

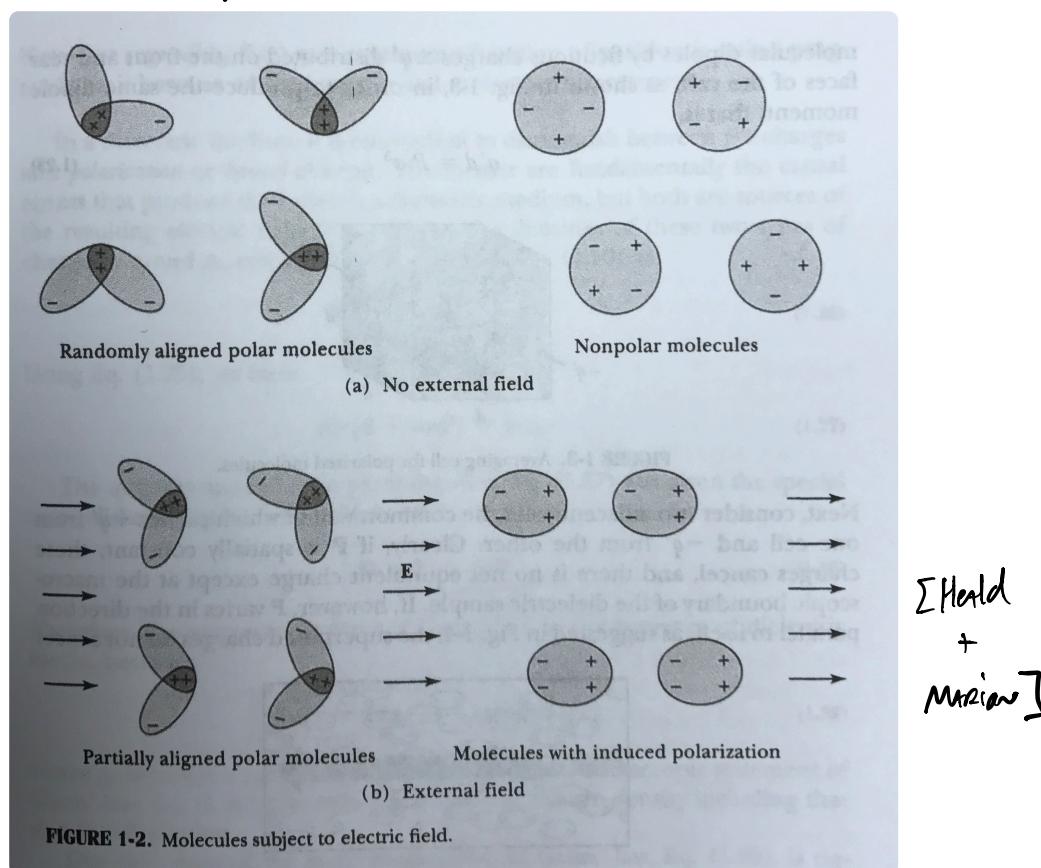
## Dielectric Media

Dielectric = electrically neutral media  
with no mobile charge carriers

Macroscopic scale of variation:

$$\sim 10^{-3} \text{ cm}$$

If we analyze the microscopic behavior,  
molecules respond to an applied field



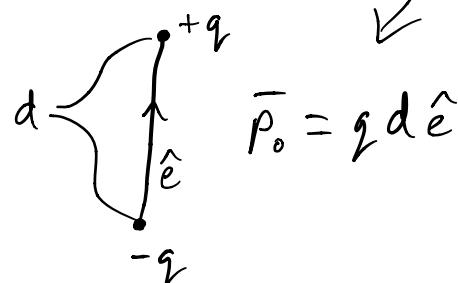
On the macroscopic scale, we use

$$\bar{P} = \text{polarization} \quad (\text{units} = \text{statvolt/cm})$$

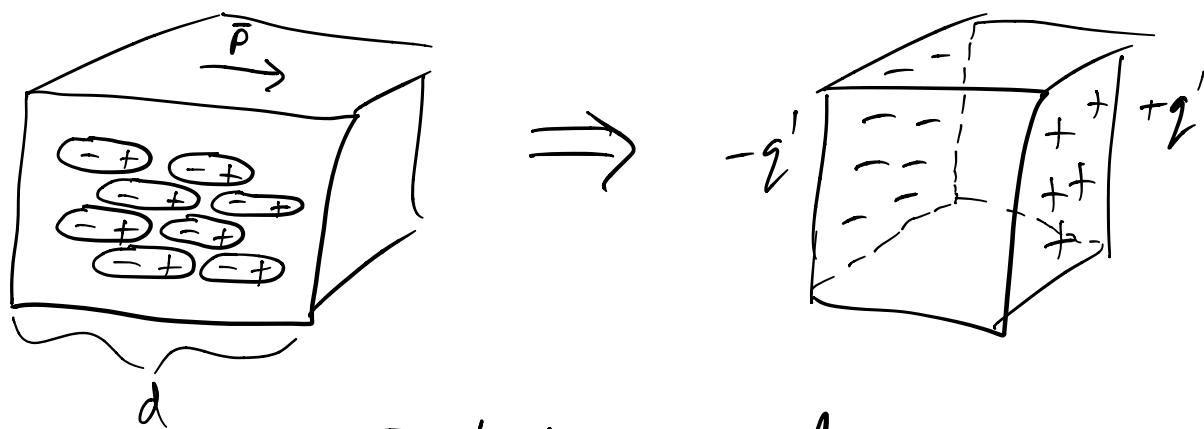
= electric dipole moment per unit volume

If each molecule has a dipole moment  
 $\bar{p}_0$  and the number density is  $N$ :

$$\bar{P}(\vec{r}) = \bar{p}_0(\vec{r})N(\vec{r})$$



Imagine a box of uniform dipoles:



Inside charges cancel

$\Rightarrow$  only boundaries contribute to total dipole moment

$$Pd^3 = \overbrace{q'}^{\text{total dipole moment}} d$$

total dipole moment  
 ↳ macroscopically averaged

Now, if the polarization is inhomogeneous

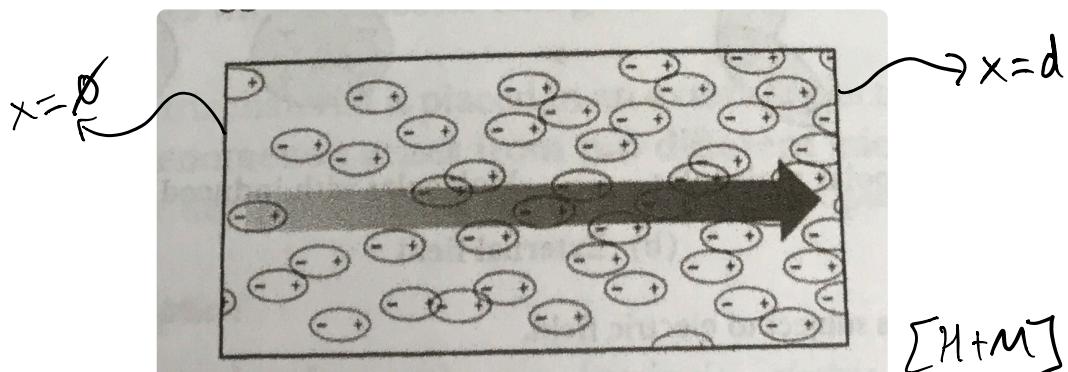


FIGURE 1-4. Inhomogeneous polarization.

$$\bar{P}(\vec{r}) = \hat{e}_x P_x(x)$$

polarization changes along  $\hat{e}_x$  direction

Analyzing the behavior at  $x=0$  and  $x=d$

$$g'd = Pd^3 \Rightarrow g' = Pd^2$$

$$\Rightarrow g'(0) = P_x(0)d^2$$

For  $x=d$ , we use a Taylor Series expansion:

$$g'(d) \approx P(0)d^2 + \left(\frac{dP_x}{dx}\right)_{x=0} d d^2$$

$$= P(0)d^2 + \left(\frac{dP_x}{dx}\right)_{x=0} d^3$$

Taking the difference in charge:

$$\Delta g' = g'(0) - g'(d) = -\left(\frac{dP_x}{dx}\right)_{x=0} d^3$$

$$\Rightarrow \frac{\Delta q'}{d^3} = \rho' = -\left(\frac{dP_x}{dx}\right)_{x=0}$$

charge density

This idea can be extended to 3-D:

$$\rho' = -\bar{\nabla} \cdot \bar{P}$$

In a dielectric medium, the charges are:

free charge: cause current, able to flow  $\Rightarrow \rho_f$

bound charge: polarization charge in medium  $\Rightarrow \rho_b$

Gauss' law can be rewritten as

$$\bar{D} \cdot \bar{E} = 4\pi(\rho_f + \rho_b)$$

But  $\bar{D} \cdot \bar{\rho} = -\rho_b$

$$\Rightarrow \bar{D} \cdot (\bar{E} + 4\pi \bar{\rho}) = 4\pi \rho_f$$

$\bar{D}$  = displacement

$$\Rightarrow \bar{D} \cdot \bar{D} = 4\pi \rho_f$$

 depends on free charge only.

$$\bar{D} \cdot \bar{E} = 4\pi \rho$$

 depends on total charge

$$\rho = \rho_f + \rho_b$$

Experimentally,  $\bar{P}$  is proportional to  $\bar{E}$   
for  $\bar{E}$  not too large

$$\bar{P} = \chi_e \bar{E}$$

$\checkmark$   
electric susceptibility

$$\Rightarrow \bar{D} = \bar{E} + 4\pi \bar{P} = (1 + 4\pi \chi_e) \bar{E}$$

$\underbrace{\phantom{0}}$   
 $\epsilon$

$$\Rightarrow \bar{D} = \epsilon \bar{E}$$

$\epsilon$  = dielectric constant (can be a tensor)

= 1 for free space

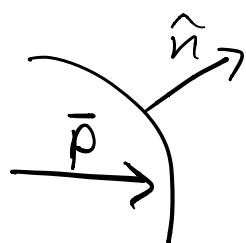
In some geometries, the polarization is not parallel to the surface normal vector, and the bound charge is

$$(\rho_s)_b = \hat{n} \cdot \bar{P}$$

$$\rho_b = -\bar{\nabla} \cdot \bar{P}$$

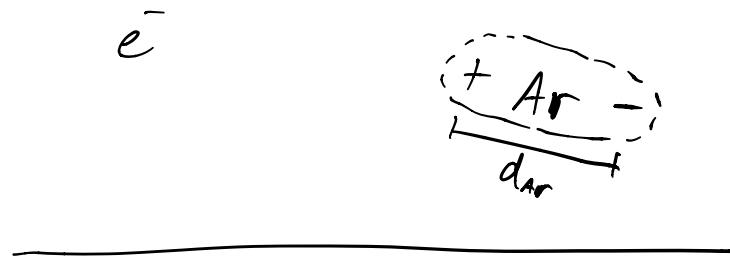
$(\rho_s)_b$  = bound surface charge density

$\hat{n}$  = outward normal vector to surface.



## Research Example

The momentum transfer cross section for Helium and Argon differ by a factor of  $\sim 3$  with Argon as the larger. This difference is due to an 8-fold increase in the polarizability of Ar compared to He.



The induced dipole moment is larger for Ar, so the induced  $\vec{E}$  is larger and the effective interaction distance is increased.