Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Growth Economics
2019-2020

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Exam. **Época Recurso** (Re-sit exam)
3.7.2020
First part: 18.00h-19.00h

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
  1. Please provide a **very short explanation** of your reasoning. In its absence, your response, even if correct, may be discounted.
  2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Points: 1(a) - 1, 1(b) - 2 , 1(c) - 1, 2(a) - 2, 2(b) -2, and 2(c) - 2.
- Your exam will only be considered if it is uploaded in Aquila between 18:00 and 19:05.

1 Assume a Solow economy in which the production technology is given by

$$Y = K^{\alpha}\, S^{\beta}\, L^{1-\alpha-\beta}$$

for $0 < \alpha < 1$ and $0 < \beta < 1-\alpha$, where K and S denote private physical capital and public infrastructures, respectively, and L denotes the labor input. The stock of public infrastructures is assumed to be constant and the initial level for the other inputs, $K(0) = k_0 > 0$ and $L(0) = l_0 > 0$ are given. The labor input is equal to the population and grows as $\dot{L} = n\,L$, for $n > 0$. The investment expenditure is $I = \dot{K} + \delta\,K$, where $\delta > 0$ is the rate of depreciation. The consumption function is $C(t) = c\,Y(t)$, where the marginal (equal to the average) propensity to consume is $0 < c < 1$.

(a) Defining the capital intensity by $k \equiv \dfrac{K}{L}$, obtain the accumulation equation for k.

(b) Characterize analytically and geometrically the solution of the accumulation equation. Solve the linearized differential equation for k.

(c) What will be the growth and level effects, on the per-capita output, of a permanent increase in the relative stock of public infrastructures $s \equiv \dfrac{S}{L}$?

2 Consider the same production function, consumption function, and population growth as in the previous question. However, now consider that the government decides to increase and

repair public infrastructures, but wants to keep a balanced budget by charging an income tax. In particular: (1) the government expenditure, G, be given by $G = \dot{S} + \delta S$, where δ is the depreciation rate of public infrastructures; and (2) the fiscal rule entails: $G = \tau Y$ at all times, where $\tau \in [0, 1-c)$ is the income tax rate.

(a) In this case, the dynamics of the economy is determined from the system of coupled differential equations over (k, s)

$$\begin{aligned} \dot{k} &= (1 - c - \tau)\, y(k, s) - (\delta + n)\, k, \\ \dot{s} &= \tau\, y(k, s) - (\delta + n)\, s \end{aligned}$$

where $y = y(k, s)$ is the per-capita product. Prove this.

(b) Find the steady state of the previous system. Characterize the dynamics of the previous model (tip: draw the phase diagram). Will there be long-run growth ?

(c) What will be the effects of increasing the tax rate, τ, over the long-run per-capita product level and the ratio s/k ? Provide an intuition for your results.