Foundations of Financial Economics
Two DSGE: introduction

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March 27, 2020

Topics

Two period General equilibrium pricing of intertemporal contracts: setting up a model

- Economic environment: information tree, real part of the economy
- Market environment: available contracts
- Models: Arrow-Debreu economy and Finance (or Radner) economy

Environments and general equilibrium

Common assumptions: regarding the economic environment

1. the time-information structure;
2. the real part of the economy, in particular regarding intertemporal preferences.

Different assumptions regarding the market environment

1. simultaneous markets' opening;
2. sequential markets' opening;

Lead to **different definitions of GE** (general equilibrium) (that may be **equivalent or not**)

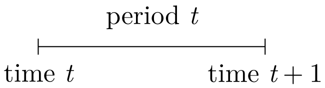
The time-information tree

This refers

- to the moments in which markets open
- to the timing of the decisions

In discrete time we have to distinguish between

- time: the timing for stocks and prices of stocks
- periods: the timing for flows and prices of flows



The time-information tree: cont

We assume:

- $t \in \mathbb{T} = \{0, 1\}$
- information changes along time, from the perspective of $t = 0$.

Most variables are 2-period random sequences

$$X = \{X_0, X_1\}$$

are determined on the basis of the information known at time $t = 0$:

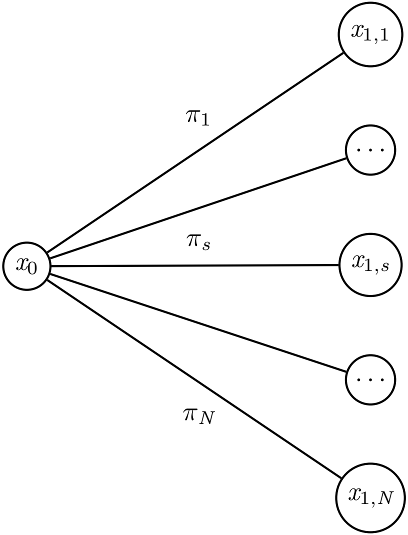
- at the end of period $t = 0$, they are observed

$$X_0 = x_0$$

- at the end of period $t = 1$, they are contingent on the information available at the end of period $t = 0$

$$X_1(\omega),\ \omega \in (\Omega, \mathcal{F}, \mathbb{P})$$

The time-information tree



Economic environment

The time-information tree

- If Ω is discrete, at $t = 1$ we have

$$X_1 = (x_{1,1}, \ldots, x_{1,s}, \ldots, x_{1,N})^\top$$

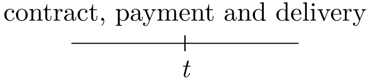
- and the sequences of outcomes and probabilities are

$$\begin{array}{ccc} x_0, 1 & & \begin{pmatrix} x_{1,1} \\ \ldots \\ x_{1,s} \\ \ldots \\ x_{1,N} \end{pmatrix}, \begin{pmatrix} \pi_1 \\ \ldots \\ \pi_s \\ \ldots \\ \pi_N \end{pmatrix} \\ \vdash & \text{———————} & \dashv \\ 0 & & 1 \end{array}$$

Timing of contracts

We distinguish:

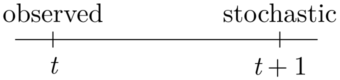
- **spot** contracts: contract, delivery and payment done in the same period



- **intertemporal or forward** contracts: contract and payment in one period, delivery in a future period

They differ along two dimensions:

- the **timing** (which may be relevant if there is , v.g., impatience, depreciation)
- the **information** set associated to the several actions (and prices) involved



The real part of the economy

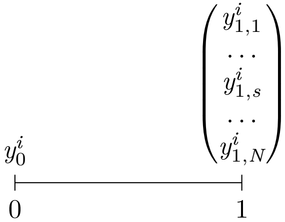
Refers to:

- the way resources are obtained, or the technology in the economy:
  - **exchange** economies: the availability of the resources is independent of decisions throughout time,
  - **production** economies: availability of resources is dependent on decisions in previous periods
- the structure of preferences: in this case preferences over 2-period random sequences
- agents distribution : homogenous or heterogenous regarding
  - endowments or technology
  - preferences
  - information

The real part of the economy

Technology

- The resource for agent i of the economy is a process $Y^i = Y^i = \{y_0^i,\, Y_1^i\}$ where $y_{t,s}^i$ is the endowment of agent i at time t for the state of nature s;



- in an exchange economy

$$Y_1^i \text{ independent of } Y_0^i$$

- in a production economy

$$Y_1^i = Y_1^i(y_0^i) \text{ dependent on } y_0^i$$

The real part of the economy

Preferences

- The consumption $C^i = \{c^i_0, C^i_1\}$ is the consumption flow for agent i

$$\begin{array}{ccc} c^i_0 & & \begin{pmatrix} c^i_{1,1} \\ \dots \\ c^i_{1,s} \\ \dots \\ c^i_{1,N} \end{pmatrix} \\ 0 & & 1 \end{array}$$

- is evaluated by an intertemporal utility functional

$$U^i(C^i) = U^i\big(c^i_0, C^i_1\big)$$

The real part of the economy

Benchmark preferences

- The most common utility functional is the discounted time-additive von-Neumman Morgenstern functional

$$U(C^i) = u^i(c_0^i) + \beta^i \mathbb{E}^i[u^i(C_1^i)] = u^i(c_0^i) + \beta^i \sum_{s=1}^{N} \pi_s^i u^i(c_{1,s}^i)]$$

  where $0 \leq \pi_s \leq 1$ and $\sum_{s=1}^{N} \pi_s^i = 1$;
- or, equivalently

$$U(C^i) = \mathbb{E}_0^i \left[\sum_{t=0}^{t=1} (\beta^i)^t u^i(c_{t,s}^i) \right]$$

- Observations
  - the utility functional $U(.)$ is doubly additive: linear as regards both time and the states of nature;
  - probabilities may be objective or subjective
  - particular relationship between the intertemporal and the risk aversion properties

The real part of the economy

Benchmark preferences

- Write is as $U(c_0, C_1) = u(c_0) + \beta \sum_s \pi_s u(c_{1,s})$
- The intertemporal marginal rate of substitution is a rendom variable

$$IMRS_{0,1_s} = \frac{\partial_{c_0} U}{\partial_{c_{1,s}} U} = \frac{u'(c_0)}{\beta\, \pi_s\, u'(c_{1,s})},$$

- The Hicks-Allen elasticities are

$$\varepsilon_{0,0} = -\frac{u''(c_0)}{u'(c_0)}\, c_0,\ \varepsilon_{1_s,1_s} = -\frac{u''(c_{1,s})}{u'(c_{1,s})}\, c_{1,s},\ s = 1, \dots N$$

- The elasticity of intertemporal substitution is also a random variable

$$IES_{0,1}(s) = \frac{c_0\, u'(c_0) + \beta\pi_s u'(c_{1,s})}{\beta\pi_s u'(c_{1,s})\, c_{1,s}\, \varepsilon_{0,0} + c_0\, u'(c_0)\, \varepsilon_{1_s,1_s}}$$

(because of the separability between c_0 and C_1)

The real part of the economy

Benchmark preferences

- If we assume that CRRA

$$CRRA = -\frac{u^{''}(c_{1,s})}{u^{'}(c_{1,s})}\, c_{1,s}$$

  is constant and equal to $\varrho > 0$, then

$$\varrho = \varepsilon_{0,0} = \varepsilon_{1_s,1_s}, \text{ for every } s = 1, \ldots, N$$

- Then

$$IES_{0,1}(s) = \frac{c_0 u^{'}(c_0) + \beta\pi_s u^{'}(c_{1,s})}{\beta\pi_s u^{'}(c_{1,s}) c_{1,s}\, \varrho + c_0 u^{'}(c_0)\, \varrho} = \frac{1}{\varrho}$$

  the elasticity of intertemporal substitution is state independent and is equal to the inverse of the coefficient of relative risk aversion

- This means that the intertemporal and the stochastic properties of preferences cannot be distinguished.

The real part of the economy

Epstein-Zin preferences

- In order to distinguish between the intertemporal preferences and the risk aversion a utility function which is becoming in macroeconomics is the Epsten-Zin utility functional (usually applied to infinite horizon models
- A two period version of the model can be the follwoing
- Let $U(c_0, C_1)$ be the intertemporal and $V(c_0, C_1) = u^{-1}\big(U(c_0, C_1)\big)$ for

$$v(c_0, C_1) = (1-\beta)u(c_0) + \beta u(c_1^c)$$

where c_1^c is the certainty equivalent of consumption at period $t = 1$:

  - intertemporal preferences are represented by $u(c)$, which is increasing and concave $u^{''}(c) < 0 < u^{'}(c)$
  - choice over uncertainty is represented by

$$v(c_1^c) = \mathbb{E}[v(C_1)]$$

is a utility function displaying risk aversion

The real part of the economy

Epstein-Zin preferences

- Therefore

$$v(c_0, C_1) = (1-\beta)u(c_0) + \beta u\left(v^{-1}\left(\mathbb{E}[v(C_1)]\right)\right)$$

- A popular version of the model assumes:
  - A generalized logarithm utility (also called iso-elastic)

$$u(c) = \frac{c^{1-\zeta} - 1}{1-\zeta}$$

  - A CRRA function

$$v(c) = \frac{c^{1-\varrho} - 1}{1-\varrho}$$

- It can be proved that, if $\zeta = \varrho$ this model reduces to the benchmark case (prove this)

The real part of the economy

Distribution

- The idiosyncratic components defining a consumer are:
  - endowments (Y^i)
  - preferences (β^i, u^i)
  - information $\mathbb{P}^i$ (only make sense with subjective probabilities)
- Agents may be homogeneous or heterogeneous regarding one or all of the previous variables and parameters

  in a homogeneous, or representative agent economy: endowments, preferences and information are equal, i.e, $Y^1 = Y^I = Y$, etc

  in a heterogeneous economy: agents can differ in one of the three dimensions: endowments ($Y^i \neq Y^j$), preferences ($\beta^i \neq \beta^j$ or $u^i(.) \neq u^j(.)$), or information ($\mathbb{P}^i \neq \mathbb{P}^j$)

The market setting

Autarky versus trade economies

The economies are distinguished by the exchanges that agents can make.

- In autarky we will have

$$c^i_{t,s} = y^i_{t,s}, \; t = 0, 1, \; s = 1, \dots, N$$

$$c^i_0 = y^i_0 \qquad \begin{pmatrix} c^i_{1,1} \\ \dots \\ c^i_{1,s} \\ \dots \\ c^i_{1,N} \end{pmatrix} = \begin{pmatrix} y^i_{1,1} \\ \dots \\ y^i_{1,s} \\ \dots \\ y^i_{1,N} \end{pmatrix}$$

0 1

The market setting

Autarky versus trade economies

- If there are markets for intertemporal transfers of contingent goods, agents can trade and be able to make

$$c^i_{t,s} \neq y^i_{t,s},\ t = 0, 1,\ s = 1, \ldots, N$$

by shifting resources across **time** and **states of nature**.

$$\begin{pmatrix} c^i_{1,1} \\ \ldots \\ c^i_{1,s} \\ \ldots \\ c^i_{1,N} \end{pmatrix} \neq \begin{pmatrix} y^i_{1,1} \\ \ldots \\ y^i_{1,s} \\ \ldots \\ y^i_{1,N} \end{pmatrix}$$

$$c^i_0 \neq y^i_0$$

0   1

The market setting

Real versus financial markets

We distinguish further:

- **real markets**: market for goods, which can be spot or forward
- **financial market**: market on financial instruments, which are always forward

Types of markets and GE

We consider next two economies which are distinguished by the type of intertemporal contracts available:

- **Arrow Debreu economies**: there are AD contingent goods $\Rightarrow$ there is simultaneous market equilibrium
- **finance economies**: Radner economies in which financial assets are traded $\Rightarrow$ there is sequential market equilibrium

  They can be **equivalent under some conditions**, i.e., have the same equilibrium allocations