Ph 22 Computational Physics **Winter 2020**

Homework 0
Philip Carr

1. Derivation:

$$x_3 = x_2 - f(x_2)\frac{x_2 - x_1}{f(x_2) - f(x_1)} \implies \epsilon_3 = \epsilon_2 - f(x_2)\frac{\epsilon_2 - \epsilon_1}{f(x_2) - f(x_1)}$$

$$\implies \epsilon_3 = \epsilon_2 - (f(x) + \epsilon_2 f'(x) + \frac{1}{2}\epsilon_2^2 f''(x))\frac{\epsilon_2 - \epsilon_1}{(\epsilon_2 - \epsilon_1)f'(x) + \frac{1}{2}(\epsilon_2^2 - \epsilon_1^2)f''(x)}$$

$$\implies \epsilon_3 = \epsilon_2 - \frac{\epsilon_2 f'(x) + \frac{1}{2}\epsilon_2^2 f''(x)}{f'(x) + \frac{1}{2}(\epsilon_2 + \epsilon_1)f''(x)}$$

$$\implies \epsilon_3 = \epsilon_2 - \frac{\epsilon_2 + \frac{1}{2}\epsilon_2^2 \frac{f''(x)}{f'(x)}}{1 + \frac{1}{2}(\epsilon_2 + \epsilon_1)\frac{f''(x)}{f'(x)}}$$

$$\implies \epsilon_3 = \epsilon_2 - (\epsilon_2 + \frac{1}{2}\epsilon_2^2\frac{f''(x)}{f'(x)})(1 - \frac{1}{2}(\epsilon_2 + \epsilon_1)\frac{f''(x)}{f'(x)})$$

(Ignoring terms of ϵ^3)

$$\implies \epsilon_3 = \frac{1}{2}\epsilon_2\epsilon_1\frac{f''(x)}{f'(x)}$$

$$\implies \epsilon_{i+2} = \frac{1}{2}\epsilon_{i+1}\epsilon_i\frac{f''(x)}{f'(x)}$$

$$\implies C\epsilon_{i+1}^r = \frac{1}{2}C\epsilon_i^r\epsilon_i\frac{f''(x)}{f'(x)}$$

$$\implies C^{1+r}\epsilon_{i+1}^{r^2} = \frac{1}{2}C\epsilon_i^{r+1}\frac{f''(x)}{f'(x)}$$

$$\implies r^2 = r + 1$$

$$\implies r = \phi = 1.618...\square$$

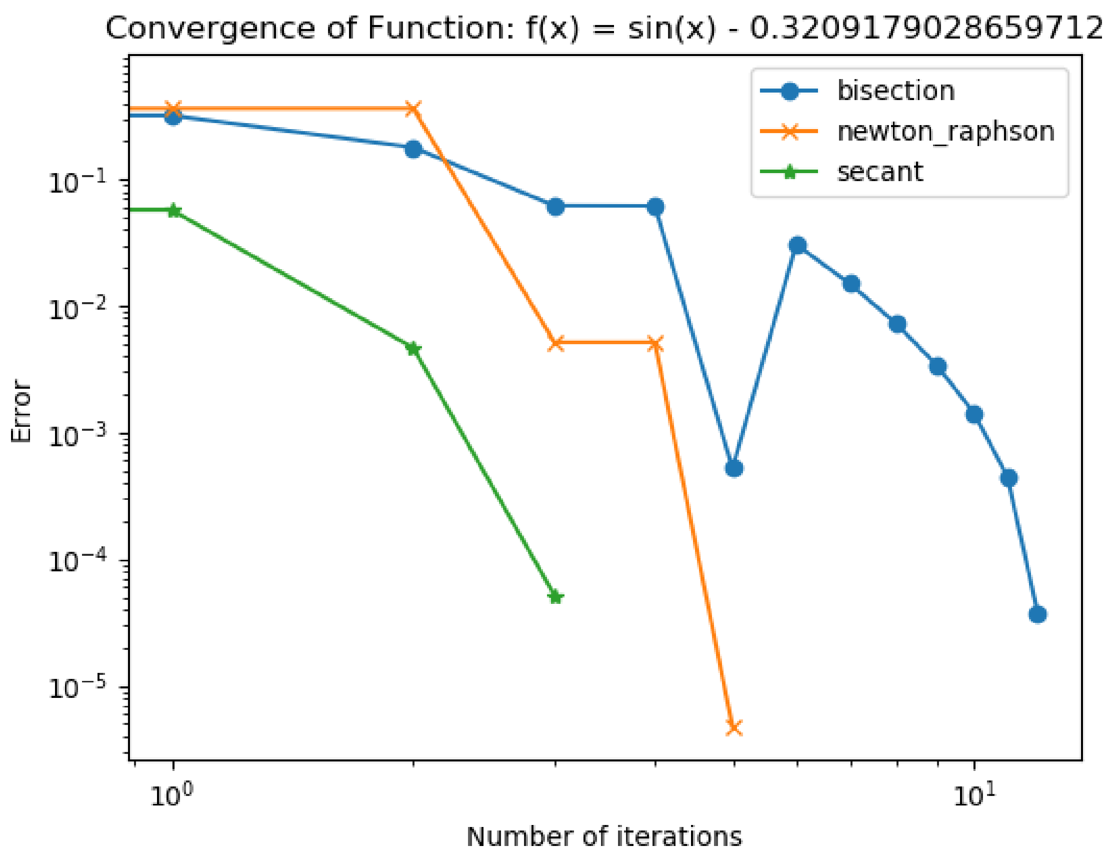
2. (See code for implementation.)
While the Newton-Raphson method and the Secant method take about the same number steps to converge on average, the bisection method takes much longer than both of those methods to converge.
Printed output for example below: (Each line after the name of the root-finding method is the current value of the guessed root at the current iteration (1 line = 1 iteration).)
Problem 2:
Function: f(x) = sin(x) - 0.20559840953709063
Bisection method
-1.0471975511965976
0.0
-0.5235987755982988
-0.2617993877991494
-0.1308996938995747

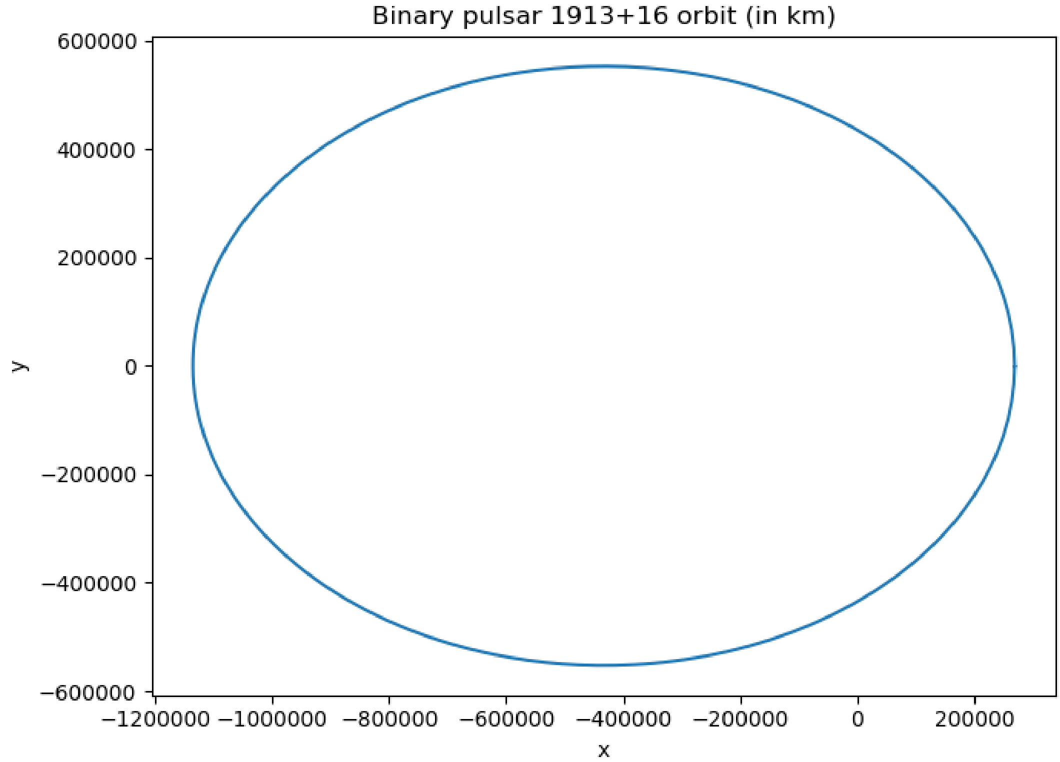
-0.19634954084936207
-0.22907446432425574
-0.2127120025868089
-0.20453077171808548
-0.20862138715244719
-0.20657607943526635
-0.20759873329385675
-0.20708740636456155

Newton-raphson method
-1.0471975511965976
0.2736564372980981
0.2736564372980981
-0.22058678688706407
-0.22058678688706407
-0.20705508348729684

Secant method
-1.0471975511965976
-0.24860950967062756
-0.20146745509384067
-0.2071011025589793

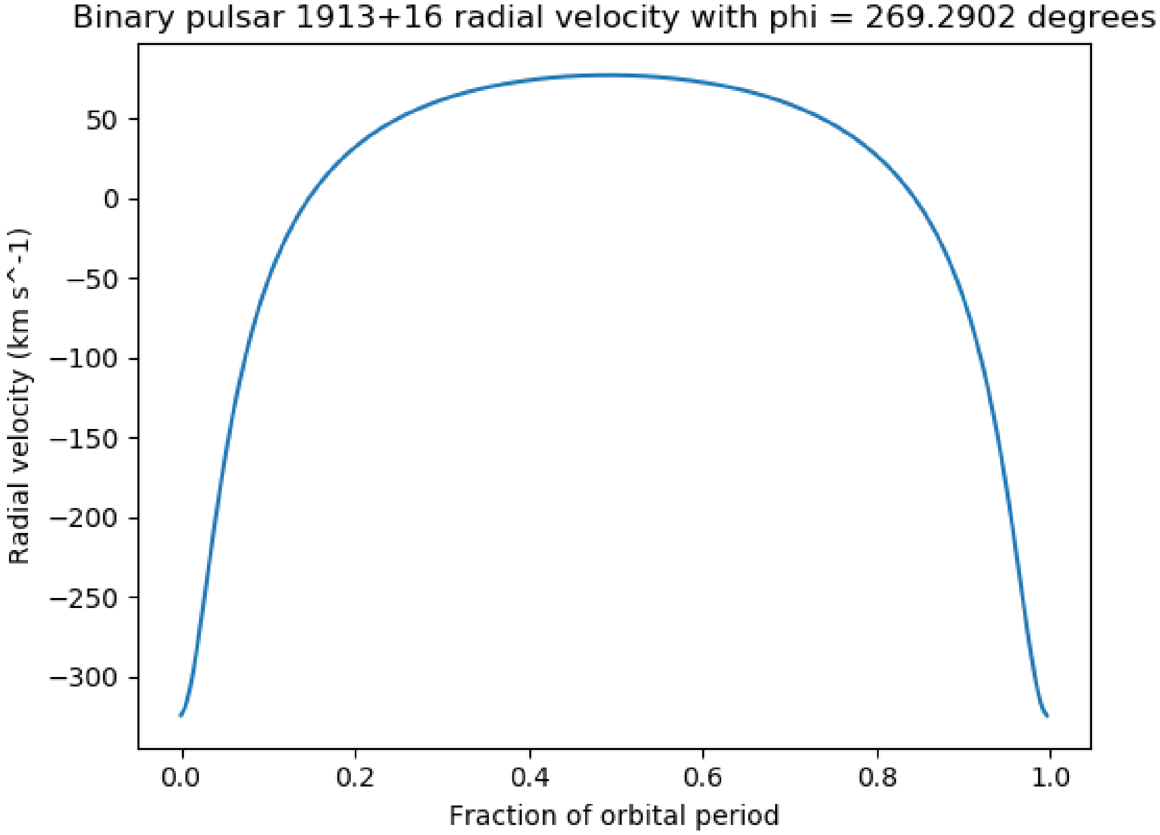
Example plot of convergence:





3.

4. Plot of best-fit radial velocity diagram below:



Binary pulsar 1913+16 radial velocity with phi = 269.2902 degrees
Radial velocity (km s^-1)
50
0
−50
−100
−150
−200
−250
−300
0.0
0.2
0.4
0.6
0.8
1.0
Fraction of orbital period