Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Crescimento Económico (Growth Economics)
2020-2021

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Exam: **Época de Recurso** (Re-sit exam)
29.6.2021 (18.00h-21.00h, 112 F1)

> Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. [6 points (3,3)] Please answer **two** of the following three questions.

   (a) The Resilience and Recovery Plan is presented as a boost to the GDP growth prospects for EU countries. Discuss, in abstract but considering the current economic situation of EU countries, its possible effects on long run economic growth. In particular, take into consideration an important dimension which is not being much debated at present: the fact that it will have to be financed by future taxes.

   (b) The Uzawa-Lucas model appears to be consistent with the experience of growing countries until they reach a maturity phase, but seem at odds with evidence after that stage. Present the main assumptions of the model and explain which assumptions lead to its inability to address the economic growth experience of mature economies at least after the 1990's.

   (c) Explain the main differences in the way R&D activities are introduced between the expansion of varieties and the Schumpeterian endogenous growth models. If you were asked to build a model with both types of activities how would you combine them ?

2. [7 points (1.5,1.5,1.5,1.5,1)] Consider a version of the Solow (1956) model in which the production function is of the VES (variable elasticity of substitution) type

$$F(K,H) = A\,K^{\alpha}\Big(H + \alpha\beta K\Big)^{1-\alpha}, \ A > 0, \ 0 < \alpha < 1, \ \beta > 0$$

where $K > 0$ is the stock of physical capital and $H > 0$ is the stock of human capital. Human capital is produced by means of a linear production function $dH(t)/dt = \gamma H(t)$, with $\gamma > 0$. The accumulation of physical capital is given by $dK(t)/dt = sF(H(t), K(t)) - \delta K(t)$ where $s > 0$ and $\delta > 0$. Both $K(0)$ and $H(0)$ are observed.

   (a) Find the characteristics of the technology, implicit in the production function, as regards: (a) the gross complementarity/substitutability between the two factors; (b) the Inada properties ; and (c) the returns to scale.

   (b) Define $k(t) \equiv K(t)/H(t)$ and determine the accumulation equation for k.

   (c) Define $\tilde{A} \equiv \frac{\delta+\gamma}{s}(\alpha\beta)^{\alpha-1}$. Prove that if $A < \tilde{A}$ there is a steady state, $k^* > 0$. Under this assumption, provide a linear approximation of the solution for $k(t)$.

   (d) Prove that if $A > \tilde{A}$ there is no steady state and the solution converges to a balanced growth path with growth factor $e^{g\,t}$ with $g > 0$ (hint: show that $\lim_{k\to\infty}\frac{\dot{k}}{k} = g$). Find the long-run growth rate g.

   (e) Discuss the growth theory which is provided by this model, by comparing your results in (c) and (d). Is your analysis in (a) helpful for explaining this difference ?

3. [7 points (0.5,1.5,1.5,1.5,2)] Consider a centralized economy in which the representative consumer has the intertemporal utility function

$$\max_{[C]_{t\geq 0}} \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt$$

where $\theta > 0$ and $\rho > 0$, and the aggregate economy constraint is

$$\dot{K} = AK(t)^\alpha - C(t)$$

where $0 < \alpha < 1$:

(a) Do the necessary conditions for the existence of a balanced growth path, with a positive growth rate, are verified ? Justify.

(b) Consider a decentralized economy in which the problem of the representative agent is as in (a), but assume there is an externality such that $A = A_0(K^a)^\beta$, $A_0 > 0$ and K^a is the aggregate level of capital for the economy, taken as exogenous by the consumer. Obtain a representation of the general equilibrium as a dynamical system in (K, C).

(c) Under which conditions a balanced growth path exists ? Assume, from now on, the conditions you arrived at. Find the equilibrium solution for capital, output and consumption. Extract the growth facts from this model.

(d) Is that equilibrium Pareto efficient ? If not, derive the Pareto efficient growth path.

(e) Consider again a decentralized economy in which the government has two fiscal instruments, a distortionary tax/subsidy and a non-distortionary tax/subsidy, and follows a balanced budget rule at all times. Can we design a fiscal policy, with those two requirements, such that we would have a Pareto efficient economic growth ?