

# Princeton QuBlitz - Hardware Prompt

Nov 15-16, 2025

## Instructions:

There are three sections, and 11 challenges in total (labeled 1a-e, 2a-c, 3a-c). You will:

- 1) **Submit a PDF File** of all your challenge responses. Challenges 1d, 2b, and 3a will have you make plots. The rest of the challenges have written responses. Submit all your plots and written responses in a single PDF.
- 2) **Prepare a Short Presentation** of about 3 minutes. The primary goal of this is to explain your plots in 1d, 2b and 3a. Afterwards, we will ask questions about your submission. This is to make sure you, not ChatGPT, understood what you were doing.
- 3) **Submit any code you have written.**

For the parts where you have to make plots, you will have to write code. We highly recommend Python, as the PSQ Officers will be able to help you only with Python.

**Generative AI Policy:** You are allowed to use LLMs and Generative AI to help you understand concepts and for code syntax. Please do not use generative AI to do *any* derivation asked in the challenges or directly answer *any* question posed in the challenges. While scoring your challenge, we will place particularly strong emphasis on explanations and answers to questions given during the presentations, to emphasize understanding.

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*Note from Author:*

**I strongly recommend you reach out for help whenever, wherever you need.** We will be a better resource for this challenge than ChatGPT is. We are more than happy to help you out in any way you need and explain anything that may be unclear, or anything you may be unsure about. The goal here is to learn some really cool physics and have discussions with all the other people involved in the event, officers and competitors included, about this really cool physics. If you at anytime want to chat during the event, you can reach me through the PSQ Discord, just give me a ping.

- Vincent

# Introduction

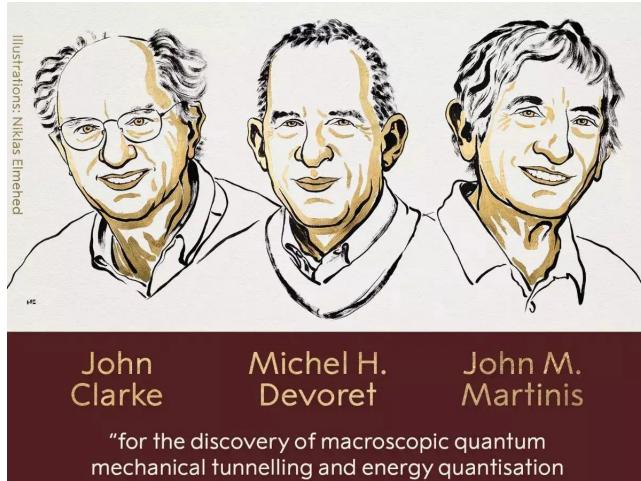


Photo Credit: Nobel Prize

Last month, John Clarke, Michel Devoret, and John Martinis won the 2025 Nobel Prize in Physics for their discoveries of “macroscopic quantum mechanical tunneling and energy quanti[z]ation in an electric circuit” [4]. This year’s Nobel Prize has particular relevance to quantum computing, as the experiments performed by Clarke, Devoret, and Martinis have laid the foundation for Circuit QED, the physical framework for modern-day superconducting qubits.

In this challenge, you will walk through some of the physics relevant to the discoveries of this year’s Nobel laureates, and hopefully be able to answer the questions of what they did, and why we care.

Although, from what I can tell, most of their discoveries stem from two papers written in 1985 [3] [2], I find this 1988 review of their work in Science to be a more accessible and complete description of their work’s impact and findings, so we will refer to this paper throughout the challenge:

**Science 1988 Paper by Clarke et.al [1]:**

<https://doi.org/10.1126/science.239.4843.992>

## Part 1: Hamiltonian Formalism

Before we dive into the physics of the Nobel Prize, it’s useful to get familiar with some formalism that will help us understand the physics.

You may be familiar with Newtonian mechanics. Essentially, it is a formulation of classical mechanics (a way to state the laws of classical mechanics) based on “forces”. System

dynamics (time evolution) are specified by what forces act on which objects, and a force, as you may know, accelerates an object with mass  $m$  according to  $F = ma$ .

Newtonian mechanics is not the only way to state the laws of classical mechanics, and here we will introduce what are called “Hamiltonian mechanics”, which is an entirely equivalent way to formulate classical mechanics. You may have solved for the dynamics of a classical system before using the total “energy” of a system rather than using “forces”, and this is exactly what the idea of Hamiltonian mechanics is. Below we will define how Hamiltonian mechanics works:

### **Hamiltonian Mechanics:**

We will first define a set of *conjugate observables*. The set of conjugate observables are physical things you can observe that come in pairs. An example of an observable is position, or momentum, or magnetic flux, or angular momentum (stuff you can observe). They are usually continuous scalar functions of time. We will define them in pairs, and call each pair “conjugates”. We will denote the set of conjugate observables as  $\{p_i, q_i\}$ , where  $p_i$  and  $q_i$  are the  $i$ th pair of conjugate observables. (Aside: you can view these conjugates as simply how the system is *defined*, although in theoretical physics, they can all be derived from a Lagrangian. This is why physicists like to state theories with Lagrangians, since you can really get *everything* you need to know about a system from it.)

Next, we will define a *Hamiltonian*, which is simply a function of the conjugate observables  $\{p_i, q_i\}$ . The Hamiltonian can be thought of as representing the *energy* of a system. An example of a Hamiltonian, denoted  $H$ , with a single set of conjugate variables  $p$  and  $q$ , is:

$$H(p, q) = \frac{p^2}{2m} + V(x)$$

Again, it is literally just any function  $H$  of the set of conjugate observables, representing the “energy” of a system.

So now we have 1) a set of conjugate observables and 2) a Hamiltonian function. If we are interested in system dynamics, or in other words, how our observables evolve over time (this is the fundamental problem of classical mechanics, and really all of physics), every observable  $O$  will follow Hamilton’s equations of motion:

$$\dot{O} = \{O, H\}_{PB} \tag{1}$$

Where we have defined the *Poisson Bracket* between any two observables  $A$  and  $B$  as follows:

$$\{A, B\}_{PB} = \sum_i \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q_i} - \frac{\partial B}{\partial p_i} \frac{\partial A}{\partial q_i} \tag{2}$$

Note here that the *definition of the Poisson Bracket depends on how you define the conjugate set of observables*. And since the system dynamics depend on the Poisson Bracket, the

way you define which observables are “conjugate” directly specifies how your system behaves.

Note that conjugate observables will always have a Poisson bracket of 1 or  $-1$  (depending on how you define order in Equation 2). Observables with nonzero Poisson brackets that are not 1 or  $-1$  are scalar multiples of conjugate observables. (You can easily check the element-wise linearity of the Poisson Bracket, due to the linearity of the partial derivative).

This is really remarkable, because we have written a single equation (Equation 1) that specifies *all* of our system dynamics. No need to write down a million different forces and see how they all act. If you can define a Hamiltonian, you know how every observable in your system behaves. This is an amazingly compact and generalized framework for literally any classical system.

Let’s apply this new formulation of classical mechanics to a system you maybe familiar with, and see that it agrees with Newtonian mechanics.

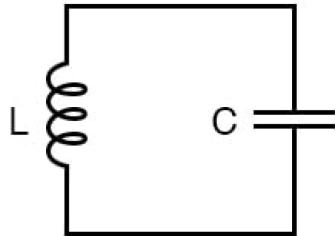
A 1D simple harmonic oscillator is a system governed by the force  $F = -kx$  (“Hooke’s Law”). Let’s solve this system with Hamiltonian mechanics. We will define a single pair of conjugate observables, position  $x$  and momentum  $p$ . I claim that the Hamiltonian of this system is the “energy” (kinetic + potential) of the system:

$$H(p, x) = \frac{p^2}{2m} + \frac{1}{2}kx^2 \quad (3)$$

where  $k$  is the same  $k$  in Hooke’s law, and  $m$  is the mass of the classical object. Verify my claim by solving for dynamics:

**Challenge 1a:** Solve for  $x(t)$ , the position as a function of time, for the 1D simple harmonic oscillator, using the Hamiltonian in Equation 3. You should find that  $x(t)$  oscillates periodically. What is the frequency (in radians / unit time) of the oscillation? Does it agree with the Newtonian approach?

Great, now let’s consider a different classical system: an LC circuit. This is an electrical circuit with an inductor and capacitor in a loop. The inductor has inductance  $L$  and the capacitor has capacitance  $C$ .



I will now define this system in the Hamiltonian formulism. I claim that we can define a single pair of conjugate variables  $Q$  and  $\Phi$ , representing the charge on the capacitor and

the flux through the inductor respectively. The energy of this circuit, you may recall from classical electromagnetism (for a single loop inductor) is:

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \quad (4)$$

Again, we'll claim this is the Hamiltonian. You may recognize this is essentially the same type of problem we solved above, so the solution should be similar. Again, verify my claim:

**Challenge 1b:** for the LC circuit, use the Hamiltonian in Equation 12 to show that voltage and current at any point in the circuit are periodic. What is the frequency (in radians/time)? Next, use arguments from classical electromagnetism to show the same, that voltage and current are periodic. What is the frequency from this calculation? Do your results agree?

We will now introduce an exotic type of circuit component that can only exist in superconducting circuits called the *Josephson Junction*. The behavior of this circuit component is very weird. The voltage ( $V$ ) and current ( $I$ ) across the junction are given by the following two equations, known as the Josephson relations:

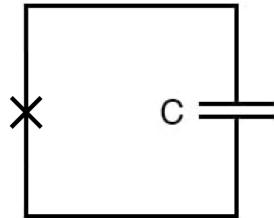
$$V = \frac{\Phi_0}{2\pi} \dot{\phi} \quad (5)$$

$$I = I_0 \sin(\phi) \quad (6)$$

Where  $I_0$  is known as the “critical current”, a characteristic of the circuit component (like capacitance, inductance, etc...), and  $\Phi_0$  is the magnetic flux quanta (just a physical constant, like Planck's constant).

Now this “ $\phi$ ” variable is something new. We will treat it as an observable and call it “phase”. The reason for this strange behavior of the Josephson Junction is due to quantum tunneling, but we will not concern ourselves with this and use the Josephson relations as given. In fact, this “phase” is really a quantum phase difference, but again, we need not concern ourselves with this.

Okay, now let's consider a circuit where we replace our inductor with a Josephson junction, denoted by a cross.



I claim that, for a single pair conjugate variables  $\phi$  and  $p = (\frac{\Phi_0}{2\pi})Q$ ,  $\phi$  being the Josephson Junction phase and  $Q$  being the charge on the capacitor, the Hamiltonian is:

$$H = \frac{Q^2}{2C} - I_0\left(\frac{\Phi_0}{2\pi}\right)\cos(\phi) \quad (7)$$

$$= \frac{p^2}{2C(\Phi_0/2\pi)^2} - I_0\left(\frac{\Phi_0}{2\pi}\right)\cos(\phi) \quad (8)$$

**Challenge 1c:** Derive an expression for the dynamics of  $\phi$ , purely using the Josephson relations and arguments from classical electromagnetism (Hint: use Kirchoff's Laws). Now verify that the Hamiltonian in Equation 8 recovers that equation you just derived.

Okay, up to now, we have purely worked with classical systems. One of the biggest advantages of using the Hamiltonian formalism, though, is that it is very easy to go to quantum mechanics, since the language of quantum mechanics is written using Hamiltonians.

We expect every fundamental building block of nature, like atoms and particles, to behave quantum mechanically, and that classical mechanics is just a large limit approximation of quantum mechanics. We will not delve into the full correspondence between classical Hamiltonian mechanics and quantum mechanics for the purposes of this challenge, especially for dynamics, but do be aware that quantum mechanics corresponds very closely with Hamiltonian mechanics save for a few key differences.

One of these differences is that energy comes in discrete packets ("quanta"), and is not continuous like in classical mechanics. The way this is formulated in quantum mechanics is that, instead of every observable being a scalar variable, they are associated with a Hermitian linear operator (a linear operator with real eigenvalues), and the values of that observable correspond to the eigenvalues of the Hermitian operator. Thus, the values an observable can take on are discrete.

So in quantum mechanics, since the Hamiltonian is a function of observables which are Hermitian operators, the Hamiltonian itself is a Hermitian operator, and thus has a spectrum of real eigenvalues. By now, you should have seen that the Hamiltonian for all the systems we have considered is the just the total energy of the system. Thus, we will say that the Hamiltonian represents the energy of a quantum mechanical system, and finding the eigenvalues of the Hamiltonian gives us our discrete energy spectrum.

Clarke et.al [1] argue in their 1988 paper about what physical conditions a classical system must obey in order to behave quantum mechanically. Let's assume those conditions are met for the LC circuit we considered. Now let's find out how the energy is quantized.

I should note that before we proceed, conjugate observables in quantum mechanics are specified by commutation relations, or essentially, how our Hermitian operators commute or don't commute. A core postulate of quantum mechanics is that classically conjugate variables, when written as quantum Hermitian operators, do not commute. In fact, we will define the *commutator* of two linear operators  $\hat{A}$  and  $\hat{B}$  as:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \quad (9)$$

And if  $A$  and  $B$  are conjugate variables:

$$[\hat{A}, \hat{B}] = i\hbar \quad (10)$$

where  $i$  is the imaginary unit and  $\hbar$  is reduced Plank's constant (a very central physical constant to quantum mechanics). For now, you do not need to worry too much about the specifics, but I will tell you that if two Hermitian operators  $A$  and  $B$  with continuous eigenvalues are conjugate (obey Equation 10), we can write  $B$  as:

$$\hat{B} = -i\hbar \frac{\partial}{\partial A} \quad (11)$$

Where what we mean by “partial with respect to  $A$ ” are the eigenvalues of  $\hat{A}$ , which we assumed to be continuous. This is very nontrivial fact that you are not expected to have intuition for, but know it can be proven only from commutation relations.

The reason I am bringing this up is that this is useful for obtaining the quantum spectrum of the LC circuit, since we have:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \quad (12)$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar \quad (13)$$

Where I am simply taking our classical system and writing the conjugate observables as operators. So our Hamiltonian can be written as:

$$\hat{H} = -\frac{\hbar^2}{2C} \frac{\partial^2}{\partial^2 \Phi} + \frac{\Phi^2}{2L} \quad (14)$$

For dynamical variable  $\Phi$ , acting on some function of  $\Phi$  we can call  $\phi(\Phi)$ .

**Challenge 1d:** Use a numerical eigensolver to find first five eigenvalues of Equation 14. Discretize your space so you can write the operator as a matrix. Set  $\hbar = 1$ . **Plot two plots:** 1) the first five eigenvalues, with respect to  $L$ , for  $C = 1$ , and 2) the first five eigenvalues, with respect to  $C$ , for  $L = 1$ .

**Challenge 1e:** You should find a uniform energy spectrum (every eigenvalue spaced evenly). What is the spacing, as a function of  $L$  and  $C$  (remember,  $\hbar = 1$ )? Clarke et.al [1] describe that the LC circuit is in what they call a “correspondence limit”, where the quantum behavior is the same as the classical behavior. Based on your findings, are Clarke et.al right?

## Part 2: The Nobel Prize Winning Experiment

Okay, there's one really big problem with what we did in the Challenges 1d and 1e. We know that very small things like subatomic particles behave quantum mechanically, so if we quantized what we did in Challenge 1a (with the harmonic oscillator of an object), we know that describes a physical system. *However*, how do we know that macroscopic variables, like flux in a circuit, behave quantum mechanically, when the system satisfies the conditions for quantum behavior outlined by Clarke et.al[1]? We can describe what happens *if* they do (in Challenge 1d), but is this what really happens in an experiment?

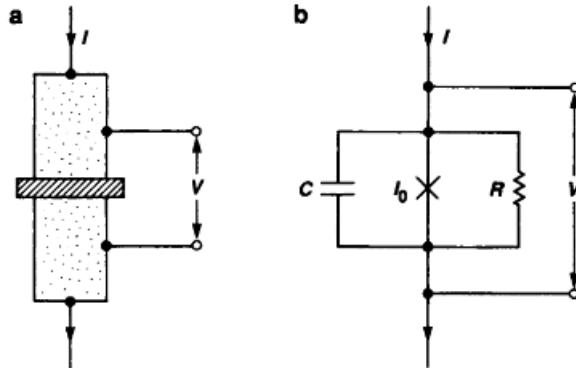
Up until the 1980s, physicists did not know, *precisely* for the reason you elaborated in Challenge 1e: systems like the LC circuit behave the same way if they were inherently quantum mechanical or not, so we had no idea if this quantum mechanical model for macroscopic variables is realistic or not.

Since the LC oscillator doesn't give us any information, Clarke, Devoret, and Martinis in 1985 thought about what *other* systems they can test that are *not* in the correspondence limit.

So they turned to Josephson Junctions and their weird behavior, and constructed an experiment that would definitively tell us whether macroscopic variables in a circuit could exhibit quantum mechanical properties. The particular observable they were interested in was not flux, but rather the Josephson phase  $\phi$  we had introduced prior.

Figure 1 from Clarke et.al [1] is copied below. The right side (Figure 1b) shows the circuit diagram of their experiment.

**Fig. 1. (a)** Schematic representation and **(b)** circuit description of Josephson tunnel junction.



What they are doing is essentially pushing a current  $I$  through a Josephson Junction and measuring the voltage across it. The junction has some internal capacitance and the environment has some loss, represented by a series capacitor and resistor, respectively.

There is also some current noise from the environment, denoted by  $I_N$  in the paper, but we will ignore this for simplicity. Their Equation 1 then reads:

$$C\left(\frac{\Phi_0}{2\pi}\right)^2 \ddot{\phi} + \frac{1}{R}\left(\frac{\Phi_0}{2\pi}\right)^2 \dot{\phi} + \frac{\partial U(\phi)}{\partial \phi} = 0 \quad (15)$$

Where:

$$U(\phi) = -(I_0\Phi_0/(2\pi))(cos\phi + (I/I_0)\phi) \quad (16)$$

*Note:* They use “ $\delta$ ” instead of “ $\phi$ ” for phase in the paper, so don’t be confused if you see “ $\delta$ ”. It is the same thing as “ $\phi$ ” we are using in this challenge.

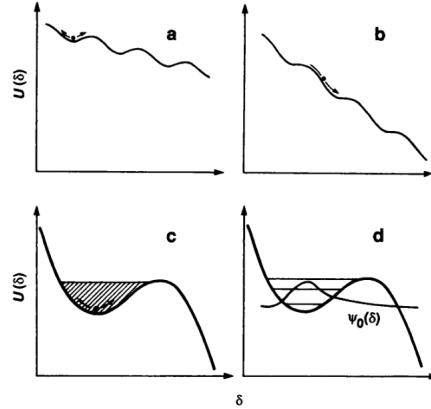
**Challenge 2a:** Derive Equations 15 and 16 from the circuit schematic above, again just using Josephson relations and classical electromagnetism arguments.

If we ignore damping (ideal lossless case), we get the following dynamical equation for  $\phi$ :

$$C\left(\frac{\Phi_0}{2\pi}\right)^2 \ddot{\phi} + \frac{\partial U(\phi)}{\partial \phi} = 0 \quad (17)$$

Which looks just like the dynamics for a harmonic oscillator, but the potential is, as they call it in the paper, a “tilted washboard” shape. It is shown in Figure 2a and 2b in the paper, shown below, and the slope is governed by the ratio  $I/I_0$ , so directly proportional to applied current  $I$ .

**Fig. 2.** Tilted washboard analog of Josephson tunnel junction: (a) stationary state ( $V = 0$ ) for  $I < I_0$ , and (b) running state ( $V \neq 0$ ) for  $I > I_0$ . In the stationary state in the classical regime (c) the particle is point-like with a continuous energy range, whereas in (d) the ground state  $\psi_0(\delta)$  of the particle is described by a wave packet and the energy is quantized into levels.



The idea of this experiment is, if  $I < I_0$ , the phase  $\phi$  will essentially behave like a particle trapped in a harmonic potential, since, as seen in the figures, there exist local minima. The expected value of the time derivative of the phase  $\langle \dot{\phi} \rangle$  is = 0.

However, this is assuming it behaves like a classical particle and cannot tunnel through the potential barrier. If it can get out of this local minimum and “slide down” the “tilted washboard”,  $\langle \dot{\phi} \rangle \neq 0$  and since  $\langle V \rangle \propto \langle \dot{\phi} \rangle$ , we will measure a voltage across the junction in the case it escapes from the local minima.

Now, it is possible that the thermal energy of the system means that there is nontrivial probability that the phase escapes the local minima. Remember saying something is “higher temperature” means that the classical probability distribution over energy levels skews more toward higher energies. It does not necessarily imply tunneling.

The paper provides a definition of an “escape temperature”  $T_{esc}$ , which can be thought of as the temperature needed for an “escape rate” (rate of nontrivial voltage, or phase “escaping” from local minima) of  $\Gamma$  to reasonably occur, *if* the phase behaved classically, according to Equation 15.  $T_{esc}$  is defined in terms of escape rate  $\Gamma$  as:

$$\Gamma = (\omega_p/(2\pi)) \exp(-\Delta U/k_B T_{esc}) \quad (18)$$

Where  $\omega_p$  and  $\Delta U$  are defined in terms of  $I$  in the paper (Equations 3 and 4 in Clarke et.al [1]).

However, if we observe that at small temperatures,  $T_{esc}$  remains constant, above the temperature  $T$ , then this implies tunneling, since even though our temperature is small such that we don’t expect escape rate  $\Gamma$ , somehow, the phase is still escaping with greater probability than can be explained by thermal energy.

## The Experiment

Now you will run the experiment! Look at the first paragraph in the paper to start on the fourth page (page 995); it begins with “To ascertain the escape rate...”. This describes, very concisely, the general gist of the experiment.

Essentially what they do is they increase  $I$  at a constant rate, with respect to  $t$ , and then observe at what time they measure a voltage. They then plot this on a histogram (time vs. counts), and from the histogram determine escape rate  $\Gamma$  and  $T_{esc}$ . They repeat this for many temperatures, and plot  $T$  vs  $T_{esc}$ .

Before I continue, I must note that **for the purposes of this challenge, we will be setting many physical constants to 1**. Specifically, set  $k_B = \Phi_0 = I_0 = C = \hbar = 1$ . This implies:

$$\omega_p = (2\pi)^{1/2} [1 - I^2]^{1/4} \quad (19)$$

$$\Delta U = 2(2^{1/2})/(3\pi)(1 - I)^{3/2} \quad (20)$$

$$\Gamma(I) = \omega_p/(2\pi) \exp(-\Delta U/T_{esc}) \quad (21)$$

Where Equations 19, 20, and 21 correspond to Equations 3, 4, and 8 in Clarke et.al [1], respectively. This admittedly makes the units of the experiment a bit nonsensical, but the core behavior of the physics still stands. Just don’t be surprised if the numbers for physical quantities you are working with in this challenge are a bit outlandish. We are setting some arbitrary units here, purely for simplicity and convenience.

**Challenge 2b:** Using the *experiment.exe* executable file provided, run the Nobel Prize winning experiment for phase tunneling. Produce a *log-log plot of T vs T<sub>esc</sub>* similar to Figure 5 in Clarke et.al [1]. Plot *T* from 0.02 and 0.10, setting all constants mentioned above to 1. **Read all the following hints before proceeding:**

**Hint 1:** The *experiment.exe* file is an executable that takes two command line arguments: the temperature *T* and the number of trials *N*. It will return *N* lines, printed to standard output, with each line having a decimal number between 0 and 100, indicating the time voltage was observed. In this experiment **the rate is 0.01 units of current / unit of time**, so if  $I_0 = 1$  as we set before, it goes from  $I = 0$  to  $I = I_0$  in 100 units of time, increasing current at a constant speed.

To run on MacOS, run:

```
chmod +x experiment.exe
./experiment.exe 0.02 1000
```

On Windows run:

```
.\experiment.exe 0.02 1000
```

Here, I have  $T = 0.02$  and  $N = 1000$ . 1000 trials takes some time to run (20s ish), and I do not suggest doing above that. You should also only run with *T* between 0.02 and 0.10.

Note: please ask for help if you are stuck with programming syntax or parsing etc... I do not want you to get stuck doing this and want to save you as much time as possible to think about the physics.

**Hint 2:** The rate of escape  $\Gamma$  is the per unit rate of an event that can happen at any point in a continuum of times (think Poisson process, if you are familiar). For these types of “continuous time rates”, the CDF (Cumulative Distribution Function) is given by the exponential distribution:

$$P(T > t) = 1 - \exp\left(-\int_0^t \Gamma(t') dt'\right) \quad (22)$$

Where  $P(T > t)$  means the probability that the first time you see the event happening (the voltage being measured) is greater than *t*.

The PDF (Probability Density Function) is the time derivative of the CDF (since our random variable is time).

Now, given this information, carefully think about exactly what your histograms represent.

**Hint 3:** When fitting my histograms, I found that setting my initial guess to  $T_{esc} = 0.04$  made fitting work better when I was having trouble getting it to work, although this depends on your implementation. *Please take this hint with a grain of salt, as what worked for my approach may not work for yours.*

**Challenge 2c:** Assuming your plot of  $T$  vs  $T_{esc}$  looks something like Figure 6 in Clarke et.al [1], what does this imply about the behavior of  $\phi$ ? If we did not know beforehand if macroscopic observables in circuits (or really any macroscopic variables) behaved quantum mechanically, why do you think this result may be considered noteworthy? (perhaps even Nobel Prize worthy??)

## Part 3: Application to Superconducting Qubits

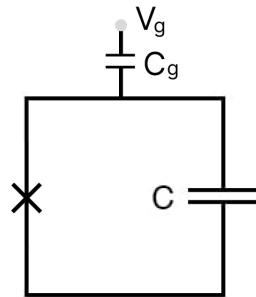
Okay, so now we know that, thanks to Clarke, Devoret, and Martinis, phase  $\phi$  in a Josephson junction can behave quantum mechanically, lets proceed to quantize the Josephson Junction circuit considered in Challenge 1c.

I will write the quantum Hamiltonian in a slightly different way:

$$\hat{H} = E_C(\hat{n} - n_g)^2 - E_J \cos(\hat{\phi}) \quad (23)$$

Where here I have lumped constants into coefficients  $E_C$  and  $E_J$ , and written  $\hat{n} = \hat{Q}/(2e)$ , denoting number imbalance of Cooper Pairs, rather than charge imbalance.

You may also notice the  $n_g$  constant, and this is because I am considering the Josephson Circuit with a gate voltage bias  $V_g$ , connected to the circuit via a gate capacitor with capacitance  $C_g$ .



We will assume:

$$[\hat{\phi}, \hat{n}] = i \quad (24)$$

so:

$$\hat{n} = -i \frac{\partial}{\partial \phi} \quad (25)$$

**Challenge 3a:** Use a numerical eigensolver to solve for the spectrum of Equation 23. For  $E_J = E_C$ , plot the first three eigenvalues vs.  $n_g$ . Now make the same plot but for  $E_J = 10E_C$ . The first example is the energy spectrum of a “Cooper Pair Box” charge qubit, and the second example is that of a charge insensitive Cooper Pair Box, known as the “transmon”. Why is the transmon “charge insensitive”?

**Challenge 3b:** On the last two pages of Clarke et.al [1], they discuss how an anharmonic energy spectrum (non-uniform) of a Josephson circuit is another way they can show macroscopic quantum behavior. Why? How is this case different than the LC circuit, and why does it take us out of the correspondence limit?

**Challenge 3c:** Why may an anharmonic energy spectrum of a system be useful for quantum computing applications?

## Bonus

Below are Bonus questions that will only be used in case of tiebreakers. They are generally listed in order of increasing difficulty.

**Bonus 1:** I claimed in Part 2 that the “tilted washboard” potential in Equation 16 does not have a local minima if  $I > I_0$ . Prove this.

**Bonus 2:** Why are the discretized matrices you used in your numerical eigensolvers Hermitian?

**Bonus 3:** Algebraically show that LC circuit Hamiltonian has a uniform energy spectrum.

**Bonus 4:** Derive Equation 23. Explain exactly what the values in the Hamiltonian, namely, your conjugate observables, represent physically. Then, find expressions for  $E_J$ ,  $E_C$ , and  $n_g$ .

**Bonus 5:** Clarke et.al [1] make a distinction between “macroscopic quantum phenomena originating in the superposition of a large number of microscopic variables” and “those displayed by a single macroscopic degree of freedom.” Explain in your own words what this means. What example do they give in the paper of the former? The latter? Why is each of those examples classified as such? You must be able to explain the underlying physics of each of these examples.

## Scoring

The challenges will be scored as follows:

**1a:** 5pts

**1b:** 5pts

**1c:** 10pts

**1d:** 15pts

**1e:** 5pts

**2a:** 10pts

**2b:** 50pts

**2c:** 5pts

**3a:** 15pts

**3b:** 5pts

**3c:** 5pts

Total is out of 130. We will determine score for each question based on both written submission and the understanding displayed during your presentation (to determine how well you understood what you were doing).

Bonus Questions are only used for tiebreakers. Therefore, do NOT prioritize bonus questions over the actual challenge. We will go off of how many bonuses you got correct.

## References

- [1] John Clarke, Andrew N. Cleland, Michel H. Devoret, Daniel Esteve, and John M. Martinis. Quantum mechanics of a macroscopic variable: the phase difference of a josephson junction. *Science*, 239:992–997, 1988.
- [2] Michel H. Devoret, John M. Martinis, and John Clarke. Measurements of macroscopic quantum tunneling out of the zero-voltage state of a current-biased josephson junction. *Phys. Rev. Lett.*, 55:1908–1911, Oct 1985.
- [3] John M. Martinis, Michel H. Devoret, and John Clarke. Energy-level quantization in the zero-voltage state of a current-biased josephson junction. *Phys. Rev. Lett.*, 55:1543–1546, Oct 1985.
- [4] The Royal Swedish Academy of Sciences. Press release: The nobel prize in physics 2025. <https://www.nobelprize.org/prizes/physics/2025/press-release/>, 10 2025. Accessed on 2025-11-11.