Foundations of Financial Economics 2017/18 Problem set 3: Two-period Arrow-Debreu economy under uncertainty

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- 1. Consider a two-period Arrow-Debreu economy with the data that follows. Define the equilibrium, determine the solution for the consumer problem, and determine the equilibrium AD prices. Interpret the results:
 - (a) assume a logaritmic utility function, $u(c) = \ln(c)$, 2, states of nature and generic probability and endowment distributions;
 - (b) assume a quadratic utility function, $u(c) = ac \frac{b}{2}c^2$, a > 0, 2, states of nature and generic probability and endowment distributions. Set conditions for the results to make sense:
 - (c) assume an exponential utility function, $u(c) = -\frac{e^{-\lambda c}}{\lambda}$, $\lambda > 0$, 2, states of nature and generic probability and endowment distributions;
 - (d) assume an isoelastic utility function, $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, $\theta > 0$, 2, states of nature and generic probability and endowment distributions;
 - (e) assume a generic HARA utility function, 2, states of nature and generic probability and endowment distributions;
 - (f) solve the same problems as before with N states of nature.
- 2. Assume the following economic environment: (1) there are N states of nature, with an uniform probability distribution, and (2) there is an endowment distribution for the period t = 1, $y_{1,s} = y_0 \Gamma^{N/2-s}$, s = 0, ..., N, for $0 < \Gamma < 1$. Consider Arrow-Debreu economies with the data that follows. Define the equilibrium, determine the solution for the consumer problem, and determine the equilibrium AD prices. Interpret the results:
 - (a) assume a logaritmic utility function, $u(c) = \ln(c)$;
 - (b) assume a quadratic utility function, $u(c) = ac \frac{b}{2}c^2$, a > 0. Set conditions for the results to make sense;
 - (c) assume an exponential utility function, $u(c) = -\frac{e^{-\lambda c}}{\lambda}, \lambda > 0$;

- (d) assume an isoelastic utility function, $u(c) = \frac{c^{1-\theta}-1}{1-\theta}, \ \theta > 0;$
- (e) assume an generic HARA utility function.
- 3. Consider Arrow-Debreu economies with the data that follows: (1) the information is given by a binomial tree with two periods two periods, $\mathbb{T} = \{0,1\}$, with probabilities, for period $1, \pi_s = \zeta \cdot (1+\zeta)^{-s}$ for state $s=1,\ldots,\infty$, where $\zeta>0$; (2) the endowment distribution for the period t=1 is $y_{1,s}=y_0\cdot (1+\zeta)^{-s/\theta}$, for state $s=1,\ldots,\infty$ and $\theta>0$; (3) agents are homogenous; (4) the representative agent has a discounted time-additive, von-Neumann-Morgenstern utility functional with a CRRA Bernoulli utility function, $u(C)=\frac{C^{1-\theta}-1}{1-\theta}$;
 - (a) Define the equilibrium, and provide an intuition for it.
 - (b) Determine the solution for the consumer problem, and provide an intuition for it.
 - (c) Determine the equilibrium AD prices. Interpret the results you have obtained
- 4. Consider Arrow-Debreu economies with the data that follows: (1) the information is given by a binomial tree with two periods, $\mathbb{T} = \{0, 1\}$ and N states of nature for period 1; (2) the endowment distribution for the period t = 1 is $y_{1,s} = y_0 \cdot (1 + \gamma_s)$, for state $s = 1, \dots, N$; (3) agents are homogenous; (4) the representative agent has a discounted time-additive, von-Neumann-Morgenstern utility functional with a CARA Bernoulli utility function,

$$u(C) = -\frac{e^{-\lambda C}}{\lambda}, \ \lambda > 0$$

- (a) Define the equilibrium, and provide an intuition.
- (b) Determine the solution for the consumer problem, and provide an intuition.
- (c) Determine the equilibrium stochastic discount factor. Assuming that $\mathbb{E}[\gamma_s] = \gamma > 0$ find a bound to the expected value of the stochastic discount factor by using Jensen's inequality. Provide an intuition for your results.
- 5. Consider endowment economy in which the information be given by a two-period binomial tree, the endowment process, $\{Y_0, Y_1\}$, verifies $Y_0 = 1$ and $Y_1 = (1 \gamma, 1 + \gamma)$ for $0 < \gamma < 1$, the intertemporal utility functional is time additive, discounted and von-Neumann-Morgenstern, with a linear Bernoulli utility function u(c) = a c, for a > 0 constant.
 - (a) Define, explicitly, the Arrow-Debreu equilibrium for this economy.
 - (b) Write the equilibrium conditions. Under which conditions an equilibrium exists? Is it unique? Justify.
 - (c) Find the stochastic discount factor and provide an economic intuition for its value.
- 6. Consider an Arrow-Debreu endowment economy in which the information tree is binomial with two periods and two states of nature at time t=1. There are two consumers (indexed by i=1,2) which have both intertemporally additive preferences, with symmetric Bernoulli functions $(u(c^i) = \ln(c^i))$, and probabilities associated to the two states of nature, but are heterogenous as regards discount factors $\beta^i = \frac{1}{1+\rho^i}$, where $\rho^i > 0$ and endowments $\{y_t^i\}_{t=0}^1$.

- (a) Define Arrow-Debreu general equilibrium for this economy. Find it explicitly. Caracterize it as regards existence and uniqueness.
- (b) Assume instead that agents are also symmetric as regards their intertemporal discount factors. Find the Arrow-Debreu equilibrium in this case.
- (c) Discuss the results you have obtained in the former two questions. Focus on their dependency as regards the symmetry of impatience and the assumptions as regards the endowments (at the individual and the aggregate levels).