Polarization

Using a right handed orthogonal coordinate system with $\hat{e}_3$ pointing in the direction of propagation, the $\bar{E}$ is

$$\bar{E} = (\hat{e}_1 E_1 + \hat{e}_2 E_2) e^{i(k\zeta - \omega t)}$$

E_1 = complex amplitude in $\hat{e}_1$ direction

E_2 = complex amplitude in $\hat{e}_2$ direction

ζ = $\perp$ distance to origin

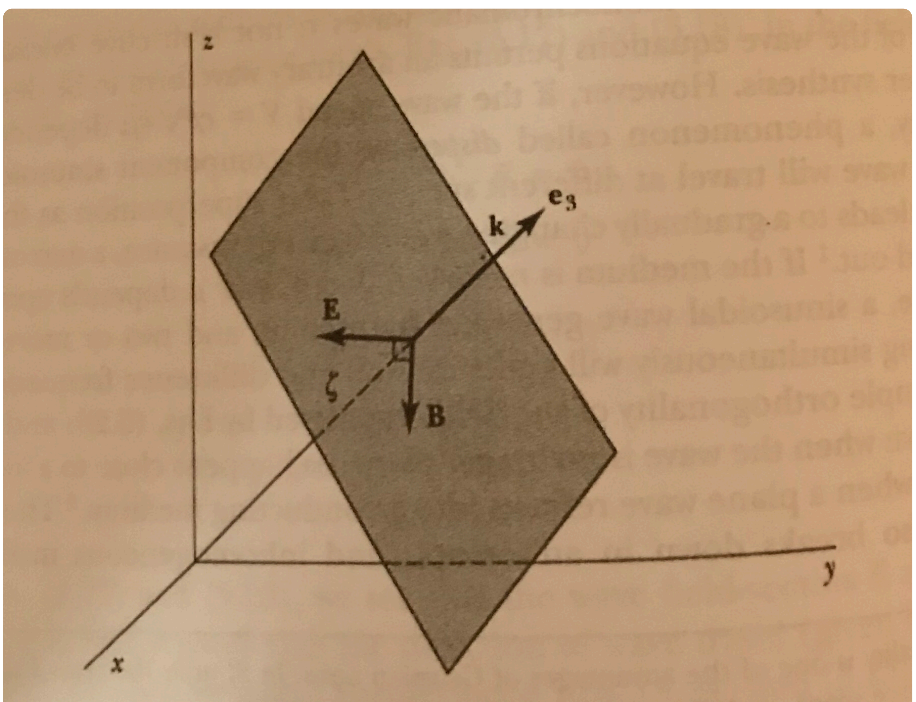


FIGURE 5-2. Mutual perpendicularity of fields and wavevector.

[H+M]

The complex constants E_1 and E_2 can be described by a <u>real</u> amplitude with a <u>phase</u>

$$E_1 = E_1^0 e^{i\alpha}$$

$$E_2 = E_2^0 e^{i\beta}$$

real amplitude　　　phase

$$\Rightarrow \qquad \bar{E}_1 = \hat{e}_1 E_1^0 e^{i(kz - \omega t + \alpha)}$$

$$\bar{E}_2 = \hat{e}_2 E_2^0 e^{i(kz - \omega t + \beta)}$$

$$\Rightarrow \qquad \bar{E} = \left(\hat{e}_1 E_1^0 e^{i\alpha} + \hat{e}_2 E_2^0 e^{i\beta}\right) e^{i(kz - \omega t)}$$

The phase difference between E_1 and E_2 is important

$$\bar{E} = \left(\hat{e}_1 E_1^0 + \hat{e}_2 E_2^0 \underbrace{e^{i[\beta - \alpha]}}_{\text{phase difference}}\right) e^{i(kz - \omega t + \alpha)}$$

Analyzing the phase difference, for the case when

$$\beta = \alpha \pm m\pi \quad \text{with} \quad m = 0, 1, 2, \ldots$$

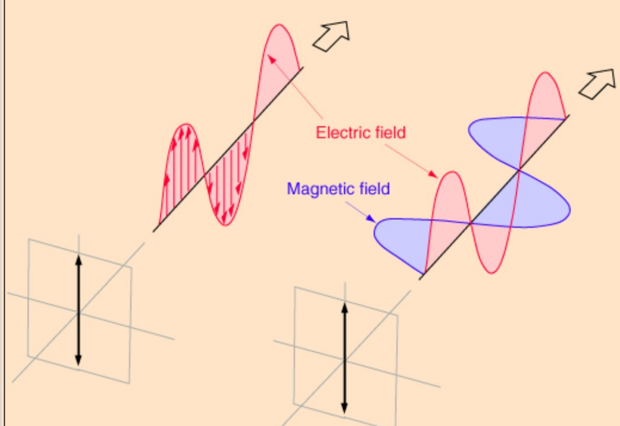
$$\Rightarrow e^{i[\beta - \alpha]} = e^{im\pi} = \pm 1$$

$$\Rightarrow \bar{E} = \underbrace{\left(\hat{e}_1 E_1^{\circ} \pm \hat{e}_2 E_2^{\circ}\right)}_{\text{direction of } \bar{E} \text{ is independent of time}} e^{i(kz - \omega t + \alpha)}$$

$\longrightarrow$ <u>linearly polarized</u>

Linear Polarization

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated.



Compare with circular and elliptical polarization

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[HyperPhysics]

If $E_1^0 = E_2^0 = E_0^0$ but the phase difference is $\pm\frac{\pi}{2}$

$$\beta = \alpha \pm \pi/2$$

$$\Rightarrow e^{i[\beta-\alpha]} = e^{\pm i\pi/2} = \pm i$$

$$\Rightarrow \bar{E} = E_0^0 \underbrace{(\hat{e}_1 \pm i\hat{e}_2)}_{} e^{i(k\zeta - \omega t + \alpha)}$$

complex, so direction of Real($\bar{E}$) changes with time

$\longrightarrow$ circularly polarized

Analyzing the real components of $\bar{E}$:

$$E_{1R} = E_0^0 \cos(k\zeta - \omega t + \alpha)$$

$$E_{2R} = \mp E_0^0 \sin(k\zeta - \omega t + \alpha)$$

If we look at the behavior over time at a fixed plane (ζ = constant)

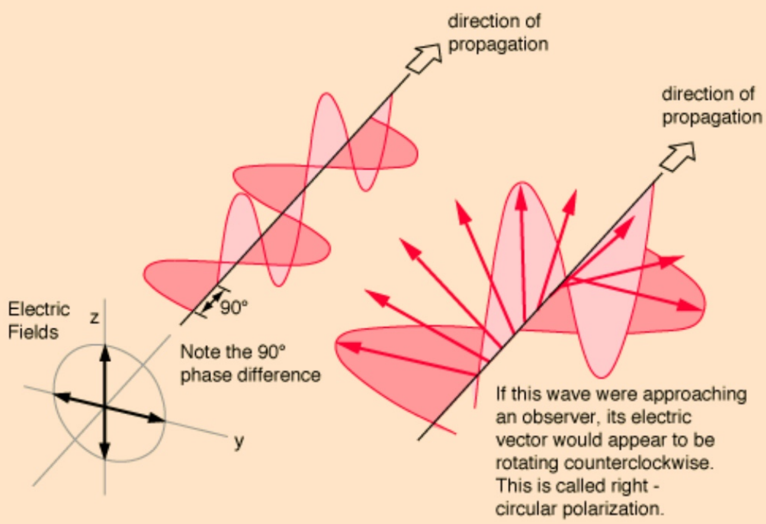
$$E_{1R} \sim \cos(-\omega t)$$

$$E_{2R} \sim \sin(-\omega t)$$

→ The $\bar{E}$ vector traces out a circle

Circular Polarization

Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized.



If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized. If you could see the tip of the electric field vector, it would appear to be moving in a circle as it approached you. If while looking at the source, the electric vector of the light coming toward you appears to be rotating counterclockwise, the light is said to be right-circularly polarized. If clockwise, then left-circularly polarized light. The electric field vector makes one complete revolution as the light advances one wavelength toward you. Another way of saying it is that if the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Circularly polarized light may be produced by passing linearly polarized light through a quarter-wave plate at an angle of 45° to the optic axis of the plate.

Compare with linear and elliptical polarization

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[Hyperphysics]

In general, $E_1^0 \neq E_2^0$ and $\alpha \neq \beta$

$$\longrightarrow \quad E_1 = E_1^0 e^{i(kz - \omega t + \alpha)}$$

$$E_2 = E_2^0 e^{i(kz - \omega t + \beta)}$$

If we define

$$\gamma \equiv \beta - \frac{\pi}{2} \implies \beta = \gamma + \frac{\pi}{2}$$

$$\implies E_2 = E_2^0 e^{i(kz - \omega t + \gamma)} e^{i\pi/2}$$

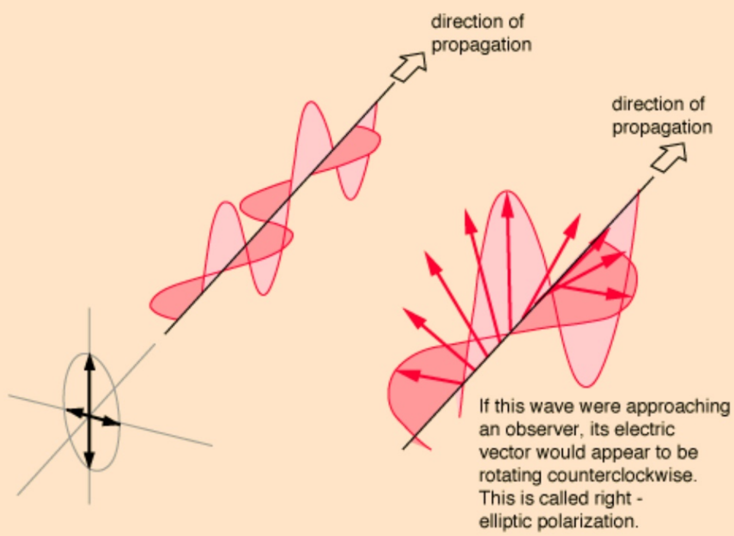
$$= i E_2^0 e^{i(kz - \omega t + \gamma)}$$

Taking the real parts of E_1 and E_2

$$\underbrace{\begin{aligned} E_{1R} &= E_1^0 \cos(kz - \omega t + \alpha) \\ E_{2R} &= -E_2^0 \sin(kz - \omega t + \gamma) \end{aligned}}_{\text{elliptically polarized}}$$

Elliptical Polarization

Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°. The illustration shows right-elliptically polarized light.



If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Compare with linear and circular polarization

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[HyperPhysics]

We've focused on $\bar{E}$ because for a plane wave, if we know $\bar{k}$ and $\bar{E}$, then we know the direction and behavior of $\bar{B}$

$$\bar{B} = n\,\hat{e}_k \times \bar{E}$$

$\bar{B}$ and $\bar{E}$ are in phase

Poynting's Vector for Complex Fields

- The math involved with waves is typically easier using <u>phasors</u> ($E_0 e^{i\phi}$) instead of sines or cosines.

- As an example let's take the <u>time average</u> of the real portion of two complex functions

$$F(t) = F_0 e^{i\alpha} e^{-i\omega t}$$

$$G(t) = G_0 e^{i\beta} e^{-i\omega t}$$

$$\Rightarrow \ [Re] \ F(t) = F_0 \cos(\alpha - \omega t)$$

$$[Re] \ G(t) = G_0 \cos(\beta - \omega t)$$

Real part

The time average is the integral over one cycle divided by the period τ

$$\langle [Re] F(t) \cdot [Re] G(t) \rangle = \frac{1}{\tau} \int_0^{\tau} [Re] F(t) \cdot [Re] G(t) \, dt$$

$$= \frac{1}{\tau} \int_0^{\tau} F_0 \cos(\alpha - \omega t) \, G_0 \cos(\beta - \omega t) \, dt$$

$$= \frac{F_0 G_0}{\tau} \int_0^{\tau} [\cos\alpha \cos\omega t + \sin\alpha \sin\omega t][\cos\beta \cos\omega t + \sin\beta \sin\omega t] \, dt$$

Common time averages that appear:

$$\langle \cos^2 \omega t \rangle = \langle \sin^2 \omega t \rangle = \frac{1}{\tau} \int_0^{\tau} \cos^2 \omega t \, dt = \frac{1}{\tau} \int_0^{\tau} \sin^2 \omega t \, dt = \frac{1}{2}$$

$$\langle \sin \omega t \cos \omega t \rangle = 0$$

$$\Rightarrow \quad = F_0 G_0 \frac{1}{2} (\cos\alpha \cos\beta + \sin\alpha \sin\beta)$$

$$= \frac{1}{2} F_0 G_0 \cos[\pm(\alpha - \beta)]$$

The previous approach is tedious. Instead, we'll repeat using phasors:

complex conjugate

$$\langle F \cdot G \rangle = [Re]\ \frac{1}{2} F_0 e^{i\alpha - \omega t} \left(G_0 e^{i\beta - \omega t} \right)^*$$

$$= [Re]\ \frac{1}{2} F_0 G_0\, e^{i(\alpha - \beta)}$$

$$= \frac{1}{2} F_0 G_0 \cos(\alpha - \beta) \longrightarrow$$ same as before! much easier

$$\Rightarrow \langle F \cdot G \rangle = \frac{1}{2} F^* G = \frac{1}{2} F G^*$$

Note: taking the real part is required and implicit using this notation

Applying this theorem to the Poynting vector

$$\bar{S} = \frac{c}{4\pi} \bar{E} \times \bar{H}$$

For propagation in the z direction:

$$\bar{E}(z,t) = \bar{E}_0 e^{i(kz-\omega t)}$$

$$\bar{H}(z,t) = \bar{H}_0 e^{i(kz-\omega t)}$$

Expanding the complex vectors $\bar{E}_0$ and $\bar{H}_0$

$$\bar{E}(z,t) = \left[E_{0x} e^{i\alpha_x} \hat{e}_x + E_{0y} e^{i\alpha_y} \hat{e}_y\right] e^{i(kz-\omega t)}$$

$$\bar{H}(z,t) = \left[H_{0x} e^{i\beta_x} \hat{e}_x + H_{0y} e^{i\beta_y} \hat{e}_y\right] e^{i(kz-\omega t)}$$

$$\Rightarrow \boxed{\langle S \rangle = \underbrace{\frac{c}{8\pi} \bar{E}_0 \times \bar{H}_0^* = \frac{c}{8\pi} \bar{E}_0^* \times \bar{H}_0}}$$

time average energy flow

easy method for calculating $\langle S \rangle$ from complex fields

↳ just need to conjugate one term

Using the wave impedance η

$$\frac{|E_0|}{|H_0|} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \langle S \rangle = \frac{c}{8\pi} \frac{E_0^2}{\eta} \hat{e}_k = \frac{c}{8\pi} \eta H_0^2 \hat{e}_k$$

The energy density of the E+M fields due to a plane wave is

$$\langle \mathcal{E} \rangle = \frac{1}{16\pi} \left(\bar{E}_0 \cdot \bar{D}_0^* + \bar{H}_0 \cdot \bar{B}_0^* \right)$$

$$= \frac{1}{16\pi} \left(\varepsilon E_0^2 + \mu H_0^2 \right) = \frac{1}{8\pi} \varepsilon E_0^2$$

In class problem

We skipped over the derivation, but show that for complex functions E and H:

$$\langle [Re]E \cdot [Re]H \rangle = \frac{1}{2}[Re]E^{*} \cdot H$$

or

$$= \frac{1}{2}[Re]\, E \cdot H^{*}$$

Start from the fact that

$$[Re]\, E = \frac{1}{2}(E + E^{*})$$

and

$$E = E_0\, e^{-i\omega t}$$

complex (has phase component)

$$H = H_0\, e^{-i\omega t}$$

ANSWER

$$\langle [Re]E \cdot [Re]H \rangle = \left\langle \tfrac{1}{2}(E+E^*)\tfrac{1}{2}(H+H^*) \right\rangle$$

$$= \left\langle \tfrac{1}{4}(EH + EH^* + E^*H + E^*H^* \right\rangle$$

$$= \left\langle \tfrac{1}{4}\left(E_0H_0e^{-2i\omega t} + E_0H_0^* + E_0^*H_0 + E_0^*H_0^*e^{2i\omega t}\right) \right\rangle$$

But

$$\tfrac{1}{2}\left(E_0H_0^* + E_0^*H_0\right) = [Re]\,E_0H_0^*$$

$$\tfrac{1}{2}\left(E_0H_0e^{-2i\omega t} + E_0^*H_0^*e^{2i\omega t}\right) = [Re]\,E_0H_0e^{-2i\omega t}$$

$$= \left\langle \tfrac{1}{2}[Re]\,E_0H_0^* + \tfrac{1}{2}[Re]\,E_0H_0e^{-2i\omega t} \right\rangle$$

$$= \underbrace{\frac{1}{\tau}\int_0^{\tau} \tfrac{1}{2}[Re]E_0H_0^*\,dt} + \underbrace{\frac{1}{\tau}\int_0^{\tau} \tfrac{1}{2}[Re]E_0H_0e^{-2i\omega t}\,dt}_{0}$$

$$= \tfrac{1}{2}[Re]E_0H_0^*$$

If we include the time term:

$$E = E_0 e^{-i\omega t} \quad \text{and} \quad H^* = H_0^* e^{i\omega t}$$

$$\Rightarrow E H^* = E_0 H_0^*$$

$$\Rightarrow \langle [Re] E \cdot [Re] H \rangle = \frac{1}{2} [Re] E \cdot H^*$$