Work:

Metric: Morris-Thorne Metric

$$ds^2 = -e^{\Phi(r)}dt^2 + \frac{1}{1-\frac{b(r)}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$\Rightarrow g_{ij} = \begin{pmatrix} -e^{\Phi(r)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b(r)}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$ w/ $ds^2 = g_{ab}(x)\,dx^a dx^b$ $\forall x^a, x^b \in \{t, r, \theta, \phi\}$

$g_{ij}\, g^{jk} = \delta_i^{\,k}$ by definition of Inverse.

$$\begin{pmatrix} -e^{\Phi(r)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b(r)}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} g^{jk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow g^{jk} = 0$ where $j \neq k$ b/c matrices are diagonal

So we have 4 equations

$$-e^{\Phi(r)}\, g^{tt} = 1 \Rightarrow g^{tt} = -e^{-\Phi(r)}$$

$$\left(1-\frac{b(r)}{r}\right)^{-1} g^{rr} = 1 \Rightarrow g^{rr} = \left(1-\frac{b(r)}{r}\right)$$

$$r^2\, g^{\theta\theta} = 1 \Rightarrow g^{\theta\theta} = r^{-2}$$

$$r^2\sin\theta\, g^{\phi\phi} = 1 \Rightarrow g^{\phi\phi} = r^2\sin^{-2}\theta$$

$$\text{in that } g^{jk} = \begin{pmatrix} -e^{-\Phi(r)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b(r)}{r}\right) & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix}$$

Next We Need the affine Connection:

$$\Gamma^i_{jk} \equiv \frac{1}{2} g^{il}(\partial_j g_{lk} + \partial_k g_{lj} - \partial_l g_{jk})$$

Next, I read off each individual component, and added them to the Display Page.

- $\Gamma^t_{tt} = \frac{1}{2} g^{tl}(\partial_t g_{lt} + \partial_t g_{lt} - \partial_l g_{tt})$

$$= \frac{1}{2} g^{tt}(\partial_t g_{tt} + \partial_t g_{tt} - \partial_t g_{tt}) = 0$$

- $\Gamma^t_{tr} = \Gamma^t_{rt} = \frac{1}{2} g^{tt}(\partial_t g_{tr} + \partial_r g_{tt} - \partial_t g_{rt})$

$$= \frac{1}{2} g^{tt}(\partial_r g_{tt}) = \frac{1}{2} e^{-\Phi(r)} \Phi'(r) e^{\Phi(r)} = \frac{1}{2}\Phi'(r)$$

- $\Gamma^t_{t\theta} = \Gamma^t_{\theta t} = \frac{1}{2} g^{tt}(\partial_t g_{t\theta} + \partial_\theta g_{tt} - \partial_t g_{\theta t}) = 0$
- $\Gamma^t_{t\phi} = \Gamma^t_{\phi t} = \frac{1}{2} g^{tt}(\partial_t g_{t\phi} + \partial_\phi g_{tt} - \partial_t g_{\phi t}) = 0$
- $\Gamma^t_{rr} = \Gamma^t_{rr} = \frac{1}{2} g^{tt}(\partial_r g_{tr} + \partial_r g_{tr} - \partial_t g_{rr}) = 0$
- $\Gamma^t_{r\theta} = \Gamma^t_{\theta r} = \frac{1}{2} g^{tt}(\partial_r g_{t\theta} + \partial_\theta g_{t\theta} - \partial_t g_{r\theta}) = 0$
- $\Gamma^t_{r\phi} = \Gamma^t_{\phi r} = \frac{1}{2} g^{tt}(\partial_r g_{t\phi} + \partial_\phi g_{tr} - \partial_t g_{r\phi}) = 0$
- $\Gamma^t_{\theta\theta} = \Gamma^t_{\theta\theta} = \frac{1}{2} g^{tt}(\partial_\theta g_{t\theta} + \partial_\theta g_{t\theta} - \partial_t g_{\theta\theta}) = 0$
- $\Gamma^t_{\theta\phi} = \Gamma^t_{\phi\theta} = \frac{1}{2} g^{tt}(\partial_\theta g_{t\phi} + \partial_\phi g_{t\theta} - \partial_t g_{\theta\phi}) = 0$
- $\Gamma^t_{\phi\phi} = \frac{1}{2} g^{tt}(\partial_\phi g_{t\phi} + \partial_\phi g_{t\phi} - \partial_t g_{\phi\phi}) = 0$
- $\Gamma^r_{rr} = \frac{1}{2} g^{rr}(\partial_r g_{rr}) = \frac{1}{2}\left(1 - \frac{b(r)}{r}\right)\left(1 - \frac{b(r)}{r}\right)^{-2} \cdot \left(-\frac{b'(r)}{r} + \frac{b(r)}{r^2}\right)$

$$= \frac{1}{2}\left(1 - \frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)$$

- $\Gamma^r_{rt} = \frac{1}{2} g^{rr}(\partial_r g_{rt} + \partial_t g_{rr} - \partial_r g_{rt}) = 0 = \Gamma^r_{tr}$
- $\Gamma^r_{tt} = \frac{1}{2} g^{rr}(\partial_t g_{rt} + \partial_t g_{rt} - \partial_r g_{tt}) = -\frac{1}{2} g^{rr} \partial_r g_{tt}$

$$= \frac{1}{2}\Phi'(r)\left(1 - \frac{b(r)}{r}\right) e^{\Phi(r)}$$

Made a Simplifying Conclusion:

Notice That: $\Gamma^i_{jk} = \frac{1}{2} g^{ii}(\partial_j g_{ik} + \partial_k g_{ij} - \partial_i g_{jk}) = 0$
if $i \neq j \neq k$ b/c g_{ij} is diagonal

$\Rightarrow \Gamma^r_{t\theta} = \Gamma^r_{\theta t} = \Gamma^r_{\phi\theta} = \Gamma^r_{\theta\phi} = \Gamma^r_{\phi t} = \Gamma^r_{\phi t} = 0$

- $\Gamma^r_{r\theta} = \Gamma^r_{\theta r} = \frac{1}{2} g^{rr}(\partial_r g_{r\theta} + \partial_\theta g_{rr} - \partial_r g_{r\theta}) = 0$
- $\Gamma^r_{r\phi} = \Gamma^r_{\phi r} = \frac{1}{2} g^{rr}(\partial_r g_{r\phi} + \partial_\phi g_{rr} - \partial_r g_{\phi r}) = 0$
- $\Gamma^r_{\theta\theta} = \frac{1}{2} g^{rr}(\partial_\theta g_{r\theta} + \partial_\theta g_{r\theta} - \partial_r g_{\theta\theta}) = -(1 - \frac{b(r)}{r}) r$
- $\Gamma^r_{\phi\phi} = \frac{1}{2} g^{rr}(\partial_\phi g_{r\phi} + \partial_\phi g_{r\phi} - \partial_r g_{\phi\phi}) = -(r - b(r))\sin^2\theta$
- $\Gamma^\theta_{tt} = \frac{1}{2} g^{\theta\theta}(\partial_t g_{\theta t} + \partial_t g_{\theta t} - \partial_\theta g_{tt}) = 0$
- $\Gamma^\theta_{tr} = \Gamma^\theta_{rt} = \Gamma^\theta_{t\phi} = \Gamma^\theta_{\phi t} = 0$
- $\Gamma^\theta_{t\theta} = \Gamma^\theta_{\theta t} = \frac{1}{2} g^{\theta\theta}(\partial_t g_{\theta\theta} + \partial_\theta g_{\theta t} - \partial_\theta g_{\theta t}) = 0$
- $\Gamma^\theta_{rr} = \frac{1}{2} g^{\theta\theta}(\partial_r g_{\theta r} + \partial_r g_{\theta r} - \partial_\theta g_{rr}) = 0$
- $\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{2} g^{\theta\theta}(\partial_r g_{\theta\theta} + \partial_\theta g_{\theta r} - \partial_\theta g_{\theta r}) = \frac{1}{2} g^{\theta\theta} \partial_r g_{\theta\theta}$
$= \frac{1}{2} r^{-2} \cdot 2r = \frac{1}{r}$
- $\Gamma^\theta_{r\phi} = \Gamma^\theta_{\phi r} = 0$
- $\Gamma^\theta_{\theta\theta} = \frac{1}{2} g^{\theta\theta}(\partial_\theta g_{\theta\theta}) = 0$
- $\Gamma^\theta_{\theta\phi} = \Gamma^\theta_{\phi\theta} = \frac{1}{2} g^{\theta\theta}(\partial_\theta g_{\theta\phi} + \partial_\phi g_{\theta\theta} - \partial_\theta g_{\theta\phi})$
$= \frac{1}{2} g^{\theta\theta} \partial_\phi g_{\theta\theta} = 0$
- $\Gamma^\theta_{\phi\phi} = \frac{1}{2} g^{\theta\theta}(\partial_\phi g_{\theta\phi} + \partial_\phi g_{\phi\theta} - \partial_\theta g_{\phi\phi})$
$= -\frac{1}{2} g^{\theta\theta} \partial_\theta g_{\phi\phi} = -\frac{1}{2} r^{-2} \cdot 2r^2 \sin\theta\cos\theta) = -\sin\theta\cos\theta$

$\Gamma^{\phi}_{tt} = \frac{1}{2} g^{\phi\phi}(\partial_t g_{\phi t} + \partial_t g_{\phi t} - \partial_\phi g_{tt}) = 0$

$\Gamma^{\phi}_{tr} = \Gamma^{\phi}_{rt} = \Gamma^{\phi}_{t\theta} = \Gamma^{\phi}_{\theta t} = 0$

$\Gamma^{\phi}_{\phi t} = \Gamma^{\phi}_{t\phi} = \frac{1}{2} g^{\phi\phi}(\partial_\phi g_{\phi t} + \partial_t g_{\phi\phi} - \partial_\phi g_{\phi t}) = 0$

$\Gamma^{\phi}_{rr} = \frac{1}{2} g^{\phi\phi}(\partial_r g_{\phi r} + \partial_r g_{\phi r} - \partial_\phi g_{rr}) = 0$

$\Gamma^{\phi}_{r\theta} = \Gamma^{\phi}_{\theta r} = 0$

$\Gamma^{\phi}_{\phi r} = \Gamma^{\phi}_{r\phi} = \frac{1}{2} g^{\phi\phi}(\partial_\phi g_{r\phi} + \partial_r g_{\phi\phi} - \partial_\phi g_{\phi r})$
$= \frac{1}{2} g^{\phi\phi} \partial_r g_{\phi\phi} = \frac{1}{2} r^{-2} \sin^{-2}\theta \cdot 2r\sin^2\theta = r^{-1}$

$\Gamma^{\phi}_{\theta\theta} = \frac{1}{2} g^{\phi\phi}(\partial_\theta g_{\phi\theta} + \partial_\theta g_{\phi\theta} - \partial_\phi g_{\theta\theta}) = 0$

$\Gamma^{\phi}_{\theta r} = \Gamma^{\phi}_{r\theta} = 0$

$\Gamma^{\phi}_{\phi\theta} = \Gamma^{\phi}_{\theta\phi} = \frac{1}{2} g^{\phi\phi}(\partial_\phi g_{\theta\phi} + \partial_\theta g_{\phi\phi} - \partial_\phi g_{\phi\theta})$
$= \frac{1}{2} g^{\phi\phi} \partial_\theta g_{\phi\phi} = \frac{1}{2} r^{-2} \sin^{-2}\theta \cdot 2r^2 \sin\theta\cos\theta$
$= \cot\theta$

$\Gamma^{\phi}_{\phi\phi} = \frac{1}{2} g^{\phi\phi}(\partial_\phi g_{\phi\phi}) = 0$

Good, Connections are Correct. Now to work on Rieman Tensor after lunch.

$$R^i{}_{jkl} = \partial_k \Gamma^i_{jl} - \partial_l \Gamma^i_{jk} + \Gamma^m_{jl}\Gamma^i_{mk} - \Gamma^m_{jk}\Gamma^i_{ml}$$

$$R^t{}_{ttt} = \underset{=0}{\partial_t \Gamma^t_{tt}} - \underset{-0}{\partial_t \Gamma^t_{tt}} + \underset{+0}{\Gamma^m_{tt}\Gamma^t_{mt}} - \underset{-0}{\Gamma^m_{tt}\Gamma^t_{mt}} = 0.$$

Note: Symmetries of $R^i{}_{jkl}$

(1) $R^i{}_{jkl} = -R^i{}_{jlk}$

(2) $R^i{}_{jkl} + R^i{}_{ljk} + R^i{}_{klj} = 0$

(3) $R_{ijkl} = R_{klij}$, Where $R_{ijkl} = g_{im} R^m{}_{jkl}$

(4) When $n=4$, R_{abcd} has 20 independent Coordinates

(a) 10 from R_{ab}

(b) 10 from Weyl tensor

Weyl tensor:

$$C_{ijkl} = R_{ijkl} + \frac{1}{2}\left(g_{il}R_{jk} + g_{jk}R_{il} - g_{ik}R_{jl} - g_{jl}R_{ik}\right) + \frac{1}{6}\left(g_{ik}g_{jl} - g_{il}g_{jk}\right)R$$

Note: $R_{jl} = R^i{}_{jil}$ $\therefore R_{jl} = R_{lj}$

$$R = g^{jl}R_{jl} = g^{jl}R^i{}_{jil}$$

$C^i{}_{jil} = 0$ & Same Symmetries as $R^i{}_{jkl}$

Now I found the components of the Ricci Tensor. They make up some components of the Riemann Tensor

$$R_{jl} = R^i{}_{jil} = \partial_i \Gamma^i_{jl} - \partial_l \Gamma^i_{ji} + \Gamma^m_{jl}\Gamma^i_{mi} - \Gamma^m_{ji}\Gamma^i_{ml}$$

$$R_{tt} = \partial_i \Gamma^i_{tt} - \partial_t \Gamma^i_{ti} + \Gamma^m_{tt}\Gamma^i_{mi} - \Gamma^m_{ti}\Gamma^i_{mt}$$

Note only $\Gamma^t_{tr} = \Gamma^t_{rt} = \frac{1}{2}\Phi'(r)$, $\Gamma^r_{tt} = \frac{1}{2}\Phi'(r)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}$

$$R_{tt} = \partial_r \Gamma^r_{tt} - 0 + \Gamma^r_{tt}\Gamma^i_{ri} - \Gamma^r_{tt}\Gamma^t_{rt} - \Gamma^t_{tr}\Gamma^r_{tt}$$

Note $\Gamma^i_{ri} = \Gamma^t_{rt} + \Gamma^r_{rr} + \Gamma^\theta_{r\theta} + \Gamma^\phi_{r\phi}$

$$= \frac{1}{2}\Phi'(r) + \frac{1}{2}\left(1-\frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right) + \frac{2}{r}$$

$$R_{tt} = \frac{1}{2}\Phi''(r)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)} + \cancel{\frac{1}{2}\Phi'(r)\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)e^{\Phi(r)}}$$
$$\cancel{+\frac{1}{2}(\Phi'(r))^2\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}}$$
$$+\frac{1}{2}\Phi'(r)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}\left[\frac{1}{2}\Phi'(r) + \cancel{\frac{1}{2}\left(1-\frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)} + \frac{2}{r}\right]$$
$$-\frac{1}{2}(\Phi'(r))^2\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}$$

$$R_{tt} = \frac{1}{2}\Phi''(r)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)} + \frac{1}{4}(\Phi'(r))^2\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)} + \frac{\Phi'(r)}{r}\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}$$

$$\boxed{R_{tt} = \left(\frac{1}{2}\Phi''(r) + \frac{1}{4}(\Phi'(r))^2 + \frac{\Phi'(r)}{r}\right)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}}$$

$$R_{tr} = R_{rt} = \partial_i \Gamma^i_{tr} - \partial_r \Gamma^i_{ti} + \Gamma^m_{tr}\Gamma^i_{mi} - \Gamma^m_{ti}\Gamma^i_{mr}$$

$$= \partial_t \Gamma^t_{tr} + \cancel{\partial_r \Gamma^r_{tr}} - \partial_r \Gamma^t_{tt} - \partial_r \Gamma^r_{tr}$$
$$+ \Gamma^t_{tr}\Gamma^i_{ti} - \cancel{\Gamma^t_{tr}\Gamma^t_{tr}} - \Gamma^r_{tt}\Gamma^t_{rt} - \Gamma^r_{tr}\Gamma^r_{rr}$$

$$= \cancel{\Gamma^t_{tr}\Gamma^t_{tt}} + \cancel{\Gamma^t_{tr}\Gamma^r_{tr}} - \Gamma^r_{tt}\Gamma^t_{rt} - \cancel{\Gamma^r_{tr}\Gamma^r_{rr}}$$

$$\boxed{R_{tr} = -\left(\frac{1}{2}\Phi'(r)\right)^2\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)} = R_{rt}}$$

$$R_{t\theta} = R_{\theta t} = \partial_i \Gamma^i_{t\theta} - \partial_\theta \Gamma^i_{ti} + \Gamma^m_{t\theta}\Gamma^i_{mi} - \Gamma^m_{ti}\Gamma^m_{i\theta}$$

Note: $\Gamma^i_{t\theta} = 0$ for any i, $\partial_\theta \Gamma^i_{ti} = 0$ for any i

$$R_{t\theta} = -[\Gamma^r_{tt}\Gamma^t_{r\theta} + \cancel{\Gamma^t_{tr}\Gamma^r_{r\theta}}] = 0$$

$$\boxed{R_{t\theta} = R_{\theta t} = 0}$$

$$R_{t\phi} = R_{t\phi} = \partial_i \Gamma^i_{t\phi} - \partial_\phi \Gamma^i_{ti} + \Gamma^m_{t\phi}\Gamma^i_{mi} - \Gamma^m_{ti}\Gamma^m_{i\phi}$$

Note: $\Gamma^i_{t\phi} = 0$ for any i, $\partial_\phi \Gamma^i_{jk} = 0$ for any i,j,k

$$R_{t\phi} = -[\Gamma^t_{tr}\Gamma^r_{t\phi} + \Gamma^r_{tt}\Gamma^t_{r\phi}] = 0$$

$$\boxed{R_{t\phi} = R_{\phi t} = 0}$$

$$R_{rr} = \partial_i \Gamma^i_{rr} - \partial_r \Gamma^i_{ri} + \Gamma^m_{rr}\Gamma^i_{mi} - \Gamma^m_{ri}\Gamma^m_{ir}$$

$$= \cancel{\partial_r \Gamma^r_{rr}} - \partial_r \Gamma^t_{rt} - \cancel{\partial_r \Gamma^r_{rr}} + \Gamma^r_{rr}\Gamma^i_{ri} - \Gamma^t_{rt}\Gamma^t_{tr} - \Gamma^r_{rr}\Gamma^r_{rr}$$

$$= \partial_r \Gamma^t_{rt} + \Gamma^r_{rr}\Gamma^t_{rt} + \cancel{\Gamma^r_{rr}\Gamma^r_{rr}} - \Gamma^t_{rt}\Gamma^t_{tr} - \cancel{\Gamma^r_{rr}\Gamma^r_{rr}}$$

$$\boxed{R_{rr} = \frac{1}{2}\Phi''(r) + \frac{1}{4}\Phi'(r)\left(1 - \frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right) - \frac{1}{4}(\Phi'(r))^2}$$

$$R_{r\theta} = \partial_i \Gamma^i_{r\theta} - \partial_\theta \Gamma^i_{ri} + \Gamma^m_{r\theta}\Gamma^i_{mi} - \Gamma^m_{ri}\Gamma^m_{i\theta}$$

$$= \cancel{\partial_\theta \Gamma^\theta_{r\theta}} - 0 + \Gamma^\theta_{r\theta}\Gamma^i_{\theta i} - \cancel{\Gamma^\theta_{rr}\Gamma^r_{\theta\theta}} - \Gamma^r_{r\theta}\Gamma^\theta_{r\theta} - \Gamma^\phi_{r\phi}\Gamma^\phi_{\phi\theta}$$

$$= \Gamma^\theta_{r\theta}\Gamma^\phi_{\theta\phi} - \Gamma^\phi_{r\phi}\Gamma^\phi_{\phi\theta}$$

$$\boxed{R_{r\theta} = R_{r\theta} = r^{-1}\cot\theta - r^{-1}\cot\theta = 0}$$

$$R_{r\phi} = \partial_i \Gamma^i_{r\phi} - \cancel{\partial_\phi \Gamma^i_{ri}} + \Gamma^m_{r\phi}\Gamma^i_{mi} - \Gamma^m_{ri}\Gamma^i_{m\phi}$$

$$= \partial_\phi \Gamma^\phi_{r\phi} + \cancel{\Gamma^\phi_{r\phi}\Gamma^i_{\phi i}} - \cancel{\Gamma^\phi_{rr}\Gamma^r_{\phi\phi} - \Gamma^\theta_{r\theta}\Gamma^\theta_{\phi\phi} - \Gamma^r_{\phi r}\Gamma^\phi_{r\phi} - \Gamma^\phi_{r\phi}\Gamma^\phi_{r\phi}}$$

$$\boxed{R_{r\phi} = R_{\phi r} = 0}$$

$$R_{\theta\theta} = \partial_i \Gamma^i_{\theta\theta} - \partial_\theta \Gamma^i_{\theta i} + \Gamma^m_{\theta\theta}\Gamma^i_{mi} - \Gamma^m_{\theta i}\Gamma^i_{m\theta}$$

$$= \partial_r \Gamma^r_{\theta\theta} - \partial_\theta \Gamma^\phi_{\theta\phi} + \Gamma^r_{\theta\theta}\Gamma^i_{ri} - \Gamma^r_{\theta\theta}\Gamma^\theta_{r\theta} - \Gamma^\theta_{\theta r}\Gamma^r_{\theta\theta} - \Gamma^\phi_{\theta\phi}\Gamma^\phi_{\phi\theta}$$

$$= b'(r) - 1 + \csc^2\theta + \Gamma^r_{\theta\theta}\Gamma^t_{rt} + \Gamma^r_{\theta\theta}\Gamma^\theta_{r\theta} + \Gamma^r_{\theta\theta}\Gamma^\phi_{r\phi} + \Gamma^r_{\theta\theta}\Gamma^r_{rr} - \cancel{\Gamma^r_{\theta\theta}\Gamma^\theta_{r\theta}} - \Gamma^\theta_{\theta r}\Gamma^r_{\theta\theta} - \Gamma^\phi_{\theta\phi}\Gamma^\phi_{\phi\theta}$$

$$= b'(r) - 1 + \csc^2\theta + (b(r)-r)\left(\tfrac{1}{2}\Phi'(r)\right) + \cancel{(b(r)-r)\tfrac{1}{r}} + (b(r)-r)\tfrac{1}{r}\left(1-\tfrac{b(r)}{r}\right)^{-1}\left(\tfrac{b(r)}{r^2} - \tfrac{b'(r)}{r}\right) - \cancel{\tfrac{1}{r}(b(r)-r)} - \cot^2\theta$$

Remember: $\csc^2\theta - \cot^2\theta = 1 \Rightarrow -1 + \csc^2\theta - \cot^2\theta = 0$

$$R_{\theta\theta} = (b'(r)-r)\tfrac{1}{2}\Phi(r) + \frac{(b(r)-r)}{(r-b(r))}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)$$

$$\boxed{R_{\theta\theta} = \tfrac{1}{2}(b'(r)-r)\Phi(r) - \left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)}$$

$$R_{\theta\phi} = \partial_i \Gamma^i_{\theta\phi} - \partial_\phi \Gamma^i_{\theta i} + \Gamma^m_{\theta\phi}\Gamma^i_{mi} - \Gamma^m_{\theta i}\Gamma^i_{m\phi}$$

$$= \cancel{\partial_\phi \Gamma^\phi_{\theta\phi}} - 0 + \cancel{\Gamma^\phi_{\theta\phi}\Gamma^i_{\phi i}} - \cancel{\Gamma^\phi_{\theta r}\Gamma^r_{\phi\phi} - \Gamma^\phi_{\theta\theta}\Gamma^\theta_{\phi\phi} - \Gamma^r_{\theta\phi}\Gamma^\phi_{r\phi} - \Gamma^\phi_{\theta\phi}\Gamma^\phi_{\theta\phi}}$$

$$\boxed{R_{\phi\theta} = 0 = R_{\theta\phi}}$$

$$\frac{(b(r)-r)}{\left(1-\frac{b(r)}{r}\right)} = \frac{r}{r}\frac{(b(r)-r)}{\left(1-\frac{b(r)}{r}\right)} = r\,\frac{(b(r)-r)}{(r-b(r))} = -r$$

$$R_{\phi\phi} = \partial_i \Gamma^i_{\phi\phi} - \partial_\phi \Gamma^i_{\phi i} + \Gamma^m_{\phi\phi}\Gamma^i_{mi} - \Gamma^i_{\phi m}\Gamma^m_{i\phi}$$

$$= \partial_r \Gamma^r_{\phi\phi} + \partial_\theta \Gamma^\theta_{\phi\phi} - 0 + \Gamma^r_{\phi\phi}\Gamma^i_{ri} + \Gamma^\theta_{\phi\phi}\Gamma^i_{\theta i} - \Gamma^r_{\phi\phi}\Gamma^\phi_{r\phi} - \Gamma^\theta_{\phi\phi}\Gamma^\phi_{\theta\phi}$$
$$- \Gamma^\phi_{\phi r}\Gamma^r_{\phi\phi} - \Gamma^\phi_{\phi\theta}\Gamma^\theta_{\phi\phi}$$

$$= (b'(r) - r)\sin^2\theta - \cos^2\theta + \sin^2\theta + \Gamma^r_{\phi\phi}\Gamma^t_{rt} + \Gamma^r_{\phi\phi}\Gamma^\theta_{r\theta}$$
$$+ \cancel{\Gamma^r_{\phi\phi}\Gamma^\phi_{r\phi}} + \Gamma^r_{\phi\phi}\Gamma^r_{rr} - \cancel{\Gamma^r_{\phi\phi}\Gamma^\phi_{r\phi}} - \Gamma^\theta_{\phi\phi}\Gamma^\phi_{\theta\phi} - \Gamma^\phi_{\phi r}\Gamma^r_{\phi\phi} - \Gamma^\phi_{\phi\theta}\Gamma^\theta_{\phi\phi}$$

$$= (b'(r) - r)\sin^2\theta + 2\cos^2\theta - 1 + \tfrac{1}{2}(b(r)-r)\sin^2\theta\,\Phi'(r) + (b(r)-r)\sin^2\theta\,\frac{1}{r}$$
$$+ \tfrac{1}{2}(b(r)-r)\sin^2\theta\left(1-\frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right) + \frac{1}{r}(b(r)-r)\sin^2\theta$$
$$+ 2\cot\theta\,\sin\theta\,\cos\theta$$

$$= \left(b'(r) - r + \frac{2b(r)}{r} - 2\right)\sin^2\theta + 2\cos^2\theta + \tfrac{1}{2}(b(r)-r)\sin^2\theta\,\Phi'(r)$$
$$+ -\frac{r}{2}\sin^2(\theta)\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right) + 2\cos^2\theta$$

$$= \left(b'(r) - r + \frac{2b(r)}{r} - 2 - \frac{b(r)}{2r} - \frac{b'(r)}{2} + \tfrac{1}{2}(b(r)-r)\Phi'(r)\right)\sin^2\theta$$
$$+ 4\cos^2\theta$$

$$\boxed{R_{\phi\phi} = \left(\frac{b'(r)}{2} + \frac{3}{2}\frac{b(r)}{r} - r - 2 + \tfrac{1}{2}(b(r)-r)\Phi'(r)\right)\sin^2\theta + 4\cos^2\theta}$$

Next I need the Ricci Scalar of a Straight Summation.

$$R = g^{jl}R_{jl}$$

$$= g^{tt}R_{tt} + g^{tr}R_{tr} + g^{rt}R_{rt} + g^{rr}R_{rr} + g^{\theta\theta}R_{\theta\theta} + g^{\phi\phi}R_{\phi\phi}$$

$$= -e^{-\Phi(r)}R_{tt} + 0 + 0 + \left(1-\frac{b(r)}{r}\right)R_{rr} + r^{-2}R_{\theta\theta} + r^{-2}\sin^{-2}\theta\,R_{\phi\phi}$$

$$R = -\left(\tfrac{1}{2}\Phi''(r) + \tfrac{1}{4}(\Phi'(r))^2 + \frac{\Phi'(r)}{r}\right)\left(1-\frac{b(r)}{r}\right)$$
$$+ \left(1-\frac{b(r)}{r}\right)\left(\tfrac{1}{2}\Phi''(r) - \tfrac{1}{4}(\Phi'(r))^2\right) + \tfrac{1}{4}\Phi'(r)\left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right)$$
$$+ \frac{1}{2r^2}(b'(r)-r)\Phi'(r) - \left(\frac{b(r)}{r^2} - \frac{b'(r)}{r}\right) + \left(\frac{b'(r)}{2} + \frac{3}{2}\frac{b(r)}{r} - r - 2 + \tfrac{1}{2}(b(r)-r)\Phi'(r)\right)$$
$$+ \frac{4}{r^2}\cot^2\theta$$

$$g_{ij} = \begin{pmatrix} -e^{\Phi(r)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b(r)}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \qquad g^{ij} = \begin{pmatrix} -e^{-\Phi(r)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b(r)}{r}\right) & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix}$$

✓ $$\Gamma^t_{jk} = \begin{array}{c} \begin{array}{cccc} t & r & \theta & \phi \end{array} \\ \begin{pmatrix} 0 & \frac{1}{2}\Phi'(r) & 0 & 0 \\ \frac{1}{2}\Phi'(r) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ t \\ r \\ \theta \\ \phi \end{array}$$

✓ $$\Gamma^r_{jk} = \begin{pmatrix} \frac{1}{2}\Phi'(r)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\left(1-\frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2}-\frac{b'(r)}{r}\right) & 0 & 0 \\ 0 & 0 & (b(r)-r) & 0 \\ 0 & 0 & 0 & (b(r)-r)\sin^2\theta \end{pmatrix}$$

✓ $$\Gamma^\theta_{jk} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & r^{-1} & 0 \\ 0 & r^{-1} & 0 & 0 \\ 0 & 0 & 0 & -\sin\theta\cos\theta \end{pmatrix}$$

✓ $$\Gamma^\phi_{jk} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r^{-1} \\ 0 & 0 & 0 & \cot\theta \\ 0 & r^{-1} & \cot\theta & 0 \end{pmatrix}$$

$$R_{ij} = \begin{pmatrix} R_{tt} & R_{rt} & 0 & 0 \\ R_{tr} & R_{rr} & 0 & 0 \\ 0 & 0 & R_{\theta\theta} & 0 \\ 0 & 0 & 0 & R_{\phi\phi} \end{pmatrix}$$

$$R_{tt} = \left(\frac{1}{2}\Phi''(r) + \frac{1}{4}(\Phi'(r))^2 + \frac{\Phi'(r)}{r}\right)\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}$$

$$R_{tr} = R_{rt} = -\left(\frac{1}{2}\Phi'(r)\right)^2\left(1-\frac{b(r)}{r}\right)e^{\Phi(r)}$$

$$R_{rr} = \frac{1}{2}\Phi''(r) + \frac{1}{4}\Phi'(r)\left(1-\frac{b(r)}{r}\right)^{-1}\left(\frac{b(r)}{r^2}-\frac{b(r)}{r}\right) - \frac{1}{4}(\Phi'(r))^2$$

$$R_{\theta\theta} = \frac{1}{2}(b'(r)-r)\Phi'(r) - \left(\frac{b(r)}{r^2}-\frac{b'(r)}{r}\right)$$

$$R_{\phi\phi} = \left(\frac{b'(r)}{2} + \frac{3}{2}\frac{b(r)}{r} - r - 2 + \frac{1}{2}(b(r)-r)\Phi'(r)\right)\sin^2\theta + 4\cos^2\theta$$

$$\cancel{G^{ij}{}_{kl} = g^{ii}g^{jj}G_{ijkl} = g^{ii}g^{jj}\left\{ g_{ik}R_{jl} + \frac{1}{2}\left(g_{il}R_{jk} + g_{jk}R_{il} - g_{ik}R_{jl} - g_{jl}R_{ik}\right) + \frac{1}{6}\left(g_{ik}g_{jl} - g_{il}g_{jk}\right)R \right\}}$$