# Analysis of the Alcubierre Shape Function f(rs)

Alcubierres' original shape function used to produce a warped spacetime capable of FTL travel is cumbersome to work with due to its definition being comprised of Tanh functions. Here I show that a better shape function can be chosen without altering the underlying physics, while also making the problem less computationally intensive.

The original function takes the form of:

$$f(r_s) = \frac{\tanh[\sigma(r_s + \rho)] - \tanh[\sigma(r_s - \rho)]}{2\tanh[\sigma\rho]}$$

$$\text{With } r_s = \sqrt{(x - x_s(t))^2 + y^2 + z^2}$$

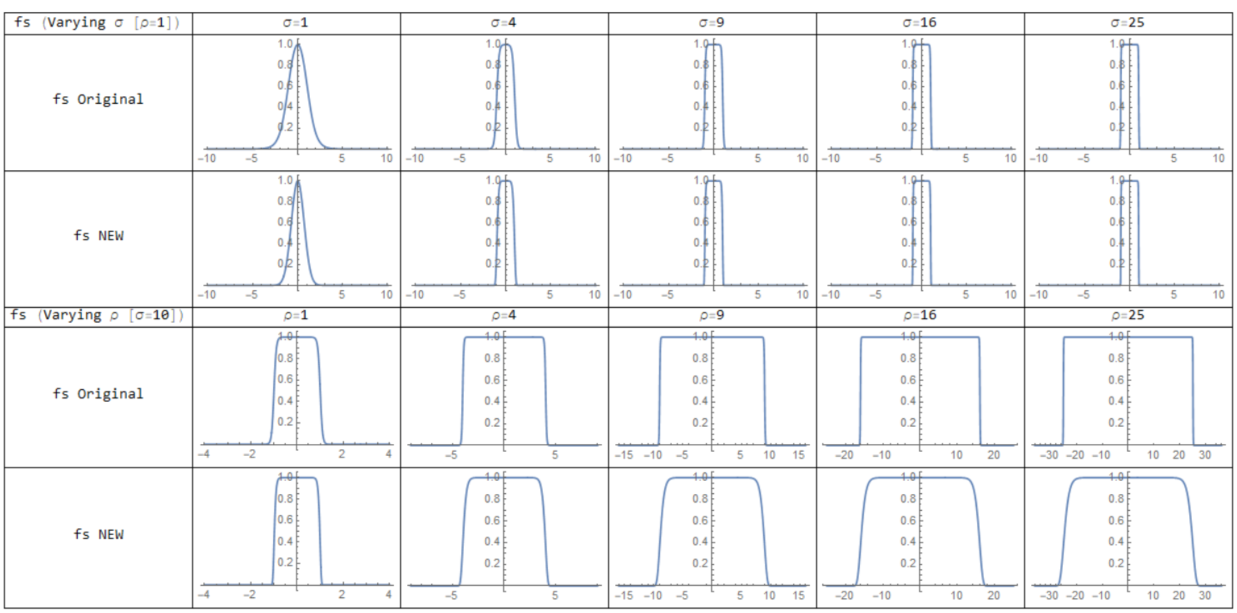
For large values of $\sigma$ the shape function $f(r_s)$ approaches the top-hat function,

$$f(r_s) = \begin{cases} 1, & rs \in [-\rho, \rho] \\ 0, & otherwise \end{cases}$$

Instead of using this complicated shape function, I propose changing it to:

$$f(r_s) = \exp\left[-\left({}^{r_s}/_{\rho}\right)^{2\sigma}\right]$$

This new shape function is a super-Gaussian function that has been matched to our original, with the same functional dependence when varying $\sigma$ and $\rho$. This new function also has the same limiting behavior as our original, except its form is now greatly simplified. **NOTE:** *the exponential is to the power of* $2\sigma$*, which is necessary to prevent odd (anti-symmetric) solutions*. The table below shows the identical nature of these two functions.




For large values of $\sigma$, the two choices of shape function are identical.

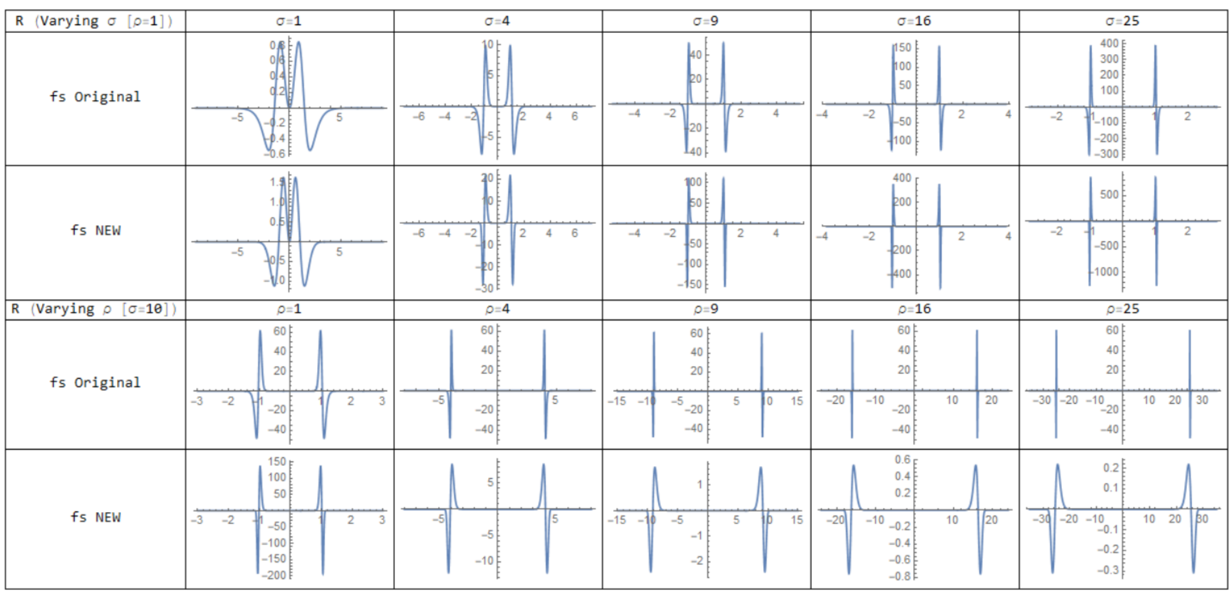
It is now important to further verify that this shape function produces the same desired physics. Our approach to analyzing the invariants of the metric will be a perfect way to confirm this. If changing the shape function has no effect on the invariants (except for maybe a re-scaling which is fine) then we an say the two metrics are functionally identical. **NOTE:** *This might be a good way of analyzing perturbed metrics (maybe quantum gravity(?)).*

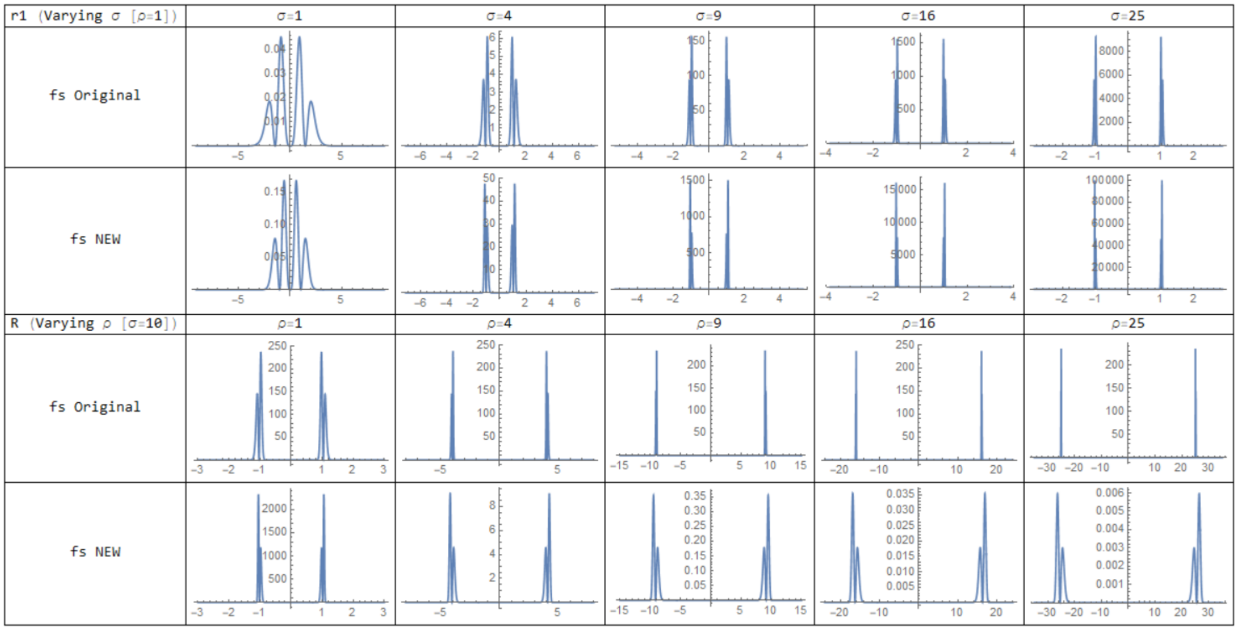
We may redefine $r_s$ by removing the dependence on *y* and *z*.

$$r_s = \sqrt{(x - x_s(t))^2 + y^2 + z^2} \rightarrow \sqrt{(x - x_s(t))^2 + r^2} \qquad (with\, r^2 = y^2 + z^2)$$

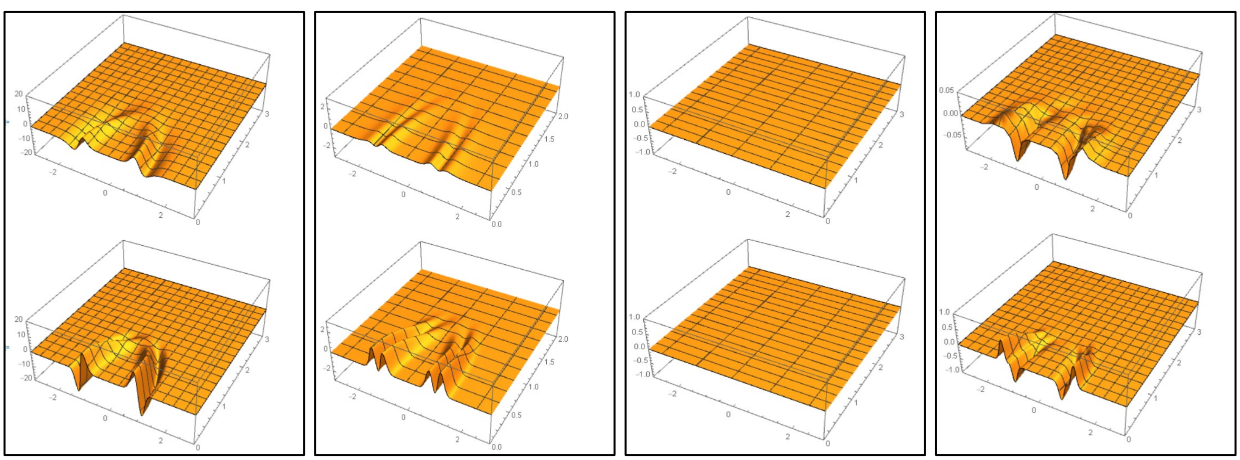
It is now important to note that there is azimuthal symmetry about the *x*-axis. The plots below will analyze the invariants along the *x*-axis. The 2D plots are taken at $r = 0$ along the path of movement and the 3D plots shows how the invariants change radially from the path of movement, from $r \geq 0$. Since each of the 3D plots represent one slice of $\varphi$ about the *x*-axis, and there is azimuthal symmetry, these slices can be rotated around the *x*-axis if one wanted to visualize the entire space. Here it is sufficient to analyze just one slice when comparing the invariants.

Here are the invariants along the *x*-axis ($r = 0$)






I only showed R and r1 since they give the best-looking results. r2 and w2 don't look fantastic since there is a loss of precision when calculating the invariants, but looking below at the last two 3D plots (R,r1,r2,w2 respectively, with the old function on top and new on bottom) we see that they are also close to each other. If anything, w2 with the new shape function produces better plots (due to it being less computationally intensive). The only real difference between the set of invariants produced by the two shape functions appears to be a simple scaling factor, which is fine.



In order to check the time dependent nature of these new shape functions I made some gifs of their evolution. I will attach them to the email this is sent in since I can't embed them in a word file. You will see that they are, again, identical.

From this analysis it is clear that the new shape function produces the same physics and is less computationally intensive. This makes it clear that a choice of shape function is relatively arbitrary as long as the limiting behavior matches our system (any small changes like scaling is merely an engineering problem and we can ignore for our analysis). With this in mind, one should choose as simple of a shape function as possible to allow for better and faster computations.

I would also like to take the time to present a possible new way of visualizing these invariant plots. Here are the density maps of the different invariants (x position on bottom and r going towards the top). Let me know what y'all think about this representation. I personally like the way these look. (the 3rd plot, r2, should be flat and identical. Just some precision rounding issues that prevent this from happening)