Foundations of Financial Economics
Financial frictions: moral hazard

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May 11, 2018

Topics

General equilibrium with moral hazard: the Holmstrom Tirole model

- The lender's problem
- Contracts in the presence of moral hazard
- Financial friction: borrowing constraint
- The borrower's problem
- Equilibrium interest rate.

The lender's problem

Assumptions

- Has liquid net worth in the amount W^b that is higher than the deserved consumption at time $t = 0$ and its return should finance consumption at time $t = 1$.
- He lends θ^b through a debt contract in which the return at time $t = 1$ is risk-free. Therefore consumption at time $t = 1$ is risk free.
- It has a concave Bernoulli utility function
- The **lender's problem** is

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \theta^l = W^l, \; c_1^l = R\theta^l$$

  where R is the return on the asset.
- Equivalently

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \frac{c_1^l}{R} = W^l$$

The lender's problem
Solution

- Solution assuming a log utility

$$c_0^l = \frac{1}{1+\beta} W^l, \; c_0^l = \frac{\beta R}{1+\beta} W^l$$

- The demand for asset, or the liquidity supply, is $\theta^l = \dfrac{c_1^l}{R}$. Therefore

$$\theta^l = \frac{\beta}{1+\beta} W^l$$

The borrowers's project

- Has net worth W^b but can invest I in a project. Needs financing if $I > W^b$. In this case borrows $\theta^b = I - W^b > 0$ to the lender.
- He can follow one of the two courses of action (**not observable by the lender**) with expected returns
  - Good project: $E[V^H] = p_H \frac{V}{p_H} + (1 - p_H)0 = V$
  - Bad project: $E[V^l] = p_L \frac{V}{p_H} + (1 - p_L)0 + B = p_L \frac{V}{p_H} + B$
- where $p_H > p_L$ (higher effort in the first case) and B diverted from the project to other purposes.

The borrowers's project

- The expected net present values, depending on the borrowers actions, are

$$NPV^H = -I + \frac{V}{R},$$

$$NPV^L = -I + \frac{p_L \frac{V}{p_H} + B}{R},$$

- We have $NPV^L < 0 < NPV^H$ if and only if

$$p_L \frac{V}{p_H} + B < RI < V$$

meaning that project L is bad and project H is good.

Contracts with moral hazard

- A contract specifies a spliting of the returns between the lender and the borrower

$$V = V^l + V^b \tag{SPL}$$

- The Holmstrom-Tirole moral hazard problem introduces two constraints
  - the **participation constraint**: the lender is only interested in doing the contract if he receives the market rate of return on the loaned funds

$$V^l = R(I - W^b) \tag{PC}$$

  - the **incentive compatibility constraint**: the lender should have the "skin in the game" (good action should be better than bad action)

$$V^b \geq p_L \frac{V^b}{p_H} + B \tag{IC}$$

The friction: borrowing constraint

- Equations (SPL) and (IC) imply a **limited pledgeability constraint**:
$$V^l \leq \bar{v} I \equiv V - \frac{p_H}{p_H} B \tag{LP}$$
this is the maximum payoff that the borrower can promise to the lender
- Implication 1 : considering equations (PC) and (LP) then $W^l = R(I - W^b) \leq \bar{v} I$ or
$$\theta^b = I - W^b \leq \frac{\bar{v} I}{R} \tag{BC}$$
that is: there is a **borrowing constraint**
- Implication 2: equivalently there is a **collateral requirement**:
$$W^b \geq \bar{W} \equiv I\left(1 - \frac{\bar{v}}{R}\right) \tag{CR}$$
the lender will only finance the project if the borrower has a minimum wealth. If $W^b < \bar{W}$ there will be no finance.

The borrower problem

The problem

- We assume that the borrower utility function is linear and that $\beta^l = 1$. This is equivalent to assuming that he maximizes the cash from from the project.
- The borrower investment problem: seeks to maximize the cash flow from investment subject to the borrowing constraint (BC)

$$\max_I \left\{ vI - R(I - W^b) : \; I - W^b \leq \frac{\bar{v}I}{R} \right\}$$

we denote $v = V/I$.

The borrower problem

Solution

- The f.o.c are

$$v - R + \lambda(\bar{v} - R) = 0$$

$$\lambda\left(\bar{v}I - R(I - W^b)\right) = 0,\ \lambda \geq 0,\ I \leq \frac{R}{R - \bar{v}}W^b$$

- It can be shown that there is only a solution if

$$\bar{v} < R < v$$

  meaning that there is need to financing $\bar{v} < R$ and the project is worthwhile ($v > R$)

- The optimal investment is

$$I^* = \frac{R}{R - \bar{v}}W^b$$

Market equilibrium

- From the lender's problem we derived the supply of liquidity

$$\theta^l = \frac{\beta}{1+\beta} W^l$$

- From the borrower's problem we have the demand for liquidity

$$\theta^b = I^* - W^b = \frac{\bar{v}}{R - \bar{v}} W^b$$

- Market equilibrium condition

$$\theta^b = \theta^l$$

Equilibrium interest rate with moral hazard

- Then the equilibrium interest rate r^* is

$$\boxed{R^* = 1 + r^* = \bar{v}\left(1 + \left(\frac{1+\beta}{\beta}\right)\frac{W^b}{W^l}\right)}$$

  - increases with W^b: more wealth from the borrower means more investment and more financing from the lender
  - decreases with W^l: higher liquidity in the economy increases the supply of funds.
- In a **frictionless** economy the equilibrium interest rate would be

$$R = \frac{1}{\beta}$$

- Interpretation: in a economy **with informational financial frictions** there is a balance sheet effect on the interest rates: they can be low if there is excess liquidity from the lenders and low net worth (v.g., because of excess leverage) from the borrowers.

References

(Holmström and Tirole, 2011, chap 1)

Holmström, B. and Tirole, J. (2011). *Inside and Outside Liquidity*. MIT Press.