Ph 22 Computational Physics **Winter 2020**

Homework 2
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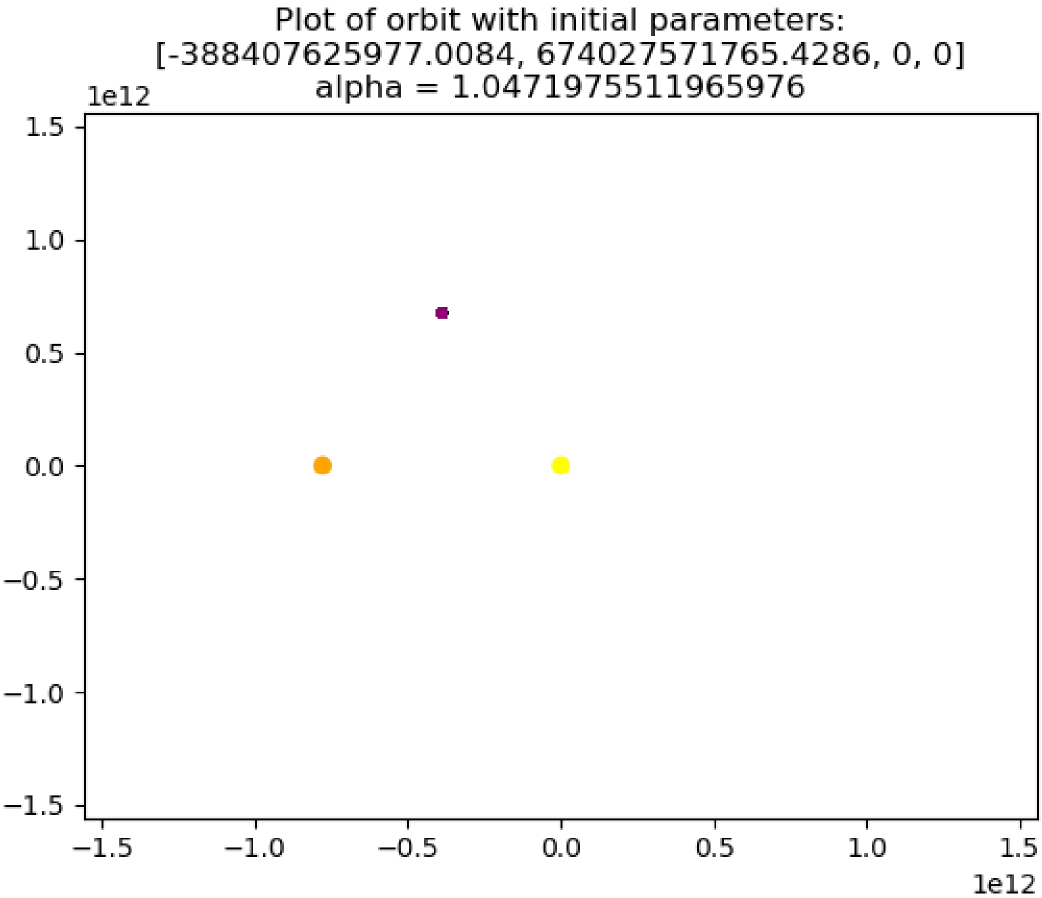
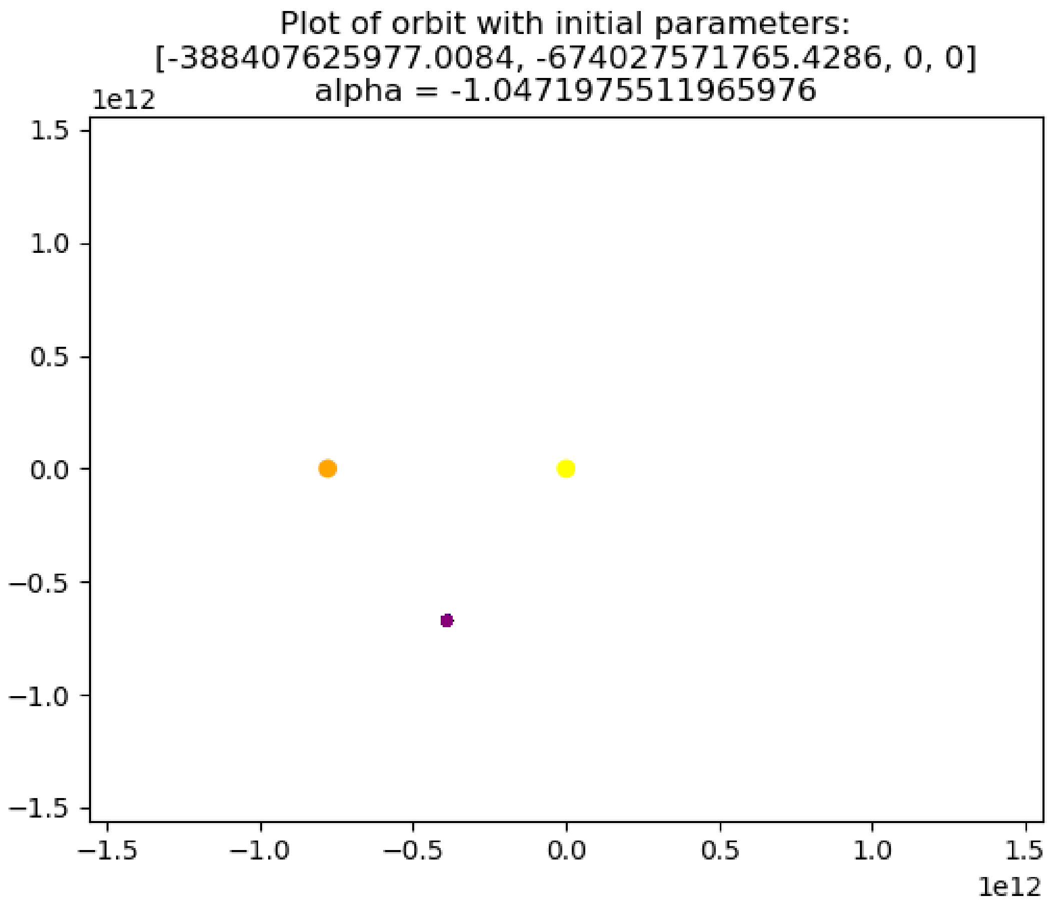
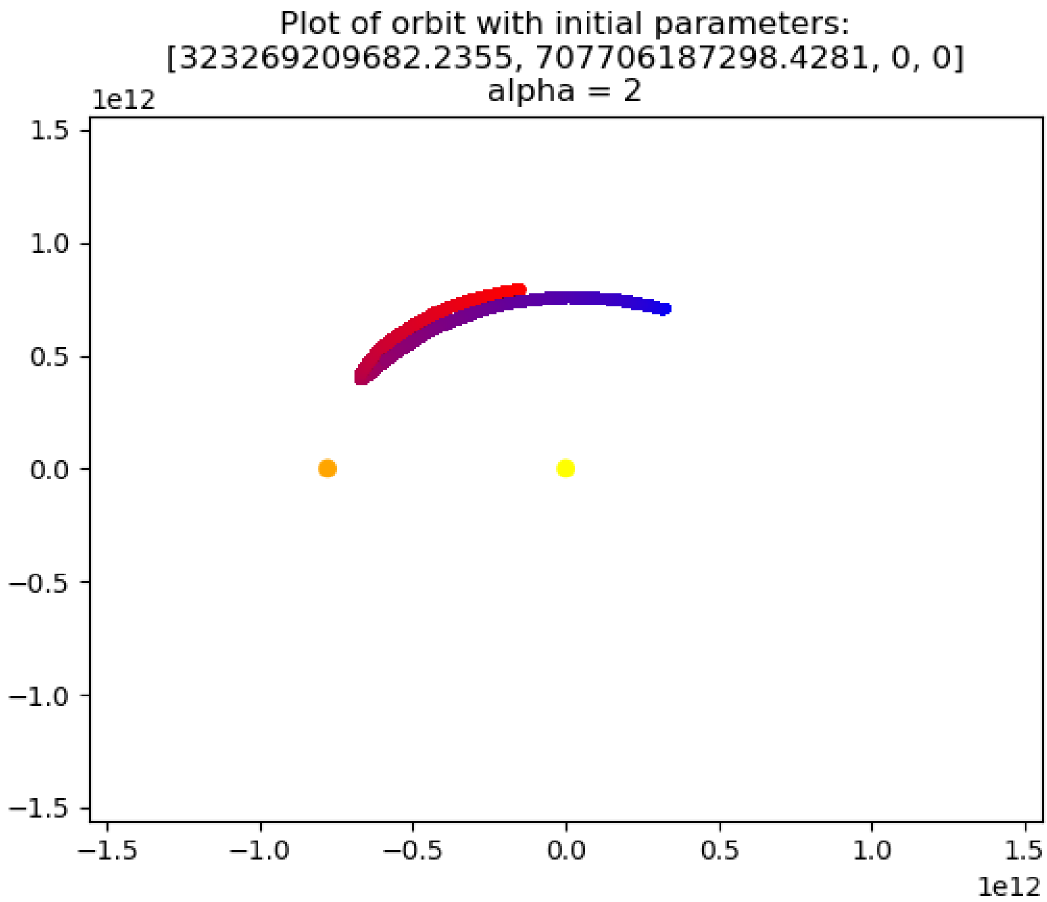
For all orbit plots, color changes from blue to red as time advances.

1. Orbit plots:

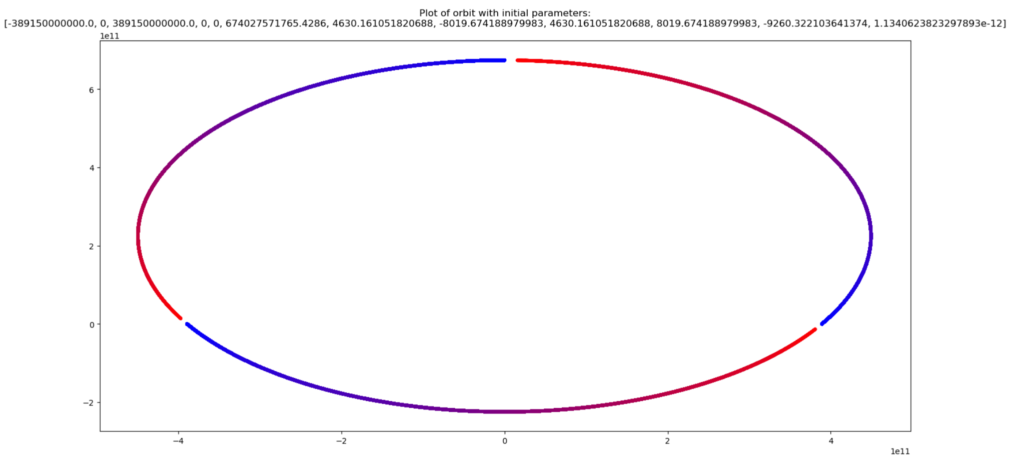



Plot of orbit with initial parameters:
[-388407625977.0084, -674027571765.4286, 0, 0]
alpha = -1.0471975511965976
1e12
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1e12



As seen in the above three plots, the Lagrange points specified by $\alpha = \pi/3$ and $\alpha = -\pi/3$ are stable.

2. Orbit of stable equilateral triangle Lagrange solution:



Equilateral Traingle Orbit Velocity Equation derivation:
Each mass $M_1 = M_2 = M_3 = M$ orbits the center of mass of the three-body system distance r away from m: Let the masses M_1, M_2, and M_3 be located in the shape of an equilateral triangle such that M_2 and M_3

are vertically colinear (along the y-direction) and M_1 is to the right of M_2 and M_3 (in the $+x$-direction). Here, the forces in the y-direction of M_2 and M_3 acting on M_1 are cancelled out, since $M_2 = M_3$, M_2 and M_3 are the same magnitude of distance away from M_1, and M_2 and M_3 are equally and oppositely as far away in the y-direction from M_1's y-position. Therefore, the only nonzero force on M_1 is in the x-direction here. Therefore,

$$F_x = M_1 a = -\frac{GM_2M_1r}{d^3} - \frac{GM_3M_1r}{d^3} = -\frac{2GM_1^2r}{d^3}$$

(a here is centripetal acceleration, since M_1 is in a circular orbit.)

$$\implies \frac{M_1v^2}{r_c^2} = \frac{2GM_1^2r}{d^3}$$

$$\implies v^2 = \frac{2GMrr_c}{d^3}$$

$$\implies v = \sqrt{\frac{2GMrr_c}{d^3}}$$

(r is the distance from M_1 to M_2 and equal to the distance from M_1 to M_3 in the x direction, which is $r = d\cos(\frac{\pi}{6}) = \frac{\sqrt{3}d}{2}$. r_c is the distance from M_1 to the center of mass of the system about which M_1 orbits, which is at the barycentric center of mass of the equilateral triangle, in which each mass in the triangle is away from this center of mass by two-thirds the distance from the mass' location to the opposite side of the triangle by the perpendicular line segment that bisects the opposite side by from the mass' location. Therefore, M_1 is colinear with the center of mass of the system along the x-direction, $r_c = \frac{2}{3}r$.)

$$\implies v = \sqrt{\frac{2GMr(\frac{2}{3}r)}{d^3}} = \sqrt{\frac{\frac{4}{3}GMr^2}{d^3}} = \sqrt{\frac{\frac{4}{3}GM(\frac{3}{4}d^2)}{d^3}} = \sqrt{\frac{GM}{d}}. \square$$

3. Figure-8 orbit: