

$$\rightarrow \rho u \frac{d}{dz} \left(c_p T + \frac{u^2}{2} \right) = \vec{j} \cdot \vec{E}$$

$$\rightarrow c_p T = \frac{\gamma}{\gamma-1} \frac{P}{\rho}$$

$$\frac{d}{dz} (c_p T) = \frac{\gamma}{\gamma-1} \frac{1}{\rho} \frac{d}{dz} \left(\frac{P}{\rho} \right) = \frac{\gamma}{\gamma-1} \left(\frac{1}{\rho} \frac{dP}{dz} - \frac{P}{\rho^2} \frac{d\rho}{dz} \right)$$

$$= \frac{\gamma}{\gamma-1} \left(\frac{1}{\rho} \frac{dP}{dz} + \frac{P}{\rho u} \frac{du}{dz} \right)$$

$$\Rightarrow \boxed{\frac{\gamma}{\gamma-1} \left(u \frac{dP}{dz} + P \frac{du}{dz} \right) + F u \frac{du}{dz} = j_r E_r + j_z E_z}$$

$$\Rightarrow \frac{\gamma}{\gamma-1} \left(u \frac{dP}{dz} + P \frac{du}{dz} \right) + F u \frac{du}{dz} = - \frac{E_r}{M_0} \frac{dB_\theta}{dz}$$

(assume $j_z \ll j_r$)

$E_r \rightarrow E$

$$u \frac{dP}{dz} + P \frac{du}{dz} + \frac{\gamma-1}{\gamma} F u \frac{du}{dz} = - \left(\frac{\gamma-1}{\gamma} \right) \frac{E}{M_0} \frac{dB_\theta}{dz}$$

$$\frac{dP}{dz} + \frac{P}{u} \frac{du}{dz} + \frac{\gamma-1}{\gamma u} F u \frac{du}{dz} = - \left(\frac{\gamma-1}{\gamma} \right) \frac{E}{M_0 u} \frac{dB_\theta}{dz}$$

$$\Rightarrow \boxed{\frac{dP}{dz} = - \left[\left(\frac{\gamma-1}{\gamma} \right) F + \frac{P}{u} \right] \frac{du}{dz} - \left(\frac{\gamma-1}{\gamma} \right) \frac{E}{M_0 u} \frac{dB_\theta}{dz}}$$

$$\Rightarrow \left(\rho \frac{P}{u} \right) \frac{du}{dz} = \left[\left(\frac{\gamma-1}{\gamma} \right) F + \frac{P}{u} \right] \frac{du}{dz} + \left(\frac{\gamma-1}{\gamma} \right) \frac{E}{M_0 u} \frac{dB_\theta}{dz} - \frac{P}{M_0} \frac{dB_\theta}{dz}$$

$$\Rightarrow \frac{du}{dz} = \left[\left(\frac{\gamma-1}{\gamma} \right) + \frac{P}{u F} \right] \frac{du}{dz} + \frac{1}{F} \left[\left(\frac{\gamma-1}{\gamma} \right) \frac{E}{M_0 u} - \frac{P}{M_0} \right] \frac{dB_\theta}{dz}$$