Michelson Interferometer

1. Objective: Learn about the Michelson Interferometer. Study the difference between the Michelson Interferometer, Fabry-Perot interferometer and grating spectrometer. Using the Michelson interferometer determine the smoothness of a mirror surface; determine the refractive index of a medium and the coherence length of a light source.

2. Preparatory Activity (Two sessions of three hours):

1. Draw an optical ray diagram of the experimental setup. Mark the distances of all optical elements.

2. Derive the expression for interference in a Michelson Interferometer.

3. What are the conditions for the light to show interference constructively or destructively. What is the difference between light from a “light bulb” and from LASER?

Solve the following problems (you can use the "Fundamentals of Physics" by Halliday and Resnick for background material).

1. If mirror M2 in a Michelson interferometer is moved through 0.233 mm, a shift of 792 bright fringes occurs. What is the wavelength of the light producing the fringe pattern?

2. A thin film with index of refraction n=1.40 is placed in one arm of a Michelson interferometer, perpendicular to the optical path. If this causes a shift of 7.0 bright fringes of the pattern produced by light of wavelength 589 nm, what is the film thickness?

3. The element sodium emits light at two wavelengths 589.1 nm and 589.59 nm. Light from sodium is being used in a Michelson interferometer. Through what distance must mirror M2 be moved if the shift in the fringe pattern for one wavelength is to be 1.00 fringes more than the shift in the fringe pattern for the other wavelength.

4. Light of wavelength 700.0nm is sent along a route of 2000nm. The route is then filled with a medium having an index of refraction of 1.400. In degrees, by how much does the medium phase shift the light? Give (a) the full shift and (b) the equivalent shift that has a value less than 360°.

5. Light of wavelength λ is used in a Michelson interferometer. Let x be the position of the movable mirror, with x=0 when arms has equal length $d_2 = d_1$. Write an expression for the intensity of the observed light as a function of x, letting I_m to be the maximum intensity.

6. When an electron moves through a medium at a speed exceeding the speed of light in that medium, the electron radiates electromagnetic energy (the Cerenkov effect). What

minimum speed must an electron have in a liquid with index of refraction 1.54 in order to radiate?

7. By the late 1800s, most scientists believed that light (any electromagnetic wave) required a medium in which to travel, that it could not travel through vacuum. One reason for this belief was that any other type of wave known to the scientist requires a medium. For example sound wave can travel through air, water or ground but not through vacuum. Thus, reasoned the scientists, when light travels from the Sun or any other star to earth, it can not be traveling through vacuum; instead, it must be traveling through a medium that fills all the space and through which Earth slips. Presumably, light has a certain speed c through this medium, which is called aether (or ether).

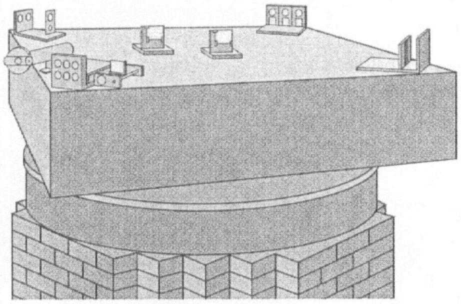
In 1887 Michelson and Edward Morely used a version of Michelson's interferometer to test for the effects of aether on the travel of light within the device. Specifically, the motion of device through aether as earth moves around the sun should affect the interference pattern produced by the device. Scientist assumed that the Sun is approximately stationary in aether; hence the speed of interferometer through aether should be Eatrth's speed v about the Sun.

The figure a shows the basic arrangement of mirrors in 1987 experiment. The mirrors were mounted on a heavy slab that was suspended on a pool of mercury so that the slab could be rotated smoothly about a vertical axis. Michelson and Morely wanted to monitor the interference pattern as they rotated the slab, changing the orientation of the arms relative to the motion through aether. A fringe shift in the interference pattern during the rotation would clearly signal the presence of aether.

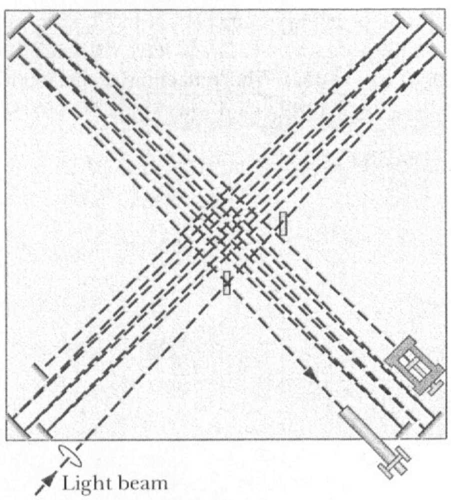
The figure b, an over head view of the equipment, shows the path of the light. To improve the possibility of the fringe shift, the light was reflected several times along the arms of the interferometer, instead of only once along each arm as indicated in a basic interferometer. This repeated reflection increased an effective length of each arm to about 10 m. Instead of the added complexity, the interferometer used in Michelson Morley experiment functions just like a simple interferometer, so we can use the basic configuration in our discussion here by merely taking the arm length d_1 and d_2 to be 10 m each.

Let us assume that there a aether through which light speed c. The figure c shows a side view of the arm of length d1 from the aether reference frame as the interferometer moves rightward through it with velocity v. (For simplicity, the beam splitter M of the Figure is drawn parallel to the mirror M1 at the far end of the arm.) The figure d shows the arm just as a particular portion of the light (represented by a dot) begins its travel along the arm. We shall follow this light to find the path length along the arm.

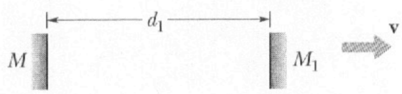
(a) As the light moves at speed c rightward through aether and towards mirror M1, that mirror moves rightward at speed v. Figure e shows the position of M and M1 when the light reaches M1, reflecting there. The light now moves leftward through aether at a speed c while M moves rightward. Figure f shows the positions M and M1 when the light has returned to M. Show that the total time of travel for this light, from M to M1 and then back to M , is



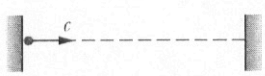
(*a*)



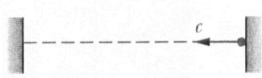
(*b*)



(*c*)

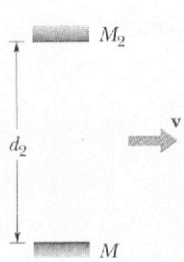


(*d*)

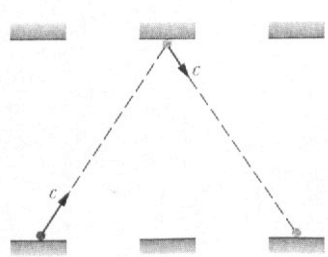


(*e*)

(*f*)



(*g*)



(*h*) (*i*) (*j*)

$$t_1 = \frac{2cd_1}{c^2 - v^2}$$

and thus the path length L1, traveled by the light along this arm is

$$L_1 = ct_1 = \frac{2c^2 d_1}{c^2 - v^2}$$

(b) Figure g shows a view of arm of length d_2, that arm also moves rightward with velocity v through the aether. For simplicity the beam splitter M is now drawn parallel to the mirror M2 at the end of this arm. Figure h shows the arm just as a particular portion of light (the dot) begins its travel along the arm. Because the arm moves rightward during the flight of the light, the path of the light is angled rightward towards the position that M2 will have when the light reaches that mirror i. The reflection of the light from M2 sends the light angled rightward towards the position that M will have when the light returns to it j. Show that the total time of travel for the light from M to M2 and then back to M is

$$t_2 = \frac{2d_2}{\sqrt{c^2 - v^2}}$$

and thus that the path length L2 traveled by the light along this arm is

$$L_2 = ct_2 = \frac{2cd_2}{\sqrt{c^2 - v^2}}$$

(c) Substitute d for d_1 and d_2 in the expression for L1 and L2. Than expand the two expressions by using the binomial expansion, retain the first two terms of the expression.

Show that path length L1 is greater than path length L2 and that their distance ΔL is

$$\Delta L = \frac{dv^2}{c^2}$$

(d) Next show that the phase difference between the light traveling along L1 and that along L2 is

$$\frac{\Delta L}{\lambda} = \frac{dv^2}{\lambda c^2}$$

where λ is the wavelength of light. This phase difference determines the fringe pattern produced by the light arriving at the telescope in the interferometer.

(e) Now rotate the interferometer by 90° so that the arm of length d_2 is along the direction of motion through the aether and the arm length d_1 is perpendicular to that direction. Show that the shift in the fringe pattern due to rotation is

$$shift = \frac{2dv^2}{\lambda c^2}$$

(f) Evaluate the shift, setting $c=3.0 \cdot 10^8$ m/s, d = 10m, and λ=500nm and earth's orbital speed 29.8 km/h.

This expected fringe shift would have been easily observable. However Michelson and Morely observed no fringe shift which cast grave doubt on the existence of aether. In fact, the idea of aether soon disappeared. Moreover, the null result of Michelson and Morely led, at least indirectly, to Einstein's special theory of relativity.

8. Gravitational wave detectors use Michelson interferometers. List the instrumental parameters of the gravitational detectors LIGO: Laser Interferometer Gravitational Wave Observatory and LISA: Laser Interferometer Space Antenna.