Fast Magnetic Field Analysis of a Deflection Yoke with a Slot Core

Shuhei Nakata and Takafumi Nakagawa

Abstract—**The cylindrical boundary element method is extended to analyze a three-dimensional configuration such as a deflection yoke with a slot core. The analyzed results are compared with the measurements and the peak field discrepancy is less than 1%. Furthermore the beam convergence characteristics on the screen surface are compared with the measured pattern and it is found that the numerical convergence error is less than 0.2 mm. As for the computation time, the matrix calculation and its solution time is reduced by a factor of 9 in comparison with conventional three-dimensional analysis. The field calculation time is also improved to 0.061 sec/point, which is 6 times faster than by conventional analysis.**

Introduction

NOWADAYS the performance of a color display monitor system requires such sophistications as fine spot diameters on the screen. For this purpose, estimations of the beam spot aberration, the raster distortion and convergent characteristics caused by the deflection yoke, are essential for the design study and optimization of the yoke shape and windings. Hence precise magnetic field calculations in a reasonable CPU time are necessary.

For field calculations, we have been applying the surface magnetic charge method [1], which is a boundary element method, to the deflection yoke design study in our group, the calculated results showing a good agreement with the measured field distribution [2]. On the other hand, when the method is applied to three-dimensional calculations such as the field analysis of a deflection yoke with a slot core, we have to split up the material surface into a large number of surface elements to guarantee acceptable precision. As a result, this takes a lot of CPU time and memory size. Therefore this method has not been so useful for the design study of deflection yokes with a complicated structure.

To save CPU time for field calculations, ordinary Fourier analysis in the azimuthal direction has been applied to the surface magnetic charge method [3]–[5]. However, the ordinary Fourier transformation requires a symmetrical boundary structure. Nowadays however the requirement of a high deflection angle has arisen and a deflection yoke with a slot core is desirable [2]. As the core has several slots on the surface and does not have a symmetrical boundary shape, ordinary Fourier analysis cannot be applied. In this article, a new method for the field calculation of the deflection yoke with a slot core is proposed.

The newly developed method combines the analytical solution of the field inside the slot region and boundary element method of the field outside the slot region to satisfy the boundary condition. By this method, the field outside the slot can be treated as a region with a symmetrical boundary and therefore the formulation of the surface charge method by Fourier transformation becomes applicable. As a result, a lot of CPU time and memory size are saved. In the following sections, the theory and the results of the field calculation and the beam orbit calculation are presented.

Theory

The schematic drawing of the slot core for a deflection yoke is shown in Fig. 1. The left and right side of Fig. 1(a) show the front view and cross sectional side view of the slot core for a deflection yoke, respectively. Fig. 1(b) is the front view and the cross-sectional side view of an ordinary symmetrical core. The shaded area in both figures represents a ferro-material.

As shown in this figure, the slot core has several slots on the inside surface and the windings of the deflection yoke can be placed inside them. On the other hand, in an ordinary core, the windings must be placed outside the inner surface of the core. Therefore, the slot core can be fabricated on the inner core surface close to the beam trajectory of the cathode ray tube, which is coincident with the core axis, as compared with an ordinary core, and a high deflection angle can be obtained with the same ampere-turns of windings. It is furthermore also recognized that the slot core has a non-rotationally symmetric boundary and hence the application of ordinary Fourier analysis becomes difficult.

In the newly developed method, the region of free space, which means outside the core itself, is divided into two regions. One is outside the slot (defined as region 1) and the other is inside the slot (defined as region 2). In the second region, the field can be calculated analytically with localized coordinates. On the other hand, the first region has a rotationally symmetric boundary and an ordinary Fourier analysis in the azimuthal direction is applicable.

Fig. 2 shows the boundary and the coordinates used in

Manuscript received June 28, 1993; revised February 19, 1994.

The authors are with Third Group, Electrotechnology Department, Central Research Laboratory, Mitsubishi Electric Corporation, 1-1, Tsukaguchi honmachi, 8 Chome Amagasaki, Hyogo, 661 Japan.

IEEE Log Number 9400797.

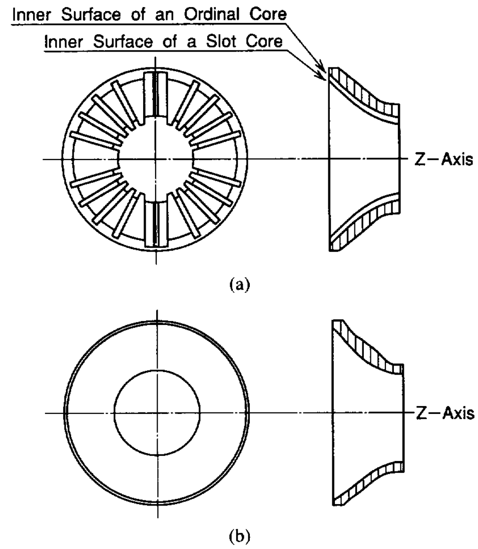

Fig. 1. Schematic drawing of the slot core and the ordinary core for a deflection yoke. (a) The front and side view of the slot core. (b) The same figure of the ordinary non-slotted core.

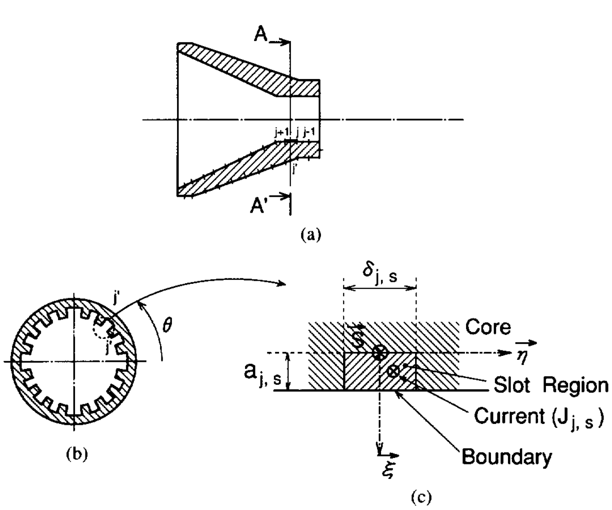

Fig. 2. Local coordinate system in the slot region. (a) is the cross-sectional side view of the core, (b) is the cross-sectional view from A–A′ side of the $r-\theta$ plane, and (c) is the expanded view of the sth slot region in jth element with the local coordinate used in this article.

this article. Figure 2(a) shows the cross-sectional side view of the slot core and the elements for the boundary element method $(\cdots, j-1, j, j+1, \cdots)$ in the region 1 which are distributed in the $r-z$ plane. Fig. 2(b) is the cross-sectional view in the $r-\theta$ plane from the dash-dotted line A–A′ of 2(a), and the outer and inner boundary are j' and jth elements for the boundary element method. Fig. 2(c) is an expanded view inside the dotted circle in Fig. 2(b) which includes the sth slot in the jth element. The localized coordinates are also defined here. In the figure $\vec{s}$ is the direction of the current flow inside the slot, $\vec{\eta}$ is the azimuthal direction of the core and $\vec{\xi}$ is defined as $\vec{s} \times \vec{\eta}$ which has an outer direction. The $a_{j,s}$ and $\delta_{j,s}$ are the depth and the width of the slot, respectively.

As is well known, the magnetic field in a free space can be expressed by the transverse and normal components of the magnetic field on the surrounding boundary surface, as in [6],

$$C_c\vec{H} = \int \vec{J} \times \nabla G \, dv - \int (\vec{n} \cdot \vec{H}) \nabla G \, dS - \int (\vec{n} \times \vec{H}) \times \nabla G \, ds, \tag{1}$$

where C_c is half on the boundary and 1 inside the space and G is a Greens function in free space $(= 1/4\pi r)$. The integration $\int dv$ is volume integration in the free space and $\int dS$ is surface integration on the surrounding.

The Greens function can be expressed by the associated Legendre functions, as in [3],

$$G(\vec{r}_s, \vec{r}_c) = \frac{1}{4\pi^2 r_s r_c} \{Q_{-1/2}(\alpha(r_s, z_s, r_c, z_c)) + 2 \sum_{n=1,2,3,\cdots} Q_{n-1/2}(\alpha(r_s, z_s, r_c, z_c)) \cdot \cos(n(\theta_c - \theta_s))\}, \tag{2}$$

$$\alpha(r_s, z_s, r_c, z_c) = 1 + \frac{(r_s - r_c)^2 + (z_s - z_c)^2}{2 r_s r_c}, \tag{3}$$

where $Q_{n-1/2}$ is the associated Legendre function of order $n - 1/2$ and the subscripts s and c represent the source point and calculated point, respectively.

Therefore, the field inside the surrounding can be calculated when the variables, $\vec{n} \cdot \vec{H}$ and $\vec{n} \times \vec{H}$, on the boundary are known. In the following, we will explain the estimation of the above variables using the flux continuity between the two regions.

The field analysis in region 1 is now considered. As the boundary of the region is symmetric, an ordinary Fourier transformation can be applied. Assuming the infinite permeability of the core, and 3rd term of the right-hand side of (1) becomes zero except out the boundary to the slot regions. Then one of the variables, $(\vec{n} \cdot \vec{H})$, on the symmetrical boundary can easily be Fourier transformed. The other variable, $(\vec{n} \times \vec{H})$, on the boundary between the slot and free space can be evaluated by using the analytical solution inside the slot and the continuity of the magnetic flux density across the boundary as follows.

In the slot region, the field can be treated as a two-dimensional problem by taking into account its structural characteristics, which are enough length in the $\vec{s}$ direction compared with its cross-section. Then the analytical solution can be applied assuming that the core material has an infinite permeability and a constant current density in the slot. The vector potential inside the slot region can be written analytically as follows,

$$A_s = \tfrac{1}{2}\mu_0 J_{j,s}\xi^2 + \sum_{m=1}^{\infty} \mu_0 \{a_{m,j,s} \cos (k^c_{m,j,s}\eta) \cdot \cosh (k^c_{m,j,s}\xi) + b_{m,j,s} \sin (k^s_{m,j,s}\eta) \cosh (k^s_{m,j,s}\xi)\} \tag{4}$$

where μ_0 and $J_{j,s}$ are the permeability of free space and the current density inside the slot, respectively. The constants $k^c_{m,j,s}$ and $k^s_{m,j,s}$ in the equation can be defined so as to satisfy the boundary condition of the magnetic field on the core surface.

$$k^c_{m,j,s} = m \frac{2\pi}{\delta_{j,s}} \tag{5}$$

$$k^s_{m,j,s} = (m - \tfrac{1}{2}) \frac{2\pi}{\delta_{j,s}}, \tag{6}$$

where m is the integer and $\delta_{j,s}$ is the width of the slot.

From the above equation, the normal (H_ξ) and the transverse (H_η) components of the magnetic field on the outer slot surface, which is the boundary between the two regions, can be described as,

$$H_\xi = \sum_{m=1}^{\infty} \{-k^c_{m,j,s} a_{m,j,s} \sin (k^c_{m,j,s}\eta) \cosh (k^c_{m,j,s} a_{j,s}) + k^s_{m,j,s} b_{m,j,s} \cos (k^s_{m,j,s}\eta) \cosh (k^s_{m,j,s} a_{j,s})\}, \tag{7}$$

$$H_\eta = -J_{j,s} a_{j,s}, - \sum_{m=1}^{\infty} \{k^c_{m,j,s} a_{m,j,s} \cos (k^c_{m,j,s}\eta) \sinh (k^c_{m,j,s} a_{j,s}) + k^s_{m,j,s} b_{m,j,s} \sin (k^s_{m,j,s}\eta) \sinh (k^s_{m,j,s} a_{j,s})\}, \tag{8}$$

where $a_{j,s}$ is the depth of the sth slot in the jth boundary element.

The coefficients ($a_{m,j,s}$ and $b_{m,j,s}$) can be represented using the variables in (1) from the continuity of normal magnetic flux on the boundary between the two regions:

$$\vec{n} \cdot \vec{H}_{\text{region1}} = \vec{n} \cdot \vec{H}_{\text{region2}} = H_\xi \tag{9}$$

From the orthogonal basis functions ($\cos (k^c_{m,j,s}\eta)$ and $\sin (k^s_{m,j,s})$), the variables ($a_{m,j,s}$, $b_{m,j,s}$) inside the slot region are obtained and are written as

$$a_{m,j,s} = -\frac{2}{k^c_{m,j,s}\delta_{j,s} \cosh (k^c_{m,j,s}, a_{j,s})} \int_{-\delta_{j,s/2}}^{\delta_{j,s/2}} (\vec{n} \cdot \vec{H}) \cdot \sin (k^c_{m,j,s}\eta)\, d\eta \tag{10}$$

$$b_{m,j,s} = \frac{2}{k^s_{m,j,s}\delta_{j,s} \cosh (k^s_{m,j,s} a_{j,s})} \int_{-\delta_{j,s/2}}^{\delta_{j,s/2}} (\vec{n} \cdot \vec{H}) \cdot \cos (k^s_{m,j,s}\eta)\, d\eta. \tag{11}$$

Then the basic equation (1) is translated and the following equation can be obtained.

$$C_c\vec{H} = -\sum_j \int \nabla G\vec{n}_j \cdot \vec{H}_j\, dS_j + \int dV(\vec{J}_0 \times \nabla G) + \sum_j \int dS_j\, J_{j,s} a_{j,s} (\vec{s} \times \nabla G) + \sum_j \int dS_j \sum_{m,s} (\vec{s} \times \nabla G) \cdot \{k^c_{m,j,s} a_{m,j,s} \cos (k^c_{m,j,s}\eta) \sinh (k^c_{m,j,s} a_{j,s}) + k^s_{m,j,s} b_{m,j,s} \sin (k^s_{m,j,s}\eta) \sinh (k^s_{m,j,s} a_{j,s})\}. \tag{12}$$

In the above equation, the first and the second terms are the surface magnetic charge effects, and the externally applied current in region 1, respectively. The third term comes from the applied current inside the slots. The final term is the contribution from the tangential component of the magnetic field on the boundary between the two regions.

By defining the variable $\vec{n} \cdot \vec{H}$ as so-called surface charge σ, linear equations for σ can be obtained from the above equation as,

$$C_c\sigma_i = -\sum_j \int \vec{n}_i \cdot \nabla G\sigma_j\, dS_j + \int dV\, \vec{n}_i \cdot (\vec{J}_0 \times \nabla G) + \sum_j \int dS_j \vec{n}_i \cdot (J_{j,s} a_{j,s} \vec{s} \times \nabla G) + \sum_j \int dS_j \sum_{m,s} \vec{n}_i \cdot (\vec{s} \times \nabla G) \{k^c_{m,j,s} a_{m,j,s} \cos (k^c_{m,j,s}\eta) \sinh (k^c_{m,j,s} a_{j,s}) + k^s_{m,j,s} b_{m,j,s} \sin (k^s_{m,j,s}\eta) \sinh (k^s_{m,j,s} a_{j,s})\}. \tag{13}$$

The above equation can be Fourier transformed and the distribution of the surface magnetic charge of the n'th order can be obtained.

$$\sigma_i = \sum_{n'} \sigma^{(n')} \cos (n'\theta) \tag{14}$$

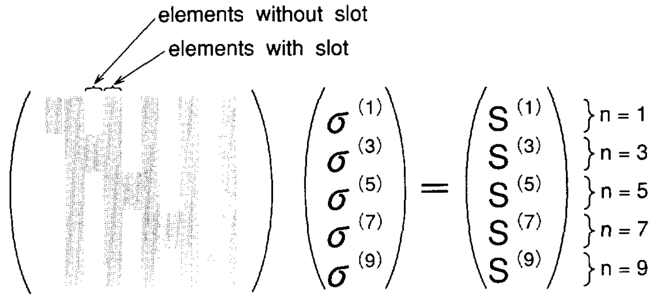

Fig. 3. Distribution of non-zero elements in the matrix.

$$
\begin{aligned}
C_c \sigma_i^{(n')} = & -\sum_j \int dS_j \vec{n}_i \cdot \nabla G^{(n')} \sigma_j^{(n')} + \int dV \, \vec{n}_i \cdot (\vec{J} \sigma_0^{(n')} \times \nabla G^{(n')}) + \sum_j \int dS_j \vec{n}_i \, (\vec{F}(J_j)^{(n')} \times \nabla G^{(n')}) \\
& + \sum_{j,s,s',n,m}^{m=\infty} \int dS_j \, (\vec{n}_i \cdot (\vec{s} \times \nabla G^{(n')})) \sigma_i^{(n)} \left\{ -\frac{2}{\delta_{j,s}} \tanh \, (k_{m,j,s}^c a_{m,j,s}) \right\} \\
& \cdot \int_{-\delta_{j,s/2}}^{\delta_{j,s/2}} \cos \, (n\theta) \sin \, (k_{m,j,s}^c \eta) \, d\eta \, \frac{1}{r_{j,s'} \pi} \int_{-\delta_{j,s'/2}}^{\delta_{j,s'/2}} \cos \, (n'\theta) \cos \, (k_{m,j,s'}^c \eta') \, d\eta' \\
& + \frac{2}{\delta_{j,s}} \tanh \, (k_{m,j,s}^s a_{m,j,s}) \int_{-\delta_{j,s/2}}^{\delta_{s/2}} \cos \, (n\theta) \cos \, (k_{m,j,s}^s \eta) \, d\eta \, \frac{1}{r_{j,s'} \pi} \int_{-\delta_{j,s'/2}}^{\delta_{j,s'/2}} \cos \, (n'\theta') \sin \, (k_{m,j,s'}^c \eta') \, d\eta'. \qquad (15)
\end{aligned}
$$

In the above equations, the constant $a_{m,j,s}$ and $b_{m,j,s}$ are transferred by using (10) and (11). The $\vec{F}(J_j)^{(n')}$ on the righthand side of the third term is the n'th coefficient of the Fourier transformed current distribution inside the slot in the jth element.

In an ordinary Fourier transformation, the coefficients of different order do not couple with each other. However, in the new method the coupling between different orders exists when the element has slots on the surface as will be recognized from (15). Fig. 3 shows the distribution of the non-zero components in the total matrix. In the figure, the shaded area represent the distribution of the non-zero elements, n is the order of the Fourier transformation. The total matrix size is $n \times j_{\text{element}}$.

By solving the above linear equations, the magnetic charge density distribution can be obtained and the field inside the free space can be calculated.

Numerical Results

The equation formulated in the above section was applied to the field calculation of a deflection yoke with a slot core. The numerical results are presented in this section. The model deflection yoke with a slot core having been developed by Mitsubishi Electric Corporation, is shown in Fig. 4. The figure shows the front view and the cross-sectional side view.

The core is 45.5 mm long, and the aperture of neck and screen side are 47 mm and 120 mm, respectively. Toroidal coils are wound for vertical deflection and saddle type coils for horizontal deflection. The vertical coils are fabricated inside the slots on the inner surface. The core has 16 slots with an aperture angle of 10 degrees and the depth is 4 mm at the neck and 3 mm at the screen.

Firstly the computational time of the field analysis by the present method and the ordinary three-dimensional calculation are compared. The details of the ordinary three-dimensional method are described in ref. [2].

The results are listed in Table I. The table shows that the new method requires only 104 elements to guarantee the residual error of less than 0.16%, the matrix calculation and solution time is 427 sec and the field calculation time is 0.061 sec/point. During the calculation, a residual errors of the matrix elements and the field were set to less than 0.1% by increasing the order of the Gaussian integration method, which is applied to surface integration in (15). The computer used in these calculations has a calculation speed of 90 MIPS. On the other hand ordinary three-dimensional analysis needs a large number of elements, i.e., about 1500, to satisfy the comparable residual error. Hence the matrix calculation and its solution time takes as long as 1 h and the field calculation time is 0.34 sec/point. The table shows a significant reduction in computational time for the field analysis and in memory size by the new method.

Next the precision of the calculated field is presented. Fig. 5 shows the vertical deflection field distribution along the z-axis when one ampere is applied to the vertical deflection coil. In the figure, the dots are the experimental values measured by a Hall probe (Bell 640 incremental gauss meter) and the solid line shows the results of the new method. The origin of the abscissa is the neck of the deflection yoke in the figure. The figure shows that the

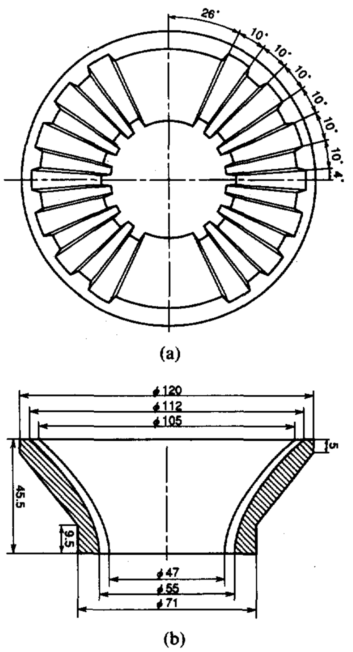

Fig. 4. The model of the deflection yoke with a slot core, (a) is the front view and (b) is the cross-section.

TABLE I
COMPARISON OF THE CPU TIME BETWEEN THE NEWLY DEVELOPED METHOD AND THE CONVENTIONAL THREE-DIMENSIONAL ANALYSIS

Item	3D Calc.	New Method
Total variables (Discretized surface element number)	1456	104
Matrix calculation and solution time	3977.2 sec	427.0 sec
Field calculation time	34.1/100 points	6.1 sec/100 points

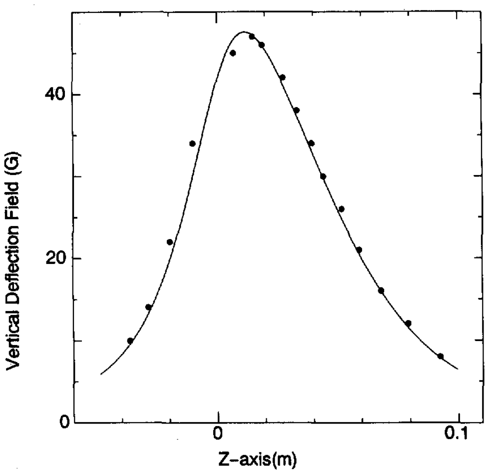

Fig. 5. Vertical deflection field distribution along the z-axis. The dots are the experimental results and the line shows the calculated result.

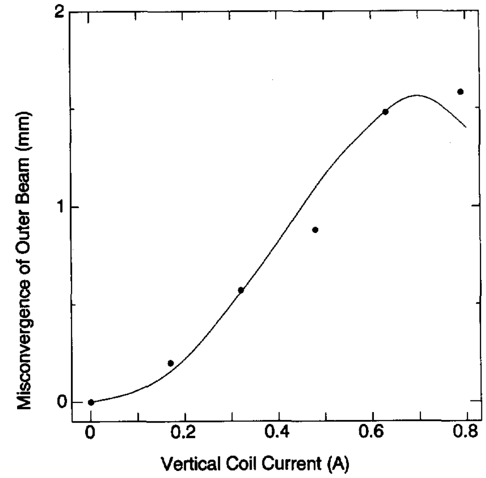

Fig. 6. Misconvergence dependence on the vertical deflection coil current. The dots are the experimental results and the line shows the calculated result.

calculated field has almost the same profile and the discrepancy of the peak field between the calculated and measured values is less than 1%. Therefore the practicability of the method is verified.

The analyzed results are applied to the beam orbit calculation and the relationship between the applied current and the misconvergence on the screen is compared with the experimentally obtained value. The misconvergence is defined here as the horizontal distance between the outer two beams of color display on the screen. For the orbit calculation, a 4th order Runge–Kutta method is applied.

The screen used in the experiment had a spherical shape with an inner curvature of 910 mm and was placed 400 mm from the deflection yoke neck. The S parameter, which is defined as the distance between the center and outer beams at the triode, was 6 mm and the triode was set 60 mm from the yoke neck. During the experiments, to avoid electrostatic interaction between the beams, the current was set to 1.0 μA.

Fig. 6 shows the results of the calculation and the misconvergence experiment. The dots are the experimental results and the solid line shows the calculated results. From the figure, it is found that the numerical error of the misconvergence from the experiment is less than 0.2 mm when the beams are deflected along the vertical axis. This result shows that the field calculated by the new method is also applied to the beam orbit analysis and can estimate the convergence characteristics of the designed deflection yoke with acceptable precision.

CONCLUSIONS

A hybrid method of analytical solution and the boundary element method with Fourier transformation has been developed and the magnetic field of a deflection yoke with a slot core, which has a non-symmetric boundary shape, was analyzed by this method.

The calculated field distribution coincides well with the measured one and the discrepancy of the peak field is less than 1%. Moreover the analyzed field data was applied to the beam orbit calculation and the convergence pattern on the screen was compared with experimental data. The result shows that the numerical error of the calculated convergence from the experimental value is less than 0.2 mm. Consequently, it is concluded that the method is particularly useful for the field analysis and estimation of the convergence characteristics of deflection yokes even when the yoke has a slot core. As for the CPU time, the matrix calculation and its solution time and the field calculation time are 472.0 sec and 0.061 sec/point, respectively. These values are 9 and 6 times faster than the ordinary three-dimensional surface charge method.

Acknowledgments

The author is deeply indebted to Messrs. M. Ogasa, H. Nishino, and T. Mitamura for their advice on the deflection yoke with a slot core and the precise measurements of the magnetic field, and to Dr. T. Yamada and Dr. S. Okuda and other members of our laboratory for their many fruitful discussions.

References

[1] S. Nakamura, T. Nomura, and M. Iwamoto, *IEE Japan*, vol. 96, p. 55, 1976.
[2] T. Nakagawa and S. Okuda, MT-11, p. 280, 1989.
[3] D. M. Fye, *J. Appl. Phys.*, vol. 50, p. 17, 1979.
[4] B. B. Dasgupta, *RCA Review*, vol. 44, p. 404, 1983.
[5] X. Sheng, *Proc. of the 12-th Int. Display Research Conf.*, p. 155, 1992.
[6] J. A. Stratton, *Electromagnetic Theory*, New York: McGraw-Hill, 1941.

Shuhei Nakata biography not available at time of publication.

Takafumi Nakagawa biography not available at time of publication.