# R&D and growth: directed technnical change

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# Technology and employment: recent developments

- There is **polarization** in the labor market in the manufacturing sector:
  - increase in wages and employment in the lower and higher levels of skill (and wages),
  - but a reduction in both employment and wages in the middle ranks
- Two possible **explanations**:
  - **automation**: effects of technology in which automation tends to make intermediate levels of labor substitutable by machines (robots);
  - **globalization**: the supply chain of multinational firms tends to de-localize segments of the production chain requiring with skills but which are costly and which can be done in countries with a relative high level of education and lower wages

# Technology and employment: recent developments

- Most empirical studies reveal that the effect of technology is dominant
- There are mixed predictions on the future impact of automation
- In any case there is evidence on the existence of a **technological bias** regarding its effects on the labor market and on growth.
- I will present next a benchmark **directed technical change model**

# Evidence on wages and technical progress

- **Evidence**: in the last century (differently from the XVIII and XIX centuries) there was a **positive** correlation between the relative wage and employment between skilled and un-skilled labor

$$\frac{\omega_H}{\omega_L} \text{ and } \frac{L_H}{L_L}$$

  where $H$=high skilled and $L$= low skilled

- **Puzzle**: the increase in education has made $L_H/L_L$ increase and therefore one would expect a decrease not an increase in $\omega_H/\omega_L$.

# Explaining the puzzle

- The puzzle can only be explained if the productivity of $L_H/L_L$ increased as a consequence of a **skill-bias in technical progress**, through the TFP $A(.)$
- Let
$$\frac{\omega_H}{\omega_L} = f\left(\frac{L_H}{L_L}, A\left(\frac{L_H}{L_L}\right)\right)$$
where $f_1 < 0$ and $f_2 > 0$ and $\frac{\partial A}{\partial (L_H/L_L)} > 0$
- then
$$\frac{d(\omega_H/\omega_L)}{d(L_H/L_L)} > 0$$
only if
$$\frac{\partial A}{\partial (L_H/L_L)} \text{ dominates}$$
- Under which conditions does this hold ?

# Directed technical change model

Acemoglu (2002) and others **extended the expansions of variety model** by considering:

- two intermediate good sectors: sectors producing **high tech** and **low tech** goods
- HT and LT sectors use machines which are complements to skilled or unskilled labor
- R&D are performed by potential producers of new machines which are skilled-complements
- **bias in technological change** is measured by the relative growth of the number of varieties of skilled-labour complementary machines versus the unskilled-labour complementary machines induced by R&D activities;
- R&D and entry are as in the expansion of varieties model

# The model

We assume a decentralized economy with:

- Consumers who work and own the firms
- A final-good production sector (competitive)
- Two intermediate-good producers for high-tech and low-tech inputs (competitive)
- A continuous of high-tech machine producers (monopolists)
- A continuous of low-tech machine producers (monopolists)
- R&D innovator who have to decide in which sector to enter (HT or LT)

# Consumers

## Problem and f.o.c

- Problem:

$$\max_{C} V[C] = \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{W} = \omega_L(t) L_L + \omega_H(t) L_H + r(t) W(t) - C(t)$$

given $W(0) = W_0$.

- f.o.c

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r(t) - \rho) \tag{1}$$

$$\dot{W} = \omega_L(t) L_L + \omega_H(t) L_H + r(t) W(t) - C(t) \tag{2}$$

# Producer of the final good

## The problem

- They are price-takers in all markets and use two types of input (they assemble two jobs)
- Their problem is, for every moment $t$

$$\max_{Y_L, Y_H} \pi(t),\ \pi(t) \equiv Y(t) - P_L(t)\, Y_L(t) - P_H(t)\, Y_H(t)$$

- the production function (constant elasticity of substitution) is

$$Y(t) = \left[ A_L\, Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + A_H\, Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$Y_j$ are intermediate goods of technology level $j \in H, L$

- $\varepsilon$ is the elasticity of substitution between $H$ and $L$ inputs

# Producer of the final good

## F.o.c

- the optimal production is

$$Y_j(t) = \left(\frac{P_j(t)}{A_j}\right)^{-\varepsilon} Y(t), \text{ for } j = L, H \tag{3}$$

- the following restriction holds

$$\left[A_L^{\varepsilon} P_L(t)^{1-\varepsilon} + A_H^{\varepsilon} P_H(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \tag{4}$$

# Producers of intermediate goods

## The problem

- Two types of goods and sectors: high-tech and low tech intermediate goods
- The problem for the competitive (i.e., price-taker) producer $j = L, H$ is

$$\max_{L_j, [x_j(v,t)]_{v \in [0, N_j(t)]}} \pi_j(t),$$

where the profit in sector $j = LH$ is

$$\pi_j(t) = P_j(t) \left( Y_j(t) - \omega_j(t) L_j \right) - \int_0^{N_j(t)} p_j^x(v,t) x_j(v,t) \, dv, \;\; j = L, H.$$

# Producers of intermediate goods

## The technology

- The production function for $j = L, H$ is

$$Y_j(t) = \frac{1}{1-\beta} \left( \int_0^{N_j(t)} x_j(v,t)^{1-\beta} dv \right) L_j^{\beta}, \; j = L, H$$

  where $x_j(v,t)$ machines of variety $v \in [0, N_j(t)]$ complement to factor $L_j$
- $\beta$ is the share of labor in the production of both goods $j = L, H$, where the elasticity of substitution is equal to one
- Assumption: $0 < \beta < 1$

# Producers of intermediate goods

## The f.o.c

- firms equalize the real wage to the marginal product of the type of labour they employ

$$\omega_j(t) = \beta \frac{Y_j(t)}{L_j}, \; j = L, H$$

- the demand for the machines of type $v \in [0, N_j(t)]$ for sector $j = L, H$ is a linear function of labour

$$x_j(v,t) = \left( \frac{P_j(t)}{p_j^x(v,t)} \right)^{1/\beta} L_j, \;\; v \in [0, N_j(t)], \; j = L, H. \tag{5}$$

# Producers of skilled-complementary machines

## Two-stage problem

- Again producers of machines $v \in [0, N_j(t)]$ have a monopoly power, but have to engage in R&D activities before they start to produce.
- Entry decision, i.e., increasing $N_j(t)$: free entry condition.
- Production phase, if they entry
- **We have to operate backwards in order to determine the benefits of entry**

# Producers of skill-complementary machines

## The problem of an incumbent in producer $v$ of sector $j = L, N$

- The problem

$$\max_{p_j^x(v,t)} \pi^{x_j}(v,t) \; v \in [0, N_j(t)], \; j = L, H$$

- **Assumption**: it has a symmetric marginal cost $\psi = 1 - \beta$
- Then

$$\pi_j^x(v,t) = (p^{x_j}(v,t) - \psi) x_j(v,t)$$

- and $x_j(v,t)$ is given by equation (5)

# Producers of skill-complementary machines

## The problem of an incumbent in producer $v$ of sector $j = L, N$

First order conditions for optimality:

- arbitrage condition

$$x_j(v,t) - (p^{x_j}(v,t) - \psi)\frac{x_j(v,t)}{\beta p^{x_j}(v,t)} = 0$$

- because there is symmetry in costs

$$p^{x_j}(v,t) = \frac{\psi}{1-\beta} = 1,\ v \in [0, N_j(t)],\ j = L, H.$$

# Implications

- If we substitute in equation (5), we get the production of intermediate R&D products

$$x_j(v,t) = x_j(t) = P_j(t)^{1/\beta} L_j, \; v \in [0, N_j(t)], \; j = L, H,$$

  which is symmetric across varieties.

- Then

$$\pi^{x_j}(v,t) = \pi^{x_j}(t) = \beta P_j(t)^{1/\beta} L_j, \; v \in [0, N_j(t)], \; j = L, H, \tag{6}$$

- and the output of $j$-complementary intermediate products

$$Y_j(t) = \frac{1}{1-\beta} P_j(t)^{(1-\beta)/\beta} N_j(t) L_j, \; j = L, H,$$

  are also symmetric across varieties.

# Value of entry in sector $j$

- The value of introducing a new variety of machines which is $j$-complementary is

$$V_j(t) = \int_t^{\infty} \pi^{x_j}(s) e^{-\int_t^s r(\tau)\, d\tau} ds, \;\; v \in [0, N_j(t)], \;\; j = L, H,$$

(because of symmetry)

- differentiating

$$\dot{V}_j(t) = -\pi^{x_j}(t) + r(t)\, V_j(t), \;\; j = L, H.$$

- Then

$$r(t) = \frac{\pi^{x_L}(t) + \dot{V}_L(t)}{V_L(t)} = \frac{\pi^{x_H}(t) + \dot{V}_H(t)}{V_H(t)} \tag{7}$$

because the real interest rate, $r(t)$ is determined for the aggregate economy, and is common to all sectors.

# Costs of entry in sector $j$

- The technology for introducing the innovation is of the lab-equipment type, that is

$$\dot{N}_j = \eta_j Z_j(t),\ j = L, H$$

  where $Z_j$ is the costs of entry

- The costs per entrant are

$$\frac{Z_j(t)}{\dot{N}_j} = \frac{1}{\eta_j},\ j = L, H$$

  are symmetric to the barriers to entry in sector $j$.

# Free entry condition in sector $j$

- If
$$V_j(t) < \frac{Z_j(t)}{\dot{N}_t} \text{ if } Z_j(t) = 0$$
then there is no entry, and
$$V_j(t) = \frac{Z_j(t)}{\dot{N}_t} \text{ if } Z_j(t) > 0$$

- **There is entry** in sector $j$ ($Z_j(t) > 0$) if and only if
$$\eta_j V_j(t) = 1, \; j = L, H.$$

# Entry in any sector

## Arbitrage condition

- If there are expenditures on R&D in both sectors, $Z_L(t) > 0$ and $Z_H(t) > 0$, then: the the arbitrage condition is

$$\eta_L V_L(t) = \eta_H V_H(t) = 1$$

- **Implications**:
  1. $\dot{V}_j = 0$ for $j = L, H$.
  2. from equations (6) and (7)

$$r = \frac{\pi_L}{V_L} = \eta_L \pi^{x_L} = \beta \eta_L L_L P_L^{1/\beta}$$

and

$$r = \frac{\pi_H}{V_H} = \eta_H \pi^{x_H} = \beta \eta_H L_H P_H^{1/\beta}$$

  3. then

$$P_j(t) = P_j = \left(\frac{r}{\beta \eta_j L_j}\right)^{\beta},\ j = L, H$$

# Interest rate and substitution between factors

- This equation together with equation (4)

$$\left[A_L^{\varepsilon} P_L(t)^{1-\varepsilon} + A_H^{\varepsilon} P_H(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1$$

  allows us to determine the interest rate as a constant

$$\boxed{r = \beta \left(A_L^{\varepsilon} (\eta_L L_L)^{\sigma-1} + A_H^{\varepsilon} (\eta_H L_H)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}} \tag{8}$$

- where

$$\boxed{\sigma \equiv 1 + (\varepsilon - 1)\beta}$$

  is the elasticity of substitution between the two factors.
- Observe that $\sigma > 1$ only if $\varepsilon > 1$ (i.e., the production of the final good is not Cobb-Douglas)

# Relative-price of skill-complementar inputs

- The relative price of the skilled-complementary relative to the unskilled-complementary input

$$\boxed{p(t) = \frac{P_H(t)}{P_L(t)} = \left(\frac{\eta_H L_H}{\eta_L L_L}\right)^{-\beta}}$$

- then, profits $\pi_j$ are constant
- the $j-$complementary intermediate inputs are

$$\boxed{Y_j(t) = \phi_j N_t(t),\ j = L, H} \tag{9}$$

where

$$\phi_j = \frac{1}{1-\beta}\left(\frac{r}{\eta_j}\right)^{1-\beta} L_j^{\beta}\ j = L, H.$$

# BGP: growth rate

- The BGP growth rate is obtained from the detrended Euler equation, (1), as

$$\boxed{\gamma^* = \frac{r^* - \rho}{\theta}}$$

where $r^* = r(L_H, L_L)$ is given in equation (8).

- in this version of the model there are scale effects (an increase in both $L_L$ and $L_H$ increases the growth rate)

$$\frac{\partial r}{\partial L_j} = A_j^{\epsilon} \eta_j \beta \left( A_L^{\epsilon} \left( \frac{\eta_L L_L}{\eta_j L_j} \right)^{\sigma - 1} + A_H^{\epsilon} \left( \frac{\eta_H L_H}{\eta_j L_j} \right)^{\sigma - 1} \right)^{\frac{2-\sigma}{\sigma-1}} > 0, \; j = L$$

# BGP: bias in technical progress

- the **bias in technical progress** can be measured by

$$\boxed{n \equiv \frac{N_H}{N_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\sigma} \left(\frac{A_H}{A_L}\right)^{\varepsilon} \left(\frac{L_H}{L_L}\right)^{\sigma-1}}$$

**Exercise**: prove this using equation (3) and (9).

- the relative wage premium for skilled workers

$$\boxed{\omega \equiv \frac{\omega_H}{\omega_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\varepsilon\beta} \left(\frac{A_H}{A_L}\right)^{\varepsilon} \left(\frac{L_H}{L_L}\right)^{\varepsilon\beta-1}}$$

- the relative price of high-tech goods

$$\frac{P_H}{P_L} = \left(\frac{\eta_H L_H}{\eta_L L_L}\right)^{-\beta}$$

# Conclusions

- there is a high-tech biased technical progress only if $\sigma > 1$:

$$\frac{\partial n}{\partial (L_H/L_L)} = (\sigma - 1)\frac{n}{(L_H/L_L)} > 0$$

- solution to our initial puzzle only if $\varepsilon\beta > 1$

$$\frac{\partial (\omega_H/\omega_L)}{\partial (L_H/L_L)} = (\varepsilon\beta - 1)\frac{(\omega_H/\omega_L)}{(L_H/L_L)} > 0$$

- reduction in the relative price of high-tech goods (for any $\sigma$)

$$\frac{\partial (P_H/P_L)}{\partial (L_H/L_L)} < 0$$

# References

- Textbooks: (Acemoglu, 2009, ch. 15), (Aghion and Howitt, 2009, ch. 8)
- Empirical application: Gil et al. (2019) the model can only be reconciled with data if we assume there is vertical innovation and there are differentiable barriers to entry, in which entry in the HT sector is costlier.


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