$$\Rightarrow \frac{du}{dz} = \frac{\left[\left(\frac{\gamma-1}{\gamma}\right)\frac{E}{\mu_0 u} - B_0/\mu_0\right] dB_0/dz}{F\left[1 - \left(\left(\frac{\gamma-1}{\gamma}\right) + \frac{\rho}{u}F\right)\right]}$$

$$dB_0/dz = -\mu_0 \sigma (E - u B_0) \qquad \rightarrow \text{known}$$

$$\Rightarrow \frac{du}{dz} = \frac{-\mu_0 \sigma \left[\left(\frac{\gamma-1}{\gamma}\right)\frac{B}{\mu_0 u} - \frac{B_0}{\mu_0}\right]}{F\left[1 - \left(\left(\frac{\gamma-1}{\gamma}\right) + \frac{\rho}{uF}\right)\right]} (E - u B_0)$$

$$M = u/a \qquad a = \sqrt{\gamma \frac{\rho}{\rho}} \qquad \rho = P/u$$

1D Ionizing MPD Flow w/ only Induced Field.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u}\right) = -\vec{\nabla} \rho + \vec{J} \times \vec{B}$$

$$\rho \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)\left(C_p T \Lambda + \frac{u^2}{2}\right) \overset{+ \varepsilon_0 u/m}{=} \frac{\partial \rho}{\partial t} + \vec{J} \cdot \vec{E}$$

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \qquad \vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\mu_0 \vec{J} = \vec{\nabla} \times \vec{B} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{d\alpha}{dt} = \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)\alpha = \frac{k_f \rho u (1-\alpha)}{m u} - \frac{k_r \rho^2 \alpha^3}{m^2 u}$$