Continuing From Previous Lecture

$$\hat{\varepsilon} = 1 + 4\pi \hat{\chi}_e = 1 + 4\pi \sum_\alpha \frac{N f_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2) - 2i\beta_\alpha \omega}$$

$$\Rightarrow \hat{n} = \sqrt{\hat{\varepsilon}} = \left(1 + 4\pi \sum_\alpha \frac{N f_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2) - 2i\beta_\alpha \omega}\right)^{1/2}$$

For $\hat{\varepsilon} \simeq 1$, we can use a Taylor series expansion

$$\sqrt{1+x} \simeq 1 + \frac{1}{2}x$$

$$\Rightarrow \hat{n} = \sqrt{\hat{\varepsilon}} \simeq 1 + 2\pi \sum_\alpha \frac{N f_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2) - 2i\beta_\alpha \omega}$$

Separating into real and imaginary

$$\sqrt{\hat{\varepsilon}} \approx 1 + 2\pi \sum_\alpha \frac{(\omega_\alpha^2 - \omega^2) N f_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2)^2 + 4\beta_\alpha^2 \omega^2}$$

$$+ i4\pi \sum_\alpha \frac{N f_\alpha \omega \beta_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2)^2 + 4\beta_\alpha^2 \omega^2}$$

The wave propagates in the medium as

$$E \sim \exp[i(\hat{k}\zeta - \omega t)]$$

$$\hat{k} = \frac{\omega}{c}\sqrt{\hat{\varepsilon}} = \frac{\omega}{c}\hat{n} = \underbrace{\frac{\omega}{c} n}_{\text{real}}(1 + i\underbrace{K}_{\text{imaginary}})$$

K = extinction coefficient

$$\Rightarrow E \sim e^{i(\hat{k}\zeta - \omega t)} = \underbrace{e^{-(\omega n/c)K\zeta}}_{\text{damping}} \; \underbrace{e^{i\left(\frac{\omega n \zeta}{c} - \omega t\right)}}_{\text{propagation}}$$

The phase velocity of the wave is

$$V_{ph} = \frac{\omega}{k} = \frac{\omega}{\omega n/c} = \frac{c}{Re[\hat{n}]}$$

real

$$\Rightarrow \boxed{n = \frac{c}{V_{ph}}}$$

Explicitly, the real and imaginary parts of $\hat{n}$ are:

$$Re(\sqrt{\hat{\varepsilon}}) = n \approx 1 + 2\pi \sum_{\alpha} \frac{(\omega_\alpha^2 - \omega^2) N f_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2)^2 + 4\beta_\alpha^2 \omega^2}$$

$$Im(\sqrt{\hat{\varepsilon}}) = n\kappa \approx 4\pi \sum_{\alpha} \frac{N f_\alpha \omega \beta_\alpha e^2/m}{(\omega_\alpha^2 - \omega^2)^2 + 4\beta_\alpha^2 \omega^2}$$

$\rightarrow$ n is fxn of ω $\rightarrow$ medium is <u>dispersive</u>

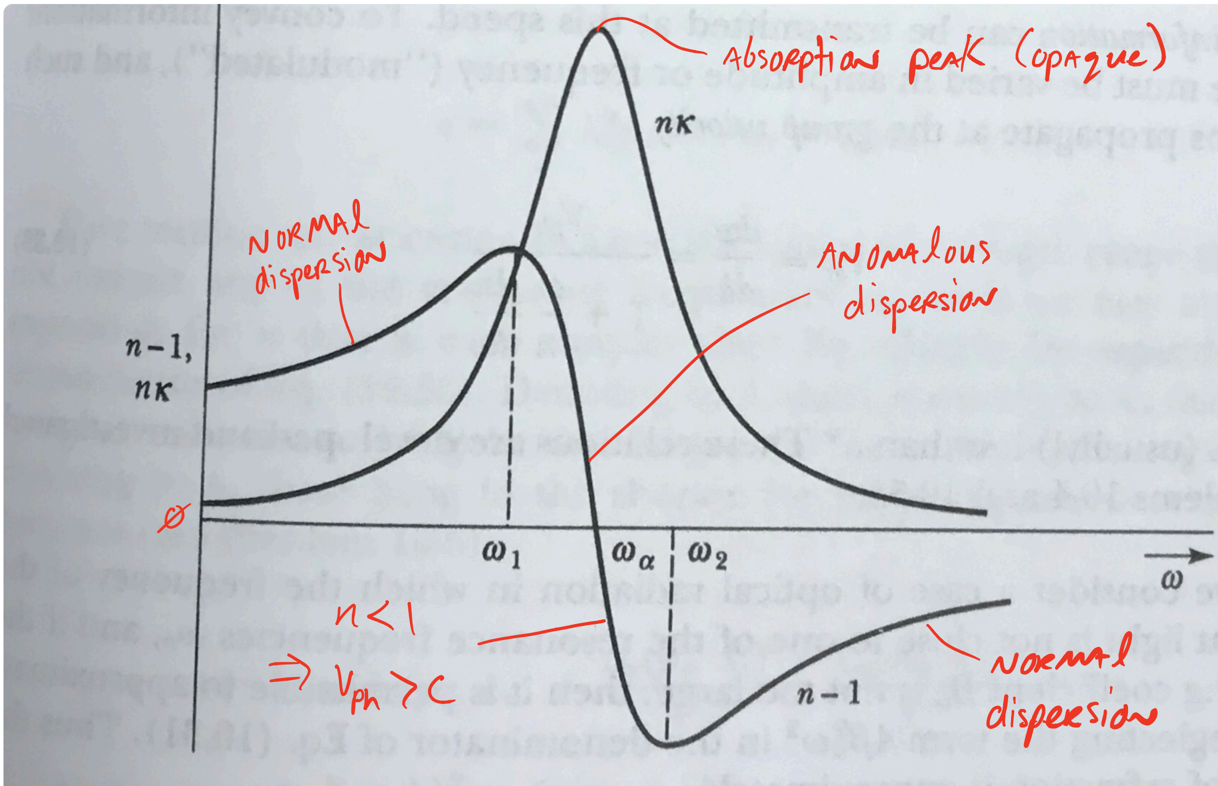

FIGURE 10-3. Dispersion $(n - 1)$ and attenuation $(n\kappa)$ near the resonance at ω_α.

[H+M]

- Normal dispersion = $n \uparrow$ as $\omega \uparrow$

  ↳ Like in a prism, blue refracts more than red

- Anomalous dispersion = $n \downarrow$ as $\omega \uparrow$

- Above the highest resonance ω_α (usually in the EUV) 10-124 nm

  $n < 1 \Rightarrow V_{ph} > c$

- No information can be transmitted at V_{ph} because the frequency or amplitude must be modulated:

$$V_{gr} \equiv \frac{d\omega}{dk} = \frac{V_{ph}}{1 + \frac{\omega}{n}\frac{dn}{d\omega}}$$

group velocity $< c$

Static Limit

Going back to our solution for a damped, driven harmonic oscillator

$$\bar{r}_\alpha(t) = \frac{-(e/m)\bar{E}_0}{(\omega_\alpha^2 - \omega^2) - 2i\beta_\alpha \omega} e^{-i\omega t}$$

If we take the steady-state limit $(\omega \to 0)$

$$\lim_{\omega \to 0} \bar{r}_\alpha = \frac{-(e/m)\bar{E}_0}{\omega_\alpha^2}$$

$$\Rightarrow \bar{p}_\alpha = -e\bar{r}_\alpha = \frac{e^2}{m\omega_\alpha^2}\bar{E}_0 = \gamma \bar{E}_0$$

$$\Rightarrow \gamma = \frac{e^2}{m\omega_\alpha^2} = \text{molecular } \underline{\text{polarizability}}$$

$\hookrightarrow$ Related to χ_e (susceptibility) from macroscopic $\bar{P} = \chi_e \bar{E}$

Conductivity of Metals

- In a metallic conductor, each atom contributes an e^- to the conduction band

- Conduction e^- are treated as a "gas"

- Apply $\bar{E}$, and we get an average motion $\longrightarrow$ drift velocity

- As the e^- move, they collide with with the lattice, producing a drag force

$$\text{effective drag force} = -\frac{m\dot{\bar{r}}}{\tau}$$

τ = mean collision time

$\dot{\bar{r}}$ = drift velocity

The equation of motion is

$$m\ddot{\bar{r}} + \frac{m\dot{\bar{r}}}{\tau} = -e\bar{E}(t)$$

no restoring force (spring)!

For $E(t) \sim e^{-i\omega t}$

$$\bar{r} = \frac{e/m}{\omega^2 + i\omega/\tau}\bar{E}$$

$$\Rightarrow \bar{u}_d \equiv \dot{\bar{r}} = -\frac{e\tau/m}{1-i\omega\tau}\bar{E}$$

drift velocity

For an e^- density N

$$\bar{J} = -eN\bar{u}_d = \frac{Ne^2\tau/m}{1-i\omega\tau}\bar{E} = \sigma\bar{E}$$

current density

conductivity

$$\Rightarrow \boxed{\tilde{\sigma} = \frac{Ne^2\tau/m}{1-i\omega\tau}}$$

In the DC limit, $\omega \to 0$

$$\boxed{\sigma_{DC} = \frac{Ne^2\tau}{m}}$$

Drude conductivity

For good conductors (copper), $\tau \sim 10^{-14}$ s

$\rightarrow$ For ω up to infrared ($\omega \sim 10^{13}$ rad/s)

$\Rightarrow \omega\tau \ll 1 \Rightarrow$ conductivity is <u>real</u>

For $\omega\tau \gtrsim 1$ (higher frequencies)

the metal acts like a plasma

Wave Propagation in Plasma

- Plasma = ionized gas
- e^- treated like a fluid
- collisions with ions/neutrals cause "drag"

$$\nu_c = \frac{1}{\tau}$$

$\llcorner$ collision frequency

Similar to a metal, the e^- displacement is:

$$\bar{r} = \frac{e/m_e}{\omega(\omega + i\nu_c)} \bar{E}$$

From $\qquad \bar{D} \equiv \bar{E} + 4\pi \bar{P} = \varepsilon \bar{E}$

$$\Rightarrow \bar{P} = \frac{\varepsilon - 1}{4\pi} \bar{E} = -Ne\bar{r} = \frac{-Ne^2/m_e}{\omega(\omega + i\nu_c)} \bar{E}$$

$$\Rightarrow \hat{\varepsilon} = 1 - \frac{4\pi N e^2/m_e}{\omega(\omega + i\nu_c)}$$

$$\Rightarrow \hat{n} = \sqrt{1 - \frac{\omega_p^2}{\omega(\omega + i\nu_c)}}$$

where

$$\omega_p \equiv \sqrt{\frac{4\pi N e^2}{m_e}}$$

plasma frequency

For a weakly damped plasma

$$\nu_c << \omega_p$$

we get 3 domains:

Dielectric Domain, $\omega > \omega_p$:

High frequency, n is mostly real

$$n \approx \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

↳ high pass filter

Evanescent Domain, $\nu_c < \omega < \omega_p$:

$$n \approx \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \text{pure imaginary}$$

↳ mostly imaginary if we include ν_c

→ wave is attenuated rapidly without phase shift or energy transport

Conductor Domain, $\omega < \nu_c$:

Low frequencies, $\hat{n}$ is complex with equal real and imaginary parts

$\rightarrow$ plasma behaves like conductor with

$$\sigma = \frac{Ne^2}{\nu_c m}$$

- In the dielectric and evanescent domains, plasma acts like a waveguide where the <u>plasma frequency</u> is the <u>cutoff frequency</u>

- For metal conductors,

$$\omega_p \rightarrow \text{ultraviolet}$$

$$\nu_c \rightarrow \text{infrared}$$

$\Rightarrow$ highly reflective until $\omega > \omega_p$ (dielectric domain) where they become transparent