Ph 22 Computational Physics **Winter 2020**

Homework 4
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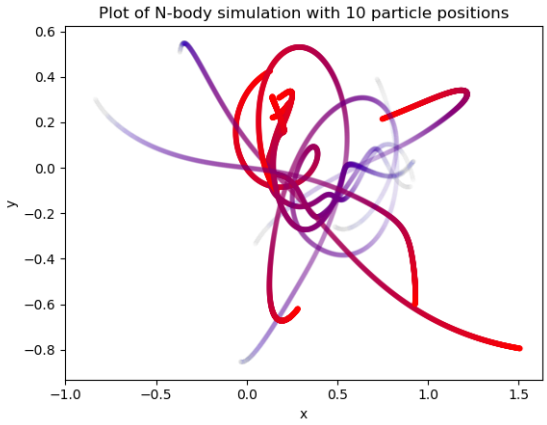
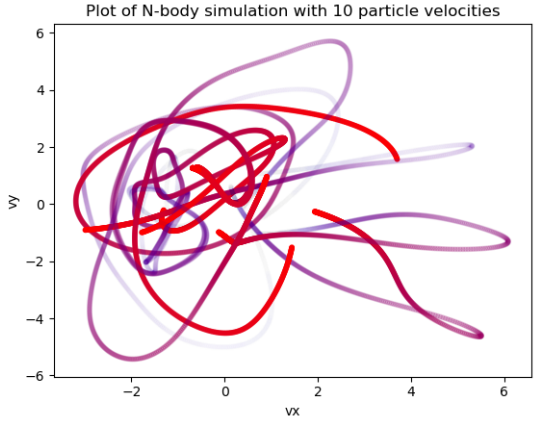
For all orbit plots, color changes from blue to red as time advances.

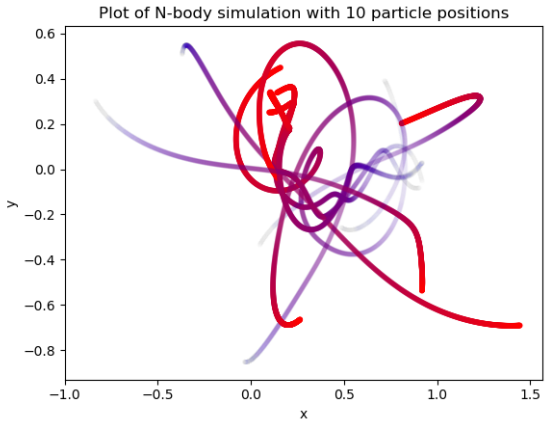
1. (See code and program output at the bottom of this document. To plot the quadtree in the program, set plot_quad_tree_node to True in the gravitational_accelerations function in the n_body_simulation_driver function in the ph22hw4.py file.)

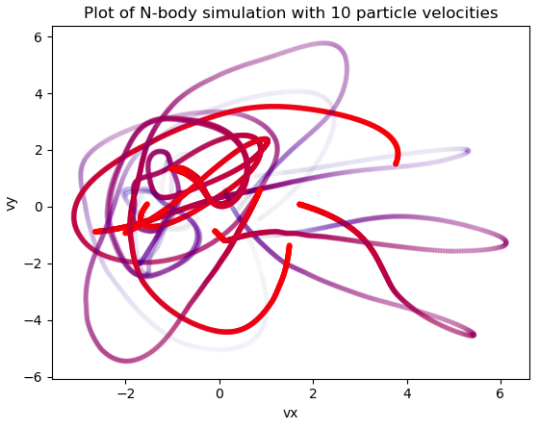
2. Below are plots of the evolution of the $\theta = 0$ system.





Below are plots of the evolution of the $\theta = 1$ system.

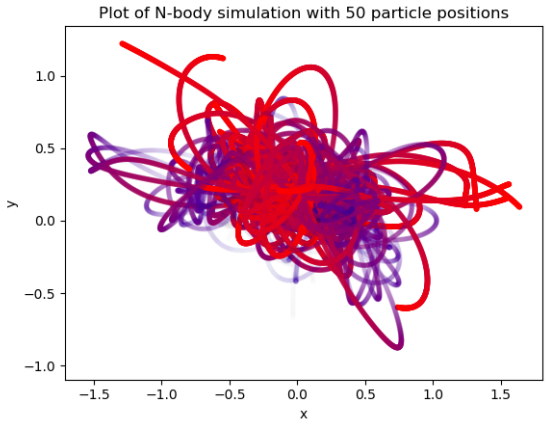


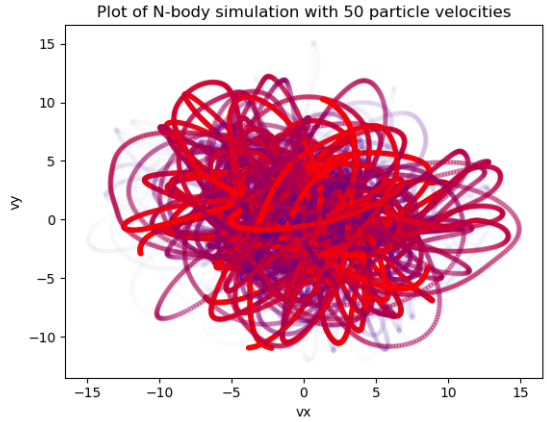


Comparing the evolution of the two systems above, the trajectories of the particles in the $\theta = 1$ case system are slightly different from those of the $\theta = 0$ case system, though not by much. The qualitative differences between $\theta < 1$ case systems and $\theta = 0$ case systems are even less apparent than those shown above since the $\theta < 1$ case system model the $\theta = 0$ system even more closely.

As θ increases from 0 to 1, the runtime decreases overall (see program output at the bottom of the document), although there are local increases from $\theta = 0.2$ to $\theta = 0.4$, and from $\theta = 0.8$ to $\theta = 1.0$.

The largest cluster that can be watched evolving with an opening angle of 0.5 is a 50-particle cluster (with each timestep taking about 0.05236 seconds for 1000 timesteps for a total of 52.36 seconds of runtime). The plots of the evolution of this system are shown below.





3. The approximation error was based on the total energy of the N-body system. Given total energy values E_θ and E_0, the approximation error formula is

$$\sqrt{\sum_{i=0}^{N-1} \frac{(E_\theta[i] - E_0[i])^2}{N E_0[i]^2}} * 100\%,$$

which is the root mean squared (rms) error of E_θ with respect to E_0, with each squared error term normalized by the respective value of E_0 squared. Normalization was used here in this form to significantly decrease the contribution of extremely large differences in total energy resulting from only one of E_θ or E_0 at a time having an extremely large-magnitude negative total energy, because at times an N-body system will have large downward spikes in total energy during the system's evolution (from particles getting very close to each other significantly increasing the system's gravitational potential energy).

For the 10-body system shown in the program output at the bottom of this document, $\theta = 0.6$ results in an approximation error of about 1%.

Program output (for reference):
Ph 22 Homework 4 Program Printout
Problems 2 and 3
Largest cluster that can be simulated in a reasonable amount of time (for opening angle of 0.5)
N-body approximation simulation with given parameters:

n_particles = 50
v_0 = 0.1
opening angle (theta) = 0.5
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 52.36203479766846 seconds to run simulation

Varying opening angle with constant cluster size
N-body approximation simulation with given parameters:
n_particles = 10
v_0 = 0.1
opening angle (theta) = 0.0
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 7.3840532302856445 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 0.0%

N-body approximation simulation with given parameters:
n_particles = 10
v_0 = 0.1
opening angle (theta) = 0.2
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 4.416036605834961 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 0.07250142950507%

N-body approximation simulation with given parameters:
n_particles = 10
v_0 = 0.1
opening angle (theta) = 0.4
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 4.528039216995239 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 0.3307714746789062%

N-body approximation simulation with given parameters:
n_particles = 10

v_0 = 0.1
opening angle (theta) = 0.6000000000000001
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 4.451035261154175 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 1.0556371110286544%

N-body approximation simulation with given parameters:
n_particles = 10
v_0 = 0.1
opening angle (theta) = 0.8
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 3.7759923934936523 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 2.354982975836179%

N-body approximation simulation with given parameters:
n_particles = 10
v_0 = 0.1
opening angle (theta) = 1.0
time_step = 0.001
n_time_steps = 1000
Progress:
done
Took 3.793023109436035 seconds to run simulation
Running theta = 0 simulation
Progress:
done
Approximation error: 13.15107509882217%