$$\Rightarrow \frac{du}{dz} = \frac{\left[\left(\frac{\gamma-1}{\gamma}\right)\frac{E}{\mu_0 u} - B_\theta/\mu_0\right] dB_\theta/dz}{F\left[1-\left(\left(\gamma-1/\gamma\right)+P/uF\right)\right]}$$

$$\boxed{dB_\theta/dz = -\mu_0\sigma(E-uB_\theta)} \rightarrow \text{known}$$

$$\Rightarrow \boxed{\frac{du}{dz} = \frac{-\mu_0\sigma\left[\left(\frac{\gamma-1}{\gamma}\right)\frac{B}{\mu_0 u} - \frac{B_\theta}{\mu_0}\right]}{F\left[1-\left(\left(\frac{\gamma-1}{\gamma}\right)+\frac{P}{uF}\right)\right]}(E-uB_\theta)}$$

$$\boxed{M = u/a \qquad a = \sqrt{\gamma\frac{P}{\rho}} \qquad \rho = F/u}$$

1D Ionizing MPD Flow w/ only Induced Field.

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0$$

$$\rho\left(\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right) = -\vec{\nabla}p + \vec{j}\times\vec{B}$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right)\left(c_pT\Lambda + \frac{u^2}{2} + \varepsilon_0 u/m\right) = \frac{\partial p}{\partial t} + \vec{j}\cdot\vec{E}$$

$$\vec{j} = \sigma(\vec{E}+\vec{u}\times\vec{B}), \quad \vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t}$$

$$\mu_0\vec{j} = \vec{\nabla}\times\vec{B}, \quad \vec{\nabla}\cdot\vec{B} = 0$$

$$\frac{d\alpha}{dt} = \left(\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right)\alpha = \frac{k_f\,\rho\alpha(1-\alpha)}{m\,u} - \frac{k_r\,\rho^2\alpha^3}{m^2 u}$$