Growth and natural resources

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Motivation

- Natural resources can be classified in several different ways,
- The one that interest us here is between renewable and non-renewable
- The non-renewable resource has a similar role as land in the Malthusian model: as it is non-reproducible it tends to force the economy to converge to a steady state. Therefore to no-growth
- However, renewable resources have a reproduction mechanism that can in part allow for growth
- The economy can perturb the natural dynamics of nature: negatively by pollution and other means and positively by ”green” policies (technical progress, de-materialization, abbatment policies, etc) .
- In the long run, maybe, the interaction with nature is the main issue for growth economies.
- We present a simple model next.

Assumptions

- the natural resource is (at least partly, renewable;
- production uses a natural resource as the only input;
- but use in production depletes its stock;
- technical progress takes the form of de-materialization;
- the natural resource has an amenity value for the consumer;

Conclusions

- the feasible growth rate is limited by the rate of technical progress and the sum of the rate of techincal progress and the rate of regeneration of the natural resource
- then growth with positive growth rates is sustainable

The structure of the economy

Product market

- production function

$$Y(t) = A(t)P(t)$$

  A is TFP and P is resource depletion
- (exogenous) technical progress

$$A(t) = A(0)e^{\gamma_A t}$$

- equilibrium in the product market

$$Y(t) = C(t)$$

- **then**

$$P(t) = \frac{C(t)}{A(0)}e^{-\gamma_A t}$$

technical progress involves dematerialization

Natrural resource dynamics

- natural resource accumulation equation

$$\dot{N} = \mu N(t) - P(t),\ \mu > 0$$

  where $N(0) = N_0$ is given
- **then**

$$\dot{N} = \mu N(t) - \frac{C(t)}{A(0)} e^{-\gamma_A t}$$

Detrending

- we assume that consumption and the stock of natural resources can be written as

$$C(t) = c(t)e^{\gamma t},\ N(t) = n(t)e^{\gamma_n t}$$

- **then** from the resource accumulation equation is

$$\dot{n} = (\mu - \gamma_n)n(t) - A(0)^{-1}c(t)e^{(\gamma - \gamma_A - \gamma_n)t}$$

- to get the autonomous ODE

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}c$$

we make $\gamma = \gamma_A + \gamma_n$

Consumers' preferecences

- the instantaneous utility function is

$$u(C, N) = \frac{(CN^{\varphi})^{1-\sigma}}{1-\sigma}$$

- observtions: φ parameterizes the amenity services provided by natural resources; observe that the utility function is homogenous of degree $(1-\sigma)(1+\varphi)$
- in detrended variables, we get

$$u(C, N) = e^{\gamma_u t} u(c, n) \equiv e^{\gamma_u t} \frac{(cn^{\varphi})^{1-\sigma}}{1-\sigma}$$

  where the rate of growth of utility is

$$\gamma_u = (1-\sigma)[\gamma_A + (1+\varphi)\gamma_n]$$

The problem

- Planner's problem written in detrended variables (c, n)

$$\max_{(c(t))_{t\in[0\infty)}} \int_0^\infty \frac{(c(t)n(t)^\varphi)^{1-\sigma}}{1-\sigma} e^{-\rho^* t}$$

  where $\rho^* \equiv \rho - \gamma_u = \rho - (1-\sigma)[\gamma_A + (1+\varphi)\gamma_n]$ and γ_n is unknown, subject to

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}c$$

  given $N(0) = N_0$ and asymptotically $\lim_{t\to\infty} N(t) \geq 0$.

- **assumption**

$$(1-\sigma)(1+\varphi)\mu < \rho - (1-\sigma)\gamma_A < (1+\varphi)\mu \qquad \text{(A)}$$

- this guarantees sustainability and positive growth

Optimality conditions

- optimal consumption

$$A(0)\, c(t)^{-\sigma}\, n(t)^{\varphi(1-\sigma)} = q(t) \tag{1}$$

- Euler equation

$$\dot{q} = q(t)(\rho^* - \mu + \gamma_n) - \varphi\, c(t)^{1-\sigma}\, n^{\varphi(1-\sigma)-1}$$

- Substituting (1) the MHDS is

$$\begin{cases} \dot{n} = (\mu - \gamma_n)n - A(0)^{-1}\, c & \\ \dot{q} = q(t)\left(\rho^* - \mu + \gamma_n - \dfrac{\varphi}{A(0)}\dfrac{c}{n}\right) & \\ \lim_{t\to\infty} q(t)n(t)e^{-\rho^* t} = 0 & \text{transversality condition} \\ n(0) = N_0 & \text{given} \end{cases}$$

The MHDS

- Taking log-derivative to (1) we get

$$-\sigma\frac{\dot{c}}{c}+\varphi(1-\sigma)\frac{\dot{n}}{n}=\frac{\dot{q}}{q}$$

- we can get the MHDS for (c, n)

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{\mu(1+\varphi(1-\sigma))+(1-\sigma)\gamma_A-\sigma\gamma_n-\rho}{\sigma}+\varphi\frac{c}{A(0)n}\\ \frac{\dot{n}}{n} &= \mu-\gamma_n-\frac{c}{A(0)n}\end{aligned}$$

Long-run rate of growth

- solving $\frac{\dot{c}}{c} = 0$ and $\frac{\dot{n}}{n} = 0$ for γ_n and c/n, we get:
- the long-run rate of growth

$$\bar{\gamma}_n = \frac{(1+\varphi)\mu + (1-\sigma)\gamma_A - \rho}{\sigma(1+\varphi)}$$

- and the long-run consumption-resources ratio

$$\frac{\bar{c}}{\bar{n}} = A(0)(\mu - \bar{\gamma}_n)$$

- as $\bar{y} = \bar{c}$ from the product market equilibrium condition then

$$\bar{y} = A(0)(\mu - \bar{\gamma}_n)\bar{n}$$

Long-run rate of growth

- **Proposition**: if assumption (A) holds then
$$0 < \bar{\gamma}_n < \mu$$
- **Intuition: the (economic) rate of growth of the natural resource is limited by the natural reproduction rate but cen still be positive.**
- Proof:
- $\bar{\gamma}_n > 0$ if and only if
$$(1+\varphi)\mu > \rho - (1-\sigma)\gamma_A$$
- $\bar{\gamma}_n < \mu$ if and only if
$$(1+\varphi)\mu + (1-\sigma)\gamma_A < \rho + \mu\sigma(1+\varphi)$$
which is equivalent to
$$\rho - (1-\sigma)\gamma_A > (1-\sigma)(1+\varphi)\mu$$

Transitional dynamics

- Defining $z(t) \equiv A(0)\dfrac{c(t)}{n(t)}$ and substituting $\gamma_n = \bar{\gamma}_n$ into the MHDS we get
$$\frac{\dot{z}}{z} = (1+\varphi)(z - \bar{z})$$
  where $\bar{z} = \mu - \bar{\gamma}_n$
- as the equation is unstable, the transversality condition only holds if $z(t) = \bar{z}$ for $t \in [0, \infty)$
- as $\bar{z} = z(0)$ we set
$$\bar{c} = \bar{y} = A(0)(\mu - \gamma_n)N_0$$

Growth facts

The long run growth rate is

$$\bar{\gamma} = \gamma_A + \bar{\gamma}_n$$

1. **Intuition: if the assumption on the parameters holds then the growth rate is limited by the natural renewal rate and the growth in dematerialization**

$$\gamma_A < \gamma < \gamma_A + \mu$$

2. the long run GDP level is

$$\bar{y} = \bar{c} = A(0)(\mu - \bar{\gamma}_n)N_0 = A(0)N_0 \frac{(\sigma - 1)((1+\varphi)\mu + \gamma_A) + \rho}{\sigma(1+\varphi)}$$

3. there is no transitional dynamics