Economic Growth Theory:

Problem set 9:

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Growth and government debt

1. Consider a growth model with a government that finances expenditures with debt. Let the consumer problem be

$$\max_{C} \int_0^{\infty} \ln\left(C(t)e^{-\rho t}dt, \rho > 0\right.$$

Â subject to $\dot{K} + \dot{B} = Y + rB - C - T$ given $B(0)$, $K(0)$ and a non-Ponzi game condition. The production function be $Y = K^{\alpha}G^{1-\alpha}$ with $0 < \alpha < 1$. (Hint: G and T are taken by the consumer as externalities). Notation: consumption, C, physical capital stock K, government debt B and r is the interest rate. The government budget constraint is $\dot{B} = G - T + rB$. There are two policy instruments: taxes are defined as $T = \tau Y$ and there is a rule of keeping the debt ratio constant as $B/Y = \bar{b} > 0$.

   (a) Write the DGE in (z, g) where $z = C/K$ and $g = G/Y$ and provide an intuition why this represents the detrended DGE dynamic system.

   (b) Find the long-run growth rate.

   (c) Study the growth facts associated to the model.

2. Consider a growth model with a government that finances expenditures with debt. Let the consumer problem be

$$\max_{C} \int_{0}^{\infty} \ln\left(C(t)e^{-\rho t}dt, \rho > 0\right.$$

Â subject to $\dot{K} + \dot{B} = Y + rB - C - T$ given $B(0)$, $K(0)$ and a non-Ponzi game condition. The production function be $Y = K^{\alpha}G^{1-\alpha}$ with $0 < \alpha < 1$. (Hint: G and T are taken by the consumer as externalities). Notation: consumption, C, physical capital stock K, government debt B and r is the interest rate. The government budget constraint is $\dot{B} = G - T + rB$. There are two policy instruments: taxes are defined as $T = \tau Y$ and there is a rule of keeping the government deficit constant relative to the GDP $\dot{B}/Y = \beta > 0$.

   (a) Write the DGE in (z, g) where $z = C/K$ and $g = G/Y$ and provide an intuition why this represents the detrended DGE dynamic system. Prove that it is

$$\dot{g} = \big((1-g)\,A(g) - z\big)\,(\beta + \tau - g)$$
$$\dot{z} = \big(z - z(g)\big)\,z$$

   where $A(g) = g^{\frac{1-\alpha}{\alpha}}$ and

$$z(g) \equiv \frac{\left(\theta(1-g) - \alpha(1-\tau)\right) A(g) + \rho}{\theta}$$

   (b) Find the steady state and draw the phase diagram.

   (c) Study the growth facts associated to the model.