The Romer model: growth and externalities

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The Romer kK model

- It has the same assumptions as the AK model with the exception of the technology
- There are **externalities in production**

$$Y = AK^{\alpha}\mathbf{K}^{\beta}$$

  where K is capital owned by firms and $\mathbf{K}$ is a production externality
- **Assumption**: there are decreasing returns at the private level but there are **constant** returns at the aggregate level $0 < \alpha < 1$ and $\alpha + \beta = 1$

The Romer kK model

- Meaning of the externality:
  - K can be seen as private capital and $\mathbf{K}$ as public (or social) capital
  - Public or social capital increases the productivity of firms, even if firms cannot decide on its level
  - Only K is relevant for deriving firm's incentives to invest
- We will see that the government can induce the internalization of externalities through a tax/subsidy policy

The Romer kK model

- This introduces a distinction between
  - decentralized economy
  - centralized economy
- Next:
  - We solve the model for the **decentralized** economy (externalities not internalized)
  - We solve the model for a **centralized** economy (externalities internalized)
  - Discuss optimal policy: is it possible to design an **optimal policy in a decentralized economy**, that is a policy that **induces internalization** of externalities ?

Decentralized economy

Decentralized economy

Representative agent's problem

- The representative agent problem

$$\max_{[C(t)]_{t\geq 0}} \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK^{\alpha}\mathbf{K}^{\beta} - C - \delta K,$$

- boundary condition $\lim_{t\to\infty} k(t)h(t) \geq 0$

Solving the representative agent's problem

Solution by the Pontriyagin's maximum principle

- The current-value Hamiltonian

$$H(C, K, Q) = \frac{C^{1-\theta} - 1}{1-\theta} + Q\left(AK^{\alpha}\mathbf{K}^{\beta} - C - \delta K\right)$$

- The first order optimality conditions

$$\frac{\partial H}{\partial C} = 0 \iff C^{-\theta} = Q$$

$$\dot{Q} = \rho\, Q - \frac{\partial H}{\partial K} \iff \dot{Q} = \left(\rho + \delta - \alpha A K^{\alpha-1}\mathbf{K}^{\beta}\right) Q$$

$$\lim_{t\to\infty} K(t)\, Q(t)\, e^{-\rho t} = 0$$

- the admissibility conditions

$$\dot{K} = A\, K^{\alpha}\mathbf{K}^{\beta} - C - \delta K,$$

$$K(0) = k_0, \;\; \text{given}, \;\; \mathrm{t} = 0$$

Decentralized economy

Macroeconomic equilibrium

- first order optimality conditions for the agent

$$\begin{aligned}
\dot{C} &= \frac{C}{\theta}\left(\alpha A K^{\alpha-1}\mathbf{K}^{\beta} - (\rho+\delta)\right), && (1)\\
\dot{K} &= A K^{\alpha}\mathbf{K}^{\beta} - C - \delta K, && (2)\\
K(0) &= k(0) && (3)\\
0 &= \lim_{t\to+\infty} C(t)^{-\theta} K(t) e^{-\rho t} && (4)
\end{aligned}$$

- macroeconomic consistency condition

$$K(t) = \mathbf{K}(t).$$

Decentralized economy

Macroeconomic equilibrium

- If we assume that $\alpha + \beta = 1$ we get

$$\begin{aligned}
\dot{K} &= (A-\delta)K - C, \\
\dot{C} &= \frac{C}{\theta}\left(\alpha A - (\rho+\delta)\right), \\
K(0) &= k(0) \\
0 &= \lim_{t \to +\infty} C(t)^{-\theta} K(t) e^{-\rho t}
\end{aligned}$$

- which is very similar to the AK model with the **exception of the rate of return for capital,** $r = \alpha A$ **instead of** $r = A$

Decentralized economy

Growth facts

- The rate of growth is

$$\bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- the consumption-capital ratio is

$$\frac{\bar{c}}{\bar{k}} = \beta_d,$$

  for

$$\beta_d \equiv A - \delta - \bar{\gamma}_d = \frac{1}{\theta}\left(A(\theta - \alpha) + \rho + \delta(1-\theta)\right) > 0$$

- the solution for output is

$$Y(t) = Ak_0 e^{\gamma_t t}$$

Centralized economy

Centralized economy

Planner's problem

- The problem:
  - the planner has the **same utility** function as the agent (we have a representative agent economy)
  - but it considers the **total capital**, and not only the privately owned as the individual agent
  - that is, externalities are internalized. This means that the production function is

$$Y = AK^{\alpha}K^{\beta} = AK, \text{ if } \alpha + \beta = 1$$

Centralized economy

Planner's problem

- Formally: the planner's problem is

$$\max_{[C(t)]_{t\geq 0}} \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK - C - \delta K,$$

- boundary condition $\lim_{t\to\infty} k(t)h(t) \geq 0$
- It is indeed the AK-model

Comparison between the decentralized and centralized economies

- the rate of growth in the centralized version is higher than in the decentralised version

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > \bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- but consumption is higher in the decentralised version

$$\left(\frac{C}{Y}\right)_d = \frac{A - \delta - \bar{\gamma}_d}{A} > \left(\frac{C}{Y}\right)_c = \frac{A - \delta - \bar{\gamma}_c}{A}$$

- rationale: in the decentralised economy the incentive to save is smaller because the private rate of return of capital is also smaller
(i.e., $r_c = \dfrac{\partial(AK)}{\partial K} = A > r_d = \dfrac{\partial(AK^{\alpha}\mathbf{K}^{\beta})}{\partial K} = \alpha A$)

Optimal government intervention

Decentralized economy with government intervention

Government intervention

- Assumption: there are two policy instruments - one distortionary (say taxes) and another non-distortionary (say expenditures)

$$T(t) = \tau\, Y(t),\;\; G = \bar{G}(t)$$

- **two possible policies**:
  distortionary taxes and lump sum transfers ($\tau > 0,\; G > 0$)
  or
  distortionary subsidies and lump sum taxes ($\tau < 0,\; G < 0$)
- assume the **government budget is balanced**

$$T(t) = G(t), \forall t \in [0, \infty)$$

Decentralized economy with government intervention

Representative agent's problem

- the representative agent problem

$$\max_{[C(t)]_{t\geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

  subject to

$$\dot{K} = (1-\tau) A K^{\alpha} \mathbf{K}^{\beta} - C - \delta K + G,$$

- boundary condition $\lim_{t\to\infty} k(t)h(t) \geq 0$

Decentralized economy with government intervention

Growth facts

- Using the same approach as before, the rate of growth is

$$\bar{\gamma}_g = \frac{\alpha(1-\tau)A - \delta - \rho}{\theta} > 0$$

- can the rate of growth of the centralized economy be reached ?

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > 0$$

- yes if the tax rate is

$$\alpha(1-\tau) = 1 \Rightarrow \tau = -\frac{1-\alpha}{\alpha} < 0$$

- Conclusion: the government intervention can introduce incentives such that the externality is internalised by agents, in this case, by a **distortionary subsidy and a lump-sum tax**.

References

- Romer (1986)
- (Acemoglu, 2009, ch. 11), (Aghion and Howitt, 2009, ch. 2) , (Barro and Sala-i-Martin, 2004, ch. 4)

Daron Acemoglu. *Introduction to Modern Economic Growth.* Princeton University Press, 2009.

Philippe Aghion and Peter Howitt. *The Economics of Growth.* MIT Press, 2009.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth.* MIT Press, 2nd edition, 2004.

Paul Romer. Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–37, October 1986.