Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Mestrado em Economia Monetária e Financeira
Fundamentos de Economia Financeira
2016-2017

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Prova de avaliação: **Época Normal**
7.6.2017 (18.00h-21.00h, 214 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. `[6 points]` Let there be uncertainty characterized by two states of nature with equal probabilities. A lottery has payoffs $Y = (y_1, y_2) = (\ e^{\epsilon}, e^{-\epsilon})$, where $\epsilon > 0$, and the behavior of an agent is charaterized by a von-Neumann Morgenstern utility functional with a logarithmic Bernoulli utility function.
   (a) Find the certainty equivalent of lottery Y.
   (b) Which is better, the lottery or a certain payoff equal to $\mathbb{E}[Y]$? Describe and give an intuition on the possible approaches to come up with an answer.
   (c) Assume you introduce an flat tax over the certain payoff $\mathbb{E}[Y]$. What would be the level of the tax such that the agent would be indifferent between the penalized certain outcome or the lottery. Provide an intuition.

2. `[8 points]` Consider a finance economy in which there are two assets with the vector of price $S = (1, 1)$ and the payoff matrix
$$V = \begin{pmatrix} 1+r-\epsilon & 1+r+\epsilon \\ 1+r+\epsilon & 1+r-\epsilon \end{pmatrix}$$
   where we assume that $r > 0$ and ϵ can take any sign. The two states of nature have equal probabilities.
   (a) Determine the state prices and characterize the finance market regarding the existence of arbitrage opportunities and completeness.
   (b) Let a risk-free bond be introduced with face value equal to one. Use arbitrage pricing theory to determine the risk free interest rate (hint: compute the replicating portfolio and use the cost of that replicating portfolio to determine the risk free interest rate). Provide an intuition to your results.
   (c) Compute the Sharpe indices for the two risky assets. Explain
   (d) Assume a general equilibrium finance economy in which the representantive consumer has a von-Neumann-Morgenstern utility functional and a logarithmic Bernoulli utility function. Write the equilibrium conditions for the risk-premia of both risky assets. Under which conditions can an equilibrium stochastic discount factor exist in this economy ? Explain.

3. `[6 points]` Consider an endowment Arrow-Debreu economy in which there is uncertainty and an infinite number of periods. The endowments are given by an exogenous process $\{Y_t\}_{t=0}^{\infty}$ and the agents are homogeneous. Assume that the representative agent has the intertemporal utility functional
$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln(C_t) \right]$$
   where $\beta \equiv \frac{1}{1+\rho}$ where $\rho > 0$ is the rate of time preference.
   (a) Discuss the assumptions underlying the utility functional. Define and find the general equilibrium for this economy.

(b) Find the recursive stochastic discount factor ($M_{t+1|t}$). Find a limiting value for $\mathbb{E}_t[M_{t+1|t}]$ associated to the Jensen inequality. Justify.

(c) If we define the one-period interest rate of this economy, say $R_{t+1|t}$, as the inverse of $M_{t+1|t}$ prove that the interest rate is a linear function of the expected value of the growth factor of the endowment, $\frac{Y_{t+1}}{Y_t}$. Briefly discuss your result.

(d) Why can we get an exact relationship for $\mathbb{E}_t[R_{t+1|t}]$, in (c), and not for $\mathbb{E}_t[M_{t+1|t}]$, in (b) ?