Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Growth Economics
2016-2017

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Exam: **Época Normal**
6.6.2017 (18.00h-21.00h, 010 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. `[6 points]` Please answer **two** of the following three questions.
   (a) Comparison of growth episodes for different countries, or different periods for the same country, is popularly done by looking at differences in observed rates of growth of GDP. Which rates of growth of GDP are relevant and what can we conclude just by observing them ? In order to compare long-run trends which other information can be relevant, and which kind of conclusions can we draw ?
   (b) In modern economies there is an increase in the relative supply of high-skilled labor relative to low-skilled labor. However, there is a puzzle: in those economies we also observe an increase in the skill premium. Why is this a puzzle ? Which explanation to that puzzle is offered by directed technical change models ? (Hint: describe the structure of those models).
   (c) In models in which government expenditures generate a productive externality and the government budget is balanced, those expenditures are financed by an income tax. In most of these models the long-term rate of growth is a function of the tax rate, but the relationship is ambiguous. The ambiguity takes the form of a inverted-U relationship between the long-run growth rate and the tax rate. Provide an explanation for this type of behavior.

2. `[6 points]` Assume that the representative consumer solves the problem: $\max_{c,b}\{u(c,b) :\ c+\rho b \leq y\}$ where c is consumption, b is the birth rate, ρ is the cost of raising children and y is per capita income. Assume that the utility function is

$$u(c,b) = \ln(c) + \phi \ln(b),\ \phi > 0$$

   and the aggregate production function is Cobb-Douglas $Y = (AX)^{\alpha}L^{1-\alpha}$, with $0 < \alpha < 1$, where X is the stock of land, A is land-specific productivity and L is population. Population growth is $\dot{L}/L = b - m$, where the mortality rate, m, is constant and exogenous, and $L(0) = L_0 > 0$ is given. Land productivity grows at a rate $\gamma > 0$.
   (a) Defining $\ell \equiv L/A$, obtain a differential equation for ℓ.
   (b) Study the qualitative dynamics of the model. Provide an intuition for your results.
   (c) Derive the growth facts (long run growth rate, long run per capita output and transition dynamics). What are the effects of an increase in γ ?

3. `[8 points]` Consider a version of the Ramsey model with constant population where the objective utility functional for the central planner is:

$$\max_{c} \int_0^{\infty} ln(c(t) - \bar{c})e^{-\rho t}dt,$$

   where $\rho > 0$ and $\bar{c} > \frac{\rho}{\alpha}$ is a minimum level of consumption, subject to

$$\dot{k} \ = Ak(t)^{\alpha} - c(t),\ 0 < \alpha < 1$$

where c and k are the per capita consumption and capital stock. We also assume that $k(0) = k_0$ is given and that the stock of capital is bounded.

(a) Apply the Pontryiagin's principle and determine the optimality conditions as a dynamic system in (c, k).

(b) Draw the phase diagram.

(c) Determine the steady states and study their local stability properties.

(d) Find an approximate solution to the problem in the neighborhood of the steady state associated with a maximum consumption.

(e) Determine the effects of a permanent increase in productivity, A.