R&D and growth: the Schumpeterian model

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Economic structure

- Technology:
  - final good production uses labor and intermediate goods
  - intermediate goods are the only reproducible inputs
  - the dynamics of output is generated by the variations in the number of of intermediate inputs (varieties)
- Environment:
  - decentralized economy
  - there are two sectors: competitive final good sector and a R&D (research and development=) sector with monopolistic competition

Core assumptions: CHANGE FROM THIS POINT ON

- technical progress takes the form of an expansion in the number (variety) of products
- it is materialized in the expansion of intermediate goods, i.e., creation of a new industry
- a new industry is created only after R&D activity takes place
- R&D is related to the production of ideas
- ideas are non-rival, i.e., cannot be made private once created
- as R&D has costs (proportional to the output generated by a new variety) it only takes place if the value of R&D is equal to the cost (free-entry condition)
- importance of the economic environment: (1) in a decentralized economy R&D can only take place if there is imperfect competition; (2) in a centralized economy R&D costs can be internalized

Results

- Without capital accumulation growth is generated by the expansion in varieties
- The rate of growth depends on the barriers to entry into R& D
- The decentralized economy is not Pareto optimal, meaning that a related centralized economy verifies a higher rate of growth
- This is because the rate of return generated by R&D activities is lower in a decentralized economy

Decentralized economy

The structure of the model

- Consumer problem
- Final producer problem
- Producers of intermediate goods (incumbents plus entrants)
- Aggregation, balance sheet and market clearing conditons

The consumer problem

- Earns labor and capital income, consumes a final product, save and own firms (final good and intermediate good producers)
- The problem

$$\begin{aligned} \max_{(C(t))_{t\in[0,\infty)}} \quad & \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \ \theta > 0 \\ \text{s.t} \quad & \\ & \dot{W} = \omega(t) L + r(t) W(t) - C(t) \end{aligned} \tag{CP}$$

- The first order conditions

$$\begin{aligned} \dot{C} &= \frac{C}{\theta} (r(t) - \rho) \\ \dot{W} &= \omega(t) L + r(t) W(t) - C(t) \end{aligned}$$

Producers of the final good

- Production function: Dixit and Stiglitz (1977)

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} x(j,t)^{\alpha} \, dj, \ 0 < \alpha < 1$$

- L labor input
- $(x(j,.))_{j \in [0,N(t)]}$ intermediate inputs, non-storable,
- $N(t)$ number of varieties
- Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^{N(t)} P(j,t)x(j,t) \, dj$$

Producers of the final good (cont)

- Buys labor and intermediate goods and sells a final good
- The problem:

$$\max_{L,(x(j,t))_{j\in[0,N(t)]}} \pi^p(t) \tag{FGPP}$$

- Obs: they are price takers in all markets
- First order conditions: demand for labor and for intermediate goods

$$L^d = (1-\alpha)\frac{Y(t)}{\omega(t)}$$

$$x^d(j,t) = \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,\ j \in [0, N(t)]$$

Producers of intermediate goods

- Perform R&D activities allowing for the production of a new variety which they sell to final producers
- Decision process for the introduction of a new variety
  - before entry: R& D
  - entry decision: free entry condition
  - after entry: decide on the price of variety j
- Solution to the problem: we work in backward order
  - first: we determine the pricing policy assuming there was entry
  - second: we determine entry (by using the free entry condition)

Producers of intermediate goods (cont)

Price decision after entry

- The profit of the producer of a variety $j \in (0, N(t)]$ is

$$\pi(j,t) = (P(j,t) - 1)x(j,t)$$

assuming a symmetric cost of production equal to 1

- where $x(j,t) = x^d(j,t)$ (solution of the FGPP)
- Then the profit after entry is

$$\pi(j,t) = (P(j,t) - 1)\left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,$$

Producers of intermediate goods (cont)

Price decision after entry

- The first order conditions $(\partial \pi(j,.)/\partial P(j,.) = 0)$
$$P^*(j,t) = \frac{1}{\alpha} \ \forall (j,t)$$
- then the demand for variety is symmetric
$$x^*(j,t) = x^* = \left(\alpha^2 A\right)^{\frac{1}{1-\alpha}} L$$
- the profit is also symmetric and constant
$$\pi^*(j,t) = \pi^* = \left(\frac{1-\alpha}{\alpha}\right) L \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}}$$
- This implies
$$\boxed{Y(t) = AL^{1-\alpha} \int_0^{n(t)} (x^*)^\alpha dj = \phi N(t)}$$

where

Entry

Value of entry

- The value for producer of a successful variety j, if it enters at time t, becomes a monopolist forever

$$v(j,t) = \max_{(P(j,s))_{s\in[t,\infty)}} \int_t^\infty \pi(j,s)\, e^{-R(s)}\, ds \tag{IGPP}$$

- where the discount factor is time-variying

$$R(s) = \int_t^s r(\tau)\, d\tau$$

- and the value of producing variety j is

$$v^*(j,t) = v^*(j) = \pi^* \int_t^\infty e^{-R(s)}\, ds$$

- differentiating we have

$$\dot{v}(t) = -\pi^* + r(t) v(t) \tag{1}$$

Entry

Cost of decision

- Costs of entry: lab-equipment assumption (doing R&D entails using the final good in proportion to the output per variety)

$$I(j,t) = \eta \frac{Y(t)}{N(t)} = \eta\phi$$

- **Free entry condition** in the market for variety j

$$v^*(j,t) = I(j,t)$$

- Then

$$\boxed{v^* = \eta\phi}$$

- Because v^* is a constant, from the (1) (and $\dot{v} = 0$)

$$\pi^* = rv^*$$

  then the interest rate is

General equilibrium

- The consumer solves (CP)
- The producer of final goods solves (FGPP)
- The intermediate producers solve problems (IGPP)
- Aggregate consistency condition hold

General equilibrium

- Consistency conditions: the rents generated by R&D distributed to consumers who own firms

$$W(t) = \int_0^{N(t)} v(j,t)\,dj = v^* N(t) = \eta\phi N(t)$$

- the budget constraint, becomes

$$\dot{W} = \omega L + rW - C \Leftrightarrow \eta\phi\dot{N} = (1-\alpha)(1+\alpha)\phi N - C$$

- because

$$\omega L = (1-\alpha)Y = (1-\alpha)\phi N \text{ and } rW = \frac{\alpha(1-\alpha)}{\eta}\eta\phi N$$

The equilibrium in the decentralized economy

- the DGE in levels

$$\boxed{\begin{aligned} \dot{C} &= \frac{C}{\theta}(r-\rho) \\ \dot{N} &= \frac{(1-\alpha^2)}{\eta}N - \frac{C}{\eta\phi} \end{aligned}} \qquad \text{(DGE)}$$

- Decomposing the variables

$$C(t) = c(t)\,e^{\gamma t},\ N(t) = n(t)\,e^{\gamma t}$$

- the DGE in detrended variables

$$\boxed{\begin{aligned} \dot{c} &= \frac{c}{\theta}(r-\rho-\theta\gamma) \\ \dot{n} &= \left(\frac{(1-\alpha^2)}{\eta} - \gamma\right) n - \frac{c}{\eta\phi} \end{aligned}} \qquad \text{(DGE detrended)}$$

The long run growth rate

Decentralized economy

- the long run growth rate is

$$\boxed{\gamma_d = \frac{1}{\theta}\left(\frac{\alpha(1-\alpha)}{\eta} - \rho\right)}$$

is a negative function of the cost of entry η (i.e, **barriers to R&D reduce growth**)

- the long run level for per capita GDP is

$$\bar{y} = \phi(A, L)\frac{n(0)}{L} = \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} n(0)$$

- there is no transitional dynamics

Centralized economy

- Consider a social planner solving the problem

$$\begin{aligned}
&\max_{(C(t))_{t\in[0,\infty)}} \quad \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \;\; \theta > 0 \\
&\text{s.t} \\
&\qquad\qquad \dot{N} = \frac{(1-\alpha^2)}{\eta} N(t) - \frac{C(t)}{\eta\phi}
\end{aligned} \tag{OP}$$

- applying the Pontriyagin principle and decomposing the variables we get

$$\begin{aligned}
\dot{c} &= \frac{c}{\theta}(r_c - \rho - \theta\gamma) \\
\dot{n} &= \left(\frac{(1-\alpha^2)}{\eta} - \gamma\right) n - \frac{c}{\eta\phi}
\end{aligned} \tag{OP detrended}$$

where

$$r_c \equiv \frac{1-\alpha^2}{\eta}$$

The long run growth rate

Centralized economy

- the long run growth rate is

$$\boxed{\gamma_c = \frac{1}{\theta}\left(\frac{1-\alpha^2}{\eta} - \rho\right) > \gamma_d = \frac{1}{\theta}\left(\frac{(1-\alpha)\alpha}{\eta} - \rho\right)}$$

- the long run growth rate in the centralized economy is higher than in the decentralized economy
- this means that the decentralized economy is not Pareto optimal: there is an externality generated by the R&D activity that is not internalized in a decentralized economy

References

- (Acemoglu, 2009, ch. 15),

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