

The Romer model: externality model of growth

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The Romer kK model

- ▶ It has the same assumptions as the AK model with the exception of the technology
- ▶ There are externalities in production

$$Y = AK^\alpha \mathbf{K}^\beta$$

where K is capital owned by firms and \mathbf{K} is an externality

- ▶ Assumption: there are decreasing returns at the private level but there are constant returns at the aggregate level
 $0 < \alpha < 1$ and $\alpha + \beta = 1$
- ▶ This introduces a distinction between
 - ▶ decentralized economy
 - ▶ centralized economy

Decentralized economy

Representative agent's problem

- ▶ the representative agent problem

$$\max_{[C(t)]_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK^{\alpha} \mathbf{K}^{\beta} - C - \delta K,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$

Decentralized economy

Macroeconomic equilibrium

- first order optimality conditions for the agent

$$\dot{C} = \frac{C}{\theta} \left(\rho + \delta - \alpha A K^{\alpha-1} \mathbf{K}^{\beta} \right), \quad (1)$$

$$\dot{K} = A K^{\alpha} \mathbf{K}^{\beta} - C - \delta K, \quad (2)$$

$$K(0) = k(0) \quad (3)$$

$$0 = \lim_{t \rightarrow +\infty} C(t)^{-\theta} K(t) e^{-\rho t} \quad (4)$$

- macroeconomic consistency condition

$$K(t) = \mathbf{K}(t).$$

Decentralized economy

Macroeconomic equilibrium

- ▶ If we assume that $\alpha + \beta = 1$ we get

$$\dot{C} = \frac{C}{\theta} (\rho + \delta - \alpha A),$$

$$\dot{K} = (A - \delta)K - C$$

$$K(0) = k(0)$$

$$0 = \lim_{t \rightarrow +\infty} C(t)^{-\theta} K(t) e^{-\rho t}$$

- ▶ which is very similar to the AK model with the exception of the rate of return for capital

Decentralized economy

Growth facts

- ▶ The rate of growth is

$$\bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- ▶ the consumption-capital ratio is

$$\frac{\bar{c}}{\bar{k}} = \beta_d,$$

for

$$\beta_d \equiv A - \delta - \bar{\gamma}_d = \frac{1}{\theta} (A(\theta - \alpha) + \rho + \delta(1 - \theta)) > 0$$

- ▶ the solution for output is

$$Y(t) = Ak_0 e^{\gamma_t t}$$

Centralized economy

Planner's problem

- ▶ The problem: the planner has the same utility function as the agent (we have a representative agent economy) but it considers the total capital, and not only the privately owned as the individual agent
- ▶ that is, externalities are internalized. This means that the production function is

$$Y = AK^\alpha K^\beta = AK, \text{ if } \alpha + \beta = 1$$

Centralized economy

Planner's problem

- ▶ Formally: the planner's problem is

$$\max_{[C(t)]_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK - C - \delta K,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$
- ▶ It is indeed the *AK*-model

Comparison between the decentralized and centralized economies

- ▶ the rate of growth in the centralized version is higher than in the decentralised version

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > \bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- ▶ but consumption is higher in the decentralised version

$$\left(\frac{C}{Y}\right)_d = \frac{A - \delta - \bar{\gamma}_d}{A} > \left(\frac{C}{Y}\right)_c = \frac{A - \delta - \bar{\gamma}_c}{A}$$

- ▶ rationale: in the decentralised economy the incentive to save is smaller because the private rate of return is also smaller

Decentralized economy with government intervention

Government intervention

- ▶ there are two policy instruments: one distortionary (say taxes) and another non-distortionary (say expenditures)

$$T(t) = \tau Y(t), \quad G = \bar{G}(t)$$

- ▶ two possibilities: distortionary taxes and lump sum transfers ($\tau > 0$, $G > 0$) or distortionary subsidies and lump sum taxes ($\tau < 0$, $G < 0$)
- ▶ assume the government budget is balanced

$$T(t) = G(t), \quad \forall t \in [0, \infty)$$

Decentralized economy with government intervention

Representative agent's problem

- ▶ the representative agent problem

$$\max_{[C(t)]_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = (1 - \tau)AK^{\alpha}\mathbf{K}^{\beta} - C - \delta K + G,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$

Decentralized economy with government intervention

Growth facts

- ▶ Using the same approach as before, the rate of growth is

$$\bar{\gamma}_g = \frac{\alpha(1 - \tau)A - \delta - \rho}{\theta} > 0$$

- ▶ can the rate of growth of the centralized economy be reached ?

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > 0$$

- ▶ **yes** if the tax rate is

$$\alpha(1 - \tau) = 1 \Rightarrow \tau = -\frac{1 - \alpha}{\alpha} < 0$$

- ▶ **Conclusion:** the government intervention can introduce incentives such that the externality is internalised by agents, in this case, by a distortionary subsidy and a lump-sum tax.

References

- ▶ Romer (1990)
- ▶ (Acemoglu, 2009, ch. 11), (Aghion and Howitt, 2009, ch. 2) , (Barro and Sala-i-Martin, 2004, ch. 4)

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