

R&D and growth: the variety expansion model

Paulo Brito
`pbrito@iseg.ulisboa.pt`

22.4.2020

R&D. ideas, competition and growth

- ▶ Technology and technical progress:
tech progress = systematic increase in TFP
- ▶ Exogenous and endogenous technical progress: endogenous
t.p = increase in TFP as a separate economic activity
- ▶ Types of endogenous technical progress: learning-by-doing,
and ideas
- ▶ Ideas: fundamental research and R&D:
R&D = ideas for profit

R&D. ideas, competition and growth

The problem with ideas: rivalry and excludability

- ▶ can be used by several people simultaneously (non-rivalry)
- ▶ once an idea is found it can be used by others (non-excludability)
- ▶ because coming up with ideas takes costs (effort, time, resources) this generates a problem of free-riding
- ▶ this is particularly serious on ideas that can have market value
- ▶ fundamental research has some characteristics of excludability and lack of market value

R&D. ideas, competition and growth

- ▶ Because of non-rivalry and non-excludability: there is a potential free-riding problem (difference between private benefits and costs for the developers of R&D)
- ▶ As R&D has costs, it can only take place under two environments:
 - ▶ market economy in which there is imperfect competition (patents or other type of rent generating mechanism)
 - ▶ in a centralized economy, where R&D costs can be internalized
- ▶ **R&D and growth:** there is a trade-off:
 - ▶ imperfect competition generates a reduction in the growth of productivity
 - ▶ but R&D generates growth
- ▶ This creates a role for policy: internalizing the externalities

R&D and endogenous growth theory

There are several strands in the literature:

- ▶ R&D takes the form of an expansion in the number of goods and industries (expansion of varieties model): goods can be producer goods or consumer goods
- ▶ R&D takes the form of an increase in quality through a process of creative destruction (Schumpeterian models): goods can be producer goods or consumer goods;
- ▶ R&D can be developed or imported
- ▶ R&D can be biased: can be complementary or substitutable with different types of inputs (v.g. labor) or tasks (job polarization)

Expansion of varieties model

Core assumptions

- ▶ Technical progress takes the form of an expansion in the number (variety) of products
- ▶ We assume that varieties relate to intermediary goods
- ▶ Technical progress it is materialized in the expansion of intermediate goods, i.e., creation of a new industry
- ▶ A new industry is created only after R&D activity takes place, and the incumbent firm is monopolist in its market
- ▶ R&D is related to the production of ideas
- ▶ As R&D has costs (proportional to the output generated by a new variety) it only takes place if the value of R&D is equal to the cost (free-entry condition)
- ▶ We consider two environments: (1) **in a decentralized economy R&D** ; (2) in a centralized economy R&D costs can be internalized

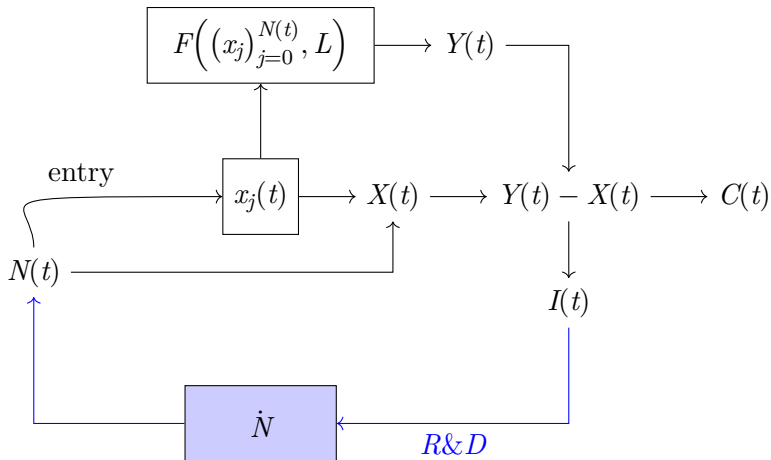
Results: implication for growth

- ▶ Without capital accumulation growth is generated by the expansion in varieties
- ▶ The rate of growth depends on the barriers to entry into R& D
- ▶ The decentralized economy is not Pareto optimal, meaning that a related centralized economy attains a higher rate of growth
- ▶ This is because the rate of return generated by R&D activities is lower in a decentralized than in the related centralized economy

Economic structure

- ▶ Environment:
 - ▶ there are two sectors: a competitive final good sector and a continuum of monopolistically competitive intermediate goods sectors
 - ▶ there is entry by creation of a new intermediate good product (=industry)
 - ▶ there is no capital accumulation
- ▶ Technology:
 - ▶ final good production uses labor and a continuum of intermediate goods
 - ▶ intermediate goods are the only reproducible inputs
 - ▶ the dynamics of output is generated by the variations in the number of intermediate inputs (varieties) which is the result of successful R&D (research and development=)

The mechanics of the model



Decentralized economy

The structure of the model

- ▶ Consumer problem
- ▶ Final producer problem
- ▶ Producers of intermediate goods (incumbents and entrants)
- ▶ Aggregation, balance sheet and market clearing conditions

The consumer problem

- ▶ Earns labor and capital income, consumes a final product and save
- ▶ they own firms (final good and intermediate good producers)
- ▶ The problem

$$\begin{array}{ll} \max_{(C(t))_{t \in [0, \infty)}} & \int_0^{\infty} \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \quad \theta > 0 \\ \text{s.t} & \end{array} \quad \text{(CP)}$$

$$\dot{W} = \omega(t)L + r(t)W(t) - C(t)$$

- ▶ The first order conditions

$$\dot{C} = \frac{C}{\theta} (r(t) - \rho)$$

$$\dot{W} = \omega(t)L + r(t)W(t) - C(t)$$

Producer of the final good

- Production function: Dixit and Stiglitz (1977)

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} x(j, t)^\alpha dj, \quad 0 < \alpha < 1$$

- L labor input
- $(x(j, \cdot))_{j \in [0, N(t)]}$ intermediate inputs, non-storable,
- $N(t)$ number of varieties
- Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^{N(t)} P(j, t)x(j, t) dj$$

Producers of the final good (cont)

- ▶ Buys labor and intermediate goods and sells a final good
- ▶ The problem:

$$\max_{L, (x(j,t))_{j \in [0, N(t)]}} \pi^P(t) \quad (\text{FGPP})$$

- ▶ Obs: they are price takers in all markets
- ▶ First order conditions: demand for labor and for intermediate goods

$$L^d = (1 - \alpha) \frac{Y(t)}{\omega(t)}$$
$$x^d(j, t) = \left(\frac{\alpha A}{P(j, t)} \right)^{\frac{1}{1-\alpha}} L, \quad j \in [0, N(t)]$$

Producers of intermediate goods

- ▶ Perform R&D activities allowing for the production of a new variety which they sell to final producers
- ▶ **Decision process** for the introduction of a new variety
 - ▶ before entry: perform R& D
 - ▶ entry decision: free entry condition
 - ▶ after entry: decide on the price of variety j , $P(j, t)$ (upon entry)
- ▶ **Solution** to the problem: work **backwards**
 - ▶ first: we determine the pricing policy assuming there was entry (incumbent's problem)
 - ▶ second: we determine entry (by using the free entry condition)

Producers of intermediate goods (cont)

Price decision after entry

- ▶ The profit of the producer of a variety $j \in (0, N(t)]$ is

$$\pi(j, t) = (P(j, t) - 1)x(j, t)$$

assuming a symmetric cost of production equal to 1

- ▶ where $x(j, t) = x^d(j, t)$ (solution of the FGPP)
- ▶ Then the profit after entry is

$$\pi(j, t) = (P(j, t) - 1) \left(\frac{\alpha A}{P(j, t)} \right)^{\frac{1}{1-\alpha}} L,$$

Producers of intermediate goods (cont)

Price decision after entry

- ▶ The first order condition:

$$\frac{\partial \pi(j, t)}{\partial P(j, t)} = 0 \Leftrightarrow P^*(j, t) = \frac{1}{\alpha} \quad \forall (j, t)$$

- ▶ then the demand for variety is **symmetric** (i.e, equal to all industries)

$$x^*(j, t) = x^* = (\alpha^2 A)^{\frac{1}{1-\alpha}} L, \text{ for any } j \in [0, N(t)]$$

- ▶ the profit is also **symmetric** across industries and constant

$$\pi^*(j, t) = \pi^* = \left(\frac{1-\alpha}{\alpha} \right) L (A \alpha^{2\alpha})^{\frac{1}{1-\alpha}} > 0$$

Producers of intermediate goods (cont)

Profits after entry

- Defining

$$\phi \equiv (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} L$$

- Then

$$\pi^* = \alpha(1 - \alpha)\phi$$

(pure-) profits are positive, symmetric and constant in time.

Entry

Value of entry

- ▶ The **value from producing a successful variety** j , if it is introduced (by entry) at time t , is a monopoly rent forever

$$v(j, t) = \max_{(P(j, s))_{s \in [t, \infty)}} \int_t^{\infty} \pi(j, s) e^{-R(s)} ds \quad (\text{IGPP})$$

- ▶ where the **market** discount factor is time-varying

$$R(s) = \int_t^s r(\tau) d\tau$$

- ▶ Introducing the previous result from an incumbent at industry j , $\pi(j, t) = \pi^*$, at the optimum we have

$$v^*(j, t) = v^*(t) = \pi^* \int_t^{\infty} e^{-R(s)} ds$$

- ▶ taking a time derivative yields

$$\dot{v}(t) = -\pi^* + r(t)v(t) \quad (1)$$

Aggregate output

- ▶ Using $x^*(j, t) = (\alpha^2 A)^{\frac{1}{1-\alpha}} L$ then

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} (x^*)^\alpha dj = \phi N(t)$$

because

$$A (\alpha^2)^{\frac{\alpha}{1-\alpha}} L = (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} L = \phi$$

- ▶ Observation: output is a linear function of the number of varieties
- ▶ We also obtain

$$X(t) = \int_0^{N(t)} x^* dj = x^* N(t) = \alpha^2 \phi N(t)$$

- ▶ Therefore net output is

$$Y(t) - X(t) = (1 - \alpha^2) \phi N(t)$$

Entry

Cost of decision

- ▶ **Lab-equipment assumption:** R&D is an activity using the final product as an input
- ▶ Costs of entry: assuming a linear and symmetric R&D technology

$$I(j, t) = \eta \frac{Y(t)}{N(t)} = \eta \phi$$

Free entry

- ▶ **Free entry condition** in the market for variety j there is entry up to the point in which benefits are equal to the costs of entry.
- ▶ Therefore, the equilibrium entry condition is

$$v(j, t) = I(j, t)$$

- ▶ Then, taking $v(j, t) = v^*(t)$ and $I(j, t) = \eta\phi$

$$\boxed{v^* = \eta\phi}$$

- ▶ Because v^* is a constant, from the (1) (and $\dot{v} = 0$)

$$\pi^* = rv^*$$

then the interest rate is constant

$$\boxed{r(t) = r^* = \frac{\pi^*}{v^*} = \frac{\alpha(1 - \alpha)}{\eta}}$$

Implication for aggregate output

- This implies

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} (x^*)^\alpha dj = \phi N(t)$$

where

$$\phi \equiv A (A\alpha^2)^{\frac{\alpha}{1-\alpha}} L = (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} L$$

- Interpretation: because $N(t)$ is the only dynamic variable (to see next) the aggregate production function has a AK (i.e. constant returns and constant marginal product) structure, where ϕ is a productivity parameter.
- Then

$$\pi^* = \alpha(1 - \alpha)\phi$$

General equilibrium

- ▶ The consumer solves (CP)
- ▶ The producer of final goods solves (FGPP)
- ▶ The intermediate producers solve problems (IGPP)
- ▶ Aggregate accounting consistency condition
- ▶ Market equilibrium:

General equilibrium

Aggregate accounting consistency

- Consistency conditions: the rents generated by R&D distributed to consumers who own firms

$$W(t) = \int_0^{N(t)} v(j, t) dj = v^* N(t) = \eta \phi N(t)$$

- the budget constraint, becomes

$$\dot{W} = \omega L + rW - C \Leftrightarrow \eta \phi \dot{N} = (1 - \alpha)(1 + \alpha) \phi N - C$$

- because

$$\omega L = (1 - \alpha) Y = (1 - \alpha) \phi N \text{ and } rW = \frac{\alpha(1 - \alpha)}{\eta} \eta \phi N$$

General equilibrium

Market equilibrium

- Equilibrium condition

$$Y(t) = C(t) + I(t) + X(t)$$

- We derived $Y(t) - X(t) = (1 - \alpha^2)\phi N(t)$
- Aggregat investment in R&D

$$I(t) = \int_0^{\dot{N}(t)} I(j, t) dj = \int_0^{\dot{N}(t)} \eta\phi dj = \eta\phi \dot{N}(t)$$

- Therefore, we get same relationship

$$(1 - \alpha^2)\phi N(t) = C(t) + \eta\phi \dot{N}(t)$$

The equilibrium in the decentralized economy

- ▶ the DGE in levels

$$\begin{aligned}\dot{C} &= \frac{C}{\theta}(r - \rho) \\ \dot{N} &= \frac{(1 - \alpha^2)}{\eta}N - \frac{C}{\eta\phi}\end{aligned}\tag{DGE}$$

- ▶ Decomposing the variables

$$C(t) = c(t)e^{\gamma t}, \quad N(t) = n(t)e^{\gamma t}$$

- ▶ the DGE in detrended variables

$$\begin{aligned}\dot{c} &= \frac{c}{\theta}(r - \rho - \theta\gamma) \\ \dot{n} &= \left(\frac{(1 - \alpha^2)}{\eta} - \gamma\right)n - \frac{c}{\eta\phi}\end{aligned}\tag{DGE detrended}$$

General equilibrium: alternative representation

- ▶ If we define the capital in this economy as $K(t) = W(t)$. Then $K(t) = \eta\phi N(t)$, $\omega L = \frac{(1-\alpha)}{\eta} K$ and $rW = \frac{\alpha(1-\alpha)}{\eta} K$
- ▶ the budget constraint becomes

$$\dot{K} = \frac{(1-\alpha)(1+\alpha)}{\eta} K - C = A^v K - C$$

- ▶ which implies that the model has a AK structure, where $A^v = A^v(\alpha, \eta) = \frac{(1-\alpha)(1+\alpha)}{\eta}$, where clearly

$$\frac{\partial A^v}{\partial \alpha} < 0, \quad \frac{\partial A^v}{\partial \eta} < 0$$

which means that A^v is a positive function of the markup, $\mu = 1/\alpha$: an increase in the markup and a reduction in the barriers to entry increase the productivity of capital

The long run growth rate

Decentralized economy

- ▶ the long run growth rate is

$$\gamma_d = \frac{1}{\theta} \left(\frac{\alpha(1-\alpha)}{\eta} - \rho \right)$$

is a negative function of the cost of entry η (i.e, **barriers to R&D reduce growth**)

- ▶ the long run level for per capita GDP is

$$\bar{y} = \phi(A, L) \frac{n(0)}{L} = (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} n(0)$$

- ▶ there is no transitional dynamics

Centralized economy

- Consider a social planner solving the problem

$$\begin{aligned} \max_{(C(t))_{t \in [0, \infty)}} \quad & \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \quad \theta > 0 \\ \text{s.t} \quad & \dot{N} = \frac{(1-\alpha^2)}{\eta} N(t) - \frac{C(t)}{\eta\phi} \end{aligned} \quad (\text{OP})$$

- applying the Pontryagin principle and decomposing the variables we get

$$\begin{aligned} \dot{c} &= \frac{c}{\theta} (r_c - \rho - \theta\gamma) \\ \dot{n} &= \left(\frac{(1-\alpha^2)}{\eta} - \gamma \right) n - \frac{c}{\eta\phi} \end{aligned} \quad (\text{OP detrended})$$

where

$$r_c \equiv \frac{1-\alpha^2}{\eta}$$

The long run growth rate

Centralized economy

- ▶ the long run growth rate is

$$\gamma_c = \frac{1}{\theta} \left(\frac{1 - \alpha^2}{\eta} - \rho \right) > \gamma_d = \frac{1}{\theta} \left(\frac{(1 - \alpha)\alpha}{\eta} - \rho \right)$$

- ▶ the long run growth rate in the centralized economy is higher than in the decentralized economy
- ▶ this means that the decentralized economy is not Pareto optimal: there is an externality generated by the R&D activity that is not internalized in a decentralized economy

Policy implications

- ▶ In the decentralized setting the government introduces a tax/subsidy on the return on capital applied/financed by a lump-sum expenditure/tax
- ▶ under a budget balanced rule we have $\tau rW = G$ in the first case (tax/expenditure) τ and G are positive and in the second (subsidy/tax) they are negative
- ▶ this implies that the rate of growth is

$$\gamma_d = \frac{1}{\theta} \left(\frac{(1 - \tau)\alpha(1 - \alpha)}{\eta} - \rho \right)$$

- ▶ to internalize fully the externality we should have $(1 - \tau)r^d = r^c$ which implies $\gamma^d = \gamma^c$, that is

$$(1 - \tau) \frac{\alpha(1 - \alpha)}{\eta} = \frac{(1 + \alpha)(1 - \alpha)}{\eta}$$

- ▶ then the optimal policy would be to introduce a subsidy whose rate should be equal to the markup $-\tau = \frac{1}{\alpha}$

References

- ▶ Grossman and Helpman (1991)
- ▶ (Barro and Sala-i-Martin, 2004, ch. 6), (Acemoglu, 2009, ch. 13), (Aghion and Howitt, 2009, ch. 3)

Daron Acemoglu. *Introduction to Modern Economic Growth*. Princeton University Press, 2009.

Philippe Aghion and Peter Howitt. *The Economics of Growth*. MIT Press, 2009.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*. MIT Press, 2nd edition, 2004.

Gene M. Grossman and Elhanan Helpman. *Innovation and Growth in the Global Economy*. MIT Press, 1991.