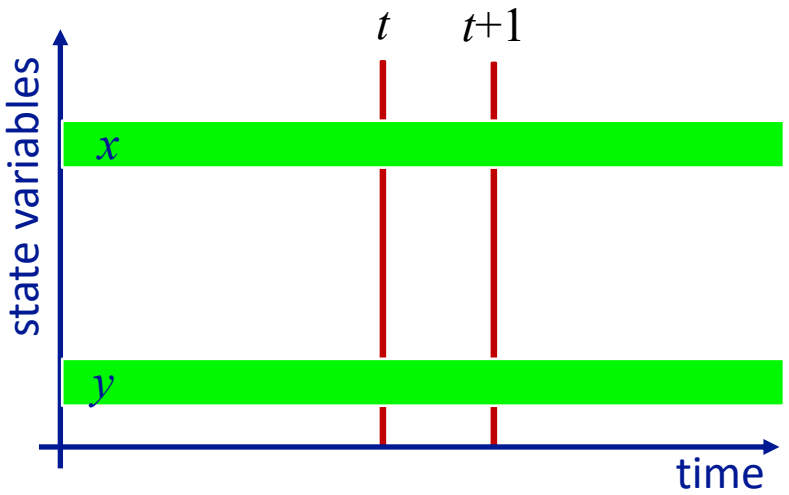
Outline

- ✓ Introduction
- **4.2 Representation**
  - Timelines
  - Actions and tasks
  - Chronicles
- 4.3 Temporal planning
- 4.4 Constraint management
- 4.5 Acting with temporal models

Timelines

- Up to now, we've used a "state-oriented view"
  - Time is a sequence of states s_0, s_1, s_2
  - Instantaneous actions transform each state into the next one
  - No overlapping actions

- Switch to a "time-oriented view"
  - Discrete: time points are integers
    - $t = 1, 2, 3, \ldots$
  - For each state variable x, a *timeline*
    - values of x during different time intervals
  - State at time t = {state-variable values at time t}



Timeline

- A pair $(\mathcal{T}, C)$
  - partially predicted evolution of one state variable
- $\mathcal{T}$: temporal assertions

  $[t_1, t_2]$ loc(r1) : (loc1, l)

  $[t_2, t_3]$ loc(r1) = l

  $[t_3, t_4]$ loc(r1) : (l, loc2)
- C : constraints

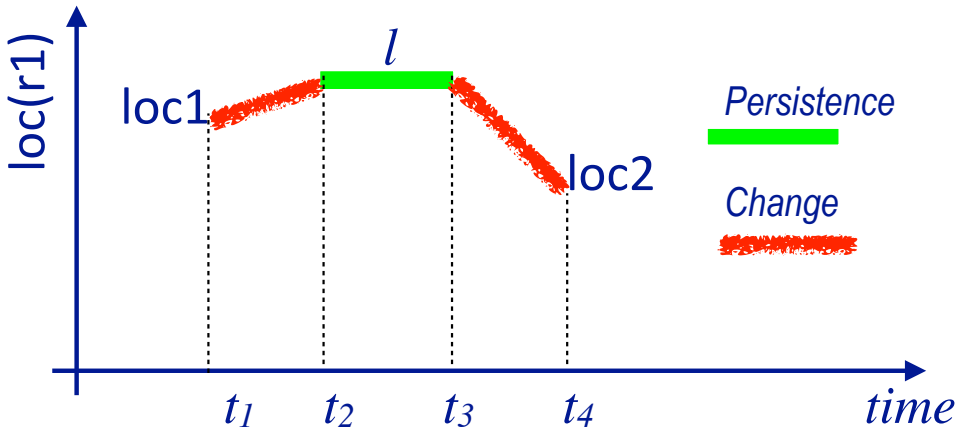
  $t_1 < t_2 < t_3 < t_4$

  $l \neq$ loc1

  $l \neq$ loc2

persistence entails $t_2 \leq t_3$

change entails $t_3 < t_4$ and $v_1 \neq v_2$



- If we want to restrict loc(r1) during $[t_1, t_2]$

  $[t_1, t_1+1]$ loc(r1) : (loc1, route)

  $[t_2-1, t_2]$ loc(r1) : (route, l)

  $[t_1+1, t_2-1]$ loc(r1) = route

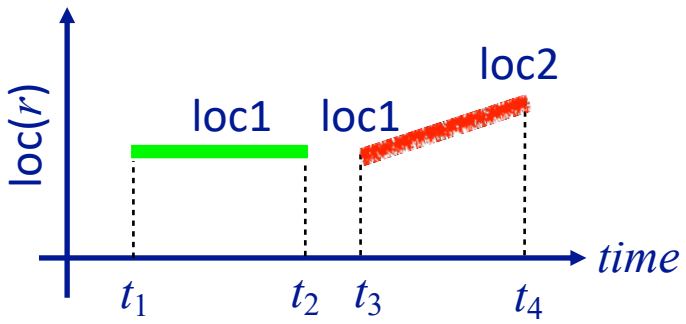
  where route is some intermediate location

Consistency

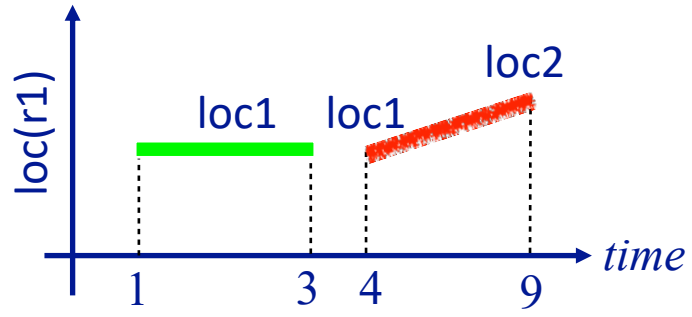
- Let $(\mathcal{T}, C)$ be a timeline
- Let $(\mathcal{T}', C')$ be a ground instance of $(\mathcal{T}, C)$
  - $(\mathcal{T}', C')$ is *consistent* if
    - $\mathcal{T}'$ satisfies C'

    and
    - no state variable in $(\mathcal{T}, C)$ has more than one value at a time
- $(\mathcal{T}, C)$ is *consistent* if it has *at least one* consistent ground instance

- A consistent timeline:
  - $\mathcal{T}_1 = \{[t_1,t_2]\ \text{loc}(r)=\text{loc1},\ [t_3,t_4]\ \text{loc}(r):(\text{loc1},\text{loc2})\}$
  - $C_1 = \{t_1 < t_2 < t_3\}$

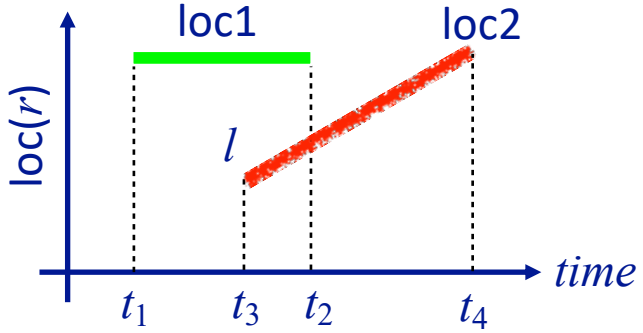


$(\mathcal{T}_1, C_1)$



a ground instance of $(\mathcal{T}_1, C_1)$

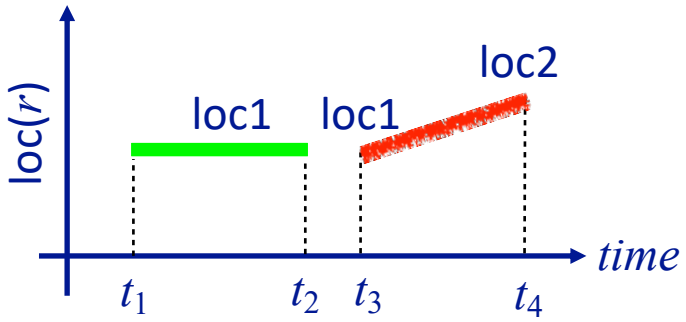
- **Poll**: is this timeline consistent?
  - $\mathcal{T}_2 = \{[t_1,t_2]\ \text{loc}(r)=\text{loc1},\ [t_3,t_4]\ \text{loc}(r):(l,\text{loc2})\}$
  - $C_2 = \{t_1 < t_3 < t_2\}$



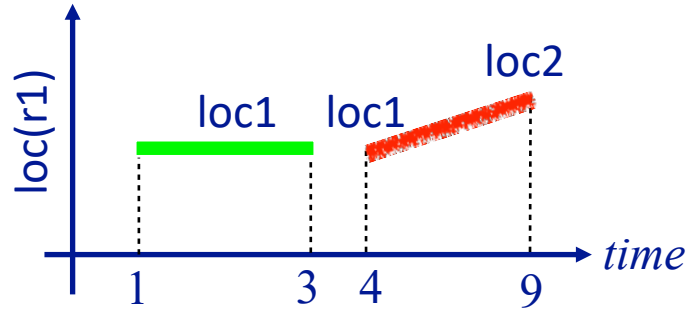
Security

- $(\mathcal{T}, C)$ is *secure* if
  - it's consistent (has *at least one* consistent ground instance)
  - *every* ground instance that satisfies the constraints is consistent
- In PSP (Chapter 2), analogous to a partial plan that has no threats
- Can make a consistent timeline secure by adding *separation constraints*:
  - $r \neq$ r1
  - $t_2 < t_3$
  - $t_4 < t_1$
  - $t_2 = t_3$, $r =$ r1 , $l =$ loc1
  - $t_4 = t_1$, $r =$ r1 , $l =$ loc1
- Analogous to resolvers in PSP

- $(\mathcal{T}_1, C_1)$ is secure
  - $\mathcal{T}_1 = \{[t_1,t_2]$ loc(r)=loc1, $[t_3,t_4]$ loc(r):(loc1,loc2)$\}$
  - $C_1 = \{t_1 < t_2 < t_3\}$

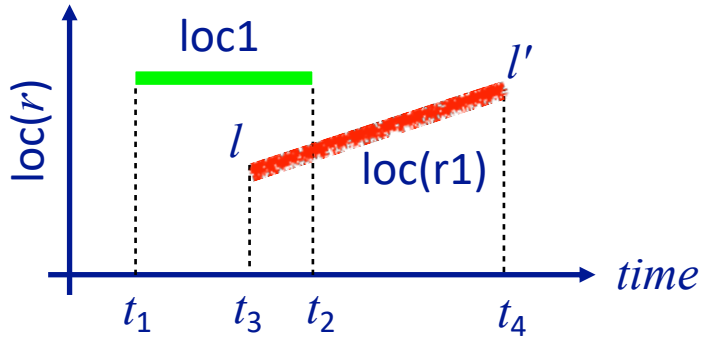


$(\mathcal{T}_1, C_1)$



a ground instance of $(\mathcal{T}_1, C_1)$

- **Poll**: is $(\mathcal{T}_3, C_3)$ secure?*
  - $\mathcal{T}_3 = \{[t_1,t_2]$ loc(r)=loc1, $[t_3,t_4]$ loc(r1):(l,l')$\}$
  - $C_3 = \{t_1 < t_2, t_3 < t_4\}$



*Not necessarily a single timeline, but we can generalize

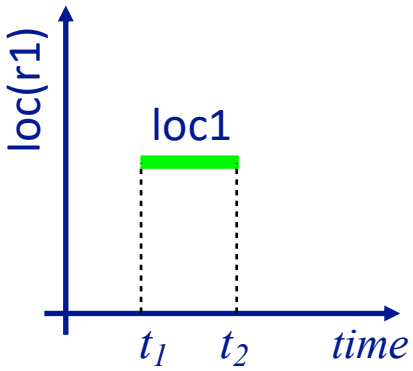
Union of Multiple Timelines

- Timelines for k different state variables
  - $(\mathcal{T}_1, C_1), \ldots, (\mathcal{T}_k, C_k)$
- Union is $(\mathcal{T}, C)$:
  - $\mathcal{T} = \mathcal{T}_1 \cup \ldots \cup \mathcal{T}_k$
  - $C = C_1 \cup \ldots \cup C_k$
- If every $(\mathcal{T}_i, C_i)$ is secure, and
  no pair of timelines $(\mathcal{T}_i, C_i)$ and $(\mathcal{T}_j, C_j)$ have any unground variables in common
  then $(\mathcal{T}_1, C_1) \cup \ldots \cup (\mathcal{T}_k, C_k)$ is also secure

book omits this part

Causal support

- Consider the assertion $[t_1,t_2]$ loc(r1) = loc1
  - How did r1 get to loc1 in the first place?



- *Causal support* for $[t_1,t_2]\ x = v_1$ or $[t_1,t_2]\ x : (v_1, v_2)$

  Either:
  - Information saying it's supported *a priori*
  - Another assertion that produces $x = v_1$ at time t_1
    - $[t_0,t_1]\ x = v_1$
    - $[t_0,t_1]\ x : (v_0, v_1)$

- A timeline is *causally supported* if every assertion has a causal support

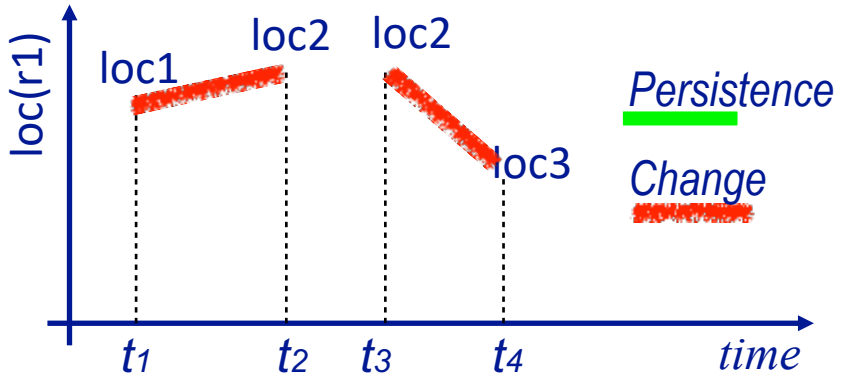
- Three ways to modify a timeline to add causal support …

Establishing causal support

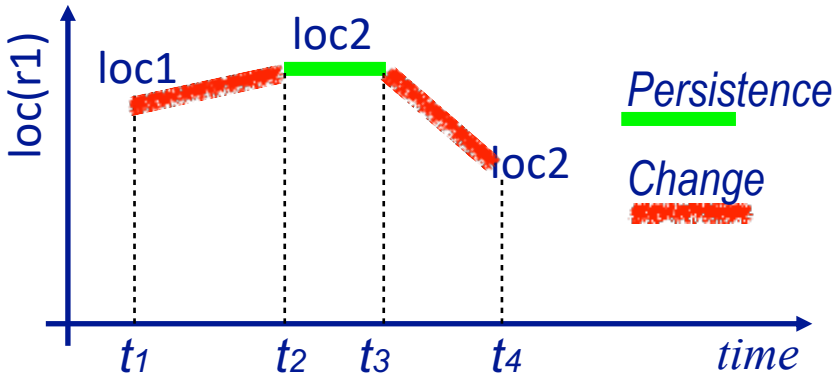
(1) Add a persistence assertion

$\mathcal{T}$ = {[t_1,t_2] loc(r1):(loc1,loc2),
[t_3,t_4] loc(r1):(loc2,loc3)}

C = {$t_1 < t_2 < t_3 < t_4$}



- Add [t_2,t_3] loc(r1) = loc2
  - Supported by the first temporal assertion
  - Supports the second one

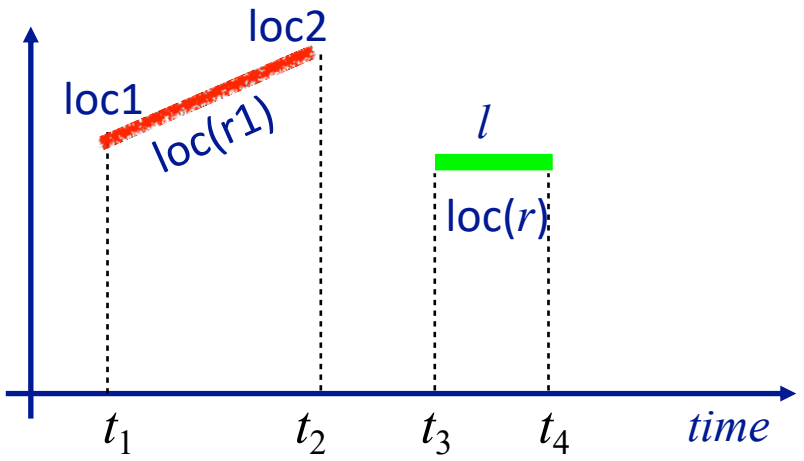


Establishing causal support

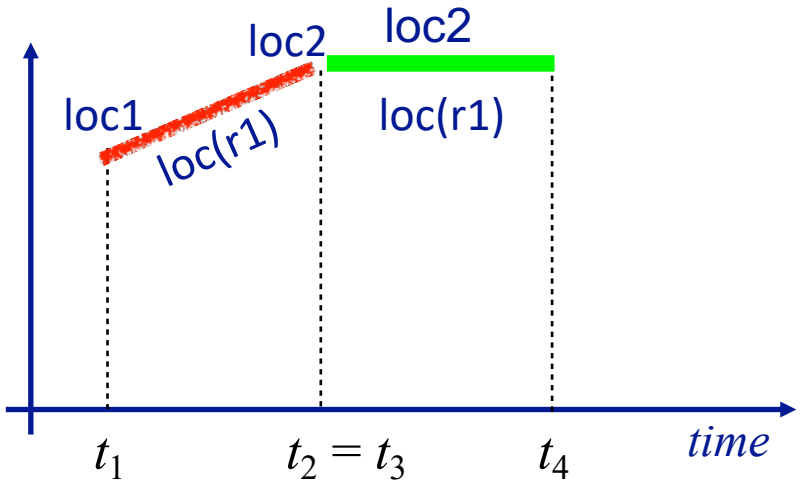
(2) Add constraints

$\mathcal{T} = \{[t_1,t_2]$ loc(r1):(loc1,loc2),
$[t_3,t_4]$ loc(r) = $l\}$

$C = \{t_1 < t_2,\ t_3 < t_4\}$



- Add $t_2 = t_3$, r = r1, l = loc2

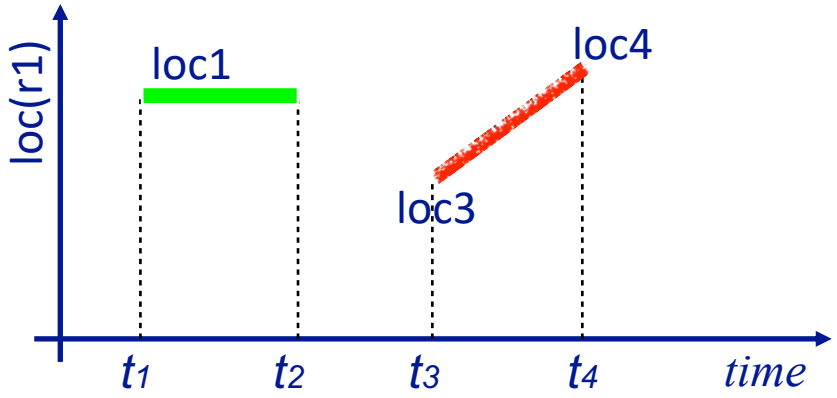


Establishing causal support

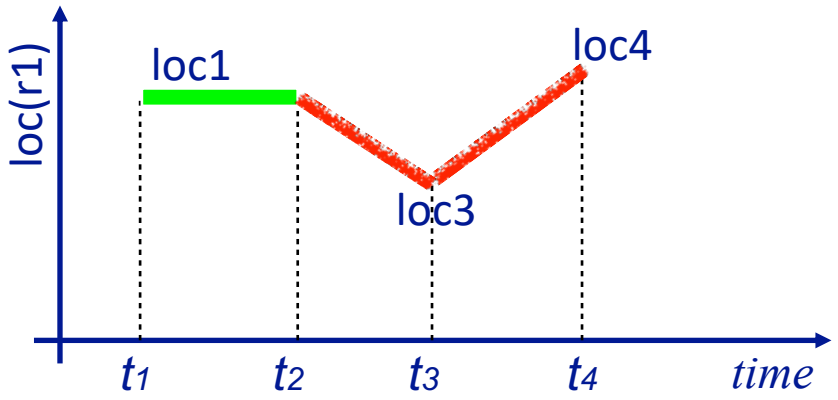
(3) Add a change assertion
(by adding an action)

$\mathcal{T}$ = {[t_1,t_2] loc(r1) = loc1,
[t_3,t_4] loc(r1):(loc3,loc4)}

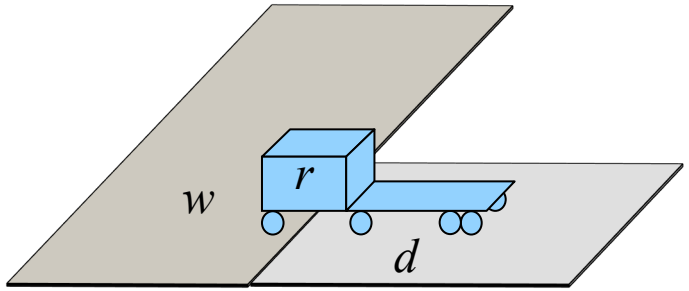
C = {$t_1 < t_2 < t_3 < t_4$}



- Add an action that includes
  [t_2,t_3] loc(r1):(loc1,loc3)



Actions



- *Action* or *primitive task* (or just *primitive*):
  - a triple $(head, \mathcal{T}, C)$
    - *head* is the name and parameters
    - $(\mathcal{T}, C)$ is the union of a set of timelines
- Always two additional parameters
  - starting time t_s , ending time t_e
- In each temporal assertion in $\mathcal{T}$,
  - left endpoint is like a precondition
    - ⇔ need for causal support
  - right endpoint is like an effect

leave(r,d,w)

// robot r goes from loading dock d to waypoint w

assertions:

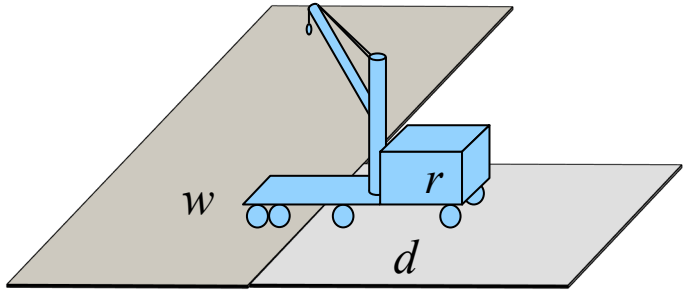
$[t_s,t_e]$ loc(r): (d,w)
$[t_s,t_e]$ occupant(d): $(r$,empty)

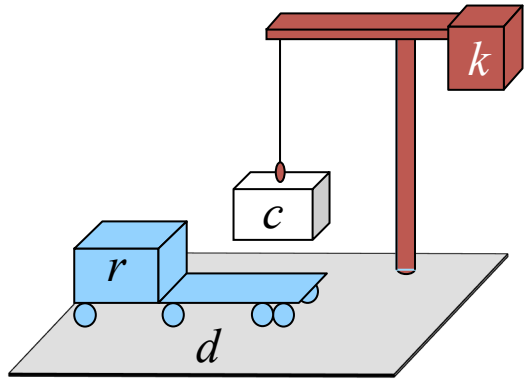
constraints:

$t_e \leq t_s + \delta_1$
adjacent(d,w)

- loc(r) changes to w with delay $\leq \delta_1$
- dock d becomes empty

Actions





enter(r,d,w)

// robot r goes from waypoint w to loading dock d

assertions:

$[t_s,t_e]$ loc(r): (w,d)
$[t_s,t_e]$ occupant(d): (empty,r)

constraints:

$t_e \leq t_s + \delta_2$
adjacent(d,w)

- loc(r) changes to d with delay $\leq \delta_2$
- dock d becomes occupied by r

take(k,c,r,d)

// crane k takes container c from robot r

assertions:

$[t_s,t_e]$ pos(c): (r, k) // where c is
$[t_s,t_e]$ grip(k): (empty, c) // what's in k's gripper
$[t_s,t_e]$ freight(r): (c,empty) // what r is carrying
$[t_s,t_e]$ loc(r) = d // where r is

constraints:

attached(k,d)

Actions

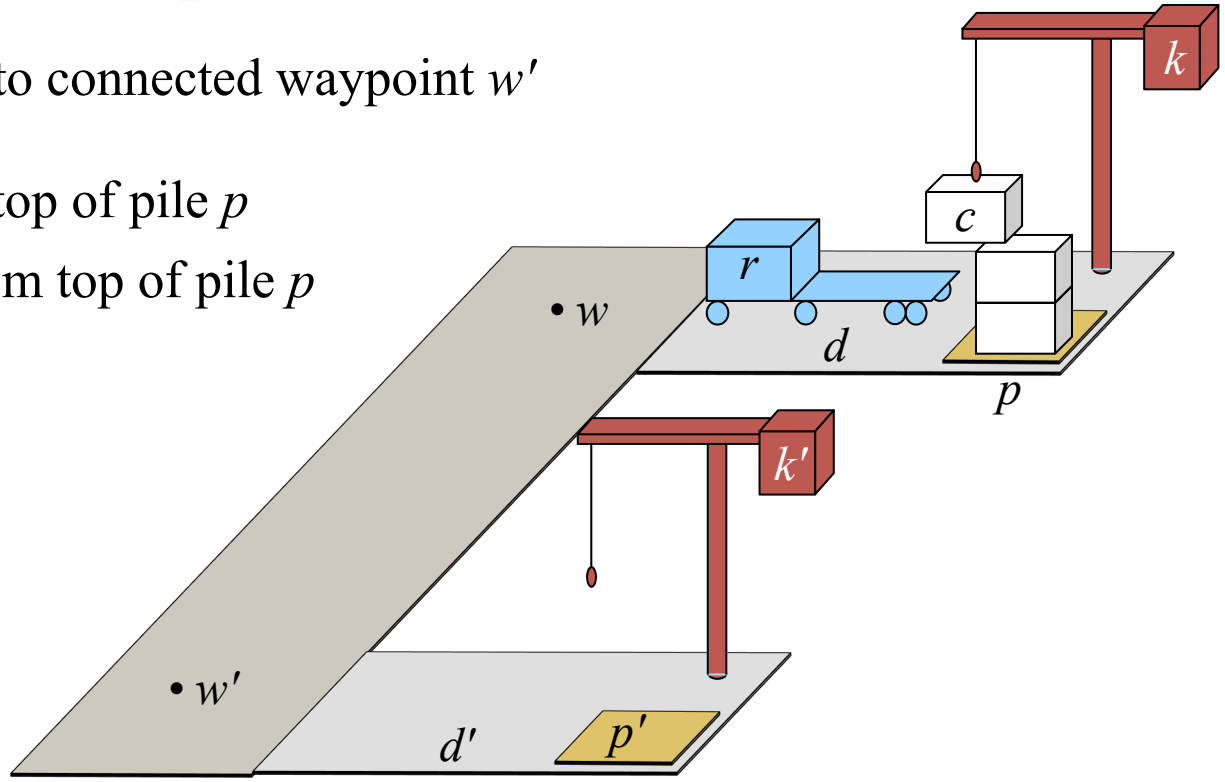
- leave(r,d,w) robot r leaves dock d to an adjacent waypoint w
- enter(r,d,w) r enters d from an adjacent waypoint w

- load(k,c,r) crane k takes container c from robot r
- unload(k,c,r) crane k puts container c onto robot r

book says take and put here, but calls them load and unload later

- navigate(r,w,w') r navigates from waypoint w to connected waypoint w'

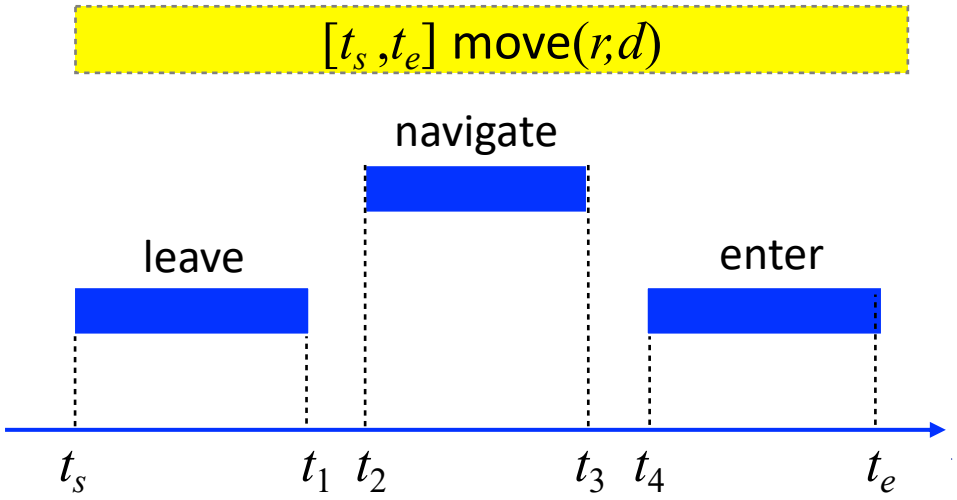
- stack(k,c,p) crane k stacks container c on top of pile p
- unstack(k,c,p) crane k takes a container c from top of pile p



c, c' - containers
d, d' - loading docks
k, k' - cranes
p, p' - piles
r - robot
w, w' - waypoints

Tasks and Methods

- Task: move robot r to dock d
  - $[t_s, t_e]$ move(r,d)



- Method:

  m-move1(r,d,d',w,w')

  task: move(r,d)

  refinement:

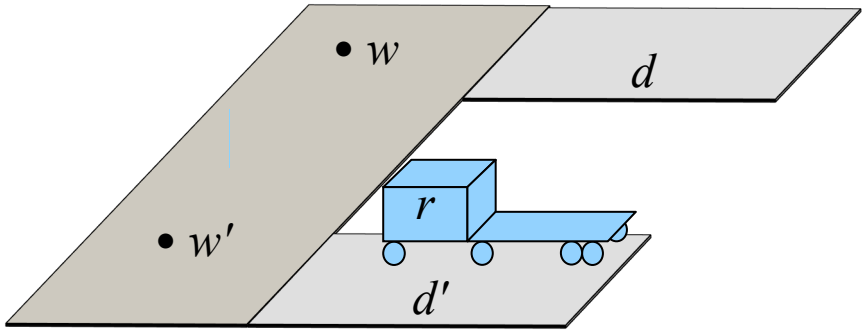
  $[t_s,t_1]$ leave(r,d',w')
  $[t_2,t_3]$ navigate(r,w',w)
  $[t_4,t_e]$ enter(r,d,w)

  assertions:

  $[t_s,t_s+1]$ loc$(r) = d'$

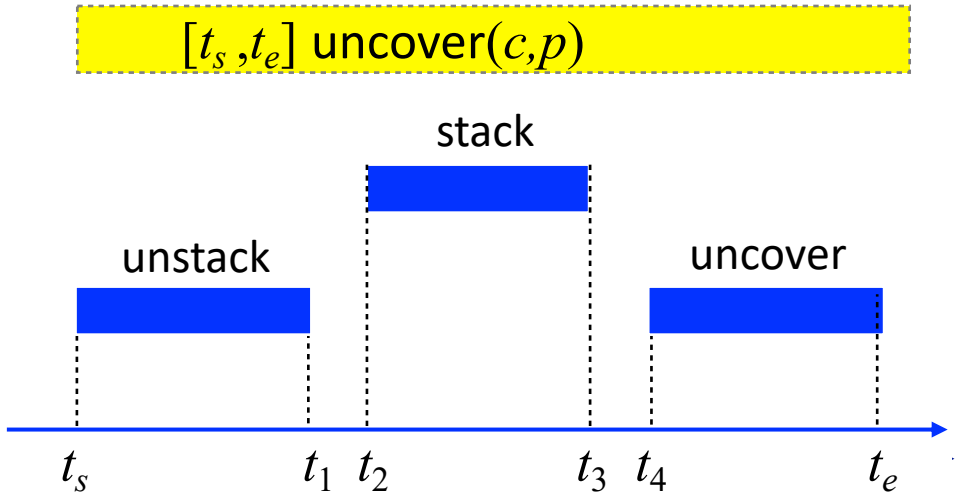
  constraints:

  adjacent(d,w),
  adjacent(d',w'), $d \neq d'$,
  connected(w,w'),
  $t_1 \leq t_2$, $t_3 \leq t_4$



Tasks and Methods

- Task: remove everything above container c in pile p
  - $[t_s, t_e]$ uncover(c,p)



c = c3 - container
p = p1 - pile it's in
k = k1 - crane
d = d1 - loading dock
p' = p2 - offload pile

- Method:

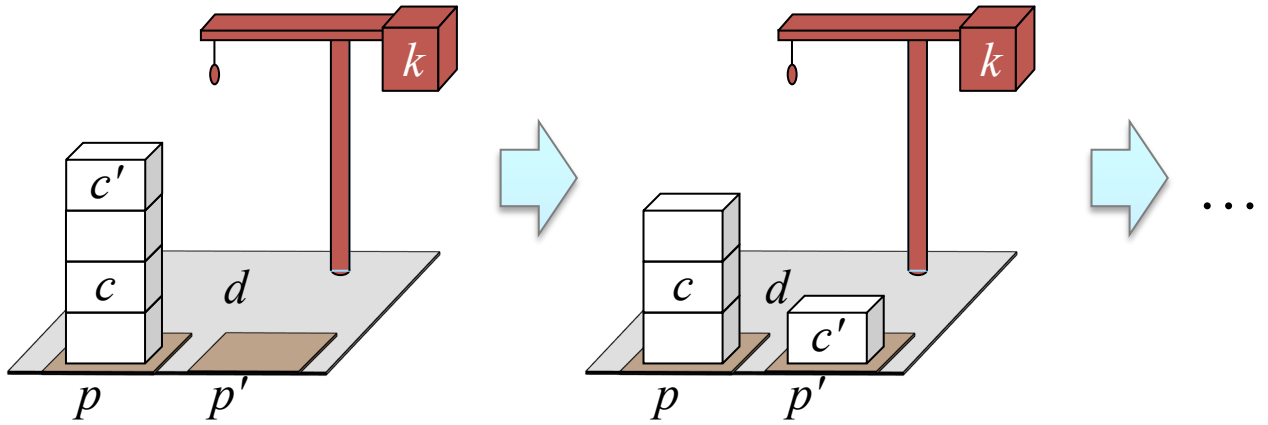
m-uncover(c,p,k,d,p')

task: uncover(c,p)

refinement: $[t_s,t_1]$ unstack(k,c',p) // action
$[t_2,t_3]$ stack(k,c',p') // action
$[t_4,t_e]$ uncover(c,p) // recursive uncover

assertions: $[t_s,t_s+1]$ pile(c) = p
$[t_s,t_s+1]$ top(p) = c'
$[t_s,\ t_s+1]$ grip(k) = empty

constraints: attached(k,d), attached(p,d),
attached(p',d),
$p \neq p'$, $c' \neq c$, $t_1 \leq t_2$, $t_3 \leq t_4$



Tasks and Methods

- Task: robot r brings container c to pile p
  - $[t_s , t_e]$ bring(r,c,p)

- Method:

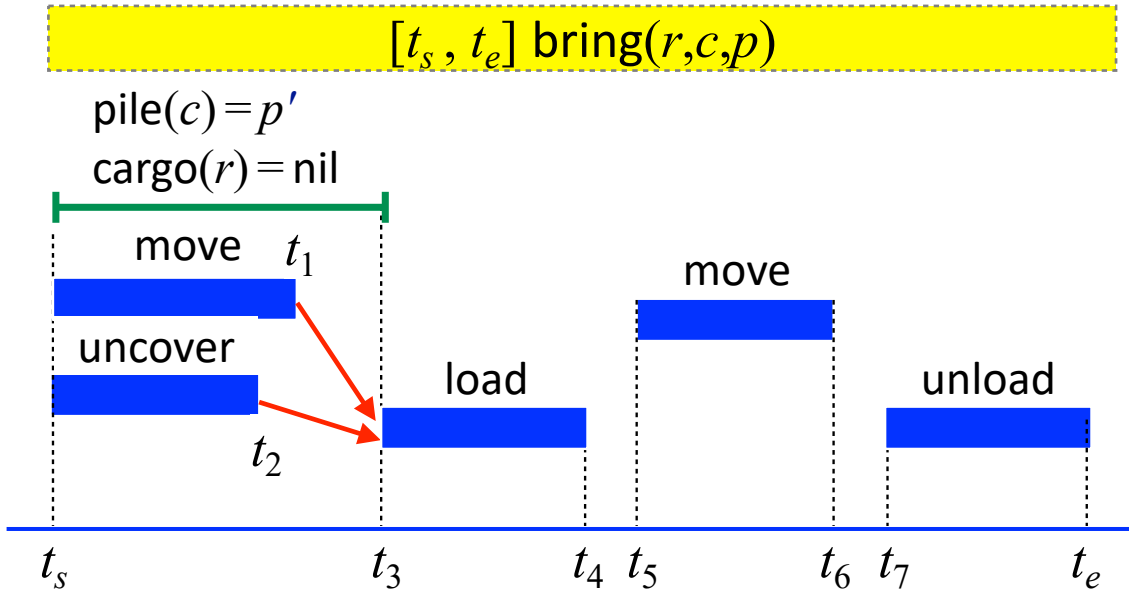
m-bring(r,c,p,p',d,d')

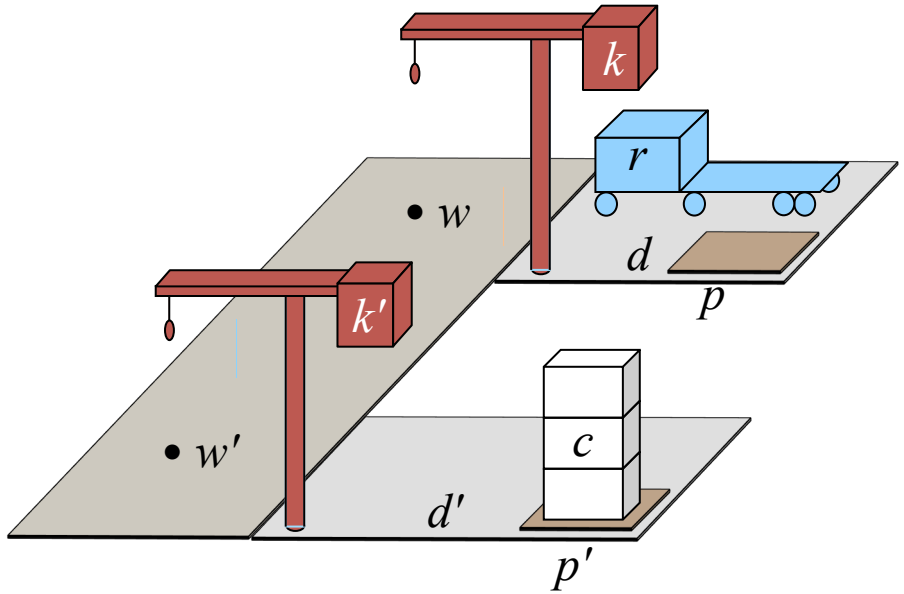
task: bring(r,c,p)

refinement:
$[t_s,t_1]$ move(r,d') // task
$[t_s,t_2]$ uncover(c,p') // task
$[t_3,t_4]$ load(k',r,c,p')
$[t_5,t_6]$ move(r,d) // task
$[t_7,t_e]$ unload(k,r,c,p)

assertions:
$[t_s,t_3]$ pile$(c) = p'$
$[t_s,t_3]$ freight(r) = empty

constraints: attached(p',d'), attached(p,d), $d \neq d'$
attached(k',d'), attached(k,d), $k \neq k'$
$t_1 \leq t_3$, $t_2 \leq t_3$, $t_4 \leq t_5$, $t_6 \leq t_7$





Chronicles

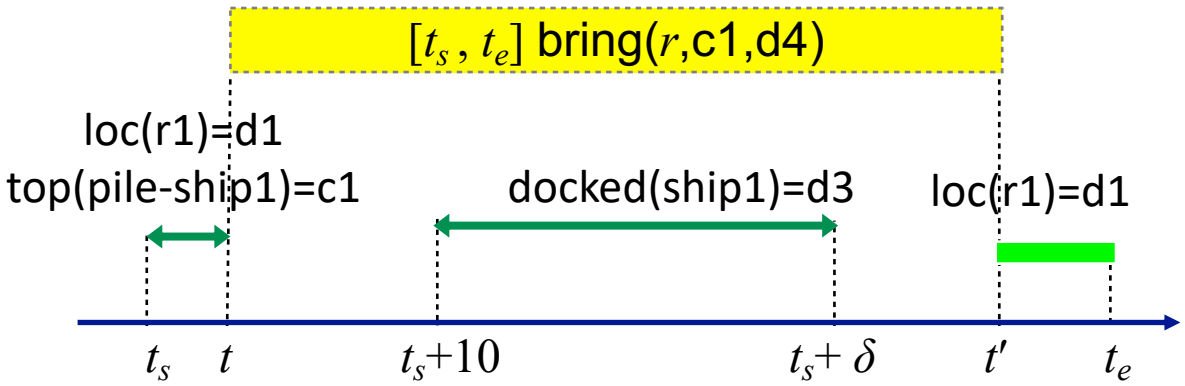
- Chronicle $\phi = (\mathcal{A}, S, \mathcal{T}, C)$
  - $\mathcal{A}$: temporally qualified actions and tasks
  - S: *a priori* supported assertions
  - $\mathcal{T}$: temporally qualified assertions
  - C: constraints

- ϕ can include
  - Current state, future predicted events
  - Tasks to perform
  - Assertions and constraints to satisfy

- Can represent
  - a planning problem
  - a plan or partial plan

ϕ_0:

tasks:	$[t_0,t_1]$ bring(r,c1,d4)
supported:	$[t_s]$ loc(r1)=d1
	$[t_s]$ loc(r2)=d2
	$[t_s+10,t_s+\delta]$ docked(ship1)=d3
	$[t_s]$ top(pile-ship1)=c1
	$[t_s]$ pos(c1)=pallet
assertions:	$[t_e]$ loc(r1)=d1
	$[t_e]$ loc(r2)=d2
constraints:	$t_s = 0 < t < t' < t_e$, $20 \le \delta \le 30$



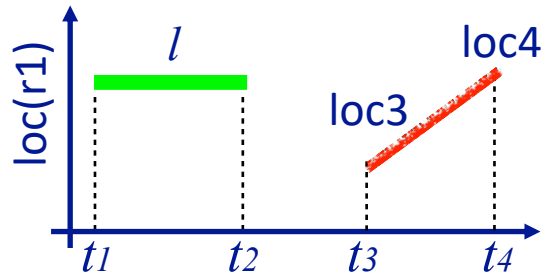
Outline

- ✓ Introduction
- ✓ Representation
- **4.3 Temporal planning**
  - Resolvers and flaws
  - Search space
- 4.4 Constraint management
- 4.5 Acting with temporal models

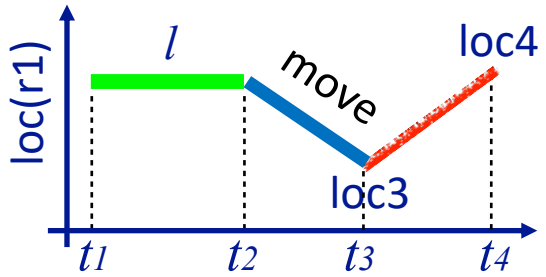
Planning

- Planning problem:
  - a chronicle ϕ_0 that has some *flaws*
    - analogous to flaws in PSP
- To resolve the flaws, add
  - assertions
  - constraints
  - actions

ϕ_0: tasks: *(none)*
supported: *(none)*
assertions: $[t_1,t_2]$ loc(r1) = l
$[t_3,t_4]$ loc(r1) : (loc3,loc4)
constraints: adjacent(loc3,w1)
adjacent(w1,loc3)
adjacent(loc4,w2)
adjacent(w2,loc4)
connected(w1,w2)



ϕ_0: tasks: $[t_2,t_3]$ move(r1,loc3)
supported: *(none)*
assertions: $[t_1,t_2]$ loc(r1) = l
$[t_3,t_4]$ loc(r1) : (loc3,loc4)
constraints: adjacent(loc3,w1)
adjacent(w1,loc3)
adjacent(loc4,w2)
adjacent(w2,loc4)
connected(w1,w2)

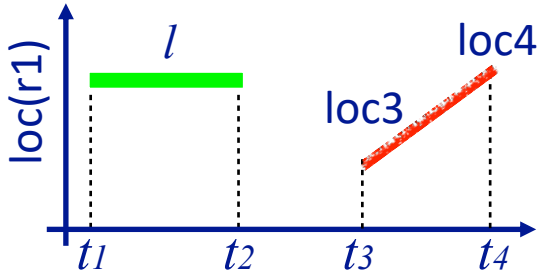


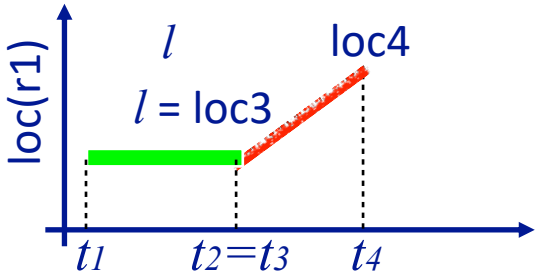
Flaws (1)

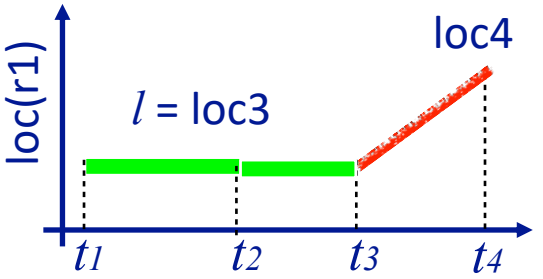
Like an open goal in PSP

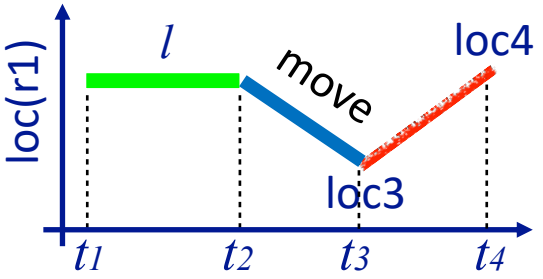
1. Temporal assertion α that isn't causally supported
   - What causes r1 to be at loc3 at time t_3?

- *Resolvers*:
  - Add constraints* to support α from an assertion in ϕ
    - l = loc3, $t_2 = t_3$
  - Add a new persistence assertion* to support α
    - l = loc3, $[t_2,t_3]$ loc(r1) = loc3
  - Add an action or task to support α
    - $[t_2,t_3]$ move(r1,loc3)
      - refining it will produce an action that supports α (see next slide)









*where the book uses equality constraints, we use substitutions

Like a task in SeRPE

Flaws (2)

2. Non-refined task

- *Resolver*: refinement method
  - Applicable if it matches the task and its constraints are consistent with ϕ's
- Applying the resolver:
  - Modify ϕ by replacing the task with m

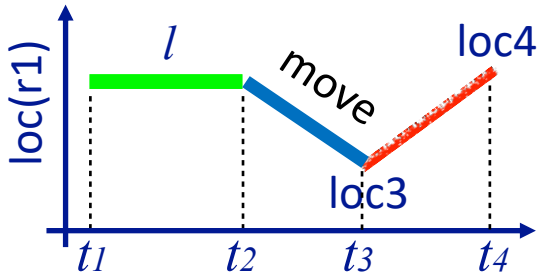
- Example: $[t_2,t_3]$ move(r1,loc3)
  - Refinement will replace it with something like

    $[t_2,t_5]$ leave(r1,l,w)

    $[t_5,t_6]$ navigate(r,w,w')

    $[t_6,t_3]$ enter(r1,loc3,w')

    plus constraints



- Method:

  m-move1(r,d,d',w,w')

  task: move(r,d)

  refinement:

  $[t_s,t_1]$ leave(r,d',w')
  $[t_2,t_3]$ navigate(r,w',w)
  $[t_4,t_e]$ enter(r,d,w)

  assertions:

  $[t_s,t_s+1]$ loc(r) = d'

  constraints:

  adjacent(d,w),
  adjacent(d',w'), $d \neq d'$,
  connected(w,w'),
  $t_1 \leq t_2$, $t_3 \leq t_4$

Like a threat in PSP

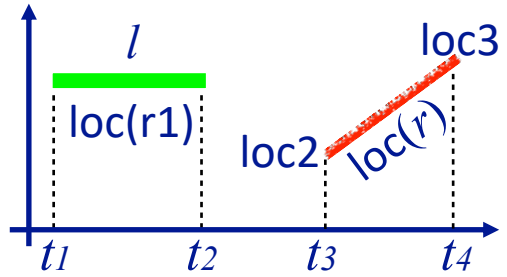
Flaws (3)

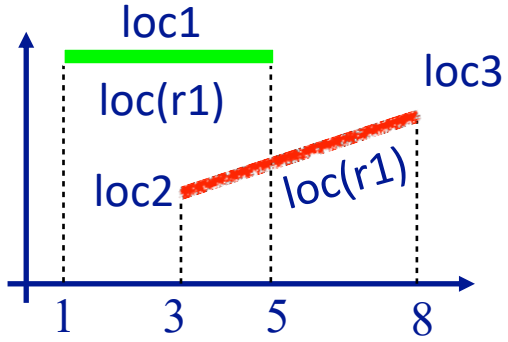
3. A pair of possibly-conflicting temporal assertions

- temporal assertions α and β *possibly conflict* if they can have inconsistent instances

e.g., $[t_1,t_2]$ loc(r1) = loc1, $[t_3,t_4]$ loc(r) : (l, l')

instance: [1,5] loc(r1) = loc1, [3,8] loc(r1) : (loc2,loc3)





- *Resolvers*: separation constraints
  - $r \neq$ r1
  - $t_2 < t_3$
  - $t_4 < t_1$
  - $t_2 = t_3$, r = r1 , l = loc1
    - Also provides causal support for $[t_3,t_4]$ loc(r) : (l, l')
  - $t_4 = t_1$, r = r1 , l = loc1
    - Also provides causal support for $[t_1,t_2]$ loc(r1) = loc1

Planning Algorithm

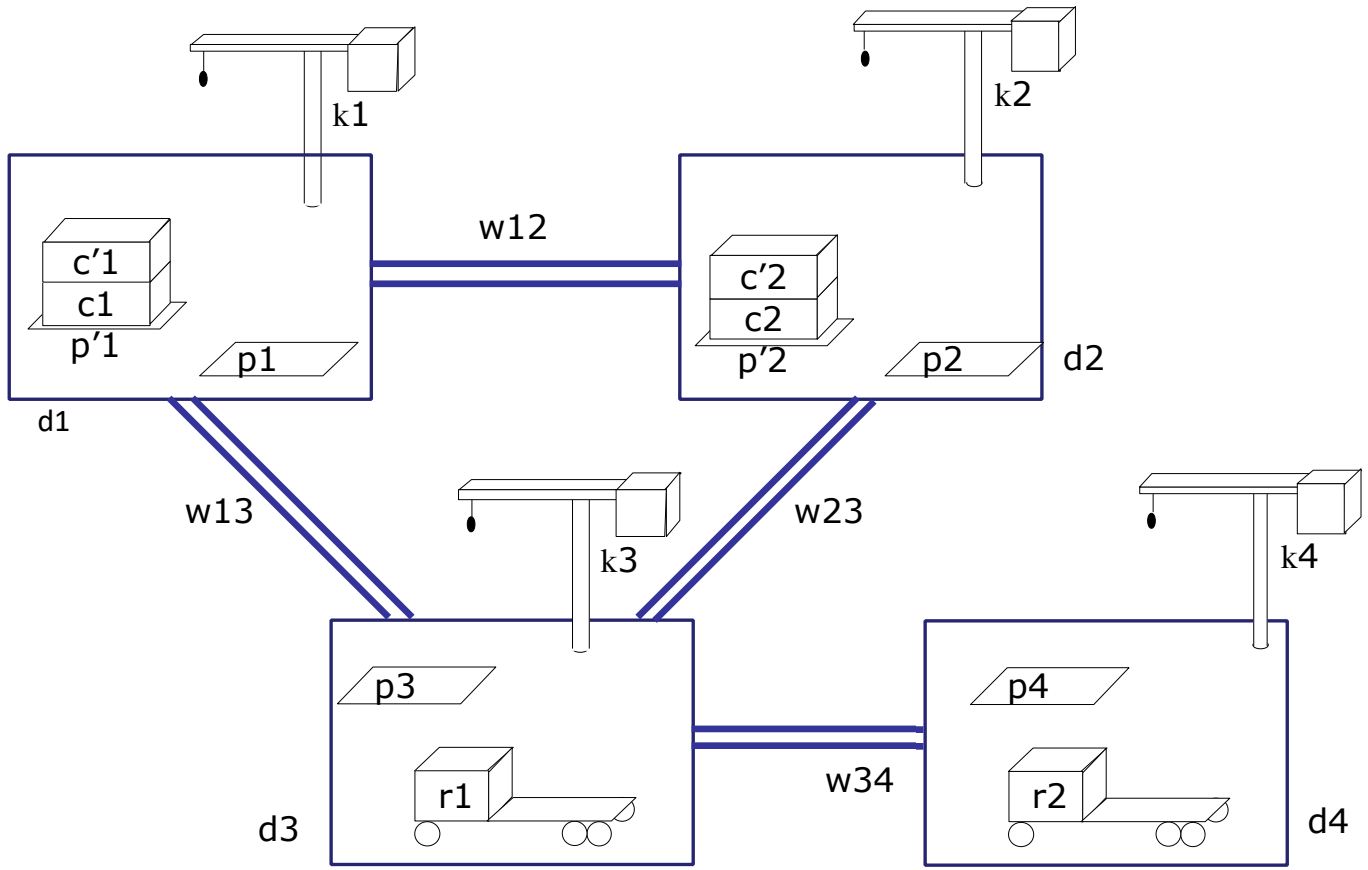
- Like PSP in Chapter 2
  - Repeatedly selects flaws and chooses resolvers
- In the book, TemPlan uses recursion
  - Can be rewritten to use a loop
  - Just programming style, equivalent either way
- In a deterministic implementation
  - Selecting a resolver ρ is a backtracking point
  - Selecting a flaw isn't
- If it's possible to resolve all flaws, at least one of the nondeterministic execution traces will do so

TemPlan(ϕ, Σ)
  $Flaws \leftarrow$ set of flaws of ϕ
  if $Flaws = \varnothing$ then return ϕ
  arbitrarily select $f \in Flaws$
  $Resolvers \leftarrow$ set of resolvers of f
  if $Resolvers = \varnothing$ then return failure
  nondeterministically choose $\rho \in Resolvers$
  $\phi \leftarrow$ Transform(ϕ, ρ)
  Templan(ϕ, Σ)

TemPlan(ϕ, Σ)
  loop:
    $Flaws \leftarrow$ set of flaws of ϕ
    if $Flaws = \varnothing$ then return ϕ
    arbitrarily select $f \in Flaws$
    $Resolvers \leftarrow$ set of resolvers of f
    if $Resolvers = \varnothing$ then return failure
    nondeterministically choose $\rho \in Resolvers$
    $\phi \leftarrow$ Transform(ϕ, ρ)

Example

- $\phi_0 = (\mathcal{A}, S, \mathcal{T}, C)$
  - Establishes state-variable values at time $t = 0$
  - Flaws: two unrefined tasks
- Select bring(r,c1,p3) to resolve first

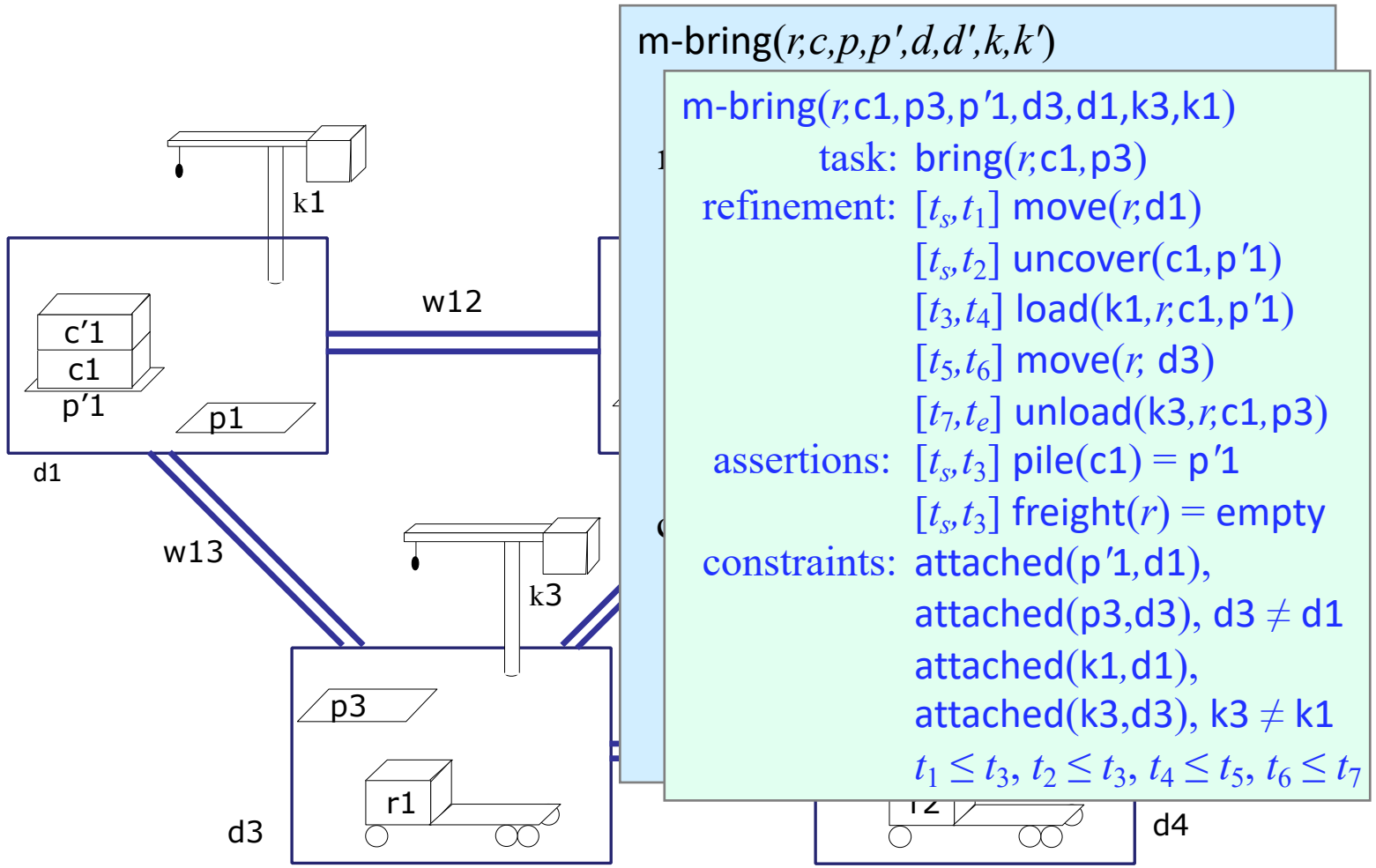


ϕ_0: tasks: bring(r,c1,p3)
bring(r',c2,p4)
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p'1
[0] pile(c2)=p'2, . . .
assertions: *(none)*
constraints: adjacent(d1,w12),
adjacent(d1,w13), . . .
connected(w12,w13), . . .

- Note:
  - [0] $x = v$ means [0,0] $x = v$

Method instance

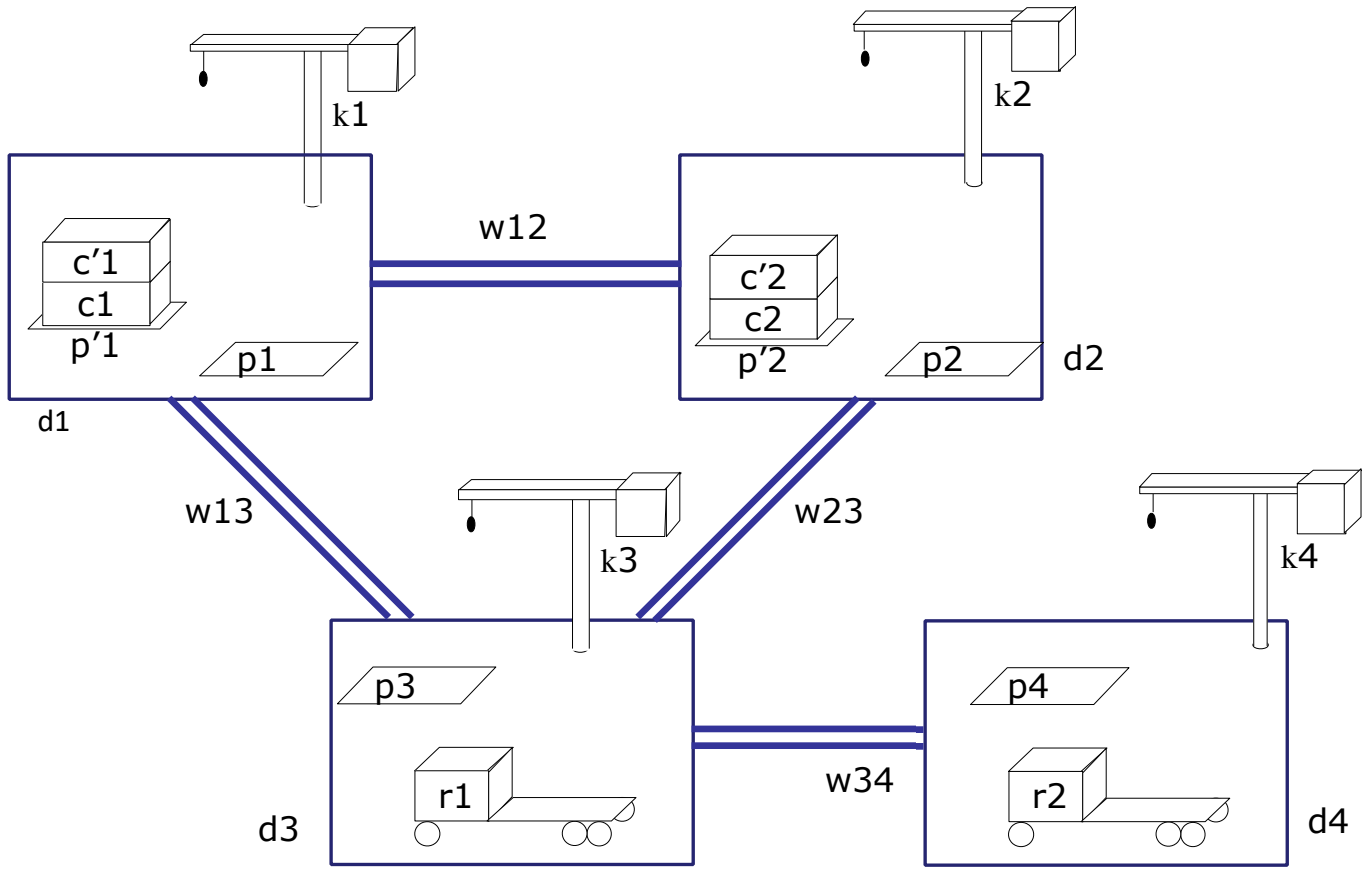
- One relevant method
  - Instantiate $c \leftarrow$ c1 and $p \leftarrow$ p3 to match bring(r,c1,p3)
- I don't think p',d,d',k,k' should be instantiated yet, but I've done so to match the example in the book



ϕ_0: tasks: bring(r,c1,p3)
bring(r',c2,p4)
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p'1
[0] pile(c2)=p'2, . . .
assertions: *(none)*
constraints: adjacent(d1,w12),
adjacent(d1,w13), . . .
connected(w12,w13), . . .

Modified Chronicle

- Changes to ϕ_0
  - Removed bring(r,c1,p3)
  - Added new tasks, assertions, constraints
- Flaws: 6 unrefined tasks, 2 unsupported assertions
- Select bring(r',c2,p4) to resolve next



ϕ_1: tasks: $[t_s,t_1]$ move(r,d1)
$[t_s,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r,c1,p′1)
$[t_5,t_6]$ move(r, d3)
$[t_7,t_e]$ unload(k3,r,c1,p3)
bring(r',c2,p4)

supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .

assertions: $[t_s,t_3]$ pile(c1) = p′1
$[t_s,t_3]$ freight(r) = empty

constraints: $t_s<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$
adjacent(d1,w12), . . .
adjacent(d1,w13), . . .
connected(w12,w13) , . . .

Method instance

- Instantiate $r \leftarrow r'$, $c \leftarrow$ c2, $p \leftarrow$ p4 to match bring(r',c2,p4)
- I don't think p',d,d',k,k' should be instantiated yet, but I've done so to match the book
- Rename timepoint variables to avoid name conflicts

m-bring(r,c,p,p',d,d',k,k')

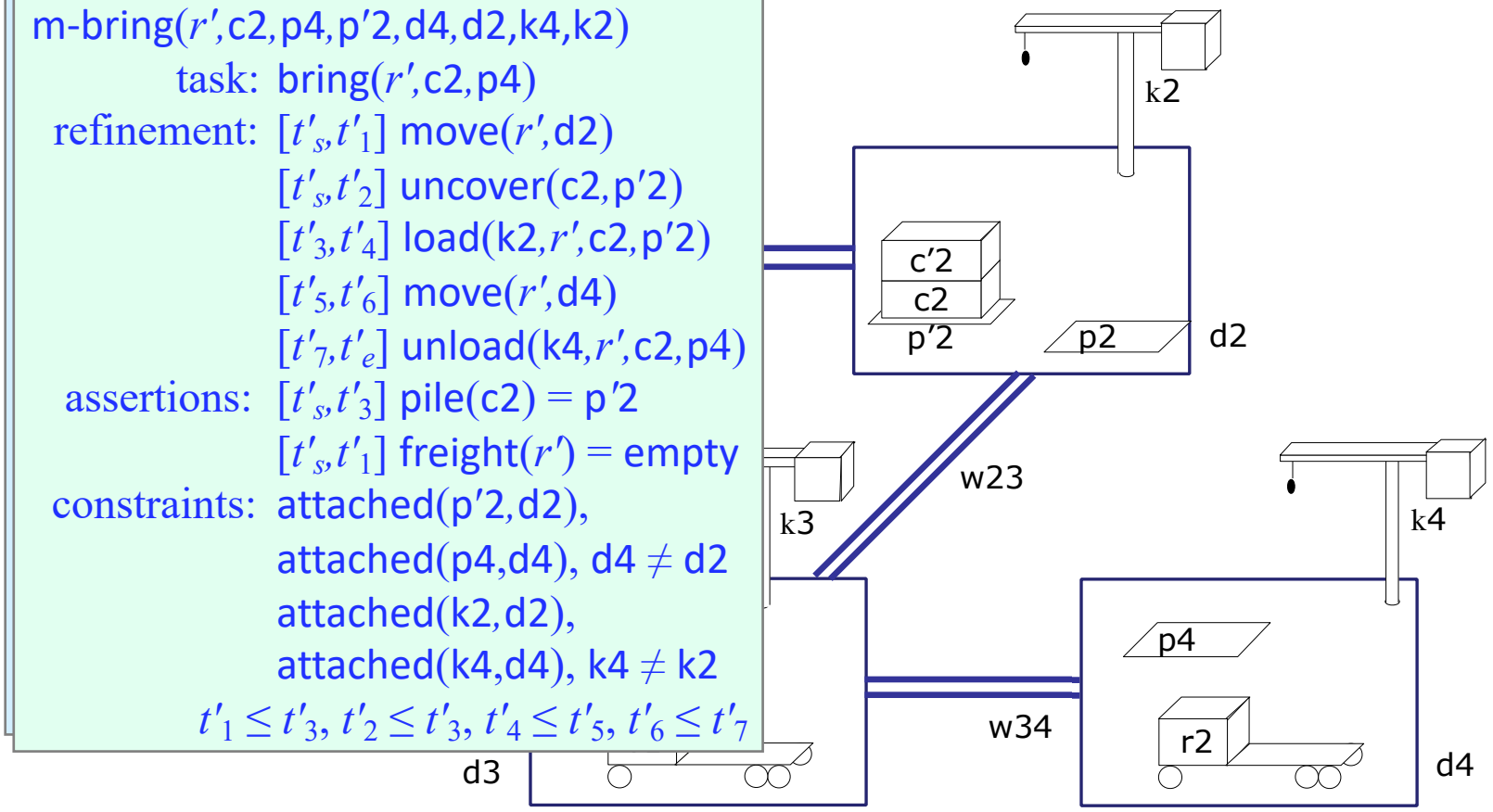
m-bring(r',c2,p4,p′2,d4,d2,k4,k2)

task: bring(r',c2,p4)

refinement:
- $[t'_s,t'_1]$ move(r',d2)
- $[t'_s,t'_2]$ uncover(c2,p′2)
- $[t'_3,t'_4]$ load(k2,r',c2,p′2)
- $[t'_5,t'_6]$ move(r',d4)
- $[t'_7,t'_e]$ unload(k4,r',c2,p4)

assertions:
- $[t'_s,t'_3]$ pile(c2) = p′2
- $[t'_s,t'_1]$ freight(r') = empty

constraints:
- attached(p′2,d2),
- attached(p4,d4), d4 ≠ d2
- attached(k2,d2),
- attached(k4,d4), k4 ≠ k2
- $t'_1 \le t'_3$, $t'_2 \le t'_3$, $t'_4 \le t'_5$, $t'_6 \le t'_7$



ϕ_1: tasks:
- $[t_s,t_1]$ move(r,d1)
- $[t_s,t_2]$ uncover(c1,p′1)
- $[t_3,t_4]$ load(k1,r,c1,p′1)
- $[t_5,t_6]$ move(r, d3)
- $[t_7,t_e]$ unload(k3,r,c1,p3)
- bring(r',c2,p4)

supported:
- [0] loc(r1)=d3
- [0] loc(r2)=d4
- [0] freight(r1)=empty
- [0] freight(r2)=empty
- [0] pile(c1)=p′1
- [0] pile(c2)=p′2, . . .
- adjacent(d1,w12), . . .
- connected(w12,w13), . . .

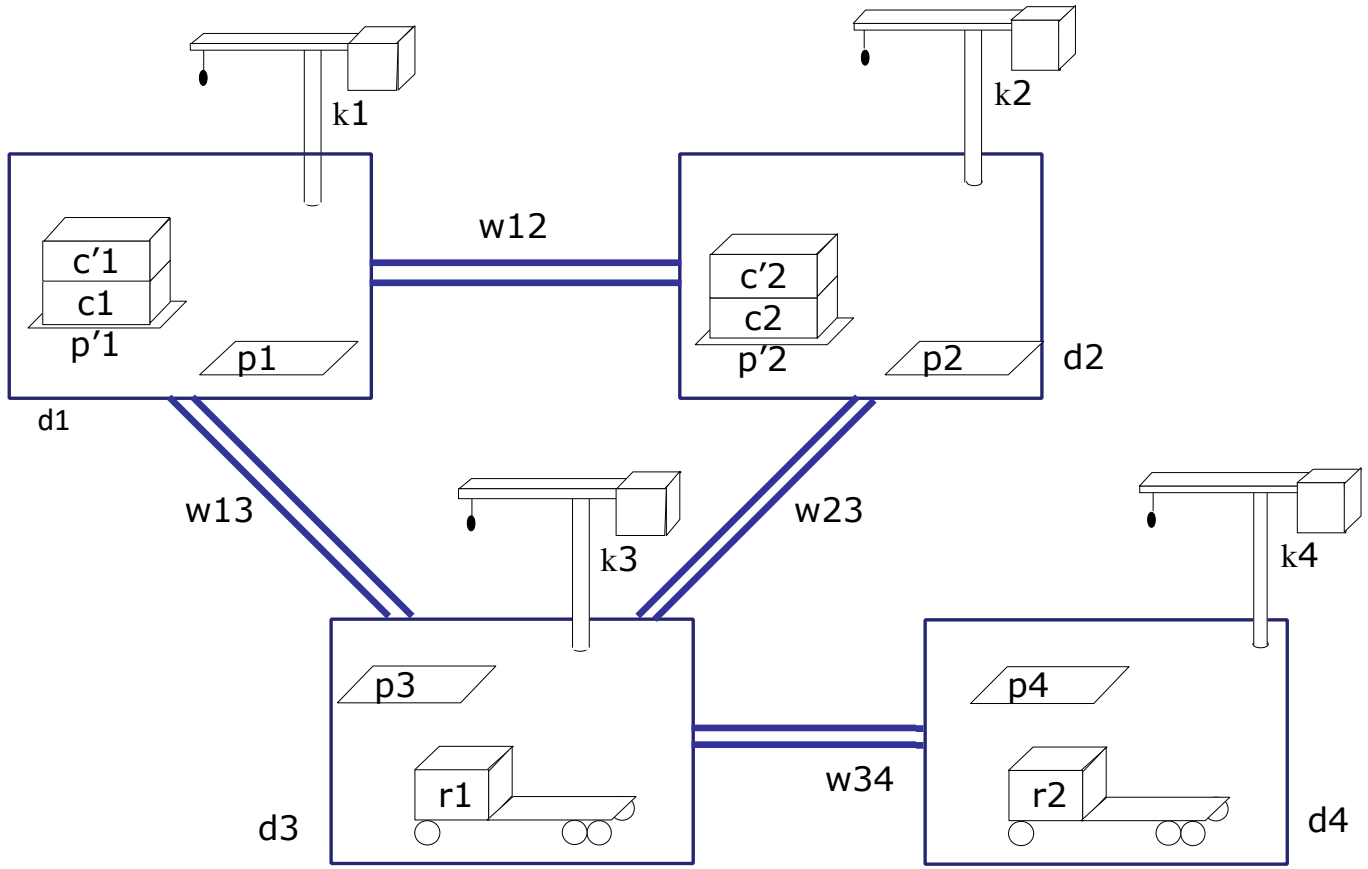
assertions:
- $[t_s,t_3]$ pile(c1) = p′1
- $[t_s,t_3]$ freight(r) = empty

constraints: $t_s<t_1\le t_3$, $0<t_2\le t_3$, $t_4\le t_5$, $t_6\le t_7$
- adjacent(d1,w12), . . .
- adjacent(d1,w13), . . .
- connected(w12,w13) , . . .

Poll: to get the method instance, I substituted $r'\leftarrow r$ in m-bring. Should I also substitute $r'\leftarrow r$ in ϕ_1?

Modified Chronicle

- Changes:
  - Removed bring(r',c2,p4)
  - Added tasks, assertions, constraints
- Flaws: 10 unrefined tasks, 4 unsupported assertions
- Select $[t_s,t_3]$ pile(c1) = p′1 to resolve next



ϕ_2: tasks: $[t_s,t_1]$ move(r,d1)
$[t_s,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r,c1,p′1)
$[t_5,t_6]$ move(r, d3)
$[t_7,t_e]$ unload(k3,r,c1,p3)
$[t'_s,t'_1]$ move(r',d2)
$[t'_s,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r',c2,p′2)
$[t'_5,t'_6]$ move(r',d4)
$[t'_7,t'_e]$ unload(k2,r',c2,p′2)

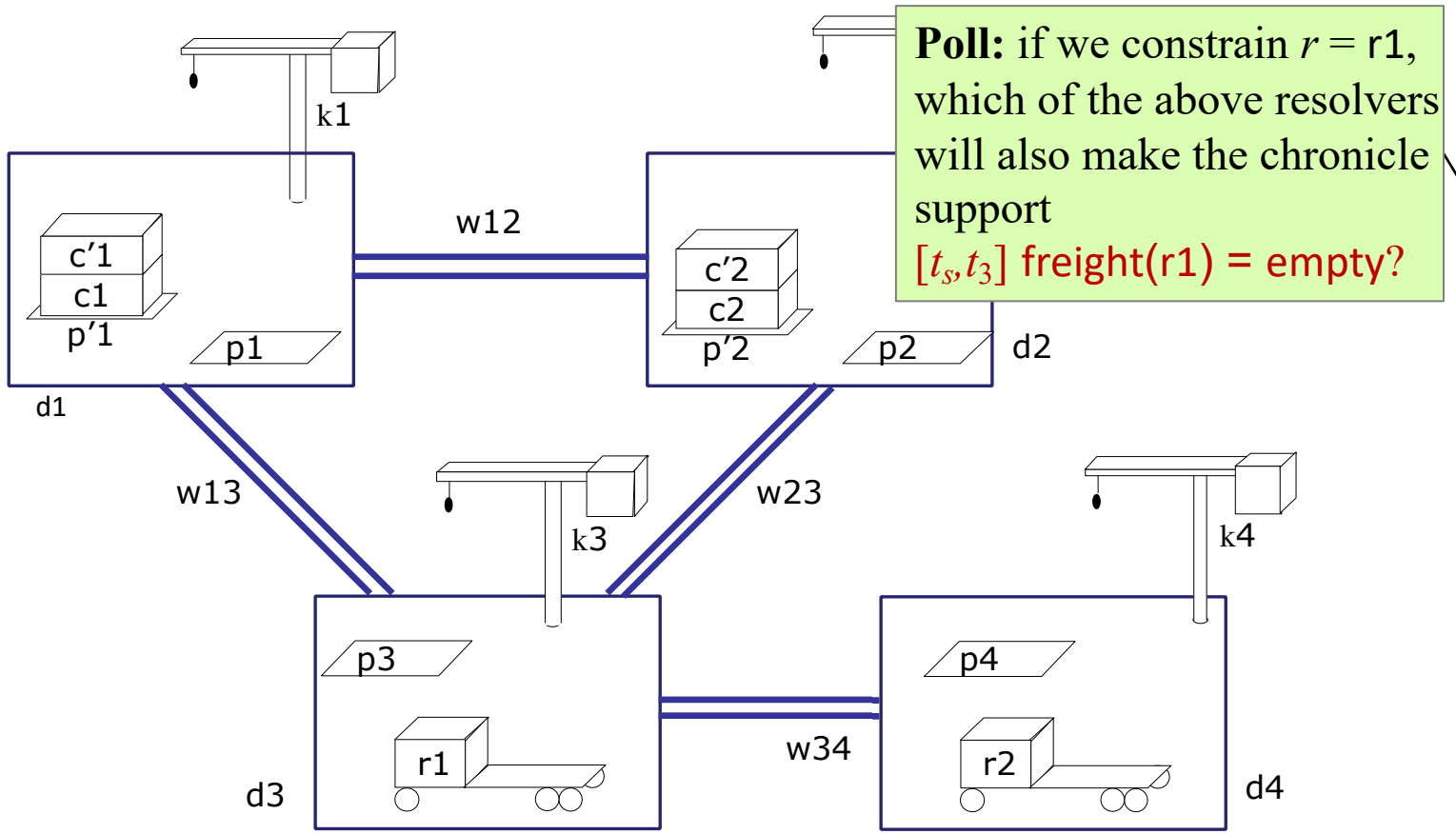
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .

assertions: $[t_s,t_3]$ pile(c1) = p′1
$[t_s,t_3]$ freight(r) = empty
$[t'_s,t'_3]$ pile(c2) = p′2
$[t'_s,t'_1]$ freight(r') = empty

constraints: $t_s<t_1\leq t_3$, $t_s<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$t'_s<t'_1\leq t'_3$, $t'_s<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13) , . . .

Flaw and Resolvers

- Three ways to support $[t_s,t_3]$ pile(c1)=p′1
  1. Support from [0] pile(c1)=p′1 by constraining $t_s = 0$
  2. Support from [0] pile(c1)=p′1 by adding persistence $[0,t_s]$ pile(c1)=p1
  3. Add new action $[t_8,t_s]$ load(r'',c1,p′1)



Poll: if we constrain r = r1, which of the above resolvers will also make the chronicle support $[t_s,t_3]$ freight(r1) = empty?

ϕ_2: tasks: $[t_s,t_1]$ move(r,d1)
$[t_s,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r,c1,p′1)
$[t_5,t_6]$ move(r, d3)
$[t_7,t_e]$ unload(k3,r,c1,p3)
$[t'_s,t'_1]$ move(r',d2)
$[t'_s,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r',c2,p′2)
$[t'_5,t'_6]$ move(r',d4)
$[t'_7,t'_e]$ unload(k2,r',c2,p′2)

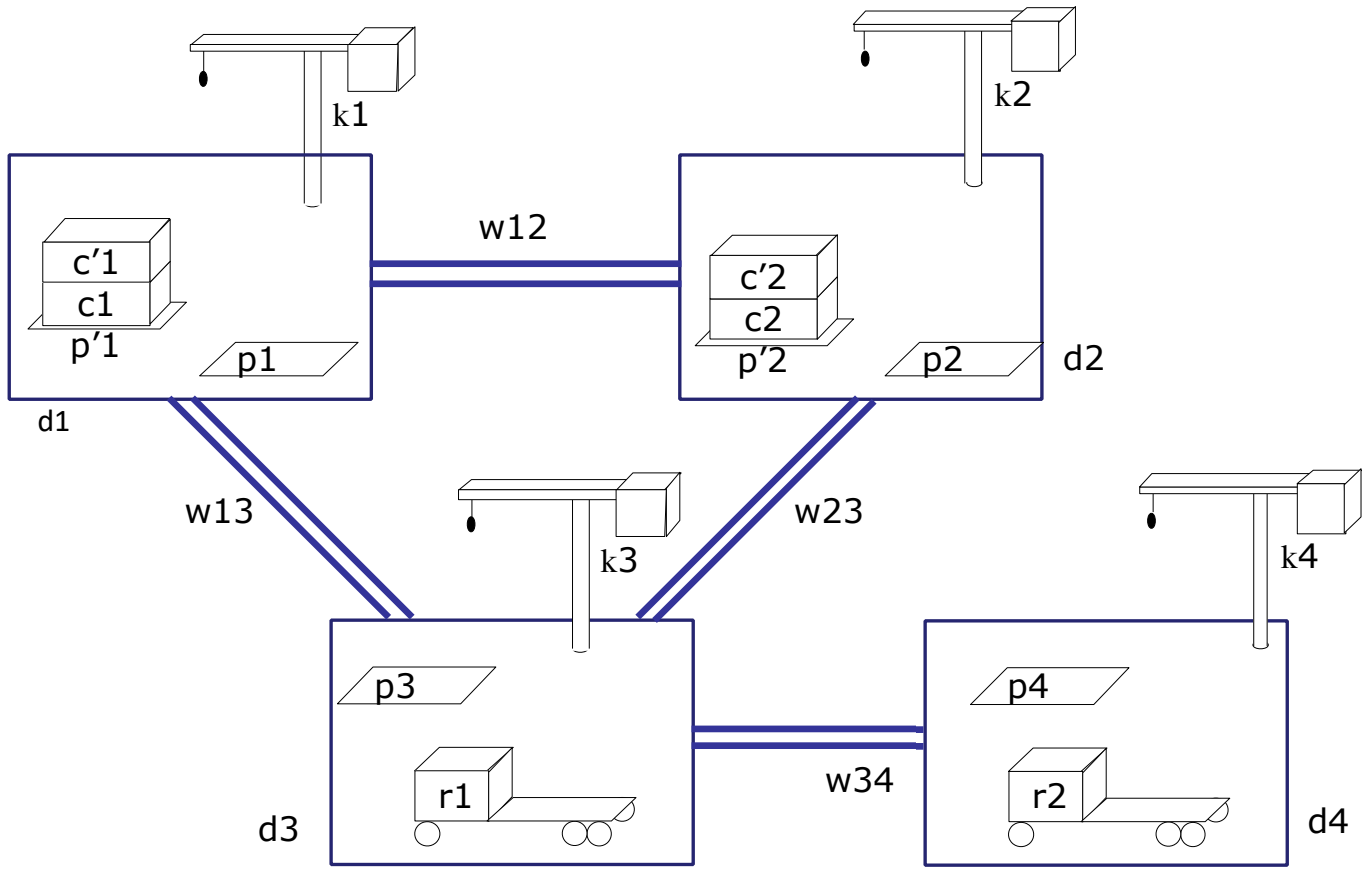
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .

assertions: $[t_s,t_3]$ pile(c1) = p′1
$[t_s,t_3]$ freight(r) = empty
$[t'_s,t'_3]$ pile(c2) = p′2
$[t'_s,t'_1]$ freight(r') = empty

constraints: $t_s<t_1\leq t_3$, $t_s<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$t'_s<t'_1\leq t'_3$, $t'_s<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Resolving Two Flaws

- Support $[t_s,t_3]$ pile(c1) = p′1 from [0] pile(c1)=p′1 by constraining $t_s = 0$
- Next, support $[0,t_3]$ freight(r) = empty from [0] freight(r2)=empty by constraining r = r2



ϕ_2: tasks: $[t_s,t_1]$ move(r,d1)
$[t_s,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r,c1,p′1)
$[t_5,t_6]$ move(r,d3)
$[t_7,t_e]$ unload(k3,r,c1,p3)
$[t'_s,t'_1]$ move(r',d2)
$[t'_s,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r',c2,p′2)
$[t'_5,t'_6]$ move(r',d4)
$[t'_7,t'_e]$ unload(k2,r',c2,p′2)

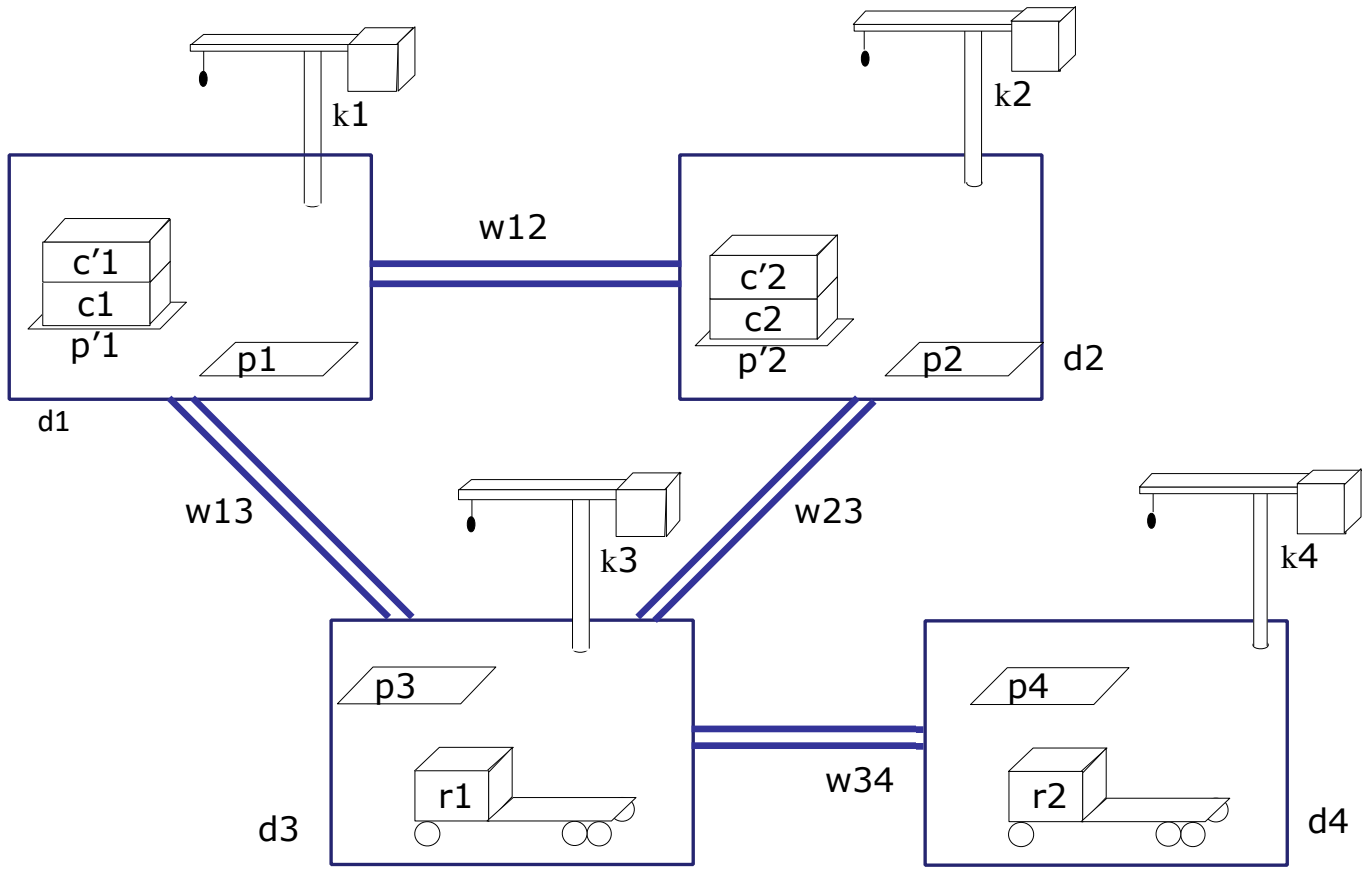
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .

assertions: $[t_s,t_3]$ pile(c1) = p′1
$[t_s,t_3]$ freight(r) = empty
$[t'_s,t'_3]$ pile(c2) = p′2
$[t'_s,t'_1]$ freight(r') = empty

constraints: $t_s<t_1\leq t_3, t_s<t_2\leq t_3, t_4\leq t_5, t_6\leq t_7,$
$t'_s<t'_1\leq t'_3, t'_s<t'_2\leq t'_3, t'_4\leq t'_5, t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Modified Chronicle

- Changes
  - Substitute $t_s \leftarrow 0$ and $r \leftarrow$ r2
  - Move $[0,t_3]$ pile(c1)=p′1 and $[0,t_3]$ freight(r) = empty to "supported" list
- Flaws: 10 unrefined tasks, 2 unsupported assertions
- Next: the two unsupported assertions



ϕ_3: tasks: $[0,t_1]$ move(r2,d1)
$[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[t'_s,t'_1]$ move(r',d2)
$[t'_s,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r',c2,p′2)
$[t'_5,t'_6]$ move(r',d4)
$[t'_7,t'_e]$ unload(k2,r',c2,p′2)

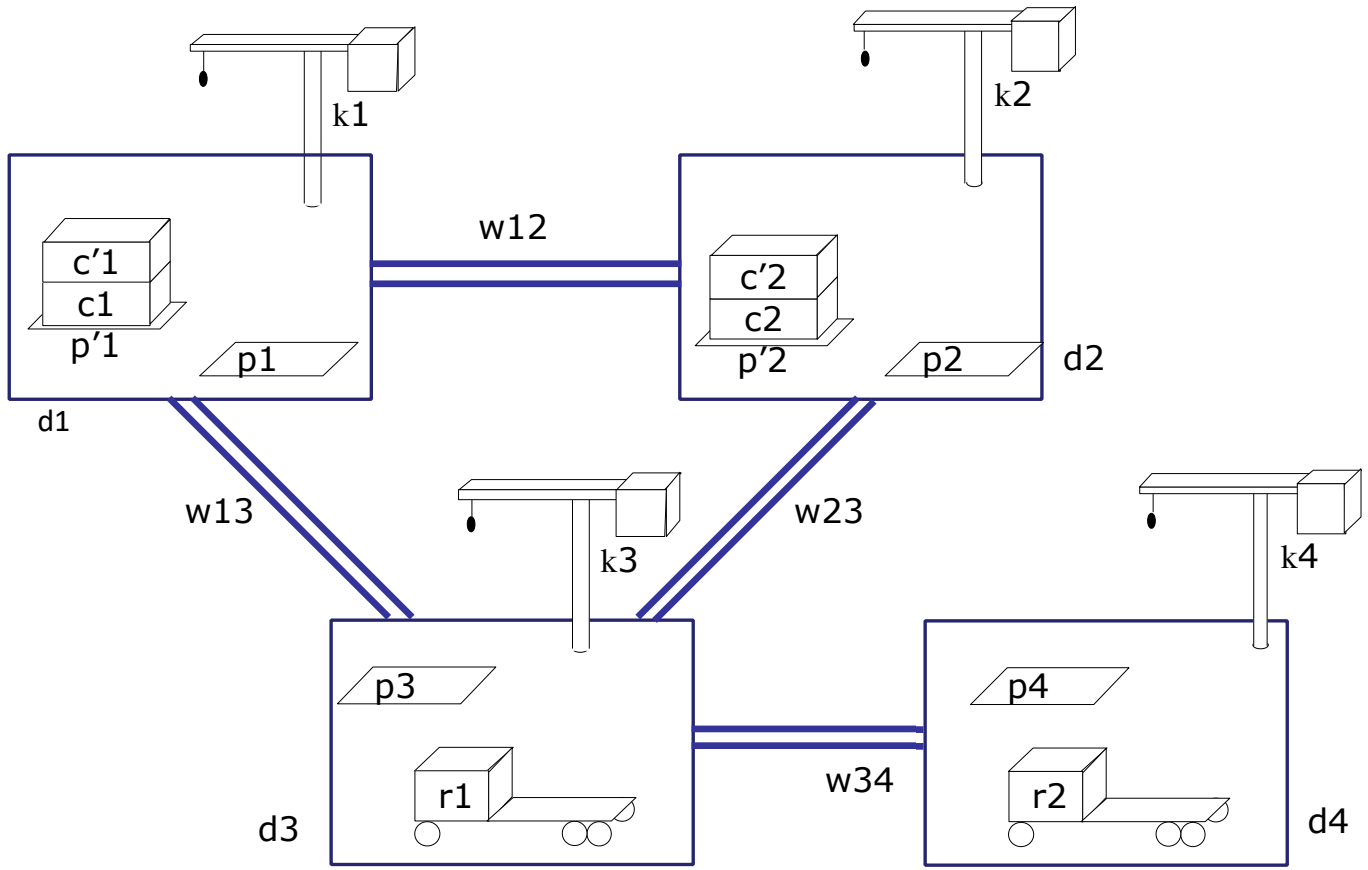
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .
$[0,t_3]$ pile(c1) = p′1
$[0,t_3]$ freight(r2) = empty

assertions: $[t'_s,t'_3]$ pile(c2) = p′2
$[t'_s,t'_1]$ freight(r') = empty

constraints: $0<t_1\leq t_3, 0<t_2\leq t_3, t_4\leq t_5, t_6\leq t_7,$
$t'_s<t'_1\leq t'_3, t'_s<t'_2\leq t'_3, t'_4\leq t'_5, t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Resolve two flaws

- Support $[t'_s,t'_3]$ pile(c2)=p′2 from [0] pile(c2)=p′2 by constraining $t'_s = 0$
- Support $[t'_s,t'_1]$ freight(r') = empty from [0] freight(r1)=empty by constraining $r' = $ r1



ϕ_3: tasks: $[0,t_1]$ move(r2,d1)
$[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[t'_s,t'_1]$ move(r',d2)
$[t'_s,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r',c2,p′2)
$[t'_5,t'_6]$ move(r',d4)
$[t'_7,t'_e]$ unload(k2,r',c2,p′2)

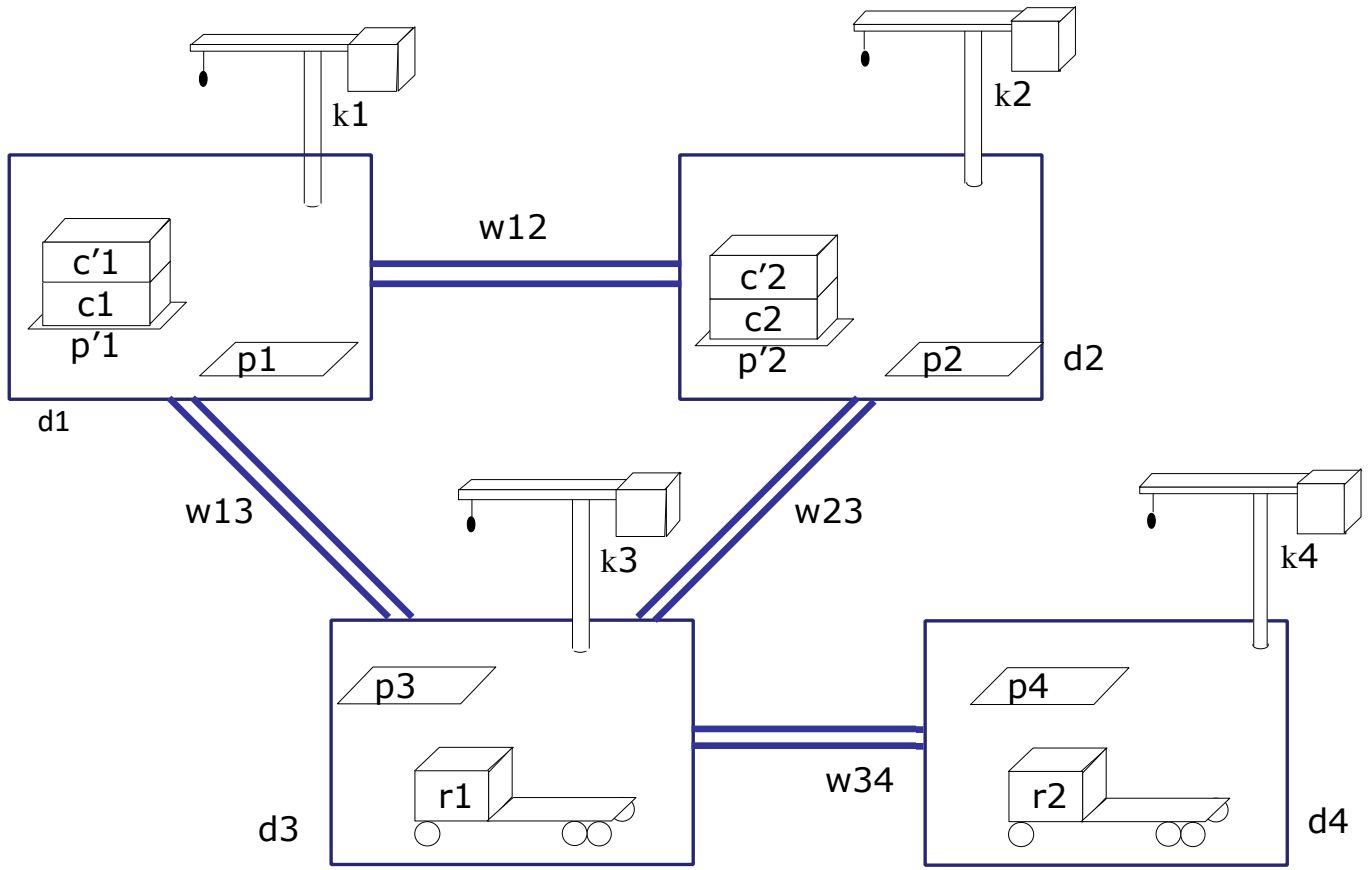
supported: [0] loc(r1)=d3
[0] loc(r2)=d4
[0] freight(r1)=empty
[0] freight(r2)=empty
[0] pile(c1)=p′1
[0] pile(c2)=p′2, . . .
$[0,t_3]$ pile(c1) = p′1
$[0,t_3]$ freight(r2) = empty

assertions: $[t'_s,t'_3]$ pile(c2) = p′2
$[t'_s,t'_1]$ freight(r') = empty

constraints: $0<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$t'_s<t'_1\leq t'_3$, $t'_s<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Modified chronicle

- Changes
  - Substituted $t'_s \leftarrow 0$ and $r' \leftarrow$ r1
  - Moved $[0,t'_3]$ pile(c2) = p′2 and $[0,t'_1]$ freight(r1) = empty to "supported" list
- Flaws: 10 unrefined tasks, 0 unsupported assertions
- Next: refine task $[0,t_1]$ move(r2,d1)



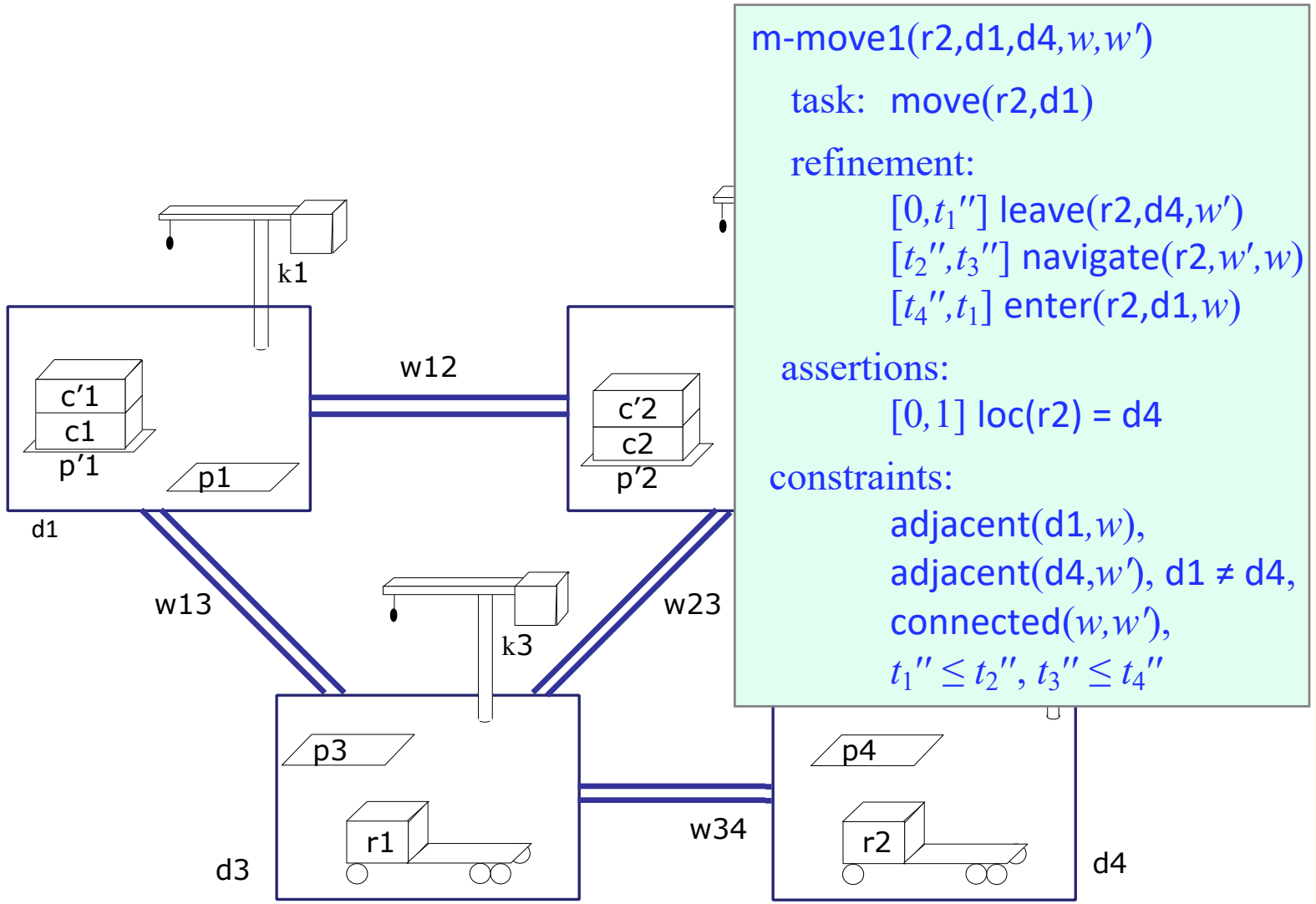
ϕ_4: tasks: $[0,t_1]$ move(r2,d1)
$[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[0,t'_1]$ move(r1,d2)
$[0,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r1,c2,p′2)
$[t'_5,t'_6]$ move(r1,d4)
$[t'_7,t'_e]$ unload(k2,r1,c2,p′2)

supported: $[0]$ loc(r1)=d3
$[0]$ loc(r2)=d4
$[0]$ freight(r1)=empty
$[0]$ freight(r2)=empty
$[0]$ pile(c1)=p′1
$[0]$ pile(c2)=p′2, . . .
$[0,t_3]$ pile(c1) = p′1
$[0,t_3]$ freight(r2) = empty
$[0,t'_3]$ pile(c2) = p′2
$[0,t'_1]$ freight(r1) = empty

constraints: $0<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$0<t'_1\leq t'_3$, $0<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Method instance

- Instantiate $r \leftarrow$ r2, $d \leftarrow$ d1, $t_s \leftarrow 0$, $t_e \leftarrow t_1$, to match $[0,t_1]$ move(r2,d1)
- Rename timepoint variables to avoid name conflicts



m-move1(r2,d1,d4,w,w')

task: move(r2,d1)

refinement:
- $[0,t_1'']$ leave(r2,d4,w')
- $[t_2'',t_3'']$ navigate(r2,w',w)
- $[t_4'',t_1]$ enter(r2,d1,w)

assertions:
- $[0,1]$ loc(r2) = d4

constraints:
- adjacent(d1,w),
- adjacent(d4,w'), d1 ≠ d4,
- connected(w,w'),
- $t_1'' \le t_2''$, $t_3'' \le t_4''$

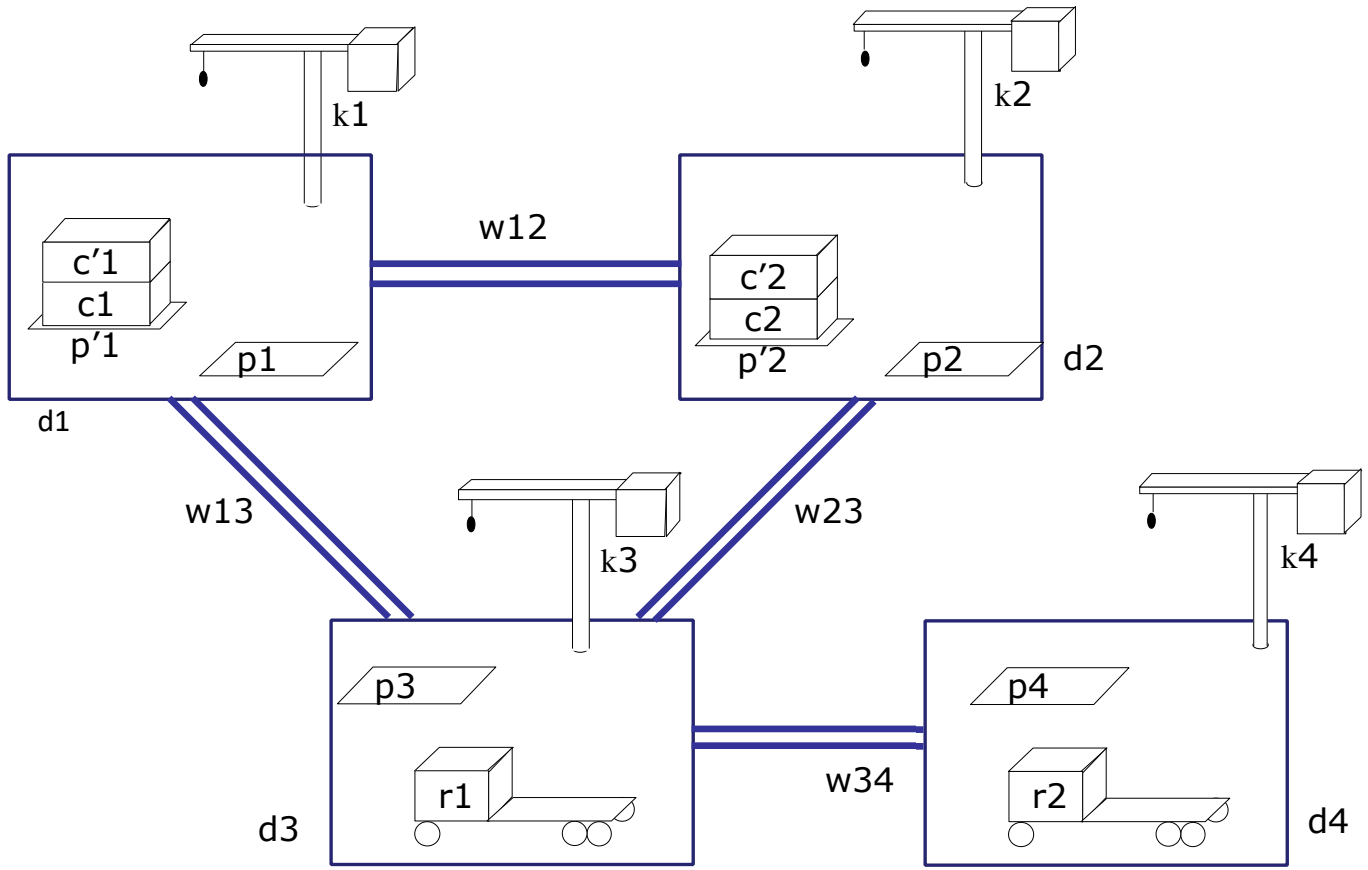
ϕ_4: tasks: $[0,t_1]$ move(r2,d1)
- $[0,t_2]$ uncover(c1,p′1)
- $[t_3,t_4]$ load(k1,r2,c1,p′1)
- $[t_5,t_6]$ move(r2,d3)
- $[t_7,t_e]$ unload(k3,r2,c1,p3)
- $[0,t'_1]$ move(r1,d2)
- $[0,t'_2]$ uncover(c2,p′2)
- $[t'_3,t'_4]$ load(k4,r1,c2,p′2)
- $[t'_5,t'_6]$ move(r1,d4)
- $[t'_7,t'_e]$ unload(k2,r1,c2,p′2)

supported: $[0]$ loc(r1)=d3, $[0]$ loc(r2)=d4,
. . .

constraints: $0<t_1\le t_3$, $0<t_2\le t_3$, $t_4\le t_5$, $t_6\le t_7$,
$0<t'_1\le t'_3$, $0<t'_2\le t'_3$, $t'_4\le t'_5$, $t'_6\le t'_7$
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Modified chronicle

- Changes:
  - Removed task $[0,t_1]$ move(r2,d1)
  - Added tasks, assertion, constraints
- 12 tasks, a supported assertion, more constraints
- Next: the first three tasks

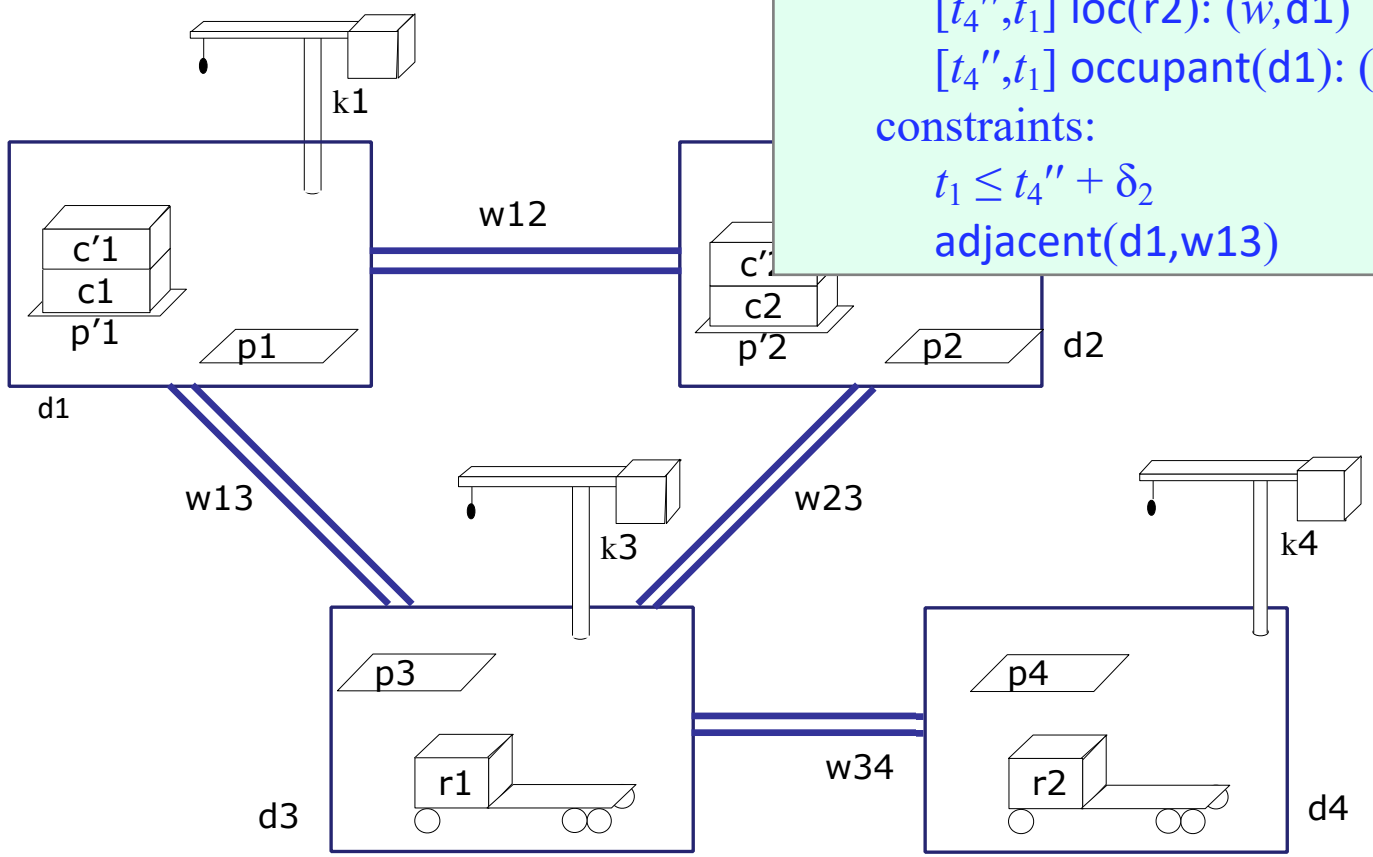


ϕ_5: tasks: $[0,t_1'']$ leave(r2,d4,w')
$[t_2'',t_3'']$ navigate(r2,w',w)
$[t_4'',t_1]$ enter(r2,d1,w)
$[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[0,t'_1]$ move(r1,d2)
$[0,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r1,c2,p′2)
$[t'_5,t'_6]$ move(r1,d4)
$[t'_7,t'_e]$ unload(k2,r1,c2,p′2)
supported: [0] loc(r1)=d3, [0] loc(r2)=d4,
. . .
[0,1] loc(r2) = d4
constraints: $0<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$0<t'_1\leq t'_3$, $0<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
$0<t_1''\leq t_2''$, $0<t_3''\leq t_4''$
adjacent(d1,w),
adjacent(d4,w'), d1 ≠ d4,
connected(w,w'),
adjacent(d1,w12), . . .
connected(w12,w13), . . .

leave(r2,d4,w34)
assertions:
$[0,t_7]$ loc(r2): (d4,w34)
$[0,t_7]$ occupant(d4): (r2,empty)
constraints:
$0 < t_7,\ t_e \le t_s + \delta_1$
adjacent(d4,w34)

navigate(r2,w34,w13)
// my guess; it's not in the book
assertions:
$[t_2'',t_3'']$ loc(r2): (w34,w13)
constraints:
connected(w34,w13)

enter(r2,d1,w13)
assertions:
$[t_4'',t_1]$ loc(r2): $(w,$d1)
$[t_4'',t_1]$ occupant(d1): (empty, r2)
constraints:
$t_1 \le t_4'' + \delta_2$
adjacent(d1,w13)



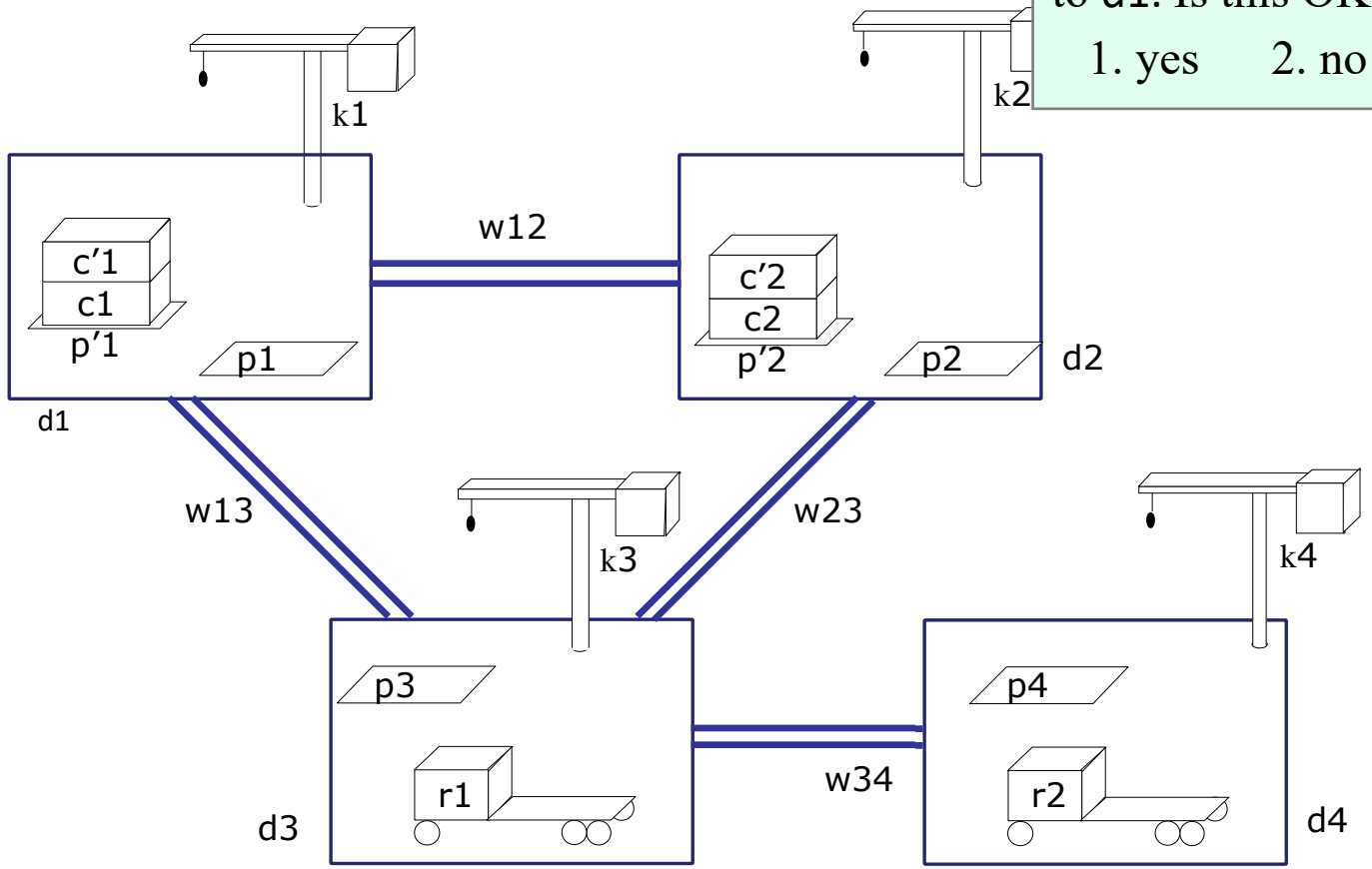
ϕ_6: tasks: $[0,t_1'']$ leave(r2,d4,w')
$[t_2'',t_3'']$ navigate(r2,w',w)
$[t_4'',t_1]$ enter(r2,d1,w)
$[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[0,t'_1]$ move(r1,d2)
$[0,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r1,c2,p′2)
$[t'_5,t'_6]$ move(r1,d4)
$[t'_7,t'_e]$ unload(k2,r1,c2,p′2)
supported: [0] loc(r1)=d3, [0] loc(r2)=d4,
. . .
[0,1] loc(r2) = d4
constraints: $0<t_1\le t_3$, $0<t_2\le t_3$, $t_4\le t_5$, $t_6\le t_7$,
$0<t'_1\le t'_3$, $0<t'_2\le t'_3$, $t'_4\le t'_5$, $t'_6\le t'_7$
$0<t_1'' \le t_2''$, $0<t_3'' \le t_4''$
(satisfied): adjacent(d1,w13),
adjacent(d4,w34), d1 ≠ d4,
connected(w34,w13),
adjacent(d1,w12), . . .
connected(w12,w13), . . .

Modified chronicle

- Changes
  - Removed three tasks, added assertions, added constraints, removed some satisfied constraints
- r2 goes from d4 to w34 to w13 to d1
- Poll: is this OK

Poll. Notice that r2 goes from d4 to w34 to w13 to d1. Is this OK?

1. yes 2. no



ϕ_7: tasks: $[0,t_2]$ uncover(c1,p′1)
$[t_3,t_4]$ load(k1,r2,c1,p′1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[0,t'_1]$ move(r1,d2)
$[0,t'_2]$ uncover(c2,p′2)
$[t'_3,t'_4]$ load(k4,r1,c2,p′2)
$[t'_5,t'_6]$ move(r1,d4)
$[t'_7,t'_e]$ unload(k2,r1,c2,p′2)

supported: [0] loc(r1)=d3, [0] loc(r2)=d4,
. . .
$[0,1]$ loc(r2) = d4

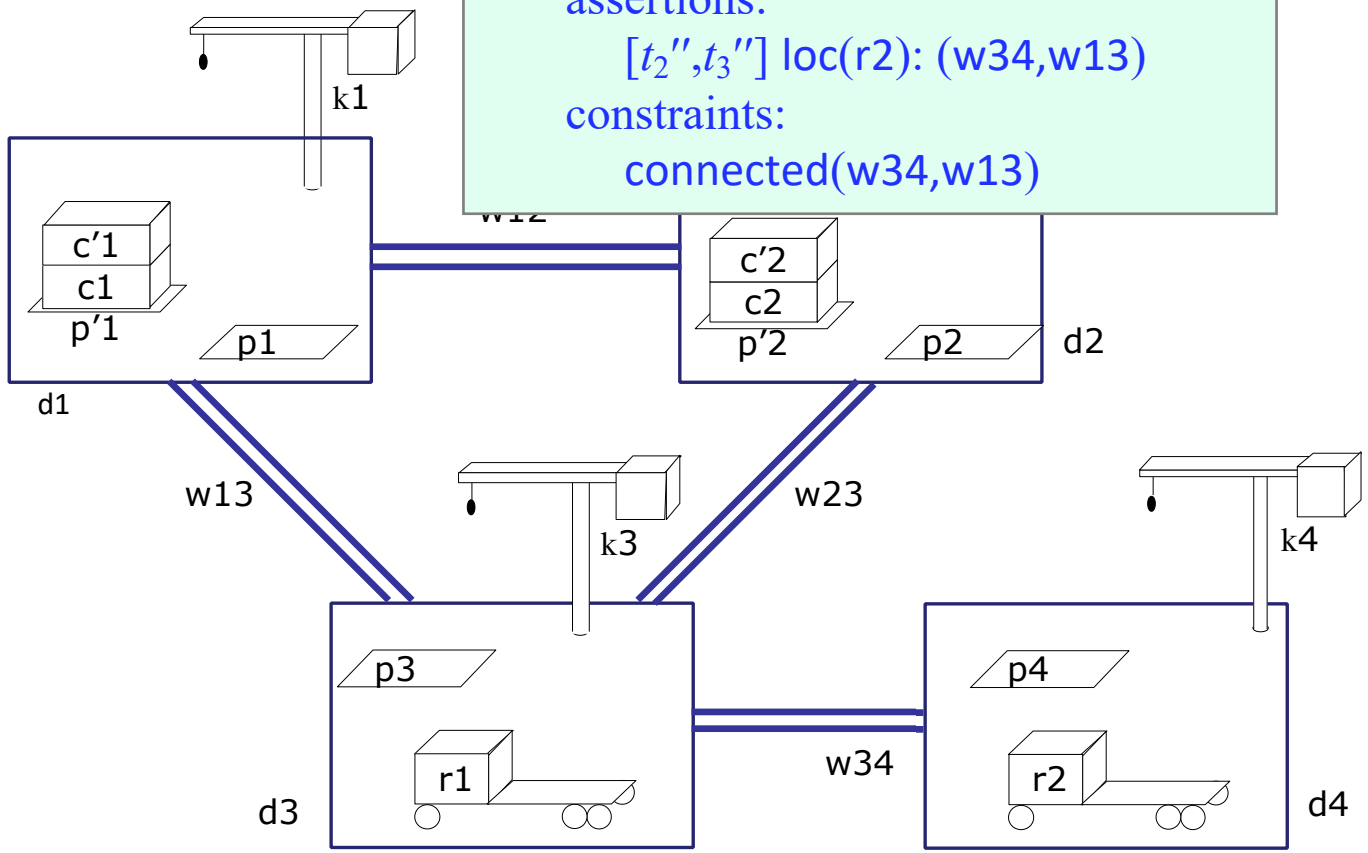
assertions: $[0,t_7]$ loc(r2): (d4,w34)
$[0,t_7]$ occupant(d4): (r2,empty)
$[t_2'',t_3'']$ loc(r2): (w34,w13)
$[t_4'',t_1]$ loc(r2): $(w,$d1)
$[t_4'',t_1]$ occupant(d1): (empty, r2)

constraints: $0<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$0<t'_1\leq t'_3$, $0<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
$0<t_1'' \leq t_2''$, $0<t_3'' \leq t_4''$,
$0 < t_7,\ t_e \leq t_s + \delta_1,\ t_1 \leq t_4'' + \delta_2$

Modified chronicle

- Problem: r2 must go through d3, which will create a conflict with r1
  - But the chronicle contains no record of it
- Here's why:

navigate(r2,w34,w13)
// my guess; it's not in the book
assertions:
$[t_2'', t_3'']$ loc(r2): (w34,w13)
constraints:
connected(w34,w13)



ϕ_7: tasks: $[0,t_2]$ uncover(c1,p'1)
$[t_3,t_4]$ load(k1,r2,c1,p'1)
$[t_5,t_6]$ move(r2,d3)
$[t_7,t_e]$ unload(k3,r2,c1,p3)
$[0,t'_1]$ move(r1,d2)
$[0,t'_2]$ uncover(c2,p'2)
$[t'_3,t'_4]$ load(k4,r1,c2,p'2)
$[t'_5,t'_6]$ move(r1,d4)
$[t'_7,t'_e]$ unload(k2,r1,c2,p'2)
supported: [0] loc(r1)=d3, [0] loc(r2)=d4,
. . .
[0,1] loc(r2) = d4
assertions: $[0,t_7]$ loc(r2): (d4,w34)
$[0,t_7]$ occupant(d4): (r2,empty)
$[t_2'', t_3'']$ loc(r2): (w34,w13)
$[t_4'', t_1]$ loc(r2): $(w,$d1)
$[t_4'', t_1]$ occupant(d1): (empty, r2)
constraints: $0<t_1\leq t_3$, $0<t_2\leq t_3$, $t_4\leq t_5$, $t_6\leq t_7$,
$0<t'_1\leq t'_3$, $0<t'_2\leq t'_3$, $t'_4\leq t'_5$, $t'_6\leq t'_7$
$0<t_1'' \leq t_2''$, $0<t_3'' \leq t_4''$,
$0 < t_7,\ t_e \leq t_s + \delta_1,\ t_1 \leq t_4'' + \delta_2$

Heuristics for Guiding TemPlan

- Flaw selection, resolver selection heuristics similar to those in PSP
  - Select the flaw with the smallest number of resolvers
  - Choose the resolver that rules out the fewest resolvers for the other flaws

- There is also a problem with constraint management
  - We ignored it when discussing PSP
  - Discuss it next

TemPlan(ϕ, Σ)
  $Flaws \leftarrow$ set of flaws of ϕ
  if $Flaws = \varnothing$ then return ϕ
  arbitrarily select $f \in Flaws$
  $Resolvers \leftarrow$ set of resolvers of f
  if $Resolvers = \varnothing$ then return failure
  nondeterministically choose $\rho \in Resolvers$
  $\phi \leftarrow$ Transform(ϕ, ρ)
  Templan(ϕ, Σ)

PSP(Σ, π)
  loop
    if $Flaws(\pi) = \varnothing$ then return π
    arbitrarily select $f \in Flaws(\pi)$
    $R \leftarrow \{$all feasible resolvers for $f\}$
    if $R = \varnothing$ then return failure
    nondeterministically choose $\rho \in R$
    $\pi \leftarrow \rho(\pi)$
  return π

Summary of Sections 4.1, 4.2, 4.3

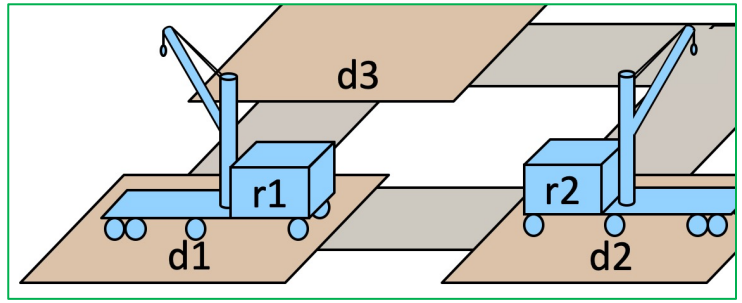
- Timelines
  - Temporal assertions (change, persistence), constraints
  - Conflicts, consistency, security, causal support
- Chronicle: union of several timelines
  - Consistency, security, causal support
  - Actions represented by chronicles; preconditions ⇔ causal support
- Planning problems
  - three kinds of flaws and their resolvers:
    - tasks, causal support, security
  - partial plans, solution plans
- Planning: TemPlan
  - Like PSP but with tasks, temporal assertions, temporal constraints

Outline

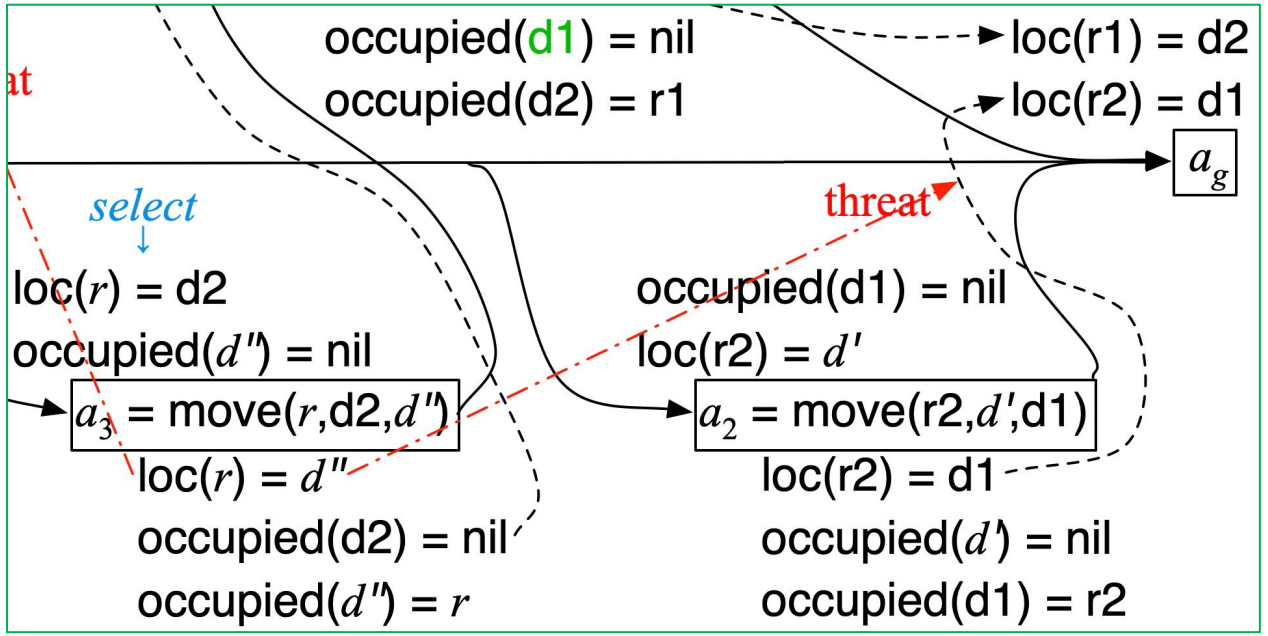
- ✓ Introduction
- ✓ Representation
- ✓ Temporal planning
- **4.4 Constraint Management**
  - Consistency of object constraints and time constraints
  - Controlling the actions when we don't know how long they'll take
- 4.5 Acting with temporal models

Constraint Management

- Each time TemPlan applies a resolver, it modifies $(\mathcal{T},C)$
  - Some resolvers will make $(\mathcal{T},C)$ inconsistent
    - No solution in this part of the search space
  - Would like to detect inconsistency, prune this part of the search space
    - Otherwise we'll waste time looking for a solution



- Analogy: PSP checked simple cases of inconsistency
  - E.g., can't create a constraint $a \prec b$ if there's already a constraint $b \prec a$
- But PSP ignored more complicated cases
  - E.g., suppose there are three threats
    - To resolve them, suppose PSP chooses $d'' \neq$ d1 , $d'' \neq$ d2 , $d'' \neq$ d3
    - No solutions in this part of the search space, but PSP searches it anyway



Constraint Management in TemPlan

$$\mathcal{T} = \{\ldots\}$$
$$C = (t_1 < t_2 < t_3 < t_1)$$

- At various points, check consistency of C
  - If C is inconsistent, then $(\mathcal{T}, C)$ is inconsistent
  - Can prune this part of the search space

- If C is consistent then $(\mathcal{T}, C)$ may or may not be consistent
  - Example:
    - $\mathcal{T} = \{[t_1,t_2]\ \mathsf{loc(r1)=loc1},$
      $[t_3,t_4]\ \mathsf{loc(r1)=loc2}\}$
    - $C = (t_1 < t_3 < t_4 < t_2)$
  - Gives loc(r1) two values during $[t_3,t_4]$

Consistency of C

- C contains two kinds of constraints
  - Object constraints
    - $\mathsf{loc}(r) \neq l_2, \quad l \in \{\mathsf{loc3}, \mathsf{loc4}\}, \quad r = \mathsf{r1}, \; o \neq o'$
  - Temporal constraints
    - $t_1 < t_3, \quad a < t, \quad t < t', \quad a \leq t' - t \leq b$

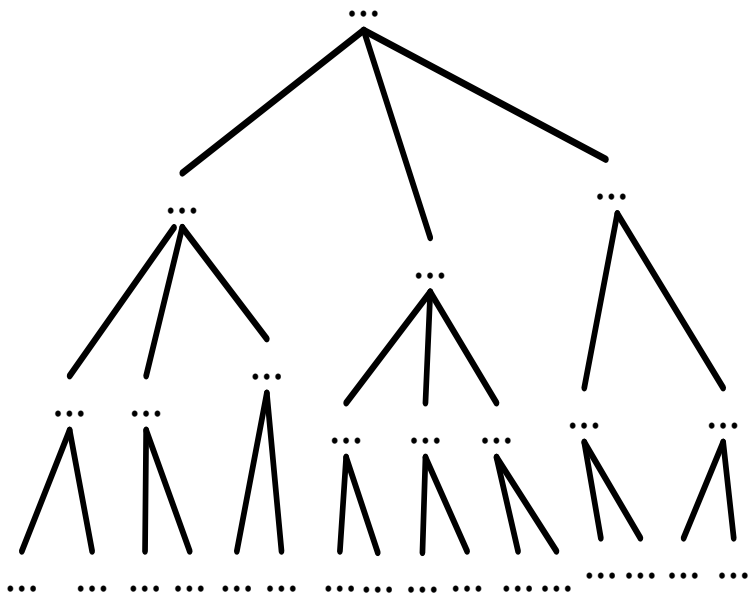
- Assume object constraints are independent of temporal constraints and vice versa
  - exclude things like $t < \mathsf{distance}(r, r')$

- Then two separate subproblems
  - (1) check consistency of object constraints
  - (2) check consistency of temporal constraints
  - C is consistent iff both are consistent

Object Constraints

- Constraint-satisfaction problem (CSP) – NP-hard
- Can write an algorithm that's *complete* but runs in exponential time
  - If there's an inconsistency, always finds it
  - Might do a lot of pruning, but spend lots of time at each node

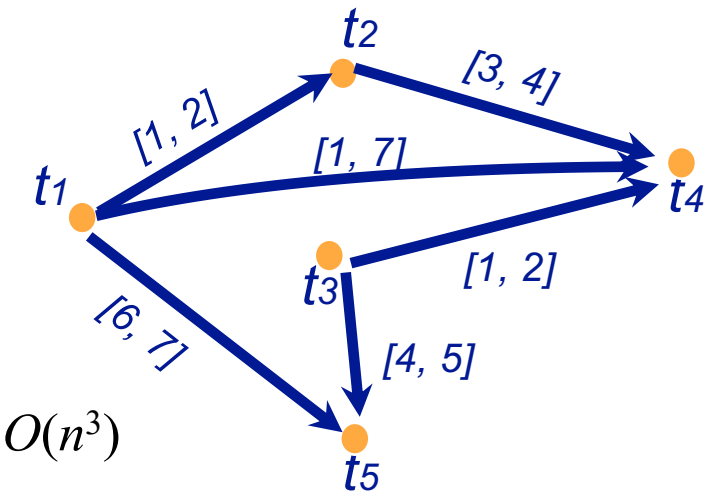
- Instead, use a technique that's incomplete but takes *polynomial* time
  - arc consistency, path consistency
- Detects some inconsistencies but not others
  - Runs much faster, but prunes fewer nodes



Time Constraints

To represent time constraints:

- Simple Temporal Networks (STNs)
  - Networks of constraints on time points

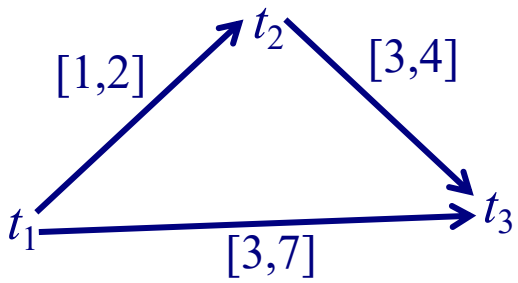


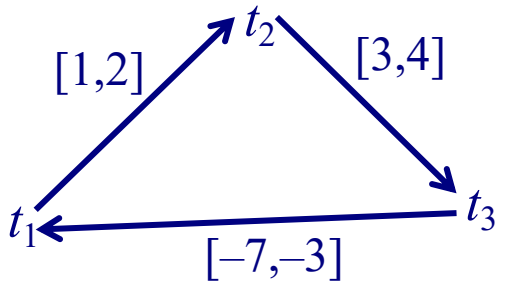
- Synthesize incrementally them starting from ϕ_0
  - TemPlan can check time constraints in time $O(n^3)$

- Incrementally instantiated at acting time
- Kept consistent throughout planning and acting

Time Constraints

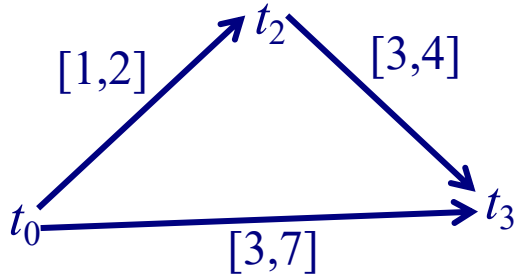
- *Simple Temporal Network* (STN):
- a pair $(\mathcal{V},\mathcal{E})$, where
  - $\mathcal{V}$ = {a set of temporal variables $\{t_1, \ldots, t_n\}$ }
  - $\mathcal{E} \subseteq \mathcal{V}^2$ is a set of arcs
- Each arc (t_i,t_j) is labeled with an interval $[a,b]$
  - Represents constraint $a \le t_j - t_i \le b$
  - Equivalently, $-b \le t_i - t_j \le -a$





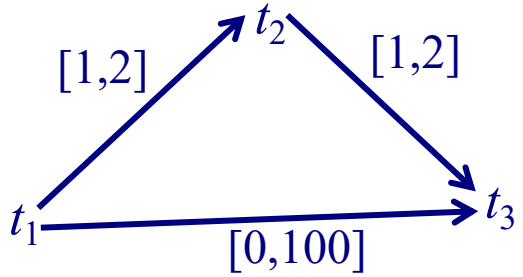
- Notation: instead of $a \le t_j - t_i \le b$, write $r_{ij} = [a_{ij},b_{ij}]$

- To represent unary constraints:
  - Dummy variable $t_0 = 0$
  - Arc $r_{0i} = (t_0,t_i)$ labeled with $[a,b]$ represents $a \le t_i - 0 \le b$



Time Constraints

- *Solution* to an STN:
  - any assignment of integer values to the time points
    such that all the constraints are satisfied
- *Consistent* STN: has a solution



- Solutions:
  - $(t_2–t_1, t_3–t_2, t_3–t_1) \in$
    $\{(1,1,2), (1,2,3), (2,1,3), (2,2,4)\}$

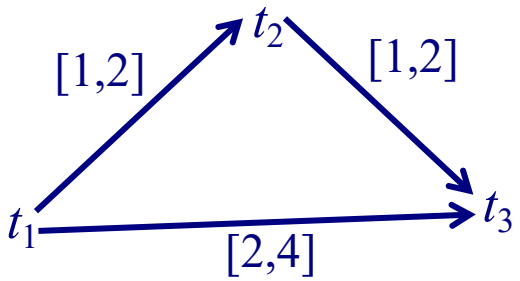
- *Minimal* STN:

  for every arc (t_i,t_j) with label $[a,b]$,

  for every $t \in [a,b]$,

  there's at least one solution such that $t_j - t_i = t$

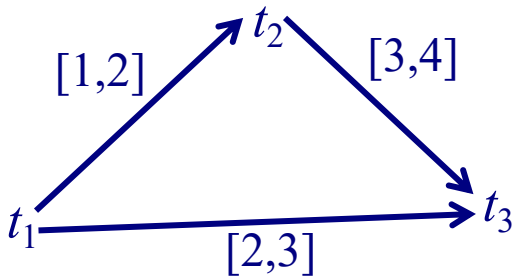
  - If we make any of the time intervals shorter, we'll exclude some solutions



Time Constraints

- *Solution* to an STN:
  - any assignment of integer values to the time points such that all the constraints are satisfied
- *Consistent* STN: has a solution

Poll: Is this network consistent?
1. yes 2. no



- *Minimal* STN:

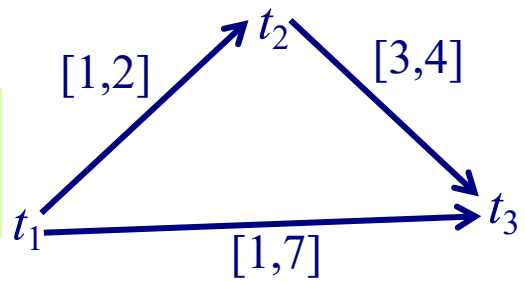
  for every arc (t_i,t_j) with label $[a,b]$,

  for every $t \in [a,b]$,

  there's at least one solution such that $t_j - t_i = t$

  - If we make any of the time intervals shorter, we'll exclude some solutions

Poll: Is this network minimal?
1. yes 2. no



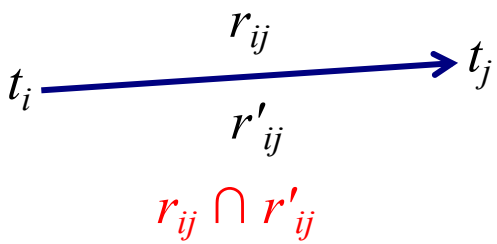
Operations on STNs

- Intersection, $\cap$

  $t_j - t_i \in r_{ij} = [a_{ij}, b_{ij}]$

  $t_j - t_i \in r'_{ij} = [a'_{ij}, b'_{ij}]$

  Infer $t_j - t_i \in r_{ij} \cap r'_{ij} = [\max(a_{ij}, a'_{ij}), \min(b_{ij}, b'_{ij})]$



$r_{ij} \cap r'_{ij}$

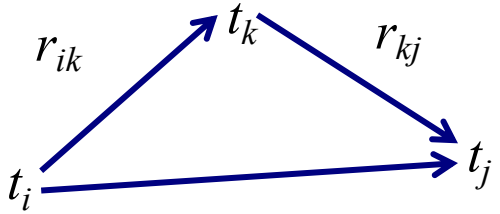
- Composition, $\bullet$

  $t_k - t_i \in r_{ik} = [a_{ik}, b_{ik}]$

  $t_j - t_k \in r_{kj} = [a_{kj}, b_{kj}]$

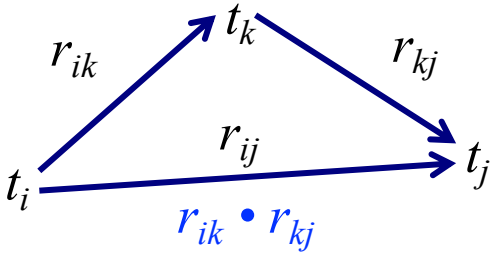
  Infer $t_j - t_i \in r_{ik} \bullet r_{kj} = [a_{ik} + a_{kj}, b_{ik} + b_{kj}]$

  Reason: shortest and longest times for the two intervals



$r_{ik} \bullet r_{kj}$

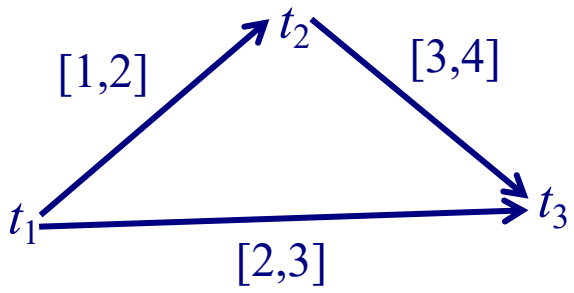
- Consistency checking
  - r_{ik}, r_{kj}, r_{ij} are consistent only if $r_{ij} \cap (r_{ik} \bullet r_{kj}) \neq \emptyset$
- Special case for networks with just three nodes
  - Consistent iff if $r_{ij} \cap (r_{ik} \bullet r_{kj}) \neq \emptyset$



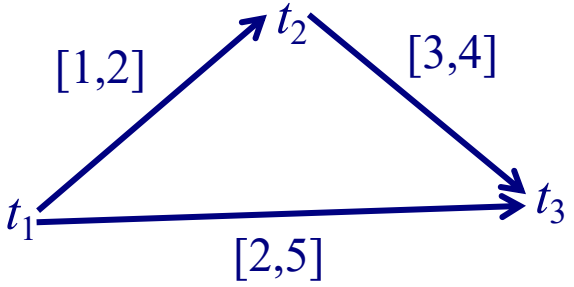
$r_{ik} \bullet r_{kj}$

$r_{ij} \cap (r_{ik} \bullet r_{kj})$

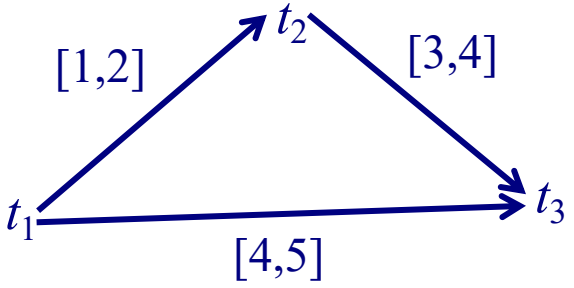
Two Examples



- STN $(\mathcal{V},\mathcal{E})$, where
  - $\mathcal{V} = \{t_1, t_2, t_3\}$
  - $\mathcal{E} = \{r_{12}=[1,2],\ r_{23}=[3,4],\ r_{13}=[2,3]\}$
- Composition:
  - $r'_{13} = r_{12} \bullet r_{23} = [4,6]$
- Can't satisfy both r_{13} and r'_{13}
  - $r_{13} \cap r'_{13} = [2,3] \cap [4,6] = \emptyset$
- $(\mathcal{V},\mathcal{E})$ is inconsistent



- STN $(\mathcal{V},\mathcal{E})$, where
  - $\mathcal{V} = \{t_1, t_2, t_3\}$
  - $\mathcal{E} = \{r_{12}=[1,2], r_{23}=[3,4], r_{13}=[2,5]\}$
- As before, $r'_{13} = [4,6]$
  - $r_{13} \cap r'_{13} = [4,5]$
- $(\mathcal{V},\mathcal{E})$ is consistent
  - $r_{13} \leftarrow [4,5]$ will make it minimal
  - Same solutions as above



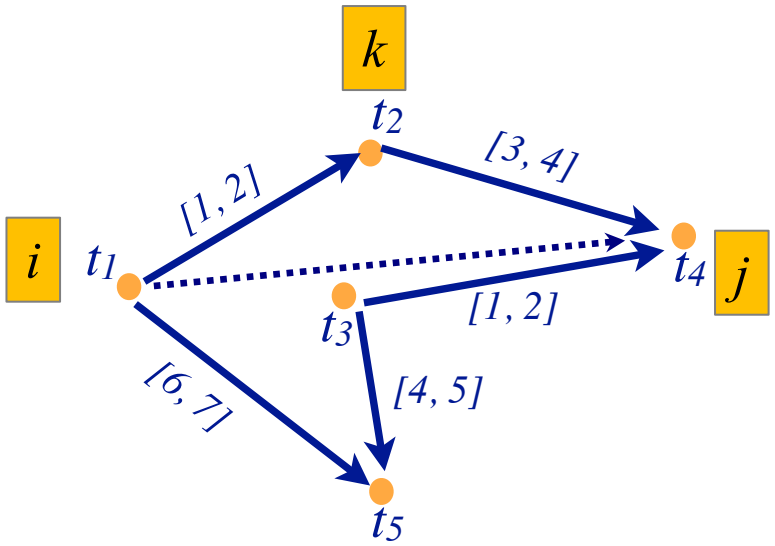
- Let $d_{12} = t_2 - t_1 \in r_{12} = [1,2]$
  - $d_{23} = t_3 - t_2 \in r_{23} = [3,4]$
  - $d_{13} = d_{12} + d_{23}$
- Solution iff $d_{13} \in r_{13} = [2,5]$

d_{12}	d_{23}	d_{13}	solution?
1	3	4	yes
1	4	5	yes
2	3	5	yes
2	4	6	no

- $d_{12} = 1, 2$
  - Can't shrink r_{12}
- $d_{23} = 3, 4$
  - Can't shrink r_{23}
- $d_{13} = 4, 5$
  - Shrink r_{13} to $[4,5]$, get the same solutions

Operations on STNs

PC($\mathcal{V}$,$\mathcal{E}$):
  for $1 \le k \le n$ do
    for $1 \le i < j \le n,\ i \ne k,\ j \ne k$ do
      $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \bullet r_{kj}]$
      if $r_{ij} = \emptyset$ then
        return inconsistent



- PC (*Path Consistency*) algorithm:
  - Iterates over all triples
    - $1 \le k \le n,\ 1 \le i < j \le n,\ i \ne k,\ j \ne k$
  - For each, shrinks interval, checks consistency
  - If an arc has no constraint, use $[-\infty, +\infty]$
  - i, j, k each go from 1 to n
  - => n^3 triples
    => time $O(n^3)$

- Example: $k = 2, i = 1, j = 4$

  $r_{12} = [1,2]$

  $r_{24} = [3,4]$

  $r_{14} = [-\infty,\infty]$

  $r_{12} \bullet r_{24} = [1+3, 2+4] = [4,6]$

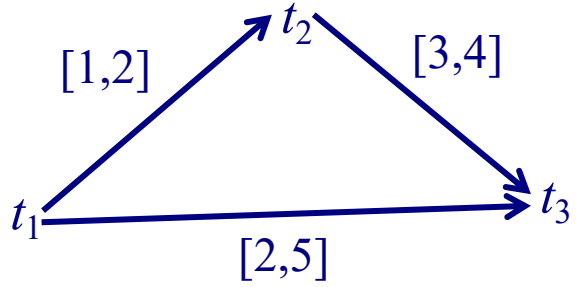
  $r_{14} \leftarrow [\max(-\infty,4), \min(\infty,6)] = [4,6]$

Operations on STNs

PC($\mathcal{V}$,$\mathcal{E}$):
  for $1 \le k \le n$ do
    for $1 \le i < j \le n$, $i \ne k$, $j \ne k$ do
      $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \bullet r_{kj}]$
      if $r_{ij} = \emptyset$ then
        return inconsistent

- If an arc has no constraint, use $[-\infty, +\infty]$
- PC (*Path Consistency*) algorithm:
  - For every $i < j$, iterates over r_{ij} once for each k
  - Each time, tries to reduce interval, checks consistency
  - i, j, k each go from 1 to n

    => n^3 triples
    => time $O(n^3)$

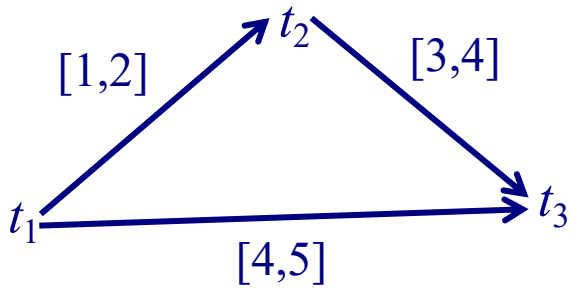


- For $n = 3$, PC tries these triples:
  - $k = 1, i = 2, j = 3$
  - $k = 2, i = 1, j = 3$
  - $k = 3, i = 1, j = 2$

- $k = 1, i = 2, j = 3$
  - $r_{ik} = r_{21} = [-2,-1]$
  - $r_{kj} = r_{13} = [2,5]$
  - $r_{ij} = r_{23} = [3,4]$
  - $r_{ik} \bullet r_{kj} = [-2+2, -1+5] = [0,4]$
  - $r_{ij} = r_{23} \leftarrow [\max(3,0), \min(4,4)] = [3,4]$
  - No change

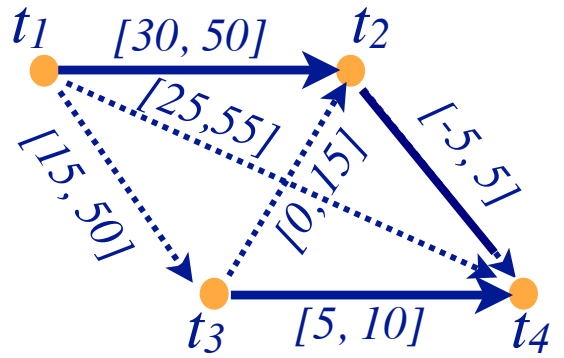
- $k = 2, i = 1, j = 3$
  - We already know PC reduces r_{13} to [2,5] to [4,5]
- $k = 3, i = 1, j = 2$
  - $r_{ik} = r_{13} = [2,5]$
  - $r_{kj} = r_{32} = [-4,-3]$
  - $r_{ij} = r_{12} = [1,2]$
  - $r_{ik} \bullet r_{kj} = [2-4, 5-3] = [-2,2]$
  - $r_{12} \leftarrow [\max(1,-2), \min(2,2)] = [1,2]$
  - No change

- Minimal network:



Operations on STNs

PC($\mathcal{V}$,$\mathcal{E}$):
  for $1 \le k \le n$ do
    for $1 \le i < j \le n,\ i \ne k,\ j \ne k$ do
      $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \bullet r_{kj}]$
      if $r_{ij} = \emptyset$ then
        return inconsistent



- PC makes network minimal
  - Reduce each r_{ij} to exclude values that aren't in any solution
- Also detects inconsistent networks
  - $r_{ij} = [a_{ij}, b_{ij}]$ empty => inconsistent

- Dashed lines:
  - constraints that were shrunk

- Can modify PC to make it incremental
  - Input:
    - a consistent, minimal STN
    - a new constraint r'_{ij}
  - Incorporate r'_{ij} in time $O(n^2)$

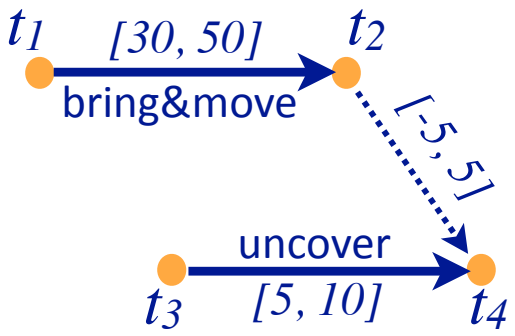
Pruning TemPlan's search space

- Take the time constraints in C
  - Write them as an STN
  - Use Path Consistency to check whether STN is consistent
  - If it's inconsistent, TemPlan can backtrack

Controllability

- Section 4.4.3 of the book
- Suppose TemPlan gives you a chronicle and you want to execute it
  - Constraints on time points
  - Need to reason about these in order to decide when to start each action

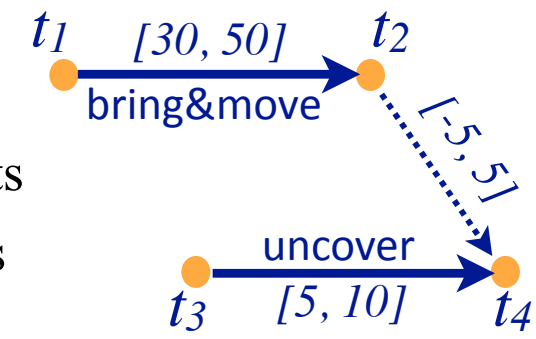
- Solid lines: duration constraints
  - Robot will do bring&move, will take 30 to 50 time units
  - Crane will do uncover, will take 5 to 10 time units
- Dashed line: synchronization constraint
  - Don't want either the crane or robot to wait long
  - At most 5 seconds between the two ending times



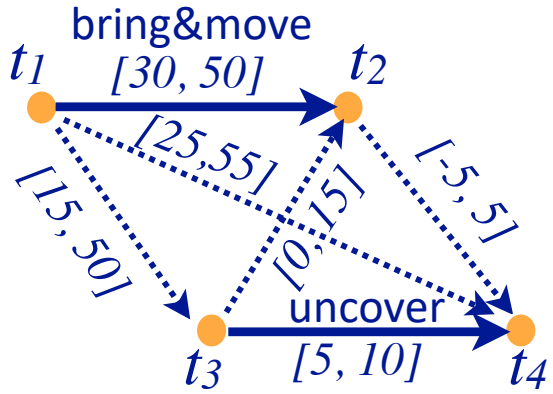
- Objective
  - Choose time points that will satisfy all the constraints

Controllability

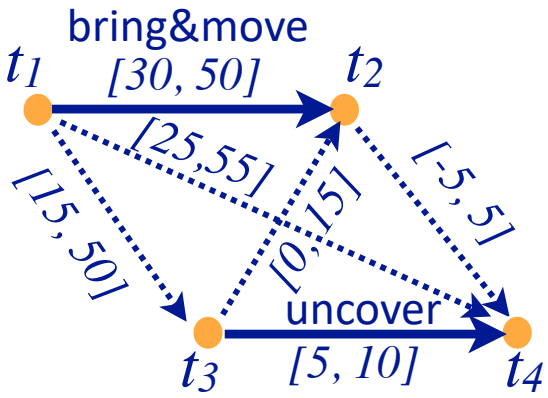
- Suppose we run PC
  - Returns a minimal and consistent network
- There *exist* time points that satisfy all the constraints
- Would work if we could choose all four time points
  - But we can't choose t_2 and t_4



- Actor can control when each action starts
  - t_1 and t_3 are *controllable*
- Can't control how long the actions take
  - t_2 and t_4 are *contingent*
  - random variables that are known to satisfy the duration constraints
    - $t_2 \in [t_1+30, t_1+50]$
    - $t_4 \in [t_3+5, t_3+10]$



Controllability



- Can't guarantee that all of the constraints will be satisfied

- Start bring&move at time $t_1 = 0$
- Suppose the durations are
  - bring&move 30, uncover 10
  - $t_2 = t_1 + 30 = 30$
  - $t_4 = t_3 + 10$
  - $t_4 - t_2 = t_3 - 20$
- Constraint r_{24}: $-5 \le t_4 - t_2 \le 5$

  $-5 \le t_3 - 20 \le 5$

  $15 \le t_3 \le 25$
- Must start uncover at $t_3 \le 25$

- But if we start uncover at $t_3 \le 25$, this is only 5 time units after t_1
  - We don't yet know how long the actions will take
- Durations might instead be

  bring&move 50, uncover 5
  - $t_2 = t_1 + 50 = 50$
  - $t_4 = t_3 + 5 \le 25 + 5 = 30$
  - $t_4 - t_2 \le 30 - 50 = -20$
    - Violates r_{34}

STNUs

- *STNU (Simple Temporal Network with Uncertainty):*
  - A 4-tuple $(V, \tilde{V}, E, \tilde{E})$
    - $V = \{$*controllable* time points$\}$, e.g., starting times of actions
    - $\tilde{V} = \{$*contingent* time points$\}$, e.g., ending times of actions
    - $E = \{$*controllable* constraints$\}$, $\tilde{E} = \{$*contingent* constraints$\}$
- Controllable and contingent constraints:
  - Synchronization between two starting times: controllable
  - Duration of an action: contingent
  - Synchronization between ending points of two actions: contingent
  - Synchronization between end of one action, start of another:
    - Controllable if the new action starts after the old one ends
    - Contingent if the new action starts before the old one ends
- Want a way for the actor to choose time points in V (starting times) that guarantee that the constraints are satisfied

Three kinds of controllability

- $(V,\tilde{V},\mathcal{E},\tilde{E})$ is *strongly controllable* if the actor can choose values for V such that success will occur for all values of $\tilde{V}$ that satisfy $\tilde{E}$
  - Actor can choose the values for V offline
  - The right choice will work regardless of $\tilde{V}$
- $(V,\tilde{V},\mathcal{E},\tilde{E})$ is *weakly controllable* if the actor can choose values for V such that success will occur for *at least one* combination of values for $\tilde{V}$
  - Actor can choose the values for V only if the actor knows in advance what the values of $\tilde{V}$ will be
- *Dynamic controllability*:
  - Game-theoretic model: actor vs. environment
  - A player's *strategy*: a function σ telling what to do in every situation
    - Choices may depend on what has happened so far
  - $(V,\tilde{V},\mathcal{E},\tilde{E})$ is *dynamically controllable* if ∃ strategy for actor that will guarantee success regardless of the environment's strategy

Two player, zero sum, extensive form, imperfect information

Dynamic Execution

For $t = 0, 1, 2, \ldots$

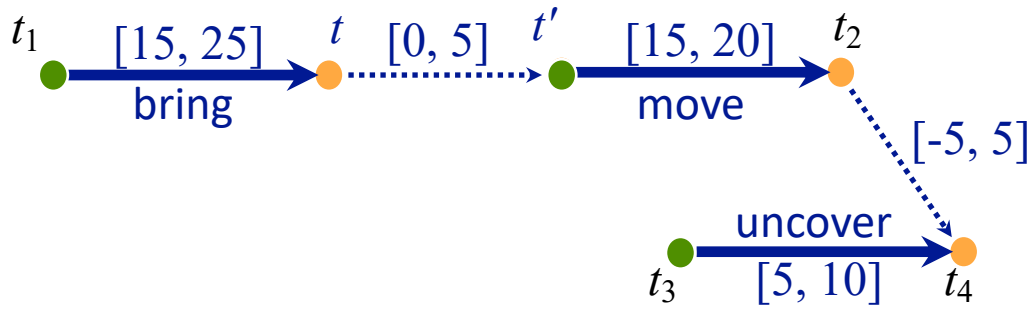
1. Actor chooses an unassigned set of variables $V_t \subseteq V$ that all can be assigned the value t without violating any constraints in C
   - ≈ actions the actor chooses to start at time t
2. Simultaneously, environment chooses an unassigned set of variables $\tilde{V}_t \subseteq \tilde{V}$ that all can be assigned the value t without violating any constraints in $\tilde{E}$
   - ≈ actions that finish at time t
3. Each chosen time point v is assigned $v \leftarrow t$
4. Failure if any of the constraints in $C \cup \tilde{E}$ are violated
   - There might be violations that neither V_t nor $\tilde{V}_t$ caused individually
5. Success if all variables in $V \cup \tilde{V}$ have values and no constraints are violated

$r_{ij} = [l,u]$ is *violated* if t_i and t_j have values and $t_j - t_i \notin [l,u]$

- *Dynamic execution strategy* σ_A for actor, σ_E for environment
  - $\sigma_A(h_{t-1})$ = {what events in V to trigger at time t, given h_{t-1}}
  - $\sigma_E(h_{t-1})$ = {what events in $\tilde{V}$ to trigger at time t, given h_{t-1}}
    - $h_t = h_{t-1} \,.\, (\sigma_A(h_{t-1}) \cup \sigma_E(h_{t-1}))$
  - $(V,\tilde{V},C,\tilde{E})$ is *dynamically controllable* if $\exists\ \sigma_A$ that will guarantee success $\forall\ \sigma_E$

Example

- Instead of a single bring&move task, two separate bring and move tasks
  - Then it's dynamically controllable



- Actor's dynamic execution strategy
  - trigger t_1 at whatever time you want
  - wait and observe t
  - trigger t' at any time from t to $t + 5$
  - trigger $t_3 = t' + 10$
  - for every $t_2 \in [t' + 15, t' + 20]$ and every $t_4 \in [t_3 + 5, t_3 + 10]$
    - $t_4 \in [t' + 15, t' + 20]$
    - so $t_4 - t_3 \in [-5, 5]$
  - So all the constraints are satisfied

Dynamic Controllability Checking

- For a chronicle $\phi = (\mathcal{A}, S\tau, \mathcal{T}, C)$
  - Temporal constraints in C correspond to an STNU
  - Put code into TemPlan to keep the STNU dynamically controllable
- If we detect cases where it isn't dynamically controllable, then backtrack
- If PC$(\mathcal{V} \cup \tilde{V}, \mathcal{E} \cup \tilde{E})$ reduces a contingent constraint then $(\mathcal{V}, \tilde{V}, \mathcal{E}, \tilde{E})$ isn't dynamically controllable

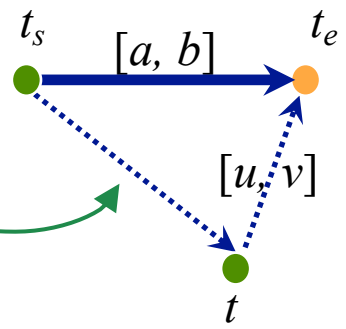
  $\Rightarrow$ can prune this branch
- If it *doesn't* reduce any contingent contraints, we don't know whether $(\mathcal{V}, \tilde{V}, \mathcal{E}, \tilde{E})$ is dynamically controllable

PC$(\mathcal{V}, \mathcal{E})$:
  for $1 \le i \le n$, $1 \le j \le n$, $1 \le k \le n$,
    $i \ne j$, $i \ne k$, $j \ne k$ do
    $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \bullet r_{kj}]$
    if $r_{ij} = \emptyset$ then
      return inconsistent

- Two options
  - Either continue down this branch and backtrack later if necessary, or
  - Extend PC to detect more cases where $(\mathcal{V}, \tilde{V}, \mathcal{E}, \tilde{E})$ isn't dynamically controllable
    - additional constraint propagation rules

Additional Constraint Propagation Rules

- Case 1: $u \geq 0$
  - t must come before t_e
- Add a composition constraint $[a',b']$
- Find $[a',b']$ such that $[a',b'] \bullet [u,v] = [a,b]$
  - $[a'+u,\ b'+v] = [a,b]$
  - $a' = a-u,\ \ b' = b-v$



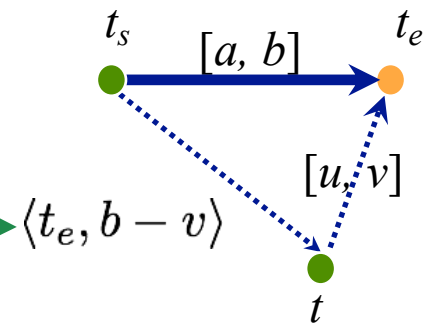
Conditions	Propagated constraint
$t_s \xRightarrow{[a,b]} t_e\ ,\ t \xrightarrow{[u,v]} t_e\ ,\ u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xRightarrow{[a,b]} t_e\ ,\ t \xrightarrow{[u,v]} t_e\ ,\ u < 0\ ,\ v \geq 0$	$t_s \xrightarrow{\langle t_e,b' \rangle} t$
$t_s \xRightarrow{[a,b]} t_e\ ,\ t_s \xrightarrow{\langle t_e,u \rangle} t$	$t_s \xrightarrow{[min\{a,u\},\infty]} t$
$t_s \xrightarrow{\langle t_e,b \rangle} t\ ,\ t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e,b' \rangle} t'$
$t_s \xrightarrow{\langle t_e,b \rangle} t\ ,\ t' \xRightarrow{[u,v]} t\ ,\ t_e \neq t$	$t_s \xrightarrow{\langle t_e,b-u \rangle} t'$

⇒ contingent
→ controllable

$a' = a - u, b' = b - v$

Additional Constraint Propagation Rules

- Case 2: $u < 0$ and $v \geq 0$
  - t may be either before or after t_e
- Add a *wait* constraint
  - Wait until either t_e occurs or current time is $t_s + b - v$, whichever comes first



Conditions	Propagated constraint
$t_s \xRightarrow{[a,b]} t_e \ , \ t \xrightarrow{[u,v]} t_e \ , \ u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xRightarrow{[a,b]} t_e \ , \ t \xrightarrow{[u,v]} t_e \ , \ u < 0 \ , \ v \geq 0$	$t_s \xrightarrow{\langle t_e,b' \rangle} t$
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$t_s \xrightarrow{\langle t_e,b \rangle} t \ , \ t' \xRightarrow{[u,v]} t \ , \ t_e \neq t$	$t_s \xrightarrow{\langle t_e,b-u \rangle} t'$

⇒ contingent
→ controllable

$$a' = a - u, b' = b - v$$

Extended Version of PC

- We want a fast algorithm that TemPlan can run at each node, to decide whether to backtrack
- There's an extended version of PC that runs in polynomial time, but it has high overhead
- Possible compromise: use ordinary PC most of the time
  - Run extended version occasionally, or at end of search before returning plan

Conditions	Propagated constraint
$t_s \xRightarrow{[a,b]} t_e \ , \ t \xrightarrow{[u,v]} t_e \ , \ u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xRightarrow{[a,b]} t_e \ , \ t \xrightarrow{[u,v]} t_e \ , \ u < 0 \ , \ v \geq 0$	$t_s \xrightarrow{\langle t_e,b' \rangle} t$
$t_s \xRightarrow{[a,b]} t_e \ , \ t_s \xrightarrow{\langle t_e,u \rangle} t$	$t_s \xrightarrow{[min\{a,u\},\infty]} t$
$t_s \xrightarrow{\langle t_e,b \rangle} t \ , \ t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e,b' \rangle} t'$
$t_s \xrightarrow{\langle t_e,b \rangle} t \ , \ t' \xRightarrow{[u,v]} t \ , \ t_e \neq t$	$t_s \xrightarrow{\langle t_e,b-u \rangle} t'$

Summary

- Managing constraints in TemPlan: like CSPs
  - Temporal constraints: STNs, PC algorithm (path consistency)