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GRAVITATIONAL EFFECTS ON A RIGID CASIMIR CAVITY

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Vacuum fluctuations produce a force acting on a rigid Casimir cavity in a weak gravitational field. Such a force is here evaluated and is found to have opposite direction with respect to the gravitational acceleration; the order of magnitude for a multi-layer cavity configuration is analyzed and experimental detection is discussed, bearing in mind the current technological resources.

Although much progress has been made in the evaluation and experimental verification of effects produced by vacuum energy in Minkowski space-time,[1–6] it remains unclear why the observed universe exhibits an energy density much smaller than the one resulting from the application of quantum field theory and the equivalence principle.[7] Furthermore, no experimental verification that vacuum fluctuations can be treated according to the equivalence principle has been obtained as yet, even though there are expectations, as we agree, that this should be the case. Motivated by these considerations, our paper, relying on Ref. 8, computes the effect of a gravitational field on a rigid Casimir cavity, evaluating the net force acting on it, so as to show that the order of magnitude of the resulting force, although not allowing an immediate experimental verification, is compatible with the current extremely sensitive force detectors, actually the interferometric detectors of gravitational waves. The Casimir cavity is rigid in that its shape and size remain unchanged under certain particular external conditions such as, for example, the absence of accelerations or impulses, and we evaluate the force by calculating the potential of a cavity fixed in a Schwarzschild field, showing that the force has opposite direction with respect to gravitational acceleration.

To evaluate the force produced by the gravitational field let us suppose that the cavity has geometrical configuration of two parallel plates of proper area $A = L^2$ separated by the proper length a. The system of plates is taken to be orthogonal to the direction of gravitational acceleration. In Minkowski space-time the zero-point energy of the system can be evaluated, in the case of perfect conductors, as

$$U = \frac{\hbar c L^2}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \sqrt{k^2 + \left(\frac{n\pi}{a}\right)^2}, \tag{1}$$

where we have introduced the normal modes labelled by the integer n and the transverse momentum k. On performing the integral by dimensional regularization, the energy takes the well known Casimir expression

$$U_{\rm reg} = -L^2 \frac{\pi^2 \hbar c}{720 a^3}, \tag{2}$$

where the final result is independent of the particular regularization method.

If the cavity is at rest in a static gravitational field, via the equivalence principle each normal mode still resonates in the cavity, with associated energy reduced from the gravitational red-shift by the factor $\sqrt{g_{00}}$. Evaluating again the integral by dimensional regularization, and requiring that $g_{00} = 1 - \frac{\alpha}{r}$, with $\alpha \equiv \frac{2GM}{c^2}$ as in a Schwarzschild geometry, we obtain $U_{\rm reg} = -L^2 \frac{\pi^2 \hbar c}{720 a^3} \left(1 - \frac{\alpha}{r}\right)^{1/2}$, assuming that $L << r$. Now minus the gradient of $U_{\rm reg}$ with respect to r yields the force on the rigid cavity; for $r >> \alpha$, the resulting expression can be approximated by

$$\vec{F} \approx \frac{\pi^2 L^2 \hbar c}{720 a^3} \frac{g}{c^2} \vec{e}_r = \frac{\pi^2 L^2 \hbar c}{720 a^4} \frac{a g}{c^2} \vec{e}_r \approx \vec{F}_C |\delta\phi|, \tag{3}$$

where $\vec{e}_r$ is the unit vector in the r-direction, g is the modulus of gravitational acceleration and $\phi(r) \equiv \frac{GM}{c^2 r}$ denotes the gravitational potential which varies by the amount $\delta\phi \approx \phi'(r)a$. Such a force has opposite direction with respect to the gravitational acceleration $\vec{g}$. The force exerted by vacuum fluctuations in a weak and static gravitational field can be interpreted as the (Newtonian) gravitational push on an object with negative energy (the Casimir energy).

In considering the possibility of experimental verification of the extremely small forces linked to this effect we point out that such measurements cannot be performed statically; this would make it necessary to compare the weight of the assembled cavity with the sum of the weights of its individual parts, a measure impossible to perform. On the contrary, the measurements we are interested in should be performed dynamically, by modulating the force in a known way; the effect will be detected if the modulation signal will be higher than the sensitivity of the detector. In this spirit we focus on the sensitivity reached by the present technology in detection of very small forces on a macroscopic body, on earth, paying particular attention to detectors of the extremely small forces induced by a gravitational wave. As an example, gravitational wave signals h of order $\approx 10^{-25}$, corresponding to forces of magnitude $\approx 5 \cdot 10^{-17} N$ at frequency of few tens of Hz, are expected to be detected with the Virgo gravitational wave detector presently under construction,[9,10] after a month of integration time.

In the course of studying experimental possibilities of verification of the force on a rigid Casimir cavity, we evaluate this force on a macroscopic body, having essentially the same dimensions of mirrors for gravitational wave detection and obtained through a multi-layer sedimentation by a series of rigid cavities. Each rigid cavity consists of two thin metallic disks, of thickness of order 100 nm[11] separated by a dielectric material, inserted to maintain the cavity sufficiently rigid: introduction of a dielectric is equivalent to enlarging the optical path length by the refractive index n ($a \mapsto na$). By virtue of presently low costs and facility of sedimentation, and low absorption in a wide range of frequencies,[12] SiO_2 can be an efficient dielectric material.

From an experimental point of view we point out that the Casimir force has so far been tested down to a distance a of about 60 nm, corresponding to a frequency $\nu_{\min}$ of the fundamental mode equal to $2.5 \cdot 10^{15} Hz$. This limit results from the difficulty to control the distance between two separate bodies, as in the case of measurements of the Casimir pressure. As stated before, in our rigid case, present technologies allow for cavities with much thinner separations between the metallic plates, of the order of few nanometers. At distances of order 10 nm, finite conductivity and dielectric absorption are expected to play an important role in decreasing the effective Casimir pressure, with respect to the case of perfect mirrors.[13,14] In this paper we discuss experimental problems by relying on present technological resources, considering cavities with plates' separation of 5 nm and estimating the effect of finite conductivity by considering the numerical results of Ref. 13; for a separation of 6.5 nm this corresponds to a decreasing factor η of about $7 \cdot 10^{-2}$ for Al. Moreover, to increase the total force and obtain macroscopic dimensions, $N_l = 10^6$ layers can be used, each having a diameter of 20 cm, and thickness of 100 nm, for a total thickness of about 10 cm. Last, one should also consider corrections resulting from finite temperature and roughness of the surfaces, although one might hope to minimize at least the former by working at low temperatures.

With these figures, the total force $\vec{F}^T$ acting on the body can be calculated with the help of Eq. (3), modified to take into account for SiO_2 the refractive index n, the decreasing factor η, the area A of disk-shape plates, and the N_l layers:

$$\vec{F}^T \approx \eta N_l \frac{A\pi^2 \hbar c}{720(na)^3} \frac{g}{c^2} \vec{e}_r. \tag{4}$$

This formula describes a static effect, while the need for a feasible experiment makes it mandatory to modulate the force, and various experimental possibilities are currently under study.[15–17] In particular, we are investigating the possibility of modulating η, by varying the temperature, so as to achieve a periodic transition from conductor to superconductor regime. By doing so one can obtain $\eta_{\max}$ of order $5 \cdot 10^{-1}$, and the magnitude of the force at the modulation frequency can reach 10^{-14} N. Even though such a force is apparently more than two orders of magnitude larger than the force which the Virgo gravitational antenna is expected to detect, we should bear in mind that the signal there is at reasonable high frequency (some tens of Hz), while our calculated signal remains at lower frequencies, i.e. tens of mHz. Moreover, the technical problems resulting from stress induced in changing temperature require careful consideration before saying that modulation is feasible.

To sum up, we have found that the gravitational field on a rigid Casimir cavity gives rise to a net force, whose direction is opposite to the one of gravitational acceleration. Experimental verification of the calculated force depends crucially on the capability of solving the signal modulation problem. On the other hand, in the authors' opinion, the absolute value of the calculated force can be already of interest to suggest that experiments involving effects of gravitation and vacuum fluctuations are not far from the reach of current technological resources.

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References

1. M. Bordag, B. Geyer, G. L. Klimchitskaya and V. M. Mostepanenko, *Phys. Rev.* **D60**, 055004 (1999); V. M. Mostepanenko and M. Novello, *Possible violation of Newtonian gravitational law at small distances and constraints on it from the Casimir effect* (HEP-PH 0008035).
2. M. T. Jaekel and S. Reynaud, *Jour. Phys.* **I3**, 1093 (1993).
3. K. Milton, *Dimensional and Dynamical Aspects of the Casimir Effect: Understanding the Reality and Significance of Vacuum Energy* (HEP-TH 0009173); G. Barton, *J. Phys.* **A34**, 4083 (2001).
4. S. K. Lamoreaux, *Phys. Rev. Lett.* **83**, 3340 (1999); U. Mohideen and A. Roy, *Phys. Rev. Lett.* **81**, 4549 (1998).
5. S. A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, Cambridge, 1989).
6. B. S. DeWitt, *Phys. Rep.* **19**, 295 (1975); M. Bordag, U. Mohideen and V. M. Mostepanenko, *Phys. Rep.* **353**, 1 (2001).
7. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
8. E. Calloni et al., QUANT-PH 0109091.
9. D. Blair, *The Detection of Gravitational Waves* (Cambridge University Press, Cambridge, 1991).
10. VIRGO Collaboration, *Nucl. Instr. Meth.* **A360**, 258 (1995); *The VIRGO Experiment* by VIRGO Collaboration, in: Verbier 2000, Cosmology and Particle Physics, pp. 138–145.
11. M. Bordag, B. Geyer, G. L. Klimchitskaya and V. M. Mostepanenko, *Phys. Rev.* **D62**, 011701 (2000).
12. R. A. Street, *Hydrogenated Amorphous Sylicon* (Cambridge University Press, Cambridge, 1991); B. Chapman, *Glow Discharge Processes* (Wiley, New York, 1980).
13. A. Lambrecht and S. Reynaud, *Eur. Phys. J.* **D8**, 309 (2000).
14. G. Jordan Maclay, *Phys. Rev.* **A61**, 052110 (2000).
15. S. Hunklinger, H. Geisselmann and W. Arnold, *Rev. Sci. Instrum.* **43**, 584 (1972).
16. P. R. Saulson, *Interferometric Gravitational Wave Detectors* (World Scientific, Singapore, 1994).
17. R. Esquivel–Sirvent, C. Villareal and G. H. Cocoletzi, *Phys. Rev.* **A64**, 052108 (2001).