Economic Growth Theory:

Problem set 4:

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Growth with expanding varieties

1. Households' utility function $U = \int_0^\infty \ln(C(t))e^{-\rho t}dt$ displaying love for variety where $C(t)$ is a composite of $j \in [0, N(t)]$ varieties of products.

$$C(t) = \left(\int_0^{N(t)} c(t,j)^{\theta/(\theta-1)} dj\right)^{(\theta-1)/\theta}, \ \theta > 1$$

   There is one firm which produces each variety. Any firm has two activities: production and R&D. Production $y(j,t) = h(j,t)L_y(j,t)$ where $h(j,t)$ and $L_y(j,t)$ denote the productivity per worker and the number of workers employed in manufacturing, respectively, in the product of good j. From the R&D activity an increase in the stock of human capital results, and we assume that there are externalities, $\dot{h}(j) = \xi \left(h(j,t)^{1-\alpha}H(t)^{\alpha}\right) L_r(j,t)$, where H measures the externality and $L_r(j)$ is the number of workers in the R&D activity. We assume that the total labor force is constant, then $L = \int_0^{N(t)} L_y(j,t) + L_r(j,t)dj$. Let us assume a symmetric equilibrium in which $L_y(j) = L_y$, $L_r(j) = L_r$, and $h(j) = h$ for any j. Let us also assume that there is a central planner which maximizes the utility of the representative consumer, and internalizes the externality.

(a) If we choose L_y, and N as control variables, prove that the central planners' problem is equivalent to

$$\max_{L_y,N} \int_0^\infty \ln\left(H(t)L_y(t)N(t)^{\theta/(\theta-1)}\right)e^{-\rho t}dt$$

subject to $\dot{H} = \xi H(t)\left(LN(t)^{-1} - L_y(t)\right)$ for $H(0) = H_0$ given:

(b) write the first order conditions (as a differential equation system in (N, H);

(c) discuss the existence of a balanced growth path, write the system in detrended variables;

(d) find the long run growth rate, and discuss the effects of changes in the productivity of R& D activities (parameter ξ). Discuss your results, and the growth of the economy according to this model.

Growth and government debt

1. Consider a growth model with a government that finances expenditures with debt. Let the consumer problem be

$$\max_C \int_0^\infty \ln\left(C(t)e^{-\rho t}dt, \rho > 0\right.$$

subject to $\dot{K} + \dot{B} = Y + rB - C - T$ given $B(0)$, $K(0)$ and a non-Ponzi game condition. The production function be $Y = K^\alpha G^{1-\alpha}$ with $0 < \alpha < 1$. (Hint: G and T are taken by the consumer as externalities). Notation: consumption, C, physical capital stock K, government debt B and r is the interest rate. The government budget constraint is $\dot{B} = G - T + rB$. There are two policy instruments: taxes are defined as $T = \tau Y$ and there is a rule of keeping the debt ratio constant as $B/Y = \bar{b} > 0$.

(a) Write the DGE in (z, g) where $z = C/K$ and $g = G/Y$ and provide an intuition why this represents the detrended DGE dynamic system.

(b) Find the long-run growth rate.

(c) Study the growth facts associated to the

Growth and the environment

1. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N - P(t)$, where μ is the renewal rate and P is the use of the resource in production. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods, which are used only in consumption. We assume a centralized economy in which the central planner maximises the utility function

$$\int_0^\infty \left(\ln\left(C(t)\right) + \varphi \ln\left(N(t)\right)\right) e^{-\rho t} dt$$

   where the rate of time preference, ρ, and the utility weight associated by consumers to the environment, φ, are both positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} e^{-\rho t} > 0$.

   (a) Write the first order conditions for optimality.

   (b) Prove that the optimal level for the natural resource is $N(t) = N_0 e^{\gamma t}$ where $\gamma = \mu - A\rho/(A+\varphi)$.

   (c) What implications on the growth facts can we draw from this model ?

2. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N - P(t)$, where μ is the renewal rate and P is the use of the resource in production. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods, which are used only in consumption. We assume a decentralised economy in which a consumer with weight $0 < \alpha < 1$ only consider her/his effect on total demand, $D(t) = C(t)^\alpha \mathbf{C}(t)^{1-\alpha}$ where C is the representative agent consumption and $\mathbf{C}$ is the aggregate consumption.

   The representative consumer maximises the utility function

$$\int_0^\infty \ln\left(C(t)\right) e^{-\rho t} dt$$

   where the rate of time preference, ρ is positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} N(t)e^{-\rho t} > 0$.

   (a) Write the first order conditions for optimality for the representative consumer as a dynamic system in (C, N).

(b) Write the dynamic system for the aggregate economy and solve it.

(c) Discuss the implications for the growth facts that we can draw from this model ?

3. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N(t)^{\alpha} X(t)^{1-\alpha} - P(t)$, where μ is the renewal rate and P is the use of the resource in production, X is the expenditure in environmental preservation and $\alpha \in (0,1)$. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods. The final good is used in consumption and environmental preservation, such that the equilibrium condition $Y = C + X$ holds. We assume a centralized economy in which the central planner has the optimality criterium

$$\max_{C,X} \int_0^{\infty} \ln\left(C(t)\right) e^{-\rho t} dt$$

where the rate of time preference, ρ, is positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} e^{-\rho t} > 0$.

(a) Write the first order conditions for optimality as a system in (Q, N), where Q is the co-state variable.

(b) Find the optimal solution for $N(t)$ (hint: reduce the dimensionality of the system by defining $V(t) = Q(t)N(t)$).

(c) What implications on the growth facts can we draw from this model ?