

Foundations of Financial Economics 2020/21

Problem set 6

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1 Production

1. Consider a two-period production economy where $Y_t = A_t K_t$, where K_t is the capital stock at the beginning of period $t = 0, 1$. Productivity is deterministic at $t = 0$ but it is stochastic at $t = 1$ such that $A_0 = a_0 > 0$, and $A_1 = (a_{1,1}, \dots, a_{1,N})$. Assume there is an Arrow-Debreu economy with homogeneous agents in which the representative agent has a intertemporally additive discounted von-Neumann-Morgenstern utility functional with $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ for $\theta > 0$.
 - (a) Specify the representative agent's problem. Find the first order conditions for an optimum. Justify.
 - (b) Define the general equilibrium for this economy and find the stochastic discount factor. Provide an intuition for your result.
2. Consider a two-period production economy where the supply of the good at $t = 0$ is $y_0 = K_0$ and in period 1 is $Y_1 = (y_{1,s})_{s=1}^N$, where $y_{1,s} = (1 + \gamma_s)K_1$ with K_t denoting the capital stock at the beginning of period $t = 0, 1$. Assume there is an Arrow-Debreu economy with homogeneous agents in which the representative agent has a intertemporally additive discounted von-Neumann-Morgenstern utility functional with a logarithmic Bernoulli utility function.
 - (a) Specify the representative agent's problem. Find the first order conditions for an optimum. Justify.
 - (b) Define the general equilibrium for this economy and find the stochastic discount factor. Provide an intuition for your result, by comparing with an endowment economy.