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PHYS 431; Dr. Taheri

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Fermi Gas Presentation Script

**First Slide and Project Introduction:**

**Personal** (Slide 1)

My name is Patrick McMillin, and I chose to do my project on the Fermi gas and its heat capacity.

**Introduction** **to the Problem** (Slide 2)

To begin, we must make several assumptions about the Fermi Gas that we are working with before we continue. The first is that the energy levels are equally spaced. This is a reasonable assumption because the original estimates and calculations for the Fermi gas presented in the course had no dependence on the spacing between energy levels, so we are free to choose the distribution. The second assumption that is made is to consider a nondegenerate Fermi gas. In this situation, only one fermion can be allowed in each energy state, which will aid in the numerical calculations greatly.

The aim of this project is to use numerical and analytical methods to develop a relationship that approximates the heat capacity of a Fermi gas as a function of temperature.

**Brief Introduction to Partitions** (Slide 3)

To proceed to calculate physical quantities about the specific Fermi gas we are working with, we must first have an understanding of a mathematical object called a partition. A partition is simply the number of ways that a positive integer can be expressed as a sum of other positive integers, excluding degeneracies (such as 1 + 2 = 3 = 2 + 1, this is counts as 1 arrangement). As stated on the slide here, I will define the function p(q) as the partition of the number 'q'.

This understanding is important, because as we will see in the next slide, the partition of a number is actually the number of ways that you can arrange the available energy in our selected Fermi gas. This is then the multiplicity of a state given the energy, and we can then derive meaningful physical observables form the partitions.

**Visualization of Partitions and their Connections to the Fermi Gas** (Slide 4)

The hand-drawn figure in this slide shows the possible arrangements of energy in a Fermi gas, given the number of available energy packets. An open circle represents an unfilled energy level, while a filled represents a filled energy level. Below the circles, I have shown the number of ways that the available energy packets can be represented as a sum of smaller positive integers. This shows that the partition of the available energy is the number of ways to arrange the energy in the Fermi gas, and thus is the number of microstate available to the system.

**Numerical Approach:**

**Generating the Partitions** (Slide 5)

The first challenge then is to generate partitions for say the first 100 natural numbers. Luckily, some of the world's greatest mathematicians have developed generating functions for such things. This particular generating function is based on the Pentagonal number theorem, which uses generalized pentagonal numbers to find partitions. The pentagonal numbers are given by the second formula on this slide, which takes both positive and negative integers. This is obviously a tedious task to perform by hand for 100 numbers, so I turned to a computer for help. Rather than using tables found online, I elected to make my own script that generates these partitions, which is presented on slide 6.

**Partition Code** (Slide 6)

Here is the aforementioned code. As stated on the slide, I used python.

**Results of the Code** (Slide 7)

After running the code, here are the results in green on the left part of the slide. These are the partitions for the numbers 0 to 100. I used a Wolfram Research website to compare my results. I included a table from that website for comparison. As can be seen, the value I obtained for the 100th partition is the accepted value. This is important because it proves that the code worked, as all partitions depend on the partitions that were calculated before.

**The Physics** (Slide 8)

Now that we have our partitions that serve as the multiplicities, we can find some useful physical observables. Firstly, we calculate the entropy. This is simply the natural logarithm of the multiplicity. From there, things get a little more complicated. To find the temperature, we must use a method known as the "centered difference" method. Instead of calculating the derivative of energy with respect to entropy, we can numerically look at the neighboring values to find an approximate derivative. The formula for this is seen in the second-to-last line, below the definition for the heat capacity. The same procedure is followed for the heat capacity once temperature is known, and it then is trivial to plot the data using a computer program, which I attached on the next slide.

**Plotting Code** (Slide 9)

This is the code I wrote to perform these calculations and plot the data.

**Numerical Result Graph** (Slide 10)

Here is the graphical result that is obtained from the analysis. It shows the proper relationship that heat capacity increases linearly with temperature, and becomes more and more precise as temperature increases.

**Analytical Result:**

**Ramanujan's Formula for Partitions** (Slide 11)

Ramanujan was one of those brilliant mathematicians who tackled the problem of finding partitions. He and Hardy developed this elegant formula to approximate the partition of large numbers 'q'. It can be very useful to use this formula, with the requirement that we concern ourselves mostly with larger energies, or large temperatures. I have included on this slide some information about the accuracy of this formula, which gives only 1.5% error as we find the partitions of numbers greater than 1000.

**The Physics** (Slide 12)

With this formula, we can apply the same ideas as discussed before to find physical observables. Finding the entropy is straightforward, and requires only the use of a logarithm. From the entropy, we can use the chain rule to find an expression for the temperature in terms of a derivative with respect to 'q'. Using this equation, we can solve for energy as a function of temperature. This is useful because we know that the heat capacity is the temperature derivative of the energy. This requires the solving of a quadratic equation in terms of 'q' and 'sqrt(q)', which can be done in a few minutes and some carful algebra tricks. Before taking the derivative to find the heat capacity, we make use of the approximation that the temperature is high to avoid taking the derivative of the square root function. We then obtain the second to last equation, which we can use in the chain rule expression listed above it to find the heat capacity, which, as expected, is linear in T. Now we must compare this to our numerical solution.

**Plotting Ramanujan's Approximation** (Slide 13)

Here is the code (which continues on the last snippet I showed). It simply plots the function on the same graph.

**Comparison:**

**Comparison of the Graphs** (Slide 14)

As we can see from the graph, these two approximations coincide with one another perfectly at higher values of available energy. Both approaches give good estimates for the heat capacity of the nondegenerate Fermi gas with evenly spaced energy levels.

**Conclusion** (Slide 15)

Here is the equation found in the test for the heat capacity of the Femri gas by using the Sommerfeld expansion. We see that the major dependence is that on temperature. The fact that the expression is linear in T is very important, as both the numerical and analytical results we obtained from approximate methods show a clear linear dependence on temperature.

**Differences** (Slide 16)

The main difference seen in the formulae is the lack of 'N' dependence in our approximate solution. However, this is expected, as we made NO assumptions about the volume of occupancy of the Fermi gas. The assumption that the energy levels are evenly spaced hides the need for volume dependence or a distribution function.

In conclusion, we see that by using approximate methods, we are able to obtain the correct relationship for the heat capacity of a nondegenerate Fermi gas through three basic assumptions and approximate methods.

**References Slide** (Slide 17)

Thank you very much for your time. Are there any questions?