# Advanced macroeconomics 2018-2019
# Problem set 1: optimal taxation

Paulo Brito
`pbrito@iseg.ulisboa.pt`

20.12.2018

## 1  Taxation: general

**1**  Compare the Chamley-Judd and the Mirrlees model of taxation.

**2**  How would you combine in a single model the two types of taxation ?

## 2  Chamley-Judd taxation

**1**  Consider a simple *AK* Chamley model in which the government finances expenditures only through capital taxation.

  (a) Define and write the equations characterizing the dynamic general equilibrium for a decentralized economy.

  (b) If the government decides to fix the tax optimally, define the Ramsey primal problem. Write the equations characterizing the optimal policy, providing an intuition for the meaning of each one.

  (c) Find the tax policy that implements the Ramsey equilibrium. Discuss your results.

**2**  Some authors, for example **?** and **?**, interpret the puzzling result that the optimal capital tax can be initially very high but should tend asymptotically to zero as an implication of the Ramsey taxation on goods. By presenting a simple version of the **?** model, discuss this result.

**3**  The **?** model of optimal taxation policy can be cast and solved in two different versions: a primal approach and a dual approach.

  (a)  Present a simple version of a general equilibrium model in which there is government and the government can finance its budget by taxation and by issuing debt.

(b)   Specify the two approaches for dealing with the optimal capital taxation policy problem.

(c) Compare the two approaches.

**4**   Consider a Chamley-Judd model with endogenous labor with the following specifications for the utility function and the production function: $u(c, \ell) = \ln(c) - \dfrac{\ell^{1+\xi}}{1+\xi}$, and $F(K, \ell) = k^\alpha \ell^{1-\alpha}$, where $\xi > 0$ and $0 < \alpha < 1$. Assume that the government finances government expenditures by levying linear labor and capital taxes.

(a) Define and write the equations characterizing the dynamic general equilibrium for a decentralized economy.

(b) If the government decides to fix the two taxes optimally, define the Ramsey dual problem. Write the equations characterizing the optimal policy, providing an intuition for the meaning of each one.

(c) Does the Chamley-Judd result of the optimality of a zero long-run capital tax holds in this case. Discuss the intuition and the conditions under which that result is obtained.

**5**   Consider a Chamley-Judd model with endogenous labor with the following specifications for the utility function and the production function: $u(c, \ell) = \ln(c) - \dfrac{\ell^{1+\xi}}{1+\xi}$, and $F(K, \ell) = k^\alpha \ell^{1-\alpha}$, where $\xi > 0$ and $0 < \alpha < 1$. Assume that the government finances government expenditures by levying capital taxes and by issuing debt.

(a) Define and write the equations characterizing the dynamic general equilibrium for a decentralized economy.

(b) Assume that the government decides to fix optimally the level of expenditure and the capital tax. Define the Ramsey primal problem. Write the equations characterizing the optimal policy, providing an intuition for the meaning of each one.

(c) Does the Chamley-Judd result of the optimality of a zero long-run capital tax holds in this case. Discuss the intuition and the conditions under which that result is obtained.

## 3   Mirrlees taxation

**1**   In a model in which there is skill heterogeneity assume that the utility function of the household is $U(c, \ell) = (1-\sigma)^{-1} c^{1-\sigma} - (1+\xi)^{-1} \ell^{1+\xi}$, where $c$ is consumption and $\ell$ is labor effort, and the social welfare function is $W = u$. All the parameters take positive values and $\ell \geq 0$.

(a)   Find the equations representing the competitive equilibrium.

(b)    Assuming perfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(c)   Discuss the properties of the tax schedule, and, in particular, the marginal income tax.

**2**   In a model in which there is skill heterogeneity assume that the utility function of the household is $U(c, \ell) = (1 - \sigma)^{-1} c^{1-\sigma} - (1 + \xi)^{-1} \ell^{1+\xi}$, where $c$ is consumption and $\ell$ is labor effort, and the social welfare function is $W = u$. All the parameters take positive values and $\ell \geq 0$.

(a)    Find the equations representing the competitive equilibrium.

(b)    Assuming imperfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(c)   Discuss the properties of the tax schedule, and, in particular, the marginal income tax.

**3**   In a model in which there is skill heterogeneity assume that the utility function of the household is $U(c, \ell) = c - (1 + \xi)^{-1} \ell^{1+\xi}$, where $c$ is consumption and $\ell$ is labor effort, and the social welfare function is $W = u$. The labor effort satisfies $\ell \geq 0$.

(a)    Find the equations representing the competitive equilibrium.

(b)    Assuming perfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(c)   Assuming imperfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(d)    Compare and discuss the properties of the two previous tax schedules.

**4**   In a model in which there is skill heterogeneity assume that the utility function of the household is $U(c, \ell) = c - (1+\xi)^{-1} \ell^{1+\xi}$, where $c$ is consumption and $\ell$ is labor effort, and the social welfare function is $W = -(u - \bar{u})^2$, where $\bar{u}$ is a constant. The labor effort satisfies $\ell \geq 0$.

(a)    Find the equations representing the competitive equilibrium.

(b)    Assuming perfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(c)   Assuming imperfect information by the tax authority, find the optimal allocation by using a primal approach. Find the tax function that implements the optimal policy.

(d)    Compare and discuss the properties of the two previous tax schedules.