Growth, taxes and public debt

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Assumptions

- production of goods involves an externality associated to the services provided by the state;
- services provided by the public sector are a function of the government expenditures;
- the government has to finance expenditures by taxes and/or debt, constrained by the government budget constraint (GBC)
- we assume that the government uses a rule of keeping the debt over the GDP ratio (B/Y) constant
- there are two extreme financing strategies: tax finance (debt ratio equal to zero), debt finance (tax rate equal to zero)

Structure of the model

- if the tax is seen as an instrument, then taxes will distort the incentives for capital accumulation
- this implies the model has externalities that are not internalized
- if the GBC does not need to be permanently balanced, this changes the rate of growth of the economy

Conclusions

- government expenditures have a positive effect on the growth rate
- however, the type of financing determines the long run level of the ratio G/Y
- tax financing has an ambiguous effect on the growth rate (it increases the long run interest before taxes but the effect on the net rate is ambiguous)
- debt financing, such that B/Y is constant, can have a negative effect on growth because the implied sustainability requires that the long-run interest rate be reduced

The model

Private sector

- chooses $(C(t))_{t\geq 0}$ to maximize

$$\int_0^\infty \frac{C(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

  C is consumption

- subject to the budget constraint

$$\dot{K} + \dot{B} = (1-\tau)\left(r(t)B(t) + Y(t)\right) - C(t)$$

  B government bonds, K private capital, τ tax rate

- non-Ponzi game condition

$$\lim_{t\to\infty} (K(t) + B(t))\, e^{\int_0^t r(s)\, ds} \geq 0.$$

- Then the Euler equation is

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(t) - \rho}{\sigma}$$

The model

Production and arbitrage in financial markets

- Production technology

$$Y(t) = K(t)^{\alpha} G(t)^{1-\alpha}, \ 0 < \alpha < 1$$

  G government expendititures

- arbitrage condition between capital and government debt

$$r(t) = \alpha K(t)^{\alpha-1} G(t)^{1-\alpha}.$$

The model

The government

- GBC: government budget constraint

$$\dot{B} = r(t)B(t) + G(t) - \tau\left(r(t)B(t) + Y(t)\right)$$

- policy rule

$$B(t) = \bar{b}\,Y(t)$$

  where b is constant

The aggregate constraint

- Consolidating the private and government constraints we get
$$\dot{K} = Y(t) - C(t) - G(t)$$

The DGE system

- the Euler equation

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(K,G) - \rho}{\sigma}$$

- the aggregate budget constraint

$$\dot{K} = Y(K,G) - C - G$$

- the GBC

$$\dot{B} = r(K,G)B + G - \tau\left(r(K,G)B + Y(K,G)\right)$$

- the policy rule

$$B = \bar{b}Y$$

The transformed DGE

The transformed Euler equation

- Defining $g \equiv \dfrac{G}{Y}$ then

$$Y = A(g)K, \text{ for } A(g) \equiv g^{\frac{1-\alpha}{\alpha}}, \; A^{'} > 0$$

  and

$$r(g) = \alpha A(g),$$

  is a positive function of g $(r^{'} > 0)$

- Then the Euler equation becomes

$$\frac{\dot{C}}{C} = \gamma(g) \equiv \frac{(1-\tau)r(g) - \rho}{\sigma}$$

  where $\gamma(g)$ is the rate of growth of consumption

The transformed DGE

The transformed aggregate constraint

- Then the rate of growth of GDP is

$$\gamma_Y = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g}$$

- Defining $z \equiv \dfrac{C}{K}$ the rate of growth for private capital is

$$\frac{\dot{K}}{K} = \gamma_K(g, z) \equiv (1-g)A(g) - z$$

where $\gamma_K(g)$ is the rate of growth of the capital stock

The transformed DGE

The transformed aggregate constraint

- Then the rate of growth of the consumption ratio is the difference between the rates of growth of consumption and

$$\frac{\dot{z}}{z} = \gamma_z(g, z) = \gamma(g) - \gamma_K(g, z) = z - z(g)$$

  where

$$z(g) \equiv \frac{\left(\sigma(1-g) - \alpha(1-\tau)\right) A(g) + \rho}{\sigma}$$

  can be rationalized as a **Laffer curve**: it embodies two effects:

  - a **positive** net effect of g on GDP (bigger for small values of g): $(1-g)A(g)$
  - a **negative** effect of taxes which finance g over r which decreases the rate of growth: $(1-\tau)\alpha A(g)$

The transformed DGE

The dynamics of b

- Defining the government debt over GDP ratio by $b = \dfrac{B}{Y}$ we get
$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{Y}}{Y} = \gamma_B - \gamma_Y$$

- where
$$\frac{\dot{B}}{B} = \gamma_B(g) = (1-\tau)r(g) + \frac{G - \tau Y}{B} = (1-\tau)r(g) + \frac{g-\tau}{b}$$

- Then
$$\frac{\dot{b}}{b} = (1-\tau)r(g) + \frac{g-\tau}{b} - \frac{\dot{K}}{K} - \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g}$$

The transformed DGE

The dynamics of g

- **Assumption**: introducing the policy rule $b = \bar{b}$ then $\dot{b} = 0$
- Then we get the growth rate of the G/Y ratio

$$\frac{\dot{g}}{g} = \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - \frac{\dot{K}}{K}\right) =$$
$$= \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - ((1-g)A(g) - z)\right) =$$
$$= \frac{\alpha}{1-\alpha}\left(z - \zeta(g)\right)$$

where

$$\zeta(g) \equiv \left((1-g) - \alpha(1-\tau)\right) A(g) + \frac{\tau - g}{\bar{b}}.$$

The DGE in detrended variables

- the DGE in detrended variables is

$$\begin{aligned}\dot{z} &= z\,(z - z(g))\\ \dot{g} &= g\frac{\alpha}{1-\alpha}\,(z - \zeta(g))\end{aligned}$$

where the long run C/K ratio for a constant g

$$\zeta(g) \equiv ((1-g) - \alpha(1-\tau))\,A(g) + \frac{\tau - g}{\bar{b}}.$$

and the Laffer-curve is

$$z(g) = \frac{(\sigma(1-g) - \alpha(1-\tau))\,A(g) + \rho}{\sigma}$$

- for $g \in (0,1)$ and $z > 0$

The long run level of g

- The long run level of g is a function of τ and $\bar{b}$: $\bar{g} = \bar{g}(\tau, \bar{b})$
- $\bar{g}$ is obtained from solving $\Phi(g) \equiv z(g) - \zeta(g) = 0$, where

$$\Phi(g, \tau, \bar{b}) \equiv \frac{(1-\tau)(\sigma-1)}{\sigma} r(g) + \frac{g-\tau}{\bar{b}} + \frac{\rho}{\sigma}$$

- The effect of increases of τ on $\bar{g}$ is

$$\left.\frac{\partial \bar{g}}{\partial \tau}\right|_{g=\bar{g}} = \frac{(1-\tau)r^{'}(\bar{g}) - r(\bar{g})}{\bar{b}\left[(1-\tau)(\sigma-1)r^{'}(\bar{g})\bar{b} + \sigma\right]} > 0$$

  if $(1-\tau)(1-\alpha) - \alpha\bar{g}$;
- The effect of increases of $\bar{b}$ on $\bar{g}$ is negative

$$\left.\frac{\partial \bar{g}}{\partial \bar{b}}\right|_{g=\bar{g}} = \frac{(\bar{g}-\tau)\sigma}{\bar{b}\left[(1-\tau)(\sigma-1)r^{'}(\bar{g})\bar{b} + \sigma\right]} < 0$$

  because $\Phi(g, \tau) = 0$ only if $\bar{g} < \tau$.

Long-run growth rate

- The long run growth rate is a positive function of the steady state g ($\bar{g}$):

$$\boxed{\bar{\gamma} = \gamma(\bar{g}) = \frac{(1-\tau)r(\bar{g}) - \rho}{\sigma}}$$

- then the long-run growth rate depends on the public financing policy

$$\boxed{\bar{\gamma} = \gamma(\tau, \bar{b})}$$

- We can prove that γ_τ is ambiguous and $\gamma_{\bar{b}} < 0$

Long-run growth rate

Policy instruments and the growth rate

- The effect on growth is **ambiguous**:

$$\frac{\partial \gamma}{\partial \tau} = -\frac{r(\bar{g})}{\sigma} + \frac{(1-\tau)}{\sigma} r^{'}(\bar{g}) \frac{\partial \bar{g}}{\partial \tau}$$

  - there is a negative direct effect
  - and a positive indirect effect through r if $(1-\tau)(1-\alpha) > \alpha \bar{g}$
- and

$$\frac{\partial \gamma}{\partial \bar{b}} = \frac{(1-\tau)}{\sigma} r^{'}(\bar{g}) \frac{\partial \bar{g}}{\partial \bar{b}} < 0$$

  the effect on the growth rate is negative

Effect of the tax rate on the long run growth rate

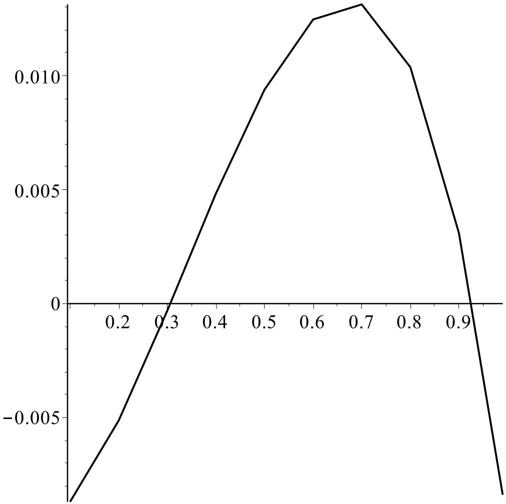

Figure: Value of γ for different levels of the tax rate τ

Transitional dynamics

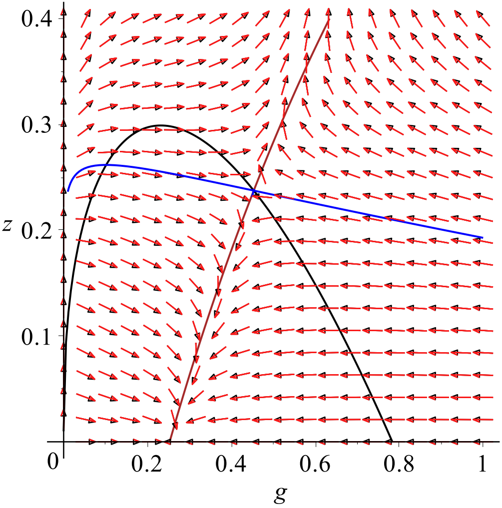

Figure: Transitional dynamics for $\sigma = 1$, $\alpha = 0.7$, $\rho = 0.02$, $\tau = 0.3$ and $\bar{b} = 0.6$.

Growth facts

Effects of policy parameters τ and $\bar{b}$

- Long-run growth rate:
  - although a higher g increases the long run growth rate
  - the need to finance it generates a countervailing effect
    Conclusion: while debt financing reduces the long run growth rate, tax finance may reduce or increase the long run growth rate, although the positive effect dominates for intermediate levels of the tax rate.
- There is transitional dynamics: $z = C/K$ and $g = G/Y$ are negatively correlated along the transition path: if $g(0)$ is too high then z increases in the transition.

References

- Barro (1990)

R. Barro. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98: S103–S125, 1990.