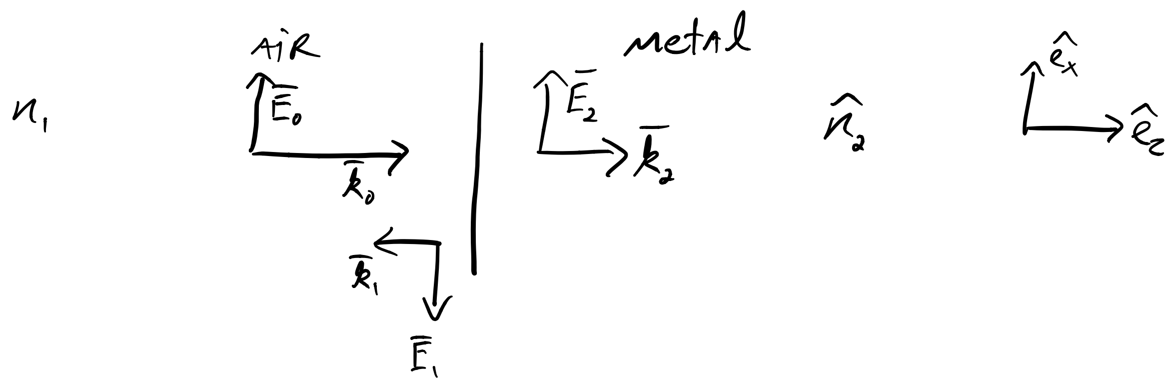
Reflection from a Metallic Surface

Extending the previous analysis with a complex $\hat{\varepsilon}$:

$$\hat{n} = \sqrt{\hat{\varepsilon}} \qquad (\mu = 1)$$

Normal Incidence



Results are the same as before but we use $n \to \hat{n}$:

<u>Reflected</u> $$E_1^0 = \frac{\hat{n}_2 - n_1}{\hat{n}_2 + n_1} E_0^0$$

<u>Transmitted</u> $$E_2^0 = \frac{2 n_1}{\hat{n}_2 + n_1} E_0^0$$

Reflected and transmitted amplitudes are now complex $\Rightarrow$ <u>shifted in phase</u>

For <u>high-conductivity</u>, $4\pi\sigma_2 >> \varepsilon_2 \omega$:

$$\hat{n}_2 = \sqrt{\varepsilon_2 + i\frac{4\pi\sigma_2}{\omega}} \simeq \sqrt{i\frac{4\pi\sigma_2}{\omega}} = \frac{c}{\omega\delta}(1+i)$$

$$\delta = \frac{c}{\sqrt{2\pi\sigma\omega}} = \text{skin-depth}$$

Introducing the <u>reduced wavelength</u>, $ƛ$

$$ƛ \equiv \frac{\lambda}{2\pi} = \frac{c}{\omega}$$

$\hookrightarrow$ in free space

$$\Rightarrow \frac{\delta}{ƛ} = \frac{\omega\delta}{c} = \sqrt{\frac{\omega}{2\pi\sigma}}$$

Reflected Amplitude

$$\Rightarrow E_1^0 = \frac{(1+i)(ƛ/\delta) - n_1}{(1+i)(ƛ/\delta) + n_1} E_0^0$$

Reflection coef

$$R = \frac{|E_1^0|^2}{|E_0^0|^2} = \frac{[1-(\delta/\lambda)n_1]^2+1}{[1+(\delta/\lambda)n_1]^2+1}$$

For a good conductor $\delta/\lambda \ll 1$

First-order Taylor series $\Rightarrow R \simeq 1 - 2\frac{\delta}{\lambda}n_1$

$$T = 1 - R \simeq 2\frac{\delta}{\lambda}n_1$$

Because $\frac{\delta}{\lambda} \ll 1$, $R \simeq 1$

☆ nearly 100% of energy is reflected at a good conductor!

$$\Rightarrow \quad \bar{E}_0 = \hat{e}_x E_0^0 e^{i(k_1 z - \omega t)}$$

$$\bar{E}_1 \simeq -\hat{e}_x E_0^0 e^{i(-k_1 z - \omega t)}$$

Adding the incident + reflected waves:

$$\bar{E} = \bar{E}_0 + \bar{E}_1 \simeq \hat{e}_x E_0^0 e^{-i\omega t}\left(e^{ik_1 z} - e^{-ik_1 z}\right)$$

The real (physical) part is

$$\underline{E} \simeq 2\,\hat{e}_x\,E_0^0 \sin\omega t \sin k_1 z$$

$\hookrightarrow$ <u>standing wave!</u>

Nodes can be found at

$$\sin k_1 z = 0 \Rightarrow k_1 z = m\pi$$

$\llcorner$ integer

$$\longrightarrow k_1 z = \frac{\omega}{v} z = n_1 \frac{\omega}{c} z = n_1 \frac{2\pi}{\lambda} z$$

$$\Rightarrow n_1 \frac{2\pi}{\lambda} z = m\pi \Rightarrow z = \frac{m\lambda}{2n_1}$$

Nodes separated by $\frac{\lambda}{2n_1}$

$\hookrightarrow$ This has been observed experimentally

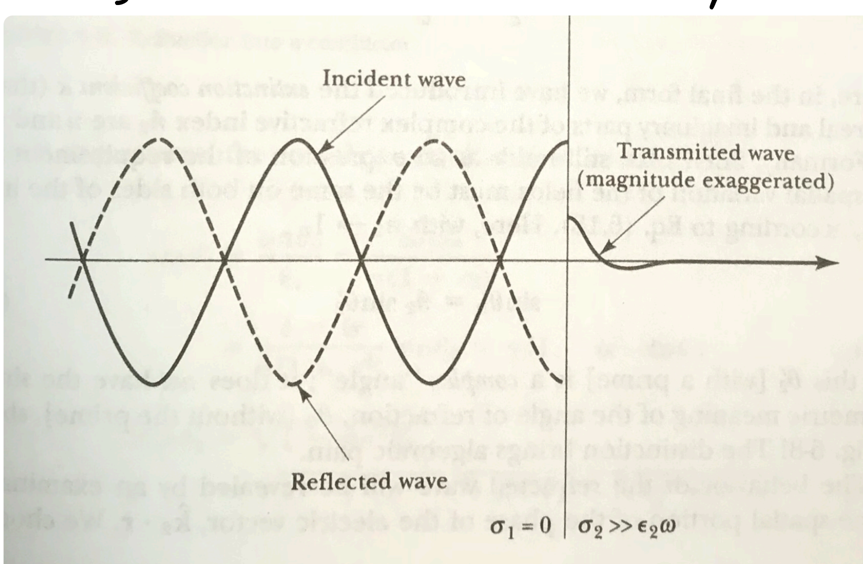


FIGURE 6-7. Reflection from a conductor.

[H+M]

Refraction into a conducting medium

For high-conductivity oblique incidence

$\hookrightarrow$ wave is absorbed rapidly

low-conductivity

$\hookrightarrow$ behaves like non-conductor with gradual absorption

medium-conductivity

$\hookrightarrow$ surprising behavior

The E in the conductor is

$$\bar{E}_2 = E_2^0 e^{i(\bar{\hat{k}}_2 \cdot \bar{r} - \omega t)}$$

$$\hat{k}_2 = \frac{\omega}{c}\hat{n}_2 = \frac{\omega}{c} n(1 + iK)$$

or

$$\hat{n}_2 = n + inK$$

K = extinction coefficient

Snell's law still holds, and for $n_1 = 1$

$$\sin\theta_0 = \underbrace{\hat{n}_2 \sin\theta_2'}_{\text{complex!}}$$

$\Rightarrow \theta_2'$ is complex, and doesn't have the same meaning as before (angle of refraction)

- Examining the phase of $\bar{k}_2 \cdot \bar{r}$ for the geometry below:

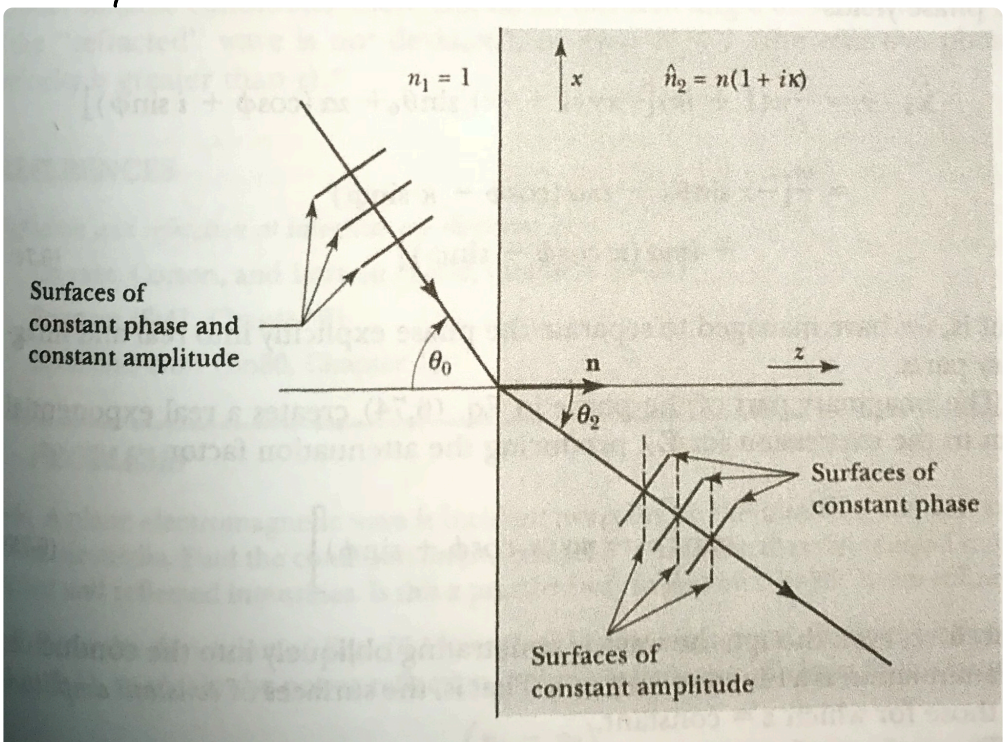


FIGURE 6-8. Refraction into a conductor.

[H+M]

$$\hat{k}_2 \cdot \bar{r} = \frac{\omega}{c} n(1 + ik)(-x \sin\theta_2' + z\cos\theta_2')$$

From Snell's

$$\sin\theta_2' = \frac{\sin\theta_0}{\hat{n}_2} = \frac{\sin\theta_0}{n(1+iK)} = \frac{(1-iK)\sin\theta_0}{n(1+K^2)}$$

$$= \gamma(1-iK)\sin\theta_0$$

Where

$$\gamma \equiv \frac{1}{n(1+K^2)}$$

Also,

$$\cos\theta_2' = \sqrt{1-\sin^2\theta_2'}$$

$$= \sqrt{1-\gamma^2(1-K^2)\sin^2\theta_0 + i2K\gamma^2\sin^2\theta_0}$$

Separating into real and complex:

$$\cos\theta_2' \equiv \alpha e^{i\phi} = \alpha(\cos\phi + i\sin\phi)$$

$\hookrightarrow$ polar form

Plugging back into our expression for $\bar{k}_2 \cdot \bar{r}$

$$\bar{k}_2 \cdot \bar{r} = \frac{w}{c} n(1+ik)\left[-x\gamma(1-ik)\sin\theta_0 + z\alpha(\cos\phi + i\sin\phi)\right]$$

$$= \frac{w}{c}\Big[\underbrace{-x\sin\theta_0 + zn\alpha(\cos\phi - k\sin\phi)}_{\text{Real}}$$

$$+ \underbrace{iz n\alpha(k\cos\phi + \sin\phi)}_{\text{imaginary}}\Big]$$

The imaginary part of the phase gives the attenuation $\left[e^{i(ix)} = e^{-x}\right]$

Attenuation factor $= \exp\left[-2\frac{w}{c}n\alpha(k\cos\phi + \sin\phi)\right]$

$\hookrightarrow$ function of z only!

$\Rightarrow$ surfaces of constant amplitude for $z =$ constant

$\rightarrow$ attenuates as a function of distance travelled in the conductor

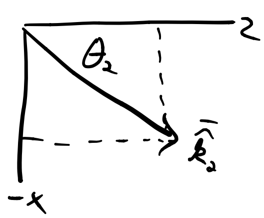
$\hookrightarrow$ function of z

The surfaces of <u>constant phase</u> are found from $Re[\bar{\hat{k}}_2 \cdot \bar{r}]$

$$\Rightarrow \underbrace{-x\sin\theta_0 + zn\alpha(\cos\phi - K\sin\phi)}_{\text{Real part of phase times } \frac{c}{\omega}} = \text{constant}$$

Recasting in the form of a <u>non-conducting</u> medium:

$$Re[\bar{\hat{k}}_2 \cdot \bar{r}] = \frac{\omega}{c} N(z\cos\theta_2 - x\sin\theta_2)$$



θ_2 = geometric refracting angle

N = <u>effective</u> (real) index of refraction

$\Rightarrow$ surfaces of <u>constant phase</u> are normal to the ray direction of θ_2

↳ <u>does not</u> correspond to surfaces of <u>constant amplitude</u>

The effective index and refraction angle are

$$N(\theta_0) = \sqrt{\sin^2\theta_0 + n^2\alpha^2(\cos\phi - K\sin\phi)^2} \quad \Big\} \text{ fxn of } \theta_0$$

$$\tan\theta_2 = \frac{\sin\theta_0}{n\alpha(\cos\phi - K\sin\phi)}$$

An <u>effective Snell's law</u> is

$$\sin\theta_0 = N(\theta_0)\sin\theta_2$$

$\Rightarrow$ can have θ_0 such that $N = 1$

and the wave is not refracted

$\Rightarrow$ can have $N < 1$ (phase velocity $> c$)