

# Advanced macroeconomics 2020-2021

## Problem set 1: optimal taxation

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### 1 Ramsey: general

- 1 Discuss the effects of the decreasing marginal returns to capital and of the intertemporal substitution in consumption on the dynamics of the Ramsey model.
- 2\* Discuss the effects of effects of anticipated permanent productivity shocks on the optimal path of the economy, according to the Ramsey model
- 3\* Discuss the effects of effects of non-anticipated temporary productivity shocks on the optimal path of the economy, according to the Ramsey model

### 2 Ramsey

- 1 Consider a Ramsey model in which there is depreciation of capital, that is  $\dot{k} = Ak^\alpha - c - \delta k$  where  $\delta > 0$  and the utility function is  $u(c) = \log(c)$  and the production function is Cobb-Douglas  $y = f(k) \equiv Ak^\alpha$ , where all the variables are in per-capita terms.
  - (a) Solve the Ramsey problem by using the PMP
  - (b) Draw the phase diagram
  - (c) In this model there is a manifold passing through two two steady states one such that both  $k$  and  $c$  are positive and another one in which  $c = 0$  and  $f(k) = \delta k$  for  $k > 0$ . Explain why any trajectory over this manifold cannot be optimal.
  - (d) Perform a comparative dynamics exercise for an increase in  $\delta$ . Provide one intuition for your results.
- 2 Consider a Ramsey model in which there is an unfunded government expenditure  $\dot{k} = Ak^\alpha - c - g - \delta k$  where  $g > 0$  is a public transfer and  $\delta > 0$  and the utility function is isoelastic  $u(c) = \frac{c^{1-\theta} - 1}{1-\theta}$ .

- (a) Solve the Ramsey problem by using the PMP.
- (b) Is it possible to solve explicitly the Ramsey using the DPP ?
- (c) Perform a comparative dynamics exercise for an increase in  $g$ . Provide one intuition for your results.

**3\*** Consider a Ramsey model with endogenous labour with additively separable preferences and Cobb-Douglas technology. That is

$$u(c, \ell) = \frac{c^{1-\theta} - 1}{1-\theta} - \psi \frac{\ell^{1+\zeta}}{1+\zeta}, \quad \theta > 0, \psi > 0, \zeta > 0$$

and

$$f(k, \ell) = A k^\alpha \ell^{1-\ell}$$

- (a) Write the MHDS
- (b) Build the phase diagram
- (c) Study the effects of a non-anticipated, permanent and constant shocks in  $A$ ,  $\psi$  and  $\rho$ . Provide one intuition for your results.

**4\*** Consider a Ramsey model with endogenous labour with KPR preferences and Cobb-Douglas technology. That is

$$u(c, \ell) = \frac{\left( c (1 - \psi \ell^\eta) \right)^{1-\theta} - 1}{1-\theta}, \quad \theta > 0, \psi > 0, \eta > 0$$

and

$$f(k, \ell) = A k^\alpha \ell^{1-\ell}$$

- (a) Write the MHDS
- (b) Build the phase diagram
- (c) Study the effects of a non-anticipated, permanent and constant shocks in  $A$ ,  $\psi$  and  $\rho$ . Provide one intuition for your results.

**5\*** Consider a Ramsey model with endogenous labour with GHH preferences and Cobb-Douglas technology. That is

$$u(c, \ell) = \frac{1}{1-\theta} \left( \left( c - \psi \frac{\ell^{1+\zeta}}{1+\zeta} \right)^{1-\theta} - 1 \right), \quad \theta > 0, \psi > 0, \zeta > 0$$

and

$$f(k, \ell) = A k^\alpha \ell^{1-\ell}$$

- (a) Write the MHDS
- (b) Build the phase diagram
- (c) Study the effects of a non-anticipated, permanent and constant shocks in  $A$ ,  $\psi$  and  $\rho$ . Provide one intuition for your results.

### 3 DGE

- 1 Consider a DGE economy in which the utility function is  $u(c) = \log(c)$ , there is a constant number of households  $N$ , there is no unemployment, and the technology for firms is CES

$$Y = A \left( \alpha K^\eta + (1 - \alpha) L^\eta \right)^{\frac{1}{\eta}}$$

- (a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.
  - (b) Build the phase diagram.
  - (c) Study the effects of a non-anticipated, permanent and constant shocks in  $A$ . Provide one intuition for your results.
- 2 Consider a DGE economy in which the utility function is  $u(c) = \log(c)$  and the production function is Cobb-Douglas in which the government raises an income tax and has a budget balanced fiscal policy. Denoting per capita government expenditure by  $g$  and the tax rate is denoted by  $\tau$  and both are constant through time. The budget balance rule is  $g = \tau (r(t)a(t) + w(t))$ . Assume that households supply labor inelastically and they have the budget constraint  $\dot{a} = (1 - \tau) (r(t)a(t) + w(t)) - c(t) + g(t)$ .
- (a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.
  - (b) Build the phase diagram.
  - (c) Study the effects of a non-anticipated, permanent and constant increase in  $g$ . Provide one intuition for your results.