## Taylor's Series

$$\tfrac{1}{2}(x-a)^T F(a)(x-a)$$

$$f(x) = f(a) + \nabla^T f(a)(x-a) + O(\|x-a\|^3)$$

## Optimization

$$\min f(x) \quad \text{where } f: \mathbb{R}^n \to \mathbb{R}$$
$$\text{s.t. } x \in \mathbb{R}^n$$

Def$^n$: $x^*$ global min if $f(x^*) \le f(x) \ \forall x \in \mathbb{R}^n$

Def$^n$: $x^*$ local min if $\exists \gamma > 0$ s.t. $f(x^*) \le f(x) \ \forall x \in \mathbb{R}^n$ s.t. $\|x - x^*\| < \gamma$

## First Order Necessary Condition ($f \in C^1$)

$$x^* = \text{local min} \implies \nabla^T f(x^*) = 0$$

Pf:


$$f(x^* + \alpha d) = f(x^*) + \alpha \nabla^T f(x^*) d + o(\alpha)$$

$$f(x^*) - f(x^* + \alpha d) \le 0$$

$$\lim_{\alpha \to 0} \frac{-\alpha \nabla^T f(x^*) d}{\alpha} - \frac{o(\alpha)}{\alpha} \le \frac{0}{\alpha}$$

$$\nabla^T f(x^*) d \ge 0$$

Consider $d, -d \implies \nabla^T f(x^*) = 0$

## Second Order Necessar Condition ($f \in C^2$)

$$x^* = \text{local min} \implies \nabla f(x^*) = 0 \ \& \ F(x^*) \ge 0$$