Automation and growth

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Automation growth and the labor market

- There is a reduction in the labor share in income in several countries (Ex: USA)
- There is also an increase in the number of robots
- Are the two processes linked ?

Robots and machines

- Robots have a peculiar nature:
  - they are generated by a production process similar to machines
  - they are financed in a similar way as machines
  - but they are more substitutable to labor than to machines
- Next we show they can generate endogenous growth in a Ramsey-like model

The macroeconomic constraint

- The production function

$$Y = M^{\alpha}\Big(A_n\,N + L\Big)^{1-\alpha},\ 0 < \alpha < 1$$

where $Y =$ output, $M =$ input of machines, $N =$ input of robots, and $L =$ labour input.

- Distribution of savings to machines

$$\theta\,S = \dot{M} + \delta M$$

- Distribution of savings to robots

$$(1-\theta)\,S = \dot{N} + \delta N$$

The macroeconomic constraint

- Then

$$S = \dot{N} + \delta N + \theta S = \dot{M} + \dot{N} + \delta(M + N) = \dot{K} + \delta K$$

- Equilibrium

$$Y = C + S$$

- Denoting fraction of machines in the total capital stock $\mu \equiv \dfrac{M}{K} \in [0, 1]$ the

$$\dot{k} = \left(\mu\, k\right)^{\alpha} \Big(A_n\,(1 - \mu)\, k + 1\Big)^{1-\alpha} - c - (\delta + n)\, k$$

where $k = K/L$ and $\dot{L} = n\, L$

A Ramsey problem

$$
\begin{aligned}
&\max_{c(\cdot),\mu(\cdot)} \int_0^{\infty} u\big(c(t)\big)\, e^{-\rho t}\, dt, \ \ \rho > 0 \\
&\qquad \text{subject to} \\
&\qquad \dot{k} = y(\mu, k) - c - (\delta + n)\, k \\
&\qquad 0 \leq \mu(t) \leq 1 \\
&\qquad k(0) = k_0 \text{ given} \\
&\qquad \lim_{t\to\infty} k(t) >\geq 0
\end{aligned}
\tag{P1}
$$

where

$$
y(\mu, k) = \big(\mu\, k\big)^{\alpha} \Big(A_n\,(1-\mu)\, k + 1\Big)^{1-\alpha}
$$

The optimal share of machines in capital

- the optimal share of machines depends on the level of the capital stock

$$\mu^* = \begin{cases} 1 & \text{if } 0 < k \leq k_m \\ \dfrac{\alpha\,(1 + A_n\,k)}{A_n\,k} & \text{if } k > k_m \end{cases}$$

where

$$k_m \equiv \frac{1}{A_n}\left(\frac{\alpha}{1-\alpha}\right).$$

- there are two regimes: no automation (for low level of capital) and automation (for higher level of capital)

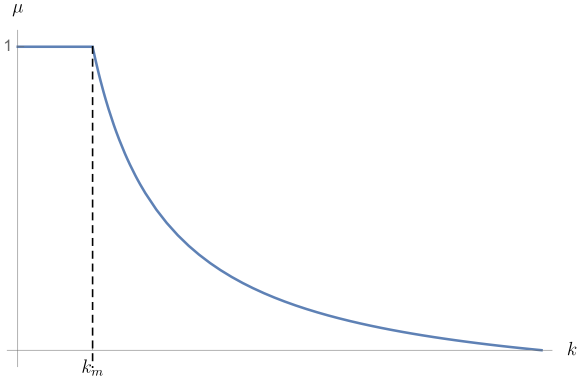

Figure: The optimal capital allocation function $\mu^*(k)$ where $M = \mu^* K$

Two regimes

- Output in the two

$$Y(k) = \begin{cases} k^{\alpha} & \text{if } 0 < k \leq k_m \\ \phi(A_n)\,\big(A_n\,k + 1\big) & \text{if } k > k_m \end{cases} \tag{1}$$

  where $\Phi(A_n) \equiv A_n^{-\alpha}\,\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}$

- and the rate of return in the two regimes

$$R(k) = \begin{cases} \alpha\,k^{\alpha-1} & \text{if } 0 < k \leq k_m \\ \tilde{r} & \text{if } k > k_m \end{cases} \tag{2}$$

  where

$$\tilde{r} = A_n\,\Phi(A_n) = A_n^{1-\alpha}\,\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}.$$

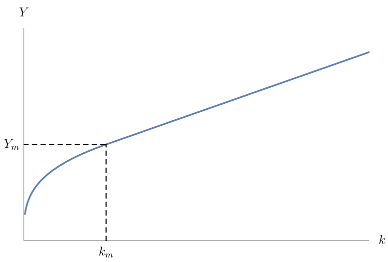

Figure: The optimal output and real return functions $Y(k)$ and $R(k)$.

The MHDS

$$\begin{cases} \dot{k} &= Y(k) - c - \delta\, k \\ \dot{c} &= \dfrac{c}{\sigma(c)} \Big(R(k) - (\rho + \delta)\, k \Big) \end{cases}$$

There is a critical value for productivity

$$A_n^* \equiv \left(\frac{\rho + \delta}{\alpha^{\alpha} \, (1 - \alpha)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

1. if $A_n < A_n^*$ then there is a steady state (k^*, c^*) in which $k^* k_m$;
2. if $A_n > A_n^*$ then there are no steady states
3. if $A_n = A_n^*$ then there is a steady state (k_m, c^*).

No automation steady state

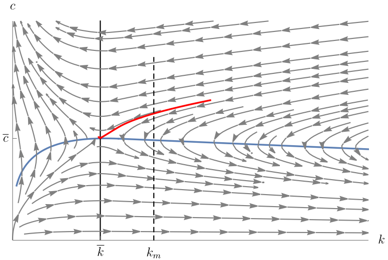

Figure: Phase diagram for $A_n < A_n^*$.

- if $A_n < A_n^*$ (high costs in using robots)
- there is no long run growth (Ramsey case)

Automation

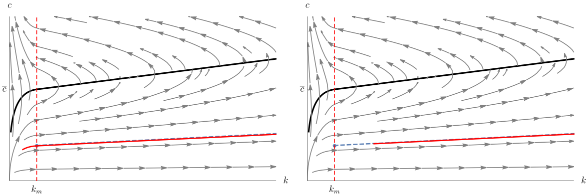

Figure: Phase diagram for $A_n > A_n^*$ starting in region 1 and 2.

- if $A_n > A_n^*$ (high costs in using robots)
- there is long run growth (as in the AK model)
- independently of the level of k_0