R&D and growth: the Schumpeterian model

Paulo Brito
pbrito@iseg.ulisboa.pt

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General idea

- growth depends on the growth of TFP (not only on human and physical capital accumulation);
- growth of TFP is the result of R&D activity (innovation);
- innovation is done by entrants in every industry which substitute previous incumbents
- innovation is non-rival and non-excludable; therefore R&D is only undertaken if, upon entry, the incumbent is monopolist in its industry;
- **creative destruction** generates growth: at the aggregate level, there is an increase in quality, and, therefore, productivity;
- this generates, as for the expansion of varieties model, a non-Paretian decentralized general equilibrium.

Assumptions

The model has a similar structure to the expansion of variety model with the following differences

- technical progress takes the form of an improvement in the quality of products (not in their number);
- we assume that quality refers to intermediate inputs;
- entry replaces an existing monopolist (not starting a new industry) by a new monopolist
- the success of inventive activity is random (follows an Exponential process) with a probability dependent on expenditures
- if R&D is successful the entrant is monopolist for a finite but unknown time (not infinitely)

Innovations

- we distinguish the physical quantity of the input, x and the quantity in efficiency units $\tilde{x}$
- quality increases the productivity of an input, which is measured in efficiency units by

$$\tilde{x} = l^{\nu} x$$

  for every unit of physical quantity of the inlut
- innovations take the form **quality ladders**: there is a quality index which evolves ias

$$1, l, l^2, l^3, \ldots, l^{\nu}$$

Innovations heterogeneity

- heterogeneity in the quality levels: different industries can be at different quality levels
- if ν_j is the quality level of industry j, we may have for any other industry k $\nu_k \neq \nu_j$
- therefore, at time t the quantity of the input produced by industry j is
$$\tilde{x}(j,t) = l^{\nu_j} x(j,t)$$

Modelling innovations and R&D

- the producer of input $x(j,t)$ is an entrant who successfully introduces a higher quality input at time $t(\nu_j)$, through creative destruction: i.e. the producer of quality l^{ν_j} displaces the incumbent which produces at levsl l^{ν_j-1}. Its instant probability of success is $\lambda(\nu_j - 1)$
- he becomes a monopolist, but only for a period lasting $T(\nu_j) = t(\nu_j + 1) - t(\nu_j)$
- the arrival of a new innovation, yielding a jump from the level ν_j to $\nu_j + 1$, is governed by an exponential process with density

$$\mathbb{P}[\nu_j + 1|\nu_j] = \lambda(\nu_j)\, e^{-\lambda(\nu_j)\, T(\nu_j)}$$

  which is decreasing in $T(\nu_j)$
- and the probability of arrival of a successful innovation, $\lambda(.)$, is **dependent on R&D effort**

Activities in industry j and quality levels

activity:	R&D	entry	production	exit
quality level:	$\nu_j - 1$	ν_j		$\nu_j + 1$
timing:		$t(\nu_j) \longleftarrow$	$T(\nu_j)$	$\longrightarrow t(\nu_j + 1)$
probabilities:		$\lambda(\nu_j - 1)$		$\mathbb{P}[\nu_j + 1 \vert \nu_j]$

- Pre-entry tech level $\nu_j - 1$
- Firm enters by raising it to ν_j (and becoming incumbent during an interval T_{ν_j})
- Firm is eventually evicted when a successful innovator raises it to $\nu_j + 1$

Results

- Even without capital accumulation, growth can be generated by the increase in quality of the goods, which increase aggregate productivity
- The rate of growth depends negatively on the cost of R& D, and positively in the quality jump

The consumer problem

- Earns labor and capital income, consumes a final product, save and own firms (final good and intermediate good producers)
- The problem

$$
\begin{aligned}
\max_{(C(t))_{t\in[0,\infty)}} \quad & \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \ \theta > 0 \\
& \text{s.t} \\
& \dot{W} = \omega(t) L + r(t) W(t) - C(t)
\end{aligned}
\tag{CP}
$$

- The first order conditions

$$
\begin{aligned}
\dot{C} &= \frac{C}{\theta} (r(t) - \rho) \\
\dot{W} &= \omega(t) L + r(t) W(t) - C(t)
\end{aligned}
$$

Producers of final goods

- Production function: Dixit and Stiglitz (1977)

$$Y(t) = \int_0^N Y(j,t)\,dj = \int_0^N AL^{1-\alpha}\left(l^{\nu_j}x(j,t)\right)^{\alpha} dj,\ 0 < \alpha < 1$$

- L labor input
- $(x(j,.))_{j\in[0,N]}$ intermediate inputs, non-storable,
- N number of inputs (number of intermediate industries)
- l^{ν_j} quality ladder of industry j
- Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^N P(j,t)x(j,t)\,dj$$

Producers of final goods (cont)

- Buys labor and intermediate goods and sells a final good
- The problem:

$$\max_{L,(x(j,t))_{j\in[0,N]}} \pi^p(t) \tag{FGPP}$$

- Competitive industry: they are price takers in all markets
- First order conditions:
  - demand for labor

$$L^d = (1-\alpha)\frac{Y(t)}{\omega(t)}$$

  - demand for intermediate goods

$$x^d(j,t) = \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} Lq(\nu_j),\; j \in [0, N(t]]$$

where $q(\nu_j) = l^{\frac{\alpha}{(1-\alpha)}\nu_j}$ is the quality level reached by industry j

Producers of intermediate goods

Production and R&D problems

- Perform R&D activities allowing for the production of a better quality input which they sell to final producers
- Decision process for the introduction of a new variety
  - R& D and entry decision: free entry condition
  - pricing of input of variety j after entry
- Solution of the problem: in backward order
  - first: we determine the pricing policy assuming there is entry
  - second: we determine entry (by using the free entry condition)

Producers of intermediate goods (cont)

Price decision if there is entry (incumbents' problem)

- The problem for the producer of a variety j is

$$\max_{P(j,t)} \pi(j,t) = (P(j,t) - 1)x(j,t) \tag{IGPP$_j$}$$

Assumption: symmetric cost of production equal to 1

- where $x(j,t) = x^d(j,t)$ (solution of the FGPP)
- then

$$\pi(j,t) = (P(j,t) - 1)\left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} Lq(\nu_j),$$

Producers of intermediate goods (cont)

Price decision if there is entry

- first order conditions (markup $= 1/\alpha$)

$$P^*(j,t) = \frac{1}{\alpha} \ \forall (j,t)$$

- then

$$x^*(j,t) = x^*(\nu_j) = \left(\alpha^2 A\right)^{\frac{1}{1-\alpha}} Lq(\nu_j)$$

  is stationary (time independent) but **not symmetric**
- Then the profit is also **not symmetric**

$$\begin{aligned} \pi^*(j,t) = \pi^*(\nu_j) &= \left(\frac{1-\alpha}{\alpha}\right) \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} Lq(\nu_j) \\ &= \pi_0 Lq(\nu_j) \\ &= \frac{1-\alpha}{\alpha} x^*(\nu_j) \end{aligned}$$

  where $\pi_0 \equiv \left(\frac{1-\alpha}{\alpha}\right) \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}}$

Producers of intermediate goods (cont)

Value of entry dependent upon duration

- The producer of a successful variety j, enters at time t_{ν_j} (when the technological level is ν_j) and becomes a monopolist in the interval $[t_{\nu_j}, t_{\nu_j} + T_{\nu_j}]$ where T_{ν_j} is stochastic
- the value entry is, while still on business

$$v^*(\nu_j, T_{\nu_j}) = \int_{t_{\nu_j}}^{t_{\nu_j}+T_{\nu_j}} \pi^*(\nu_j) e^{-r(t-t_{\nu_j})} dt =$$

$$= \frac{\pi^*(\nu_j)}{r}(1 - e^{-rT_{\nu_j}})$$

- then the value of entry is a stochastic variable, dependent upon the duration of the monopoly T_{ν_j}

Producers of intermediate goods (cont)

Expected benefit of entry

- The success of an innovation arriving at industry j (with the present level ν_j) is governed by the exponential process

$$g(\nu_j, T) = \mathbb{P}[\nu_j + 1 | \nu_j] = \lambda(\nu_j) e^{-\lambda(\nu_j) T}, \;\; T \in [0, \infty)$$

  where T is the period of incumbency (i.e., after entry)

- Then the **expected benefit for introducing quality ladder** ν_j depends on the distribution of incumbency time (T_{ν_j})

$$\begin{aligned}
\mathbb{E}[v^*(\nu_j)] &= \int_0^\infty v^*(\nu_j, T_{\nu_j})\, g(\nu_j, T_{\nu_j})\, dT_{\nu_j} = \\
&= \frac{\pi^*(\nu_j)\lambda(\nu_j)}{r} \int_0^\infty (1 - e^{-rT_{\nu_j}})\, e^{-\lambda(\nu_j)\, T_{\nu_j}}\, dT_{\nu_j} = \\
&= \frac{\pi^*(\nu_j)}{r + \lambda(\nu_j)} = \\
&= \frac{1-\alpha}{\alpha} \frac{x^*(\nu_j)}{r + \lambda(\nu_j)}
\end{aligned}$$

Producers of R& D

R&D: benefits and costs

- An entrant firm, needs to raise present technological level from $\nu_j - 1$ to ν_j by performing R&D;
- Two cases can occur
  - with probability $\lambda(\nu_j - 1)$ the innovation is successful allowing to becoming incumbent and having the expected value $\mathbb{E}[v^*(\nu_j)]$
  - with probability $1 - \lambda(\nu_j - 1)$ the innovation is not successful and having the value 0
- Therefore:
  - the value of doing R&D

$$\lambda(\nu_j - 1)\mathbb{E}[v^*(\nu_j)]$$

  - the cost is $Z(\nu_j)$

Producers of R& D

R&D technology

- **Assumption: production function of innovations**: the probability of success of introducing innovation level ν_j (from $\nu_j - 1$ to ν_j) is linear function of the expenditure $Z(\nu_j)$ ()

$$\lambda(\nu_j - 1) = Z(\nu_j)\psi(\nu_j)$$

- efficiency is an inverse function of the marginal increase in output produced by the innovation

$$\psi(\nu_j) = \frac{1}{\zeta\, Y(\nu_j)}$$

where ζ is a efficiency parameter, and

$$Y(\nu_j) = AL^{1-\alpha} l^{\alpha \nu_j} x(\nu_j)^{\alpha} = x^{*}(\nu_j)\alpha^{-2} = A_y q(\nu_j)$$

where $A_y \equiv \left((\alpha^{2\alpha})A\right)^{\frac{1}{1-\alpha}} L$

Free entry condition

- Free entry in the R&D sector:

$$\underbrace{\lambda(\nu_j - 1)E[v^*(\nu_j)]}_{\text{expected benefit}} = \underbrace{Z(\nu_j)}_{\text{cost}} \Leftrightarrow E[v^*(\nu_j)] = \zeta Y(\nu_j)$$

- this is equivalent to

$$\frac{1-\alpha}{\alpha}\frac{x^*(\nu_j)}{r+\lambda(\nu_j)} = \frac{\zeta x^*(\nu_j)}{\alpha^2}$$

- then there is an arbitrage equation for entry

$$\boxed{\lambda(\nu_j) = r_0 - r \text{ where } r_0 \equiv \frac{\alpha(1-\alpha)}{\zeta}}$$

- this implies that $\lambda(\nu_j) = \lambda$ the **probability should be the same for all sectors** , and depends on the market interest rate r.

Aggregate evolution of quality

- Let at time t exist a particular distribution of quality levels $\Big(\nu_j(t)\Big)_{j=0}^{N}$
- We define the aggregate quality as

$$Q(t) \equiv \int_0^N q(\nu_j(t))\, dj$$

- Let the quality change in a small interval of time be

$$q(\nu_j + 1(t + dt)) - q(\nu_j(t)) = (l^{\frac{\alpha}{1-\alpha}} - 1)\, q(\nu_j(t))\, dt$$

- The instantaneous variation of quality at time t is

$$\begin{aligned} \frac{dQ(t)}{dt} &= \int_0^N \lambda q(\nu_j)(l^{\frac{\alpha}{1-\alpha}} - 1)\, dj = \\ &= \lambda \Xi Q(t) \end{aligned}$$

- where $\Xi = l^{\frac{\alpha}{1-\alpha}} - 1$ is the "jump" in quality.

Aggregate wealth

- The aggregate wealth is equal to the sum of the rents from R&D production

$$
\begin{aligned}
W(t) = \int_0^N E[v^*(\nu_j)]\,dj = \\
&= \frac{1-\alpha}{\alpha(r+\lambda(\nu_j))}\int_0^N x^*(\nu_j)\,dj \\
&= \zeta A_Y L Q(t) \\
&= \zeta Y(t)
\end{aligned}
$$

where

$$
Y(t) = \int_0^N Y(\nu_j)\,dj = A_y Q(t)
$$

- then

$$
X(t) = \int_0^N x^*(\nu_j)\,dj = \alpha^2 Y(t)
$$

Aggregate consistency

- Then

$$\frac{\dot{W}}{W} = \frac{\dot{Q}}{Q}$$

- Using the consumer's budget constraint and the fact that $W = \zeta A_Y L Q$

$$\frac{\dot{W}}{W} = \frac{\dot{Q}}{Q} \Leftrightarrow \frac{\omega L + rW - C}{W} = \Xi\lambda$$

- then using $\omega L = (1-\alpha)\, Y = \frac{(1-\alpha)}{\zeta} W$, and $r + \lambda = r_0$,

$$\frac{\dot{W}}{W} = \frac{\dot{Q}}{Q} \Leftrightarrow \Xi(r_0 - r) = \frac{(1-\alpha)\, Y - C}{W} + r = \frac{(1-\alpha)}{\zeta} - \frac{C}{W} + r$$

- We obtain an expression for the **market interest rate**

$$\boxed{r = r_0 - \lambda, \text{ for } \lambda = \Lambda(C/W) \equiv \frac{1}{l^{\frac{\alpha}{1-\alpha}}} \left(\frac{1-\alpha^2}{\zeta} - \frac{C}{W} \right)}$$

The equilibrium in the decentralized economy

- the DGE system in levels becomes

$$\begin{aligned}\dot{C} &= \frac{C}{\theta}(r_0 - \Lambda(C/W) - \rho) \\ \dot{W} &= \Xi\Lambda(C/W)\,W\end{aligned} \tag{DGE}$$

- Decompossing the variables

$$C(t) = c(t)\,e^{\gamma t},\;\; W = We^{\gamma t}$$

- the DGE in detrended variables

$$\begin{aligned}\dot{c} &= \frac{c}{\theta}(r_0 - \Lambda(c/w) - \rho - \theta\gamma) \\ \dot{w} &= (\Xi\Lambda(c/w) - \gamma)\,w\end{aligned} \tag{DGE detrended}$$

The long run growth rate

Decentralized economy

- Solving the steady state jointly to r and γ we obtain
- the long run growth rate is equal to the (endogeneous) probability of arrival on innovations

$$\boxed{\gamma_d = \Xi\bar{\lambda} = \frac{r_0 - \rho}{\Xi^{-1} + \theta}}$$

- where $r_0 = \alpha(1-\alpha)/\zeta$
- the growth rate is a negative function of the cost of entry ζ (i.e, barriers to R&D reduce growth) and a positive function of the quality jump (Ξ)
- the long-run per capita GDP level is

$$\boxed{\bar{y} = A_Y Q = A_Y \int_0^N l^{\frac{\alpha\nu_j}{1-\alpha}}\, dj}$$

is higher the higher the "quality ladders" for all sectors;

- there is no transitional dynamics

References

- (Barro and Sala-i-Martin, 2004, ch. 7), (Acemoglu, 2009, ch. 14)

Daron Acemoglu. *Introduction to Modern Economic Growth*. Princeton University Press, 2009.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*. MIT Press, 2nd edition, 2004.