Polarization Laboratory Exercises

1) Malus' Law: $I = \frac{1}{2} I_o \cos^2(\theta)$ θ is angle between the polarizers.

$$20^\circ = \theta = 90^\circ - 70^\circ \qquad I_o = 43 \, W/m^2$$

$$I = \frac{1}{2} 43 \frac{W}{m^2} \cos^2(20^\circ) = \frac{1}{2}\left(37.97 \frac{W}{m^2}\right) = \boxed{18.98 \frac{W}{m^2} = I}$$

2) $E_H = 2.3 E_V$

a)
$$I^2 = I_V^2 + I_H^2$$

$$I^2 = E_V^2 + (2.3E_V)^2 \qquad I = \sqrt{E_V^2(1+(2.3)^2)} = E_V\sqrt{6.29}$$

When the glasses are on, we have only E_V.

So the fraction of transmitted light will be $\frac{E_V}{E_V\sqrt{6.29}} = \boxed{\frac{1}{\sqrt{6.29}}}$

b) Similarly: $\frac{2.3E_V}{E_V\sqrt{6.29}} = \boxed{\frac{2.3}{\sqrt{6.29}}}$

3) We don't have the figure mentioned in the problem, but Malus' law states:

$$I = \frac{1}{2} I_o \cos^2(|\theta_2 - \theta_1|)\cos^2(|\theta_3 - \theta_2|)$$

$$\frac{I}{I_o} = \frac{1}{2}\cos^2[|90^\circ - \theta_1|]\cos^2[|\theta_3 - 90^\circ|]$$

So presumably, if we have θ_1 and θ_3 we can solve this problem

$$\boxed{\frac{I}{I_o} = \frac{1}{2}\sin^2(\theta_1)\sin^2(\theta_3)}$$

4) a) This scenario is easily achievable with two polarizers.

One oriented at any angle from $(0^\circ, 90^\circ)$ and another oriented at 90° will achieve the desired polarization. $\boxed{\text{2 Polarizers}}$

b) This is a little more tricky...

With two polarizers, one at 45° and one at 90°, we expect only 25 % to be transmitted. So with say 6 polarizers each at 15° to one another:

$$I = I_o(\cos^2(15^\circ))^6 = 0.66 I_o \qquad \boxed{\text{6 Polarizers } (\theta = 15^\circ)}$$

6) $I = I_0 \left(\cos^2(30)\right)^4 = \boxed{0.32\, I_0}$

7) $\frac{1}{3} I_0 = \frac{1}{2} I_0 \cos^2(x)$

$\frac{2}{3} = \cos^2(x)$

$x = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ $\boxed{x = 35.3^\circ}$

8) $I = \frac{1}{2} I_0 \cos^2(45-0) \cos^2(90-45) = \frac{1}{2} I_0 \cos^4(45)$

$\frac{I}{I_0} = 0.125$ or $\boxed{\frac{I}{I_0} = \frac{1}{8}}$

9) $I = \frac{1}{2} I_0 \cos^2(22.5^\circ)$ $\frac{I}{I_0} = \frac{1}{2}\cos^2(22.5^\circ) = 0.43$

$\frac{I}{I_0} = 0.43$ or $\boxed{43\%}$

10) $I_0 = 1000\ W/m^2$

$I = \frac{1}{2} I_0 \cos^2(40^\circ) = 500\ W/m^2 \cos^2(40^\circ) = \boxed{293\ W/m^2}$

with a polarizer between them:

$I = \frac{1}{2} I_0 \cos^2(20^\circ)\cos^2(20^\circ) = 500 \frac{W}{m^2} \cos^2(20^\circ)\cos^2 20^\circ)$

$= 500 \frac{W}{m^2} \cos^4(20^\circ) = \boxed{390.\ W/m^2}$

11) Brewster angle given by:

$\tan(\theta_b) = \frac{n_2}{n_1}$ since it comes from air, $n_1 = 1$

$n_2 = 1.52$

so $\theta_b = \arctan(1.52)$

$\boxed{\theta_b = 56.7^\circ}$