PHYSICS 757 MIDTERM QUANTUM COMPUTING

Name ____________________

1. (16 pts) Consider the Pauli matrices given by,

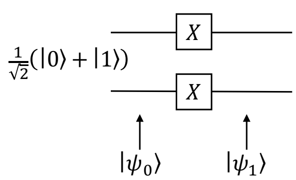
$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Using the Pauli matrices, show that $XZ = -iY$ and complete the multiplication table,

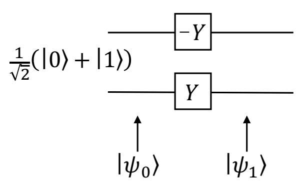
	1	X	Y	Z
1				
X				$-iY$
Y				
Z		iY		

2) (16 pts) The input state for the following three circuits is given by the Bell state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Perform a state analysis and matrix analysis for each circuit.

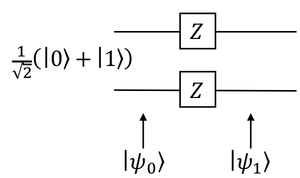
a)



b)



c)



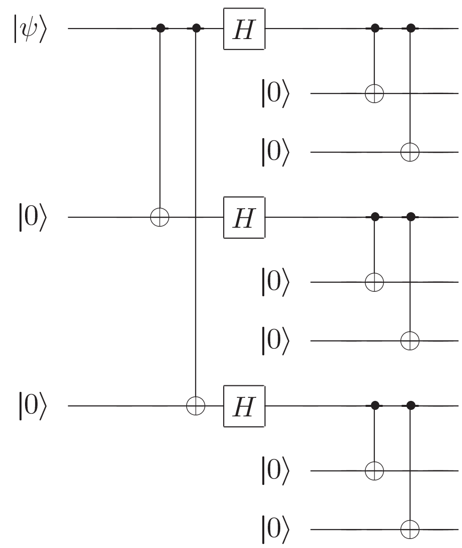
d) Is the Bell state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ an eigenstate of the two qubit operators, $1 \otimes 1$, $X \otimes X$, $Y \otimes Y$, and $Z \otimes Z$? e) If so, what are the corresponding eigenvalues?

3) (16 pts) Complete the multiplication table for the two qubit operators from part (2) where, $A \otimes B \cdot C \otimes D = A \cdot B \otimes C \cdot D$ and " $\cdot$ " denotes matrix multiplication.

	$1 \otimes 1$	$X \otimes X$	$-Y \otimes Y$	$Z \otimes Z$
$1 \otimes 1$				
$X \otimes X$				
$-Y \otimes Y$				
$Z \otimes Z$				

4) (16 pts) Do the operators in the set $G = \{1 \otimes 1, X \otimes X, -Y \otimes Y, Z \otimes Z\}$ form a group (hint: see appendix 2 in your textbook)? The set of all operators g_i for which $g_i|\psi\rangle = |\psi\rangle$ is called the stabilizer group of $|\psi\rangle$ and plays an important role in error control algorithms. Do think the group G is or is not the stabilizer group for the Bell state $|\beta_{00}\rangle$? Provide an argument to support your answer.

5) (16 pts) Encoding the state of a single qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ into several qubits is an important aspect in quantum error control algorithms. The circuit below is the nine qubit Shor encoding circuit (Figure 10.4 in your textbook).



Using a state analysis at each layer confirm that this circuit will encode the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ into the logical state $|\psi_L\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$ where,

$$|0_L\rangle = \frac{1}{\sqrt{8}}\{|000000000\rangle + |000000111\rangle + |000111000\rangle + |000111111\rangle + |111000000\rangle + |111000111\rangle + |111111000\rangle + |111111111\rangle\}$$

$$|1_L\rangle = \frac{1}{\sqrt{8}}\{|000000000\rangle - |000000111\rangle - |000111000\rangle + |000111111\rangle - |111000000\rangle + |111000111\rangle + |111111000\rangle - |111111111\rangle\}$$

6) (16 pts) Show that the states $|0_L\rangle$ and $|1_L\rangle$ are eigenstates of the eight operators g_i given by table 10.11 in your textbook,

Name	Operator
g_1	$Z\,Z\,I\,I\,I\,I\,I\,I\,I$
g_2	$I\,Z\,Z\,I\,I\,I\,I\,I\,I$
g_3	$I\,I\,I\,Z\,Z\,I\,I\,I\,I$
g_4	$I\,I\,I\,I\,Z\,Z\,I\,I\,I$
g_5	$I\,I\,I\,I\,I\,I\,Z\,Z\,I$
g_6	$I\,I\,I\,I\,I\,I\,I\,Z\,Z$
g_7	$X\,X\,X\,X\,X\,X\,I\,I\,I$
g_8	$I\,I\,I\,X\,X\,X\,X\,X\,X$

where for example, $ZZIIIIIII$ is short hand for $Z \otimes Z \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I$ and $I = 1$. What are the corresponding eigenvalues?