Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Growth Economics
2016-2017

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Exam: **Época Recurso**
3.7.2017 (18.00h-21.00h, 006 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. [6 points] Please answer **two** of the following three questions.
   (a) Describe the most common structure of a Malthusian growth model. Which kind of growth facts can these models account for ? Why can't they account for the "modern economic growth"?
   (b) The Ramsey model is the workhorse for modern macroeconomics and growth theory. Describe its main assumptions and give an intuition for the behaviour of its solution.
   (c) The Uzawa-Lucas model displays transition dynamics while the AK model only displays long-run growth. After presenting a short description of the two models, provide a justification for that difference in the growth behavior which can be obtained from the two models.

2. [7 points] Consider a version of the Solow model, in which there are two types of labor: skilled L_s, and unskilled labor L_u. The proportion of population with each skill is constant, such that $\ell = L_u/L$ and $1 - \ell = L_s/L$, where $0 < \ell < 1$. The total population, L, grows at a constant rate $n > 0$. The technology of production involves a complementarity between capital and unskilled labor and a substitution between them and skilled labor. It is represented by the production function
$$Y(t) = (K(t) + L_u(t))^{\alpha}(AL_s(t))^{1-\alpha}$$
where $0 < \alpha < 1$ and $A > 1$ measures the specific productivity of skilled labor. The savings function is $S(t) = sY(t)$, with $0 < s < 1$, and there is no depreciation of capital.
   (a) Derive the accumulation equation for the detrended capital stock $k(t) \equiv K(t)/L(t)$.
   (b) Prove there is a unique long run level for k. Is uniqueness related to the Inada properties , for k , of the production function ?
   (c) Describe the properties of the model regarding the existence of a balanced growth path, of transition dynamics and of endogenous growth.
   (d) Assume there is a permanent increase in the proportion of unskilled labour ℓ. Determine the effects over long run growth, the level effects, and the transitional dynamics. (Hint: assume that $\ell < \alpha$ and $s\alpha^{\alpha}(A(1-\alpha))^{1-\alpha} > n$).

3. [7 points] Consider an economy in which physical and human capital are perfect substitutes in production and investment. We denote aggregate physical and human capital by K^a and H^a, respectively, and aggregate total capital by $W^a = K^a + H^a$. The production function is $Y^a = AW^a$, where Y^a is aggregate output, and the accumulation equation is $\dot{W}^a = Y^a - C^a$. We assume that total population follows the equation $N(t) = e^{nt}$ with $n > 0$. Consider a centralized economy model in where the central planner has the utility functional
$$\max_{(C^a(t))_{t\geq 0}} \int_0^{\infty} \frac{C^a(t)^{1-\theta}}{1-\theta}\, e^{-\rho t} dt,$$
where $\theta > 1$ and $\rho > 0$, given $W^a(0) = W_0 > 0$ and $\lim_{t\to\infty} e^{-At} W^a(t) \geq 0$.

(a) Determine the optimality conditions as an initial-terminal value problem for per capita consumption and total wealth.

(b) Discuss the verification of the necessary conditions for the existence of a balanced growth path.

(c) Specify the model in detrended variables, and determine the long-run (endogenous) growth rate.

(d) Solve the planner's problem. Determine the solution for the optimal per capita output.

(e) Discuss the growth properties of the model, and, in particular, the implications of changes in parameter A.