

Gradient Descent

Iterative

$$x^{t+1} = x^t + \eta d$$

↗ step size
↘ direction

f convex, $x, y \in \mathbb{R}^n$

$$\begin{aligned} f(y) &= f(x) + \nabla^T f(x)(y-x) + o(\|x-y\|) \\ &\geq f(x) + \nabla^T f(x)(y-x) \end{aligned}$$

$$x^{t+1} = \arg \min_x \left(\frac{1}{2} \|x - x^t\|^2 + \eta \left(f(x^t) + \nabla^T f(x^t)(x - x^t) \right) \right)$$

$$\nabla^T g(x) = (x - x^t) + \eta \nabla^T f(x^t) = 0$$

$$x = x^t - \eta \nabla^T f(x^t)$$

Local min \Leftrightarrow global min

$$\Rightarrow x^* \text{ local min} \rightarrow \nabla^T f(x^*) = 0$$

$$\begin{aligned} f(y) &\geq f(x^*) + \nabla^T f(x^*)(y-x^*) \quad \forall y \in \mathbb{R}^n \\ f(y) &\geq f(x^*) \end{aligned}$$