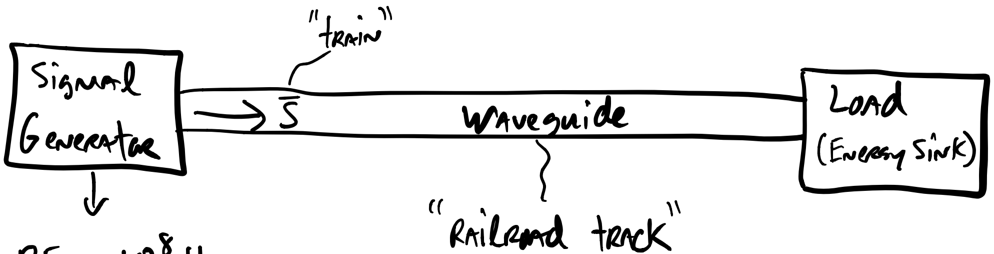
Waveguides

- Guide E+M energy (Poynting vector) through a structure



RF ~ 10^8 Hz

microwave ~ 10^{10} Hz

near-infrared ~ 10^{14} Hz

Two-Conductor Transmission Lines

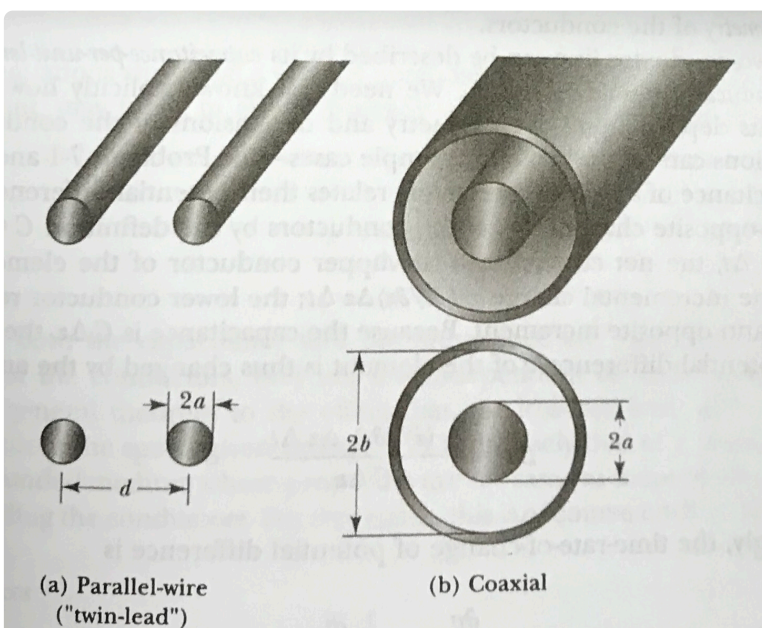


FIGURE 7-1. Common two-conductor transmission lines.

[H+M]

Consider two conductors extending to ∞ in $\hat{e}_z$ as in Figure above

- Resistance $= \emptyset$
- In free space

Analyzing the behavior in terms of voltage and current

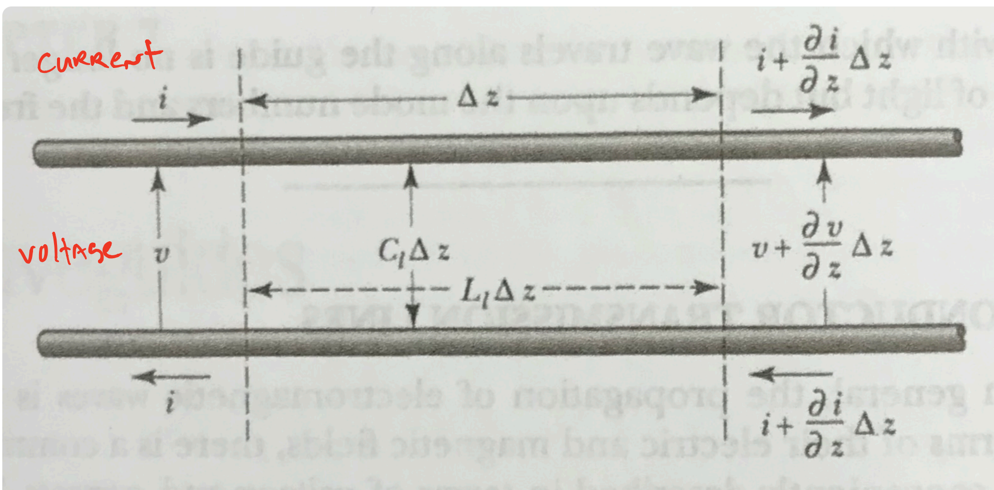


FIGURE 7-2. Schematic element of a two-conductor line.

[H+M]

We can calculate the

- Capacitance per unit length, C_l
- Inductance per unit length, L_l

} Functions of geometry

The capacitance is defined by

$$C \equiv \frac{q}{V}$$

q – charge

V – voltage (potential)

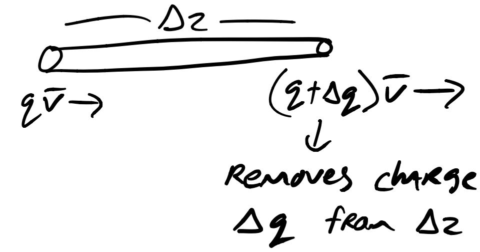
$+q$

$-q$

For the top line, the <u>net current</u> is

$$\frac{dq}{dt} = \underbrace{I}_{\text{left}} - \underbrace{I + \frac{\partial I}{\partial z}\Delta z}_{\text{Right}} = -\frac{\partial I}{\partial z}\Delta z$$

net current



$$\Rightarrow \Delta q = \frac{\partial I}{\partial z}\Delta z \Delta t$$

"extra" charge delivered

$$\Rightarrow C_\ell \Delta z = \frac{\Delta q}{\Delta V} = \frac{-\frac{\partial I}{\partial z}\Delta z \Delta t}{\Delta V}$$

$\frac{\Delta C}{\Delta z}$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{-1}{C_\ell}\frac{\partial I}{\partial z} \Rightarrow \frac{\partial v}{\partial t} = \frac{-1}{C_\ell}\frac{\partial I}{\partial z}$$

time rate of change of potential

The self-inductance L relates an <u>induced EMF</u> to $\frac{\partial I}{\partial t}$:

$$L \equiv \frac{-\text{EMF}}{\frac{\partial I}{\partial t}}$$

The induced EMF across an element Δz:

$$\frac{\partial V}{\partial z}\Delta z = -\underset{\substack{\uparrow \\ L/\Delta z}}{L_\ell}\,\Delta z\,\frac{\partial I}{\partial t}$$

$$\Rightarrow \frac{\partial I}{\partial t} = -\frac{1}{L_\ell}\frac{\partial V}{\partial z}$$

Combining the two equations:

$$\frac{\partial V}{\partial t} = \frac{-1}{C_\ell}\frac{\partial I}{\partial z} \Rightarrow \frac{\partial^2 V}{\partial t^2} = \frac{-1}{C_\ell}\frac{\partial^2 I}{\partial z\partial t} \Rightarrow \frac{\partial^2 V}{\partial z\partial t} = \frac{-1}{C_\ell}\frac{\partial^2 I}{\partial z^2}$$

$$\frac{\partial I}{\partial t} = \frac{-1}{L_\ell}\frac{\partial V}{\partial z} \Rightarrow \frac{\partial^2 I}{\partial t^2} = \frac{-1}{L_\ell}\frac{\partial^2 V}{\partial z\partial t} \Rightarrow \frac{\partial^2 I}{\partial z\partial t} = \frac{-1}{L_\ell}\frac{\partial^2 V}{\partial z^2}$$

$$\Rightarrow \quad \frac{\partial^2 V}{\partial t^2} = \frac{1}{C_\ell L_\ell} \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial^2 I}{\partial t^2} = \frac{1}{C_\ell L_\ell} \frac{\partial^2 I}{\partial z^2}$$

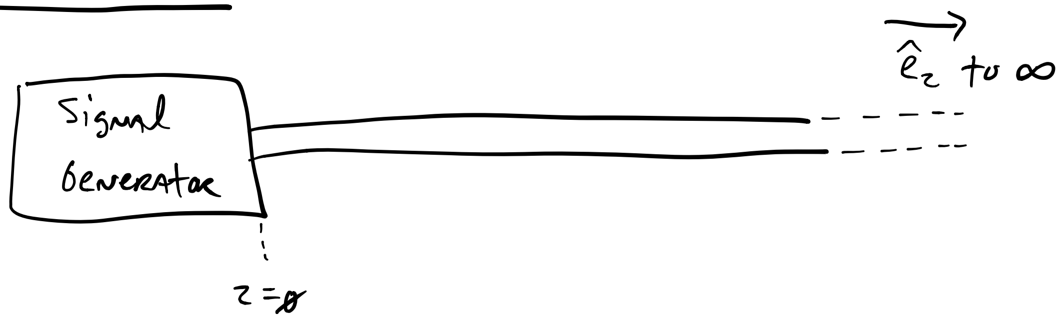
$$\Rightarrow \boxed{\begin{aligned} \frac{\partial^2 V}{\partial z^2} + C_\ell L_\ell \frac{\partial^2 V}{\partial t^2} &= 0 \\ \frac{\partial^2 I}{\partial z^2} + C_\ell L_\ell \frac{\partial^2 I}{\partial t^2} &= 0 \end{aligned}}$$

→ Look familiar?
<u>wave equations!</u>

wave speed $\Rightarrow \quad c = \dfrac{1}{\sqrt{L_\ell C_\ell}}$

- C_ℓ and L_ℓ depend on geometry but are dependent on each other.
- For loss-free lines with constant cross-section, c = speed of plane wave $= 3 \times 10^{10}$ cm/s in free space

Impedences



$$V(0,t) = V_0 e^{-i\omega t}$$

NO RETURN SIGNAL SO

$$V(z,t) = V_0 e^{i(kz-\omega t)}$$

↳ solution to wave eqn

In-class Problem:

Calculate $I(z,t)$ from $V(z,t)$

Solution

$$\frac{\partial I}{\partial t} = -\frac{1}{L_\ell}\frac{\partial V}{\partial z} \Rightarrow I = \frac{-ik}{L_\ell}\int V_0\, e^{i(kz-\omega t)}\, dt$$

$$\Rightarrow I = \frac{k}{\omega}\frac{V_0}{L_\ell}\, e^{i(kz-\omega t)} = \frac{1}{c}\frac{V_0}{L_\ell}\, e^{i(kz-\omega t)}$$

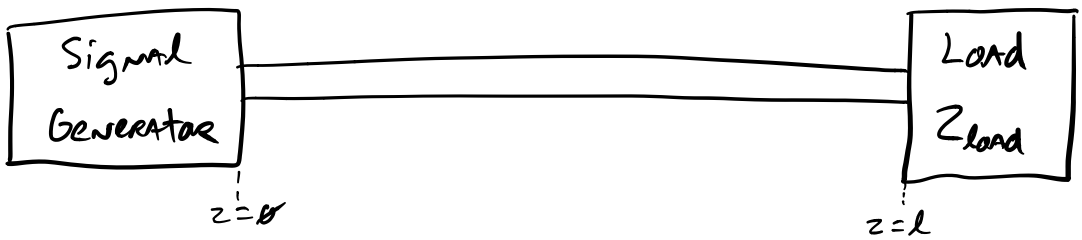
But $\quad c = \dfrac{1}{\sqrt{L_\ell C_\ell}} \quad \Longrightarrow \quad \underline{I(z,t) = \sqrt{\dfrac{C_\ell}{L_\ell}}\, V_0\, e^{i(kz-\omega t)}}$

$\Rightarrow$ The current is <u>in phase</u> with the voltage

The <u>characteristic impedence</u> Z_0 is defined by:

$$Z_0 \equiv \frac{V}{I} = \sqrt{\frac{L_\ell}{C_\ell}} \qquad \left(\text{Think } V = I\underset{Z_0}{R}\right)$$

For a lossless line, Z_0 = Resistance, and is independent of frequency



- If $Z_{load} = Z_0 = \sqrt{\frac{L_l}{C_l}}$ no wave is reflected and the line transfers power from the signal generator to the load

- If $Z_{load} \neq Z_0$, there is an impedence mismatch and the wave is partially reflected

$\Rightarrow$ standing wave is produced

Superposition of $\xrightarrow{+}$ and $\xleftarrow{-}$ waves

$$V(z,t) = V_+ e^{i(kz-\omega t)} + V_- e^{i(-kz-\omega t)}$$

$$I(z,t) = I_+ e^{i(kz-\omega t)} + I_- e^{i(-kz-\omega t)}$$

The load impedence at $z = l$:

$$Z_{load} = \frac{V(l,t)}{I(l,t)} = \frac{V_+ e^{ikl} + V_- e^{-ikl}}{I_+ e^{ikl} + I_- e^{-ikl}}$$

Note: In general, the impedence is complex

with $\underset{\text{resitance}}{Z = R} - i \underset{\text{reactance}}{X}$

(Electrical Engineer notation $Z = R + iX$) $\llcorner$ from $e^{+i\omega t}$

The $\xrightarrow{+}$ and $\xleftarrow{-}$ voltages and currents obey

$$\frac{V_+}{I_+} = -\frac{V_-}{I_-} = Z_0$$

The generator (input) impedence is

$$Z_{gen} \equiv \frac{V(0,t)}{I(0,t)} = \frac{V_+ + V_-}{I_+ + I_-}$$

Comparing Z_{load}, Z_{gen}, and Z_0 we find

$$Z_{gen} = Z_0 \frac{Z_{load} - i Z_0 \tan kl}{Z_0 - i Z_{load} \tan kl}$$

→ provides complex impedence as a fxn of load impedence Z_{load}, characteristic impedence Z_0, and distance l

→ Z_{gen} is periodic wrt l

↳ period of π from tan

↳ complete cycle in $\frac{1}{2}\lambda$

The amplitude of reflection is

$$r \equiv \frac{V_- e^{-ikl}}{V_+ e^{ikl}} = \frac{Z_{load} - Z_0}{Z_{load} + Z_0}$$

Power refection coef:

$$R = \left| \frac{Z_{load} - Z_0}{Z_{load} + Z_0} \right|^2$$

- r and R are analogs of plane wave reflection from an interface for normal incidence

- As $\omega \to 0$, $\lambda \to \infty$ (low frequency) the properties reduce to elementary circuit theory ($V = IR$, Kirchhoff's rules)