

R&D and growth: the Schumpeterian model

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Some empirical research on firms' demographics

- ▶ Firm entry and survival [Bartelsman an all: see page 41 and after](#)
- ▶ Firm entry and survival [see Figures 2, 3 and 7](#)

General idea

- ▶ growth depends on the growth of TFP (not only on human and physical capital accumulation);
- ▶ growth of TFP is the result of R&D activity (innovation);
- ▶ innovation is done by **entrants** in every industry which substitute previous incumbents
- ▶ innovation is non-rival and non-excludable; therefore R&D is only undertaken if, upon entry, the incumbent is monopolist in its industry;
- ▶ **creative destruction** generates growth: at the aggregate level, there is an increase in quality, and, therefore, productivity;
- ▶ this generates, as for the expansion of varieties model, a non-Paretian decentralized general equilibrium.

Assumptions

The model has a similar structure to the expansion of variety model with the following differences

- ▶ technical progress takes the form of an improvement in the quality of products (not in their number);
- ▶ we assume that quality refers to intermediate inputs;
- ▶ **entry replaces an existing monopolist (not starting a new industry) by a new monopolist**
- ▶ the success of inventive activity is random with a probability dependent on expenditures
- ▶ if R&D is successful the entrant is monopolist for a finite but unknown time (not infinitely)

Innovations

- ▶ we distinguish the physical quantity of the input, x and the quantity in efficiency units \tilde{x}
- ▶ quality increases the productivity of an input, which is measured in efficiency units by

$$\tilde{x} = l^\nu x$$

for every unit of physical quantity of the input

- ▶ innovations take the form **quality ladders**: there is a quality index which evolves as

$$1, l, l^2, l^3, \dots, l^\nu$$

Innovations heterogeneity

- ▶ heterogeneity in the quality levels: different industries can be at different quality levels
- ▶ if ν_j is the quality level of industry j , we may have for any other industry k $\nu_k \neq \nu_j$
- ▶ therefore, at time t the quantity of the input produced by industry j is

$$\tilde{x}(j, t) = l^{\nu_j} x(j, t)$$

Survival arithmetics

- Survival function

$$S(T) = P[\textit{duration} \geq T]$$

- Hazard function

$$\lambda(T) = \lim_{dt \rightarrow 0} \frac{P[T \leq \textit{duration} \leq T + dt | \textit{duration} > T]}{dt}$$

- if λ is constant

$$S[T] = e^{-\lambda t}$$

- defining

$$f(T) = \lambda S(t) \Rightarrow f(T) = \lambda e^{-\lambda T}$$

instantaneous frequency of non-survival

Modelling innovations and R&D

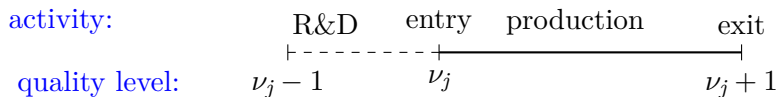
- ▶ the producer of input $x(j, t)$ is an entrant who successfully introduces a higher quality input at time $t(\nu_j)$, through creative destruction: i.e. the producer of quality l^{ν_j} displaces the incumbent which produces at level l^{ν_j-1} . Its instant probability of success is $\lambda(\nu_j - 1)$
- ▶ it becomes a monopolist, but only for a period lasting $T(\nu_j) = t(\nu_j + 1) - t(\nu_j)$
- ▶ the arrival of a new innovation, yielding a jump from the level ν_j to $\nu_j + 1$, is governed by an exponential process with density

$$\mathbb{P}[\nu_j + 1 | \nu_j] = \lambda(\nu_j) e^{-\lambda(\nu_j) T(\nu_j)}$$

which is decreasing in $T(\nu_j)$

- ▶ and the probability of arrival of a successful innovation, $\lambda(\cdot)$, is **dependent on R&D effort**

Activities in industry j and quality levels



timing: $t(\nu_j) \longleftarrow T(\nu_j) \longrightarrow t(\nu_j + 1)$

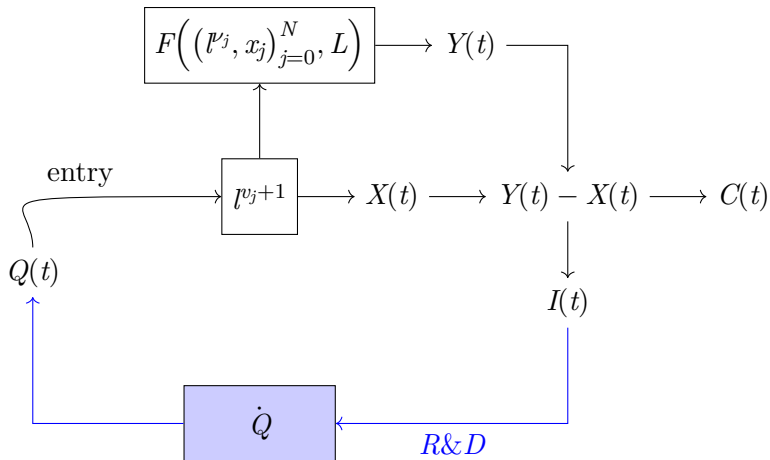
probabilities: $\mathbb{P}[\nu_j + 1 | \nu_j]$

- ▶ Pre-entry tech level $\nu_j - 1$
- ▶ Firm enters by raising it to ν_j (and becoming incumbent during an interval T_{ν_j})
- ▶ Firm is eventually evicted when a successful innovator raises it to $\nu_j + 1$

Results

- ▶ Even without capital accumulation, growth can be generated by the increase in quality of the goods, which increase aggregate productivity
- ▶ The rate of growth depends negatively on the cost of R&D, and positively in the quality jump

The mechanics of the model



Decentralized (market) economy

The consumer problem

- ▶ Earns labor and capital income, consumes a final product, save and own firms (final good and intermediate good producers)
- ▶ The problem

$$\begin{aligned} \max_{(C(t))_{t \in [0, \infty)}} \quad & \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad \theta > 0 \\ \text{s.t} \quad & \\ & \dot{W} = \omega(t)L + r(t)W(t) - C(t) \end{aligned} \quad (\text{CP})$$

- ▶ The first order conditions

$$\begin{aligned} \dot{C} &= \frac{C}{\theta} (r(t) - \rho) \\ \dot{W} &= \omega(t)L + r(t)W(t) - C(t) \end{aligned}$$

Producers of final goods

- ▶ Production function: Dixit and Stiglitz (1977)

$$Y(t) = \int_0^N Y(j, t) dj = \int_0^N AL^{1-\alpha} (\ell^j x(j, t))^\alpha dj, \quad 0 < \alpha < 1$$

- ▶ L labor input
- ▶ $(x(j, \cdot))_{j \in [0, N]}$ intermediate inputs, non-storable,
- ▶ N number of inputs (number of intermediate industries)
- ▶ ℓ^j quality ladder of industry j
- ▶ Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^N P(j, t)x(j, t) dj$$

Producers of final goods (cont)

- ▶ Buys labor and intermediate goods and sells a final good
- ▶ The problem:

$$\max_{L, (x(j,t))_{j \in [0, N]}} \pi^p(t) \quad (\text{FGPP})$$

- ▶ Competitive industry: they are price takers in all markets
- ▶ First order conditions:
 - ▶ demand for labor

$$L^d = (1 - \alpha) \frac{Y(t)}{\omega(t)}$$

- ▶ demand for intermediate goods

$$x^d(j, t) = \left(\frac{\alpha A}{P(j, t)} \right)^{\frac{1}{1-\alpha}} Lq(\nu_j), \quad j \in [0, N(t)]$$

where $q(\nu_j) = l^{\frac{\alpha}{1-\alpha}} \nu_j$ is the quality level reached by industry j

Producers of intermediate goods

Production and R&D problems

- ▶ Perform R&D activities allowing for the production of a better quality input which they sell to final producers
- ▶ Decision process for the introduction of a new variety
 - ▶ R& D and entry decision: free entry condition
 - ▶ pricing of input of variety j after entry
- ▶ Solution of the problem: in backward order
 - ▶ first: we determine the pricing policy assuming there is entry
 - ▶ second: we determine entry (by using the free entry condition)

Producers of intermediate goods (cont)

Price decision if there is entry (incumbents' problem)

- ▶ The problem for the producer of a variety j is

$$\max_{P(j,t)} \pi(j,t) = (P(j,t) - 1)x(j,t) \quad (\text{IGPP}_j)$$

Assumption: symmetric cost of production equal to 1

- ▶ where $x(j,t) = x^d(j,t)$ (solution of the FGPP)
- ▶ then

$$\pi(j,t) = (P(j,t) - 1) \left(\frac{\alpha A}{P(j,t)} \right)^{\frac{1}{1-\alpha}} Lq(\nu_j),$$

Producers of intermediate goods (cont)

Price decision if there is entry

- first order conditions (markup = $1/\alpha$)

$$P^*(j, t) = \frac{1}{\alpha} \forall (j, t)$$

- then

$$x^*(j, t) = x^*(\nu_j) = (\alpha^2 A)^{\frac{1}{1-\alpha}} Lq(\nu_j)$$

is stationary (time independent) but **not symmetric**

- Then the profit is also **not symmetric**

$$\begin{aligned} \pi^*(j, t) &= \pi^*(\nu_j) = \left(\frac{1-\alpha}{\alpha} \right) (A\alpha^2)^{\frac{1}{1-\alpha}} Lq(\nu_j) \\ &= \frac{1-\alpha}{\alpha} x^*(\nu_j) \end{aligned}$$

- The incorporation in the final good is also **not symmetric**

$$y(j, t) = A L^{1-\alpha} x^*(j, t) = \frac{x^*(j, t)}{\alpha^2}$$

Producers of intermediate goods (cont)

Value of entry dependent upon duration

- ▶ The producer of a successful variety j , enters at time t_{ν_j} (when the technological level is ν_j) and becomes a monopolist in the interval $[t_{\nu_j}, t_{\nu_j} + T_{\nu_j}]$ where T_{ν_j} is stochastic
- ▶ the value entry is, while still on business (assuming r is constant)

$$\begin{aligned} v^*(\nu_j, T_{\nu_j}) &= \int_{t_{\nu_j}}^{t_{\nu_j} + T_{\nu_j}} \pi^*(\nu_j) e^{-r(t - t_{\nu_j})} dt = \\ &= \frac{\pi^*(\nu_j)}{r} (1 - e^{-rT_{\nu_j}}) \end{aligned}$$

- ▶ then the value of entry is a stochastic variable, dependent upon the duration of the monopoly T_{ν_j}

Producers of intermediate goods (cont)

Expected benefit of entry

- ▶ The success of an innovation arriving at industry j (with the present level ν_j) is governed by the exponential process

$$g(\nu_j, T) = \mathbb{P}[\nu_j + 1 | \nu_j] = \lambda(\nu_j) e^{-\lambda(\nu_j)T}, \quad T \in [0, \infty)$$

where T is the period of incumbency (i.e., after entry)

- ▶ Then the **expected benefit for introducing quality ladder** ν_j depends on the distribution of incumbency time (T_{ν_j})

$$\begin{aligned} \mathbb{E}[v^*(\nu_j)] &= \int_0^\infty v^*(\nu_j, T_{\nu_j}) g(\nu_j, T_{\nu_j}) dT_{\nu_j} = \\ &= \frac{\pi^*(\nu_j)\lambda(\nu_j)}{r} \int_0^\infty (1 - e^{-rT_{\nu_j}}) e^{-\lambda(\nu_j)T_{\nu_j}} dT_{\nu_j} = \\ &= \frac{\pi^*(\nu_j)\lambda(\nu_j)}{r} \left(\frac{1}{\lambda(\nu_j)} - \frac{1}{\lambda(\nu_j) + r} \right) \end{aligned}$$

Producers of intermediate goods (cont)

Expected benefit of entry

- Therefore the expected benefit from entry is

$$\begin{aligned}\mathbb{E}[v^*(\nu_j)] &= \frac{\pi^*(\nu_j)}{r + \lambda(\nu_j)} = \\ &= \frac{1 - \alpha}{\alpha} \frac{x^*(\nu_j)}{r + \lambda(\nu_j)} = \\ &= \alpha(1 - \alpha) \frac{y^*(\nu_j)}{r + \lambda(\nu_j)}\end{aligned}$$

Producers of R& D

R&D: benefits and costs

- ▶ An entrant firm, needs to raise present technological level from $\nu_j - 1$ to ν_j by performing R&D;
- ▶ Two cases can occur
 - ▶ with probability $\lambda(\nu_j - 1)$ the innovation is successful allowing to becoming incumbent and having the expected value $\mathbb{E}[v^*(\nu_j)]$
 - ▶ with probability $1 - \lambda(\nu_j - 1)$ the innovation is not successful and having the value 0
- ▶ Therefore:
 - ▶ the value of doing R&D

$$\lambda(\nu_j - 1)\mathbb{E}[v^*(\nu_j)]$$

- ▶ the cost is $Z(\nu_j)$

Producers of R& D

R&D technology: lab equipment

- ▶ **Assumption: production function of innovations:**
the probability of success of introducing innovation level ν_j (from $\nu_j - 1$ to ν_j) depends on the probability of introducing an innovation
- ▶ we assume a lab-equipment production function
- ▶ the expenditure for introducing an innovation is

$$Z(\nu_j) = \lambda(\nu_j - 1) \zeta Y(\nu_j)$$

where ζ is a barrier to entry , and

$$y(\nu_j) = x^*(\nu_j)\alpha^{-2} = A_y q(\nu_j)$$

where $A_y \equiv ((\alpha^{2\alpha})A)^{\frac{1}{1-\alpha}} L$

Free entry condition

- ▶ Free entry in the R&D sector:

$$\underbrace{\lambda(\nu_j - 1)E[v^*(\nu_j)]}_{\text{expected benefit}} = \underbrace{Z(\nu_j)}_{\text{cost}} \Leftrightarrow E[v^*(\nu_j)] = \zeta Y(\nu_j)$$

- ▶ this is equivalent to

$$\frac{1 - \alpha}{\alpha} \frac{x^*(\nu_j)}{r + \lambda(\nu_j)} = \frac{\zeta x^*(\nu_j)}{\alpha^2}$$

- ▶ then there is an arbitrage equation for entry

$$\lambda(\nu_j) = \lambda = r_0 - r \text{ where } r_0 \equiv \frac{\alpha(1 - \alpha)}{\zeta}$$

- ▶ this implies that $\lambda(\nu_j) = \lambda$ the **probability should be the same for all sectors** , and depends on the market interest rate r .

Aggregate evolution of quality

- ▶ Let at time t exist a particular distribution of quality levels $\left(\nu_j(t)\right)_{j=0}^N$
- ▶ Change in quality in sector j

$$q(\nu_j) = l^{\frac{\alpha \nu_j}{1-\alpha}} \Rightarrow q(\nu_j + 1) = l^{\frac{\alpha}{1-\alpha}} q(\nu_j)$$

- ▶ assume that it takes place in a small interval of time, write

$$q(j, t) = q(\nu_j), \quad q(j, t + dt) = q(\nu_j + 1) = l^{\frac{\alpha}{1-\alpha}} q(t)$$

- ▶ then the instantaneous change in quality is

$$\frac{dq(j, t)}{dt} = \lim_{h \rightarrow \text{zero}} \frac{q(j, t + h) - q(j, t)}{h} = q(j, t) \left(l^{\frac{\alpha}{1-\alpha}} - 1 \right)$$

Aggregate evolution of quality

- ▶ Writing $\Xi = l^{\frac{\alpha}{1-\alpha}} - 1$ is the "jump" in quality,

$$dq(j, t) = \Xi q(j, t) dt$$

- ▶ We define the aggregate quality as

$$Q(t) \equiv \int_0^N q(\nu_j(t)) dj = \int_0^N q(j, t) dj$$

- ▶ Then expected value for the existence of a variation of quality at time t is

$$\begin{aligned} \frac{dQ(t)}{dt} &= \int_0^N \lambda \frac{dq(j, t)}{dt} dj = \int_0^N \lambda \Xi q(j, t) dj \\ &= \lambda \Xi Q(t) \end{aligned}$$

Aggregate wealth

- ▶ The aggregate wealth is equal to the sum of the rents from R&D production

$$\begin{aligned} W(t) &= \int_0^N E[v^*(\nu_j)] dj = \\ &= \zeta \int_0^N y^*(\nu_j) dj \\ &= \zeta Y(t) \end{aligned}$$

- ▶ the aggregate production of the final good is linear in aggregate quality

$$Y(t) = \int_0^N Y(\nu_j) dj = A_y Q(t)$$

- ▶ then

$$W(t) = \zeta A_y Q(t)$$

Aggregate consistency

- ▶ Then

$$\frac{\dot{W}}{W} = \frac{\dot{Q}}{Q}$$

- ▶ Using the consumer's budget constraint

$$\frac{\dot{W}}{W} = \frac{\dot{Q}}{Q} \Leftrightarrow \frac{\omega L + rW - C}{W} = \Xi \lambda$$

- ▶ We know

- ▶ from the optimality condition for the FP firm

$$\omega L = (1 - \alpha) Y = \frac{(1 - \alpha)}{\zeta} W$$

- ▶ from the free entry condition

$$r + \lambda = r_0 = \frac{\alpha (1 - \alpha)}{\zeta}$$

Aggregate consistency

► Then

$$\begin{aligned}\Xi(r_0 - r) &= \frac{(1 - \alpha)Y - C}{W} + r = \frac{(1 - \alpha)}{\zeta} - \frac{C}{W} + r \\ &= \frac{1}{\zeta} \left(1 - \alpha - \frac{C}{A_y Q} \right) + r\end{aligned}$$

► We obtain an expression for the **market interest rate**

$$r = r_0 - \lambda, \text{ for } \lambda = \Lambda(C/W) \equiv \frac{1}{l^{\frac{\alpha}{1-\alpha}}} \left(\frac{1 - \alpha^2}{\zeta} - \frac{C}{A_y Q} \right)$$

The equilibrium in the decentralized economy

- ▶ the DGE system in levels becomes

$$\begin{aligned}\dot{C} &= \frac{C}{\theta}(r_0 - \Lambda(C/Q) - \rho) \\ \dot{Q} &= \Xi\Lambda(C/Q)Q\end{aligned}\tag{DGE}$$

- ▶ Decomposing the variables between trend and transition

$$C(t) = c(t)e^{\gamma t}, \quad Q = Qe^{\gamma t}$$

- ▶ the DGE in detrended variables

$$\begin{aligned}\dot{c} &= \frac{c}{\theta}(r_0 - \Lambda(c/q) - \rho - \theta\gamma) \\ \dot{q} &= (\Xi\Lambda(c/q) - \gamma)q\end{aligned}\tag{DGE detrended}$$

The long run growth rate

Decentralized economy

- ▶ Solving the steady state jointly to r and γ we obtain
- ▶ the long run growth rate is equal to the (endogenous) probability of arrival on innovations

$$\gamma_d = \Xi \bar{\lambda} = \frac{r_0 - \rho}{\Xi^{-1} + \theta}$$

- ▶ where $r_0 = \alpha(1 - \alpha)/\zeta$
- ▶ the growth rate is a negative function of the cost of entry ζ (i.e, barriers to R&D reduce growth) and a positive function of the quality jump (Ξ)
- ▶ the long-run per capita GDP level is

$$\bar{y} = A_Y Q = A_Y \int_0^N l^{\frac{\alpha \nu_j}{1-\alpha}} dj$$

is higher the higher the "quality ladders" for all sectors;

- ▶ there is no transitional dynamics

Central planner economy economy

The problem

- The problem

$$\max_{(C(t))_{t \in [0, \infty)}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad \theta > 0$$

subject to

$$\dot{Q} = \Xi \Lambda(C/Q) Q$$

$$Q(0) = Q_0 \text{ given}$$

- Exercise: find the long-run growth rate
- Exercise: Compare with the growth rate for a decentralized economy
- How do creative destruction compare n the two models ?

References

- ▶ (?, ch. 7), (?, ch. 14)

Daron Acemoglu. *Introduction to Modern Economic Growth*.
Princeton University Press, 2009.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*.
MIT Press, 2nd edition, 2004.