Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Mestrado em Economia Monetária e Financeira
Fundamentos de Economia Financeira
2016-2017

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Prova de avaliação: **Época Recurso**
3.7.2017 (18.00h-21.00h, 006 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. `[5 points]` Consider two potential investments projects, labelled A and B, that generate contingent profits. Let $\Pi^i_s = p^i y^i_s - c^i$ be the profit of project $i \in \{A, B\}$, at state of nature $s \in \{1, 2\}$, where p^i is the selling price, y^i_s is the contingent output and c^i is the cost. Consider the following data: and

Investments		$i = A$	$i = B$
selling price	p^i	$1 + \psi$	1
cost	c^i	1	1
output in state $s = 1$	y^i_1	$1 - a$	$1 + \epsilon\, a$
output in state $s = 2$	y^i_2	$1 + a$	$1 - \epsilon\, a$

assume that the two states of nature have equal probabilities, and that $0 < a < 1$, $0 < \varepsilon < 1$, $\psi > -1$ and $a^2 \left(3 - (1 + a^2)\varepsilon^2\right) < 1$. The projects are ranked by their value, with the value of project i be determined by $V^i = \mathbb{E}[u(\Pi^i)]$.

   (a) Assume that the agent has the utility function $u(\Pi) = \Pi$. How would the investor rank the projects ?

   (b) Now assume that the agent values the projects with the utility function $u(\Pi) = \Pi - \frac{1}{2}\,\Pi^2$. How would the investor rank the projects in this case?

   (c) Provide an intuition for the results you obtained in (a) and (b).

2. `[8 points]` Let information be given by a two-period binomial tree with probabilities

$$P = (\pi_1, \pi_2) = (\frac{3}{4}, \frac{1}{4})$$

for the two states of nature. Consider an homogeneous-agent endowment finance economy in which there are no arbitrage opportunities. The endowment sequence is $\{y_0, Y_1\}$ with

$$Y_1 = (y_{1,1}, y_{1,2}) = (y_0\,(1 - \gamma), y_0\,(1 + \gamma))$$

where $0 < \gamma < 1$. Further, assume that agent has the Bernoulli utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$, with $\theta > 0$. There are two financial assets with returns

$$R^1 = \begin{pmatrix} R^1_1 \\ R^1_2 \end{pmatrix} = \begin{pmatrix} 1 + i \\ 1 + i \end{pmatrix},\ R^2 = \begin{pmatrix} R^2_1 \\ R^2_2 \end{pmatrix} = \begin{pmatrix} 1 + i + \epsilon \\ 1 + i - \epsilon \end{pmatrix}$$

where $i > 0$ and $\epsilon > 0$.

   (a) Find the Sharpe index. Justify your reasoning.

   (b) Find the stochastic discount factor.

   (c) Would the Hansen-Jagannathan bound be satisfied ? Provide an intuition for your results.

3. [7 points] Consider an Arrow-Debreu (AD) economy with an information tree with two periods and $N > 1$ states of nature for the last period. There are $I > 1$ agents in the economy who are heterogeneous as regards the subjective probabilities associated to the two states of nature, π_s^i, and are homogeneous as regards preferences and endowments. The problem for agent $i \in \{1, \dots, I\}$ is to choose the optimal consumption sequence $\{c_0^i, C_1^i\}$, with $C_1^i = (c_{1,1}^i, \dots, c_{1,N}^i)$, to maximise the intertemporal utility functional

$$U^i(c_0^i, C_1^i) = \ln(c_0^i) \ + \beta \sum_{s=1}^{N} \pi_s^i \ln(c_{1,s}^i)$$

subject to the constraint $c_0^i + \sum_{s=1}^N q_s c_{1,s}^i = y_0 + \sum_{s=1}^N q_s y_{1,s}$, where q_s are the AD prices, and $y_{t,s}$ denotes the endowment for agent i at time t for the state of nature s.

(a) Define the AD general equilibrium for this economy.

(b) Solve agent i's problem.

(c) Find the equilibrium AD prices. Provide an intuition for your results (hint: compare with an analogous model in which there is homogeneity in information).

(d) Discuss the consequences of introducing heterogeneity in the endowments of both agents. Focus on both micro and aggregate consequences.