Foundations of Financial Economics *2019/20*
Problem set 1: two-period DGE

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1 Intertemporal utility functions

1. Consider the following intertemporal utility functions (IUF)

$$U(c_0,c_1)=\ln(c_0)\ +\beta\ln(c_1),\ \text{for } 0<\beta<1, \tag{1}$$

$$U(c_0,c_1)=\frac{c_0^{1-\phi}-1}{1-\phi}\ +\beta\frac{c_1^{1-\phi}-1}{1-\phi},\ \text{for } 0<\beta<1,\ \phi>0, \tag{2}$$

$$U(c_0,c_1)=-\frac{e^{-\zeta c_0}}{\zeta}\ +\beta\frac{e^{-\zeta c_1}}{\zeta},\ \text{for } 0<\beta<1,\ \zeta>0, \tag{3}$$

$$U(c_0,c_1)=\ln(c_0-\bar{c})\ +\beta\ln(c_1-\bar{c}),\ \text{for } 0<\beta<1,\ \bar{c}>0 \tag{4}$$

$$U(c_0,c_1)=\frac{c_0^{1-\phi}-1}{1-\phi}\ +\frac{\beta}{1-\phi}\Big(\frac{c_1}{c_0}\Big)^{\zeta(1-\phi)},\ \text{for } 0<\beta<1,\ \zeta>0,\ \phi>0 \tag{5}$$

$$U(c_0,c_1)=\ln(c_0)\ +\beta\ln\left(\Big(\frac{c_1}{c_0}\Big)^{\zeta}\right),\ \text{for } 0<\beta<1,\ \zeta>0 \tag{6}$$

$$U(c_0,c_1)=\ln(c_0)\ +\beta\ln(c_1-\eta\,c_0),\ \text{for } 0<\beta<1,\ 0<\eta<1 \tag{7}$$

$$U(c_0,c_1)=(1-\beta)\ln(c_0)\ +\beta\ln(c_1),\ \text{for } 0<\beta<1, \tag{8}$$

$$U(c_0,c_1)=\Big((1-\beta)c_0^{\eta}\ +\beta c_1^{\eta}\Big)^{\frac{1}{\eta}},\ \text{for } 0<\beta<1,\ \eta>0 \tag{9}$$

$$U(c_0,c_1)=\ln(c_0)\ +\beta\begin{cases}\dfrac{c_1^{1-\phi}-1}{1-\phi}, & \text{if } 0<c_1<c_0,\\ \ln(c_1), & \text{if } c_1\geq c_0,\end{cases}\ \text{for } 0<\beta<1,\ \phi>0 \tag{10}$$

For each utility function:

(a) Find the intertertemporal marginal rate of substitution, $IMRS_{0,1}$.

(b) Find the intertemporal elasticity of substitution, $IES_{0,1}$

(c) Discuss the implicit properties of the IUF concerning patience and intertemporal dependence. If needed, impose conditions for the existence of impatience and intertemporal substitution.

(b) Provide an economic intuition for your results.

2 Arrow-Debreu economy

1. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \log c_1,\ 0 < \beta < 1$$

   a) Specify the agent's problem. Define the general equilibrium.

   b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

   c) Find the equilibrium AD price. Provide an intuition.

2. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{c_0^{1-\theta}}{1-\theta} + \beta \frac{c_1^{1-\theta}}{1-\theta},\ 0 < \beta < 1,\ \theta > 0$$

   a) Specify the agent's problem. Define the general equilibrium.

   b) Characterize the implicit behavioral assumptions Solve the representative agent problem.

   c) Find the equilibrium AD price. Provide an intuition.

3. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = -\frac{e^{-\zeta c_0}}{\zeta} + \beta \left(-\frac{e^{-\zeta c_1}}{\zeta} \right),\ 0 < \beta < 1,\ \zeta > 0$$

   a) Specify the agent's problem. Define the general equilibrium.

   b) Characterize the implicit behavioral assumptions Solve the representative agent problem.

   c) Find the equilibrium AD price. Provide an intuition.

4. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log (c_0 - \bar{c}) + \beta \log (c_1 - \bar{c}),\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Provide one condition for the existence of an equilibrium. Find the equilibrium AD price. Provide an intuition.

5. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{(c_0 - \bar{c})^{1-\theta}}{1-\theta} + \beta \frac{(c_1 - \bar{c})^{1-\theta}}{1-\theta},\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Provide one condition for the existence of an equilibrium. Find the equilibrium AD price. Provide an intuition.

6. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \log \left(\left(\frac{c_1}{c_0} \right)^{\zeta} \right),\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium AD price. Provide an intuition.

7. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{c_0^{1-\theta}}{1-\theta} + \frac{\beta}{1-\theta} \frac{\left(\left(\frac{c_1}{c_0} \right)^{\zeta} \right)^{1-\theta}}{1-\theta},\ 0 < \beta < 1,\ \theta > 0,\ \zeta > 0.$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium AD price. Provide an intuition.

8. Consider a deterministic, two-period, representative-agent Arrow-Debreu endowment economy where the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \begin{cases} \log c_1 & \text{if } c_1 \geq c_0 \\ \dfrac{c_1^{1-\theta}}{1-\theta} & \text{if } 0 < c_1 < c_0 \end{cases}$$

for $0 < \beta < 1$ and $\theta > 1$.

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium AD price. Provide an intuition.

3 Finance economy

1. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \log c_1,\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.

2. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{c_0^{1-\theta}}{1-\theta} + \beta \frac{c_1^{1-\theta}}{1-\theta},\ 0 < \beta < 1,\ \theta > 0$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.

3. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = -\frac{e^{-\zeta c_0}}{\zeta} + \beta \left(-\frac{e^{-\zeta c_1}}{\zeta} \right),\ 0 < \beta < 1,\ \zeta > 0$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.

4. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log(c_0 - \bar{c}) + \beta \log(c_1 - \bar{c}),\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Provide one condition for the existence of an equilibrium. Find the equilibrium asset return. Provide an intuition.

5. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{(c_0 - \bar{c})^{1-\theta}}{1-\theta} + \beta \frac{(c_1 - \bar{c})^{1-\theta}}{1-\theta},\ 0 < \beta < 1.$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Provide one condition for the existence of an equilibrium. Find the equilibrium asset return. Provide an intuition.

6. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \log\left(\left(\frac{c_1}{c_0}\right)^{\zeta}\right),\ 0 < \beta < 1$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.

7. Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is

$$U(c_0, c_1) = \frac{c_0^{1-\theta}}{1-\theta} + \frac{\beta}{1-\theta} \frac{\left(\left(\frac{c_1}{c_0}\right)^{\zeta}\right)^{1-\theta}}{1-\theta},\ 0 < \beta < 1,\ \theta > 0,\ \zeta > 0.$$

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.

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$$U(c_0, c_1) = \log c_0 + \beta \begin{cases} \log c_1 & \text{if } c_1 \geq c_0 \\ \dfrac{c_1^{1-\theta}}{1-\theta} & \text{if } 0 < c_1 < c_0 \end{cases}$$

for $0 < \beta < 1$ and $\theta > 1$.

a) Specify the agent's problem. Define the general equilibrium.

b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.

c) Find the equilibrium asset return. Provide an intuition.