Economic Growth
Problem set 1: Toolkit

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Disclaimer: This problem set is provided as a help to self study, in an open academic spirit of providing potentially interesting problems. Solving them is not mandatory, however it is advisable because exams' questions will be in a large part similar to some of them. The instructor does not commit himself to provide the solutions of them all, but is available to help solving specific difficulties arising in efforts to actually solving them. Not all questions have been completely verified.

1. Consider the production function

$$Y = F(K) = AK,$$

   for $A > 0$ constant and K is the stock of capital

   (a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for K.

   (b) Assume that a producer problem is $\max_K \{AK - rK\}$ where r is the rate of return of capital. What can we say about the demand for capital for the firm ?

2. Consider the production function

$$Y = F(K, L) = AK^\alpha L^{1-\alpha}$$

   for $0 < \alpha < 1$, where A is constant, and K and L denote the stocks of physical and human capital.

(a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for each input, and substitution/complementarity of the inputs.

(b) Study the relationship between the two factors of production.

(c) Assume that a producer problem is $\max_K \{F(K,L) - rK - wL\}$ where r is the rate of return of capital and w is the wage rate. Determine the Hicksian demand functions and classify the two inputs.

3. Consider the production function

$$Y = F(K,L) = \left(\alpha \, (AK)^\varepsilon + (1-\alpha)(AL)^\varepsilon \right)^{\frac{1}{\varepsilon}}$$

for $\varepsilon < 1$, $0 < \alpha < 1$, where A is constant, and K and L denote the stocks of physical and human capital.

(a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for each input, and substitution/complementarity of the inputs.

(b) Study the relationship between the two factors of production.

(c) Assume that a producer problem is $\max_K \{F(K,L) - rK - wL\}$ where r is the rate of return of capital and w is the wage rate. Determine the Hicksian demand functions and classify the two inputs.

4. Consider the production function

$$Y(t) = \left[K^\alpha \, L^{1-\alpha} \right]^\beta K^{1-\beta}$$

for $0 < \alpha < 1$ and $0 < \beta < 1$ and K and L denote the stocks of physical and human capital.

(a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for each input, and substitution/complementarity of the inputs.

(b) Study the relationship between the two factors of production.

(c) Assume that a producer problem is $\max_K \{F(K,L) - rK - wL\}$ where r is the rate of return of capital and w is the wage rate. Determine the Hicksian demand functions and classify the two inputs.

5. Consider the production function

$$Y = F(K, L_u, L_s) = \left(K + L_u\right)^{\alpha} \left(A\, L_s\right)^{1-\alpha}$$

where $0 < \alpha < 1$ and K, L_u and L_s denote the stocks of physical, unskilled and skilled labor, and $A > 1$ is a measure of specific productivity of skilled labor.

(a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for each input, and substitution/complementarity of the inputs.

(b) Study the relationship between the two factors of production.

(c) Assume that a producer problem is $\max_K \{F(K, L) - r\, K - w\, L\}$ where r is the rate of return of capital and w is the wage rate. Determine the Hicksian demand functions and classify the two inputs.

6. Consider the production function

$$Y = F(K, H) = A K^{\alpha} \left[H + \alpha \beta K\right]^{1-\alpha}, \quad A > 0, \ 0 < \alpha < 1, \ \beta > -1$$

where K and H denote the stocks of physical and human capital.

(a) Classify the production function and study the properties of the production function, in particular regarding returns to scale, marginal returns for each input, and substitution/complementarity of the inputs.

(b) Study the relationship between the two factors of production.

(c) Assume that a producer problem is $\max_K \{F(K, L) - r\, K - w\, H\}$ where r is the rate of return of capital and w is the wage rate. Determine the Hicksian demand functions and classify the two inputs.