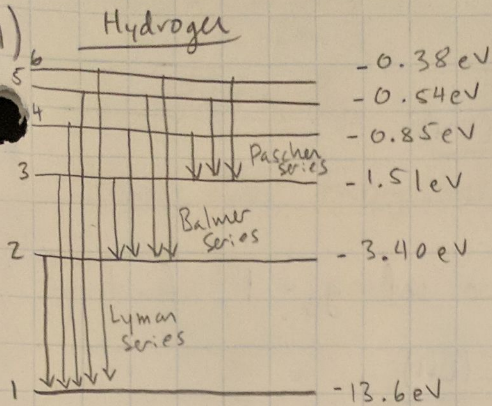
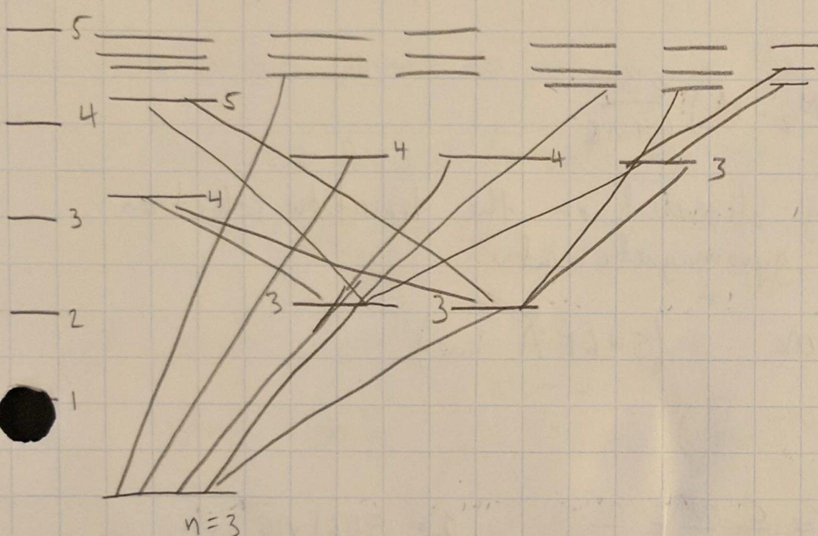
Fabry-Perot Interferometer and Zeeman Effect Laboratory Exercises

1) Hydrogen

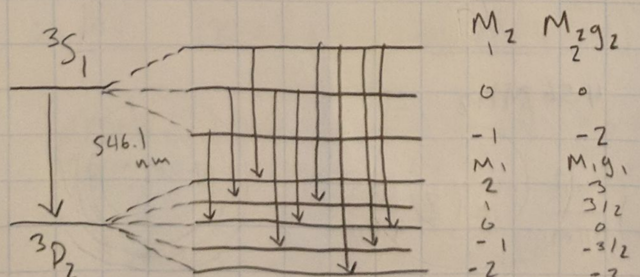


Spectral lines appear due to the emission of photons when electrons transition from a higher energy state to a lower energy state due to conservation of energy. The photon's energy is related to it's wavelength, so the transition energy is associated with a color for the resultant photon.

Sodium



2) Magnetic fields will split the spectral lines:



$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

for $^3S_1$: $J = 1$, $L = 0$, $S = 1$

$$g_{^3S_1} = 1 + \frac{1(1+1) - 0(0+1) + 1(1+1)}{2(1(1+1))}$$

$$= 1 + \frac{2+2}{2\,2} = 1 + \frac{4}{4} = 1 + 1 = \boxed{2}$$

for $^3P_2$: $J = 2$, $L = 1$, $S = 1$

$$g_{^3P_2} = 1 + \frac{2(2+1) - 1(1+1) + 1(1+1)}{2(2(2+1))}$$

$$= 1 + \frac{6 - 2 + 2}{12} = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$

3) Frequency difference for spectral lines:

$$\Delta\nu = (M_2 g_2 - M_1 g_1)\frac{eB}{4\pi mc}$$

$$g = g_L \cdot \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)} + g_s \cdot \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

for electron, $g_s \sim 2$, $g_L = 1$. In this scenario, we set $g_s = 1$, finding

$$g = \frac{J(J+1) - S(S+1) + L(L+1)}{2(J(J+1))} + \frac{J(J+1) + S(S+1) - L(L+1)}{2(J(J+1))}$$

$$= \frac{2J(J+1)}{2J(J+1)} = 1$$

thus $g_1 = g_2 = 1$ and $\Delta\nu = (M_2 - M_1)\frac{eB}{4\pi mc}$

Thus the Zeeman effect is only dependent on the transition selection rules for the atom, not the gyromagnetic ratio.

4) $\frac{1}{2}mv^2 = \frac{3}{2}kT$ $\quad v = \sqrt{\frac{3kT}{m}}$ $\quad T = 500K$ $\quad$ (5461 Å line)

$$\nu' = \nu\left(1 + \frac{v}{c}\right)$$

$$\nu' - \nu = \nu\frac{v}{c} \qquad \Delta\nu = \nu\frac{v}{c} = \frac{c}{\lambda}\frac{v}{c} = \frac{v}{\lambda} \qquad \lambda = 5461 \times 10^{-10} m$$

$$v = \sqrt{\frac{3(500K)(1.38 \times 10^{-23} J/K)}{(200.59)amu\,(1.66 \times 10^{-27} kg/amu)}} \cong 249 \frac{m}{s}$$

$$\Delta\nu = \frac{249 \frac{m}{s}}{5461 \times 10^{-10} m} = 4.56 \times 10^{8} Hz = 456 MHz$$

For Zeeman splitting: $\Delta\nu = (M_2 g_2 - M_1 g_1)\frac{eB}{4\pi m} = 2\frac{eB}{4\pi m}$

$$\Delta\nu = \frac{(1T)(1.602 \times 10^{-19} C)}{2\pi(9.1 \times 10^{-31} kg)} = 2.79 \times 10^{10} Hz$$

$$= 28 GHz$$

This is several orders of magnitude away from the doppler shift expectations so the Zeeman effect should be very obvious.

5)

$$\Delta E = \frac{e\hbar B}{2m}\left(M_2 g_2 - M_1 g_1\right)$$

additionally, we know $\Delta E = hc\frac{\Delta\lambda}{\lambda^2}$

And the Fabry-Perot Interferometer gives us: $\Delta\lambda = \lambda^2\frac{\delta D}{2D\Delta D}$

with some re-arranging, we find simply:

$$\frac{e}{m} = \frac{4\pi Z}{\cancel{\hbar}B} hc \frac{\cancel{\lambda^2}}{\cancel{\lambda^2}}\frac{\delta D}{2D\Delta D} \Rightarrow \boxed{\frac{e}{m} = \frac{2\pi c}{(M_2 g_2 - M_1 g_1)Bd}\left(\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2}\right)}$$

6) Non-colinear points do not lie on the same line.

Thus, no two points will be the same:

$(x_1, y_1), (x_2, y_2), (x_3, y_3)$

A circle's equation is found as: $x^2+y^2=r^2$

Whoes center is: $(x-a), (y-b)$ where (a,b) is the center.

So if these points satisfy only one circle:

$((x_1-a),(y_1-b)), ((x_2-a),(y_2-b)), ((x_3-a),(y_3-b))$

and: $(x_1-a)^2+(y_1-b)^2 = (x_2-a)^2+(y_2-b)^2 = (x_3-a)^2+(y_3-b)^2$

Which impies they all have some radius r, and a common center (a,b), thus

Three colinear points satisfy one and only one circle.

7) Given that when θ becomes small, no diffrence is observed, and will dissapear if $\theta = \pi/2$,

we gather

$$\delta\theta \propto \cos\theta.$$

We expect $\delta\theta$ to be directly proportional to both the length of the etalon 'd' and the index of refraction.

To remain dimensionless, we require a factor of $\frac{1}{\text{length}}$. and since the only unused length parameter is the wavelength, we have

$$\delta\theta \propto \frac{n_f d}{\lambda_0} \cos\theta$$

finally due to the circular nature of the rings being projections of conical sections, our normalization is found by integrating over ϕ and $\sin\theta$ to obtain 4π.

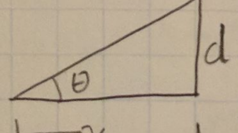
$$\boxed{\delta\theta = 4\pi n_f \frac{d}{\lambda_0} \cos\theta}$$

8) We require destructive interference:

$$2nd = \frac{\lambda}{2}$$

$$d = \frac{\lambda}{4n} = \frac{589.3 \times 10^{-9}\,m}{4(1.38)} = \boxed{1.067 \times 10^{-5}\,cm}$$

9)



we have: $2nd\cos\theta = (2n+1)\frac{\lambda}{2}$

$$d = \frac{(2n+1)\lambda}{n2\cos\theta 2} = \frac{7}{2}(630) = 2.205 \times 10^{-3}\,m$$

$$= \boxed{2.2\,mm}$$

10)

a) Resolving Power: $R = \frac{\lambda}{\Delta\lambda_m} = \frac{656 \times 10^{-9} m}{0.14 \times 10^{-10} m} = 4.8 \times 10^4$

$$\boxed{R = 4.8 \times 10^4}$$

b)

$$F = \frac{\pi r}{1 - r^2} = \frac{\pi (0.9)}{1 - (0.9)^2} = \frac{2.83}{.19} = 14.9$$

$$m = \frac{R}{F} = \frac{4.8 \times 10^4}{14.9} = 3.2 \times 10^3$$

$$m = \frac{2d}{\lambda} \Rightarrow d = \frac{m\lambda}{2} = \frac{(3.2 \times 10^3)}{2} \cdot 6.56 \times 10^{-9}$$

$$= \boxed{1.05 \times 10^{-5} m}$$

11) a) Order $m = \frac{2(\text{distance traveled})}{\lambda}$

$$= \frac{2(0.00325 cm)}{6.5 \times 10^{-7}} = 100 \qquad \boxed{m = 100}$$

b) Maxima: $2d \cos\theta = (m' + \frac{1}{2})\lambda$

$\frac{2d}{\lambda} = 100 = m$ from above, thus $\cos\theta = \frac{(m' + \frac{1}{2})}{100}$

$m = 99, 98, 97$, and 96

$$\cos^{-1}\left(\frac{99 + 1/2}{100}\right) = 5.7$$

$$\cos^{-1}\left(\frac{98 + 1/2}{100}\right) = 8.8$$

$$\cos^{-1}\left(\frac{97 + 1/2}{100}\right) = 12.8$$

$$\cos^{-1}\left(\frac{96 + 1/2}{100}\right) = 15.2$$