Lunar Meteoroid Ejecta Engineering Model

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2.1 Porosity

The lunar regolith porosity is related to the amount of free space between individual grains. The greater the porosity, the more void space is present. Table 3.4.2.3.4-1 of the DSNE gives values of the porosity as a function of depth down to 60 cm derived from Apollo core measurements (copied from Table 9.5 of the Lunar Sourcebook, *Heiken et al.* [1991]) and shown here in Table 1.

Table 1: Porosity for various depths.

Depth Range (cm)	Average Porosity, n (%)
$0-15$	52 ± 2
$0-30$	49 ± 2
$30-60$	44 ± 2
$0-60$	46 ± 2

2.2 Density

The bulk density (ρ) of the lunar regolith is defined as the mass of material in a given volume, which relates the particle density (ρ_p) and porosity (n) to the bulk density as (see Section 3.4.2.3.1 of the DSNE or Chapter 9 of the Lunar Sourcebook)

$$\rho = \rho_p(1-n). \tag{2.1}$$

The DSNE suggests using $\rho_p = 3.1$ g/cm^3 for the average particle density over the entire Moon. Otherwise, the typical highlands particle density is $\rho_p = 2.75 \pm 0.1$ g/cm^3 whereas the typical mare particle density is $\rho_p = 3.35 \pm 0.1$ g/cm^3.

The bulk density[1] as a function of depth, fit to Apollo data, is given by

$$\rho(z) = 1.92\frac{z+12.2}{z+18}, \tag{2.2}$$

where z is the depth in cm and ρ is in units of g/cm^3. At the surface ($z=0$), the density is 1.30 g/cm^3, and increases to 1.92 g/cm^3 for large depths. This expression is fairly reasonable down to 3 m (the limit reached by Apollo drill core samples). In order to get an up-to-depth average of the bulk density, take

$$\rho_{avg,depth}(z) = \frac{1}{z}\int_0^z dz'\rho(z'), \tag{2.3}$$

[1] Follows the average particle density of 3.1 g/cm^3 for all depths with a porosity depth dependence following Table 1, see the *porosity of lunar soil* paragraph on page 492 in the Lunar Sourcebook.

which gives (compare with the equation for d_m on page 494 of the Lunar Sourcebook)

$$\rho_{avg,depth}(z) = 1.92\left[1 - \frac{5.8\ln\left(\frac{z+18}{18}\right)}{z}\right]. \tag{2.4}$$

For example, the average bulk density of the regolith with a depth range of $0 - 60$ cm would be $\rho_{avg,depth}(60)$ = 1.65 g/cm^3.

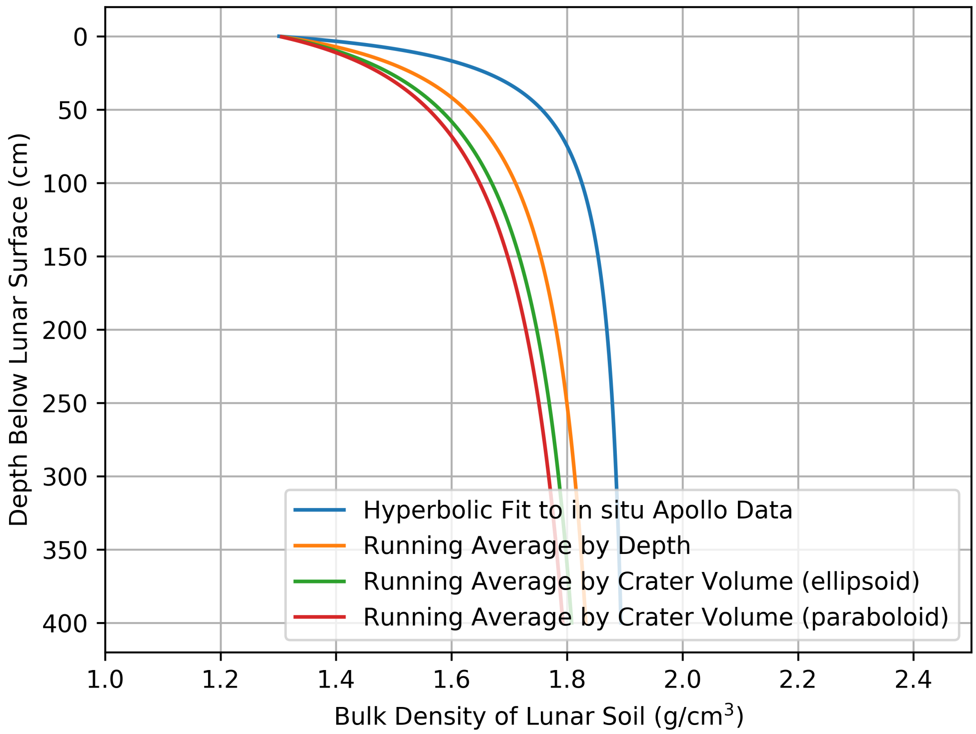

Figure 1: A comparison of the regolith bulk density for a certain depth depth (blue), the depth-averaged bulk density (orange), and the volume-averaged bulk density (green). See also, Figure 9.16 of the Lunar Sourcebook [*Heiken et al.*, 1991].

For a higher-fidelity estimate of the average bulk density sampled by the crater, a volume-average can be used instead of a depth-average, given by

$$\rho_{avg,volume}(z) = \frac{\int dV \rho(z')}{\int dV}. \tag{2.5}$$

Expanding the integral in a cylindrical coordinate system and assuming an ellipsoidal crater shape, Equation (2.5) becomes

$$\rho_{avg,ellipsoid}(z) = \frac{\int_0^z \int_0^{R\sqrt{1-z'^2/z^2}} \int_0^{2\pi} d\phi r dr dz' \rho(z')}{\int_0^z \int_0^{R\sqrt{1-z'^2/z^2}} \int_0^{2\pi} d\phi r dr dz'} \tag{2.6}$$

$$= \frac{1.92}{4z^3}\left[z(6ab - 6b^2 - 3az + 3bz + 4z^2) + 6(a-b)(b^2 - z^2)\ln\left(\frac{b}{z+b}\right)\right], \tag{2.7}$$

for the volume-averaged density in g/cm^3 with z in cm, where $a = 12.2$ and $b = 18$. Following the example from earlier, the average bulk density of the regolith with a depth range of $0 - 60$ cm would be $\rho_{avg,ellipsoid}(60) = 1.60$ g/cm^3, which is $\sim 3\%$ less than $\rho_{avg,depth}(60) = 1.65$ g/cm^3.

On the other hand, if a paraboloid crater shape is assumed, Equation (2.5) becomes

$$\rho_{avg,paraboloid}(z) = \frac{\int_0^z \int_0^{R\sqrt{1-z'/z}} \int_0^{2\pi} d\phi r dr dz' \rho(z')}{\int_0^z \int_0^{R\sqrt{1-z'/z}} \int_0^{2\pi} d\phi r dr dz'} \tag{2.8}$$

$$= \frac{1.92}{z/2}\left[b - a + \frac{z}{2} - \frac{(a-b)(b+z)\ln\left(\frac{b}{z+b}\right)}{z}\right], \tag{2.9}$$

using the same values for a and b as before. Again, with the prior example, the average bulk density of the regolith with a depth range of $0 - 60$ cm would be $\rho_{avg,paraboloid}(60) = 1.58$ g/cm^3, which is $\sim 4\%$ less than $\rho_{avg,depth}(60) = 1.65$ g/cm^3, see Figure 2. The expression given in Equation (2.9) is useful for computing the ejected mass from a crater[2], given a crater depth z.

The expressions for the regolith density at a certain depth z, weighted by depth, and weighted by crater volume (ellipsoid and paraboloid) are given by Equations (2.2), (2.4), and (2.7), (2.9), respectively, are compared in Figure 1. The crater volume is approximated as a half-ellipsoid with two of the dimensions scaled by the crater radius R and one dimension scaled by the crater depth z, sliced such that the half-ellipsoid is symmetric about the surface normal for Equation (2.7). On the other hand, a paraboloid-shaped crater is used for Equation (2.9). For a given crater, more of the volume is near the surface so that more weight is given by bulk densities that originate near the surface. In contrast, the depth-averaged bulk density takes the bulk density at each depth equally. This results in the volume-averaged bulk density to be slightly less than the depth-averaged bulk density, as shown in Figure 1. In addition, comparing an ellipsoidal crater vs. a parabolic crater, the parabolic crater (typically used in literature, see *Singer et al.* [2020]) exhibits the softest increase of the average bulk density as a function of depth.

[2] In an iterative fashion, since the crater radius depends on the regolith density.

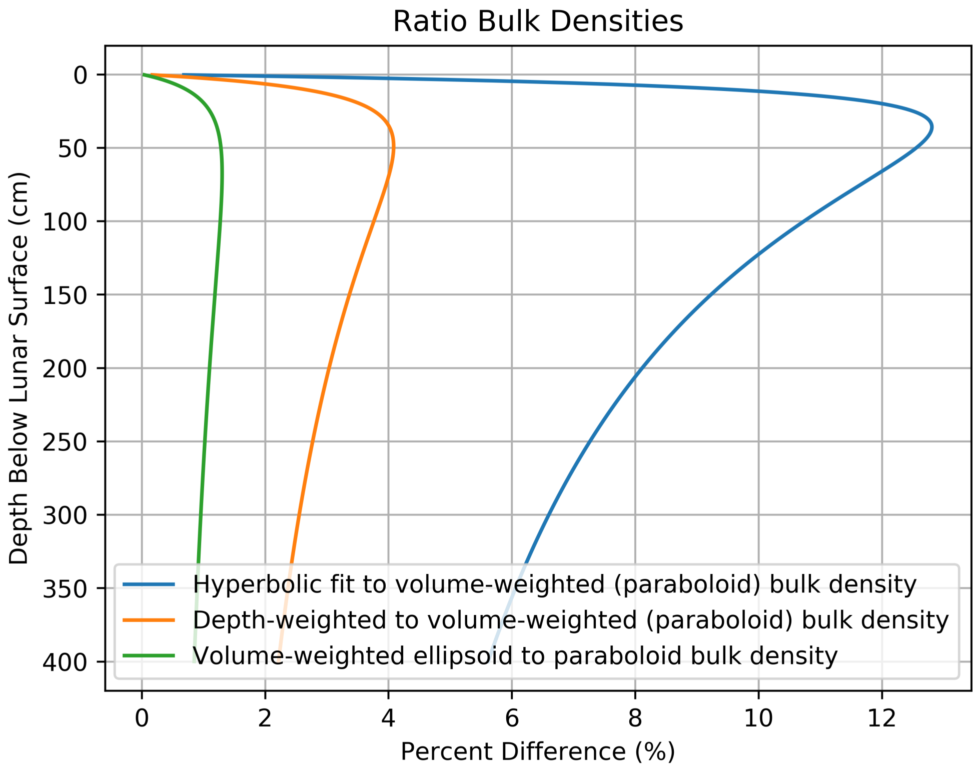

Figure 2: Relative error of using the density at a certain depth (blue) and using the depth-weighted average (orange) of the regolith bulk density.

2.3 Strength

The strength of the regolith can be measured in different ways, depending on the use. In the case of modeling impacts, the shear strength of loose regolith and tensile strength of solid rock can be used. The top 10 m or so consists of fine-grained material called regolith. From 10 m to about 2 km is the large scale ejecta that is course grained and ballistically transported (Fig 4.22 of the Lunar Sourcebook, *Heiken et al.* [1991]). All of the impacts studied in this model will create craters less than 2 km, so it is expected that the shear strength will be used for the loose regolith and large scale ejecta material.

The shear strength of the lunar soil increases with the depth. As an example, both Figure 9.26 of the Lunar Sourcebook, reproduced in Figure 3, and Table 12 of *Slyuta* [2014] depict this characteristic, shown in Table 2.

The average shear strength (green curve) in Figure 3 can be expressed by the

Table 2: The change of the shear strength of the lunar soil with depth.

Depth (cm)	Shear strength (kPa)
5	$0.1 - 2.5$
50	$1 - 3.5$
100	$2 - 4$
200	$4 - 8$

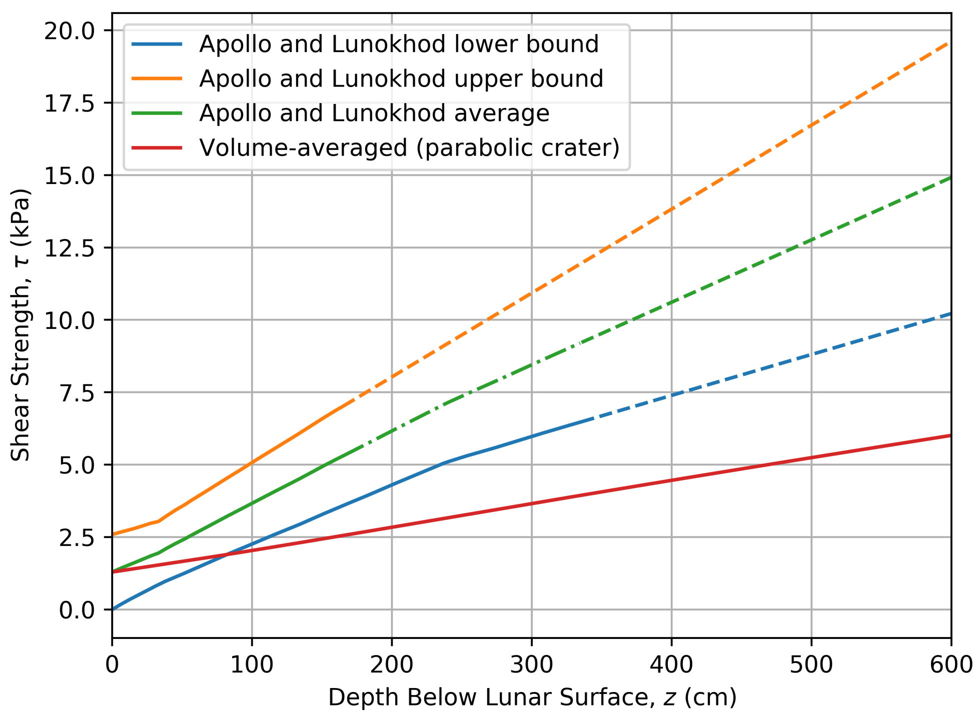

Figure 3: The range of regolith shear strength as a function of depth below the lunar surface taken from Figure 9.26 of the Lunar Sourcebook. The average shear strength is also calculated (green). Points that are extrapolated beyond what is available are shown by the dashed lines. The volume-averaged shear strength (red) assumes a parabolic-shaped crater of depth z.

piece-wise form

$$\tau(z) = \begin{cases} \frac{z}{45.55} + 1.288, & 0 \leqslant z < 50 \text{ cm} \\ \frac{z}{40.21} + 1.144, & 50 \text{ cm} \leqslant z < 250 \text{ cm} \\ \frac{z}{46.06} + 1.942, & z > 250 \text{ cm} \end{cases} . \tag{2.10}$$

Assuming the crater has a parabolic shape and that the effective shear strength is the volume-average, Equation (2.10) becomes (as shown by the red curve in Figure 3)

$$\tau(z) = \begin{cases} \frac{z}{136.65} + 1.288,\ 0 \leqslant z < 50 \text{ cm} \\ \frac{z}{120.63} + 1.144 + \frac{7.163}{z} - \frac{118.33}{z^2},\ 50 \text{ cm} \leqslant z < 250 \text{ cm} \\ \frac{z}{138.18} + 1.942 - \frac{194.81}{z} + \frac{16902}{z^2},\ z > 250 \text{ cm} \end{cases} . \tag{2.11}$$

2.4 Particle Size Distribution

2.5 Scaling Law Properties

3 Primary Flux Environment

3.1 Space-Time Dependence of Environment

3.1.1 Ephemeris Definition

3.1.2 Selenographic Extent

3.2 Sporadic Meteoroid Complex

3.2.1 Angular Distribution

3.2.2 Density Distribution

3.2.3 Mass Distribution

3.3 Near-Earth Objects

3.3.1 Speed Distribution

3.3.2 Mass Distribution

3.4 Meteoroid Showers

4 Secondary Flux Environment

4.1 Ejecta Distribution

4.1.1 Speed Distribution

4.1.2 Zenith Distribution

4.1.3 Azimuth Distribution

4.2 Mass/Particle Size Distribution

4.3 Orbital Mechanics

4.3.1 Crater on Surface to Observer at Surface

4.3.2 Crater on Surface to Observer at or above Surface

4.4 Selenographic Distance & Bearing

5 Scaling Laws

5.1 Crater Size – Strength & Gravity Regime

5.2 Minimum & Maximum Ejected Speed

5.3 Maximum Ejected Particle Mass

5.4 Mass Ejected from Crater

References

Heiken, G. H., D. T. Vaniman, and B. M. French, *Lunar Sourcebook, a user's guide to the Moon*, Cambridge University Press, 1991.

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Slyuta, E., Physical and mechanical properties of the lunar soil (a review), *Solar System Research*, *48*(5), 330–353, 2014.