R&D and growth: directed technnical change

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Technology and employment: recent developments

- There is **polarization** in the labor market in the manufacturing sector: with an increase in wages and employment in the lower and higher levels of skill (and wages), and a reduction in both employment and wages in the middle ranks
- there are two possible explanations:
  - automation: effects of technology in which automation tends to make intermediate levels of labor substitutable by machines (robots);
  - globalization: the supply chain of multinational firms tends to de-localize segments of the production chain requiring with skills but which are costly and which can be done in countries with a relative high level of education and lower wages

Technology and employment: recent developments

- Most empirical studies reveal that the effect of technology is dominant
- There are mixed predictions on the future impact of automation
- In any case there is evidence on the existence of a **technological bias** regarding its effects on the labor market and on growth.
- I will present next a benchmark **directed technical change model**

Evidence on wages and technical progress

- Evidence: in the last century (differently from the XVIII and XIX centuries) there was a **positive** correlation between the relative wage and employment between skilled and un-skilled labor

$$\frac{\omega_H}{\omega_L} \text{ and } \frac{L_H}{L_L}$$

  where H=high skilled and L= low skilled

- **Puzzle**: the increase in education has made L_H/L_L increase and therefore one would expect a decrease in ω_H/ω_L

Accounting for the puzzle

- The puzzle can only be explained if the productivity of L_H/L_L increased as a consequence of a **skill-bias in technical progress**, through the TFP $A(.)$
- Let
$$\frac{\omega_H}{\omega_L} = f\left(\frac{L_H}{L_L}, A\left(\frac{L_H}{L_L}\right)\right)$$
where $f_1 < 0$ and $f_2 > 0$ and $\frac{\partial A}{\partial(L_H/L_L)} > 0$
- then
$$\frac{d(\omega_H/\omega_L)}{d(L_H/L_L)} > 0$$
only if
$$\frac{\partial A}{\partial(L_H/L_L)} \text{ dominates}$$
- Under which conditions does this hold ?

Directed technical change model

Acemoglu (2002) and others extend the expansions of variety model considering:

- two intermediate good sectors: sectors producing high tech and low tech goods
- HT and LT sectors use machines which are complements to skilled or unskilled labor
- R&D are performed by potential producers of new machines which are skilled-complements
- **bias in technological change** is measured by the relative growth of the number of varieties of skilled-labour complementary machines versus the unskilled-labour complementary machines induced by R&D activities;
- R&D and entry are as in the expansion of varieties model

The model

We assume a decentralized economy with:

- Consumers who work and own the firms
- A final-good production sector (competitive)
- Two intermediate-good producers for high-tech and low-tech inputs (competitive)
- A continuous of high-tech machine producers (monopolists)
- A continuous of low-tech machine producers (monopolists)
- Performers of R&D who have to decide which sector to enter

Consumers

Problem and f.o.c

- Problem:

$$\max_{C} V[C] = \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{W} = \omega_L(t) L_L + \omega_H(t) L_H + r(t) W(t) - C(t)$$

given $W(0) = W_0$.

- f.o.c

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r(t) - \rho) \quad (1)$$

$$\dot{W} = \omega_L(t) L_L + \omega_H(t) L_H + r(t) W(t) - C(t) \quad (2)$$

Producer of the final good

The problem

- They are price-takers in all markets and use two types of input (they assemble two jobs)
- Their problem is, for every moment t

$$\max_{Y_L, Y_H} \pi(t), \; \pi(t) \equiv Y(t) - P_L(t)\, Y_L(t) - P_H(t)\, Y_H(t)$$

- the production function (constant elasticity of substitution) is

$$Y(t) = \left[A_L\, Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + A_H\, Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Y_j are intermediate goods of technology level $j \in H, L$

- ε is the elasticity of substitution between H and L inputs

Producer of the final good

F.o.c

- the optimal production is

$$Y_j(t) = \left(\frac{P_j(t)}{A_j}\right)^{-\varepsilon} Y(t), \text{ for } j = L, H \tag{3}$$

- the following restriction holds

$$\left[A_L^{\varepsilon} P_L(t)^{1-\varepsilon} + A_H^{\varepsilon} P_H(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \tag{4}$$

Producers of intermediate goods

The problem

- Two types of goods and sectors: high-tech and low tech intermediate goods
- The problem for the competitive (i.e., price-taker) producer $j = L, H$ is
$$\max_{L_j, [x_j(v,t)]_{v \in [0, N_j(t)]}} \pi_j(t),$$
where
$$\pi_j(t) = P_j(t)\left(Y_j(t) - \omega_j(t) L_j\right) - \int_0^{N_j(t)} p_j^x(v,t) x_j(v,t)\, dv, \; j = L, H.$$
- for $0 < \beta < 1$: the degree of man-machine substitution
$$Y_j(t) = \frac{1}{1-\beta}\left(\int_0^{N_j(t)} x_j(v,t)^{1-\beta} dv\right) L_j^{\beta}, \; j = L, H$$
where $x_j(v,t)$ machines of variety $v \in [0, N_j(t)]$ complement to factor L_j

Producers of intermediate goods

The f.o.c

- firms equalize the real wage to the marginal product of the type of labour they employ

$$\omega_j(t) = \beta \frac{Y_j(t)}{L_j},\ j = L, H$$

- the demand for the machines of type $v \in [0, N_j(t)]$ for sector $j = L, H$ is a linear function of labour

$$x_j(v,t) = \left(\frac{P_j(t)}{p_j^x(v,t)} \right)^{1/\beta} L_j,\ \ v \in [0, N_j(t)],\ j = L, H. \qquad (5)$$

Producers of skilled-complementary machines

Two-stage problem

- Again producers of machines $v \in [0, N_j(t)]$ have a monopoly power, but have to engage in R&D activities before they start to produce.
- Entry decision, i.e., increasing $N_j(t)$: free entry condition.
- Production phase, if they entry
- **We have to operate backwards in order to determine the benefits of entry**

Producers of skill-complementary machines

The problem of an incumbent in producer v of sector $j = L, N$

- The problem

$$\max_{p_j^x(v,t)} \pi^{x_j}(v,t) \; v \in [0, N_j(t)], \; j = L, H$$

- **Assumption**: it has a symmetric marginal cost $\psi = 1 - \beta$
- Then

$$\pi_j^x(v,t) = (p^{x_j}(v,t) - \psi) x_j(v,t)$$

- and $x_j(v,t)$ is given by equation (5)

Producers of skill-complementary machines

The problem of an incumbent in producer v of sector $j = L, N$

First order conditions for optimality:

- arbitrage condition

$$x_j(v,t) - (p^{x_j}(v,t) - \psi)\frac{x_j(v,t)}{\beta p^{x_j}(v,t)} = 0$$

- because there is symmetry in costs

$$p^{x_j}(v,t) = \frac{\psi}{1-\beta} = 1, \; v \in [0, N_j(t)], \; j = L, H.$$

Implications

- If we substitute in equation (5), we get the production of intermediate R&D products

$$x_j(v,t) = x_j(t) = P_j(t)^{1/\beta} L_j, \; v \in [0, N_j(t)], \; j = L, H,$$

  which is symmetric across varieties.
- Then

$$\pi^{x_j}(v,t) = \pi^{x_j}(t) = \beta P_j(t)^{1/\beta} L_j, \; v \in [0, N_j(t)], \; j = L, H, \tag{6}$$

- and the output of j-complementary intermediate products

$$Y_j(t) = \frac{1}{1-\beta} P_j(t)^{(1-\beta)/\beta} N_j(t) L_j, \; j = L, H,$$

  are also symmetric across varieties.

Value of entry in sector j

- The value of introducing a new variety of machines which is j-complementary is

$$V_j(t) = \int_t^{\infty} \pi^{x_j}(s) e^{-\int_t^s r(\tau)\, d\tau} ds, \;\; v \in [0, N_j(t)], \;\; j = L, H,$$

(because of symmetry)

- differentiating

$$\dot{V}_j(t) = -\pi^{x_j}(t) + r(t)\, V_j(t), \;\; j = L, H.$$

- Then

$$r(t) = \frac{\pi^{x_L}(t) + \dot{V}_L(t)}{V_L(t)} = \frac{\pi^{x_H}(t) + \dot{V}_H(t)}{V_H(t)} \tag{7}$$

because the real interest rate, $r(t)$ is determined for the aggregate economy, and is common to all sectors.

Costs of entry in sector j

- The technology for introducing the innovation is of the lab-equipment type, that is

$$\dot{N}_j = \eta_j Z_j(t),\ j = L, H$$

  where Z_j is the costs of entry

- The costs per entrant are

$$\frac{Z_j(t)}{\dot{N}_j} = \frac{1}{\eta_j},\ j = L, H$$

  are symmetric to the barriers to entry in sector j.

Free entry condition in sector j

- If

$$V_j(t) < \frac{Z_j(t)}{\dot{N}_t} \text{ if } Z_j(t) = 0$$

then there is no entry, and

$$V_j(t) = \frac{Z_j(t)}{\dot{N}_t} \text{ if } Z_j(t) > 0$$

- **There is entry** in sector j ($Z_j(t) > 0$) if and only if

$$\eta_j V_j(t) = 1,\ j = L, H.$$

Entry in any sector

Arbitrage condition

- If there are expenditures on R&D in both sectors, $Z_L(t) > 0$ and $Z_H(t) > 0$, then: the the arbitrage condition is

$$\eta_L V_L(t) = \eta_H V_H(t) = 1$$

- **Implications**:
  1. $\dot{V}_j = 0$ for $j = L, H$.
  2. from equations (6) and (7)

$$r = \frac{\pi_L}{V_L} = \eta_L \pi^{x_L} = \beta \eta_L L_L P_L^{1/\beta}$$

and

$$r = \frac{\pi_H}{V_H} = \eta_H \pi^{x_H} = \beta \eta_H L_H P_H^{1/\beta}$$

  3. then

$$P_j(t) = P_j = \left(\frac{r}{\beta \eta_j L_j}\right)^{\beta}, \; j = L, H$$

Interest rate and substitution between factors

- This equation together with equation (4)

$$\left[A_L^{\varepsilon} P_L(t)^{1-\varepsilon} + A_H^{\varepsilon} P_H(t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1$$

  allows us to determine the interest rate as a constant

$$\boxed{r = \beta \left(A_L^{\varepsilon}(\eta_L L_L)^{\sigma-1} + A_H^{\varepsilon}(\eta_H L_H)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}} \tag{8}$$

- where

$$\boxed{\sigma \equiv 1 + (\varepsilon - 1)\beta}$$

  is the elasticity of substitution between the two factors.
- Observe that $\sigma > 1$ if $\varepsilon > 1$ (i.e., the production of the final good is not Cobb-Douglas)

Relative-price of skill-complementar inputs

- The relative price of the skilled-complementary relative to the unskilled-complementary input

$$\boxed{p(t) = \frac{P_H(t)}{P_L(t)} = \left(\frac{\eta_H L_H}{\eta_L L_L}\right)^{-\beta}}$$

- then, profits π_j are constant
- the $j-$complementary intermediate inputs are

$$\boxed{Y_j(t) = \phi_j N_t(t),\ j = L, H} \tag{9}$$

where

$$\phi_j = \frac{1}{1-\beta}\left(\frac{r}{\eta_j}\right)^{1-\beta} L_j^{\beta}\ j = L, H.$$

BGP: growth rate

- The BGP growth rate is obtained from the detrended Euler equation, (1), as

$$\boxed{\gamma^* = \frac{r^* - \rho}{\theta}}$$

  where $r^* = r(L_H, L_L)$ is given in equation (8).

- in this version of the model there are scale effects (an increase in both L_L and L_H increases the growth rate)

$$\frac{\partial r}{\partial L_j} = A_j^{\epsilon} \eta_j \beta \left(A_L^{\epsilon} \left(\frac{\eta_L L_L}{\eta_j L_j} \right)^{\sigma - 1} + A_H^{\epsilon} \left(\frac{\eta_H L_H}{\eta_j L_j} \right)^{\sigma - 1} \right)^{\frac{2-\sigma}{\sigma - 1}} > 0, \; j = L$$

BGP: bias in technical progress

- the **bias in technical progress** can be measured by

$$\boxed{n \equiv \frac{N_H}{N_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\sigma} \left(\frac{A_H}{A_L}\right)^{\varepsilon} \left(\frac{L_H}{L_L}\right)^{\sigma-1}}$$

Exercise: prove this using equation (3) and (9).

- the relative wage premium for skilled workers

$$\boxed{\omega \equiv \frac{\omega_H}{\omega_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\varepsilon\beta} \left(\frac{A_H}{A_L}\right)^{\varepsilon} \left(\frac{L_H}{L_L}\right)^{\varepsilon\beta-1}}$$

- the relative price of high-tech goods

$$\frac{P_H}{P_L} = \left(\frac{\eta_H L_H}{\eta_L L_L}\right)^{-\beta}$$

Conclusions

- there is a high-tech biased technical progress only if $\sigma > 1$:

$$\frac{\partial n}{\partial(L_H/L_L)} = (\sigma - 1)\frac{n}{(L_H/L_L)} > 0$$

- solution to our initial puzzle only if $\varepsilon\beta > 1$

$$\frac{\partial(\omega_H/\omega_L)}{\partial(L_H/L_L)} = (\varepsilon\beta - 1)\frac{(\omega_H/\omega_L)}{(L_H/L_L)} > 0$$

- reduction in the relative price of high-tech goods (for any σ)

$$\frac{\partial(P_H/P_L)}{\partial(L_H/L_L)} < 0$$

References

- (Acemoglu, 2009, ch. 15),
- (Aghion and Howitt, 2009, ch. 8)

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