Advanced macroeconomics 2020-2021
Problem set 1: Ramsey model

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1 Ramsey: general

1 Discuss the effects of the decreasing marginal returns to capital and of the intertemporal substitution in consumption on the dynamics of the Ramsey model.

2* Discuss the effects of effects of anticipated permanent productivity shocks on the optimal path of the economy, according to the Ramsey model

3* Discuss the effects of effects of non-anticipated temporary productivity shocks on the optimal path of the economy, according to the Ramsey model

2 Ramsey

1 Consider a Ramsey model in which there is depreciation of capital, that is $\dot{k} = Ak^{\alpha} - c - \delta\, k$ where $\delta > 0$ and the utility function is $u(c) = \log(c)$ and the production function is Cobb-Douglas $y = f(k) \equiv A\, k^{\alpha}$, where all the variables are in per-capita terms.

(a) Solve the Ramsey problem by using the PMP

(b) Draw the phase diagram

(c) In this model there is a manifold passing through two two steady states one such that both k and c are positive an another one in which $c = 0$ and $f(k) = \delta k$ for $k > 0$. Explain why any trajectory over this manifold cannot be optimal.

(d) Perform a comparative dynamics exercise for an increase in δ. Provide one intuition for your results.

2 Consider a Ramsey model in which there an unfunded government expenditure $\dot{k} = Ak^{\alpha} - c - g - \delta\, k$ where $g > 0$ is a public transfer and $\delta > 0$ and the utility function is isoelastic $u(c) = \dfrac{c^{1-\theta} - 1}{1 - \theta}$.

(a) Solve the Ramsey problem by using the PMP.

(b) Is it possible to solve explictly the Ramsey using the DPP ?

(c) Perform a comparative dynamics exercise for an increase in g. Provide one intuition for your results.

3* Consider a Ramsey model with endogenous labour with additively separable preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{c^{1-\theta} - 1}{1-\theta} - \psi \frac{\ell^{1+\zeta}}{1+\zeta},\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

and

$$f(k,\ell) = A\, k^{\alpha}\, \ell^{1-\ell}$$

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

4* Consider a Ramsey model with endogenous labour with KPR preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{\Big(c\,(1 - \psi\,\ell^{\eta}\Big)^{1-\theta} - 1}{1-\theta}\ ,\ \theta > 0,\ \psi > 0,\ \eta > 0$$

and

$$f(k,\ell) = A\, k^{\alpha}\, \ell^{1-\ell}$$

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

5* Consider a Ramsey model with endogenous labour with GHH preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{1}{1-\theta}\Bigg(\Big(c - \psi \frac{\ell^{1+\zeta}}{1+\zeta}\Big)^{1-\theta} - 1\Bigg),\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

and

$$f(k,\ell) = A\, k^{\alpha}\, \ell^{1-\ell}$$

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

3 DGE

1 Consider a DGE economy in which the utility function is $u(c) = \log(c)$, there is a constant number of households N, there is no unemployment, and the technology for firms is CES

$$Y = A\left(\alpha K^{\eta} + (1-\alpha) L^{\eta}\right)^{\frac{1}{\eta}}$$

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant shocks in A. Provide one intuition for your results.

2 Consider a DGE economy in which the utility function is $u(c) = \log(c)$ and the production function is Cobb-Douglas in which the government raises an income tax and has a budget balanced fiscal policy. Denoting per capita government expenditure by g and the tax rate is denoted by τ and both are constant through time. The budget balance rule is $g = \tau\left(r(t)a(t) + w(t)\right)$. Assume that households supply labor inelastically and they have the budget constraint $\dot{a} = (1-\tau)\left(r(t)a(t) + w(t)\right) - c(t) + g(t)$.

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant increase in g. Provide one intuition for your results.