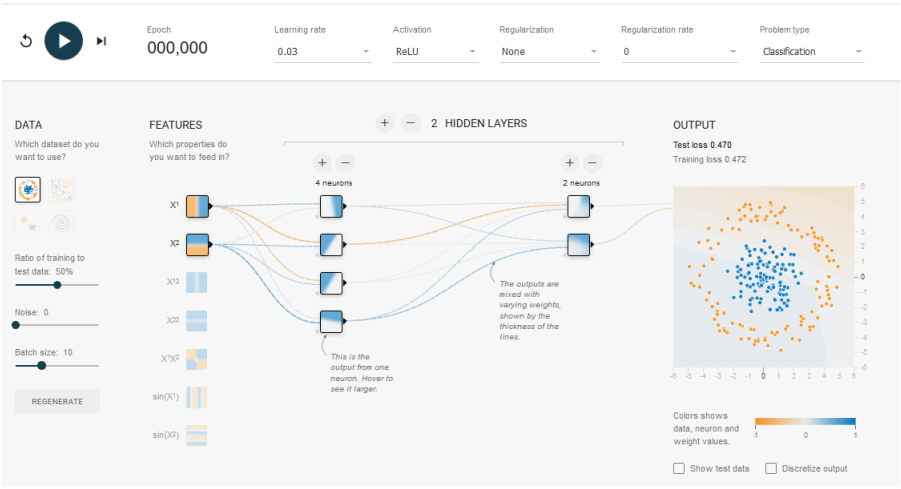
Philip Carr
January 2019.

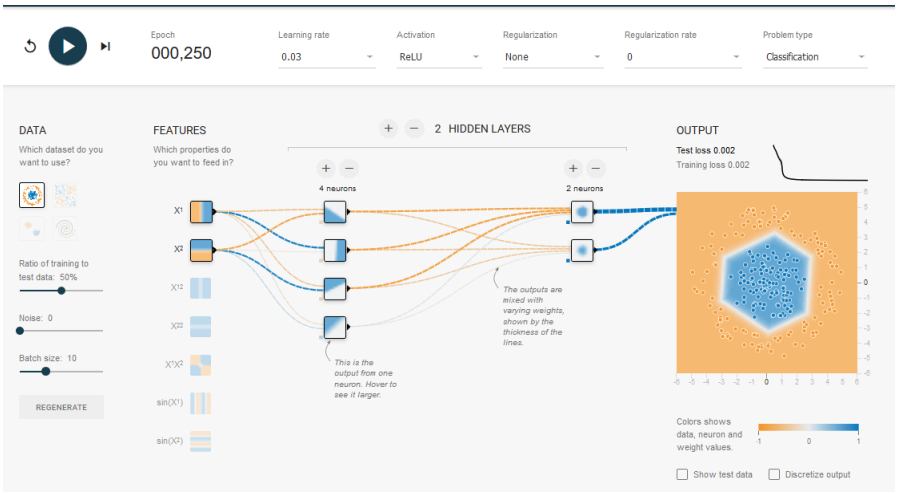
1 Deep Learning Principles [35 Points]

Relevant materials: lectures on deep learning

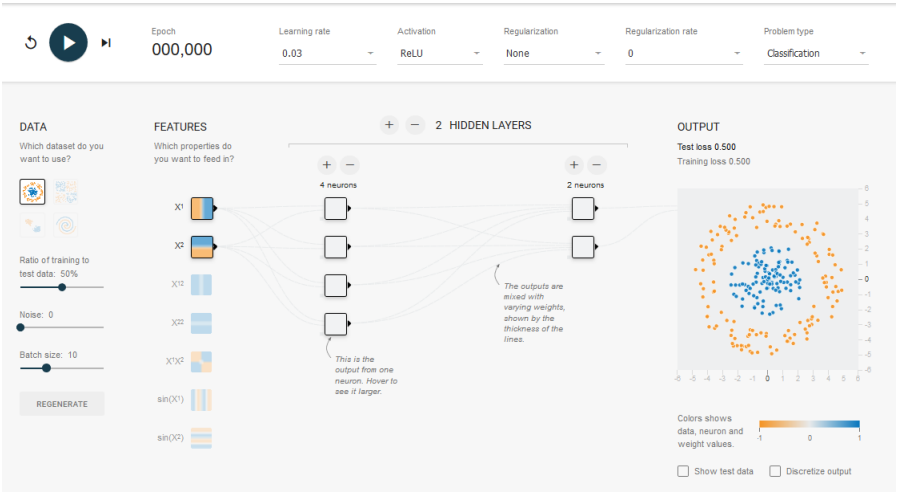
Problem A [5 points]: Backpropagation and Weight Initialization Part 1

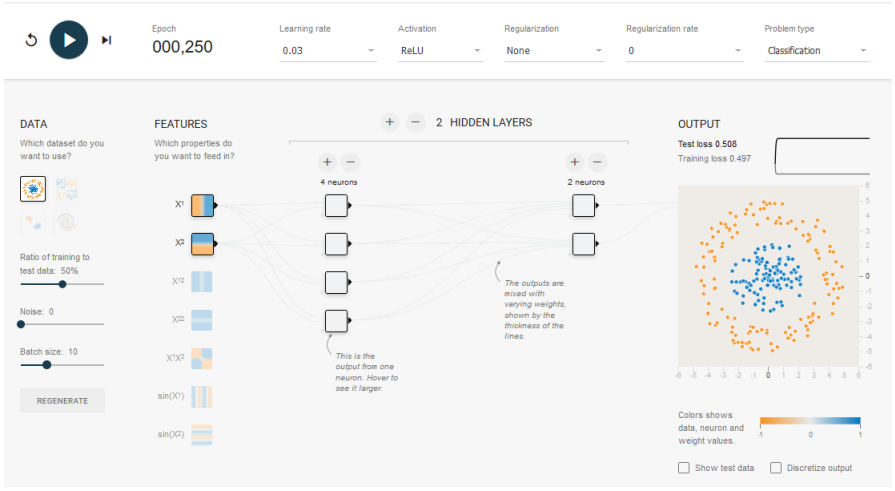
Solution A:





Since the weights for this model are initialized to be non-zero, the backpropagation algorithm is able to update the weights, along with the ReLU activation function being able to become nonzero from the weights. Thus, the model is able to converge to a solution with minimum trianing error.



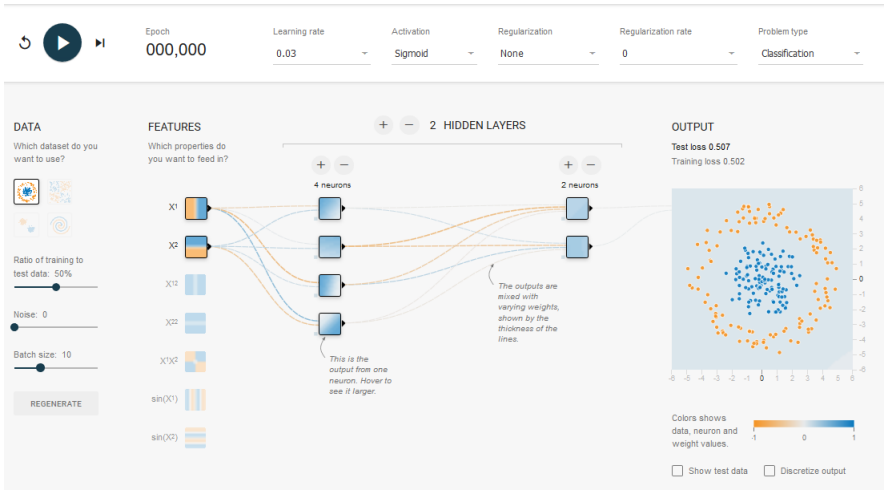


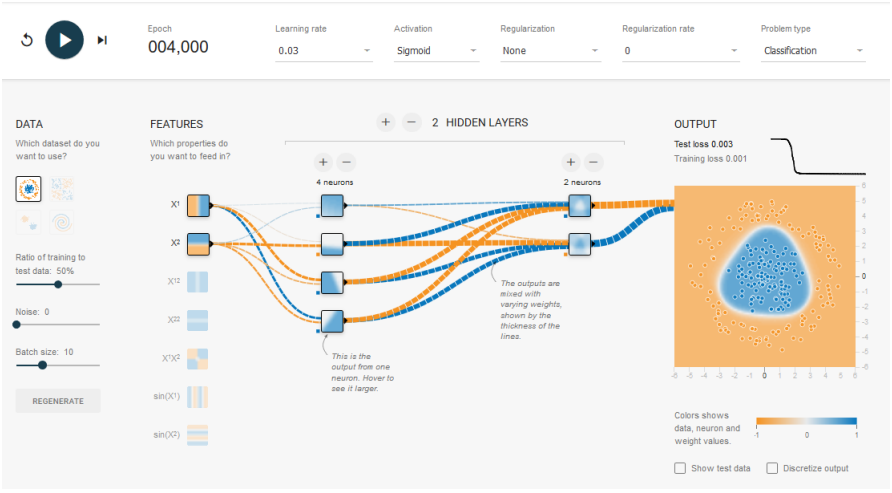
Since the weights for this model are initialized to be zero, the backpropagation algorithm is not able to update the weights from 0, along with the ReLU activation function always staying 0 from the weights which are all 0. Thus, the model is not able to converge to a solution with minimum trianing error.

Philip Carr
January 2019.

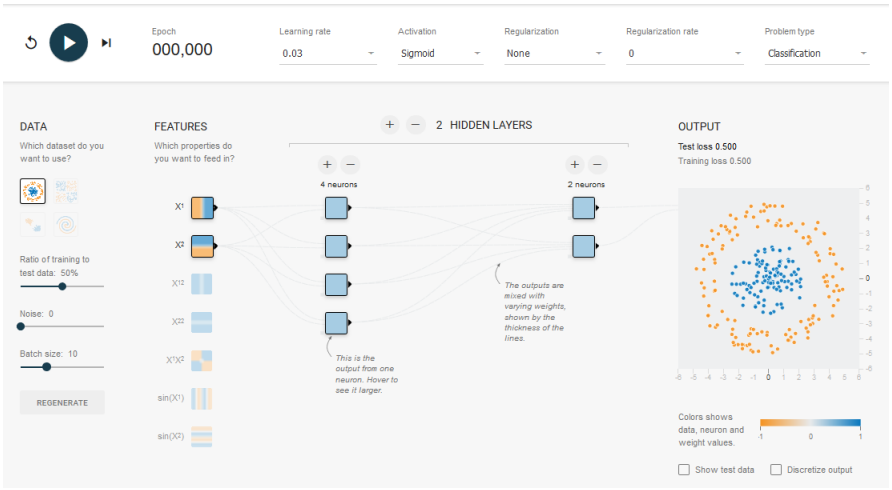
Problem B [5 points]: Backpropagation and Weight Initialization Part 2

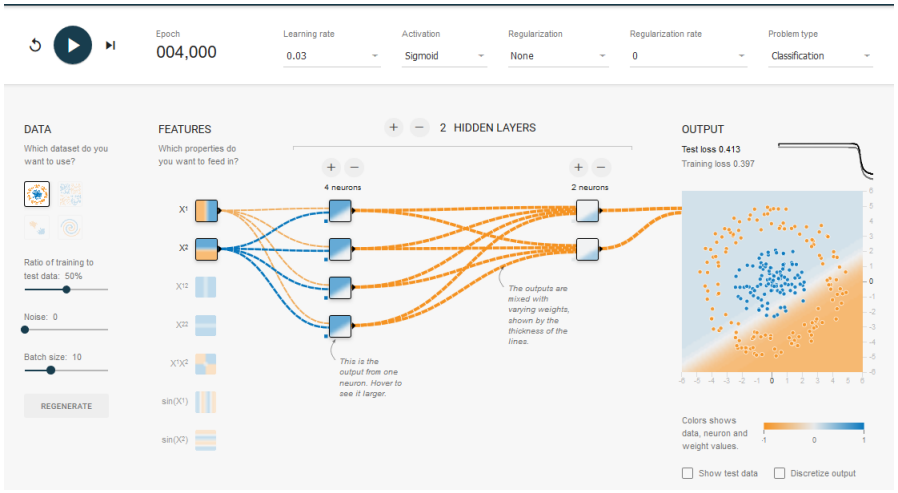
Solution B:





The neural network with nonzero-value initial weights learns the same as the corresponding neural network in Problem A, but this neural network learns at a much slower rate, since the sigmoid activation function $\sigma(x)$ approaches a derivative of 0 for large values of x.





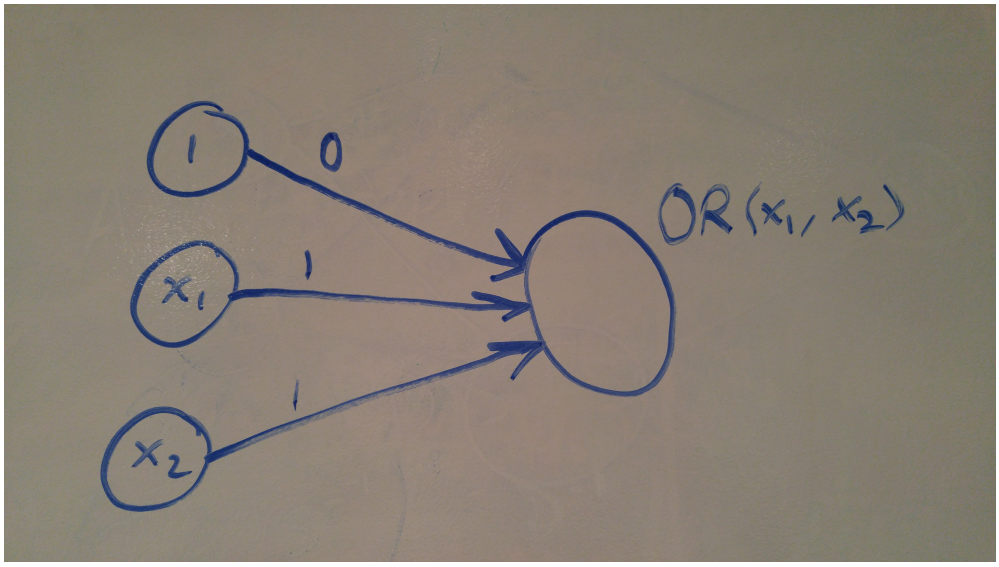
The neural network with zero-value initial weights learns a different model from that corresponding neural network in Problem A, as this model fits a line across the data, while the corresponding model does not fit any particular function to the data. This neural network learns at a slow rate, because even though the sigmoid function is nonzero at $x = 0$, the backpropagation algorithm updates weights close to 0 at a slow rate, along with the sigmoid function which converges slowly in general.

Problem C: [10 Points]

Solution C: If a fully-connected network with ReLU activations is trained using SGD looping through all the negative examples before any of the positive examples, then while the neural network is training on only negative examples first, the neural network will converge to fit only negative examples, which will cause some of the weights to become 0 and become impossible to update because ReLU activations cannot change zero-value weights. This makes the neural network unable to fit the positive examples next, because some of the weights of the neural network will be zero-value where they normally would not be to fit both the positive and negative examples (this is described by the dying ReLU problem).

Problem D [7 points]: Approximating Functions Part 1

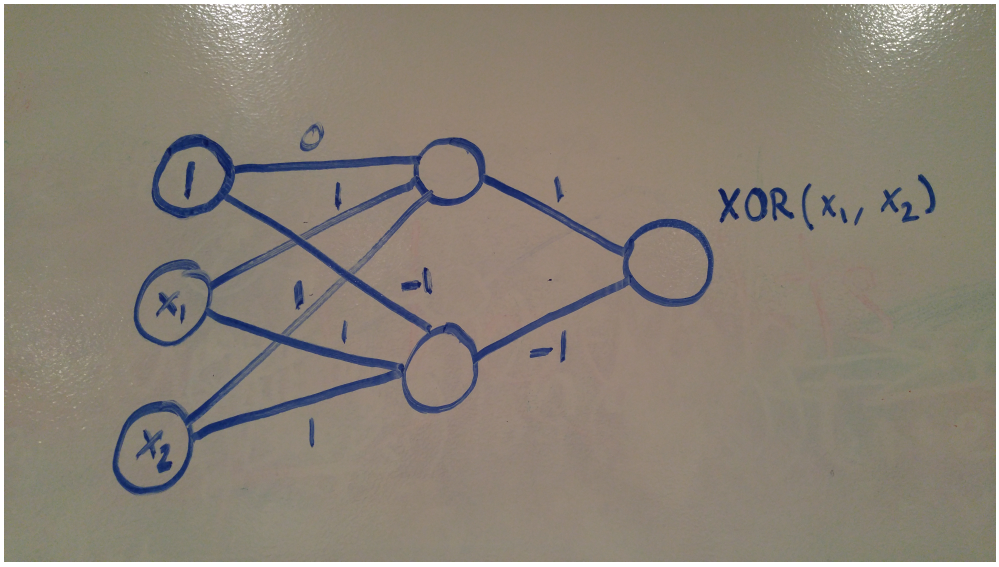
Solution D:



The above neural network produces the OR function on two $0/1$-valued inputs x_1 and x_2 using 0 hidden units, so this neural network uses the minimum number of hidden units to compute the OR function as described.

Problem E [8 points]: Approximating Functions Part 2

Solution E:



The above neural network produces the XOR function on two 0/1-valued inputs x_1 and x_2 using 2 hidden units. This neural network computes XOR by feeding the inputs and bias into the OR function (the top hidden unit), feeding the inputs and bias into the AND function (the bottom hidden unit), then subtracts the AND output from the OR output (this subtraction acts like applying NOT to the AND function output, so $(1, 1)$ returns 0 from the neural network). For the $(0, 0)$ input, the weights make AND function output -1, but the ReLU activation sets this value to 0. Since both hidden units are need for the calculation of XOR (one to compute OR and the other to compute AND), this neural network uses the minimum number of hidden units to compute the XOR function as described, so a network with fewer layers than 2 cannot compute XOR.

2 Depth vs Width on the MNIST Dataset [25 Points]

Problem A: Installation [2 Points]

Solution A:

Keras: 2.2.4

Tensorflow: protobuf-3.6.1 tensorflow-gpu-1.12.0

Problem B: The Data [1 Points]

Solution B: The input data are images of handwitten numerical digits. The dimensions of the images are 28x28 pixels. High pixel values indicate where the digit is written in the image.

Problem C: One-Hot Encoding [2 Points]

Solution C: Shape of the training input: (60000, 784).

Problem D: Modeling Part 1 [8 Points]

Solution D: (See 2_notebook for the model code.)

Test accuracy: 0.9771.

Problem E: Modeling Part 2 [6 Points]

Solution E: (See 2_notebook for the model code.)

Test accuracy: 0.9801.

Problem F: Modeling Part 3 [6 Points]

Solution F: (See 2_notebook for the model code.)

Test accuracy: 0.9836.

3 Convolutional Neural Networks [40 Points]

Problem A: Zero Padding **[5 Points]**

Solution A: One advantage of zero-padding is that zero-padding can preserve the size of the input that the convolution operates on, which can make the convolution output easier to feed into another layer of the neural network (such as a dense layer of a neural network that works best with input sizes of powers of 2).

One disadvantage of zero-padding is that zero-padding introduces noise into the convolution output, since the original data are augmented with the zero-padding to allow that convolution to preserve the size of the input with convolution.

5 x 5 Convolutions

Problem B [2 points]:

Solution B: For each filter, there are $5 * 5 * 3$ parameters plus 1 bias term for a total of 76 parameters. Since there are 8 filters, there are $76 * 8 = 608$ parameters in total.

Problem C [3 points]:

Solution C: Since there is no zero-padding, the convolution runs on a tensor (the RGB image) the size of $28 \times 28 \times 3$, and since there are 8 filters, the shape of the output tensor is $28 \times 28 \times 8$.

Max/Average Pooling

Problem D [3 points]:

Solution D: The average pooling matrices are as follows respectively to the original matrices:
$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}$, $\begin{bmatrix} 0.5 & 1 \\ 0.25 & 0.5 \end{bmatrix}$, $\begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0.5 & 0.25 \\ 1 & 0.5 \end{bmatrix}$.

Problem E [3 points]:

Solution E: The max pooling matrices are as follows respectively to the original matrices: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Problem F [4 points]:

Solution F: Pooling might be advantageous given these distortions in the dataset because pooling helps get rid of noise by smoothing the image slightly (in the case of average pooling), and by removing some pixels entirely (in the case of max pooling). Thus, pooling may help prevent a model from fitting to the noise of data, thus reducing overfitting for the model.

Problem G [20 points]:

i. [6, each 2 points]:

Solution G:

2_notebook

February 6, 2019

1 Problem 2

2 Philip Carr

Use this notebook to write your code for problem 2.

```
In [370]: import numpy as np
          import matplotlib.pyplot as plt
          %matplotlib inline
```

2.1 Dense network

Load, preprocess, and deal with the MNIST data.

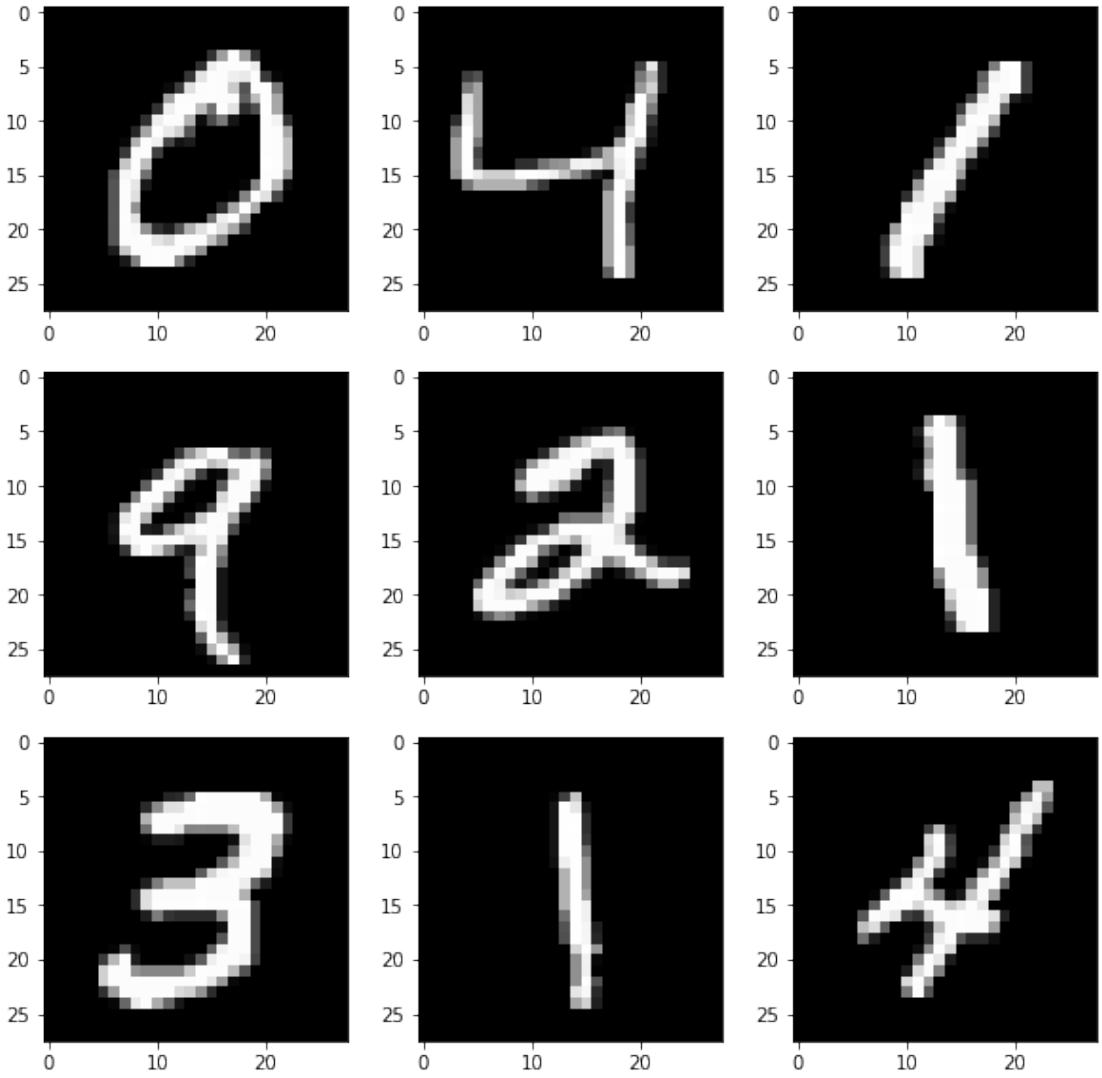
```
In [371]: # load MNIST data into Keras format
          import keras
          from keras.datasets import mnist

          (x_train, y_train), (x_test, y_test) = mnist.load_data()
```

```
In [372]: # look at the shapes
          print(x_train.shape)
          print(x_test.shape)
```

```
(60000, 28, 28)
(10000, 28, 28)
```

```
In [373]: fig = plt.figure(figsize=(10,10))
          size = 3
          for i in range(1, size*size+1):
              fig.add_subplot(size, size, i)
              plt.imshow(x_train[i], cmap=plt.get_cmap('gray'))
```



2.2 Problem C

In [374]:
```
# One-hot encode the labels.
y_train = keras.utils.np_utils.to_categorical(y_train)
y_test = keras.utils.np_utils.to_categorical(y_test)
```

In [375]:
```
# Normalize the input data.
x_train = np.divide(x_train, 255)
x_test = np.divide(x_test, 255)
```

In [376]:
```
# we must reshape the X data (add a channel dimension)
x_train = np.reshape(x_train, (len(x_train), len(x_train[0])
                                              * len(x_train[0][0])))
x_test = np.reshape(x_test, (len(x_test), len(x_test[0])
                                            * len(x_test[0][0])))
```

```
In [377]: # Shape of the training input.
          print(x_train.shape)
          print(x_test.shape)
```

```
(60000, 784)
(10000, 784)
```

2.3 Problem D

```
In [378]: from keras.models import Sequential
          from keras.layers.core import Dense, Activation, Flatten, Dropout
```

```
In [379]: ## Create the model here.
          modelD = Sequential()
          modelD.add(Dense(100))
          modelD.add(Activation('relu'))
          modelD.add(Dense(10))
          modelD.add(Activation('softmax'))
```

```
In [380]: modelD.compile(loss='categorical_crossentropy',
                        optimizer='adadelta', metrics=['accuracy'])
```

```
In [381]: fit = modelD.fit(x_train, y_train, batch_size=64, epochs=10,
              verbose=1)
```

```
Epoch 1/10
60000/60000 [==============================] - 8s 128us/step - loss: 0.3205 - acc: 0.9103
Epoch 2/10
60000/60000 [==============================] - 5s 91us/step - loss: 0.1578 - acc: 0.9553
Epoch 3/10
60000/60000 [==============================] - 5s 90us/step - loss: 0.1155 - acc: 0.9671
Epoch 4/10
60000/60000 [==============================] - 5s 90us/step - loss: 0.0931 - acc: 0.9733
Epoch 5/10
60000/60000 [==============================] - 5s 91us/step - loss: 0.0779 - acc: 0.9782
Epoch 6/10
60000/60000 [==============================] - 6s 94us/step - loss: 0.0676 - acc: 0.9806
Epoch 7/10
60000/60000 [==============================] - 6s 94us/step - loss: 0.0583 - acc: 0.9833
Epoch 8/10
60000/60000 [==============================] - 5s 90us/step - loss: 0.0514 - acc: 0.9857
Epoch 9/10
60000/60000 [==============================] - 5s 90us/step - loss: 0.0455 - acc: 0.9871
Epoch 10/10
60000/60000 [==============================] - 5s 90us/step - loss: 0.0406 - acc: 0.9888
```

```
In [382]: # Printing a summary of the layers and weights in the model.
          modelD.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_235 (Dense)            (None, 100)               78500
_________________________________________________________________
activation_234 (Activation)  (None, 100)               0
_________________________________________________________________
dense_236 (Dense)            (None, 10)                1010
_________________________________________________________________
activation_235 (Activation)  (None, 10)                0
=================================================================
Total params: 79,510
Trainable params: 79,510
Non-trainable params: 0
_________________________________________________________________
```

```
In [383]: # Printing the accuracy of the model, according to the loss function specified in mode
          score = modelD.evaluate(x_test, y_test, verbose=0)
          print('Test score:', score[0])
          print('Test accuracy:', score[1])
```

```
Test score: 0.07948249847483821
Test accuracy: 0.9771
```

2.4 Problem E

```
In [561]: ## Create the model here given the constraints in the problem
          modelE = Sequential()

          modelE.add(Dense(100))
          modelE.add(Activation('relu'))
          modelE.add(Dense(100))
          modelE.add(Activation('relu'))
          modelE.add(Dense(10))
          modelE.add(Activation('softmax'))
```

```
In [562]: modelE.compile(loss='categorical_crossentropy',
                        optimizer='rmsprop', metrics=['accuracy'])
```

```
In [563]: fit = modelE.fit(x_train, y_train, batch_size=64, epochs=20,
              verbose=1)
```

```
Epoch 1/20
60000/60000 [==============================] - 10s 166us/step - loss: 0.2702 - acc: 0.9215
Epoch 2/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.1182 - acc: 0.9646
Epoch 3/20
```

```
60000/60000 [==============================] - 6s 106us/step - loss: 0.0850 - acc: 0.9743
Epoch 4/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0663 - acc: 0.9802
Epoch 5/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.0537 - acc: 0.9833
Epoch 6/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0454 - acc: 0.9860
Epoch 7/20
60000/60000 [==============================] - 7s 110us/step - loss: 0.0394 - acc: 0.9883
Epoch 8/20
60000/60000 [==============================] - 6s 107us/step - loss: 0.0335 - acc: 0.9899
Epoch 9/20
60000/60000 [==============================] - 6s 108us/step - loss: 0.0296 - acc: 0.9909
Epoch 10/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0243 - acc: 0.9926
Epoch 11/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0211 - acc: 0.9934
Epoch 12/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.0191 - acc: 0.9945
Epoch 13/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0172 - acc: 0.9949
Epoch 14/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0151 - acc: 0.9954
Epoch 15/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.0129 - acc: 0.9959
Epoch 16/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.0109 - acc: 0.9967
Epoch 17/20
60000/60000 [==============================] - 6s 107us/step - loss: 0.0102 - acc: 0.9967
Epoch 18/20
60000/60000 [==============================] - 7s 112us/step - loss: 0.0094 - acc: 0.9970
Epoch 19/20
60000/60000 [==============================] - 6s 105us/step - loss: 0.0088 - acc: 0.9975
Epoch 20/20
60000/60000 [==============================] - 6s 106us/step - loss: 0.0081 - acc: 0.9977
```

In [564]:
```
## Printing a summary of the layers and weights in the model.
modelE.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_359 (Dense)            (None, 100)               78500
_________________________________________________________________
activation_351 (Activation)  (None, 100)               0
_________________________________________________________________
dense_360 (Dense)            (None, 100)               10100
```

```
_________________________________________________________________
activation_352 (Activation)  (None, 100)               0
_________________________________________________________________
dense_361 (Dense)            (None, 10)                1010
_________________________________________________________________
activation_353 (Activation)  (None, 10)                0
=================================================================
Total params: 89,610
Trainable params: 89,610
Non-trainable params: 0
_________________________________________________________________
```

```
In [565]: ## Printing the accuracy of the model, according to the loss function specified in mod
          score = modelE.evaluate(x_test, y_test, verbose=0)
          print('Test score:', score[0])
          print('Test accuracy:', score[1])
```

```
Test score: 0.125881357786508
Test accuracy: 0.9801
```

2.5 Problem F

```
In [632]: ## Create the model here given the constraints in the problem.
          modelF = Sequential()
          modelF.add(Dense(400))
          modelF.add(Activation('relu'))
          modelF.add(Dropout(0.1))
          modelF.add(Dense(200))
          modelF.add(Activation('relu'))
          modelF.add(Dropout(0.1))
          modelF.add(Dense(400))
          modelF.add(Activation('relu'))
          modelF.add(Dense(10))
          modelF.add(Activation('softmax'))
```

```
In [633]: modelF.compile(loss='categorical_crossentropy',
                        optimizer='rmsprop', metrics=['accuracy'])
```

```
In [634]: fit = modelF.fit(x_train, y_train, batch_size=64, epochs=20,
              verbose=1)
```

```
Epoch 1/20
60000/60000 [==============================] - 12s 207us/step - loss: 0.2322 - acc: 0.9296
Epoch 2/20
60000/60000 [==============================] - 8s 141us/step - loss: 0.1086 - acc: 0.9684
Epoch 3/20
60000/60000 [==============================] - 8s 132us/step - loss: 0.0872 - acc: 0.9758
```

```
Epoch 4/20
60000/60000 [==============================] - 8s 128us/step - loss: 0.0727 - acc: 0.9805
Epoch 5/20
60000/60000 [==============================] - 8s 131us/step - loss: 0.0678 - acc: 0.9834
Epoch 6/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0631 - acc: 0.9848
Epoch 7/20
60000/60000 [==============================] - 8s 128us/step - loss: 0.0655 - acc: 0.9848
Epoch 8/20
60000/60000 [==============================] - 8s 126us/step - loss: 0.0623 - acc: 0.9858
Epoch 9/20
60000/60000 [==============================] - 8s 131us/step - loss: 0.0585 - acc: 0.9866
Epoch 10/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0596 - acc: 0.9874
Epoch 11/20
60000/60000 [==============================] - 8s 129us/step - loss: 0.0597 - acc: 0.9880
Epoch 12/20
60000/60000 [==============================] - ETA: 0s - loss: 0.0606 - acc: 0.988 - 8s 126us/st
Epoch 13/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0600 - acc: 0.9886
Epoch 14/20
60000/60000 [==============================] - 8s 129us/step - loss: 0.0572 - acc: 0.9889
Epoch 15/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0569 - acc: 0.9897
Epoch 16/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0571 - acc: 0.9896
Epoch 17/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0570 - acc: 0.9903
Epoch 18/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0563 - acc: 0.9908
Epoch 19/20
60000/60000 [==============================] - 8s 127us/step - loss: 0.0647 - acc: 0.9902
Epoch 20/20
60000/60000 [==============================] - 8s 126us/step - loss: 0.0541 - acc: 0.9915
```

In [635]:
```
## Printing a summary of the layers and weights in the model.
modelF.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_446 (Dense)            (None, 400)               314000
_________________________________________________________________
activation_429 (Activation)  (None, 400)               0
_________________________________________________________________
dropout_120 (Dropout)        (None, 400)               0
_________________________________________________________________
```

```
dense_447 (Dense)            (None, 200)               80200
_________________________________________________________________
activation_430 (Activation)  (None, 200)               0
_________________________________________________________________
dropout_121 (Dropout)        (None, 200)               0
_________________________________________________________________
dense_448 (Dense)            (None, 400)               80400
_________________________________________________________________
activation_431 (Activation)  (None, 400)               0
_________________________________________________________________
dense_449 (Dense)            (None, 10)                4010
_________________________________________________________________
activation_432 (Activation)  (None, 10)                0
=================================================================
Total params: 478,610
Trainable params: 478,610
Non-trainable params: 0
_________________________________________________________________
```

```
In [636]: ## Printing the accuracy of the model, according to the loss function specified in mod
          score = modelF.evaluate(x_test, y_test, verbose=0)
          print('Test score:', score[0])
          print('Test accuracy:', score[1])
```

```
Test score: 0.13775982027360675
Test accuracy: 0.9836
```