The least-square method for
Bermudan swaptions
Advanced topics in Finance

1 Bermudan swaption pricing

Interest-bearing liabilities and interest rate derivates are the financial asset class with by far the highest annual trading volume in the bilateral market. This can be attributed to the fact that most corporates participating in the market need to attract funding to finance their activities. These corporates are therefore exposed to interest rate risk, which can be hedged by trading interest rate derivatives, such as swaps, caps or swaptions. Not surprisingly, the field of research evolving around interest rate modelling is therefore very active.
Depending on the model of choice, most of the vanilla interest rate derivatives admit analytical or semi-closed form valuation routines. Such expressions are important for efficient model calibration or for risk-management purposes. For exotic derivatives however, practitioners typically resort to numerical approximations to price a contract. An example of such a product, which is frequently traded in the market, is the Bermudan swaption. A vanilla swap settles the exchange of floating versus fixed interest rate payments between two parties. A swaption is a European option written on a swap, which gives the holder the right to enter a swap at a future date at a pre-specified fixed rate. A Bermudan swaption gives the holder the right to enter the underlying swap at any of the interest payment dates of the swap tenor. Due to the path-dependent nature of the latter, the valuation of a Bermudan is complex.
This topic concerns the least-square method (LSM) popularized by F. Longstaff and E. Schwartz [3], applied in an interest rate modelling environment. In their paper, they proposed a Monte Carlo (MC) based regression method to approximate the value of callable derivatives. To date, this remains one of the most commonly used methods for this purpose.

1.1 Project description

For this project you will familiarize yourself with interest rate (IR) modelling and short-rate models in particular. A general and thorough introduction to IR models and IR derivatives can be found in [1] and [4]. A classic method for pricing Bermudan-style options in a MC environment is the least-square method (LSM) popularized by F. Longstaff and E. Schwartz [3]. Their paper a good introduction to the valuation of callable derivatives.
The first objective is to set up a short-rate model to simulate the instantaneous interest rate r_t. The Hull-White model [2] is a good starting point. You may choose any programming language.
Second you will implement a numerical pricing routine for a Bermudan swaption. The LSM routine presented in [3] will be your starting point. The main objective will be to convert the methodology to an IR framework. Test your implementations and experiment with the algorithm. Don't forget to consider the greeks.

References

[1] Damiano Brigo, Fabio Mercurio, et al. *Interest rate models: theory and practice*. Vol. 2. Springer, 2001.

[2] John Hull and Alan White. "Pricing interest-rate-derivative securities". In: *The review of financial studies* 3.4 (1990), pp. 573–592.

[3] Francis A Longstaff and Eduardo S Schwartz. "Valuing American options by simulation: a simple least-squares approach". In: *The review of financial studies* 14.1 (2001), pp. 113–147.

[4] Antoon Pelsser. *Efficient methods for valuing interest rate derivatives*. Springer Science & Business Media, 2000.