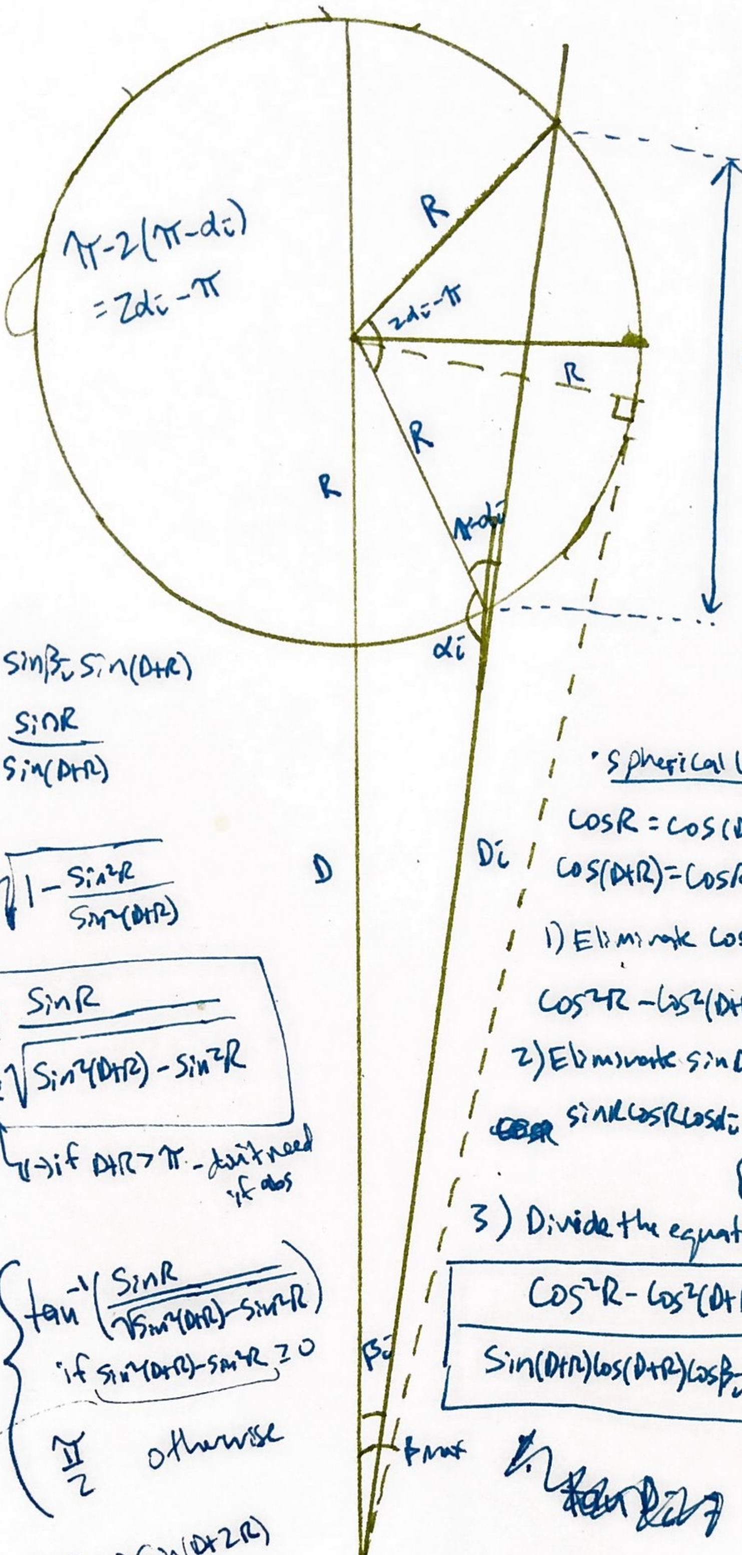
12/30/21

Lunar Ejecta

R13



• Planar law of cosines

$$C^2 = 2R^2\left(1-\underbrace{\cos(2\alpha_i-\pi)}_{=\cos(\pi-2\alpha_i)\,=\,-\cos(2\alpha_i)}\right)$$

$\therefore$ $C^2 = 4R^2\cos^2\alpha_i$

$$\boxed{C = 2R|\cos\alpha_i|}$$

• Spherical law of sines

$$\frac{\sin\alpha_i}{\sin(D+R)} = \frac{\sin\beta_i}{\sin R}, \quad \sin\alpha_i = \frac{\sin(D+R)}{\sin R}\sin\beta_i$$

$\therefore$

$$\boxed{\cos\alpha_i = -\sqrt{1-\frac{\sin^2(D+R)}{\sin^2 R}\sin^2\beta_i}}$$

- Need (-) since $\alpha_i \geq \pi/2$

• Spherical law of cosines

$\cos R = \cos(D+R)\cos D_i + \sin(D+R)\sin D_i\cos\beta_i$

$\cos(D+R) = \cos R\cos D_i + \sin R\sin D_i\cos\alpha_i$

1) Eliminate $\cos D_i$, × top by $\cos R$ & bottom by $-\cos(D+R)$ & add

$\cos^2 R - \cos^2(D+R) = [\cos R\sin(D+R)\cos\beta_i - \sin R\cos(D+R)\cos\alpha_i]\sin D_i$

2) Eliminate $\sin D_i$, × top by $\sin R\cos\alpha_i$ & bottom by $-\sin(D+R)\cos\beta_i$

$\sin R\cos R\cos\alpha_i - \sin(D+R)\cos(D+R)\cos\beta_i = [\sin R\cos(D+R)\cos\alpha_i - \cos R\sin(D+R)\cos\beta_i]\cos D_i$

3) Divide the equations

$$\boxed{\frac{\cos^2 R-\cos^2(D+R)}{\sin(D+R)\cos(D+R)\cos\beta_i - \sin R\cos R\cos\alpha_i} = \tan D_i}$$

$\therefore$ For $\beta_i=0, \alpha_i=\pi$ $\tan D_i = \tan D \Rightarrow D_i = D$ ✓

1/3/22

$\sin R = \sin\beta_i\sin(D+R)$

$\sin\beta_{max} = \frac{\sin R}{\sin(D+R)}$

$\cos\beta_{max} = \sqrt{1-\frac{\sin^2 R}{\sin^2(D+R)}}$

$$\boxed{\tan\beta_{max} = \frac{\sin R}{\pm\sqrt{\sin^2(D+R)-\sin^2 R}}}$$

(-) if $D+R>\pi$ - don't need if abs

$$\beta_{max} = \begin{cases} \tan^{-1}\left(\frac{\sin R}{\sqrt{\sin^2(D+R)-\sin^2 R}}\right) & \text{if } \underbrace{\sin^2(D+R)-\sin^2 R}_{=\sin D\sin(D+2R)} \geq 0 \\ \frac{\pi}{2} & \text{otherwise}\end{cases}$$