R&D and growth: the variety expansion model

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Economic structure

- Environment:
  - there are two sectors: a competitive final good sector and a continuum of monopolistically competitive intermediate goods sectors
  - there is entry by creation of a new intermediate good product (=industry)
  - there is no capital accumulation
  - decentralized economy
- Technology:
  - final good production uses labor and a continuum of intermediate goods
  - intermediate goods are the only reproducible inputs
  - the dynamics of output is generated by the variations in the number of of intermediate inputs (varieties) which is the result of successful R&D (research and development=)

Core assumptions

- technical progress takes the form of an expansion in the number (variety) of products
- a new industry is created only after R&D activity takes place
- R&D is related to the production of ideas
- ideas are non-rival, i.e., cannot be made private once created
- as R&D has costs (proportional to the output generated by a new variety) it only takes place if the value of R&D is equal to the cost (free-entry condition)
- importance of the economic environment: (1) **in a decentralized economy R&D can only take place if there is imperfect competition**; (2) in a centralized economy R&D costs can be internalized

Results: implication for growth

- Without capital accumulation growth is generated by the expansion in varieties
- The rate of growth depends on the barriers to entry into R& D
- The decentralized economy is not Pareto optimal, meaning that a related centralized economy attains a higher rate of growth
- This is because the rate of return generated by R&D activities is lower in a decentralized than in the related centralized economy

Decentralized economy

The structure of the model

- Consumer problem
- Final producer problem
- Producers of intermediate goods (incumbents and entrants)
- Aggregation, balance sheet and market clearing conditons

The consumer problem

- Earns labor and capital income, consumes a final product and save
- they own firms (final good and intermediate good producers)
- The problem

$$\begin{aligned} &\max_{(C(t))_{t\in[0,\infty)}} && \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \ \theta > 0 \\ &\text{s.t} && \\ & && \dot{W} = \omega(t) L + r(t) W(t) - C(t) \end{aligned} \tag{CP}$$

- The first order conditions

$$\begin{aligned} \dot{C} &= \frac{C}{\theta}(r(t) - \rho) \\ \dot{W} &= \omega(t) L + r(t) W(t) - C(t) \end{aligned}$$

Producer of the final good

- Production function: Dixit and Stiglitz (1977)

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} x(j,t)^{\alpha} dj, \; 0 < \alpha < 1$$

- L labor input
- $(x(j,.))_{j \in [0, N(t)]}$ intermediate inputs, non-storable,
- $N(t)$ number of varieties
- Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^{N(t)} P(j,t)x(j,t) dj$$

Producers of the final good (cont)

- Buys labor and intermediate goods and sells a final good
- The problem:

$$\max_{L,(x(j,t))_{j\in[0,N(t)]}} \pi^p(t) \tag{FGPP}$$

- Obs: they are price takers in all markets
- First order conditions: demand for labor and for intermediate goods

$$L^d = (1-\alpha)\frac{Y(t)}{\omega(t)}$$

$$x^d(j,t) = \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,\ j \in [0, N(t)]$$

Producers of intermediate goods

- Perform R&D activities allowing for the production of a new variety which they sell to final producers
- Decision process for the introduction of a new variety
  - before entry: R& D
  - entry decision: free entry condition
  - after entry: decide on the price of variety j
- Solution to the problem: we work in backward order
  - first: we determine the pricing policy assuming there was entry (incumbent's problem)
  - second: we determine entry (by using the free entry condition)

Producers of intermediate goods (cont)

Price decision after entry

- The profit of the producer of a variety $j \in (0, N(t)]$ is

$$\pi(j,t) = (P(j,t) - 1)x(j,t)$$

  assuming a symmetric cost of production equal to 1
- where $x(j,t) = x^d(j,t)$ (solution of the FGPP)
- Then the profit after entry is

$$\pi(j,t) = (P(j,t) - 1)\left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,$$

Producers of intermediate goods (cont)

Price decision after entry

- The first order condition:

$$\frac{\partial \pi(j,t)}{\partial P(j,t)} = 0 \Leftrightarrow P^*(j,t) = \frac{1}{\alpha} \; \forall (j,t)$$

- then the demand for variety is **symmetric** (i.e, equal to all industries)

$$x^*(j,t) = x^* = \left(\alpha^2 A\right)^{\frac{1}{1-\alpha}} L$$

- the profit is also symmetric and constant

$$\pi^*(j,t) = \pi^* = \left(\frac{1-\alpha}{\alpha}\right) L \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} > 0$$

(pure-) profits are positive, symmetric and constant in time.

Implication for aggregate output

- This implies

$$\boxed{Y(t) = AL^{1-\alpha} \int_0^{N(t)} (x^*)^\alpha dj = \phi N(t)}$$

where

$$\phi \equiv A\left(A\alpha^2\right)^{\frac{\alpha}{1-\alpha}} L = (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} L$$

- Interpretation: because $N(t)$ is the only dynamic variable (to see next) the aggregate production function has a AK (i.e. constant returns and constant marginal product) structure, where ϕ is a productivity parameter.
- Then

$$\pi^* = \alpha(1-\alpha)\phi$$

Entry

Value of entry

- The value from producing a successful variety j, if it is introduced (by entry) at time t, is a monopoly rent forever

$$v(j,t) = \max_{(P(j,s))_{s\in[t,\infty)}} \int_t^\infty \pi(j,s)\, e^{-R(s)}\, ds \qquad \text{(IGPP)}$$

- where the discount factor is time-variying

$$R(s) = \int_t^s r(\tau)\, d\tau$$

- Introducing the previous result from an incumbent at industry j, $\pi(j,t) = \pi^*$, at the optimum we have

$$v^*(j,t) = v^*(t) = \pi^* \int_t^\infty e^{-R(s)}\, ds$$

- taking a time derivative yields

$$\dot{v}(t) = -\pi^* + r(t)v(t) \qquad (1)$$

Entry

Cost of decision

- **Lab-equipment assumption**: R&D is an activity using the final product as an input
- Costs of entry: assuming a linear and symmetric R&D technology

$$I(j,t) = \eta \frac{Y(t)}{N(t)} = \eta\phi$$

Free entry

- **Free entry condition** in the market for variety j there is entry up to the point in which benefits are equal to the costs of entry.
- Therefore, the equilibrium entry condition is

$$v(j,t) = I(j,t)$$

- Then, taking $v(j,t) = v^*(t)$ and $I(j,t) = \eta\phi$

$$\boxed{v^* = \eta\phi}$$

- Because v^* is a constant, from the (1) (and $\dot{v} = 0$)

$$\pi^* = rv^*$$

then the interest rate is constant

$$\boxed{r(t) = r^* = \frac{\alpha(1-\alpha)}{\eta}}$$

General equilibrium

- The consumer solves (CP)
- The producer of final goods solves (FGPP)
- The intermediate producers solve problems (IGPP)
- Aggregate consistency condition hold

General equilibrium

- Consistency conditions: the rents generated by R&D distributed to consumers who own firms

$$W(t) = \int_0^{N(t)} v(j,t)\,dj = v^* N(t) = \eta\phi N(t)$$

- the budget constraint, becomes

$$\dot{W} = \omega L + rW - C \Leftrightarrow \eta\phi\dot{N} = (1-\alpha)(1+\alpha)\phi N - C$$

- because

$$\omega L = (1-\alpha)\,Y = (1-\alpha)\phi N \text{ and } rW = \frac{\alpha(1-\alpha)}{\eta}\eta\phi N$$

The equilibrium in the decentralized economy

- the DGE in levels

$$\boxed{\begin{aligned} \dot{C} &= \frac{C}{\theta}(r-\rho) \\ \dot{N} &= \frac{(1-\alpha^2)}{\eta}N - \frac{C}{\eta\phi} \end{aligned}} \tag{DGE}$$

- Decomposing the variables

$$C(t) = c(t)e^{\gamma t},\ N(t) = n(t)e^{\gamma t}$$

- the DGE in detrended variables

$$\boxed{\begin{aligned} \dot{c} &= \frac{c}{\theta}(r-\rho-\theta\gamma) \\ \dot{n} &= \left(\frac{(1-\alpha^2)}{\eta} - \gamma\right)n - \frac{c}{\eta\phi} \end{aligned}} \tag{DGE detrended}$$

General equilibrium: alternative representation

- If we define the capital in this economy as $K(t) = W(t)$. Then $K(t) = \eta\phi N(t)$, $\omega L = \frac{(1-\alpha)}{\eta} K$ and $rW = \frac{\alpha(1-\alpha)}{\eta} K$
- the budget constraint becomes

$$\dot{K} = \frac{(1-\alpha)(1+\alpha)}{\eta} K - C = A^v K - C$$

- which implies that the model has a AK structure, where $A^v = A^v(\alpha, \eta) = \dfrac{(1-\alpha)(1+\alpha)}{\eta}$, where clearly

$$\frac{A^v}{\alpha} < 0, \ \frac{A^v}{\eta} < 0$$

which means that A^v is a positive function of the markup, $\mu = 1/\alpha$: an increase in the markup and a reduction in the barriers to entry increase the productivity of capital

The long run growth rate

Decentralized economy

- the long run growth rate is

$$\boxed{\gamma_d = \frac{1}{\theta}\left(\frac{\alpha(1-\alpha)}{\eta} - \rho\right)}$$

  is a negative function of the cost of entry η (i.e, **barriers to R&D reduce growth**)

- the long run level for per capita GDP is

$$\bar{y} = \phi(A, L)\frac{n(0)}{L} = \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} n(0)$$

- there is no transitional dynamics

Centralized economy

- Consider a social planner solving the problem

$$\begin{aligned} &\max_{(C(t))_{t\in[0,\infty)}} \quad \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \ \theta > 0 \\ &\text{s.t} \\ &\qquad\qquad \dot{N} = \frac{(1-\alpha^2)}{\eta} N(t) - \frac{C(t)}{\eta\phi} \end{aligned} \tag{OP}$$

- applying the Pontriyagin principle and decomposing the variables we get

$$\begin{aligned} \dot{c} &= \frac{c}{\theta}(r_c - \rho - \theta\gamma) \\ \dot{n} &= \left(\frac{(1-\alpha^2)}{\eta} - \gamma \right) n - \frac{c}{\eta\phi} \end{aligned} \tag{OP detrended}$$

where

$$r_c \equiv \frac{1-\alpha^2}{\eta}$$

The long run growth rate

Centralized economy

- the long run growth rate is

$$\boxed{\gamma_c = \frac{1}{\theta}\left(\frac{1-\alpha^2}{\eta} - \rho\right) > \gamma_d = \frac{1}{\theta}\left(\frac{(1-\alpha)\alpha}{\eta} - \rho\right)}$$

- the long run growth rate in the centralized economy is higher than in the decentralized economy
- this means that the decentralized economy is not Pareto optimal: there is an externality generated by the R&D activity that is not internalized in a decentralized economy

Policy implications

- In the decentralized setting the government introduces a tax/subsidy on the return on capital applied/financed by a lump-sum expenditure/tax
- under a budget balanced rule we have $\tau r W = G$ in the first case (tax/expenditure) τ and G are positive and in the second (subsidy/tax) they are negative
- this implies that the rate of growth is

$$\gamma_d = \frac{1}{\theta}\left(\frac{(1-\tau)\alpha(1-\alpha)}{\eta} - \rho\right)$$

- to internalize fully the externality we should have $(1-\tau)r^d = r^c$ which implies $\gamma^d = \gamma^c$, that is

$$(1-\tau)\frac{\alpha(1-\alpha)}{\eta} = \frac{(1+\alpha)(1-\alpha)}{\eta}$$

- then the optimal policy would be to introduce a subsidy whose rate should be equal to the markup $-\tau = \frac{1}{\alpha}$

References

- Grossman and Helpman (1991)
- (Barro and Sala-i-Martin, 2004, ch. 6), (Acemoglu, 2009, ch. 13), (Aghion and Howitt, 2009, ch. 3)

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