

# Multiagent decision making and Games in Normal Form



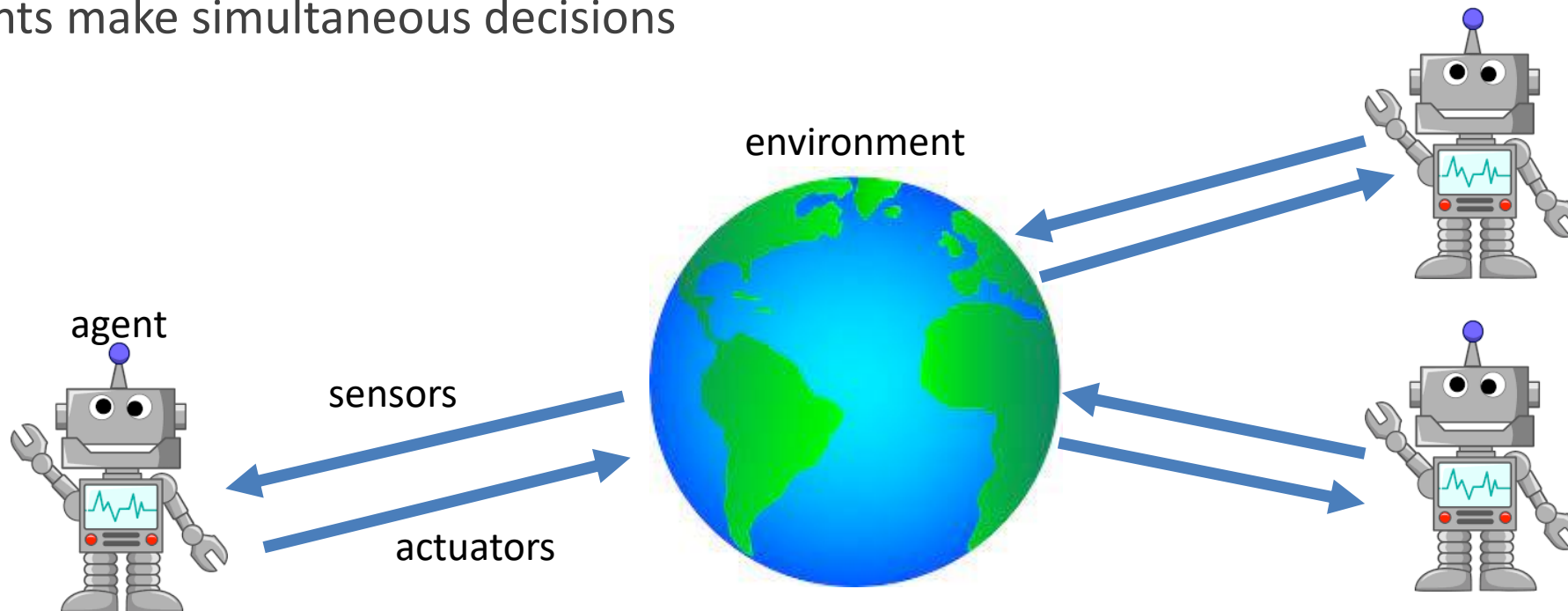
# Outline

- **Multiagent decision making with Game theory**
- Normal-form games
  - Example – Prisoner's dilemma
- Strictly dominated action
- Solution Concepts
  - Iterated elimination of strictly dominated actions
  - Nash Equilibrium



# Multiagent decision making

- We are now going to study **multiagent decision making**
  - Group of agents coexist within an environment
  - Agents make simultaneous decisions



# Game Theory

- What is game theory?
  - Mathematical study of **interactions** among **independent, self-interested agents**
  - It has been **applied in many disciplines**, such as:
    - Economics, political science, biology, psychology, linguistics, and **computer science (multiagent decision making)**

# Game Theory

- What is game theory?
  - Agents are considered **self-interested**
    - Each agent has a **description of states (outcomes)** it likes
    - The dominant theory for modeling agent's interests is **utility theory**

# Game Theory

The success of an agent depends on the decisions  
of all agents



# Game Theory

- Game theory was **originally designed for modelling economical interaction**
- Game theory is now an **independent field**
  - Solid mathematical foundation
  - Many applications

# Game Theory

- Game theory is based on **two assumptions**:
  - Participating agents are **rational**
  - Agents reason **strategically**
    - They **take into account the other agent's decision** in their own decision making process



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# Normal-Form Games

- Can be viewed as the **multiagent extension** of utility-based decision making
- Also known as **static game or strategic games** in some game theory books

# Normal-Form Games

- **Definition (Normal-form game):** A (finite,  $n$ -person) normal-form game is a **tuple**  $(N, A, u)$ , where:
  - $N$  is a **finite set of  $n$  players**, indexed by  $i$ ;
  - $A = A_1 \times \cdots \times A_n$ , where  $A_i$  is a **finite set of actions available to player/agent  $i$** . Each vector  $\mathbf{a} = (a_1, \dots, a_n) \in A$  is called an **action profile (or joint action)**
  - $u = (u_1, \dots, u_n)$ , where  $u_i : A \mapsto \mathbb{R}$  is a **real-valued utility (or payoff) function for player  $i$** .

# Strategic game

- In summary:
  - **Each agent chooses a single action** and then **receives a payoff that depends on the joint action**
  - The joint action is the **outcome** of the game
  - Although payoffs are common knowledge, an **agent does not know the actions of the other agents**
    - The best an agent can do is to **predict** the actions of others
  - A game's **solution** is a **prediction of the outcome** using the assumptions that all agents are rational and strategic

# Notation in Economics

- The **normal-form representation** of a game specifies:
  - *an  $n$ -player game*
  - the **strategies** available to each player
    - Let  $s_i \in S_i$  denote a strategy for player  $i$ , where  $S_i$  is the strategy space
    - Let  $(s_1, \dots, s_n)$  denote a combination of strategies
  - the **payoff** received by each player for each combination of strategies that could be chosen by the players
    - Let  $u_i(s_1, \dots, s_n)$  denote player  $i$ 's payoff function
- Hence, we denote a **game** by  $G = \{S_1, \dots, S_n, u_1, \dots, u_n\}$

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# Prisoner's dilemma

- The **prisoner's dilemma** is one of the **oldest and most studied model** in game theory
- Information about the game:
  - **Two suspects** are arrested and charged for a crime
  - The **police lack sufficient evidence** to convict the suspects, unless at least one confesses
  - The police hold the suspects in **separate cells**



# Prisoner's dilemma

- The **policemen explain the consequences** that will follow from the following actions:
  - **If neither confesses then both will be convicted** of a minor offense and sentenced to **one month** in jail
  - **If both confess then both will be sentenced** to jail for **six months**
  - **if one confesses but the other does not**, then the **confessor will be released** immediately but the **other will be sentenced to nine months** in jail





# Prisoner's dilemma

- Elements of this normal-form game:
  - $N = \{\text{Prisoner 1, Prisoner 2}\}$  (i.e., there are **2 agents** within the environment)
  - Each agent  $i$  can select an **action**  $a_i$  (or **strategy**) from his own action set  $A_i$ 
    - $A_1 = A_2 = \{not\ confess, confess\}$
    - And  $a = (a_1, a_2)$  is the **joint action** (or action/strategy profile)
      - e.g.,  $a = (a_1 = confess, a_2 = not\ confess)$
  - $u = (u_1(a_1, a_2), u_2(a_1, a_2))$ , where  $u_i$  is the payoff function for each agent (next slide)

# Prisoner's dilemma

- Elements of this normal-form game :
- In the special case of two agents, the normal-form game can be represented by a **payoff matrix**

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

- E.g.:
  - $u_1(a_1 = confess, a_2 = not\ confess) = 0$
  - $u_2(a_1 = confess, a_2 = not\ confess) = -9$

# Prisoner's dilemma

- In its most general form, the Prisoner's Dilemma is any normal-form game as follows:

		Agent 2	
		<i>Cooperate</i>	<i>Defect</i>
Agent 1	<i>Cooperate</i>	$a, a$	$b, c$
	<i>Defect</i>	$c, b$	$d, d$

In which:  $c > a > d > b$

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# Strictly dominated action

- **In game theory, we assume** that a **rational agent will never choose a suboptimal action** (or play a strictly dominated action/strategy)
- **Suboptimal action** will always result in lower payoffs for the agent than some other action, no matter what the other agents do

# Strictly dominated action

- We formalize this concept as follows:
  - **Definition:** *We will say that an action  $a_i$  of agent  $i$  is **strictly dominated** by another action  $a_i'$  of agent  $i$  if*

$$u_i(a_i', a_{-i}) > u_i(a_i, a_{-i})$$

*for all actions  $a_{-i}$  of the other agents*

# Strictly dominated action

- Example in the prisoner's dilemma:
  - for prisoner 1, the *not confess* is a **strictly dominated action** by the action *confess*

$$u_1(a_1' = \text{confess}, a_2 = \text{not confess}) > u_1(a_1 = \text{not confess}, a_2 = \text{not confess})$$
$$u_1(a_1' = \text{confess}, a_2 = \text{confess}) > u_1(a_1 = \text{not confess}, a_2 = \text{confess})$$

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

# Strictly dominated action

- Example in the prisoner's dilemma:
- similarly, **for prisoner 2**, the *not confess* is a **strictly dominated action** by the action *confess*

$$u_2(a_2' = \text{confess}, a_1 = \text{not confess}) > u_2(a_2 = \text{not confess}, a_1 = \text{not confess})$$
$$u_2(a_2' = \text{confess}, a_1 = \text{confess}) > u_2(a_2 = \text{not confess}, a_1 = \text{confess})$$

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6



# Outline

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- Strictly dominated action
- **Solution Concepts**
  - Iterated elimination of strictly dominated actions
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# Solution Concepts

- Now that we have defined games in normal form:
  - How can we reason about these games?
  - How can we predict the outcomes of a game?
  - How can we predict the actions of a game?

# Solution Concepts

- The problem of **reasoning about games** and **identifying certain subsets of outcomes** is called

## Solution Concept

# Outline

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- Strictly dominated action
- Solution Concepts
  - **Iterated elimination of strictly dominated actions**
  - Nash Equilibrium



# Iterated elimination of strictly dominated actions

- The **iterated elimination of strictly dominated actions** is a **first solution concept** for games
  - This solution concept is based on the **assumption** that a **rational agent will never choose a suboptimal action** (or play a strictly dominated action/strategy)

# Iterated elimination of strictly dominated actions

- The **iterated elimination of strictly dominated actions** is a solution technique that:
  - **iteratively eliminates strictly dominated actions** from all agents, until no more actions are strictly dominated
- This technique is based on **two assumptions**:
  - A rational agent would never take a strictly dominated action
  - It is *common knowledge* that all agents are rational

# Iterated elimination of strictly dominated actions

- Example (Game 2):
  - Imagine 2 agents with the following payoff matrix
  - Are there strictly dominated actions?

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	1, 0	1, 2	0, 1
	<i>Down</i>	0, 3	0, 1	2, 0

# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
  - Let us first check agent 1
  - For agent 1, **neither Up nor Down is strictly dominated**

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	1, 0	1, 2	0, 1
	<i>Down</i>	0, 3	0, 1	2, 0



# Iterated elimination of strictly dominated actions

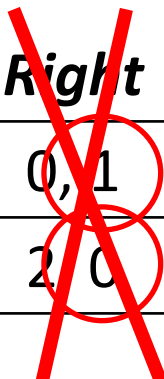
- Example (**Game 2**):
  - Let us now check agent 2
  - For agent 2, *Right* is strictly dominated by *Middle*

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	1, 0	1, 2	0, 1
	<i>Down</i>	0, 3	0, 1	2, 0

# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
  - A **rational** agent 2 will **not** choose *Right*
  - Thus, if agent 1 knows that agent 2 is rational, then agent 1 can **eliminate** *Right* from the **action set**

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	1, 0	1, 2	0, 1
	<i>Down</i>	0, 3	0, 1	2, 0



# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
  - Hence, if agent 1 knows that agent 2 is rational, then **agent 1 can play the game as if it were the following:**

		Agent 2	
		<i>Left</i>	<i>Middle</i>
Agent 1	<i>Up</i>	1, 0	1, 2
	<i>Down</i>	0, 3	0, 1

**Are there strictly dominated actions?**

# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
  - *Down* is now strictly dominated by *Up* for agent 1.

		Agent 2	
		<i>Left</i>	<i>Middle</i>
Agent 1	<i>Up</i>	1, 0	1, 2
	<i>Down</i>	0, 3	0, 1

# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
  - If **agent 1 is rational** (and agent 1 knows that agent 2 is rational, so that the game below applies) then **agent 1 will not choose *Down***
  - if agent 2 knows that agent 1 is rational, then **agent 2 can eliminate *Down* from agent 1's action set**
    - Note that we assume that agent 2 knows that agent 1 knows that agent 2 is rational (so that agent 2 knows that the game below applies)

Agent 2

		<i>Left</i>	<i>Middle</i>
<i>Up</i>		1, 0	1, 2
<i>Down</i>		0, 3	0, 1

# Iterated elimination of strictly dominated actions

- Example (**Game 2**):
- Hence, considering the rationality assumption, **the agents can play the game as if it were the following:**

		Agent 2	
		<i>Left</i>	<i>Middle</i>
Agent 1	<i>Up</i>	1, 0	1, 2

**Are there strictly dominated actions?**

# Iterated elimination of strictly dominated actions

- Example (Game 2):
  - *Left* is strictly dominated by *Middle* for agent 2
  - Thus, the solution (outcome) of this game is **(Up, Middle)**

		Agent 2	
		<del><i>Left</i></del>	<i>Middle</i>
Agent 1	<i>Up</i>	<del>1, 0</del>	1, 2
	<i>Down</i>	0, 1	0, 0

# Iterated elimination of strictly dominated actions

Can we use this solution concept to find the outcome in the Prisoner's Dilemma?



# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - Are there strictly dominated actions? Yes!
  - For prisoner 1, *Not confess* is strictly dominated by *Confess*

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, 1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - A **rational** prisoner 1 will **not choose *Not confess***
  - Thus, if prisoner 2 knows that prisoner 1 is rational, then prisoner 2 can **eliminate *Not confess*** from the action set

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	<del>-1, -1</del>	<del>-9, 0</del>
	<i>Confess</i>	0, -9	-6, -6

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - Hence, if prisoner 2 knows that prisoner 1 is rational, then **prisoner 2 can play the game as if it were the following:**

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Confess</i>	0, -9	-6, -6

**Are there strictly dominated actions?**

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - *Not confess* is strictly dominated by *Confess* for prisoner 2
  - Thus, the solution (outcome) of this game is (*Confess*, *Confess*)

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	0, 0	-6, 0
	<i>Confess</i>	<del>0, -9</del>	-6, -6

# Iterated elimination of strictly dominated actions

- Does the order matter in this algorithm?
- Will we end up with different outcomes if we change the order?

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - Recall that we started with this:
  - For prisoner 1, *Not confess* is strictly dominated by *Confess*

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - **However, we could have started with this:**
    - For prisoner 2, *Not confess* is strictly dominated by *Confess*

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - A **rational** prisoner 2 will **not choose *Not confess***
  - Thus, if prisoner 1 knows that prisoner 2 is rational, then prisoner 1 can **eliminate *Not confess* from the action set**

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	<del>-1, -1</del>	-9, 0
	<i>Confess</i>	<del>0, -9</del>	-6, -6



# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
- Hence, if prisoner 1 knows that prisoner 2 is rational, then **prisoner 1 can play the game as if it were the following:**

		Prisoner 2	
		<i>Confess</i>	
Prisoner 1	<i>Not confess</i>	-9, 0	
	<i>Confess</i>	-6, -6	

# Iterated elimination of strictly dominated actions

- Example with the Prisoner's Dilemma:
  - *Not confess* is strictly dominated by *Confess* for prisoner 1
  - Thus, the solution (outcome) of this game is (*Confess*, *Confess*)

		Prisoner 2	
		<i>Confess</i>	
Prisoner 1	<i>Not confess</i>	<del>-2, 0</del>	
	<i>Confess</i>	<del>-6, -6</del>	

# Iterated elimination of strictly dominated actions

- **Appealing idea** that rational agents do not play strictly dominated actions
- **First drawback:**
  - Each step requires a further assumption about what the agents know about each other's rationality
    - We need to assume that it is *common knowledge* that the **agents are rational**

# Iterated elimination of strictly dominated actions

- Second drawback:
  - the process often produces a **very imprecise prediction** about the play of the game
    - For instance, if no actions are eliminated then anything could happen

# Iterated elimination of strictly dominated actions

- Second drawback:
  - Does the game (i.e., **Game 3**) below have any strictly dominated actions?

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

# Iterated elimination of strictly dominated actions

- Second drawback:

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

**For agent 1, there are  
no strictly  
dominated actions**

# Iterated elimination of strictly dominated actions

- Second drawback:

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6

**For agent 2, there are  
no strictly  
dominated actions**

# Iterated elimination of strictly dominated actions

- Second drawback:
  - Hence, **Game 3** (below) does not have any strictly dominated actions
    - And the technique does not produce a prediction whatsoever about this game, thus we are unsure about the outcome

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	0, 4	4, 0	5, 3
<i>M</i>	4, 0	0, 4	5, 3
<i>B</i>	3, 5	3, 5	6, 6



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  - **Nash Equilibrium**



# Nash equilibrium

- The **Nash equilibrium** is a **stronger solution concept** than the iterated elimination of strictly dominated actions
- Hence, it produces **more accurate predictions in a wider class of games**

# Nash equilibrium

- **Defintion:** A **Nash equilibrium** is a joint action  $a^*$  with the property that the following holds for every agent  $i$ :

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$$

for all action  $a_i \in A_i$

- In other words, a Nash equilibrium is a **joint action from where no agent can *unilaterally* improve his payoff**
  - Hence, **no agent has any incentive to deviate**

# Nash equilibrium

- Note that:
  - The **iterated elimination of strictly dominated actions** produces a solution of a game by means of an **algorithm**
  - However, the previous definition of **Nash equilibrium** describes a solution in terms of the **conditions that hold at that solution**

# Nash equilibrium

- An alternative for the previous definition makes use of the **best-response function**
- **Definition:** *The **best-response function** for agent  $i$  is*

$$B_i(a_{-i}) = \{a_i \in A_i: u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \text{ for all } a_i' \in A_i\}$$

- Note that  $B_i(a_{-i})$  can be a set containing many actions

# Nash equilibrium

- For instance, in the prisoner's dilemma:
  - If **prisoner 2** takes the action *Not confess*
  - The **best response** of **prisoner 1** is the action *Confess*
    - $B_1(a_2 = \text{Not confess}) = \text{Confess}$

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

# Nash equilibrium

- We can compute the **best-response function** for each agent:

- $B_1(a_2 = \text{Not confess}) = \text{Confess}$

- $B_1(a_2 = \text{Confess}) = \text{Confess}$

- $B_2(a_1 = \text{Not confess}) = \text{Confess}$

- $B_2(a_1 = \text{Confess}) = \text{Confess}$

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, <u>0</u>
	<i>Confess</i>	<u>0</u> , -9	<u>-6</u> , <u>-6</u>

# Nash equilibrium

- Using the definition of best-response function, we can now formulate an **equivalent definition** (to the first definition):
- **Definition 2:** A **Nash equilibrium** is a joint action  $a^*$  with the property that the following holds for every agent  $i$ :

$$a_i^* \in B_i(a_{-i}^*)$$

- In other words, in a Nash equilibrium, **each agent's actions is an optimal response to the other agents' actions**



# Nash equilibrium

- We can compute the **best-response function** for each agent:

- $B_1(a_2 = \text{Not confess}) = \text{Confess}$
- $B_1(a_2 = \text{Confess}) = \text{Confess}$
- $B_2(a_1 = \text{Not confess}) = \text{Confess}$
- $B_2(a_1 = \text{Confess}) = \text{Confess}$

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	<u>0</u> , -9	<u>-6</u> , <u>-6</u>

Nash Equilibrium

# Nash equilibrium

- In the prisoner's dilemma, note that:

		Prisoner 2	
		<i>Not confess</i>	<i>Confess</i>
Prisoner 1	<i>Not confess</i>	-1, -1	-9, 0
	<i>Confess</i>	0, -9	-6, -6

No incentives to deviate

# Nash equilibrium

- So, we could use a **brute-force algorithm** to finding a game's **Nash equilibria** (NE):
  - Check whether each possible joint action satisfies the condition in the definition

# Nash equilibrium

- Let us try this approach in **Game 2**:

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	1, 0	1, 2	0, 1
	<i>Down</i>	0, 3	0, 1	2, 0

# Nash equilibrium

- In a two-agent game, this approach has the following steps:
  - for each agent and for each action of that agent, **determine the other agent's best response** to that action

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	<u>1</u> , 0	<u>1</u> , <u>2</u>	0, 1
	<i>Down</i>	0, <u>3</u>	0, 1	<u>2</u> , 0

# Nash equilibrium

- In a two-agent game, this approach has the following steps:
- A **pair of actions satisfies condition in the definition of a NE** if each agent's action is a best response to the other's
- Thus, we have **NE** if **both payoffs are underlined** in the corresponding cell of the payoff matrix

		Agent 2		
		<i>Left</i>	<i>Middle</i>	<i>Right</i>
Agent 1	<i>Up</i>	<u>1</u> , 0	<u>1</u> , <u>2</u>	0, 1
	<i>Down</i>	0, <u>3</u>	0, 1	<u>2</u> , 0

Nash equilibrium

# Nash equilibrium

- Let us try this approach in **Game 3**:

		Agent 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Agent 1	<i>T</i>	0, 4	4, 0	5, 3
	<i>M</i>	4, 0	0, 4	5, 3
	<i>B</i>	3, 5	3, 5	6, 6

# Nash equilibrium

- In a two-agent game, this approach has the following steps:
  - for each agent and for each action of that agent, **determine the other agent's best response** to that action

		Agent 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Agent 1	<i>T</i>	0, <u>4</u>	<u>4</u> , 0	5, 3
	<i>M</i>	<u>4</u> , 0	0, <u>4</u>	5, 3
	<i>B</i>	3, 5	3, 5	<u>6</u> , <u>6</u>



# Nash equilibrium

- In a two-agent game, this approach has the following steps:
- A pair of actions satisfies condition in the definition of a NE if each agent's action is a best response to the other's
- Thus, we have NE if both payoffs are underlined in the corresponding cell of the payoff matrix

		Agent 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Agent 1	<i>T</i>	0, <u>4</u>	<u>4</u> , 0	5, 3
	<i>M</i>	<u>4</u> , 0	0, <u>4</u>	5, 3
	<i>B</i>	3, 5	3, 5	<u>6</u> , <u>6</u>

Nash equilibrium

# Final remarks

- Although the two definitions suggest a **brute-force method** for finding the **Nash equilibrium** of a game
  - the **cost** of such an algorithm is **exponential in the number of agents**

# Final remarks

- Addressing the **relation** between **Nash equilibrium (NE)** and **iterated elimination of strictly dominated actions (IESDA)**
  - Recall the **NE joint actions** in the Prisoners' Dilemma (*Confess, Confess*) and Game 2 (*Up, Middle*)
    - These are the only joint actions that “*survived*” IESDA
- In fact, this **result can be generalized**:
  - If IESDA eliminates all but a single joint action  $a$ , then this joint action is the unique NE of the game

# Final remarks

- Addressing the **relation** between **NE** and **IESDA**:
  - Since IESDA **frequently does not eliminate** all but a single joint action
    - We say that the **Nash equilibrium is a stronger solution concept** than iterated elimination of strictly dominated actions
- For example in Game 3:
  - any outcome is possible with IESDA
  - while NE gives a unique prediction

# Thank You



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