Growth and natural resources

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Assumptions

- the natural resource is renewable;
- production uses a natural resource, but that use depletes its stock;
- technical progress takes the form of dematerialization
- the natural resource has an amenity value for the consumer

Conclusions

- the feasible growth rate is limited by the rate of technical progress and the sum of the rate of techincal progress and the rate of regeneration of the natural resource
- then growth with positive growth rates is sustainable

The structure of the economy

Product market

- production function

$$Y(t) = A(t)P(t)$$

  A is TFP and P is resource depletion

- (exogenous) technical progress

$$A(t) = A(0)\, e^{\gamma_A t}$$

- equilibrium in the product market

$$Y(t) = C(t)$$

- **then**

$$P(t) = \frac{C(t)}{A(0)}\, e^{-\gamma_A t}$$

  technical progress involves dematerialization

Natrural resource dynamics

- natural resource accumulation equation

$$\dot{N} = \mu N(t) - P(t),\ \mu > 0$$

  where $N(0) = N_0$ is given
- **then**

$$\dot{N} = \mu N(t) - \frac{C(t)}{A(0)} e^{-\gamma_A t}$$

Detrending

- we assume that consumption and the stock of natural resources can be written as

$$C(t) = c(t)e^{\gamma t},\ N(t) = n(t)e^{\gamma_n t}$$

- **then** from the resource accumulation equation

$$\dot{n} = (\mu - \gamma_n)n(t) - A(0)^{-1}c(t)e^{(\gamma - \gamma_A - \gamma_n)t}$$

- if we make $\gamma = \gamma_A + \gamma_n$ then we get the autonomous ODE

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}c$$

Consumers' preferecences

- the instantaneous utility function

$$u(C,N) = \frac{(CN^{\varphi})^{1-\sigma}}{1-\sigma}$$

- observtions: φ parameterizes the amenity services provided by natural resources; observe that the utility function is homogenous of degree $(1-\sigma)(1+\varphi)$
- in detrended variables, we get

$$u(c,n) = e^{\gamma_u t}\frac{(cn^{\varphi})^{1-\sigma}}{1-\sigma}$$

where

$$\gamma_u = (1-\sigma)[\gamma_A + (1+\varphi)\gamma_n]$$

The problem

- Planner's problem

$$\max_{(c(t))_{t\in[0\infty)}} \int_0^\infty \frac{(c(t)n(t)^\varphi)^{1-\sigma}}{1-\sigma} e^{-\rho^* t}$$

where

$$\rho^* \equiv \rho - \gamma_u = \rho - (1-\sigma)[\gamma_A + (1+\varphi)\gamma_n]$$

subject to

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}c$$

given $N(0) = N_0$ and asymptotically $\lim_{t\to} N(t) \geq 0$.

- **assumption**

$$(1-\sigma)(1+\varphi)\mu < \rho - (1-\sigma)\gamma_A < (1+\varphi)\mu \qquad \text{(A)}$$

- this guarantees sustainability and positive growth

Optimality conditions

- optimal consumption

$$A(0)\,c(t)^{-\sigma}\,n(t)^{\varphi(1-\sigma)} = q(t)$$

- Euler equation

$$\begin{aligned}\dot{q} &= q(t)(\rho^* - \mu + \gamma_n) - \varphi\, c(t)^{1-\sigma}\, n^{\varphi(1-\sigma)-1} \\ &= q(t)\left(\rho^* - \mu + \gamma_n - \frac{\varphi}{A(0)}\frac{c}{n}\right)\end{aligned}$$

- transversality equation

$$\lim_{t\to\infty} q(t)\,n(t)\,e^{-\rho^* t} = 0$$

- contraints

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}\,c$$

and $N(0) = N_0$

The MHDS

- as
$$-\sigma \frac{\dot{c}}{c} + \varphi(1-\sigma)\frac{\dot{n}}{n} = \frac{\dot{q}}{q}$$
- we can get the MHDS for (c, n)
$$\begin{aligned} \frac{\dot{c}}{c} &= \frac{\mu(1+\varphi(1-\sigma)) + (1-\sigma)\gamma_A - \sigma\gamma_n - \rho}{\sigma} + \varphi \frac{c}{A(0)n} \\ \frac{\dot{n}}{n} &= \mu - \gamma_n - \frac{c}{A(0)n} \end{aligned}$$

Long-run rate of growth

- solving $\frac{\dot{c}}{c} = 0$ and $\frac{\dot{n}}{n} = 0$ for γ_n and c/n, we get:
- the long-run rate of growth

$$\bar{\gamma}_n = \frac{(1+\varphi)\mu + (1-\sigma)\gamma_A - \rho}{\sigma(1+\varphi)}$$

- and the long-run consumption-resources ratio

$$\frac{\bar{c}}{\bar{n}} = A(0)(\mu - \bar{\gamma}_n)$$

- as $\bar{y} = \bar{c}$ from the product market equilibrium condition then

$$\bar{y} = A(0)(\mu - \bar{\gamma}_n)\bar{n}$$

Long-run rate of growth

- **Proposition**: if assumption (**??**) holds then

$$0 < \bar{\gamma}_n < \mu$$

- proof:
- $\bar{\gamma}_n > 0$ iff and only iff

$$(1+\varphi)\mu > \rho - (1-\sigma)\gamma_A$$

- $\bar{\gamma}_n < \mu$ iff and only if

$$(1+\varphi)\mu + (1-\sigma)\gamma_A > \rho + \mu\sigma(1+\varphi)$$

  which is equivalent to

$$\rho - (1-\sigma)\gamma_A > (1-\sigma)(1+\varphi)\mu$$

Transitional dynamics

- Defining $z(t) \equiv A(0)\dfrac{c(t)}{n(t)}$ and substituting $\gamma_n = \bar{\gamma}_n$ into the MHDS we get
$$\frac{\dot{z}}{z} = (1+\varphi)(z-\bar{z})$$
where $\bar{z} = \mu - \bar{\gamma}_n$
- as the equation is unstable, the transversality condition only holds if $z(t) = \bar{z}$ for $t \in [0, \infty)$
- as $\bar{z} = z(0)$ we set
$$\bar{c} = \bar{y} = A(0)(\mu - \gamma_n)N_0$$

Growth facts

- the long run growth rate is

$$\bar{\gamma} = \gamma_A + \bar{\gamma}_n$$

- if the assumption on the parameters holds then the growth rate is limited by the natural renewal rate and the growth in dematerialization

$$\gamma_A < \gamma < \gamma_A + \mu$$

- the long run GDP level is

$$\bar{y} = \bar{c} = A(0)(\mu - \bar{\gamma}_n)N_0 = A(0)N_0 \frac{(\sigma - 1)((1+\varphi)\mu + \gamma_A) + \rho}{\sigma(1+\varphi)}$$

- there is no transitional dynamics