Laser-Induced Anomalous Heating of a Plasma

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It is shown that a sufficiently intense laser beam can drive low-frequency instabilities in a fully ionized plasma; these instabilities may cause considerable enhancement of the high-frequency resistivity of the plasma around certain frequencies and thus lead to an anomalous heating effect.

When a tiny solid pellet is irradiated with a giant pulsed laser, it is converted into a blob of dense plasma.[1-8] Because of the violent manner in which the plasma is produced, strong nonthermal density fluctuations can be excited in it. The well-known fine structure (in time) of nanosecond laser pulses is quite likely to aid the excitation of such nonthermal fluctuations. If the incident laser pulse is sufficiently intense, it drives low-frequency instabilities in the plasma which will amplify any nonthermal (or even thermal!) fluctuations to produce large-amplitude ion waves. Such ion waves can be responsible for a considerable enhancement of the high-frequency resistivity of the plasma.[9] One thus obtains an anomalous heating effect that should be taken into account, over and above the conventional collisional heating. In this paper we investigate this effect in some detail for typical laser plasma situations. It is concluded that laser powers which are currently being used or envisaged for the near future are sufficiently large to cause this effect to be important in some experiments.

In their analysis of the high-frequency conductivity of a fully ionized plasma, Dawson and Oberman[9] have shown that if there are large-amplitude ion waves present in a plasma (so that the ion correlations can depart significantly from thermal), its resistivity near the plasma frequency may be enhanced by many orders of magnitude. Qualitatively, the enhanced resistivity arises because of excitation of longitudinal plasma oscillations through an interaction of the oscillatory electron current and the ion density fluctuations. A similar enhancement in the resistivity around twice the plasma frequency, arising due to electron–electron interactions, has been investigated by Dubois and Gilinsky.[10] Following Dawson and Oberman,[9] the factor β, by which the high-frequency resistivity around plasma frequency is enhanced in the presence of large-amplitude ion waves, is given by

$$\beta = \frac{(2\pi)^3}{n_i}\,|n_i(k_0)|^2 = \frac{1}{(2\pi)^3 n_i}\sum_{l,j}\exp\,[i\mathbf{k}_0\cdot(\mathbf{r}_l - \mathbf{r}_j)],$$

where $k_0 \simeq (\omega^2 - \omega_p^2)^{1/2}/\sqrt{3}u_0$, n_i is the equilibrium ion density, and u_0 is the electron thermal velocity. For an ion density spectrum peaked at $|\mathbf{k}| = \langle k\rangle$ and having a width $\Delta k \sim \langle k\rangle$, it can be shown[9] that

$$\beta \simeq \frac{\delta^2\langle n_i\rangle\langle\lambda\rangle^3}{4\pi(2\pi)^3}\,, \tag{1}$$

where $\delta^2 = (\langle n_{i,\Delta k}^2(r)\rangle/\langle n_i^2\rangle)$ is the ratio of the mean square density fluctuations to the mean density n_i squared, and $\langle\lambda\rangle = (2\pi/\langle k\rangle)$. Now $\langle n_i\rangle\langle\lambda\rangle^3$ (which is the number of ions per cubic wavelength) can readily be 10^6 or more, so that β can be significantly large ($\gtrsim 10$) even if δ^2 is as low as 10^{-2}. If it is assumed that the perturbation grows from random thermal fluctuations, i.e., $(n_{\text{initial}}/n) \sim 1/(n\lambda^3)^{1/2}$ then, typically less than 10 e-folding times are required before the perturbation grows enough ($\delta \sim 0.1$) to cause a significant modification of the high-frequency resistivity; on the other hand, if the initial fluctuation is nonthermal, considerably shorter times may be required.

We shall now investigate low-frequency insta-

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bilities that a laser beam may readily excite in a plasma. The first instability we study, and whose possible existence was suggested to us by Kidder,[11] will be called thermal instability. When energy is fed into a certain volume of the plasma, at a rate higher than that at which it can diffuse away by thermal conduction and other dissipative processes, one can expect the ion acoustic waves to become unstable. It is quite clear that in the presence of an intense laser beam the Joule heating of the plasma by the laser beam itself can provide the energy input necessary for this instability. A reasonably detailed investigation of this effect can be carried out starting from the linearized one-fluid equations for the plasma (assuming no flows or gradients in equilibrium), viz., the fluid continuity equation

$$\dot{\rho}_1 + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \qquad (2)$$

(where the dot on top of ρ_1 denotes the partial time derivative $\partial/\partial t$), the fluid momentum conservation equation

$$\rho_0 \dot{v}_1 = -\nabla p_1, \qquad (3)$$

and the energy conservation equation (it is instructive to write this equation in its complete nonlinear form)

$$\rho \dot{T} + \tfrac{2}{3}\rho T \nabla \cdot \mathbf{v} = \tfrac{2}{3} H = \tfrac{2}{3} A \rho^2 T^n. \qquad (4)$$

Here, T has been used in place of KT, where K is the Boltzmann constant, $p = (\rho T/M)$ is the fluid pressure, and $H = \eta j^2 \equiv A\rho^2 T^n$ is the Joule heating rate of the plasma; the functional dependence of the resistivity η on temperature will determine the value of n (thus, for a fully ionized gas, $n = -1.5$). The value of A depends on the intensity of the laser beam. It has been assumed that the plasma is reasonably transparent to the laser light, so that the variation of its intensity with distance in the plasma can be neglected. Note also that we have not retained any dissipative terms like the thermal conductivity or viscosity on the right side of the energy equation; our treatment, therefore, will not predict any threshold powers for this instability. The equations governing the time dependence of the average temperature T_0 and the perturbation T_1 are readily obtained from Eq. (4):

$$\dot{T}_0 = \tfrac{2}{3} A \rho_0 T_0^n \qquad (5a)$$

and

$$\dot{T}_1 + \tfrac{2}{3} T_0 \nabla \cdot \mathbf{v}_1 = \tfrac{2}{3} A \rho_0 T_0^n \left(\frac{\rho_1}{\rho_0} + n \frac{T_1}{T_0} \right). \qquad (5b)$$

[11] R. E. Kidder (private communication).

Fourier-analyzing in space, and eliminating all dependent variables except ρ_1 from Eqs. (2), (3), and (5a, b), one obtains

$$\ddot{\rho}_1 - n \frac{\dot{T}_0}{T_0} \dot{\rho}_1 + k^2 s^2 \rho_1 + \tfrac{6}{5} k^2 s \dot{s} (2-n) \rho_1 = 0, \qquad (6)$$

where $s = (5T_0/3M)^{1/2}$ is the acoustic speed and k is the wave vector. To get a correct dispersion relation from Eq. (6) it is absolutely essential to take account of the fact that the mean temperature is varying on a time scale comparable to the growth or damping rate of these waves; erroneous conclusions can be drawn if this important fact is ignored.[12] Accordingly, we assume that

$$\rho_1 \propto \exp\left(\int (-i\omega + \gamma)\, dt \right),$$

where terms of order $(\dot{\omega}/\omega)$ and γ are comparable in magnitude. Substituting in Eq. (6) and assuming weakly unstable modes, i.e., γ, $(\dot{\omega}/\omega) \ll \omega$, one obtains

$$\omega = ks \quad \text{and} \quad \gamma = \left(\frac{4n-3}{10} \right) \frac{\dot{\omega}}{\omega}. \qquad (7)$$

It is of interest to mention here that since there are two distinct time scales in the problem, viz., the fast time scale governed by the frequency of the sound wave and the slow time scale determined by the rate at which the frequency changes with time, one can use the adiabatic equation of state, $p\rho^{-5/3} =$ const, to eliminate (T_1/T_0) from the right side of Eq. (5b), which is essentially a fast time scale equation. This simplifies Eq. (6) somewhat and the results given in Eq. (7) can be derived more easily.

One immediately notes from Eq. (7) that acoustic waves can become unstable only if $n > \frac{3}{4}$ (i.e., if the resistivity η increases with temperature faster than $T^{3/4}$); otherwise, the waves damp away and decay in amplitude. It is thus concluded that the thermal instability cannot be excited in a fully ionized gas, since $n = (-1.5)$. It is interesting to observe that for this value of n the heating term on the right side of Eq. (5b) vanishes (note above that ρ_1 and T_1 are connected by the adiabatic equation of state). Note also that one can find situations for a partially ionized gas where $n > 0.75$; it is conceivable, therefore, that in laser plasma experiments these waves are excited and grow in amplitude in the early stages of formation of the plasma, and then persist as large-amplitude ion waves even when the plasma is fully ionized.

[12] We are indebted to Dr. R. M. Kulsrud and Dr. F. W. Perkins for driving this point home to us and helping to clarify a lot of the subsequent analysis of this instability.

A second instability that we have investigated in some detail is a modified form of the two-stream instability, excited by the high-frequency field. As the following analysis will show, the threshold powers for this modified two-stream instability are considerably reduced when the incident frequency is sufficiently close (not too close, however!) to the plasma resonant frequency; fortunately, this is also one of the regions most appropriate for the anomalous heating effect, since it is the point of reflection of the laser wave.

The basic equations with which one begins are the electron and ion continuity equations

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot(n_\alpha \mathbf{v}_\alpha) = 0, \tag{8}$$

the electron and ion momentum conservation equations

$$\begin{aligned} m_\alpha n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t} + \mathbf{v}_\alpha\cdot\nabla\mathbf{v}_\alpha\right) &= -\nabla p_\alpha + n_\alpha e_\alpha(\mathbf{E}_0 \sin\omega_0 t + \mathbf{E}) \\ &\quad - m_\alpha n_\alpha v_\alpha \nu_\alpha, \end{aligned} \tag{9}$$

and finally, the Poisson's equation

$$\nabla\cdot\mathbf{E} = 4\pi \sum_\alpha e_\alpha n_\alpha. \tag{10}$$

Here, the subscript α denotes one of the two species (electrons or ions), the external field $\mathbf{E}_0 \sin\omega_0 t$ has been assumed to have an infinite wavelength (the dipole approximation—this is good at the point of reflection, since the laser wavelength approaches infinity there), the ν's are the relevant collision frequencies, and the rest of the symbols have their usual meanings. Because the external laser field has a very high frequency and the ions are heavy, they respond only weakly to the high-frequency field. One can, therefore, ignore their high-frequency motion in the lowest order. [This is essentially equivalent to making an expansion in the mass ratio m_e/m_i and retaining only the lowest-order terms; further, this approximation can be made rigorous by going to a frame oscillating with the ions. In this frame, the driving field on the electrons should be replaced by $(1 + m_e/m_i)\mathbf{E}_0 \sin\omega_0 t$.] The velocities and the self-consistent field $\mathbf{E}$ may now be eliminated from the linearized form of Eqs. (8)–(10) to give

$$\frac{\partial^2 \tilde{n}_i}{\partial t^2} + \nu_i \frac{\partial \tilde{n}_i}{\partial t} - s_i^2 \nabla^2 \tilde{n}_i + \omega_{pi}^2 \tilde{n}_i = \omega_{pi}^2 \tilde{n}_e \tag{11a}$$

and

$$\left(\frac{\partial}{\partial t} + v_{e0}\cdot\nabla\right)^2 \tilde{n}_e + \nu_e \left(\frac{\partial}{\partial t} + v_{e0}\cdot\nabla\right)\tilde{n}_e - s_e^2 \nabla^2 \tilde{n}_e + \omega_{pe}^2 \tilde{n}_e = \omega_{pe}^2 \tilde{n}_i, \tag{11b}$$

where $\omega_{p\alpha}^2 = (4\pi n_{\alpha 0} e_\alpha^2/m_\alpha)$, $s_\alpha^2 = (\gamma_\alpha T_\alpha/m_\alpha)$, the equation of state for the αth species is $p/n^{\gamma_\alpha} =$ const, all quantities with the $\sim$ over them denote first-order perturbations, and

$$\mathbf{v}_{e0} = (e\mathbf{E}_0/m_e\omega_0)\cos\omega_0 t$$

is the undisturbed oscillating electron velocity.

In general it is difficult to solve the coupled system of Eqs. (11a) and (11b). One method, which is quite appropriate for low-frequency instabilities, is to solve Eq. (11b) quasistatically; i.e., take $\tilde{n}_i$ to be a slowly varying function of time and treat it as static in Eq. (11b) for the electrons (the solution is quite readily obtained in the oscillating electron frame[9]). Substituting $\tilde{n}_e$ back into Eq. (11a), retaining only the low-frequency contribution to the resulting driving term on the right side, we obtain the following dispersion relation:

$$(\omega^2 + i\omega\nu_i) = k^2 s_i^2 + \omega_{pi}^2 \cdot\left(1 - \omega_{pe}^2 \sum_{p=-\infty}^{p=\infty} \frac{J_p^2(\mathbf{k}\cdot\boldsymbol{\varepsilon})}{\omega_R^2 - p^2\omega_0^2 - i\nu_e p\omega_0}\right), \tag{12}$$

where $\omega_R^2 = \omega_{pe}^2 + k^2 s_e^2$ and $\boldsymbol{\varepsilon} = (e\mathbf{E}_0/m\omega_0^2)$ is the electron excursion length in the external field. When ω_0 is close to ω_R, one need only retain terms with $p = 0, \pm 1$ in the sum on the right side of Eq. (12). The resulting equation is a quadratic in ω and can be solved to give

$$\omega = \frac{1}{2}\left\{- i\nu_i \pm \left[- \nu_i^2 + 4\left(\omega_L^2 + \frac{2\beta(\omega_0^2 - \omega_R^2)}{(\omega_0^2 - \omega_R^2)^2 + \nu_e^2\omega_0^2}\right)\right]^{1/2}\right\}, \tag{13}$$

where

$$\omega_L^2 = k^2 s_i^2 + \omega_{pi}^2\left(1 - \frac{\omega_{pe}^2}{\omega_R^2} J_0^2\right)$$

and

$$\beta = \omega_{pi}^2 \omega_{pe}^2 J_1^2.$$

For moderately high values of the external field, such that the electron excursion length in the field is small compared with the wavelength of the excited wave, one can expand the Bessel functions and retain only the lowest-order terms; thus, if J_0 is replaced by unity, ω_L is the typical frequency of ion acoustic waves. We note from Eq. (13) that instability can occur only if the terms within the square brackets add up to a quantity more negative than $-\nu_i^2$, for then ω can have a positive imaginary part; this requires that the inner parentheses be a negative quantity. Hence, instability occurs only if $\omega_0 < \omega_R$ and a typical threshold field is exceeded; for moderate amplitudes of the external field (so that $\mathbf{k}\cdot\boldsymbol{\varepsilon} \ll 1$), the threshold is given by

$$v_E \equiv \frac{eE_0}{m\omega_0} \gtrsim \sqrt{2}\, s_e \left(1 + \frac{T_i}{T_e}\right)^{1/2} \left(1 - \frac{\omega_0^2}{\omega_R^2}\right)^{1/2} \cdot \left(1 + \frac{\nu_e^2}{\omega_R^2(1 - \omega_0^2/\omega_R^2)^2}\right)^{1/2}. \tag{14}$$

We find that if ω_0 is very close to ω_R, so that $(\nu_e/\omega_R)^2 \gg (1 - \omega_0^2/\omega_R^2)^2$, the threshold goes up as ω_0 approaches ω_R; on the other hand, if the opposite inequality holds, i.e., if $(\nu_e/\omega_R)^2 \ll (1 - \omega_0^2/\omega_R^2)^2$, then the threshold goes down as ω_0 approaches ω_R. The latter is the regime of interest and in this regime one may roughly replace $(1 - \omega_0^2/\omega_R^2)$ with $(3\nu_e/\omega_R)$ to get the least value of the threshold field. This is given by

$$v_E \gtrsim 6^{1/2} s_e \left(1 + \frac{T_i}{T_e}\right)^{1/2} \left(\frac{\nu_e}{\omega_R}\right)^{1/2}. \tag{15}$$

Note that our analysis, being based on fluid equations, ignores any ion Landau damping of these waves; this should be a good approximation for $T_e \gg T_i$. Equation (15) shows that for sufficiently low collision frequencies, the threshold fields for this instability can be quite small; in particular, these thresholds are considerably lower than the usual two-stream instability threshold velocity, which would require the directed component of electron velocity v_E to be comparable to the electron thermal velocity s_e.

Using Eq. (15) and an expression for the electron-ion collision frequency for ν_e, one can derive a formula for the threshold power directly in terms of the density and temperature; thus, for $T_i \ll T_e$, one obtains for the power density P in watts per square centimeter

$$P \simeq 10^2 \left(\frac{N_{12}^3}{T}\right)^{1/2}, \tag{16}$$

where N_{12} is the particle density per cubic centimeter in units of 10^{12} and T is the temperature in electron volts. For a typical plasma produced by a ruby laser beam, one has $N_{12} = 10^9$ (this is determined by the condition $\omega_0 \approx \omega_R$); thus, for $T = 10^3$ V, we get

$$P = 10^{14}\ \mathrm{W/cm^2}.$$

This requires that 10^{10} W of ruby laser power be focused into an area roughly 100 μ by 100 μ; one can readily achieve these power densities with the present-day nanosecond and subnanosecond pulses.

Another quantity of interest is the minimum absorption length that one can hope to achieve after this instability has been excited and the anomalous collisional (and heating) effect is operating. The maximum collision frequency that one can hope to get before the instability is shut off is given by Eq. (15). [One should, of course, verify that the maximum ν_e still satisfies the requirement $(\nu_e/\omega_R)^2 \ll (1 - \omega_0^2/\omega_R^2)^2$.] Thus,

$$\nu_{e\,\max} \simeq (e^2E^2/6m^2s_e^2\omega_0)$$

and the minimum absorption length

$$\lambda_{am} = \frac{C}{\nu_{e\,\max}} \simeq 3 \times 10^4 \left(\frac{TN_{12}^{1/2}}{P}\right)\ \mathrm{cm}, \tag{17a}$$

where T, P, and N_{12} have the same meaning and units as before. For an incident power of 10^{15} W/cm^2 (10 times the threshold), for the above example, we get

$$\lambda_{am} \simeq 10\ \mu,$$

so that the laser beam will be completely absorbed in the 100 μ thickness of the plasma. The classical absorption length due to the usual electron ion collisions can be written as

$$\lambda_c \simeq \frac{5 \times 10^2\, T^{3/2}}{N_{12}}. \tag{17b}$$

(This equation holds only if $\omega \approx \omega_{pe}$; if ω is different from ω_{pe}, we have to multiply the above λ_c by ω^2/ω_{pe}^2.) We note that for the example given above

$$\lambda_c \simeq 150\ \mu,$$

so that classical absorption would not be very efficient. It is of interest to note that the ratio (λ_{am}/λ_c) is improved for hotter plasmas and by using higher-power densities.

A third quantity of considerable interest is the growth rate of these instabilities; this can be readily obtained from Eq. (13). For fields much higher than the threshold, one need only retain the electric field term in Eq. (13) to obtain

$$\gamma = \mathrm{Im}\,(\omega) \approx \left(\frac{k^2v_E^2\omega_{pi}^2}{2(\omega_R^2 - \omega_0^2)}\right)^{1/2} \simeq \left(\frac{k}{k_D}\right)\omega_{pi}\,\frac{v_E}{\sqrt{2}\,s_e}\left(1 - \frac{\omega_0^2}{\omega_R^2}\right)^{-1/2} \gtrsim \frac{k}{k_D}\,\omega_{pi}.$$

The last inequality results because the field is above the threshold value. Thus, the growth rate is at least of the order of $(k/k_D)\omega_{pi}$. The important point

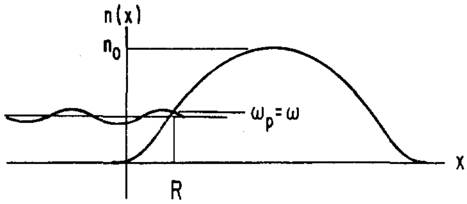

FIG. 1. Laser radiation incident on an overdense plasma.

to be established is that the waves do have a sufficient number of e-folds before they convect out of the unstable region. The position of the unstable region in a laser-produced plasma, having a density profile of the type shown in Fig. 1, is near the $\omega_0 \approx \omega_{pe}$ layer and its typical width is about $(R/10)$, where R is the scale length (because for regions further removed from the $\omega_0 \approx \omega_{pe}$ layer, the condition ω_0 close to ω_R cannot be satisfied, and so the thresholds are much higher). Therefore, the time that the ion waves take to convect out of the unstable region is roughly

$$\tau \simeq (R/10s_i);$$

the number of e-folds in amplitude that the waves undergo in this time is given by

$$(\gamma\tau) = \frac{R\omega_{pi}}{10s_i}\left(\frac{k}{k_D}\right). \tag{18}$$

This number, which is essentially the number of wavelengths that can be accommodated in the unstable region, is easily 100 or more and may be verified by direct substitution. Thus, the waves have ample time to grow sufficiently to cause anomalous heating before they convect out of the unstable region.

A third low-frequency instability, called the "parametric instability," may also be readily excited in the plasma by the laser beam. This instability was first discussed by DuBois and Goldman[13]; it arises if we feed energy into the plasma oscillations at a rate higher than that at which they can damp it away. Now, the plasma resonance frequency is roughly given by $\omega_R \simeq (\omega_{pe}^2 + 3k^2s_e^2)^{1/2}$. Since the laser pulse is very intense, the external frequency ω_0 interacts nonlinearly with ω_R to pump the plasma at the beat frequency $\omega_L = \omega_0 - \omega_R$ (if ω_0 and ω_R are sufficiently close to each other, then ω_L is a low frequency, characteristic of the ion acoustic waves); similarly, ω_0 and ω_L mix and pump the plasma at ω_R. Thus, if energy can be fed to the plasma oscillations preferentially (i.e., at a rate higher than that at which they damp away), then the plasma can become unstable both at ω_R and ω_L. DuBois and Goldman[13] estimate that the threshold power for this instability is roughly given by

$$P \simeq 5 \times 10^3\, N_{12} T \left(\frac{\gamma}{\omega_{pe}}\right)$$
$$\simeq (8.3) \times 10^{-8}\, N_{12}^{1/2}\, T\gamma, \tag{19}$$

where P, N_{12}, and T have the same units as before and γ is the linear plasma wave damping rate. Taking $\gamma/\omega_{pe} \simeq 0.02$ (a typical value–DuBois and Goldman[13] and Jackson[14]), $N_{12} = 10^9$ and $T = 10^3$ V, one obtains

$$P \simeq 10^{14}\ \mathrm{W/cm^2},$$

which is well within the reach of present-day laser technology. The maximum growth rate is typically[14] of the order of $(\omega_{pe}/20)$, so that the number of e-folds in amplitude that the waves undergo before convecting out of the unstable region is again very large [see Eq. (18)].

There are, however, some other important requirements for the excitation of the parametric instability. Thus, the wavenumber k should satisfy the condition

$$(\omega_0 - \omega_R) \simeq (1.7 \pm 0.5)kv_i;$$

otherwise, the growth rates are negligibly small. This puts restrictions on the nonmonochromaticity of the laser and the inhomogeneity of the plasma. The former is not a very restrictive requirement; the requirements on the inhomogeneity can be written as

$$\frac{\Delta\omega_p}{\omega_p} \simeq \frac{\Delta n}{2n} \simeq \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{k}{k_D}\right).$$

For weak damping, $(k/k_D) \ll 1$ (typically of order 0.1), so that $(\Delta n/n) \sim (1/200)$. If $L = [(1/n)\cdot(dn/dx)]^{-1}$ is the scale length, then $(X/L) \sim (1/200)$, where X is the width of the unstable region; thus, the plasma should only be very weakly inhomogeneous. This seems to be the most important restriction that could inhibit the excitation of this instability. The most favorable region for exciting this instability is perhaps the peak of the density profile, because the spatial variation of the density is much smaller there.

For maximum growth rate, the k value is given by[13]

$$\frac{k}{k_D} = \left[\frac{2}{3}\left(\frac{\omega_0}{\omega_p} - 1\right) + \left(\frac{1.7}{3}\right)^2 \frac{m_e}{m_i}\right]^{1/2} - \frac{1.7}{3}\left(\frac{m_e}{m_i}\right)^{1/2}.$$

[13] D. F. DuBois and M. V. Goldman, Phys. Rev. Letters **14**, 544 (1965); Phys. Rev. **164**, 207 (1967).

[14] E. A. Jackson, Phys. Rev. **153**, 235 (1967).

This gives $(k/k_D) = (0.25)$ for $(\omega_0/\omega_p) = 1.1$, $(k/k_D) = 0.18$ for $(\omega_0/\omega_p) = 1.05$, etc. For a typical laser plasma with a density of 10^{21} cm^{-3} and a temperature of 1 kV, $\lambda_D \simeq 10^{-6}$ cm; the lower bound on (k/k_D) is thus $(k_{\min}/k_D) \sim (\lambda_D/R) \sim 10^{-4}$. The above values of (k/k_D) are, therefore, readily realized.

Thus, if the requirements on the inhomogeneity are met, the parametric instability can be excited and can grow sufficiently to cause significant modifications of the high-frequency resistivity around the plasma frequency, and hence the anomalous heating effect.

Another parametric instability that may be excited by high-frequency fields with the above range of power densities is the one around $\omega_0 \approx 2\omega_{pe}$, investigated by Jackson.[14] The threshold for this instability is given by

$$P \simeq (1.5) \times 10^{11} N_{12} \left(\frac{\gamma}{\omega_{pe}}\right)^2 \simeq 4 \times 10^{-11} \gamma^2. \qquad (20)$$

For the ruby laser plasma with $N_{12} = 10^9$, $(\gamma/\omega_{pe}) \approx 0.02$, this gives $P \simeq 6 \times 10^{16}$ W/cm^2, which is between 2 and 3 orders of magnitude higher than the threshold for DuBois–Goldman instability. We pointed out in the introduction that there is also a bump in the resistivity–frequency curve near $\omega \approx 2\omega_{pe}$ (due to electron–electron collisions); this bump can, therefore, be enhanced manyfold by the above instability and may also lead to anomalous heating. It should be noted that in typical laser-produced plasmas (such as those shown in Fig. 1) the region with $\omega \approx 2\omega_{pe}$ will be in the low-density region. Thus, if this instability is excited most of the energy may go into the nearer low-density regions and the bulk of the plasma may remain cold. It appears, therefore, that this instability can be largely beneficial only after the plasma has expanded so that its density is rather low (ω_0/ω_{pm} lying roughly between 2 and 2.8, which is the relevant frequency regime for this instability; ω_{pm} is the plasma frequency at the peak of the density profile), since in this regime the normal absorption would be falling off quite rapidly.

Finally, it seems to us that it may be possible to verify some of the conclusions of the theory presented above for the modified two-stream instability, by experiments on low-density plasmas ($\approx 10^{12}$ cm^{-3}), using high-intensity microwaves. Thus, for a 1-V plasma with a density of 10^{12} cm^{-3}, one needs only 10^2 W/cm^2 at 3 cm wavelength to excite that instability. Since the focal spot size in this case cannot be less than 10 cm^2, at least 1 kW of power is required. To get reasonable absorption lengths, however, higher incident powers are necessary. Thus to get an absorption length of 30 cm, about 10 kW of incident power are needed; the classical absorption length for this example would be 500 cm.

Note added in proof: The type of modified two-stream instability discussed by us in this paper has also been recently investigated by Nishikawa[15] and Sanmartin.[16] We have also carried out some computer simulation experiments which show that these instabilities are excited and that they lead to an anomalous increase in the high-frequency resistivity of the plasma; these results will be presented elsewhere.

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