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Superluminal warp drive

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Abstract

In this Letter we consider a warp drive spacetime resulting from that suggested by Alcubierre when the spaceship can only travel faster than light. Restricting to the two dimensions that retains most of the physics, we derive the thermodynamic properties of the warp drive and show that the temperature of the spaceship rises up as its apparent velocity increases. We also find that the warp drive spacetime can be exhibited in a manifestly cosmological form.

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Colonizing the solar system and its close neighborhood appears to be one of the main aims of human race. There are many rather compelling reasons for seriously pursuing that endeavor. Besides the Hawking's argument that such a policy would protect the human race as a whole from disasters like the impact of an asteroid, an extreme climate change or a global nuclear war which will sooner or later menace the humanity with full extinction [1], I think that the main deepest argument in favor of cosmic colonization is for it to give the largest possible number of potential living beings the chance of come into existence in the future.

Essentially there are two ways through which one can envisage that colonization to be made. On one hand, by using a technology able to practically implement spaceships moving at speeds very close to the speed of light, so preserving the most sacrosanct principle of relativity. On the other hand, one can also make recourse to the existence of the negative energy amounts allowed by quantum theory that give rise to superluminal phenomena. Spaceships are so theoretically allowed that may travel at 'speeds faster than light' in the sense that the spacetime permits effective 'superluminal travel' even though the speed of light is not *locally* surpassed. A typical example of this kind of travelling procedures is what is known as the Alcubierre warp drive [2] and its improved alternatives [3].

In this Letter we shall restrict ourselves to consider some physical aspects of a special kind of warp drives which are characterized by producing travelling speeds which are defined to necessarily be faster than that of light. We derive a set of results that admits the interpretation that such spacetimes: (i) will give rise to thermal emission processes analogous to those taking place in black holes and de Sitter universes, and (ii) can be extended to contain a central black hole whose horizon is enclosed within the event horizon of the warp drive, so leaving a sort of ergo sphere where observers can be placed.

In an Alcubierre warp drive spacetime (t', x, y, z) which is apparently moving along trajectory x_s with velocity $v = dx_s(t')/dt'$, most of the physics depending on the spaceship worldline is concentrated on the two-dimensional spacetime resulting from setting the coordinates $y = z = 0$, which define the axis about which a cylindrically symmetric space develops. The spacetime of this two-dimensional warp drive in this way contains the entire worldline of the spaceship. However, it is worth noticing that, e.g., the three-dimensional warp drive

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structure may have important consequences which are not contemplated by its two-dimensional description, such as it happens with the energy density measured by Eulerian observers which becomes zero in two dimensions. Restricting ourselves to the two-dimensional case, if the apparent velocity of the spaceship is constant, $v = v_0$, then the two-dimensional metric can be written as [4]

$$ds^2 = -A(r)\left[dt' - \frac{v_0(1-f(r))}{A(r)}dr\right]^2 + \frac{dr^2}{A(r)}, \tag{1}$$

where $r = \sqrt{(x - v_0 t')^2}$ (note that in the past of the spaceship, $x > v_0 t'$, $r = x - v_0 t'$), $dx = dr + v_0\, dt'$, and

$$A(r) = 1 - v_0^2\big(1 - f(r)\big)^2. \tag{2}$$

The function $f(r)$ describes the main kinematic characteristic of the warp drive. In order for the above spacetime to behave like a warp drive, all what is required is that the function $f(r)$ be subjected to the boundary conditions that $f = 1$ at $r = 0$ (the location of the spaceship) and $f = 0$ at $r = \infty$. A general choice for that function which is able to describe any value of the constant apparent velocity was provided by Alcubierre himself and reads as [2]

$$f(r) = \frac{\tanh[\sigma(r+R)] - \tanh[\sigma(r-R)]}{2\tanh(\sigma R)}, \tag{3}$$

where σ and R are positive arbitrary constant. Another simpler and more restrictive choice for $f(r)$ which only allows for real, constant spaceship apparent velocities which, if we want to keep the existence of an event horizon, are $v_0 \geqslant 1$ and also satisfies the above boundary conditions is

$$f(r) = 1 - \tanh(\sigma r), \tag{4}$$

with σ again any positive constant. Because it is simple enough to enable us to retain all warp drive properties in the most interesting case that $v_0 \geqslant 1$, this is the choice for $f(r)$ that we shall consider throughout this Letter. This new form for $f(r)$ means, furthermore, that inside the bubble spacetime is no longer flat nor even regular at $r = 0$ since $f(r)$ cannot be differentiated there.

Metric (1) can finally be given a comoving, manifestly static form if we introduce the proper time $dt = dt' - v_0(1 - f(r))\,dr/A(r)$. In this case, we in fact derive a two-dimensional spacetime with line element [5]

$$ds^2 = -A(r)\,dt^2 + \frac{dr^2}{A(r)}. \tag{5}$$

This metric shows an apparent singularity at the event horizon that occurs whenever $A(r_0) = 0$. For a function $f(r)$ as given by Eq. (4) such an event horizon takes place at

$$r_0 = \frac{\operatorname{arccotanh}(v_0)}{\sigma}. \tag{6}$$

We note that, in fact, if $v_0 < 1$ then all the event horizon structure is lost, and that if the spaceship moves at the speed of light then the event horizon is shifted to infinity, and finally that if it does at unboundedly high speeds then the event horizon tends to concentrate onto the very position of the spaceship. It is also worth noticing that the above warp drive can also be converted into be a time machine for the travelers in the spaceship interior which becomes multiply connected by using the usual identification properties of the three-dimensional Misner space, such as it is described in Ref. [5].

In what follows we shall derive the spacetime metric of a warp drive spaceship which contains a Schwarzschild black hole in its interior. We shall start with a general d-dimensional line element given by $ds^2 = -e^{\nu}\,dt^2 + e^{\lambda}\,dr^2 + r^2\,d\Omega_{d-2}^2$, where we have specialized in warp drive bubbles preserving the Schwarzschild spherical symmetry, so that $d\Omega_{d-2}^2$ is the metric on the unit $(d-2)$-sphere. In this case, we obtain for the two main 00 and rr components of the Einstein equations,

$$\frac{\lambda'}{r}e^{-\lambda} - \frac{(d-3)}{r^2}\big(e^{-\lambda} - 1\big) = T_{00}, \tag{7}$$

$$\frac{\nu'}{r}e^{-\lambda} + \frac{(d-3)}{r^2}\big(e^{-\lambda} - 1\big) = T_{11}. \tag{8}$$

Now, since metric (5) has a similar form as that for a two-dimensional de Sitter space, with $v_0^2\tanh^2(\sigma r)$ playing the role of $\Lambda r^2/3$ (note that for σr small enough $3v_0^2\sigma^2$ becomes like a cosmological constant), and because for the de Sitter and Schwarzschild–de Sitter spaces the components of the stress-energy tensor must satisfy $T_{00} = -T_{11}$, it appears a reasonable assumption keeping here this relation. Thus, if we take $T_{00} = -T_{11}$ and sum up Eqs. (7) and (8), then we derive $\lambda = -\nu$, and hence, e.g., from Eq. (7) we finally have for the general d-dimensional case

$$e^{-\lambda} = e^{\nu} = 1 - \frac{\int dr\, T_{00} r^{d-2}}{r^{d-3}} - \frac{K}{r^{d-3}}, \tag{9}$$

where K is a constant to be related with the positive mass of the black hole. In the two-dimensional case for which the time component of the stress tensor, T_{00}, is chosen to be

$$T_{00} = \rho = \frac{v_0^2\tanh^2(\sigma r)}{r^2}\left(\frac{4\sigma r}{\sinh(2\sigma r)} - 1\right), \tag{10}$$

with ρ the energy density, we have for our static two-dimensional Schwarzschild—warp drive metric

$$e^{-\lambda} = e^{\nu} = 1 - v_0^2\tanh^2(\sigma r) - Kr. \tag{11}$$

It is worth noting that although near the position of the spaceship one can have that the energy density $\rho \simeq v_0^2\sigma^2 > 0$, ρ will steadily decrease with r to vanish at $r = \sinh(2\sigma r)/(4\sigma)$, becoming increasingly negative thereafter until it reaches a new extremal value, to finally again vanish as $r \to \infty$, such as one should expect for a warp drive spacetime [2–5].

On the other hand, the transformation from t to t' which can be obviously done in two dimensions is not so obvious in higher dimensions, so that a general d-dimensional solution obtained following the above procedure would become suspect. Thus, that calculation can be only regarded to be a mathematical procedure which leads to a solution that is valid only in the two-dimensional case (11).

The presence of an event horizon in the static form of the two-dimensional warp drive metric makes it possible that astronauts inside the warp drive spaceship would experience a thermal bath of particles. In what follows we shall show in detail that this is in fact the case by using the Euclidean formulation. First of all, it must be noted that the existence of an apparent event horizon in the warp drive metric (5) makes it necessary to maximally extend that metric. This can be achieved by first defining the quantity

$$r^* = \int \frac{dr}{A(r)}, \tag{12}$$

which yields either

$$r^* = \frac{\operatorname{arccotanh}[v_0 \tanh(\sigma r)]}{v_0\sigma} \tag{13}$$

if $\sinh^2(\sigma r) > (v_0^2 - 1)^{-1}$, or

$$r^* = \frac{\operatorname{arctanh}[v_0 \tanh(\sigma r)]}{v_0\sigma} \tag{14}$$

if $\sinh^2(\sigma r) < (v_0^2 - 1)^{-1}$. Introducing then advanced and retarded coordinates such that $V = t + r^*$, $W = t - r^*$, we can derive the metric

$$ds^2 = -\left[1 - v_0^2 \tanh^2(\sigma r)\right] dV\, dW, \tag{15}$$

where we have implicitly defined the radial coordinate as

$$r = \frac{1}{\sigma} \operatorname{arctanh}\left\{\frac{\operatorname{cotanh}[v_0\sigma(V-W)/2]}{v_0}\right\}. \tag{16}$$

Metric (15) cannot still be the maximally extended line element. This should be achieved by first introducing new coordinates V'' and W'', such that either $V'' = \arctan(V/r_0)$, $W'' = \arctan(W/r_0)$, from which we get

$$ds^2 = -\left[1 - v_0^2 \tanh^2(\sigma r)\right] r_0^2\, dV''\, dW'' \operatorname{cosec}^2 W'' \operatorname{cosec}^2 V'', \tag{17}$$

or $V'' = \arctan[\exp(V/r_0)]$, $W'' = \arctan[\exp(W/r_0)]$, with which

$$ds^2 = 4\left[1 - v_0^2 \tanh^2(\sigma r)\right] r_0^2 \times \operatorname{cosec}(2V'') \operatorname{cosec}(2W'')\, dW''\, dV'', \tag{18}$$

where r is implicitly defined to be

$$r = \frac{1}{\sigma} \operatorname{arctanh}\left\{\frac{\operatorname{cotanh}[v_0\sigma r_0(\tan V'' - \tan W'')/2]}{v_0}\right\}. \tag{19}$$

Finally, with the re-definition $V' = \exp(V/r_0)$, $W' = -\exp(-W/r_0)$, we can get

$$ds^2 = \left[1 - v_0^2 \tanh^2(\sigma r)\right] \frac{r_0^2\, dW'\, dV'}{V'W'}, \tag{20}$$

where

$$\frac{W'}{V'} = -\exp\left(-\frac{2t}{r_0}\right) \tag{21}$$

with either

$$r = \frac{1}{\sigma} \operatorname{arctanh}\left\{\frac{\operatorname{cotanh}[v_0\sigma r_0 \operatorname{arccotanh}(\frac{W'V'+1}{W'V'-1})]}{v_0}\right\} \tag{22}$$

if $\sinh^2(\sigma r) > (v_0^2 - 1)^{-1}$, or

$$r = \frac{1}{\sigma} \operatorname{arctanh}\left\{\frac{\tanh[v_0\sigma r_0 \operatorname{arctanh}(\frac{W'V'+1}{W'V'-1})]}{v_0}\right\} \tag{23}$$

if $\sinh^2(\sigma r) < (v_0^2 - 1)^{-1}$.

Following the Hartle–Hawking Euclidean method [6], let us now construct the propagator for a generic scalar field with mass m between two points (x, x') through the warp drive spacetime by using the path integral

$$G(x, x') = \lim_{\epsilon \to 0} \int_0^\infty dQ\, F(Q, x, x') e^{-(imQ + \epsilon Q^{-1})}, \tag{24}$$

where

$$F(Q, x, x') = \int \delta x[Q] \exp\left[\frac{i}{4} \int_0^Q \delta(\hat{x}, \hat{x}')\, dQ\right], \tag{25}$$

with the integral being taken over all paths $x(Q)$ from x to x' and Q is a parameter. In order to analytically continuing Q to the negative imaginary values required for giving the path integral a well-defined meaning and making the metric positive-definite, we shall now embed our two-dimensional warp drive spacetime as the three-hyperboloid [7]

$$-T^2 + S^2 + X^2 = r_0^2 = \frac{\operatorname{arccotanh}^2(v_0)}{\sigma^2} \tag{26}$$

with Lorentzian metric

$$ds^2 = -dT^2 + dS^2 + dX^2. \tag{27}$$

This embedding can here be achieved by exhibiting the new coordinates in the form,

$$T = \sqrt{A(r)} r_0 \sinh(t/r_0),$$
$$S = \sqrt{A(r)} r_0 \cosh(t/r_0),$$
$$X = \Theta(r), \tag{28}$$

with

$$\left[d\Theta(r)/dr\right]^2 = -\left[\frac{r_0^2 (\frac{d\chi}{dr})^2 - 4}{4(1+\chi)}\right], \tag{29}$$

in which $\chi = -v_0^2(1 - f(r))^2$. This coordinate transformation can be readily seen to convert metric (5) into metric (27).

For the sake of completeness, we shall now show that our warp drive spacetime can be also described as the two-dimensional version of a cosmological Friedmann–Robertson–Walker space if we exhibit the hyperboloid coordinates in terms of new coordinates $[\theta \epsilon (-\infty, \infty), p \epsilon (0, \pi/2)]$, such that

$$T = r_0 \sinh(\theta/r_0),$$
$$S = r_0 \cosh(\theta/r_0) \sin p,$$
$$X = r_0 \cosh(\theta/r_0) \cos p. \tag{30}$$

We then in fact obtain

$$ds^2 = -d\theta^2 + r_0^2 \cosh^2(\theta/r_0)\, dp^2. \tag{31}$$

These coordinates entirely cover the whole of the two-dimensional warp drive space which, like in de Sitter space, would first contract until $\theta = 0$ and expands thereafter to infinity. If we define as usual the conformal time as $\eta = \int dt/a(t)$, which in the present case is $\eta = \arccos(\cosh(\theta/r_0)^{-1})$, then metric (31) becomes

$$ds^2 = a(\eta)^2\left(-d\eta^2 + dp^2\right), \tag{32}$$

with

$$a(\eta) = \frac{r_0}{\cos\eta}. \tag{33}$$

Solutions (31) and (32) show that the physics of a warp drive spacetime can be regarded as that of an isolated universe with cosmological horizon at r_0 where there is a distribution of space-like separated positive and negative energy density.

Coming back to our thermodynamic analysis we note now that the horizons at r_0 are at the intersection of the hyperplanes $T = \pm S$ with the hyperboloid. Then, on the complexified horizons, which are defined by $r = \sigma(\operatorname{arccotanh}(v_0))^{-1}$ real, X is real and either $T = S = r_0 W'$, $V' = 0$, or $T = -S = r_0 V'$, $W' = 0$. Following then exactly the same line of reasoning as in Ref. [6], we keep r fixed on real values and let t to be continued to $\tau + i\xi$. Using finally Eq. (21) we can uncover that the warp drive spaceship is filled with a thermal bath of radiation at temperature

$$T = \frac{1}{2\pi r_0} = \frac{\sigma}{2\pi \operatorname{arccotanh}(v_0)}. \tag{34}$$

Such a thermal radiation appears thus to be actually originated from the superluminal character of the spaceship motion, as for $v_0 = 1$, $T = 0$. Thus, as v_0 increases beyond unity, the temperature inside the ship rises up, tending to infinity as $v_0 \to \infty$. This result can indeed be interpreted by resorting to the presence of space-like separated peaks of positive and negative energies in the warp drive spacetime which is somehow reminiscent of the Hawking interpretation on the evaporation of black holes in terms of an escaping peak of positive energy and an incoming peak of negative energy, both being created from the vacuum near the event horizon as a pair [8].

The above result can be interpreted by considering that the crew of the spaceship will measure an isotropic background of thermal radiation inside the ship, at a temperature that rises up as its apparent motion is being accelerated. Because all geodesics must be equivalent inside the spaceship, any other crew will also detect an isotropic bath with the same temperature even though it is moving relative to the first crew. Finally, one may interpret the result of an increasing temperature for rising velocities by simply invoking the analogy with the de Sitter space where the thermal bath temperature depends on the square root of the cosmological constant which also goes like the inverse of the radius of the event horizon.

At first sight, the above calculation appears to suffer from an interesting shortcoming. Even in two dimensions a superluminal warp drive has in fact two classically different horizons. The horizon in the back of the bubble is like that of a black hole in that the bubble can send signals to the back but they cannot come back. The horizon in front is nevertheless like that of a white hole since the bubble can receive signals from the outside but cannot send them back. However, according to Hawking [9], even though a white hole is geometrically different from a black hole classically, they are physically indistinguishable quantum-mechanically, and therefore in our quantum mechanical calculation for two-dimensional warp drives all distinction between the back (black hole like) and front (white hole like) horizons should vanish as the considered quantum process is essentially time-symmetric [9]. A caveat arises nevertheless when one takes into account the Penrose contrary opinion that the boundary conditions for the universe must make quantum gravity time-asymmetric [10]. Although such a controversy has produced a long fruitful debate [11] it appears that for the kind of semiclassical process considered here one should adhere to the Hawking view, specially if quantum coherence is preserved during such a process, such as Hawking himself has recently acknowledged for black holes [12].

In summary, this Letter deals with some physical aspects of a special kind of two-dimensional Alcubierre warp drive which always describes spaceships moving at the speed of light or at superluminal apparent velocities. We first reviewed the spacetime of such a construct and extended it to contain a black hole. Next we considered the thermodynamic properties of the warp drive by employing the semiclassical Euclidean method. We thus maximally extended the static metric and exhibited it as a three-dimensional hyperboloid. The use made of such a representation was twofold. On the one hand, it allowed us to analytically continue the time and use a propagator formalism for a generally massive scalar field that finally led to obtain a thermal bath filling the space bubble at a temperature which was zero at the luminal limit and increased with the apparent spaceship velocity. On the other hand, the hyperboloid metric was also two-dimensionally exhibited as a cosmic Friedmann–Robertson–Walker metric, so indicating that our current universe could well be regarded to be an accelerating warp drive immersed in another larger cosmological space.

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